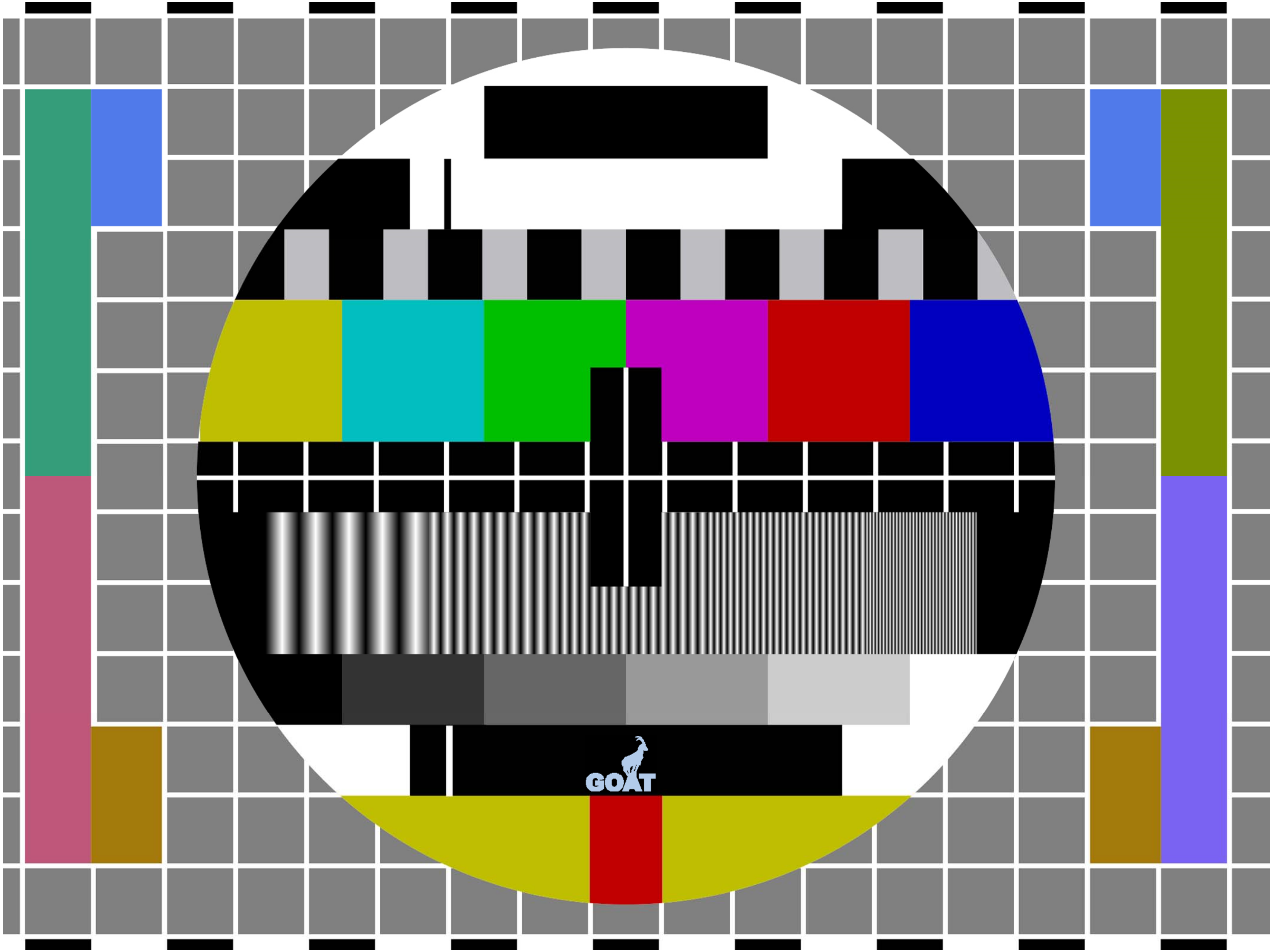
GOAT

Quantum Optimal Control for Superconducting Qubits with GOAT

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מכון ויצמן למדע

Overview

- Quantum Optimal Control (QOC)

- New QOC algorithm - GOAT

- Application to superconducting qubits

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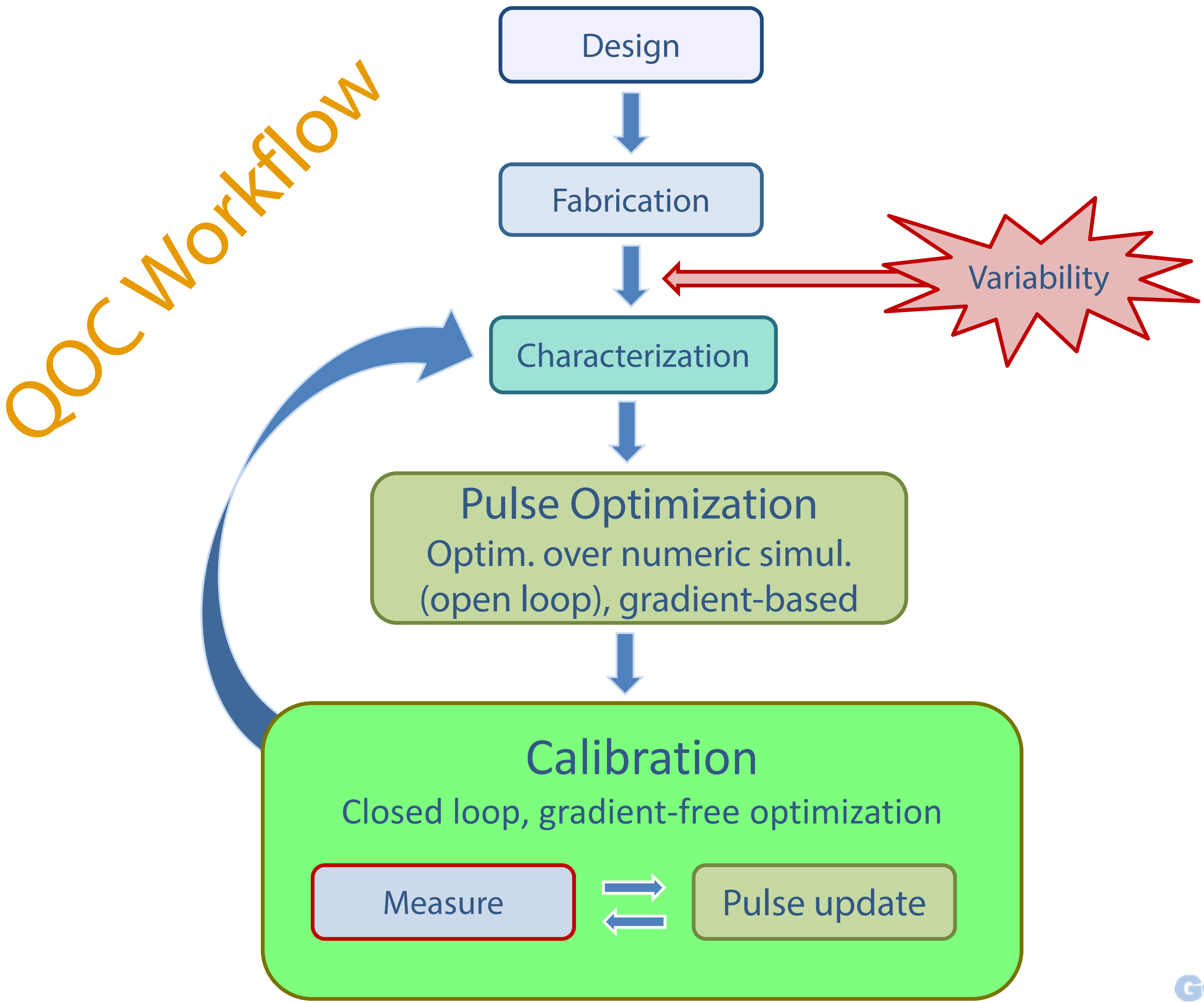
- New QOC algorithm - GOAT
- Application to superconducting qubits

S. J. Glaser, et al., *The European Physical Journal* D 69.12 (2015): 1-24
C. P. Koch, *Journal of Physics: Condensed Matter* 28.21 (2016): 213001


QOC Workflow
Design
Fabrication
Variability
Characterization
Pulse Optimization
Optim. over numeric simul.
(open loop), gradient-based
Calibration
Closed loop, gradient-free optimization
Measure
Pulse update
GOAT

Requirements from QOC

- Optimize state transfer, gate generation, etc.
- Unitary, Lindblad, non-Markovian, etc.

- Flexible
  - Everything is constrained
  - Ansatz + AWG + Filter + IQ mixer + Filter
  - Multi-goal
  - more

- Quick
- Accurate

Quantum Optimal Control

Given

$$H(t) = H_0 + \sum_{k=1}^{K} c_k(t) H_k$$

Find control fields

$$c_k(t)$$

such that we have very accurate

- State preparation, or
- Gate generation, or
- Cooling (super-operator generation), or …

Quantum Optimal Control

Given

$$H(\bar{\alpha}, t) = H_0 + \sum_{k=1}^{K} c_k(\bar{\alpha}, t) H_k$$

$$c_k(\bar{\alpha}, t) = \sum_{j=1}^{m} A_{k,m} \exp\left(-(t - \tau_{k,m})^2 / \sigma_{k,m}^2\right)$$

$$\bar{\alpha} = \{A_{k,m}, \tau_{k,m}, \sigma_{k,m}\}_{k=1...K, j=1...m}$$

$$\left|A_{k,m}\right| \leq 500\text{MHz}$$

$$U(\bar{\alpha}, T) = \mathbb{T} \exp\left(\int_0^T -\frac{i}{\hbar} H(\bar{\alpha}, t)\, dt\right)$$

$$g(\bar{\alpha}) = 1 - \frac{1}{dim(U)} \left| U_{goal}^{\dagger} U(\bar{\alpha}, T) \right|$$

Task: Find $\bar{\alpha}$ such that $g(\bar{\alpha})$ is minimal

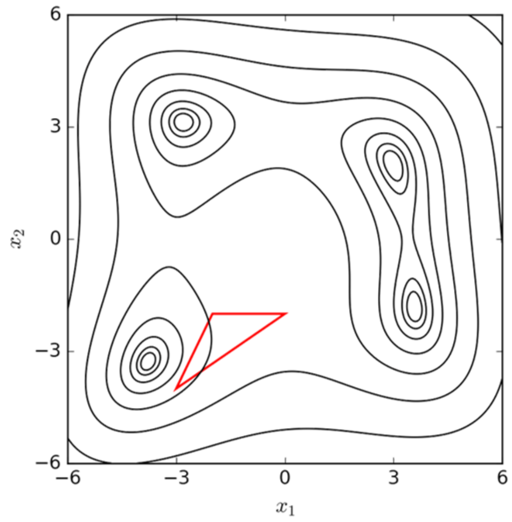
QOC Algorithms

- Direct methods - No gradient information
- Variational methods - Gradient based

QOC Algorithms – Direct

Direct methods - No gradient information

- Pro
  - Only require black-box goal function evaluators
  - Great for closed loop calibration
  - Relatively simple



- Con
  - Slow
  - Very slow in high dimension search spaces

QOC Algorithms - Direct

Direct methods - No gradient information

- Deterministic - CRAB:
  - Arbitrary analytic controls (usually Fourier basis)
  - Simplex search (e.g. Nelder-Mead)
  - Deterministic

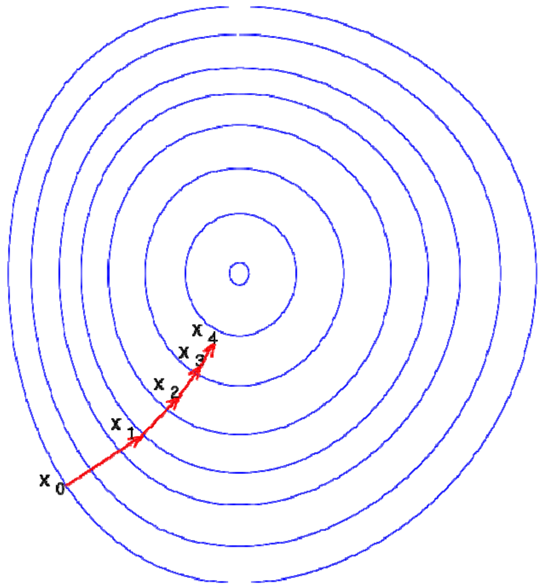
T. Caneva, T. Calarco, and S. Montangero. Phys. Rev. A, 84(2):022326, 2004
P. Doria, T. Calarco, and S. Montangero. Phys. Rev. Lett., 106:190501 (2011)

- Stochastic
  - CMA-ES (Covariance Matrix Adaptation Evolution Strategy)
  - NES (Natural Evolution Strategy)
  - Genetic algorithms, Particle swarms

GOAT

QOC Algorithms - Variational

Variational methods- Gradient based



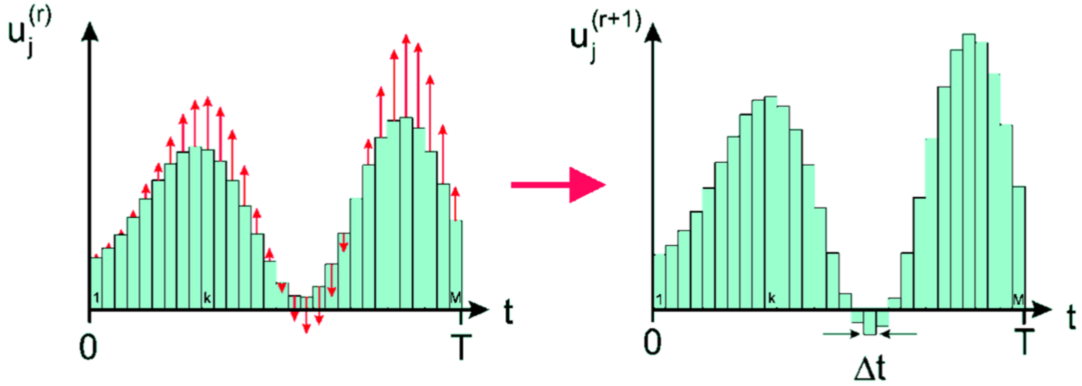
- Pro
  - Fast
  - Can handle large parameter spaces
  - Optional: Monotonic convergence
- Con
  - Requires gradient of goal w.r.t. control params
  - Usually based on the Pontryagin Max. Principle (PMP)
    - Non-trivial mathematically
    - Requires backwards-in-time propagation of an adjoint state

L. S. Pontryagin, V. G. Bol'tanskii, R. S. Gamkre-lidze, and E. F. Mischenko. The Mathematical Theory of Optimal Processes. Pergamon Press, New York (1964)

QOC Algorithms - GRAPE

Variational methods- Gradient based

- GRAPE
  - PWC parametrization of control fields
  - Fast gradient-based optimization (L-BFGS)
  - Arbitrary ansatz, constraints, filters: non-trivial



N. Khaneja, et al., J. Magn. Reson., 172(2) 296 (2005)

P. de Fouquieres, S. G. Schirmer, S. J. Glaser, and I. Kuprov, J. Magn. Reson., 212(2) 412 (2011)

S. Machnes, et. al.. PRA, 84 022305 (2011)

QOC Algorithms - Krotov

Variational methods- Gradient based

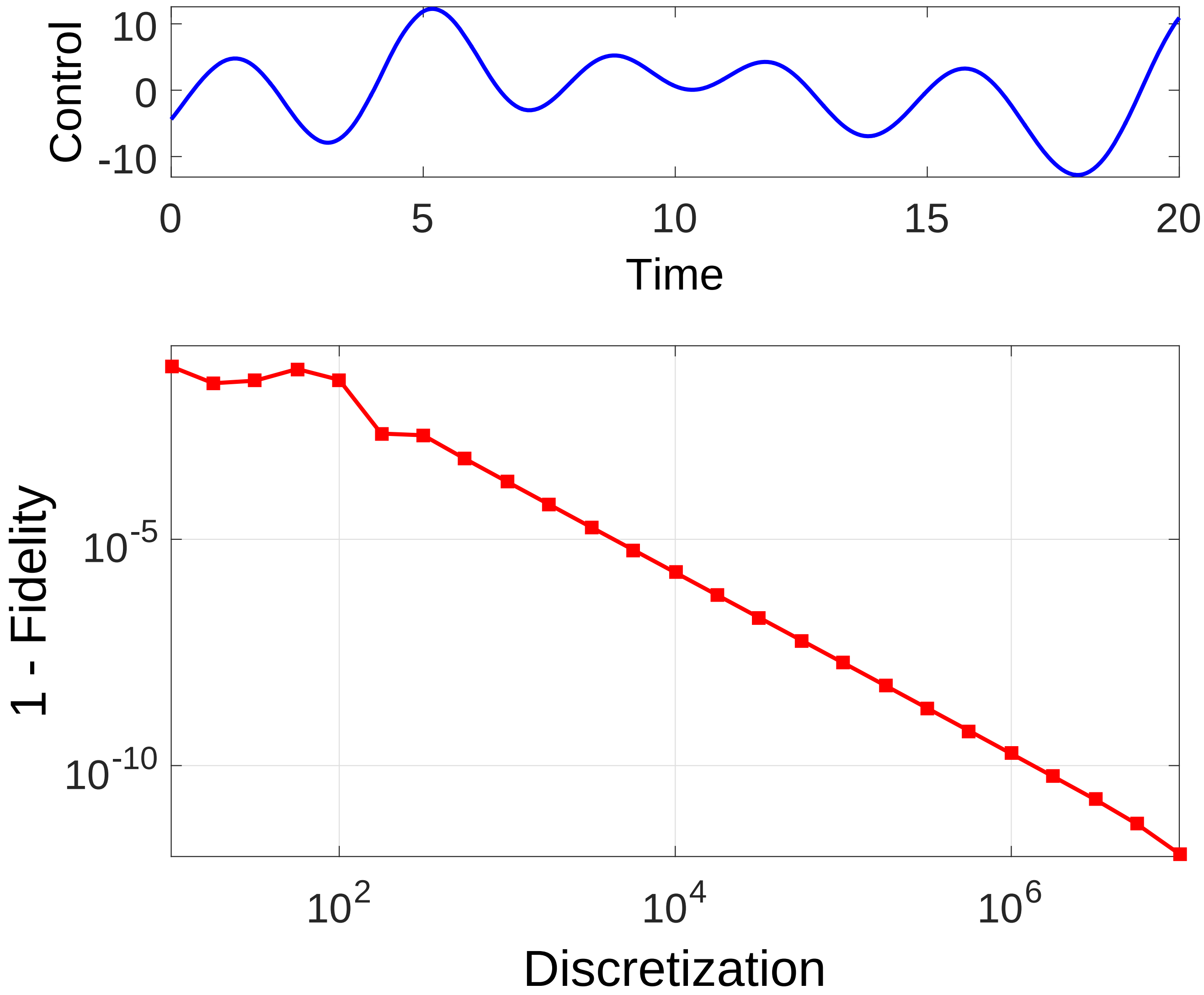
- Krotov
  - Based on PMP (i.e. required backwards propagation)
  - Monotonic convergence
  - In theory flexible - both PWC and analytic ansatz
  - In practice … mathematically non-trivial

V. F. Krotov, Global Methods in Optimal Control Theory. Dekker, New York, 1996
D. M. Reich, M. Ndong, and C. P. Koch, JCP 136, 104103 (2012)
R. Eitan, M. Mundt, and D. J. Tannor, Phys. Rev. A, 83(5):053426, 2011

QOC– Dos and Don’ts

- Don’t
  - Don’t use Piece-Wise Constant (PWC) – it is a poor approximation for bandwidth-limited controls.
  - Don’t use an approximate Hamiltonian
- Do
  - Gradient based optimization is much faster
  - Use analytic controls
    - You only need a few parameters
    - You only want a few parameters
    - Flexible: ansatz and constraints

PWC is a poor approx. of smooth controls



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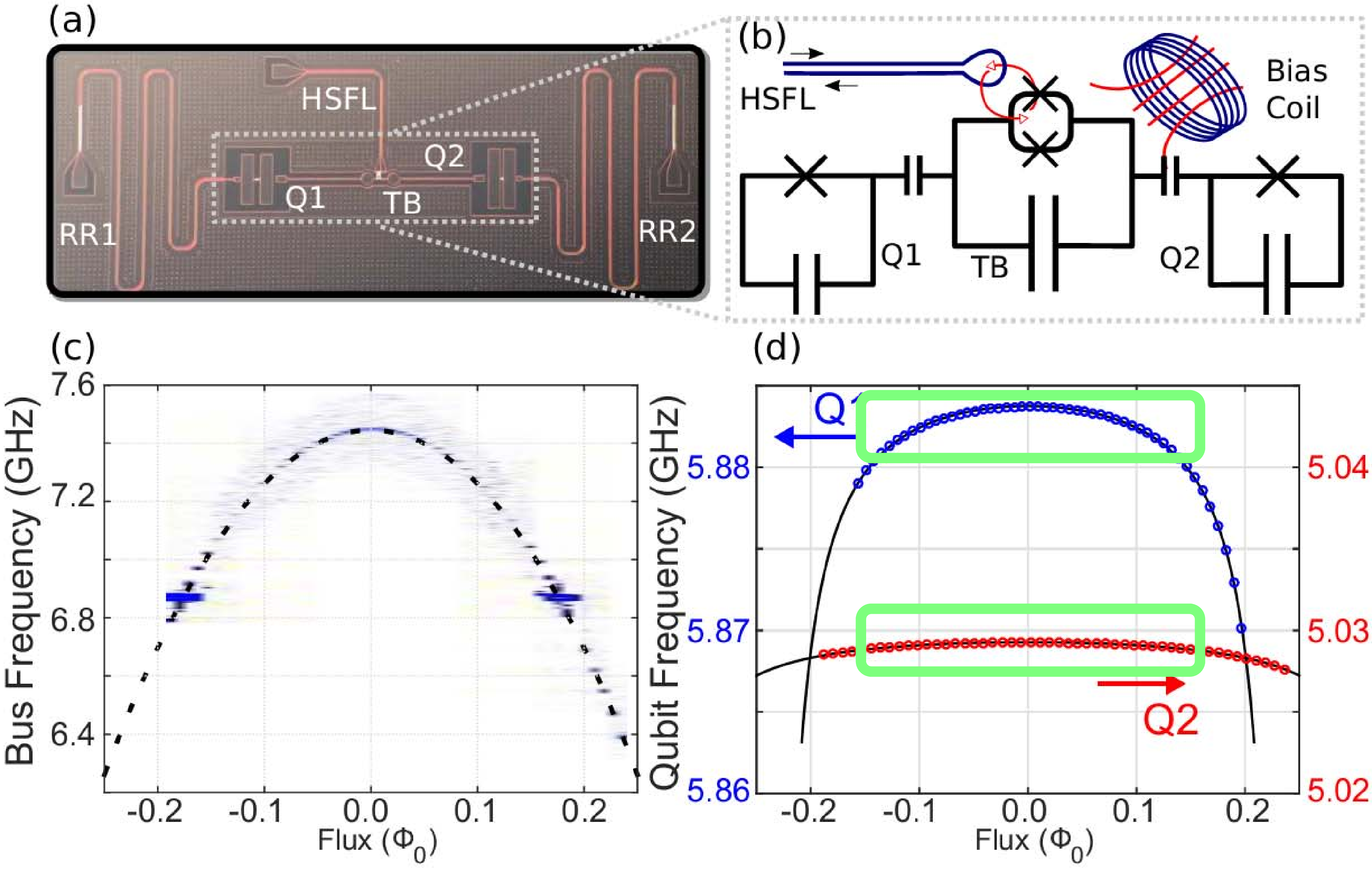
Get the Model Right

Get the Model Right

Get the Model Right

Simplified models needed for analytics

Detrimental for optimal control



D. McKay, S. Filipp, A. Mezzacapo, E. Magesan, J. Chow, and J. Gambetta, arXiv 1604.0307 (2016)

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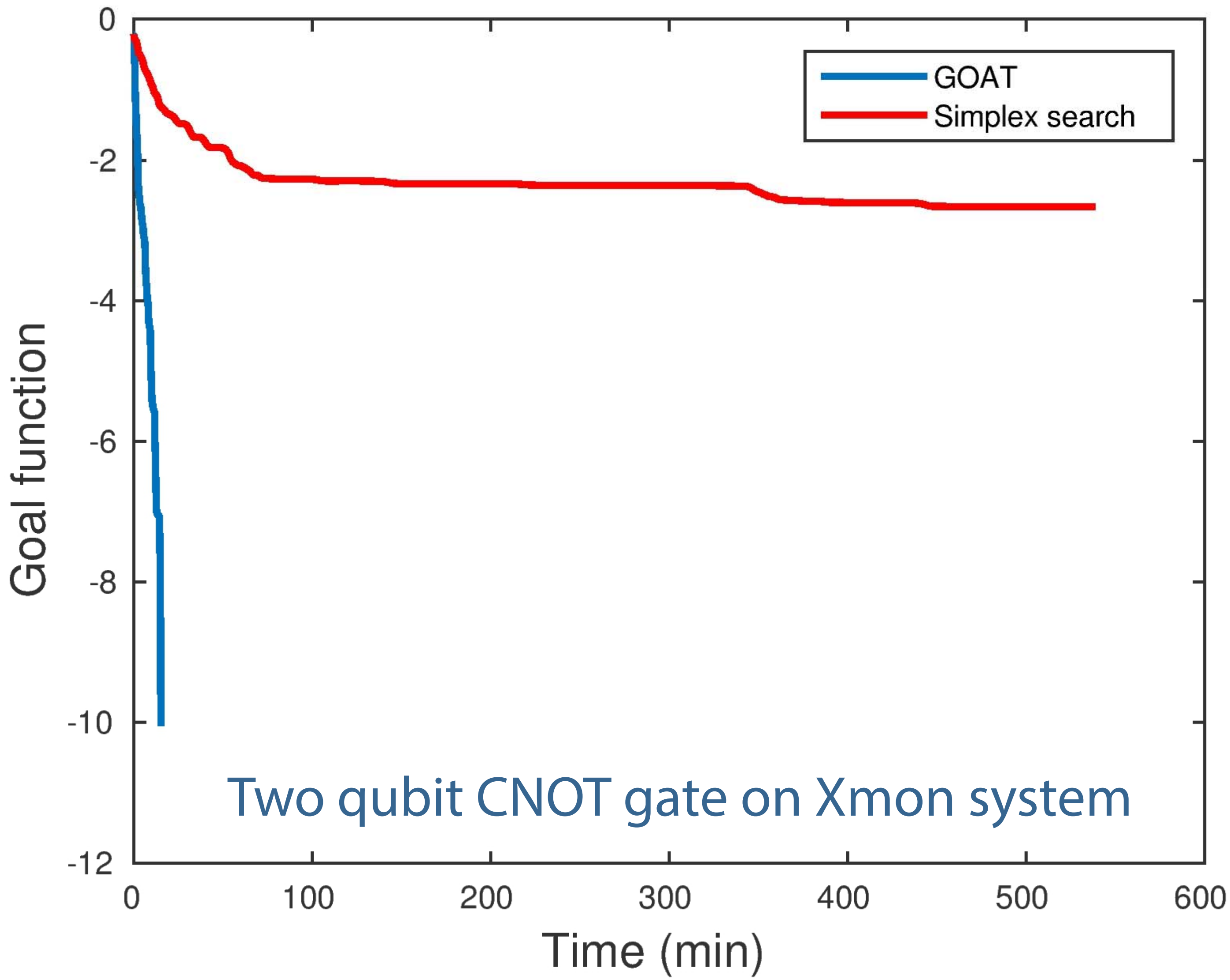
Gradients allow faster search



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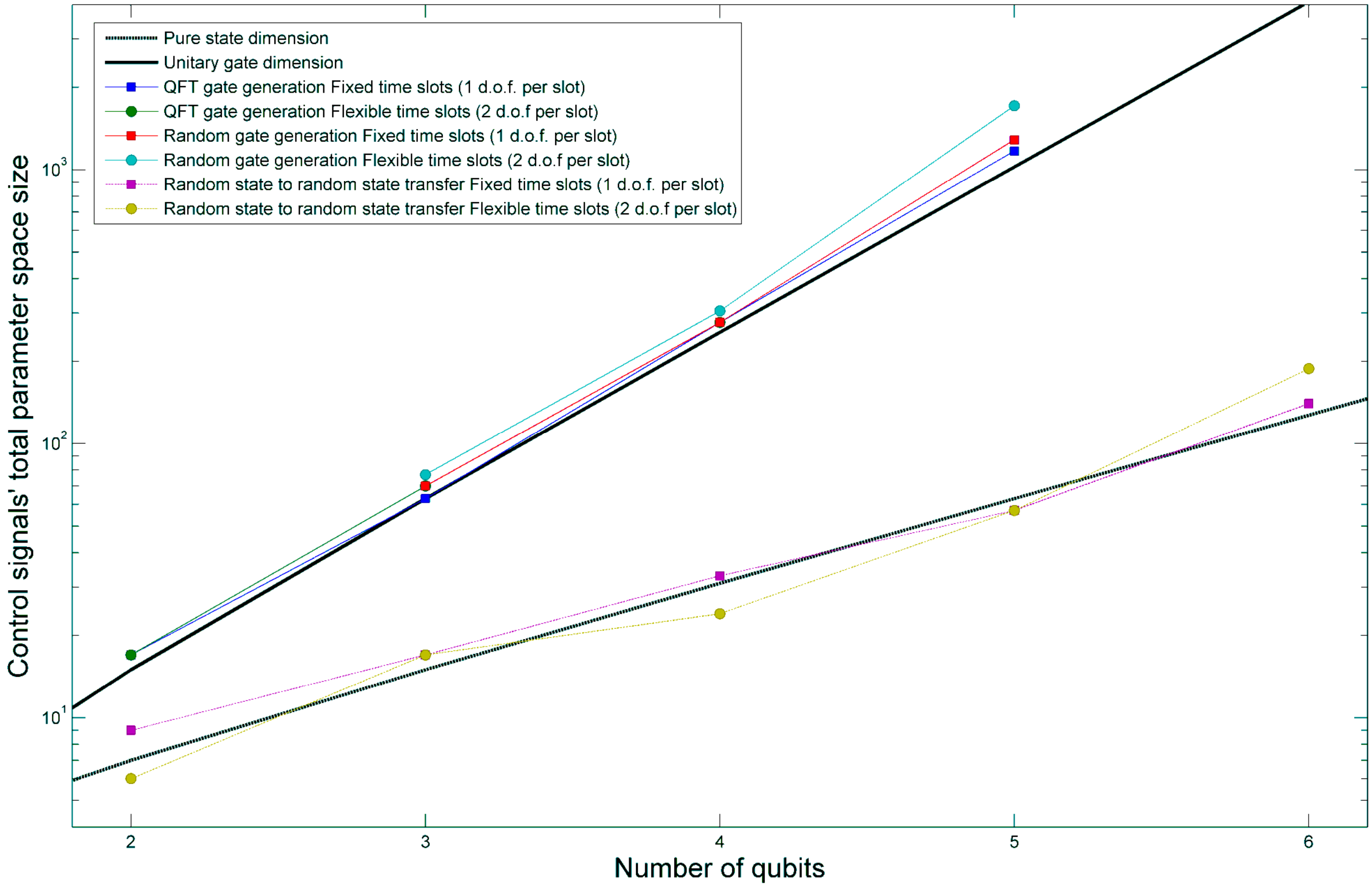
Number of params = Hilbert space size



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  - Don't use an approximate Hamiltonian
- Do
  - Gradient based optimization is much faster
  - Use analytic controls
    - You only need a few parameters
    - You only want a few parameters
    - Flexible: ansatz, constraints, filters, etc

Overview

- Quantum Optimal Control (QOC)
- New optimization algorithm - GOAT
- Application to superconducting qubits

Typical optimal control setup

$$g(\bar{\alpha}) = 1 - \frac{1}{dim(U)} \left| U^{\dagger}_{goal} U(\bar{\alpha}, T) \right|$$

$$U(\bar{\alpha}, T) = \mathbb{T} \exp\left(\int_0^T -\frac{i}{\hbar} H(\bar{\alpha}, t)\, dt \right)$$

$$H(\bar{\alpha}, t) = H_0 + \sum_{k=1}^{K} c_k(\bar{\alpha}, t) H_k$$

$$c_k(\bar{\alpha}, t) = \sum_{j=1}^{m} A_{k,m} \exp\left(-(t - \tau_{k,m})^2 / \sigma^2_{k,m} \right)$$

$$\bar{\alpha} = \{A_{k,m}, \tau_{k,m}, \sigma_{k,m}\}_{k=1...K, j=1...m}$$

Find $\partial_{\bar{\alpha}} g(\bar{\alpha})$ and follow the gradient.

Deriving GOAT (very simple)

$$g(\bar{\alpha}) = 1 - \frac{1}{dim(U)} \left| U^{\dagger}_{goal} U(\bar{\alpha}, T) \right|$$

$$\partial_{\bar{\alpha}} g(\bar{\alpha}) =? -real\left(\frac{g^*}{|g|} \frac{1}{dim(U)} Tr\left(U^{\dagger}_{goal} \partial_{\bar{\alpha}} U(\bar{\alpha}, t) \right) \right)$$

$$g = Tr\left(U^{\dagger}_{goal} U(\bar{\alpha}, t) \right)$$

$$\partial_{\bar{\alpha}} U(\bar{\alpha}, t) = ?$$

$\partial_{\bar{\alpha}} U(\bar{\alpha}, t)$ via modified Schrödinger

$$\partial_t U = -\frac{i}{\hbar} H U$$

$$\partial_{\bar{\alpha}} \partial_t U = -\frac{i}{\hbar} \left(\left(\partial_{\bar{\alpha}} H \right) U + H \partial_{\bar{\alpha}} U \right)$$

GOAT

$\partial_{\bar{\alpha}} U(\bar{\alpha}, t)$ via modified Schrödinger

$$\partial_t U = -\frac{i}{\hbar} H U$$

$$\partial_t \partial_{\bar{\alpha}} U = -\frac{i}{\hbar} \left((\partial_{\bar{\alpha}} H) U + H \partial_{\bar{\alpha}} U \right)$$

$$\partial_t \begin{pmatrix} U \\ \partial_{\bar{\alpha}} U \end{pmatrix} = -\frac{i}{\hbar} \begin{pmatrix} H & 0 \\ \partial_{\bar{\alpha}} H & H \end{pmatrix} \begin{pmatrix} U \\ \partial_{\bar{\alpha}} U \end{pmatrix}$$

This is the GOAT core idea

Deriving GOAT (very simple)

$$\partial_t \begin{pmatrix} U \\ \partial_{\bar{\alpha}} U \end{pmatrix} = -\frac{i}{\hbar} \begin{pmatrix} H & 0 \\ \boxed{\partial_{\bar{\alpha}} H} & H \end{pmatrix} \begin{pmatrix} U \\ \partial_{\bar{\alpha}} U \end{pmatrix}$$

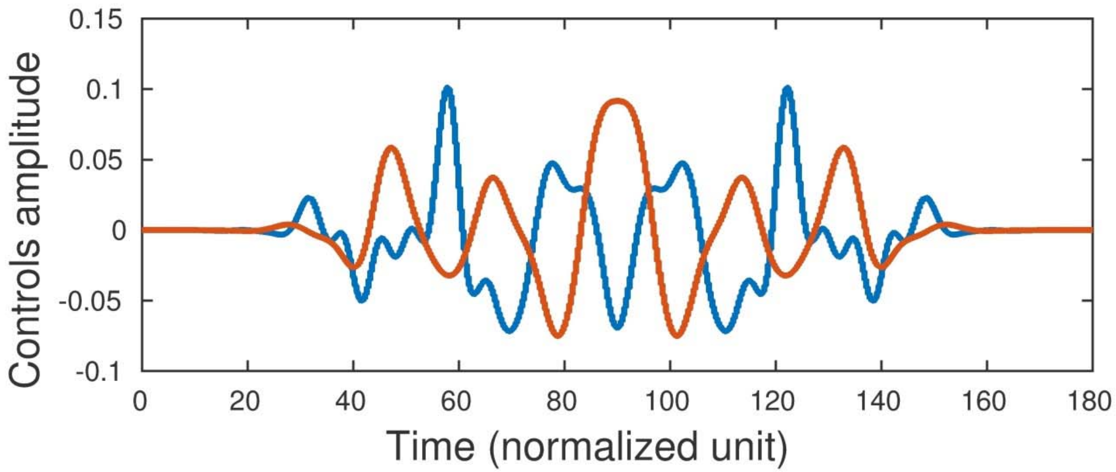
$$H(\bar{\alpha}, t) = H_0 + \sum_{k=1}^{K} c_k(\bar{\alpha}, t) H_k$$

$$\partial_{\bar{\alpha}} H(\bar{\alpha}, t) = \sum_{k=1}^{K} \partial_{\bar{\alpha}} c_k(\bar{\alpha}, t) H_k$$

Constraints?

$$c_k(t) = Ae^{-\frac{(t-\tau)^2}{\sigma^2}} \qquad A = 500\,\mathrm{MHz}\ \frac{\sin(p)+1}{2}$$

Smooth start / finish $\qquad c_k(\bar{\alpha}, t) \longrightarrow w(t)\, c_k(\bar{\alpha}, t)$



I don’t have time tell you

how GOAT can do

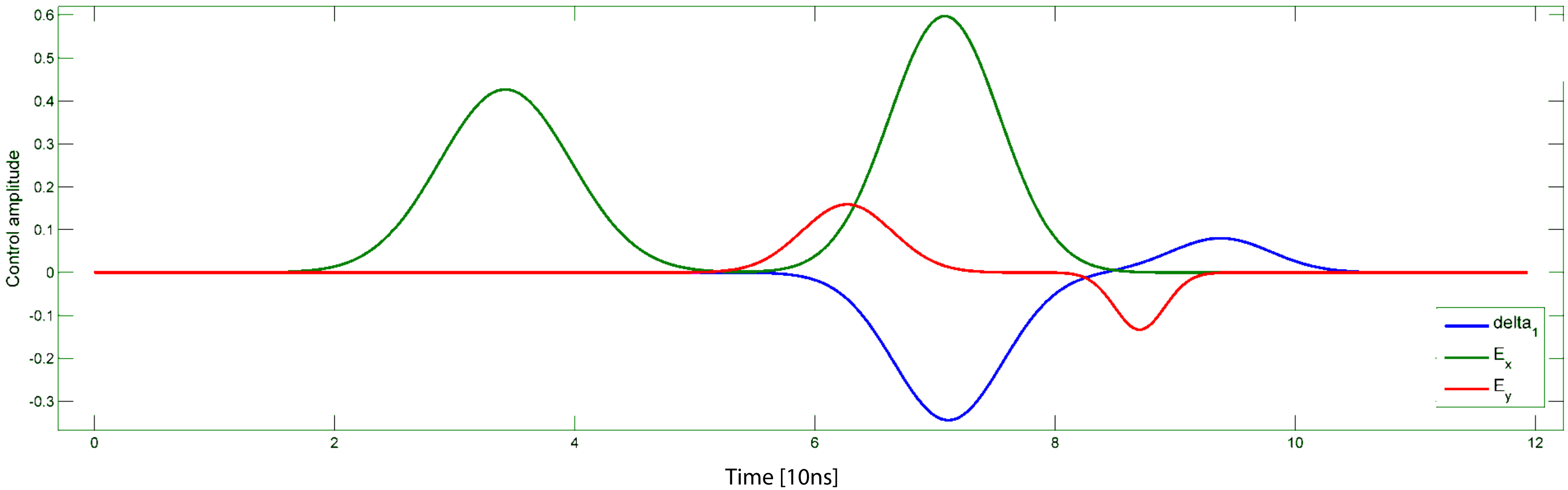
- Filters
- Perfect entangler, without knowing which
- Optimal gate duration for best fidelity
- Multi-goal optimization
- Robustness
- Open systems

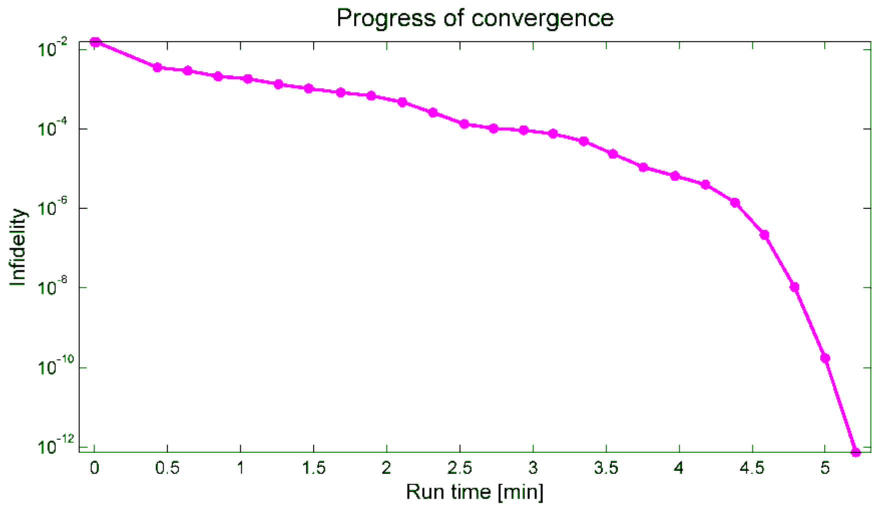
And all of these with full gradient information

Overview

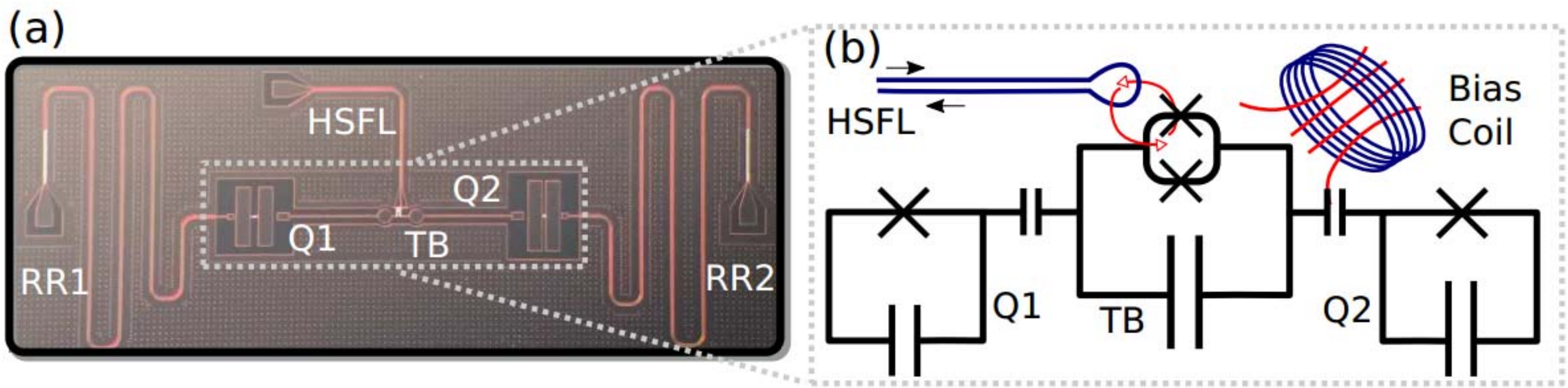
- The challenges of quantum optimal control
- New optimization algorithm - GOAT
- Application to superconducting qubits

Example: Transmon gates





Flux Tunable Coupler



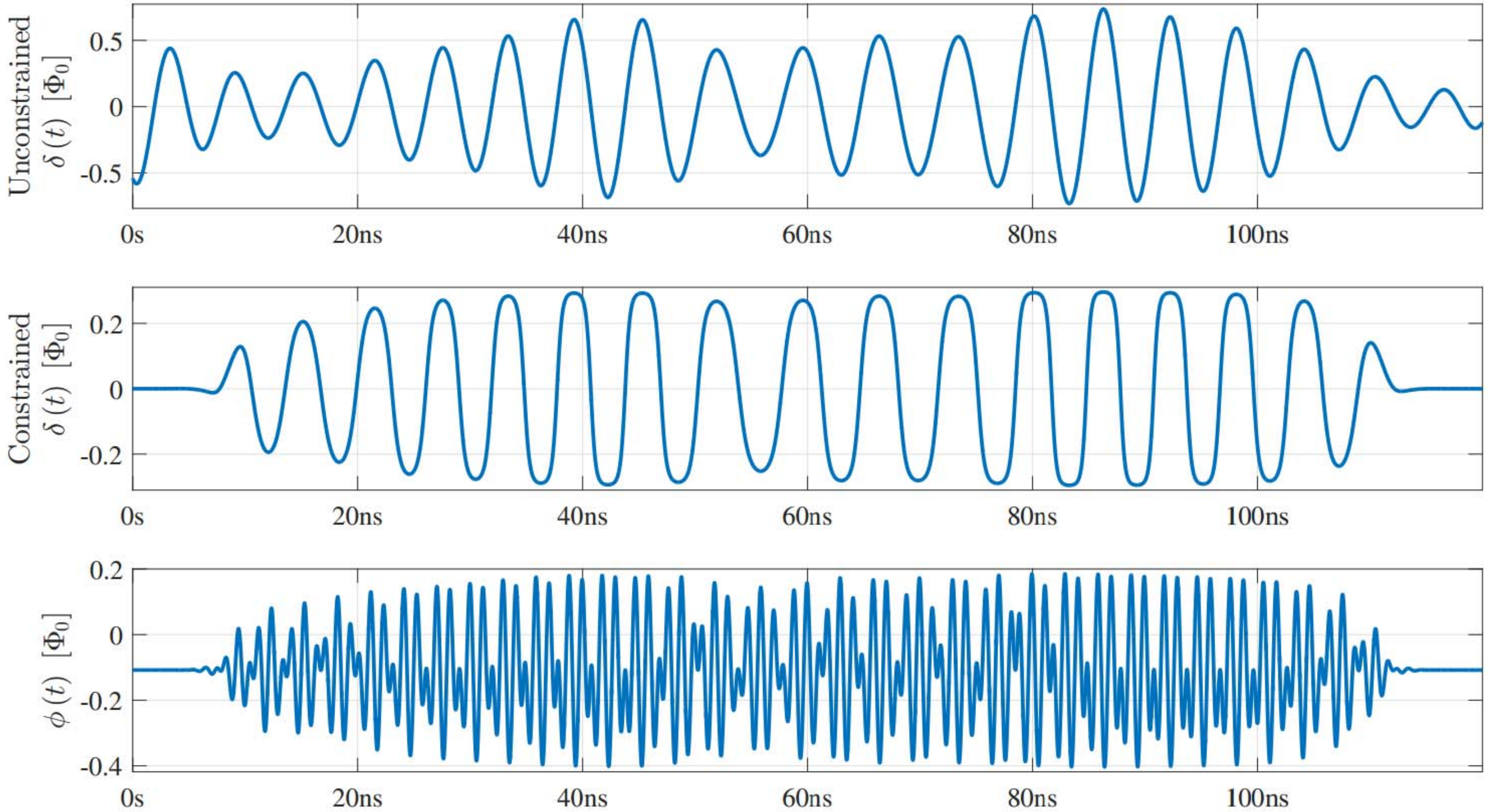
D. McKay, S. Filipp, A. Mezzacapo, E. Magesan, J. Chow, and J. Gambetta, arXiv 1604.0307 (2016)

$$H = \sum_k \omega_k a^\dagger a - \alpha |2\rangle \langle 2|_k + \omega_{TB}(\Phi) b^\dagger b + g_k (a_k^\dagger b + b^\dagger a_k)$$

$$\omega_{TB}(\Phi) = \omega_{\mathrm{TB},0} \sqrt{|\cos(\pi\Phi/\Phi_0)|}$$

$$\Phi = \Theta + \delta(t) \cos(\omega_\Phi t)$$

Flux Tunable Coupler



6 Fourier components, full system model

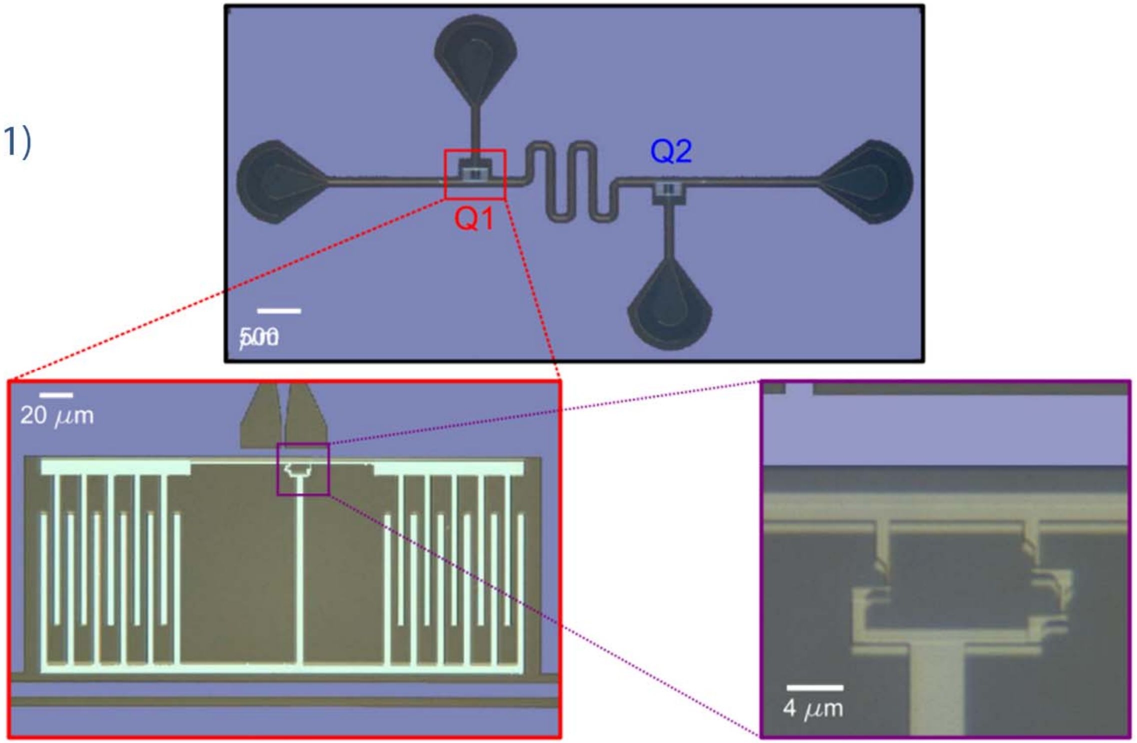
10^{-12} infidelity (excluding dephasing)

Cross-Resonance Gate

[1] Rigetti, Devoret, PRB (2010)

[2] J. Chow et al., PRL 107, 080502 (2011)

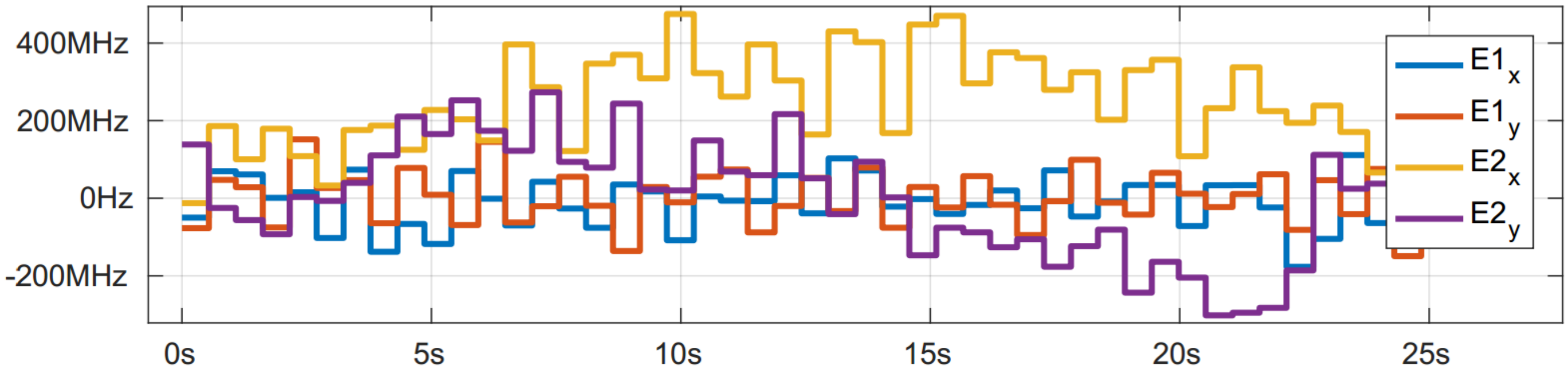
[3] S. Sheldon et al., arXiv 1603.04821



Preliminary work:

- Speed limit
- Fast & simple pulses enabling calibration

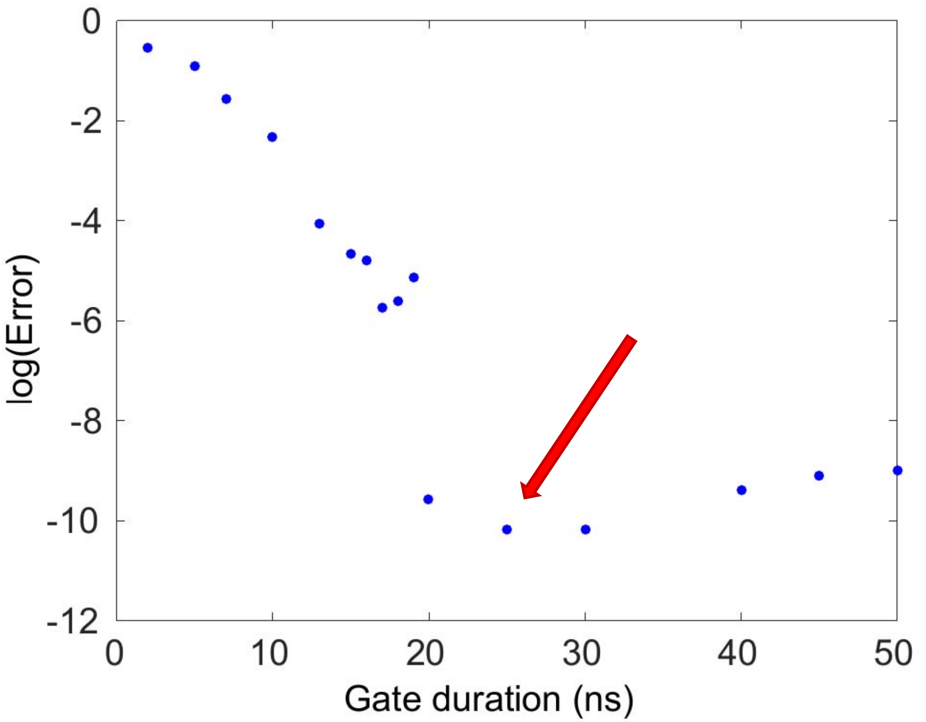
Speed Limit

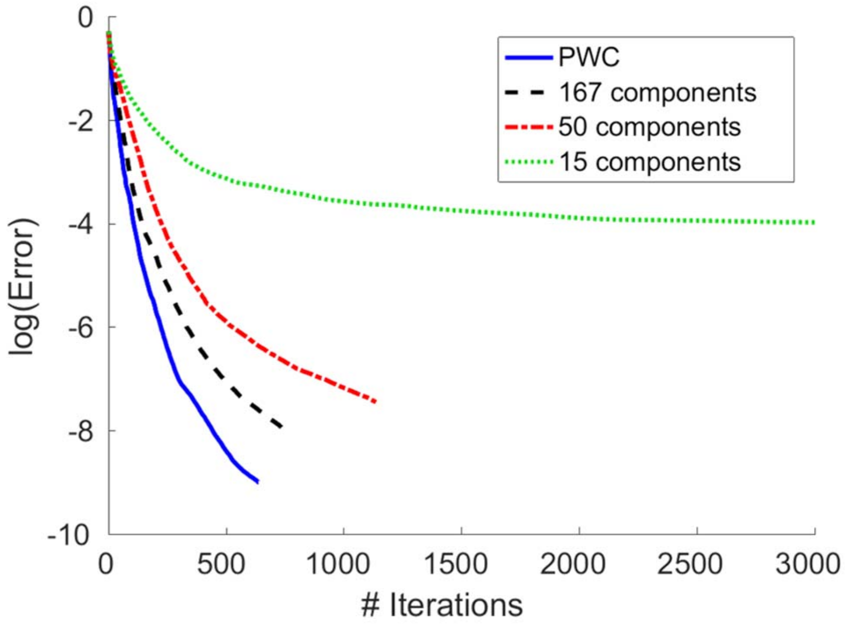


- XY controls for the two qubits
- AWG 50x0.5ns
- Infidelity $1.6x10^{-5}$

by Susanna Kirchhoff

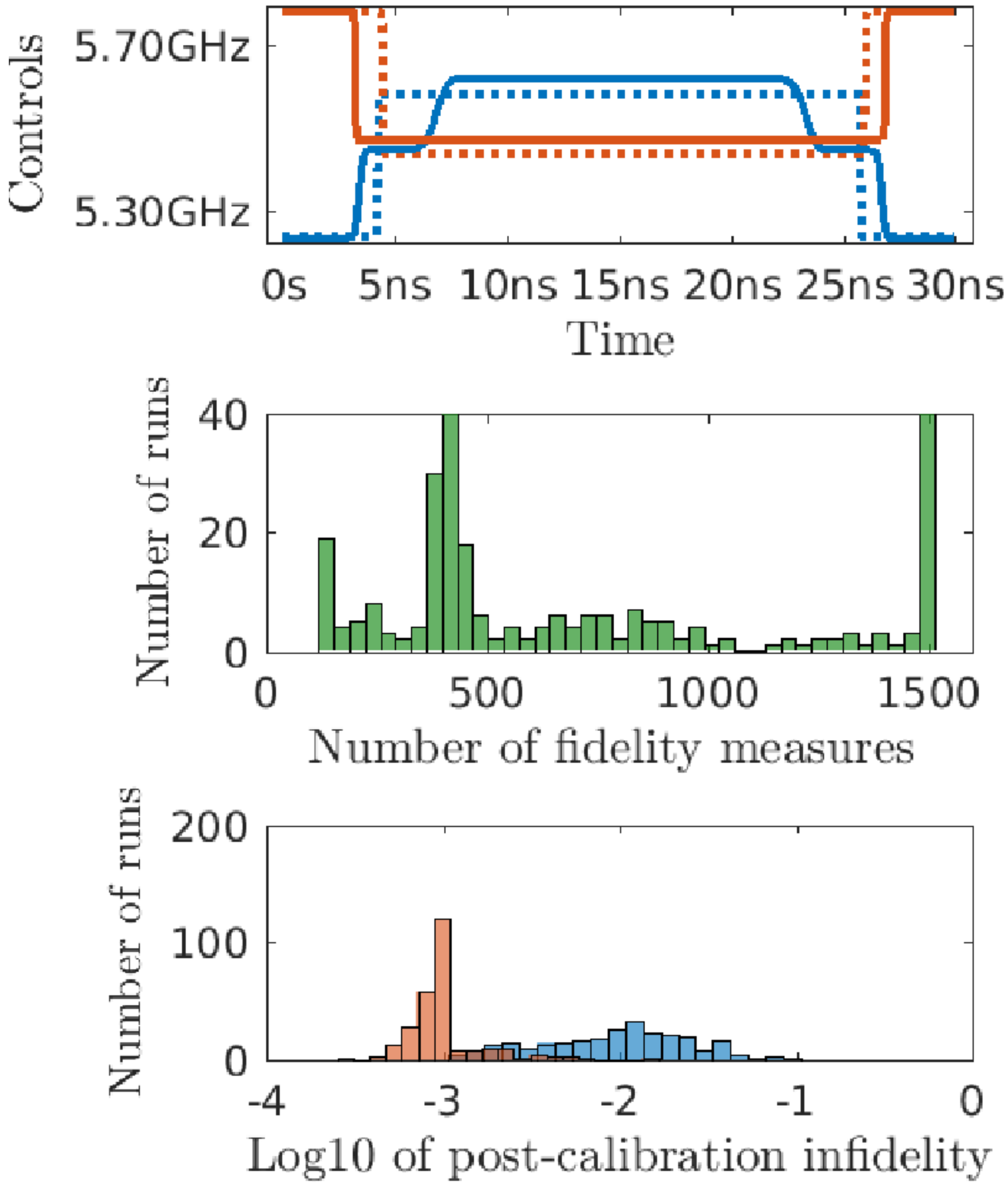
Speed limits & optimal speed





- Too short: Leakage (non-adiabatic)
- Too long: Markov
- Optimal speed for best fidelity

- Simple pulses are hard



Few Params

=

Easy experimental Calibration

Summary

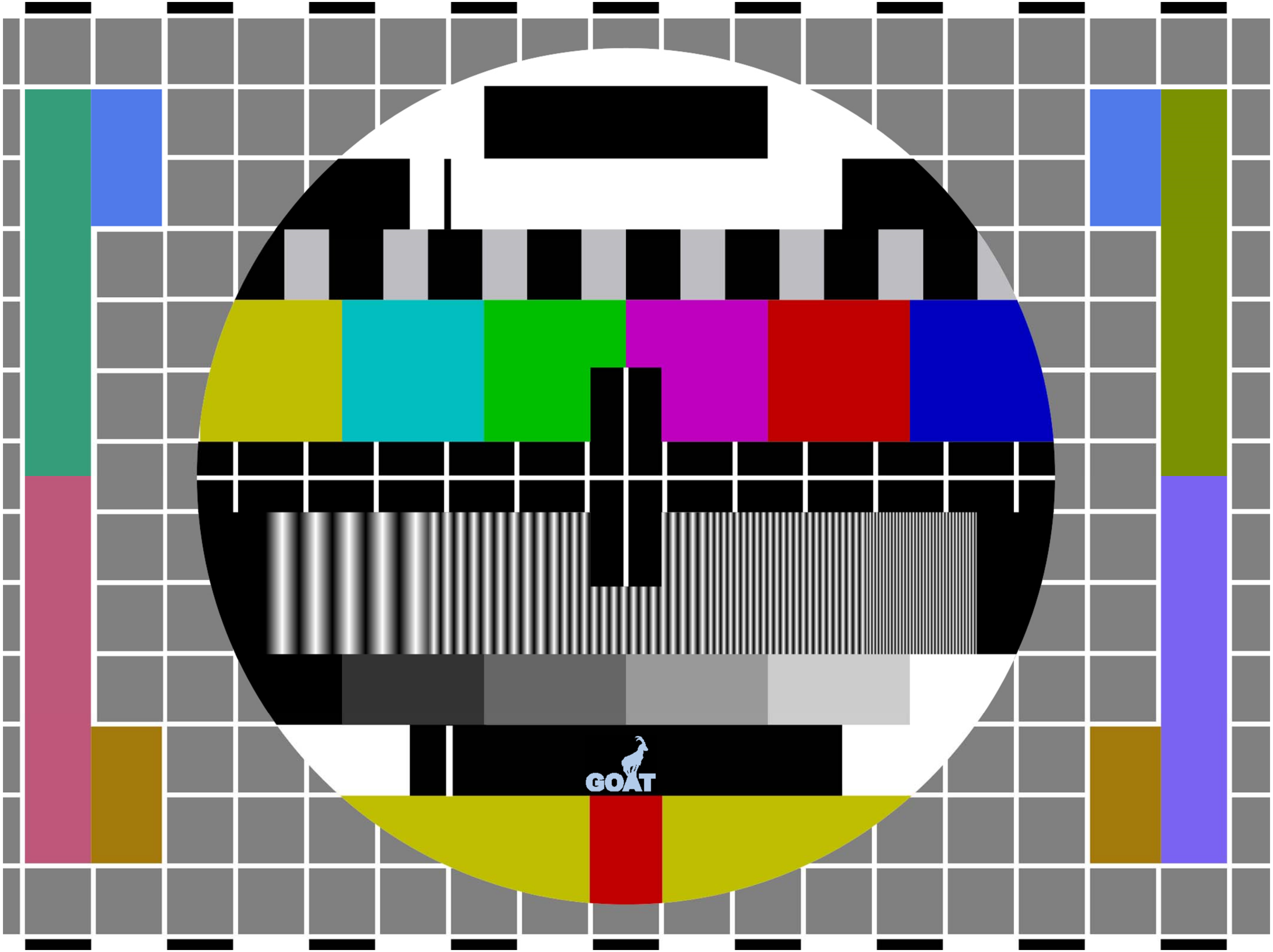
- Optimal control requires the model to be as accurate as possible
- Current optimal control techniques suffer from either low-accuracy, mathematical complexity, insufficient flexibility or slow convergence rates.
- GOAT is a novel, highly flexible, conceptually simple and easy-to-implement gradient-based optimal control algorithm.
- GOAT is extremely well-suited for optimization of calibration-ready pulses for real-world experiments.

Thank you !

GOAT

arXiv 1507.04261

Looking for experiments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