THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Probing contextuality with superconducting circuits

Arkady Fedorov

Superconducting Quantum Device Laboratory
Centre for Engineered Quantum Systems
University of Queensland

Experiment:
Markus Jerger, Yarema Reshitnyk, Andres Rosario, Pascal Macha (UQ),
Nathan Langford (TU Delft, Netherlands)

Sample: Kristinn Juliusson, Denis Vion (Sacley, France)
Parametric amplifier: Markus Oppliger, Anton Potočnik, Andreas Wallraff (ETHZ, Switzerland)

P-value calculation: Kenneth Goodenough, Stephanie Wenher (Delft TU)

Quantum randomness, entanglement and non-contextuality

Quantum randomness

Quantum mechanics prescribes possible outcomes and probabilities of the measurement but no predictions on an individual experimental run

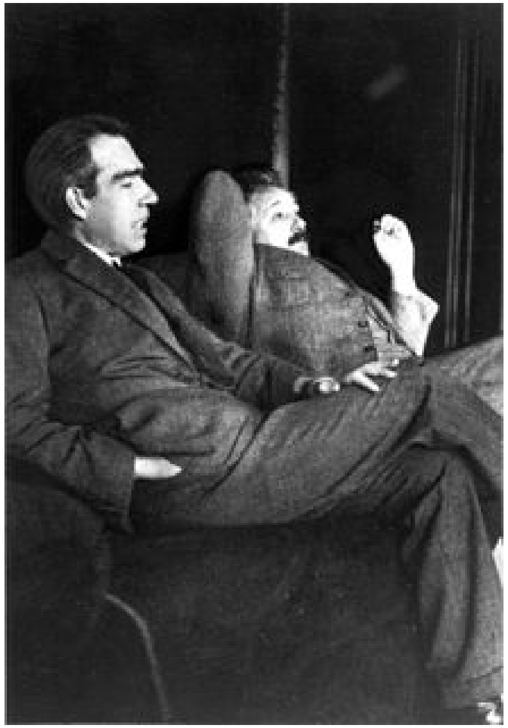
Example: prepare an eigenstate of σ_X: $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and measure σ_z
QM: each measurement returns -1 or +1, $\langle\sigma_z\rangle = 0$

Different to deterministic classical mechanics.

- Accept is as physical reality (Bohr)

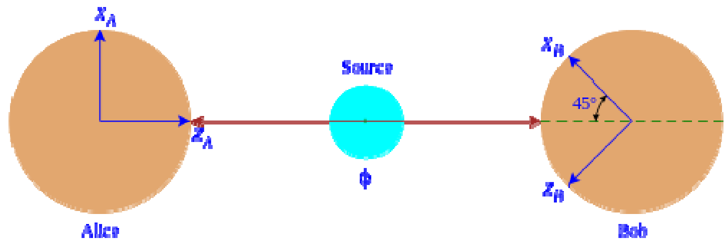
- Claim that QM is incomplete (Einstein) ->

Need to supplement QM with hidden variables



Entanglement and local realism

1935 Einstein, Podolsky and Rosen: EPR paradox for 2 spin-1/2 particles (qubits).



$$|\psi\rangle = (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$$

Local hidden variables (HV) were expected to resolve the paradox

1967 Bell theorem: no local hidden variables can reproduce the outcomes of QM

1969 John, Horner, Simony, Holt (CHSH) inequality: experimentally testable inequality

$$-2 \le \langle A_A A_B \rangle + \langle A_{A'} A_B \rangle + \langle A_{A'} A_{B'} \rangle - \langle A_A A_{B'} \rangle \le 2$$

Experimental verification: A. Aspect (1982),…, R. Hanson (2015),...

Divide between classical and quantum is deeper than quantum entanglement.

Quantum mechanics and non-contextual

Local realism is a part of non-contextual realism:
outcome of a measurement depends only on the current state of the system, and not on which other measurements, if any, are performed in conjunction with it (the measurement context).

1967 Kochen and Specker: proved that non-contextual realism is in contradictions with outcomes of QM (proven for spin-1 no entanglement)

2008 Klyachko, Can, Binicioglu and Shumovsky (KCBS) found the simplest recipe to demonstrate contextuality with a qutrit (no entanglement but state dependent)

$$\langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle + \langle A_5 A_1 \rangle \geq -3$$

2012 Yu and Oh, state independent test for a qutrit

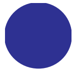
- One of the most fundamental property of quantum mechanics not requiring composite systems, entanglement, non-locality and specific state

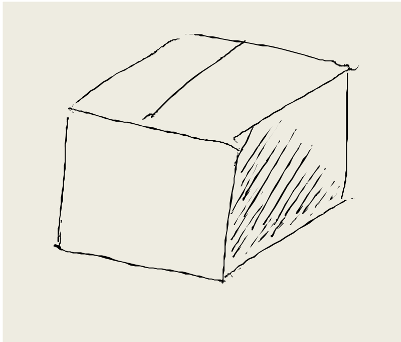
2014 Howard, Wallman, Veitch & Emerson: contextuality – responsible for exponential speedup of a quantum computer

Compatible measurements and non-contextuality

Binary-outcome classical measurement

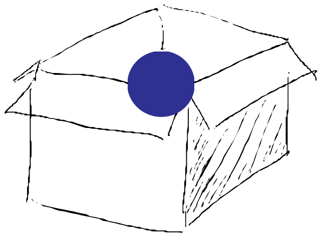
THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Consider a closed box with  or  inside

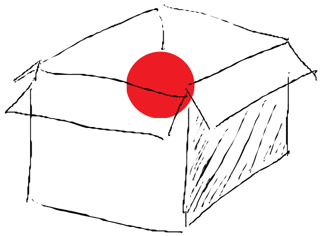


Define the measurement:

A = +1



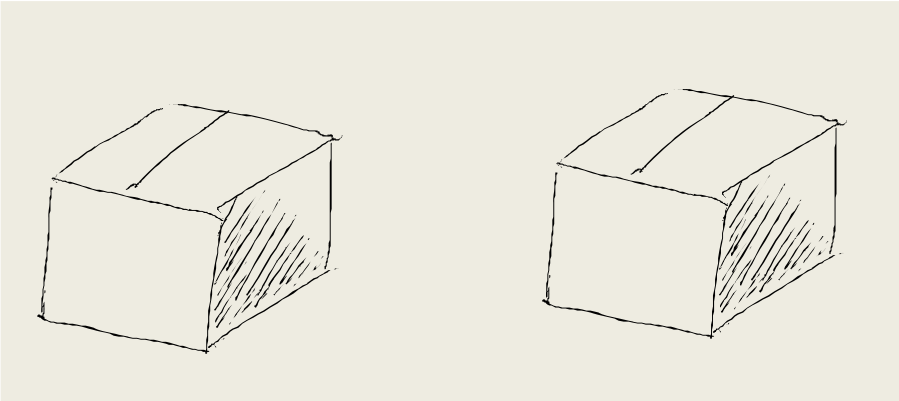
A = -1



Opening the box merely reveals the predetermined values

Compatible measurements

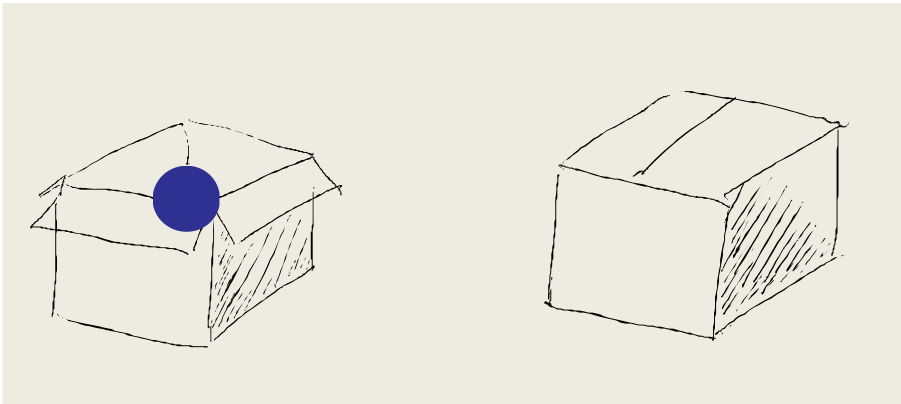
Can be measured jointly on the same individual system without disturbing each other



Compatible measurements

Can be measured jointly on the same individual system without disturbing each other (the outcomes and expectation values agree for any sequence of measurements)

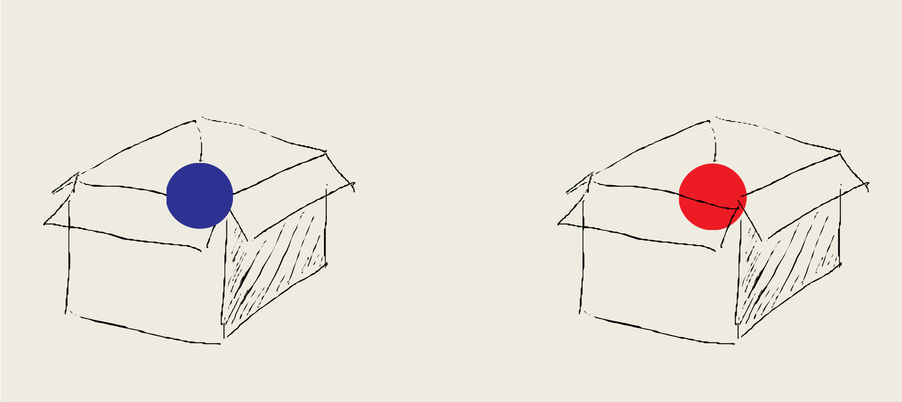
$\boldsymbol{A_1} = +1$



Compatible measurements

Can be measured jointly on the same individual system without disturbing each other (the outcomes and expectation values agree for any sequence of measurements)

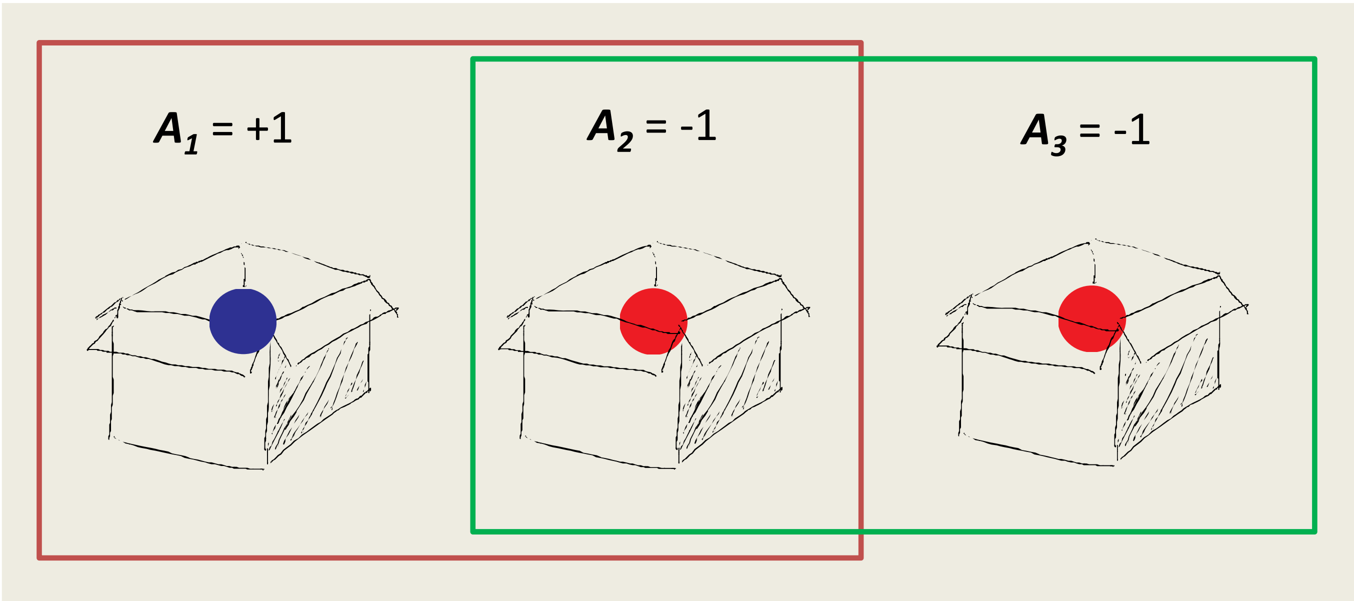
$\boldsymbol{A_1} = +1$ $\boldsymbol{A_2} = -1$



Can be measured simultaneously or in any order

Non-contextuality

Context: a set of *compatible* measurements
Outcomes are independent of the measurement context.



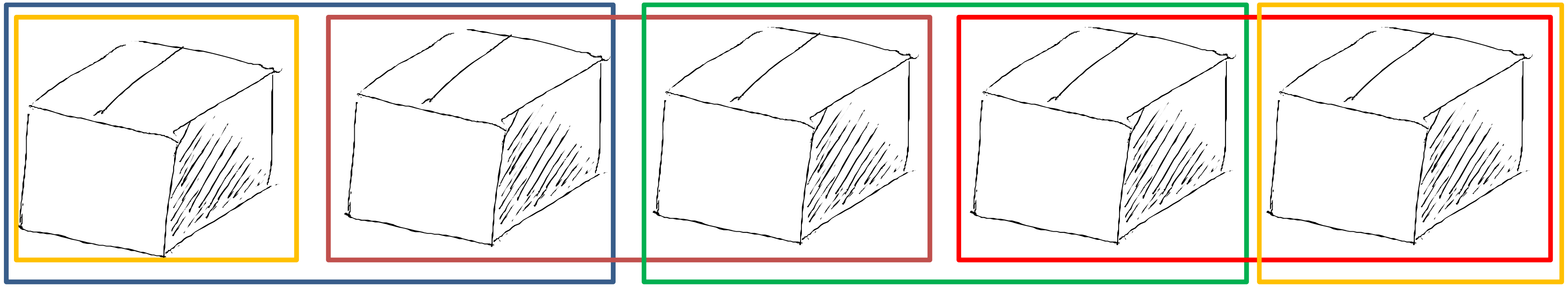
Noncontextual realism: outcome are independent of which other compatible measurements are carried out with the measurement -> follows our classical intuition about deterministic world

Does QM follows similar intuition?

Klyachko, Can, Binicioglu and Shumovsky (KCBS) inequality

Klyachko, Can, Binicioglu and Shumovsky (KCBS) scenario

Consider five boxes



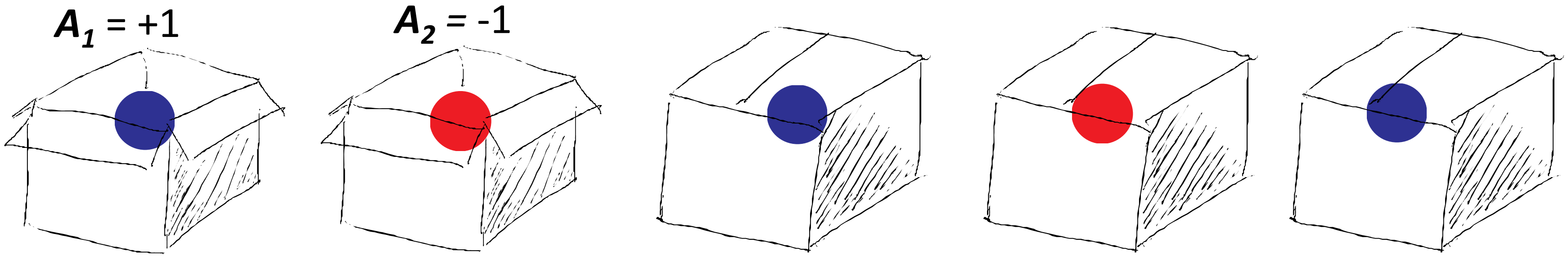
Open five pairs and based on the outcomes construct the number:

$$A_1 A_2 + A_2 A_3 + A_3 A_4 + A_4 A_5 + A_5 A_1$$

How to put marbles to reach the minimum possible value for this number?

Answer: the marbles should alternate the colours

Classical case



Open five pairs and based on the outcomes construct the number:

$$\underbrace{A_1 A_2}_{-1} + \underbrace{A_2 A_3}_{-1} + \underbrace{A_3 A_4}_{-1} + \underbrace{A_4 A_5}_{-1} + \underbrace{A_5 A_1}_{+1} = -3$$

In general, for any predetermined distribution classical functions one can find

$$\langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle + \langle A_5 A_1 \rangle \geq -3$$

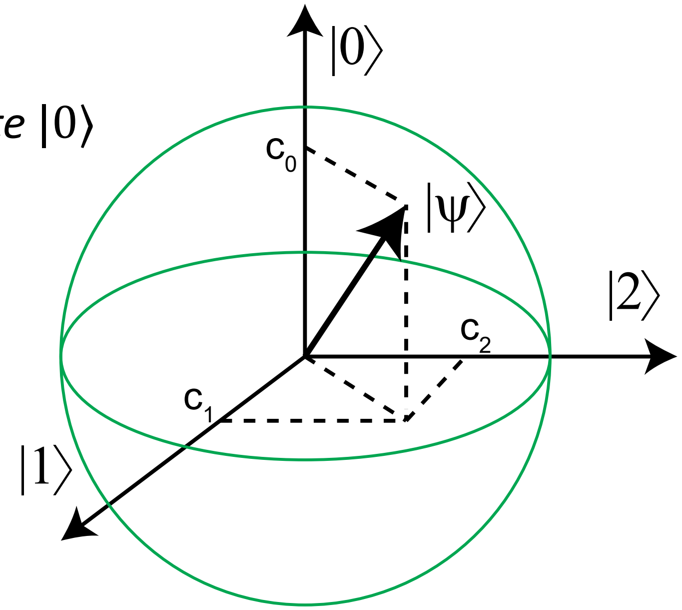
Why compatible: to reveal correlations, not cross-talk of the measurements

Quantum case: dichotomic measurement for a qutrit

Consider three-level quantum system (qutrit):

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$$

Define measurement along $|0\rangle$: *is the system in state* $|0\rangle$ *or not?*



Quantum case: dichotomic measurement for a qutrit

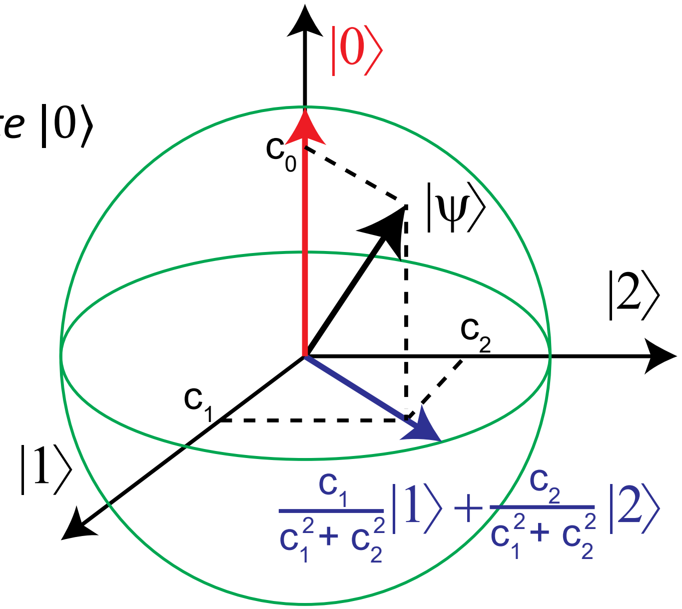
Consider three-level quantum system (qutrit):

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$$

Define measurement along $|0\rangle$: *is the system in state* $|0\rangle$ *or not?*

Described by
$A_0 = 2|0\rangle\langle 0| - \mathrm{I},\ \{M_{|0\rangle}\} = \{|0\rangle\langle 0|, I - |0\rangle\langle 0|\}$:

The result of the measurement returns
A = -1 if the state found along $|0\rangle$ and
A = +1 otherwise.



Note: that coherence between $|1\rangle$ and $|2\rangle$ is preserved (see later)

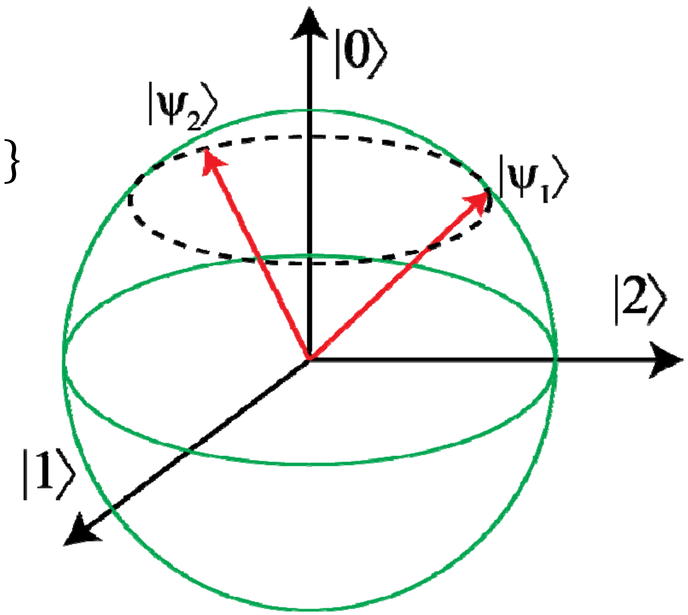
Quantum case: dichotomic measurement for a qutrit

Consider two states: $|\psi_1\rangle, |\psi_2\rangle$

Can define two observables and measurements

$\mathrm{A}_i = 2|\psi_i\rangle\langle\psi_i| - \mathrm{I}, \{\mathrm{M}_{|\psi_i\rangle}\} = \{|\psi_i\rangle\langle\psi_i|, I - |\psi_i\rangle\langle\psi_i|\}$
$\mathrm{i} = 1,2$

If $\langle\psi_1|\psi_2\rangle = 0$ then $[\mathrm{A}_1, \mathrm{A}_2] = 0$ and QM predicts that the measurements will be compatible

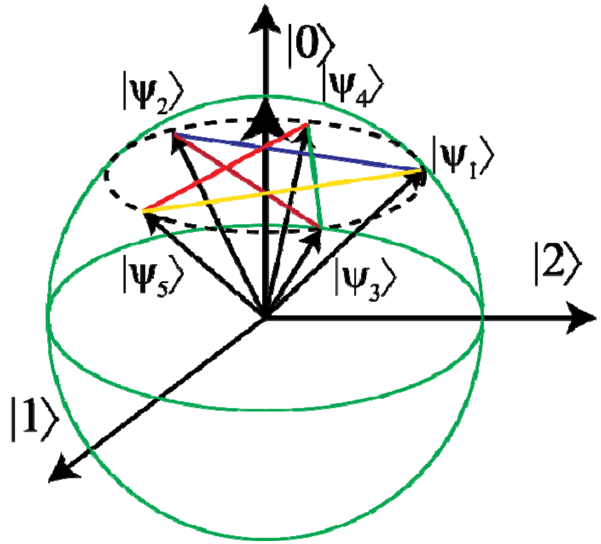


Note: that coherence in the orthogonal subspace is necessary for compatibility

KCBS inequality

The University of Queensland
AUSTRALIA

Define five sequentially pair-wise orthogonal measurement directions (not possible for a qubit)



If we prepare the system in $|0\rangle$ the result of the five pairs of measurements give

$$\langle A_1A_2\rangle + \langle A_2A_3\rangle + \langle A_3A_4\rangle + \langle A_4A_5\rangle + \langle A_5A_1\rangle = -3.994 < -3$$

The outcomes are not predetermined: cannot be explained by any non-contextual hidden variable theory

No non-locality, composite system, or entanglement are involved

Requires compatibility test

Extended KCBS inequality

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Orthogonality predicts that measurements are compatible
Need some additional test to check that.
Following the recipe of O. Guehne et. al. Phys. Rev. A 81, 022121 (2010)

What happens if the measurements are not compatible?

$$\langle \boldsymbol{A_1} \rangle \text{ (for } \boldsymbol{A_1 A_2}) - \langle \boldsymbol{A_1} \rangle \text{ (for } \boldsymbol{A_2 A_1}) = \varepsilon_{12}$$

The threshold to rule out non-contextual models is higher

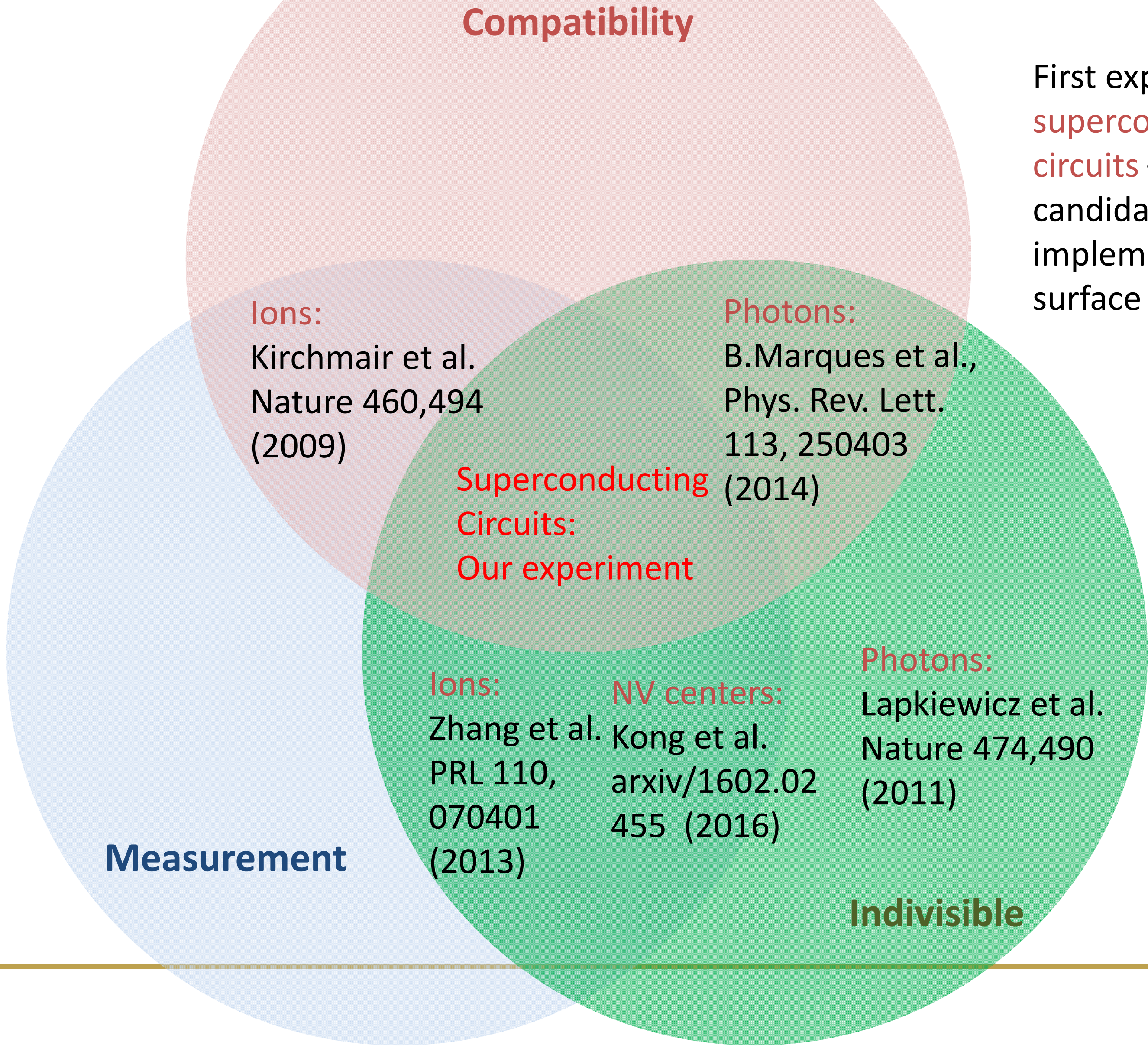
$$\langle \boldsymbol{A_1 A_2} \rangle + \langle \boldsymbol{A_2 A_3} \rangle + \langle \boldsymbol{A_3 A_4} \rangle + \langle \boldsymbol{A_4 A_5} \rangle + \langle \boldsymbol{A_5 A_1} \rangle < -(3 + |\varepsilon_{12}| + |\varepsilon_{23}| + |\varepsilon_{34}| + |\varepsilon_{45}| + |\varepsilon_{15}|)$$

Possible reasons for incompatability:

- Not perfect orthogonality (control errors)
- Wrong measurement apparatus
- Measurement cross-talk

Being non-contextual in experiment

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA



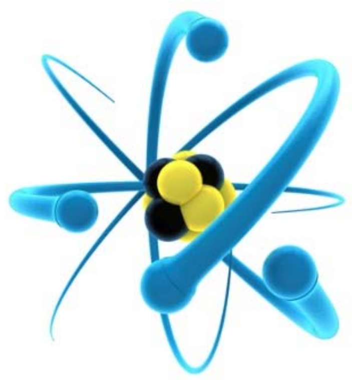
First experiment with superconducting circuits – the primary candidate for implementation surface codes

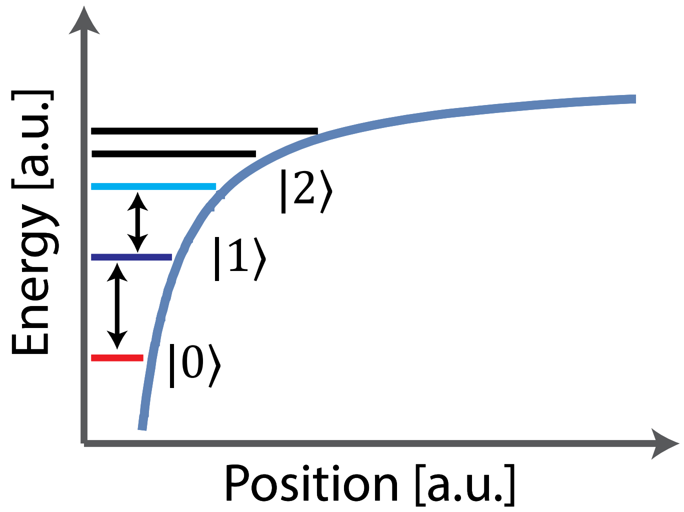
Wei, L. F., Maruyama, K., Wang, X. B., You, J. Q. & Nori, F. Testing quantum contextuality with macroscopic superconducting circuits. Phys. Rev. B 81, 174513 (2010).

Superconducting qutrit

Natural qutrit

Atom: anharmonic spectrum

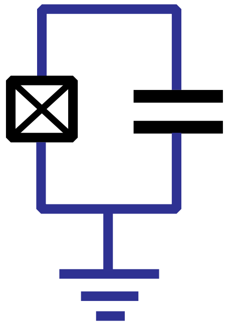


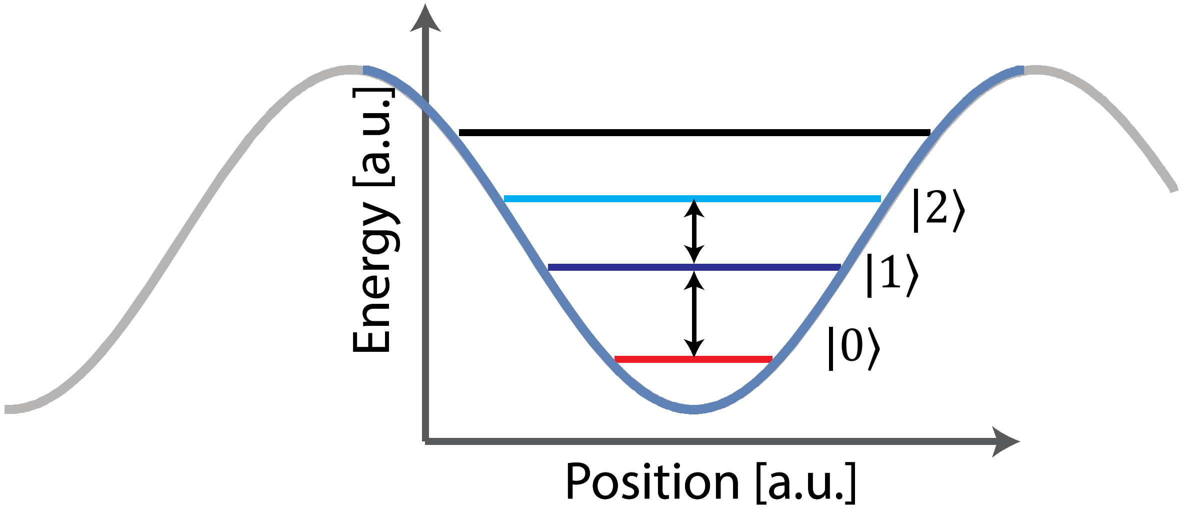


Good approximation for a qutrit

Superconducting qubit

Josephson junction shunted by a capacitor

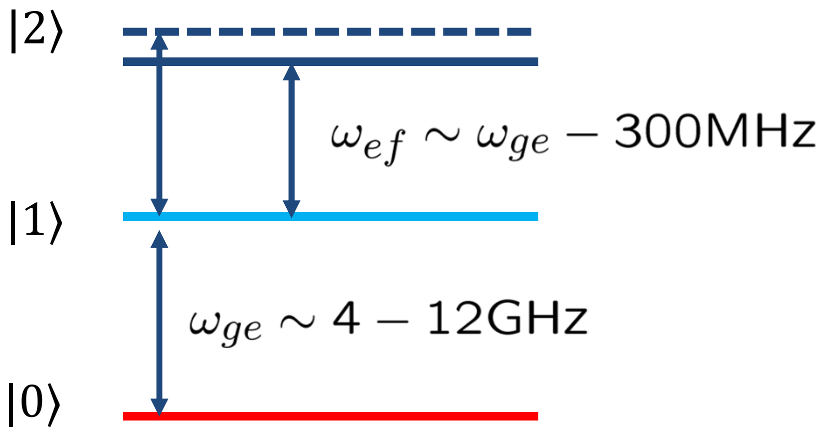


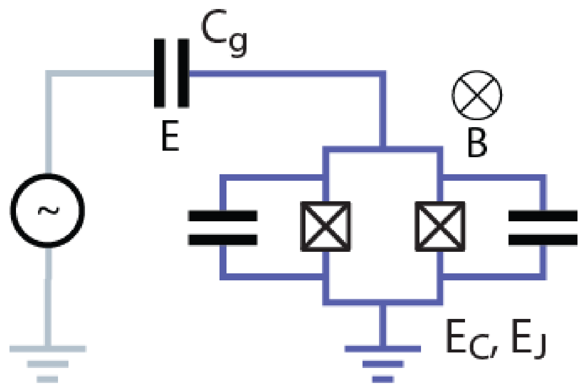


Artificial atom: superconducting qutrit

Superconducting qubit

Spectrum: ladder type





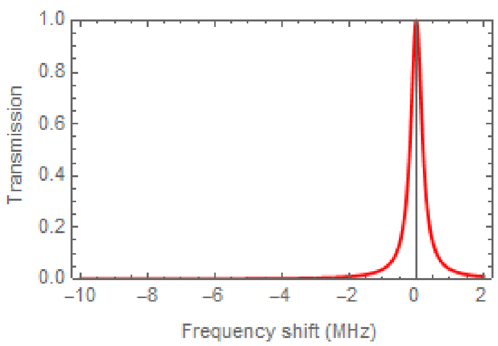
- Control:
- Flux: modulates energy splitting
- Charge: induces transition

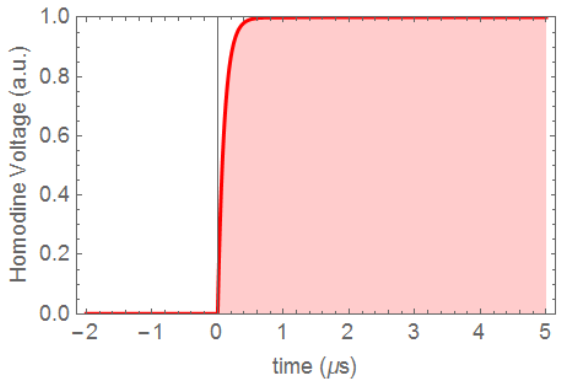
  between levels

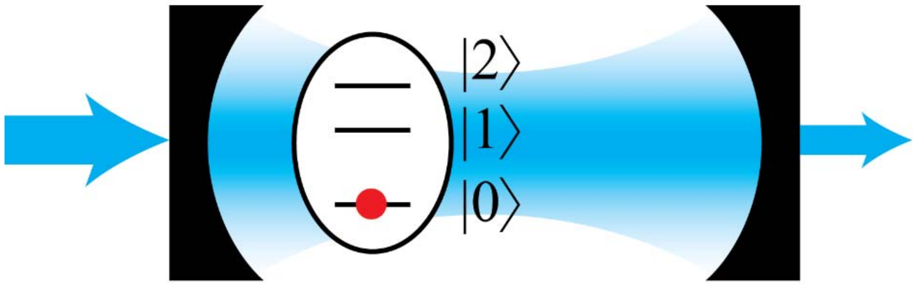
Generating compatible measurements with superconducting qutrit

Dispersive readout

Superconducting qutrit in a cavity: measure transmission to determine the state

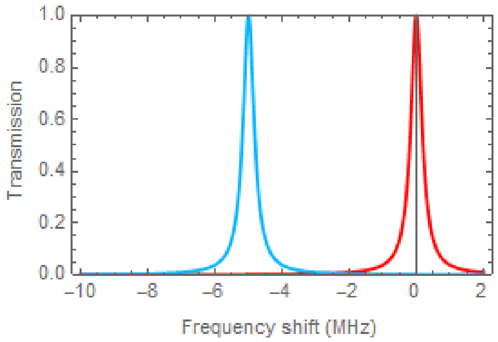


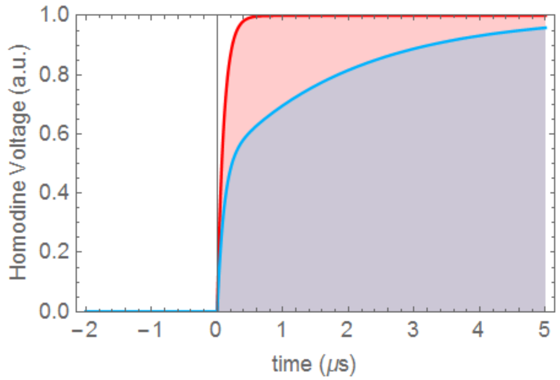


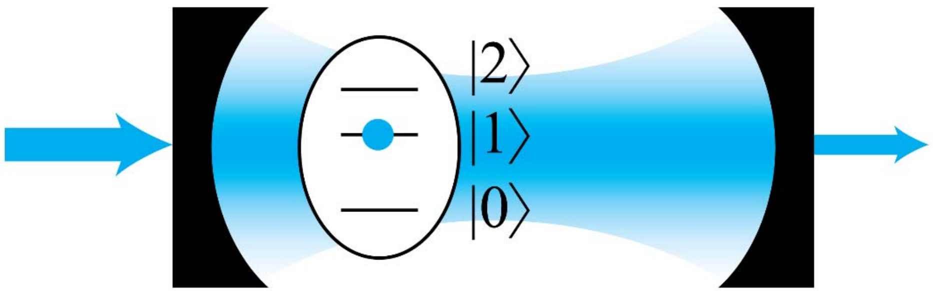


Dispersive readout

Superconducting qutrit in a cavity: measure transmission to determine the state

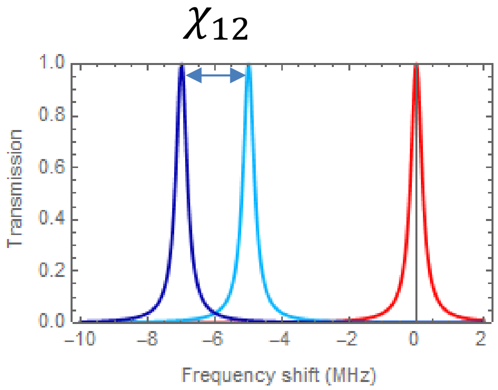


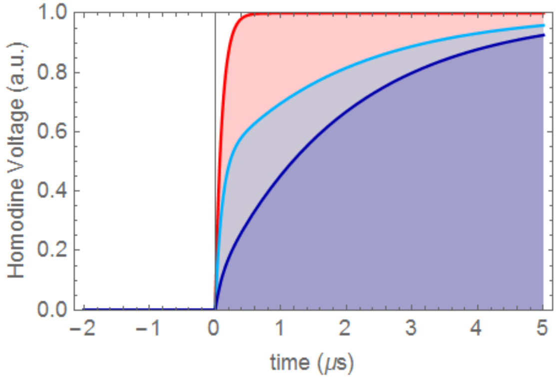


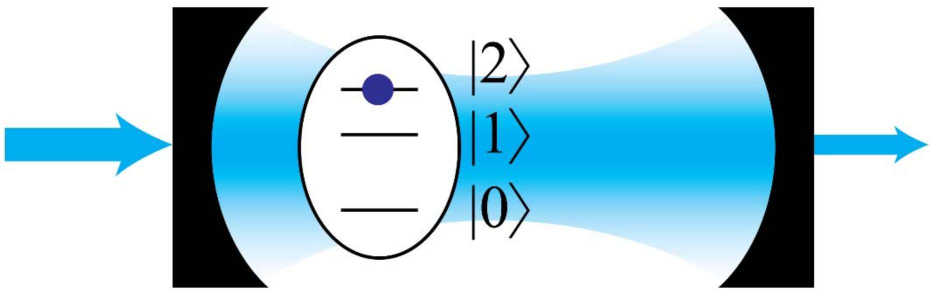


Dispersive readout

Superconducting qutrit in a cavity: measure transmission to determine the state







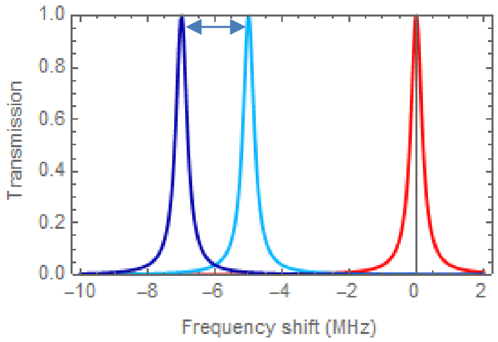
Measurement always provides information about all states destroying the coherence between $|1\rangle$ and $|2\rangle$

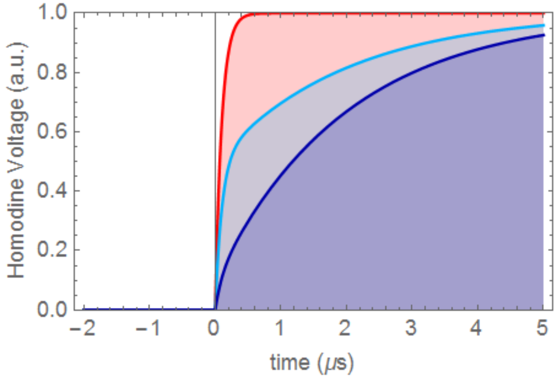
Dispersive readout

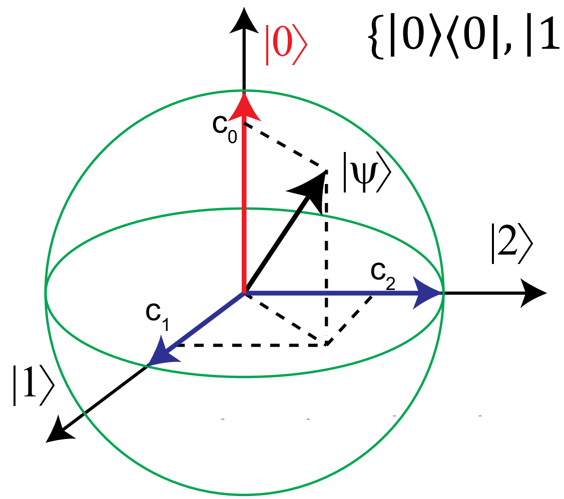
The University Of Queensland
AUSTRALIA

Superconducting qutrit in a cavity: measure transmission to determine the state

χ_{12}







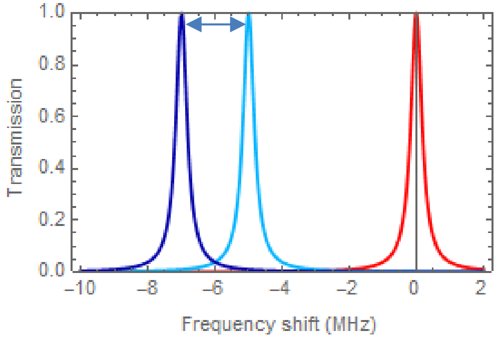
$\{|0\rangle\langle 0|, |1\rangle\langle 1|, |2\rangle\langle 2|\}$

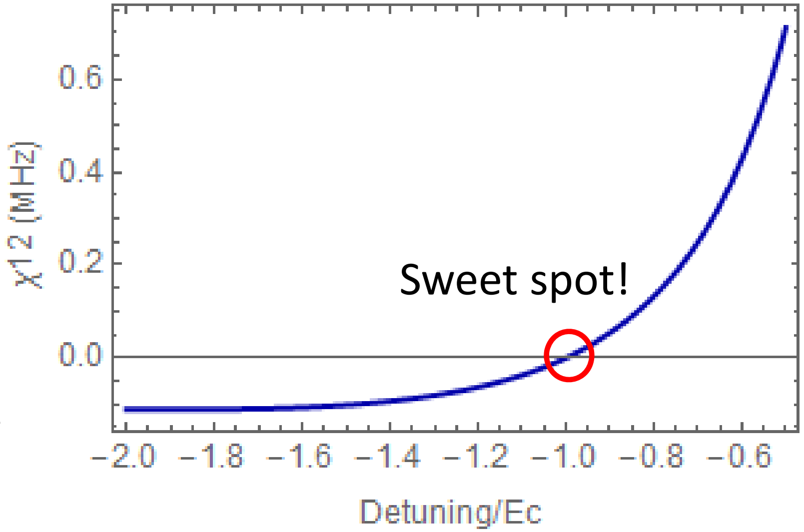
Measurement always provides information about all states destroying the coherence between $|1\rangle$ and $|2\rangle$

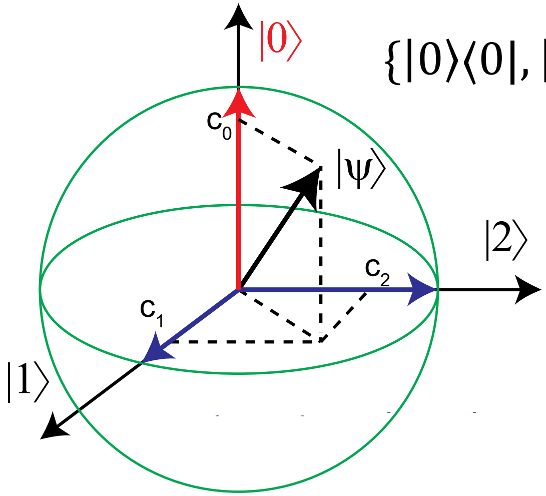
Dispersive readout

Relative dispersive shift as function of qubit detuning

χ_{12}





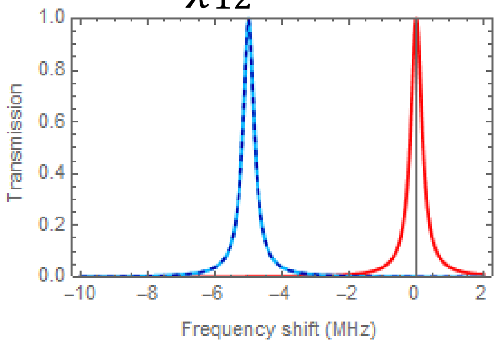


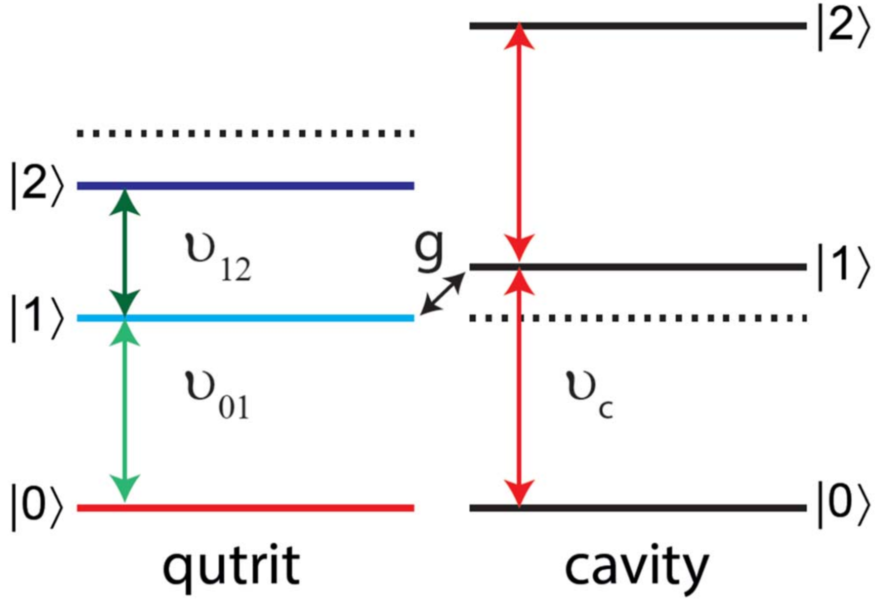
$\{|0\rangle\langle 0|, |1\rangle\langle 1|, |2\rangle\langle 2|\}$

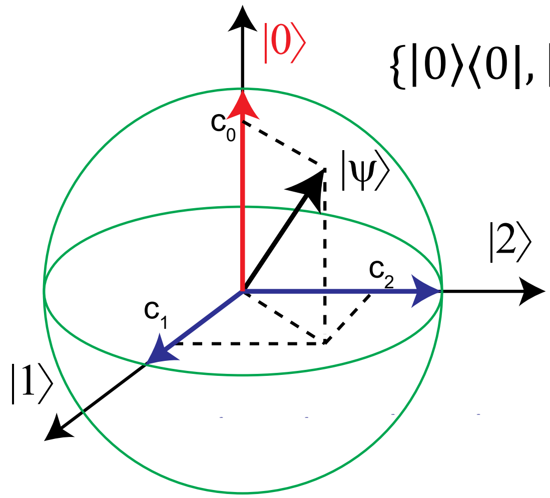
Dispersive readout: sweet spot

Relative dispersive shift as function of qubit detuning

$\chi_{12} = 0$





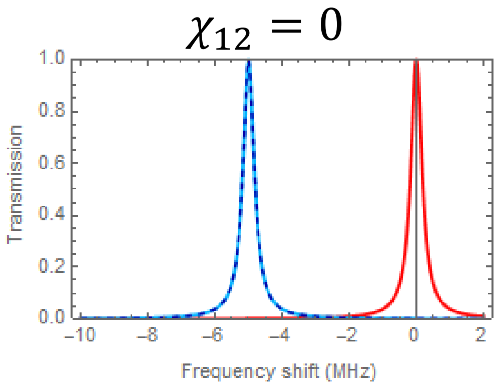


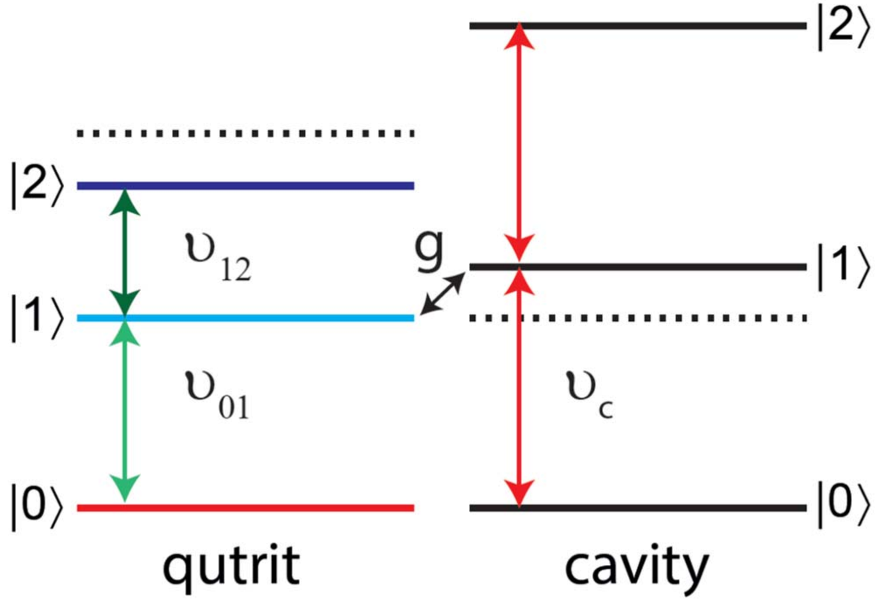
$\{|0\rangle\langle 0|, |1\rangle\langle 1|, |2\rangle\langle 2|\}$

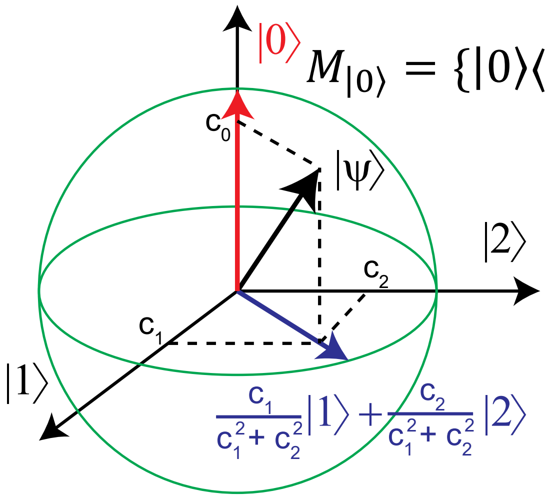
At sweet spot it is not possible to distinguish $|1\rangle$ from $|2\rangle$.

Dispersive readout: sweet spot

Relative dispersive shift as function of qubit detuning







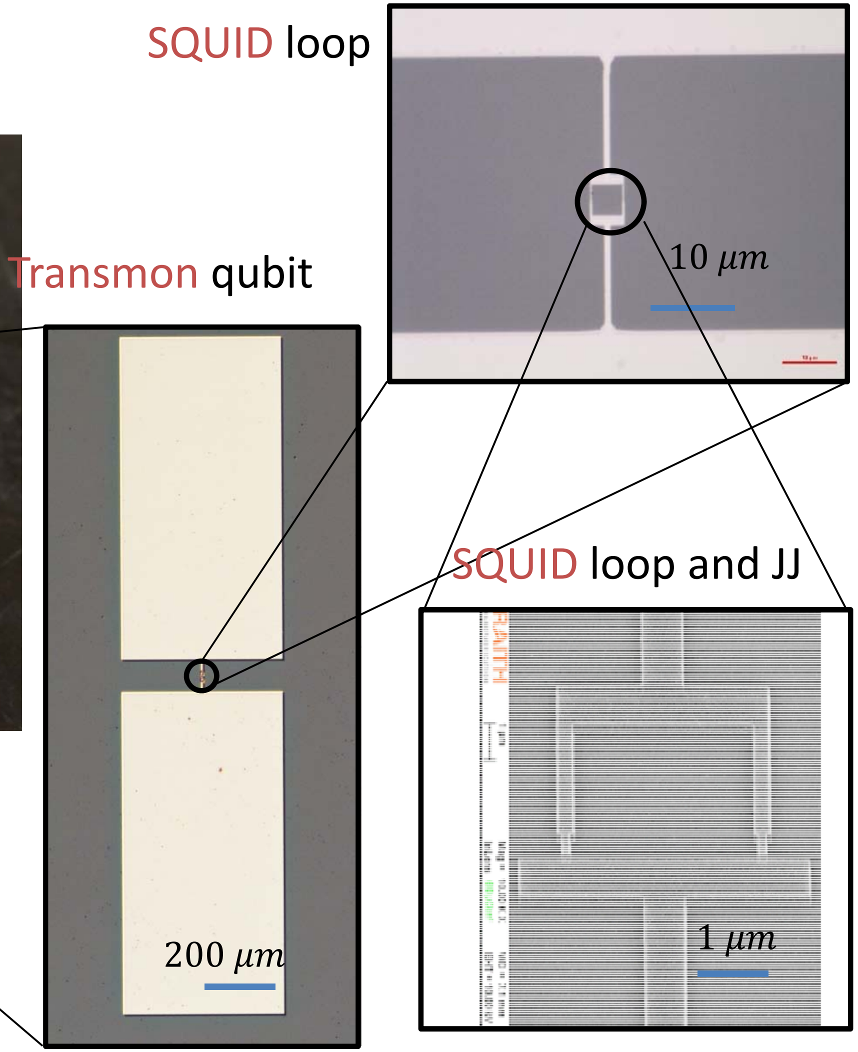
$$M_{|0\rangle} = \{|0\rangle\langle 0|, I - |0\rangle\langle 0|\}$$

At sweet spot it is not possible to distinguish $|1\rangle$ from $|2\rangle$. Measurement retains coherence between $|1\rangle$ and $|2\rangle$

Experimental setup

Transmon in 3D microwave cavity

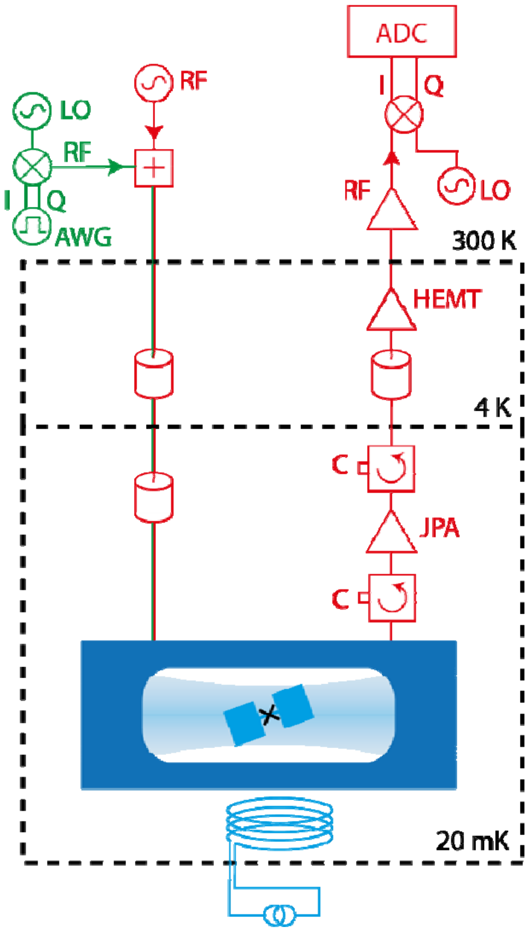
3D microwave cavity and a chip



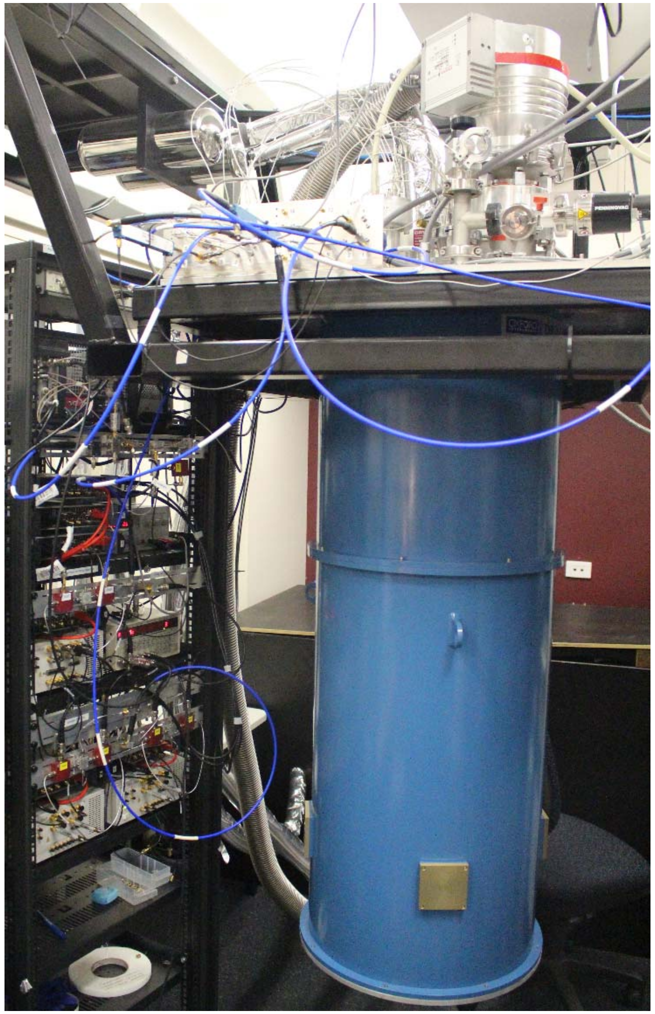
Sample fabricated by Kristinn Juliusson (CEA Sacley, France)
Cavity: designed and machined at UQ

Setup

Simplified experiment scheme



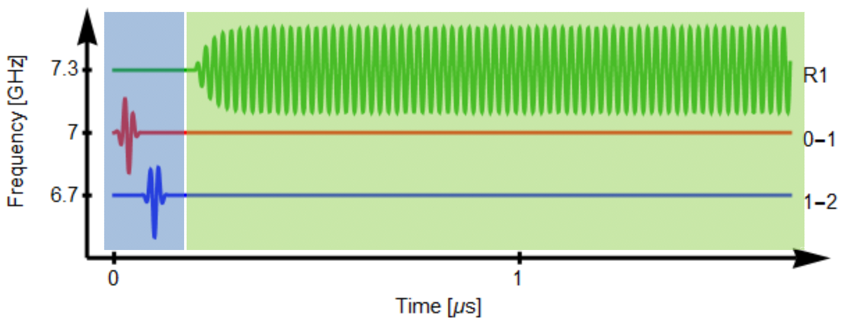
How it actually looks in the lab

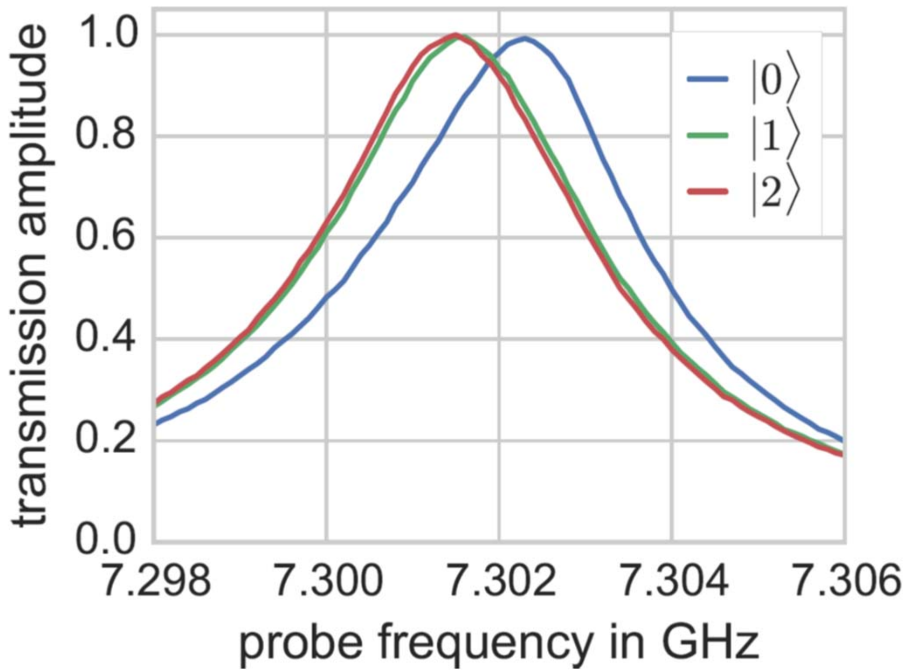


Measuring dispersive shifts

The University of Queensland
AUSTRALIA

Prepare $|0\rangle, |1\rangle, |2\rangle$ and measure transmission through the cavity.
Plot integrated signal as function of frequency



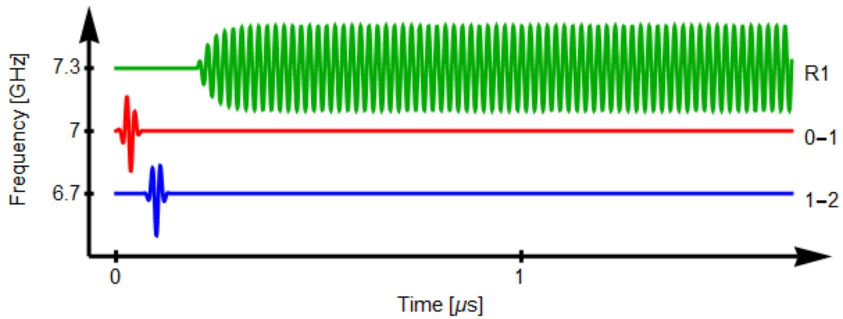


Dispersive shifts for $|1\rangle$ and $|2\rangle$ are identical

Measuring cavity response in time domain

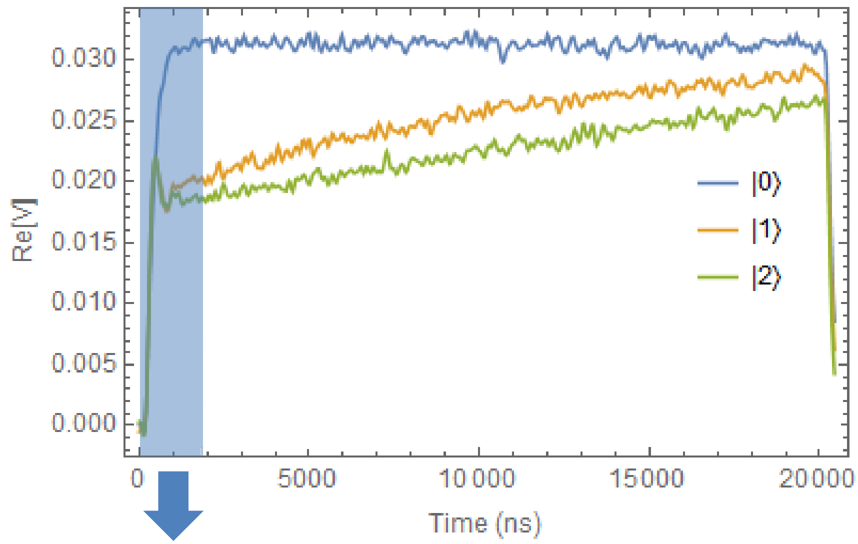
THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

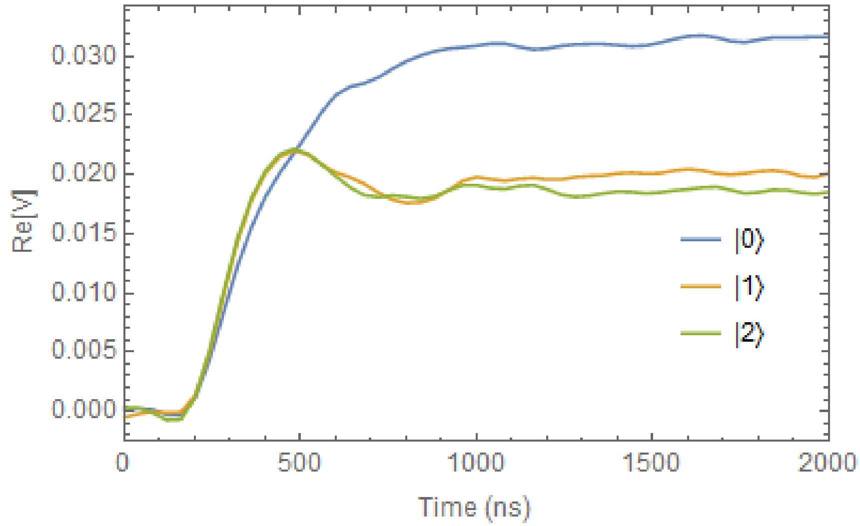
Prepare $|0\rangle, |1\rangle, |2\rangle$ and measure transmission through the cavity at fixed frequency as function of time



Averaged: 16384 times

Need to distinguish states with certainty within first hundreds of nanosecond!

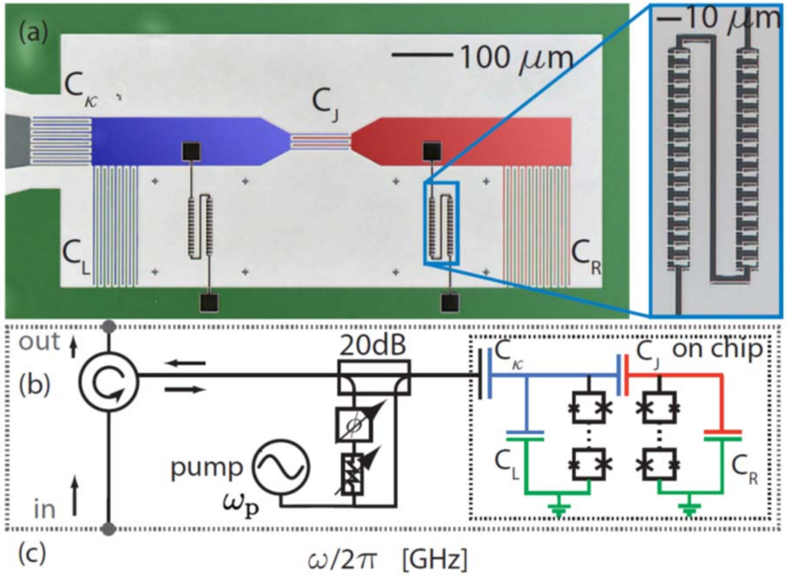




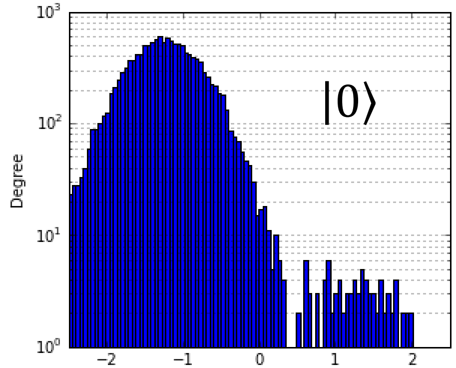
Single-shot measurement

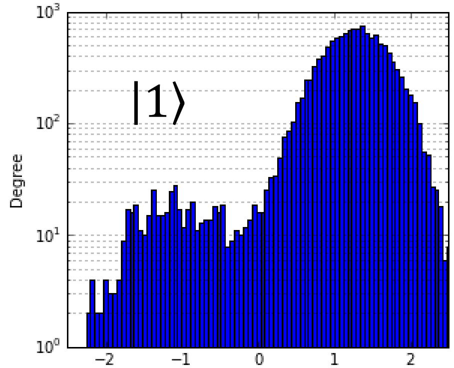
Fix readout length to ~300-400 ns. Use parametric amplifier for near quantum limited amplification. Prepare $|0\rangle$, $|1\rangle$, $|2\rangle$ and plot histograms for integrated voltage

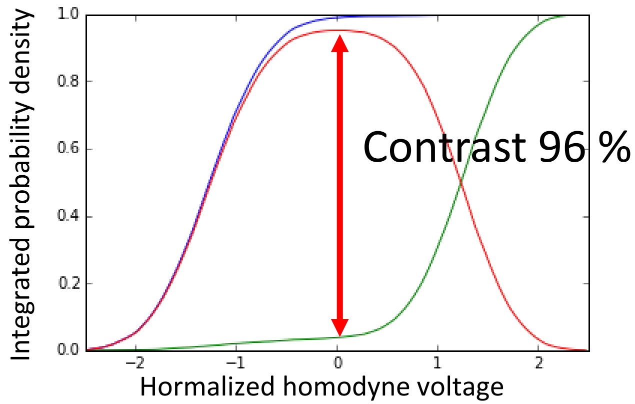
Paramp fabricated by Marcus Oppliger, Anton Potochnik, Andreas Wallraff ETH Zurich



C. Eichler, Y. Salathe, J. Mlynek, S. Schmidt, and A. Wallra, Phys. Rev. Lett. , 110502 (2014).

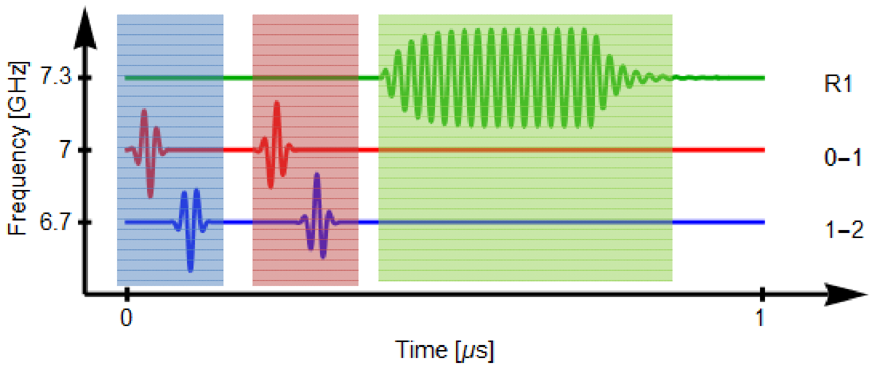


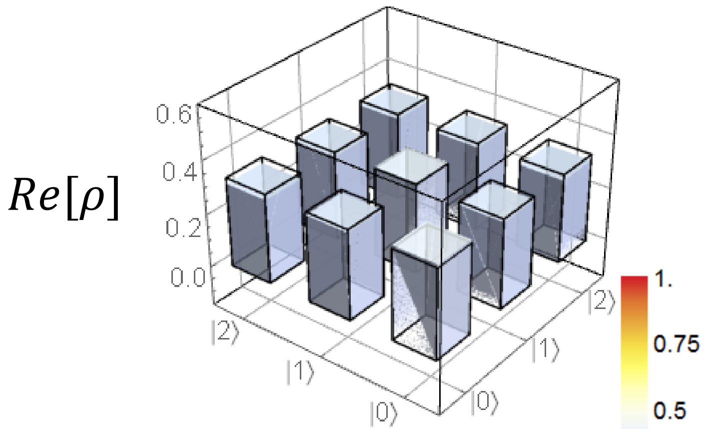




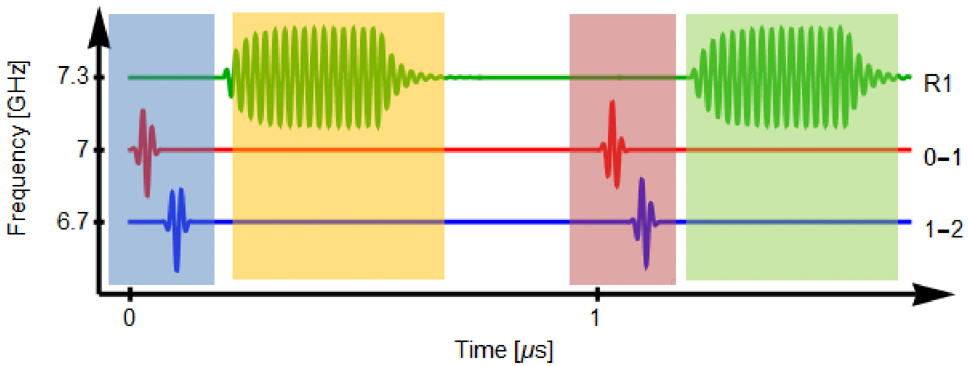
Testing readout on a state

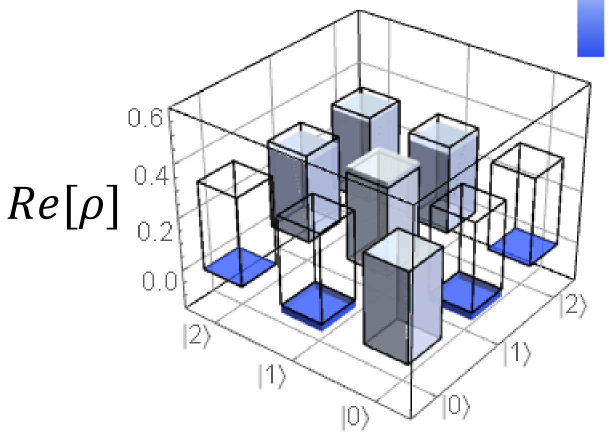
Prepare a superposition: $|\psi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle$
Do tomography of a state (measure prepared state for 9 tomography pulses):





Measure and do tomography again:





Process tomography of the readout

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

Arbitrary quantum process: $\boxed{\rho' = \mathcal{E}(\rho)}$

decomposed into: $\boxed{\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}}$

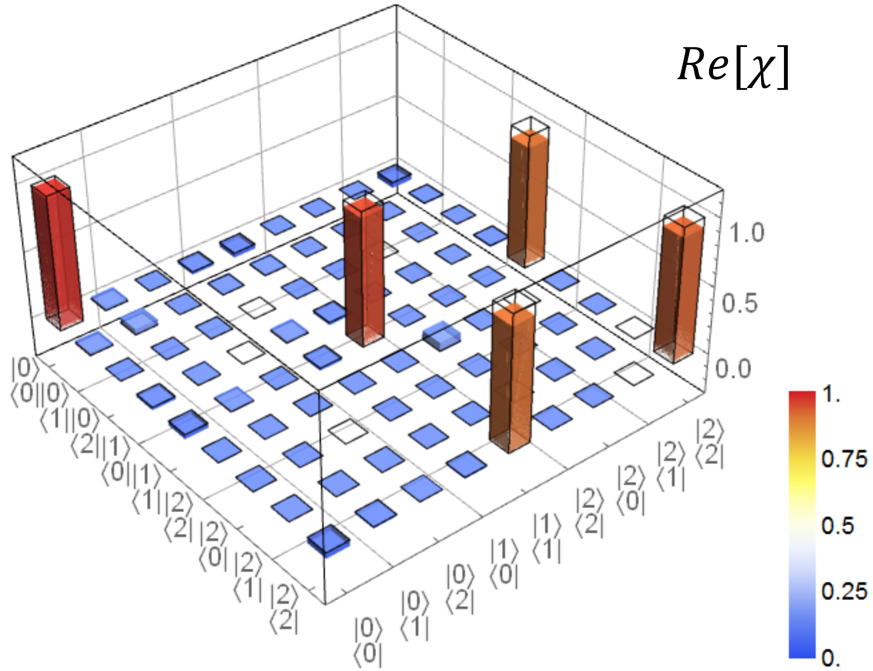
$\{\tilde{E}_k\}$ is an operator basis
χ is a positive semi definite Hermitian matrix characteristic for the process

Prepare 9 superpositions and do tomography for each -> reconstruct χ-matrix of the process

F = 97% to the binary projective measurement described $\{\mathrm{M}_{|0\rangle}\} = \{|0\rangle\langle 0|, I - |0\rangle\langle 0|\}$

Can be also used for leakage detection

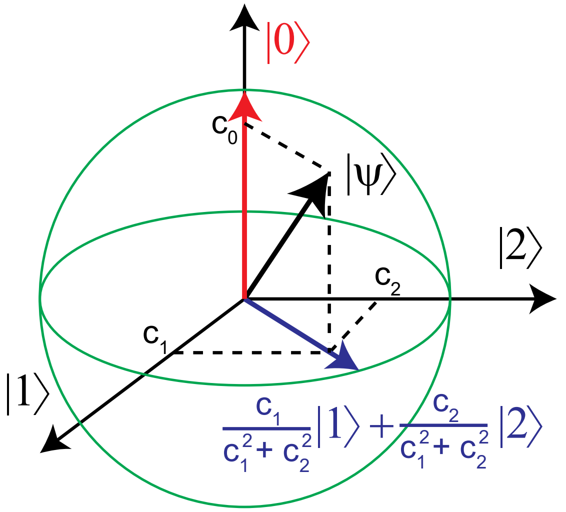
Jerger *et al. Phys. Rev. Applied* 6, 014014 (2016)



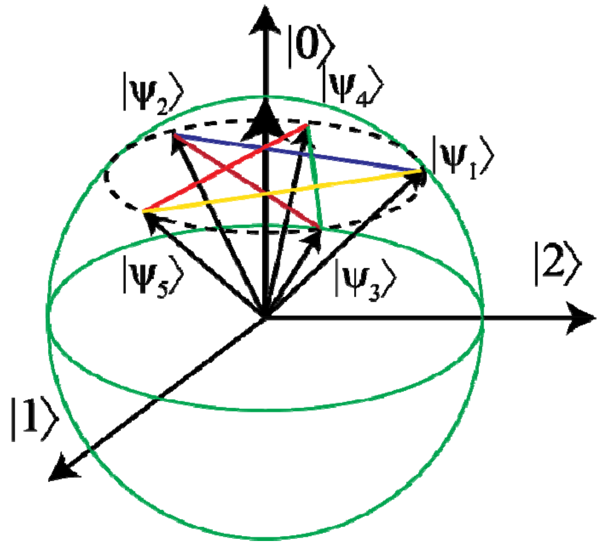
Testing KCBS inequality

Creating five pairs of compatible measurements

Dispersive read out at seeet spot: measurement along $\mathrm{M}_{|0\rangle}$



Needed: measurement along $\mathrm{M}_{|\psi_1\rangle}$, $\mathrm{M}_{|\psi_2\rangle}$, $M_{|\psi_3\rangle}$, $M_{|\psi_4\rangle}$, $\mathrm{M}_{|\psi_5\rangle}$

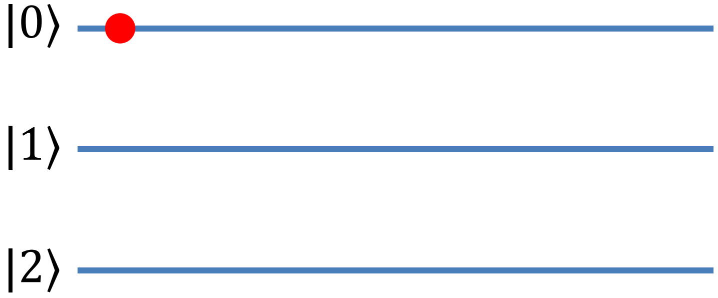


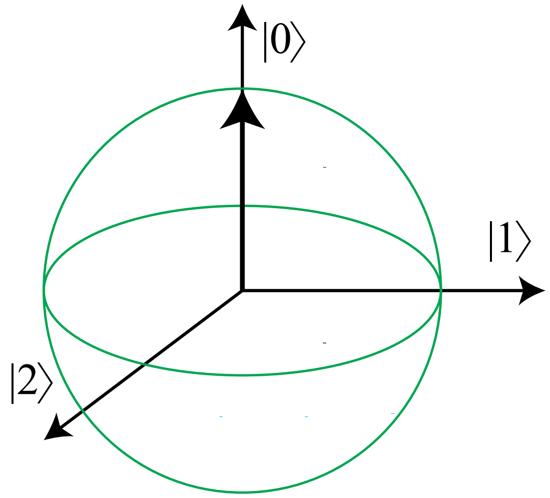
Solution: rotating a state not the measurement basis

Generating KCBS states

How to generate $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$?

Start from the ground state

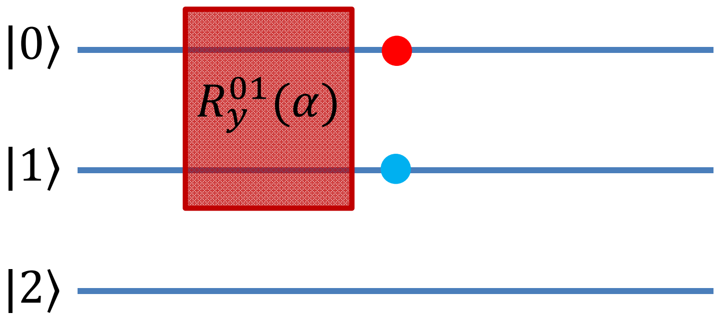


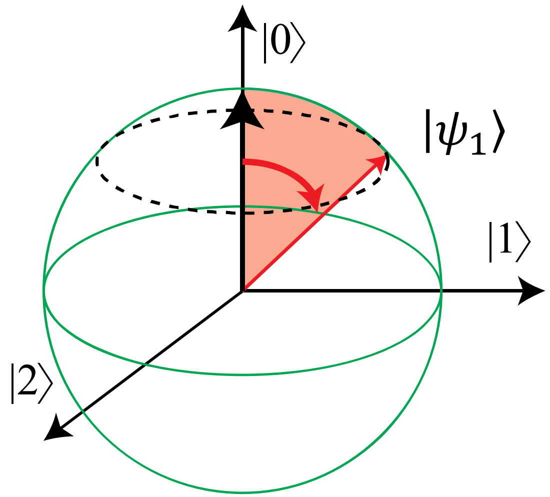


Generating KCBS states

How to generate $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$?

Apply rotation for 0-1 transition to get $|\psi_1\rangle$

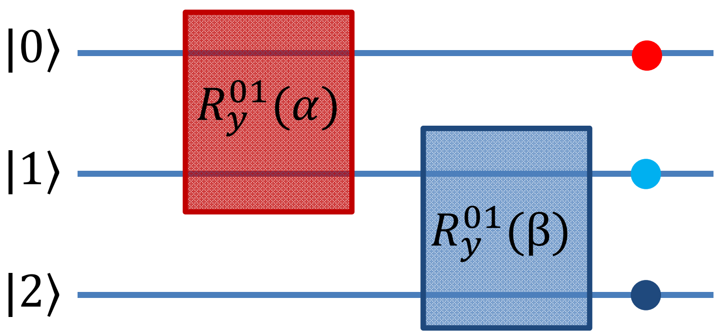


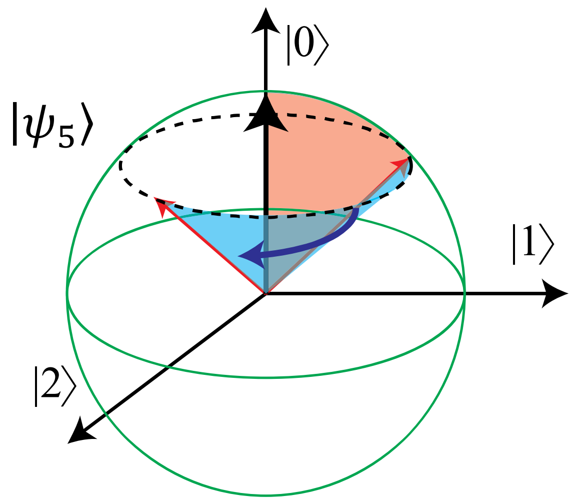


Generating KCBS states

How to generate $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$?

Apply rotation for 0-1 transition to all other states $|\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$

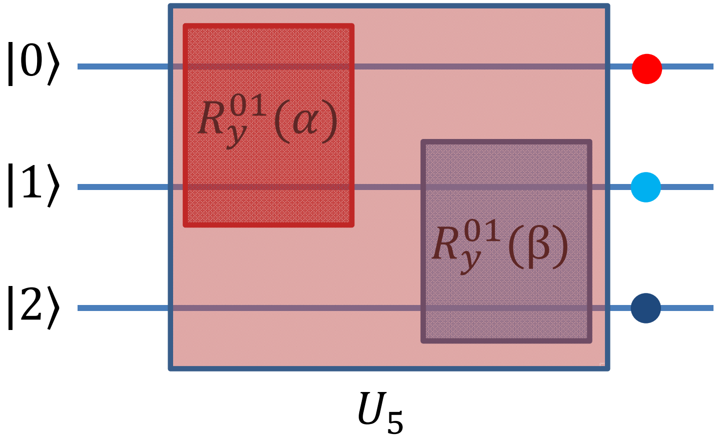


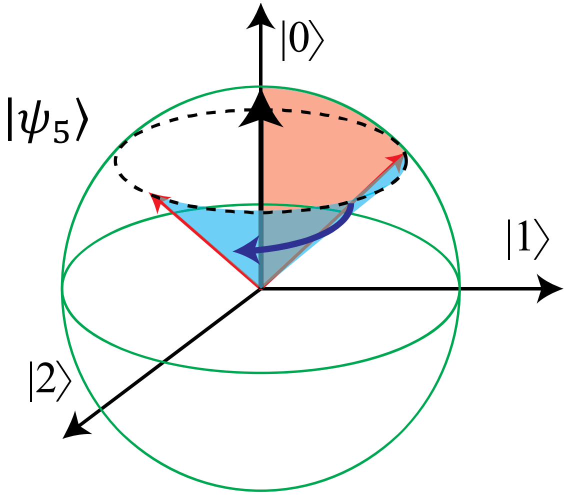


Generating KCBS states

How to generate $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$?

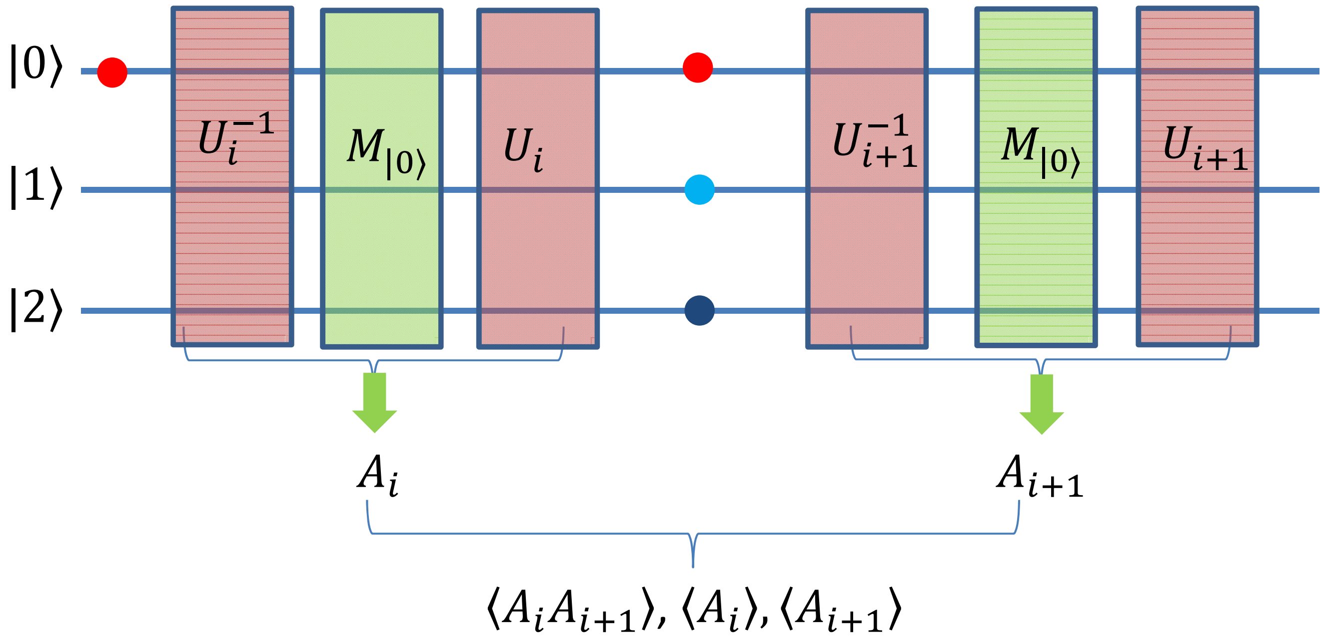
Apply rotation for 0-1 transition to all other states $|\psi_2\rangle, |\psi_3\rangle, |\psi_4\rangle, |\psi_5\rangle$

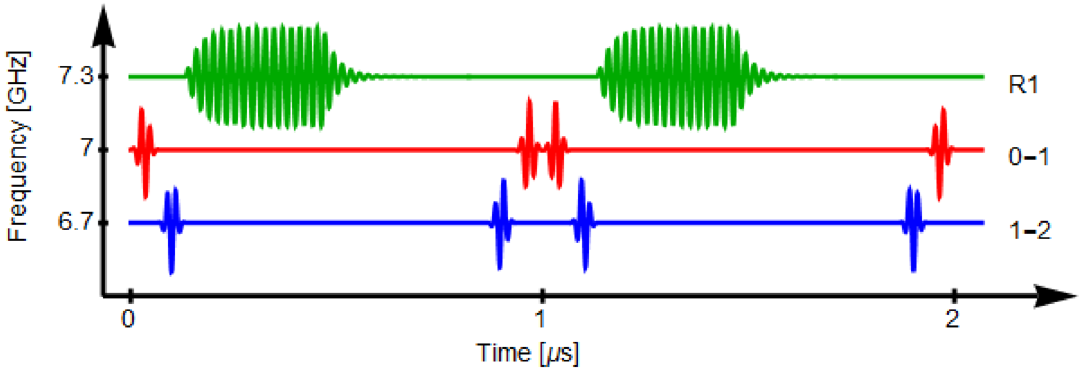




State	Rotations	U	F
$\|\psi_1\rangle$	$R_y^{01}(0.53\pi)$	U_1	~0.99
$\|\psi_2\rangle$	$R_y^{01}(0.53\pi)\ R_y^{02}(1.6\ \pi)$	U_2	~0.99
$\|\psi_3\rangle$	$R_y^{01}(-0.53\pi)\ R_y^{02}(1.2\ \pi)$	U_3	~0.99
$\|\psi_4\rangle$	$R_y^{01}(0.53\pi)\ R_y^{02}(0.8\ \pi)$	U_4	~0.99
$\|\psi_5\rangle$	$R_y^{01}(-0.53\pi)\ R_y^{02}(0.4\ \pi)$	U_5	~0.99

Measuring correlations $\langle A_i A_{i+1} \rangle$





Violation of the KCBS Inequality

Measured correlations:

$$\langle \boldsymbol{A_1 A_2} \rangle + \langle \boldsymbol{A_2 A_3} \rangle + \langle \boldsymbol{A_3 A_4} \rangle + \langle \boldsymbol{A_4 A_5} \rangle + \langle \boldsymbol{A_5 A_1} \rangle = -3.51(2)$$

Adjusted threshold:

$$-(3 + |\varepsilon_{12}| + |\varepsilon_{32}| + |\varepsilon_{34}| + |\varepsilon_{54}| + |\varepsilon_{51}|) = -3.38(7)$$

KCBS inequality violated (by more than 49 standard deviations).

(i, j)	$\langle A_i A_i \rangle$	$\langle A_j \rangle$ (1st)	$\langle A_j \rangle$ (2nd)	ε_{ij}
(1, 2)	-0.70(3)	0.10(0)	0.18(5)	0.08(5)
(2, 3)	-0.70(2)	0.10(8)	0.17(8)	0.07(0)
(3, 4)	-0.69(5)	0.10(8)	0.18(5)	0.07(8)
(4, 5)	-0.70(5)	0.10(6)	0.18(3)	0.07(8)
(5, 1)	-0.70(9)	0.10(3)	0.17(9)	0.07(6)
$\sum$	-3.51(2)			0.38(7)

P-value

Experimental tests of HV models can be formulated as a hypothesis test that the measurement statistics can be modelled using HV subject to compatibility bound

Non-contextual hidden variable theories rejected with P-value
$$< 3 \cdot 10^{-575}$$

Separate test of the compatibility condition rejects the hypothesis that the observables are more incompatible with P-value
$$< 4 \cdot 10^{-4}$$

The only assumptions used in the analysis are i.i.d. (device perform the same in the each run) and no memory

The most comprehensive experimental evidence in a scenario without entanglement.

M. Jerger et al., Nature Comm. 7, 7, 12930 (2016)

Summary

Several important elements combined in one experiment:

- Highly coherent yet tunable multi-level quantum system
- High fidelity single-shot readout, yet maximally non-invasive

Summary:

- Realized degenerate binary outcome projective measurement for a superconducting qutrit using engineered dispersive shifts
- Tested quantum contextuality with a superconducting circuit
- Used minimum possible assumptions in an scenario without entanglement