Bridging the Theory and Experiment for Device-Indepedent Quantum Information

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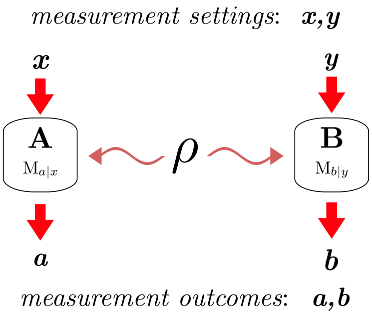
Quantum experiment



- Shared quantum state ρ, e.g., $\rho = |\psi^-\rangle\langle\psi^-|$
- Perform (e.g., spin) measurements described by POVM, $\{M_{a|x}\}$, $\{M_{b|y}\}$
- Register outcomes
- Born's rule: $\vec{P}_{\mathcal{Q}}(a, b|x, y) = \{\mathrm{tr}\left(M_{a|x} \otimes M_{b|y}\, \rho\right)\}_{x,y,a,b}$
- Estimate expectation value from data, e.g., distribution of measurement outcome

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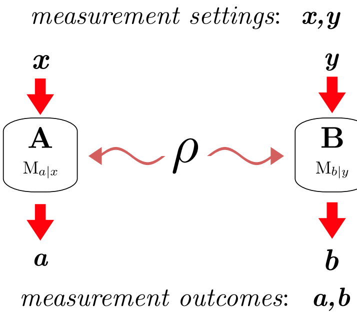

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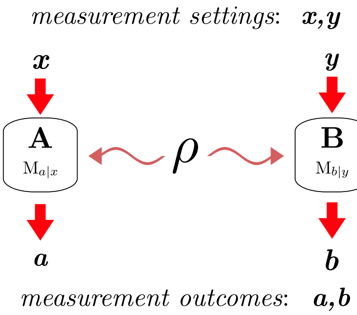

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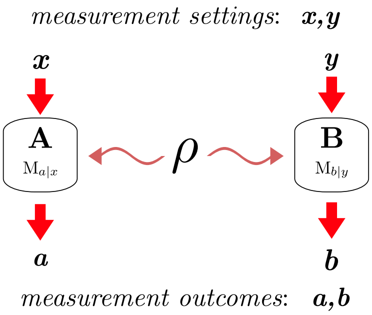

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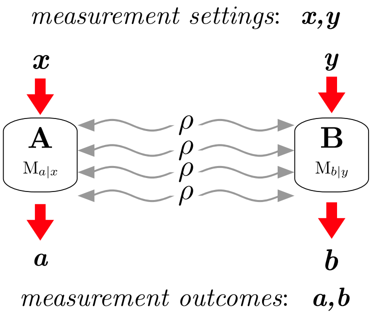

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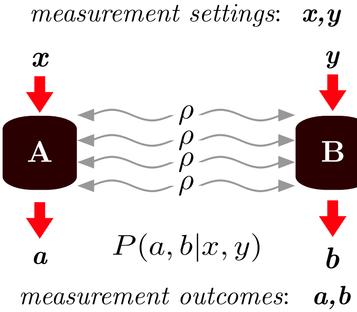

Device-independent paradigm

- Drop assumptions on devices
- Keep label of measurement settings by x, y; and outcomes by a and b
- Goal: use correlation $\{P(a, x|x, y)\}$ to learn something nontrivial about ρ and the measurements

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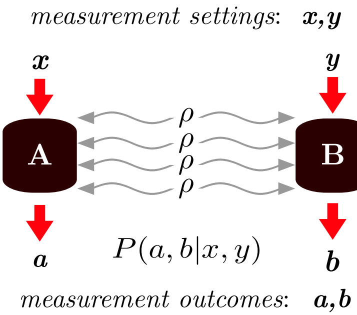

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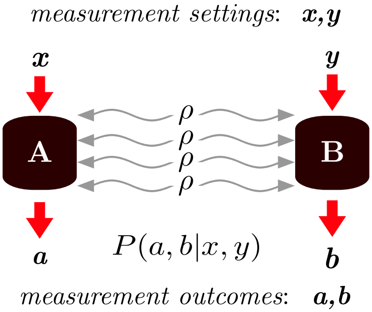

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Relativistic causality

Non-Signaling conditions

For all a, b,

$$P(a|x,y) = \sum_b P(a,b|x,y) = \sum_b P(a,b|x,y') = P(a|x,y'),\ \forall\ y \neq y'$$

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Physical significance:

- Probability distribution cannot be affected by each other's measurement choice
- If violated, we can use the difference in correlations to send signals faster than light

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Statistical fluctuations due to finite statistics

An ideal coin,

$$P_{\text{Ideal}}(H) = 0.5,\ P_{\text{Ideal}}(T) = 0.5$$

In experiment, correlation is estimated by:

$$P_{\text{Obs}}(H) = \frac{N(H)}{N(H)+N(T)},\ P_{\text{Obs}}(T) = \frac{N(T)}{N(H)+N(T)}$$

- Due to finite statistics, $P_{\text{Obs}} \neq P_{\text{Ideal}}$
- When sample size $(N) \to \infty$, $P_{\text{Obs}} \to P_{\text{Ideal}}$
- The same kind of deviation applies to the observed distribution $\vec{P}_{\text{Obs}}(a,b|x,y)$ and the ideal quantum distribution $\vec{P}_{Q}(a,b|x,y)$

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Dealing with the difference between theory and practice

- In experiment, due to finite statistics,

$$P_{\mathrm{Obs}}(a, b|x, y) = \frac{N(a, b, x, y)}{N(x, y)}$$

  violates the non-signaling condition, even if equipped with ideal setup
- Device-independent quantum information (DIQI) utilizes only the correlations, $\vec{P}(a, b|x, y)$, to arrive at conclusions
- Theoretical tools developed for DIQI assume that $\vec{P}(a, b|x, y)$ obeys the non-signaling condition
- A gap between raw experimental data and theoretical tools
- Our goal: To bridge this gap

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- Numerical simulation:
  - Consider different $\vec{P}_{\mathcal{Q}}$, e.g., $\vec{P}_{\mathcal{Q}}^{\mathrm{CHSH}}$
  - Simulate the outcomes of the experiment according to $\vec{P}_{\mathcal{Q}}$
  - Estimate $\vec{P}_{\mathrm{Obs}}$
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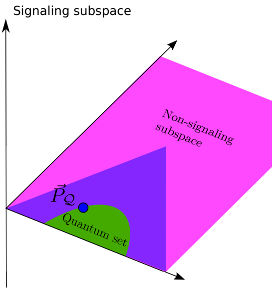

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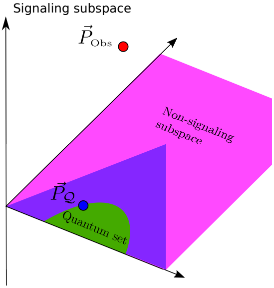

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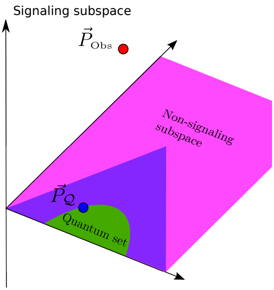

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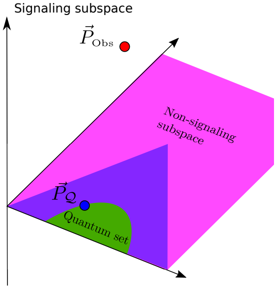

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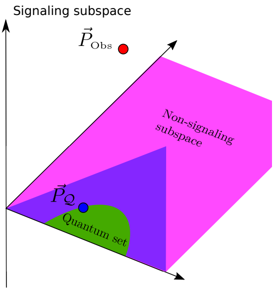

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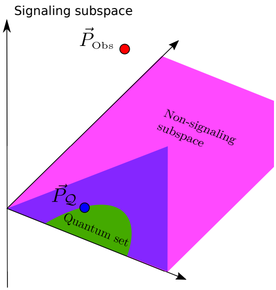

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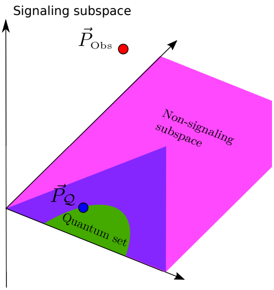

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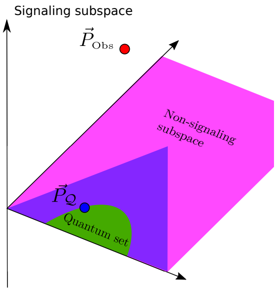

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1. Projection[1] : Project $\vec{P}_{\text{Obs}}$ to non-signaling subspace
   - $\vec{P}_{\text{Obs}} = \vec{P}_{\text{NS}} \oplus \vec{P}_{\text{S}}$
2. Nearest quantum approximation[2] (NQA)
   - $||\vec{P}_{\mathcal{Q}_n} - \vec{P}_{\text{Obs}}||_{L_2}$ is minimal
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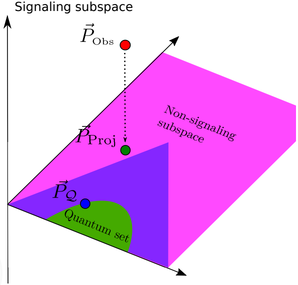

[1] Renou *et al.*, arXiv:1610.01833
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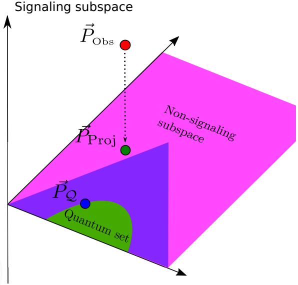

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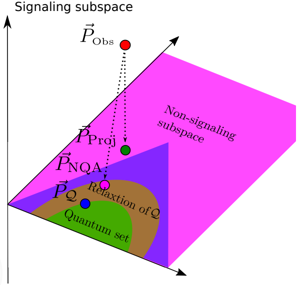

[1] Renou *et al.*, arXiv:1610.01833
[2] Schwarz *et al.*, NJP (2016)
[3] Zhang *et al.*, PRA (2013)

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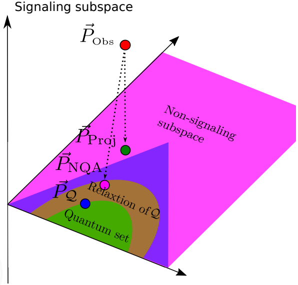

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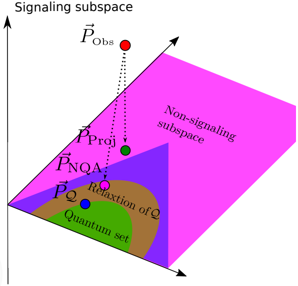

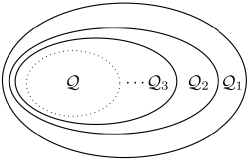

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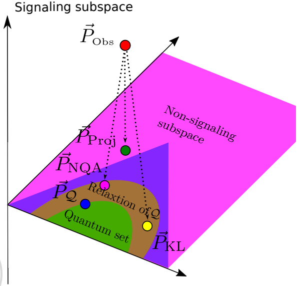

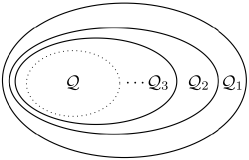

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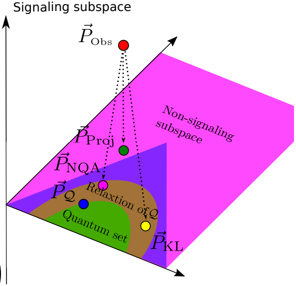

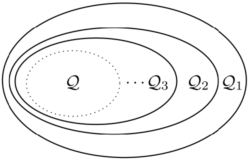

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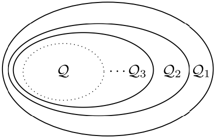

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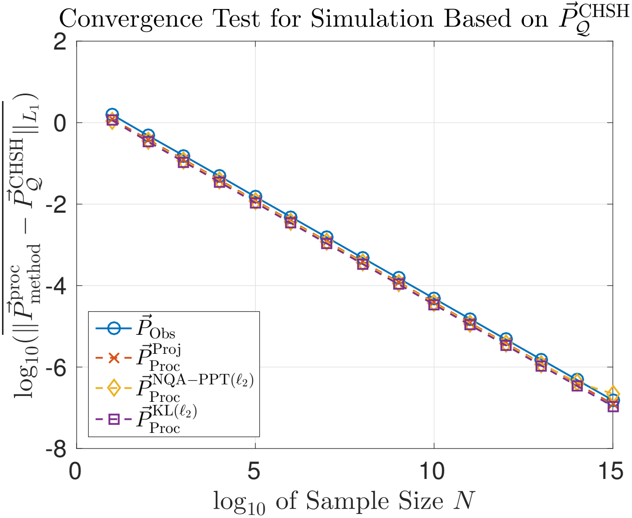
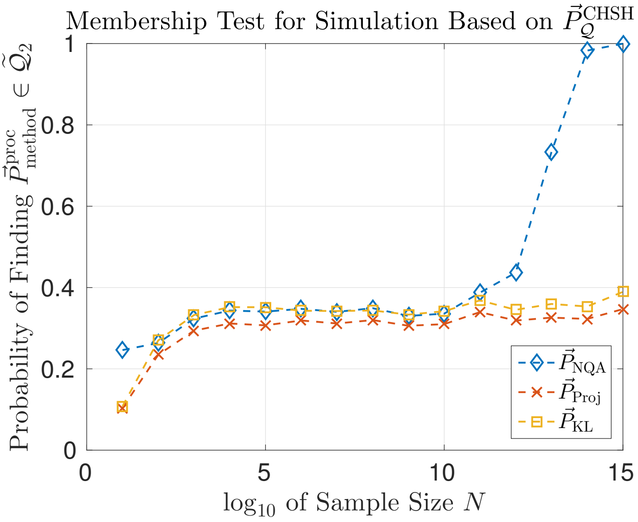
Convergence criterion-$\vec{P}_{\mathrm{Q}}^{\mathrm{CHSH}}$



Membership criterion-$\vec{P}_{\mathrm{Q}}^{\mathrm{CHSH}}$



Take home messages

Big picture

1. Finite statistics leads to “signaling” correlations, it’s unavoidable even if equiped with perfect system.
2. A gap between experimental data and usage of theoretical tools, due to violation of the non-signaling condition.

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