Non-Markovian dynamics of two-time correlation functions for open quantum systems

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Md. Manirul Ali

Physics Division, National Center for Theoretical Sciences, NTHU, Hsinchu, Taiwan

Open quantum systems

- Realistic quantum systems interact to its environment, essentially they are many body quantum systems

- We are interested in: how does the interaction influence the open system causing dissipation, decoherence

- dynamics of density matrix, equation of motion of physical observables of the system

Open system dynamics (System plus reservoir model)

Total system Closed: ρ

Open system: $\rho_S = Tr_E\{\,\rho\,\}$

Environment: $\rho_E = Tr_S\{\,\rho\,\}$

Total Hamiltonian: $H = H_S + H_E + H_I$

Open system dynamics:

$$\rho_S(t) = Tr_E\left[\,U(t)\,\rho(0)\,U^{\dagger}(t)\right]$$

Perturbative method (Born-Markov) master equation

$$\boxed{\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[\,\mathrm{H}, \rho(t)]}$$

$$H = H_S + H_E + H_I = H_0 + H_I$$

Interaction Picture:

$$\tilde{\rho}(t) = e^{\frac{i H_0 t}{\hbar}}\, \rho(t)\, e^{-\frac{i H_0 t}{\hbar}}$$

$$\widetilde{H_I}(t) = e^{\frac{i H_0 t}{\hbar}}\, H_I\, e^{-\frac{i H_0 t}{\hbar}}$$

$$\boxed{\frac{d}{dt}\tilde{\rho}(t) = -\frac{i}{\hbar}[\widetilde{H_I}(t), \tilde{\rho}(t)]}$$

Born-Markov master equation

Initial State Factorized:

$$\tilde{\rho}(0) = \rho(0) = \rho_S(0)\rho_E(0)$$

2nd Order Born Approximation Weak Coupling :

$$\tilde{\rho}(t) = \widetilde{\rho_S}(t)\rho_E(0) + \mathcal{O}(H_I)$$

Markov Approximation – I No dependence on intermediate states

$$\widetilde{\rho_S}(t') \rightarrow \widetilde{\rho_S}(t)$$

Markov Approximation – II Environment Correlation functions decay very fast

$$\mathrm{C}(t\text{-}t') \rightarrow \delta(t - t')$$

$$\frac{d}{dt}\rho_S(t) = -\frac{\boldsymbol{i}}{\hbar}\left[H_S, \rho_S(t)\right] + \mathcal{D}(\rho_S(t))$$

Feynman-Vernon influence functional approach in the framework of coherent-state path-integral representation

$$H = \hbar\omega_0\, a^\dagger a + \sum_k \hbar\omega_k\, b_k^\dagger b_k + \sum_k \hbar\left(V_k\, a^\dagger b_k + V_k^*\, a\, b_k^\dagger \right)$$

Reduced density matrix elements in the coherent state representation

$$\langle \boldsymbol{\alpha_f} |\, \boldsymbol{\rho(t)}\, | \boldsymbol{\alpha'_f} \rangle = \langle \boldsymbol{\alpha_f} |\, \boldsymbol{Tr_E\{\, U(t)\rho_S(t_0)\rho_E(t_0)U^\dagger(t)\}}\, | \boldsymbol{\alpha'_f} \rangle$$

$$= \int \boldsymbol{d\mu(\alpha_0)\, d\mu(\alpha'_0)\, \langle \alpha_0 |\, \rho_S(t_0)\, | \alpha'_0 \rangle}\, \boldsymbol{\mathcal{J}(\alpha_f^*,\ \alpha'_f, t \mid \alpha_0, \alpha_0'^*, t_0)}$$

Feynman & Vernon, Ann. Phys. 1963

The propagator

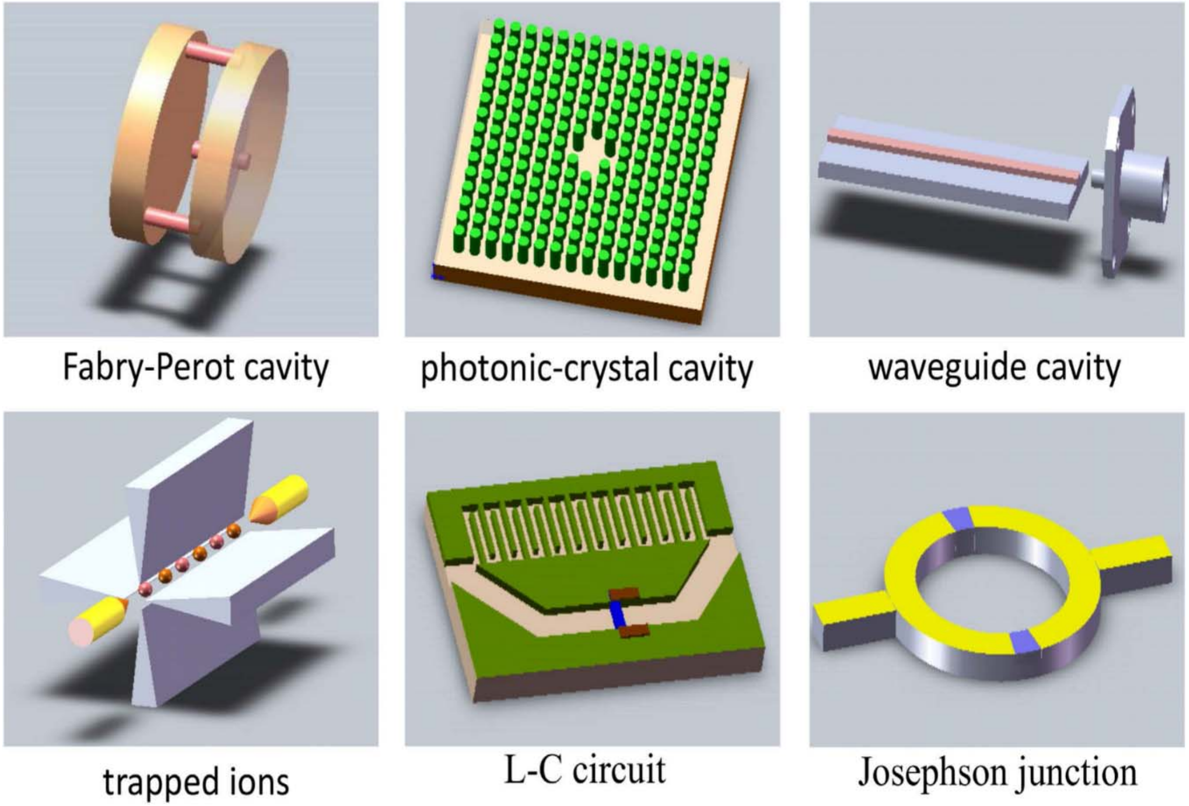
action of the system

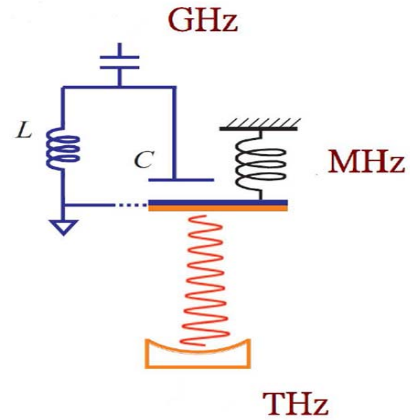
influence functional

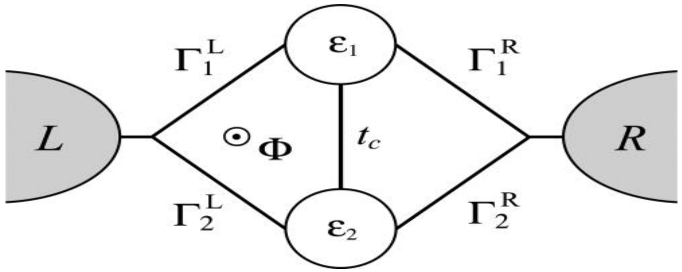
$$\boldsymbol{\mathcal{J}\left(\alpha_f^*, \alpha'_f, t \mid \alpha_0, \alpha_0'^*, t_0\right)} = \int \boldsymbol{\mathfrak{D}[\alpha^*\alpha;\ \alpha'^*\alpha']\, e^{i\left(S_c[\alpha^*, \alpha] - S_c^*[\alpha'^*, \alpha'] \right)}\, \mathcal{F}[\alpha^*\alpha;\ \alpha'^*\alpha']}$$

path integral

Open Quantum Systems: beyond weak coupling







$$S_c[\alpha^*, \alpha\,] = -\frac{i}{2}[\,\alpha_f^*\,\alpha(t) + \alpha^*(t_0)\alpha_0]$$

$$+\int_{t_0}^{t} d\tau\,\{\,\frac{i}{2}\,[\,\alpha^*(\tau)\dot{\alpha}(\tau) - \dot{\alpha}^*(\tau)\alpha(\tau)] - \omega_0\,\alpha^*(\tau)\,\alpha(\tau)\,\}$$

$$\mathcal{F}[\alpha^*\alpha;\ \alpha'^*\alpha'] = exp\left\{-\int_{t_0}^{t} d\tau\,\int_{t_0}^{\tau} d\tau'\,[\,g(\tau,\tau')\,\alpha^*(\tau)\,\alpha(\tau') + g(\tau',\tau)\alpha'^*(\tau')\,\alpha'(\tau)]\right\}$$

$$exp\left\{\int_{t_0}^{t} d\tau\,\int_{t_0}^{t} d\tau'\,[\,g(\tau,\tau')\,\alpha'^*(\tau)\,\alpha(\tau') - \big(\alpha^*(\tau) - \alpha'^*(\tau)\big)\tilde{g}(\tau,\tau')\big(\alpha(\tau') - \alpha'(\tau')\big)]\right\}$$

$$\mathcal{J}\big(\alpha_f^*, \alpha_f', t \mid \alpha_0, \alpha_0'^*, t_0\big) = A(t)\,exp\left\{\alpha_f^*\,J_1(t)\,\alpha_0 + \alpha_0'^* J_1^*\,\alpha_f' + \alpha_f^*\,J_2(t)\alpha_f' + \alpha_0'^*\,J_3(t)\alpha_0\right\}$$

$$J_1(t) = w(t)u(t,t_0), \quad J_2\,(t) = 1 - w(t), \quad J_3\,(t) = 1 - u^*(t,t_0)\,w(t)\,u(t,t_0)$$

$$A(t) = w(t) = \frac{1}{1 + v(t,t)}$$

Propagator

$$\mathcal{J}\left(\alpha_f^*, \alpha_f', t \mid \alpha_0, \alpha_0'^*, t_0\right)$$

$$= A(t)\, exp\left\{\alpha_f^* J_1(t)\, \alpha_0 + \alpha_0'^* J_1^*\, \alpha_f' + \alpha_f^* J_2(t)\alpha_f' + \alpha_0'^* J_3(t)\alpha_0\right\}$$

$$J_1(t) = w(t)u(t, t_0),$$

$$J_2(t) = 1 - w(t),$$

$$J_3(t) = 1 - u^*(t, t_0)\, w(t)\, u(t, t_0)$$

$$A(t) = w(t) = \frac{1}{1 + v(t, t)}$$

Exact Master Equation

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}\left[H_s', \rho(t)\right] + \gamma(t)\left[2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a\right] + \tilde{\gamma}(t)\left[a^\dagger\rho(t)a + a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)aa^\dagger\right]$$

$$H_s' = \hbar\omega_0'(t)a^\dagger a, \quad \omega_0'(t) = -Im\left[\frac{\dot{u}(t,t_0)}{u(t,t_0)}\right], \quad \gamma(t) = -Re\left[\frac{\dot{u}(t,t_0)}{u(t,t_0)}\right]$$

$$\tilde{\gamma}(t) = \dot{v}(t,t) - 2v(t,t)\, Re\left[\frac{\dot{u}(t,t_0)}{u(t,t_0)}\right]$$

W. M. Zhang et al. Phys. Rev. Lett. 109, 170402 (2012)

The functions $\mathbf{u}(\mathrm{t}, t_0)$ and $v(\mathrm{t}, t)$ are analogous to the nonequilibrium Green's functions of the Schwinger-Keldysh theory

$$\mathbf{u}(\mathbf{t}, \mathbf{t}_0) = \langle [a(t), a^\dagger(t_0)] \rangle = i\, G^r(t, t_0), \qquad u^*(\mathbf{t}, \tau) = -i\, G^a(\tau, t)$$

$$v(\tau, t) = \langle a^\dagger(t) a(\tau) \rangle = -i\, G^<(\tau, t)$$ (neglecting an initial state dependent term)

$$\frac{d}{dt}\, \mathbf{u}(\mathbf{t}, \mathbf{t}_0) + \mathrm{i}\, \omega_0 u(t, t_0) + \int_{t_0}^{t} d\tau g(t, \tau) u(\tau, t_0) = 0$$

$$\frac{d}{dt} v(\mathbf{t}, \mathbf{t}) + \mathrm{i}\, \omega_0 v(t, t) + \int_{t_0}^{t} d\tau g(t, \tau) v(\tau, t) = \int_{t_0}^{t} d\tau \tilde{g}(t, \tau) u^*(\tau, t_0)$$

$$v(\mathbf{t}, \mathbf{t}) = \int_{t_0}^{t} d\tau_1 \int_{t_0}^{t} d\tau_2\, u(t, \tau_1) \tilde{g}(\tau_1, \tau_2) u^*(t, \tau_2)$$

The time correlation functions characterize the non-Markovian back-action memory effect between the system and the environment

$$g(t,\tau) = \int_0^\infty d\omega\, J(\omega)\, e^{-i\omega(t-\tau)} = i\,\Sigma^r(t,\tau)$$

$$\tilde{g}(t,\tau) = \int_0^\infty d\omega\, J(\omega)\, \overline{n}(\omega,T) e^{-i\omega(t-\tau)} = -i\Sigma^<(t,\tau)$$

$$J(\omega) = \sum_k |V_k|^2\, \delta(\omega - \omega_k) = \varrho(\omega)|V(\omega)|^2 \textit{ is the spectral density}$$

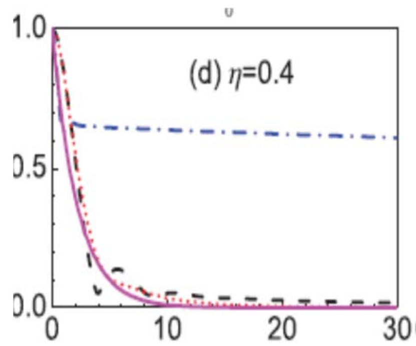
$$\overline{n}(\omega,T) = \frac{1}{e^{\hbar\omega/k_B T} - 1} \textit{ is the BE distribution of the reservoir}$$

Solutions:

$$u(t,t_0) = \frac{1}{1-\Sigma'(\omega_b)} e^{-i\omega_b(t-t_0)} + \int_0^\infty d\omega \, \frac{J(\omega)e^{-i\omega(t-t_0)}}{[\omega-\omega_0-\Delta(\omega)]^2+\pi^2 J^2(\omega)}$$

$$\Sigma(\omega) = \int_0^\infty d\omega' \, \frac{J(\omega')}{\omega-\omega'}$$ is the reservoir induced self-energy correction

$$\Sigma'(\omega_b) = \frac{\partial\Sigma(\omega)}{\partial\omega}\Big|_{\omega=\omega_b}, \qquad \Delta(\omega) = Re\,\{\Sigma(\omega)\}$$



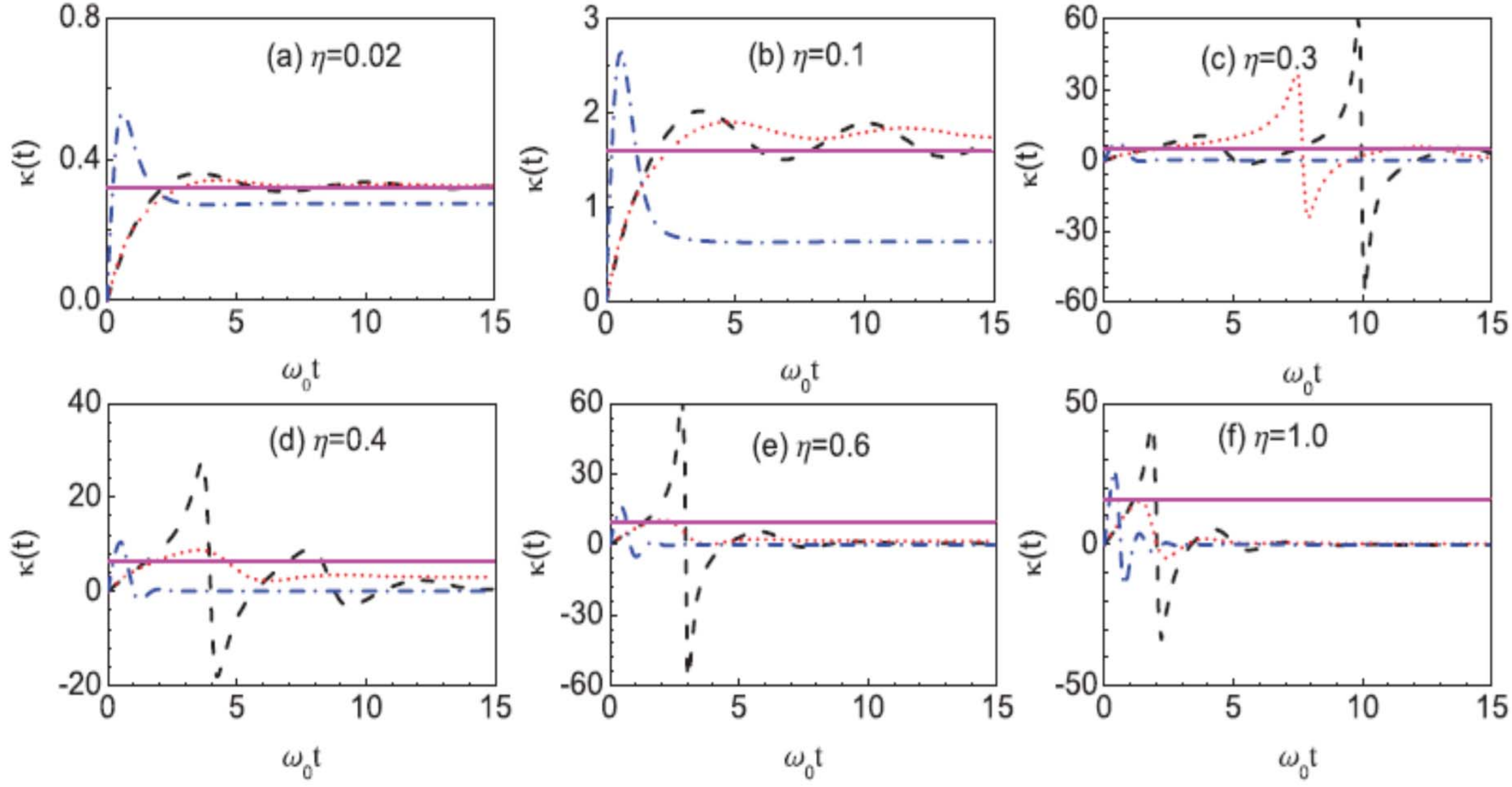
Fourier transform:

$$\mathcal{D}(\omega) = \frac{1}{1-\Sigma'(\omega_b)}\delta(\omega-\omega_b) + \frac{J(\omega)}{[\omega-\omega_0-\Delta(\omega)]^2+\pi^2 J^2(\omega)}$$

The environment modifies the system spectrum as a combination of localized modes (dissipationless process) plus a continuum spectrum part

Dissipation coefficient γ(t)

$$J(\omega) = \eta\omega\left(\frac{\omega}{\omega_c}\right)^{s-1} e^{-\omega/\omega_c}$$



sub-Ohmic s=1/2 (dashed black line), Ohmic s=1 (dotted red line), super-Ohmic s=3 (dash-dotted blue line) with the corresponding BM limit (solid magenta line)

Initial State:

$$\rho(t_0) = |n_0\rangle\langle n_0|$$

Final State:

$$\rho(t) = \sum_{n=0}^{\infty} \mathcal{P}_n^{n_0}(t)\,|n\rangle\langle n|$$

$$\mathcal{P}_n^{n_0}(t) = \frac{[v(t,t)]^n}{[1+v(t,t)]^{n+1}}[1-\Omega(t)]^{n_0} \sum_{k=0}^{\min\{n_0,n\}} \binom{n_0}{k}\binom{n}{k}\left[\frac{1}{v(t,t)}\,\frac{\Omega(t)}{1-\Omega(t)}\right]^k$$

where

$$\Omega(t) = \frac{|u(t,t_0)|^2}{1+v(t,t)}$$

$$n(t) = |u(t,t_0)|^2\,n_0 + v(t,t)$$

Born-Markov

$$\boldsymbol{u(t \to \infty, t_0) \to 0}$$

$$\boldsymbol{n(t \to \infty) = v(t, t \to \infty) = \bar{n}(\omega_0, T)} \ , \qquad \boldsymbol{\bar{n}(\omega_0, T) = \frac{1}{e^{\hbar\omega_0/k_B T} - 1}}$$

$$\boldsymbol{\mathcal{P}_n^{n_0}(t \to \infty) = \frac{[v(t, t \to \infty)]^n}{[1 + v(t, t \to \infty)]^{n+1}} = \frac{[\bar{n}(\omega_0, T)]^n}{[1 + \bar{n}(\omega_0, T)]^{n+1}}}$$

Bose-Einstein statistical distribution

Strong non-Markov (exact)

$$\boldsymbol{n(t) = |u(t, t_0)|^2 \, n_0 + v(t, t)}$$

Bose-Einstein statistical Distribution is not obeyed

P. Y. Lo, H. N. Xiong, W. M. Zhang, Sci. Rep. 5, 9423 (2015)

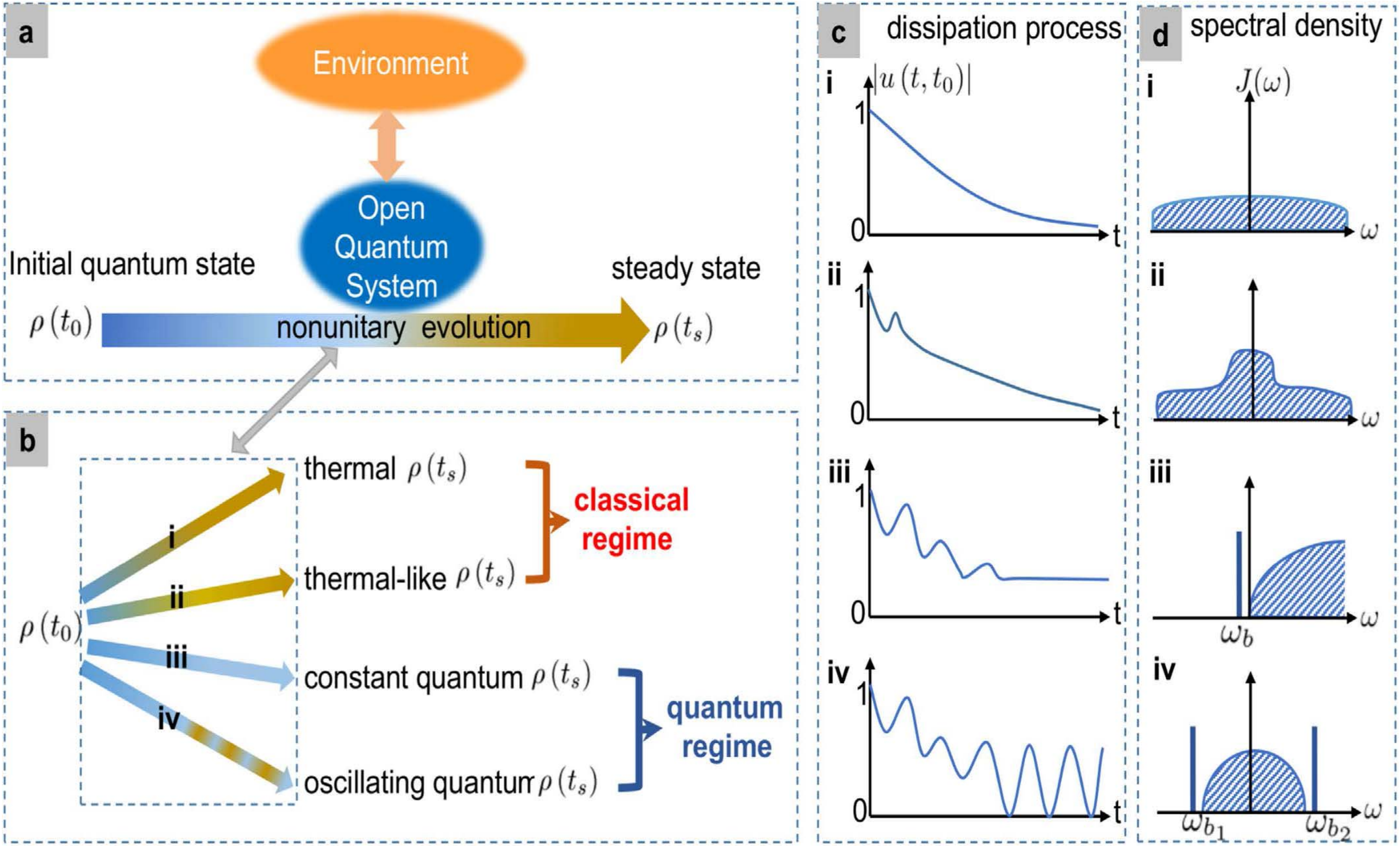
$$n(t_s) = |u(t_s,t_0)|^2\, n(t_0) + v(t_s,t_s)$$

$$\boxed{u(t_s,t_0) \nrightarrow 0}$$

$$u(t_s,t_0) = \mathcal{Z}_b\, exp\{-i\,\omega_b(t_s - t_0)\}, \qquad \mathcal{Z}_b = \frac{1}{1-\Sigma'(\omega_b)}$$

$$\boxed{v(t_s,t_s) \nrightarrow \bar{n}(\omega_0,T)}$$

$$= \int_0^\infty d\omega\, \bar{n}(\omega,T) \left\{ \frac{{\mathcal{Z}_b}^2\, J(\omega)}{(\omega_b-\omega)^2} + \frac{J(\omega)}{[\omega-\omega_0-\Delta(\omega)]^2 + \pi^2 J^2(\omega)} \right\}$$



H.-N. Xiong, P. Y. Lo, W. M. Zhang, D. H. Feng, F. Nori, Sci. Rep. 5, 13353 (2015)

Nonequilibrium transient dynamics of photon statistics

$$g^{(2)}(t,t+\tau) = \frac{\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle}{\langle a^\dagger(t)a(t)\rangle\langle a^\dagger(t+\tau)a(t+\tau)\rangle}$$

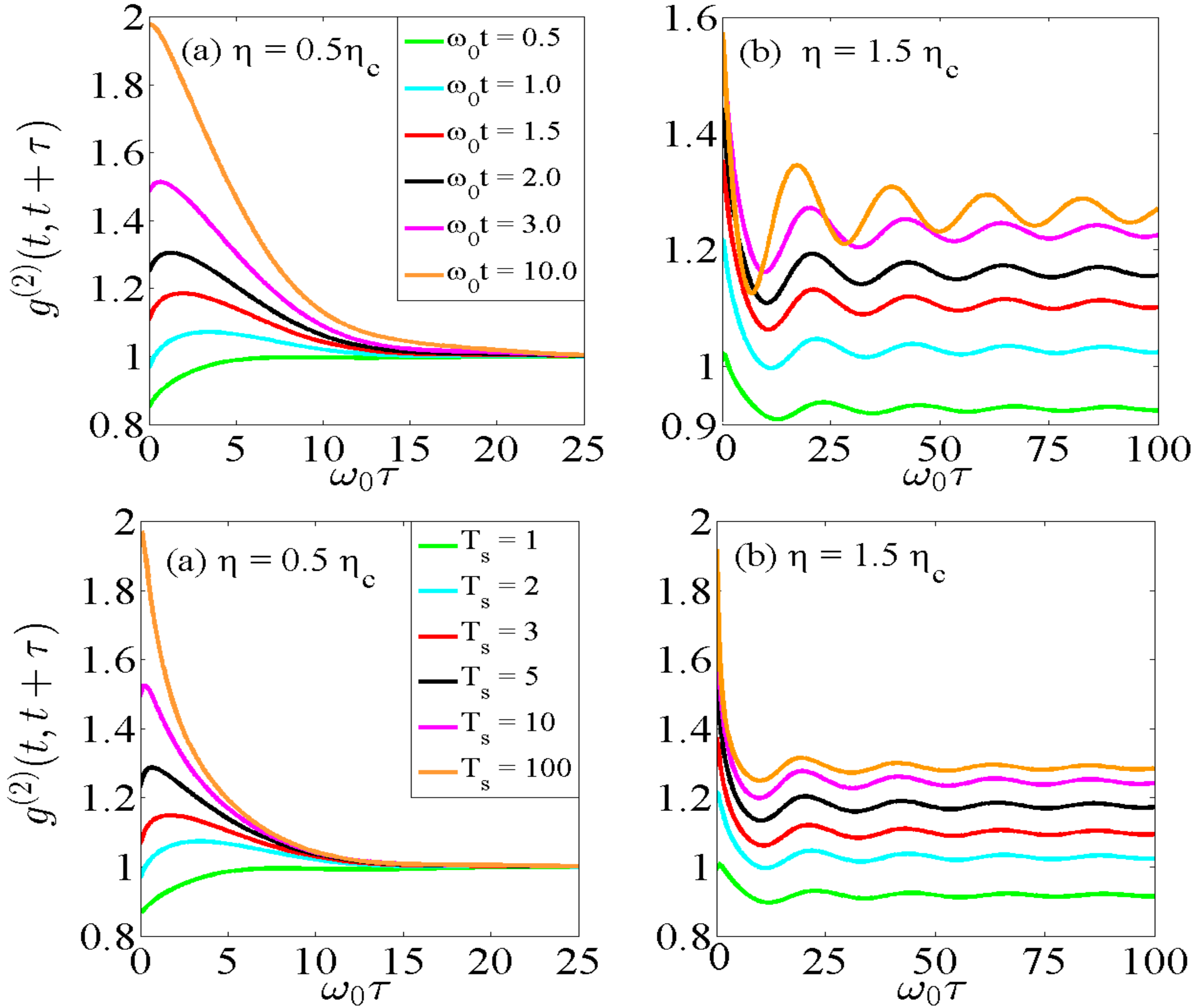
$$\dot{a}(t) = -i\omega_0 a(t) - i\sum_k V_k\, b_k(0)e^{-i\omega_k t} - \int_0^t d\tau\, g(t,\tau)\, a(\tau)$$

$$\langle a^\dagger(t)a^\dagger(t')a(t')a(t)\rangle = v(t)\,v(t') + |v(t,t')|^2 + |u(t)|^2\,|u(t')|^2\beta \\ + \{\, \mathrm{v}(\mathrm{t})|\mathrm{u}(\mathrm{t}')|^2 + v(t')|u(t)|^2 + 2Re[v(t,t')u^*(t)u(t')]\,\}\,\alpha$$

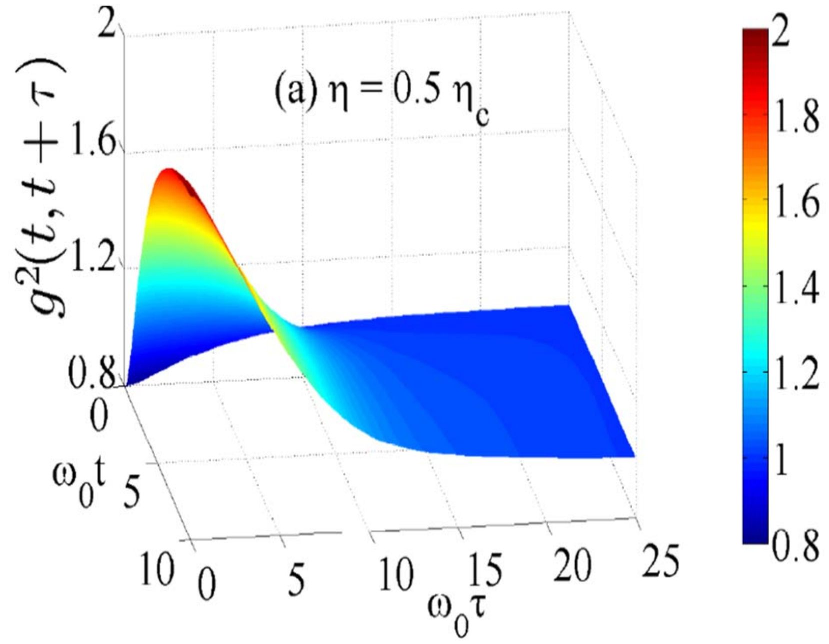
$$\beta = \langle a^\dagger(0)a^\dagger(0)a(0)a(0)\rangle$$

$$\alpha = \langle a^\dagger(0)a(0)\rangle$$

M. M. Ali and W. M. Zhang, Phys. Rev. A (Comm. 2016)



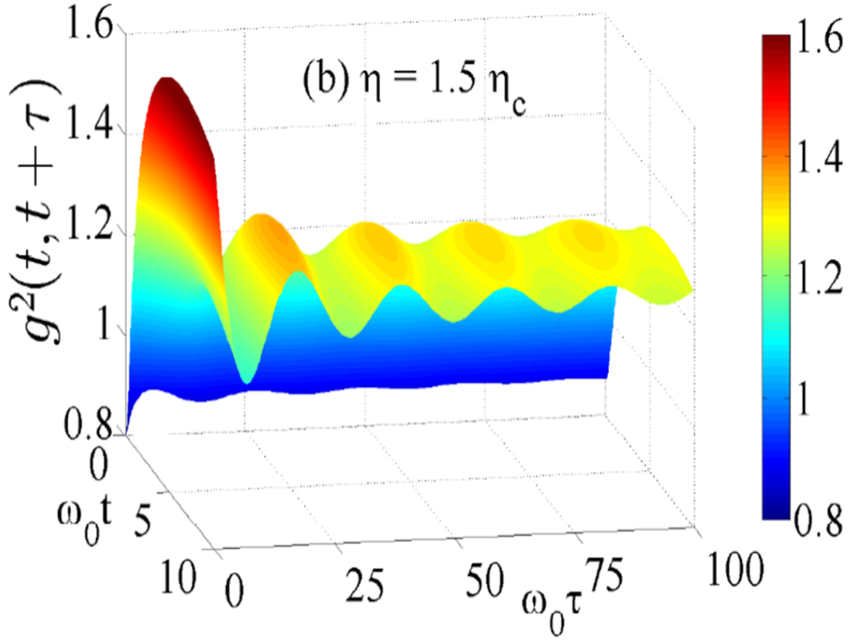
(a) η = 0.5η_c
ω_0t = 0.5
ω_0t = 1.0
ω_0t = 1.5
ω_0t = 2.0
ω_0t = 3.0
ω_0t = 10.0
g^(2)(t, t + τ)
ω_0τ
(b) η = 1.5 η_c
(a) η = 0.5 η_c
T_s = 1
T_s = 2
T_s = 3
T_s = 5
T_s = 10
T_s = 100
(b) η = 1.5 η_c

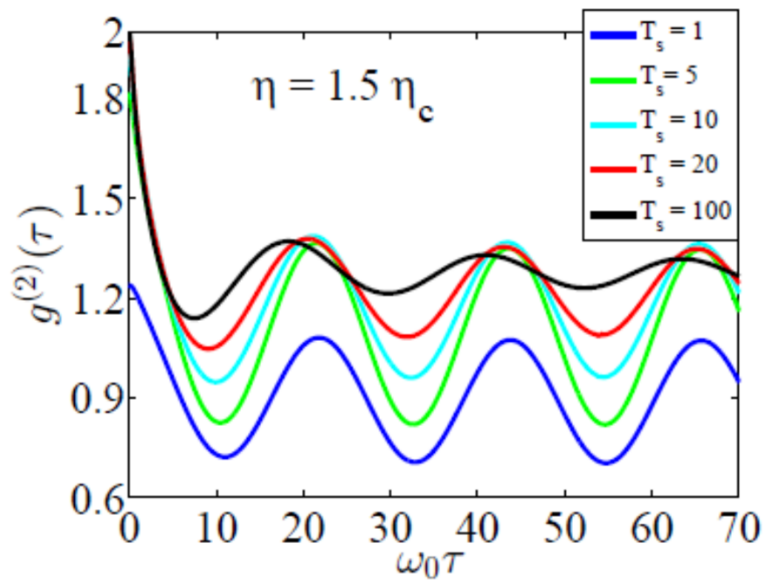


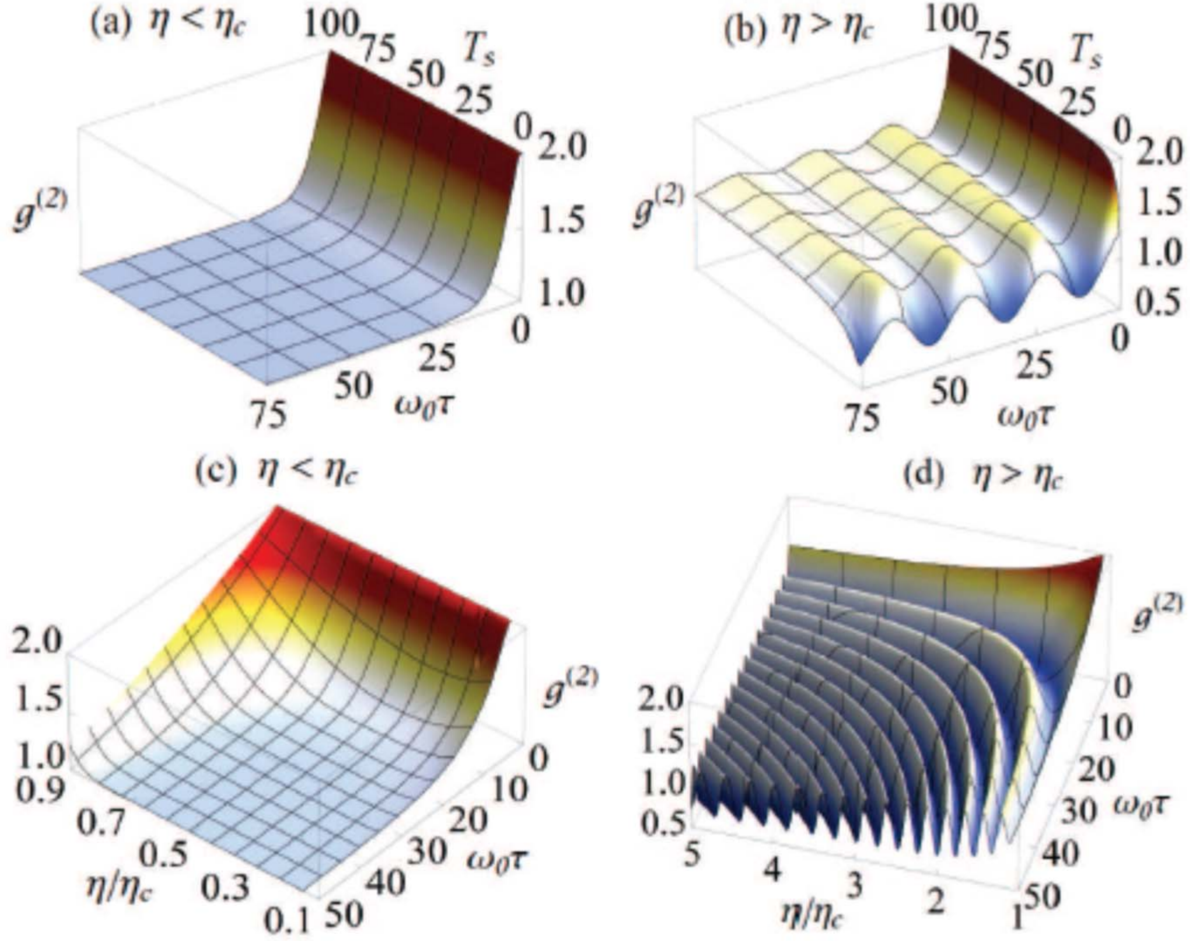
$$g_{ss}^{(2)}(\tau) = \lim_{t\to\infty} g^{(2)}(t, t+\tau) = 1 + e^{-2\kappa\tau}$$

$$\rho(t_s) = \sum_{n=0}^{\infty} \frac{[v(t_s)]^n}{[1+v(t_s)]^{n+1}} |n\rangle\langle n|$$

Nonequilibrium steady state







New type of phase transition of photon statistics occurs
at a critical value while passing through weak to strong system-reservoir coupling

arXiv: 1606.02397

Exact decoherence dynamics of 1/f noise

M. M. Ali, P. Y. Lo, W. M. Zhang, New J. Phys. 16, 103010 (2014)

$$H(t) = \epsilon\,\sigma_z + \nu\, c(t)\,\sigma_z$$

Decoherence of flux qubits due to 1/f noise

Phys. Rev. Lett. 97, 167001

$$S(\omega,\nu) = \int_{-\infty}^{+\infty} \mathrm{d}t\, e^{\mathrm{i}\omega t}\,\overline{c(t)c(0)}$$

Model for 1/f flux noise in SQUIDS and Qubits

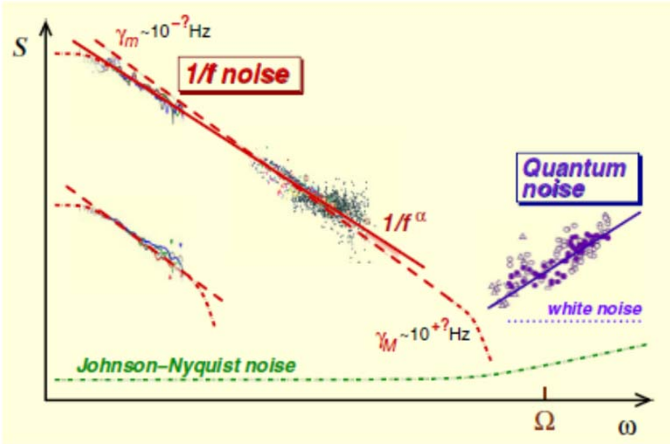
$$S_{1/f^{\alpha}}(\omega) = \int_{\nu_1}^{\nu_2} S(\omega,\nu)\,p_{\alpha}(\nu)\,\mathrm{d}\nu$$

Phys. Rev. Lett. 98, 267003

1/f flux noise in Josephson phase qubits

Phys. Rev. Lett. 99, 187006

$$H = \hbar\omega_0\, a^{\dagger} a + \sum_k \hbar\omega_k\, b_k^{\dagger} b_k + \sum_k \hbar\left(V_k\, a^{\dagger} b_k + V_k^{*}\, a\, b_k^{\dagger}\right)$$



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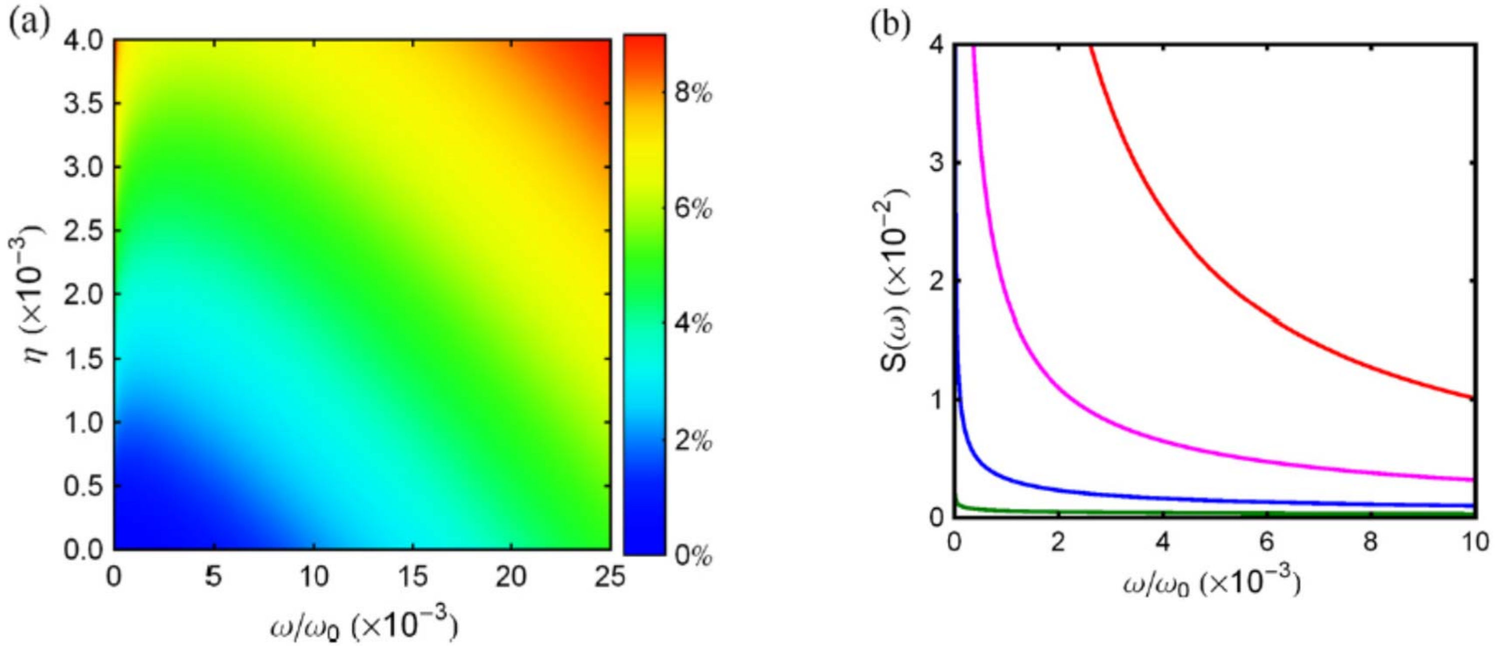
$$S(\omega) = \lim_{t\to\infty} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega\tau} \langle a^{\dagger}(t+\tau)a(t)\rangle \, \mathrm{d}\tau.$$

$$S(\omega) \sim \frac{1}{\omega^{x}}$$



$$S(\omega) = \mathcal{Z}^{2}\delta(\omega - \omega_{b})\langle a^{\dagger}(t_0)a(t_0)\rangle + \left[\frac{\mathcal{Z}^{2}J(\omega)\bar{n}(\omega, T)}{(\omega - \omega_{b})^{2}} + \frac{J(\omega)\bar{n}(\omega, T)}{\left[\omega - \omega_0 - \Delta(\omega)\right]^{2} + \gamma^{2}(\omega)} \right]$$

$$J(\omega) = \eta\, \omega \left(\frac{\omega}{\omega_c}\right)^{-x} e^{-\omega/\omega_c}$$

$$x \approx 1$$

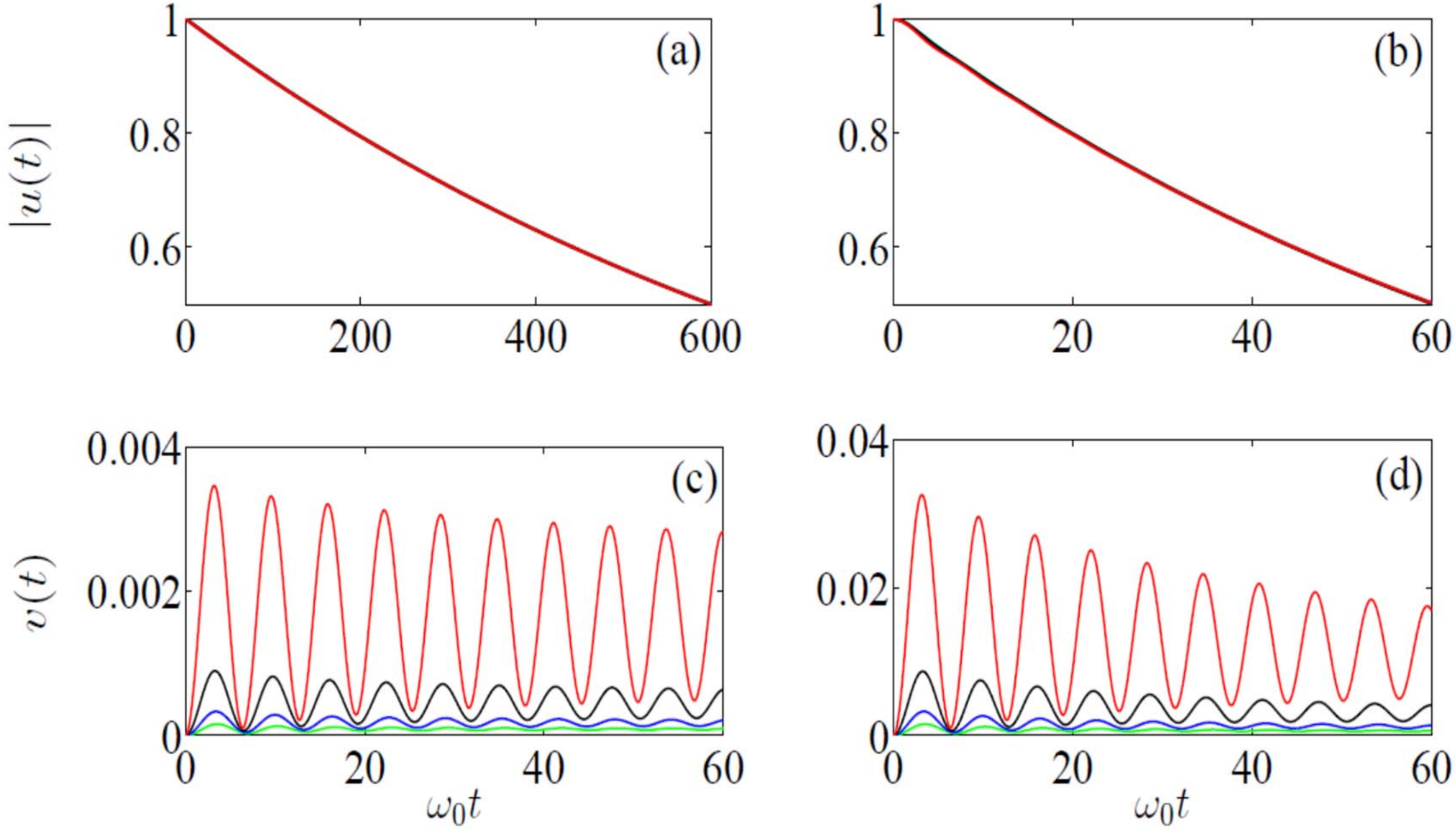


Figure 2. Comparison of the exact solution of $|u(t)|$ and $v(t)$ for $1/f^x$ noise with $x = 0.25$ (greren), $x = 0.5$ (blue), $x = 0.75$ (black), $x = 0.9999$ (red). We plot the exact $|u(t)|$ and $v(t)$ for different system–environment coupling η: (a, c) $\eta = 10^{-3}$ and (b, d) $\eta = 10^{-2}$. The other parameters are taken as $\omega_c = \omega_0 = 5\,\text{GHz}$, and $T = 25\,\text{mK}$.

New J. Phys. **16** (2014) 103010

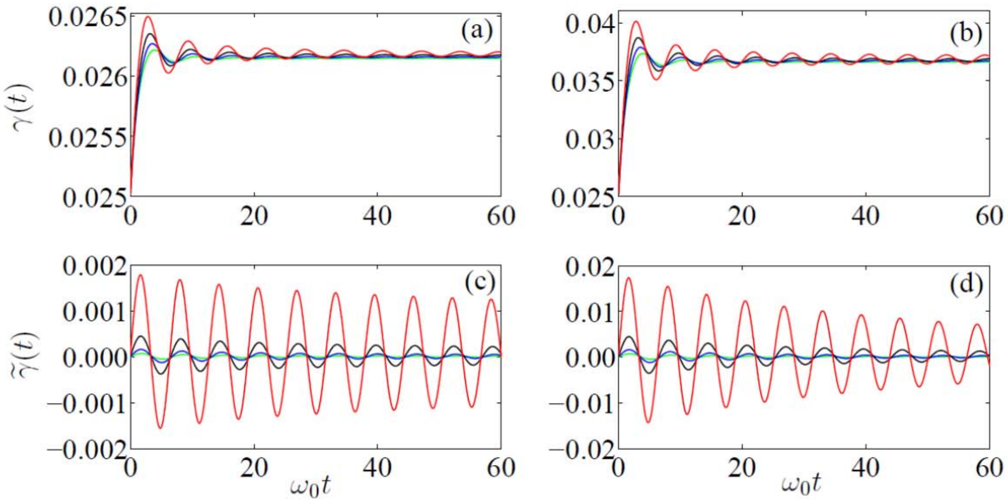


Figure 3. The dissipation and fluctuation coefficients $\gamma(t)$ and $\tilde{\gamma}(t)$ are calculated from the exact solution of $u(t)$ and $v(t)$ for $1/f^x$ noise with $x = 0.25$ (green), $x = 0.5$ (blue), $x = 0.75$ (black), $x = 0.9999$ (red). We plot exact $\gamma(t)$ for different system–environment coupling η (a) $\eta = 10^{-3}$ and (b) $\eta = 10^{-2}$. We next plot exact $\tilde{\gamma}(t)$ at (c) $\eta = 10^{-3}$ and (d) $\eta = 10^{-2}$. The values of other parameters are $\omega_c = \omega_0 = 5\,\text{GHz}$, and $T = 25\,\text{mK}$.

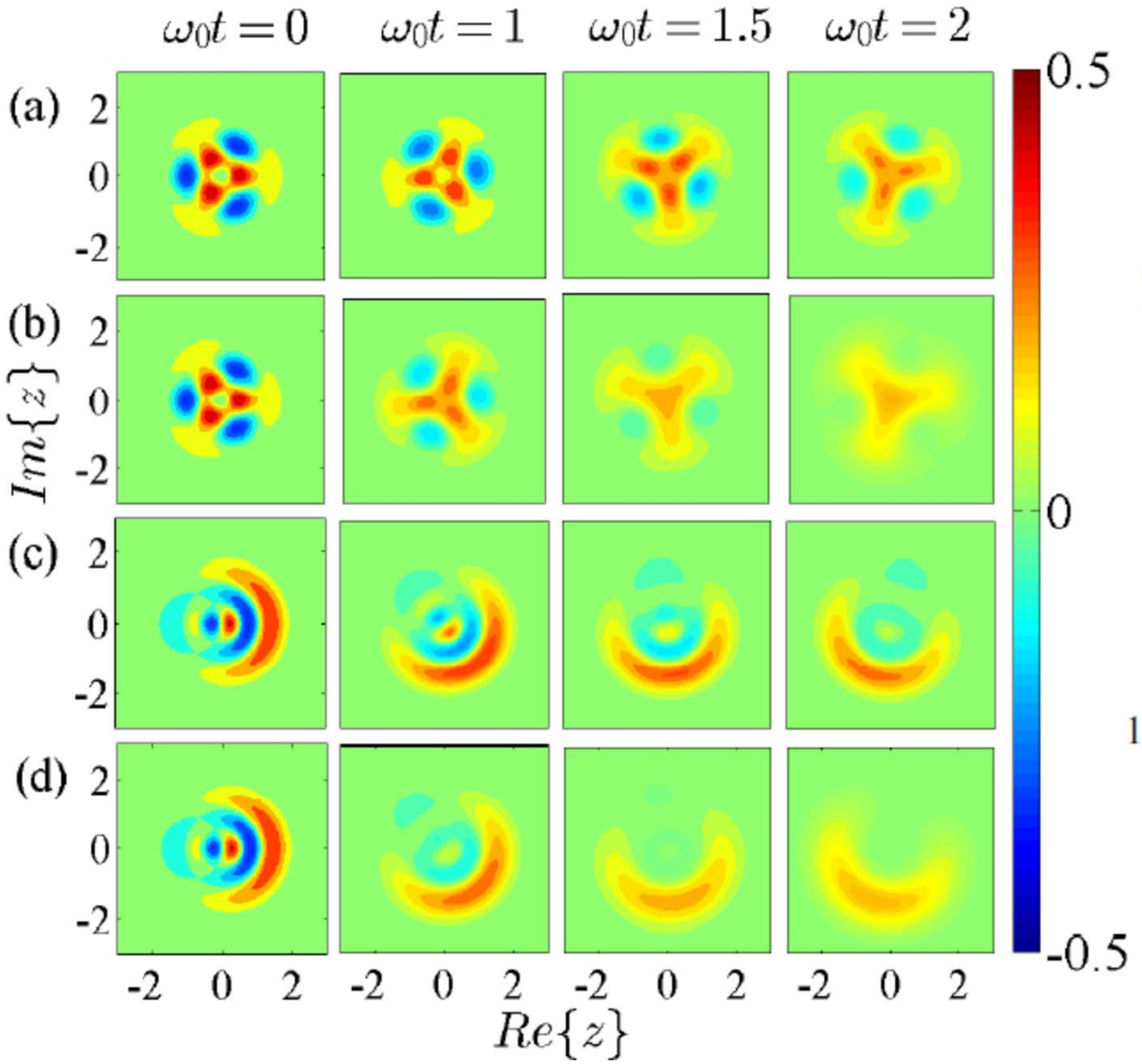
Exact decoherence dynamics of 1/f noise using Wigner distribution

$$W(z,t) = \int \mathrm{d}\mu(\alpha_0)\mathrm{d}\mu(\alpha_0')\left\langle \alpha_0 \left| \rho(t_0) \right| \alpha_0' \right\rangle \mathcal{T}\left(z, t \middle| \alpha_0, \alpha_0'^*, t_0\right)$$

$$\begin{aligned}
W_n^m(z,t) = {} & \frac{1}{2}\left[W_n^n(z,t) + W_m^m(z,t) \right] \\
& + \frac{1}{2} W_0^0(z,t) \sum_{p=0}^{\min(n,m)} \frac{\sqrt{n!}\,\sqrt{m!}}{p!(n-p)!(m-p)!} \\
& \times \left\{ \left(z^* \Omega(t) u(t) \right)^{n-p} \left(z \Omega(t) u^*(t) \right)^{m-p} \left(1 - |u(t)|^2 \Omega(t) \right)^p \right. \\
& \left. + \left(z^* \Omega(t) u(t) \right)^{m-p} \left(z \Omega(t) u^*(t) \right)^{n-p} \left(1 - |u(t)|^2 \Omega(t) \right)^p \right\}
\end{aligned}$$

$$\Omega(t) = \frac{2}{1 + 2\nu(t)} \qquad W_0^0(z,t) = \frac{2\exp\left(-\Omega(t)\,|z|^2\right)}{\pi[1 + \nu(t)]}$$

New J. Phys. **16** (2014) 103010



$1/\sqrt{2}\,(|0\rangle + |3\rangle)$

$1/\sqrt{2}\,(|2\rangle + |3\rangle)$.

Non-Markovianity measure using two-time correlation function

Markovianity: Divisibility of dynamical map, monotonic decrease of distinguishability of states using trace distance, monotonic decrease of entanglement between the system and an ancilla, negative decay rate, using mutual information

Non-Markovian dynamics due to strong back action (memory effects), significant in the short transient regime, strong SE coupling, low temperature, finite size structured environment

Rev. Mod. Phys. 88, 021002 (2016)

Two-time correlation function using Quantum regression theorem

$$\frac{\partial}{\partial t}\langle\widehat{O_i}(t)\rangle = Tr_S[\,\widehat{O_i}(0)\frac{\partial}{\partial t}\rho(t)\,] = \sum_j M_{ij}(t)\langle\widehat{O_j}(t)\rangle$$

QRT is valid under Born-Markov approx.

$$\frac{\partial}{\partial \tau}\langle\widehat{O_1}(t)\,\widehat{O_i}(t+\tau)\rangle = \sum_j M_{ij}(\tau)\langle\widehat{O_1}(t)\,\widehat{O_j}(t+\tau)\rangle$$

M. M. Ali, P. Y. Lo, M.W.Y. Tu, W. M. Zhang, Phys. Rev. A 92, 062306 (2015)

Non-Markovianity measure using exact two-time correlation function

$$\mathcal{N}(t,\tau) = |C(t,\tau) - C_{BM}(t,\tau)|$$

$$C(t,\tau) = \frac{\langle a^\dagger(t)a(t+\tau)\rangle_e}{\sqrt{\langle a^\dagger(t)a(t)\rangle_e \langle a^\dagger(t+\tau)a(t+\tau)\rangle_e}}$$

$$C_{BM}(t,\tau) = \frac{\langle a^\dagger(t)a(t+\tau)\rangle_{BM}}{\sqrt{\langle a^\dagger(t)a(t)\rangle_{BM} \langle a^\dagger(t+\tau)a(t+\tau)\rangle_{BM}}}$$

PHYSICAL REVIEW A **92**, 062306 (2015)

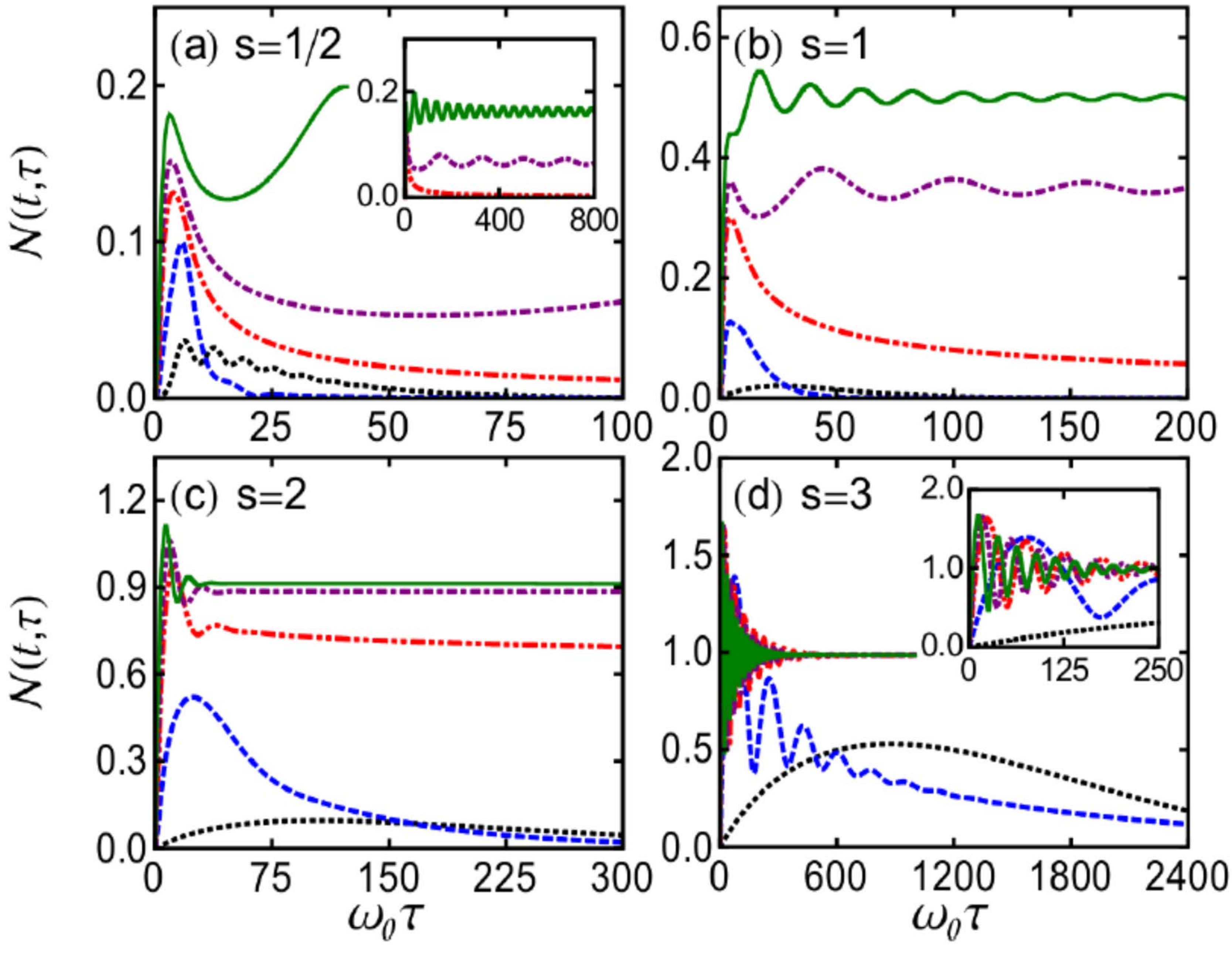
Exact Two-time correlation function using Quantum Langevin equation

$$\dot{a}(t) = -i\omega_0 a(t) - i\sum_k V_k\, b_k(0) e^{-i\omega_k t} - \int_0^t d\tau\, g(t,\tau)\, a(\tau)$$

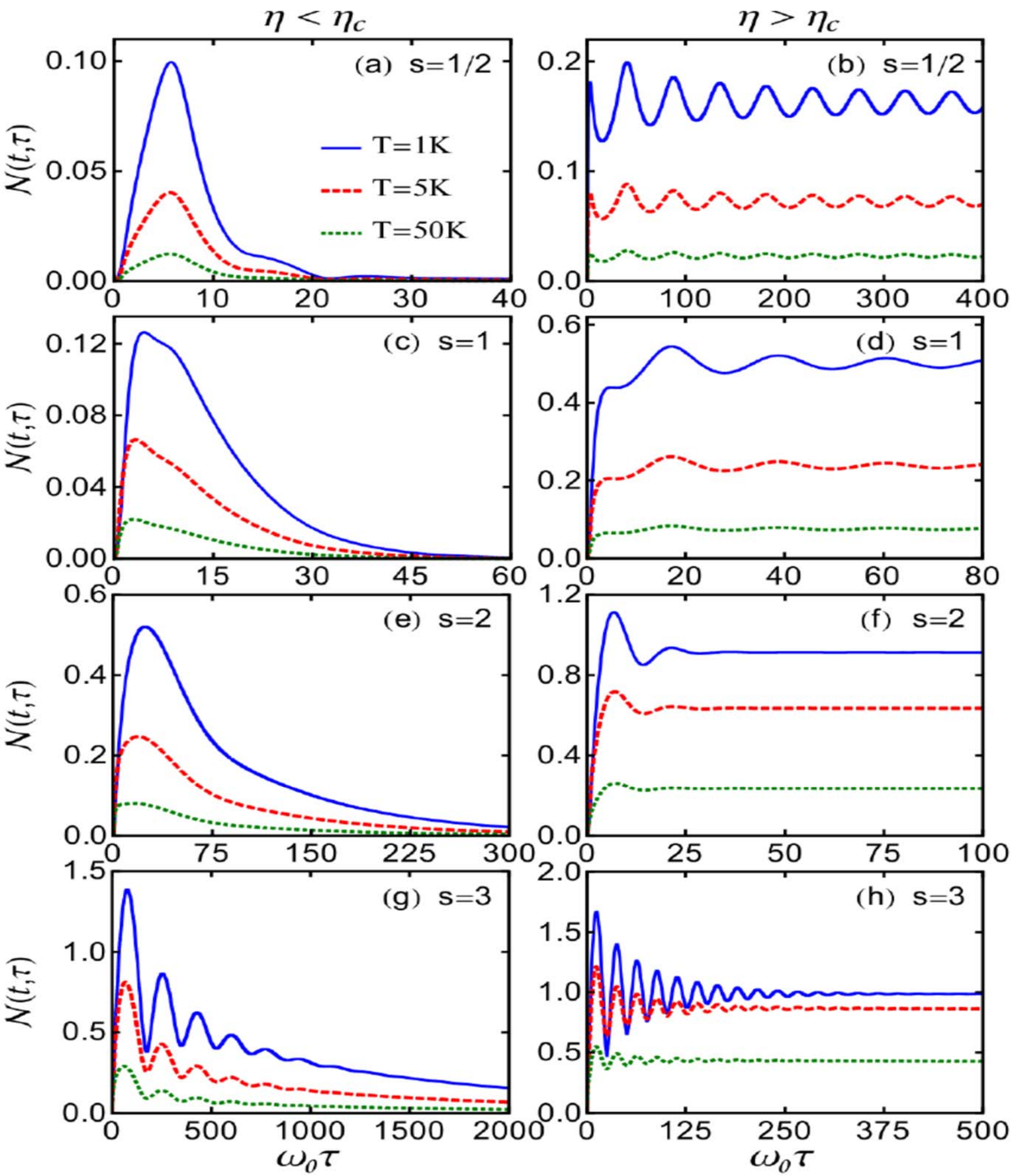
$$\langle a^\dagger(t) a(t+\tau)\rangle_e = u^*(t,0) u(t+\tau,0)\, \langle a^\dagger(0) a(0)\rangle + v^*(t,t+\tau)$$

$$v(\mathrm{t},\mathrm{t}+\tau) = \int_0^t d\tau_1 \int_0^{t+\tau} d\tau_2\, u(t,\tau_1)\tilde{g}(\tau_1,\tau_2) u^*(t+\tau,\tau_2)$$

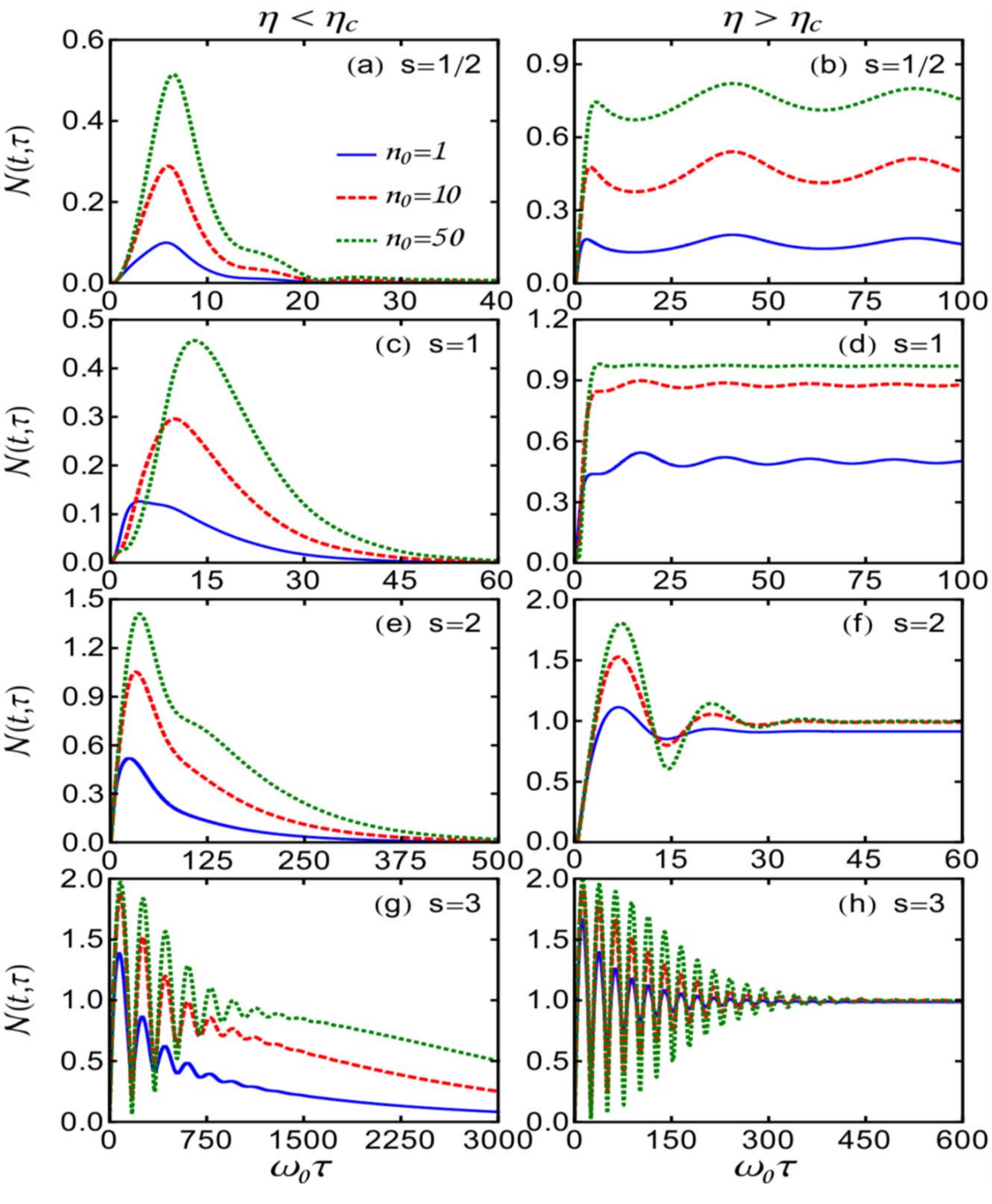
PHYSICAL REVIEW A **92**, 062306 (2015)



(a) s=1/2
(b) s=1
(c) s=2
(d) s=3
N(t,τ)
N(t,τ)
ω0τ
ω0τ
0.0
0.1
0.2
0
25
50
75
100
0.2
0.0
0
400
800
0.0
0.2
0.4
0.6
0
50
100
150
200
0.0
0.3
0.6
0.9
1.2
0
75
150
225
300
0.0
0.5
1.0
1.5
2.0
0
600
1200
1800
2400
2.0
1.0
0.0
0
125
250



$\eta < \eta_c$
$\eta > \eta_c$
(a) s=1/2
(b) s=1/2
(c) s=1
(d) s=1
(e) s=2
(f) s=2
(g) s=3
(h) s=3
T=1K
T=5K
T=50K
$\mathcal{N}(t,\tau)$
$\omega_0\tau$



$\eta < \eta_c$
$\eta > \eta_c$
(a) s=1/2
(b) s=1/2
(c) s=1
(d) s=1
(e) s=2
(f) s=2
(g) s=3
(h) s=3
n_0=1
n_0=10
n_0=50
$\mathcal{N}(t,\tau)$
$\omega_0\tau$

Thank You