Quantum Coherence Effects in Thouless Adiabatic Pump and Their Possible Observation with One Qubit

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H.L. Wang, L.W. Zhou, and J.B. Gong, Interband coherence induced correction to adiabatic pumping in periodically driven systems, **Physical Review B** 91, 085420 (2015)

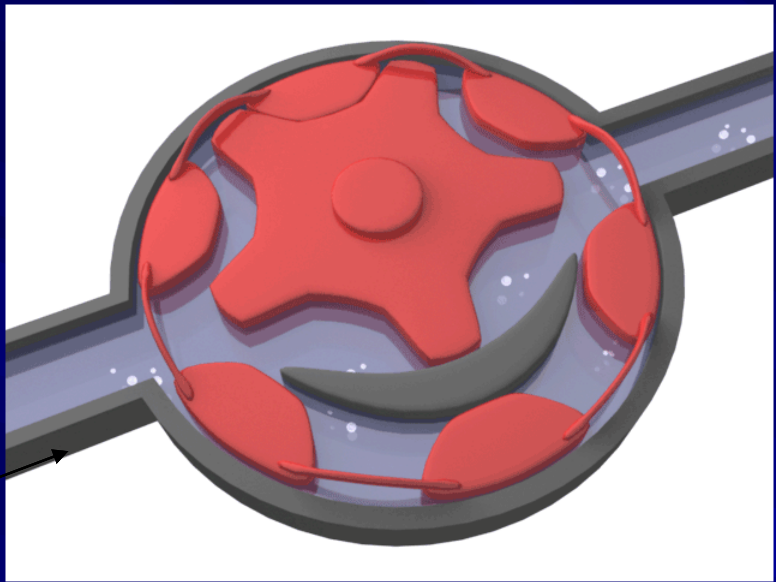
L.W. Zhou, D.Y. Tan, and J.B. Gong, Effects of Dephasing on Quantum Aiabatic Pumping with Nonequilibrium Initial states, **Physical Review B** 92, 245409 (2015).

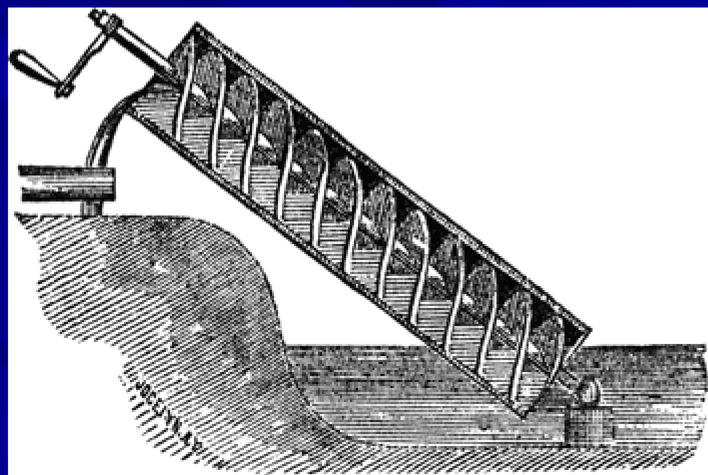
Experiment ongoing at USTC

Outline

- **Thouless topological pumping**
- **Remarks on quantum adiabatic theorem**
- **Theory: Influence of interband coherence on adiabatic pumping**
- **1D model with nontrivial topological phases**
- **Results and experimental proposal**

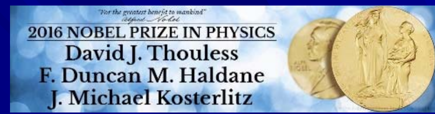
The operation of a (classical) pump:
Transport of fluid under periodic modulations

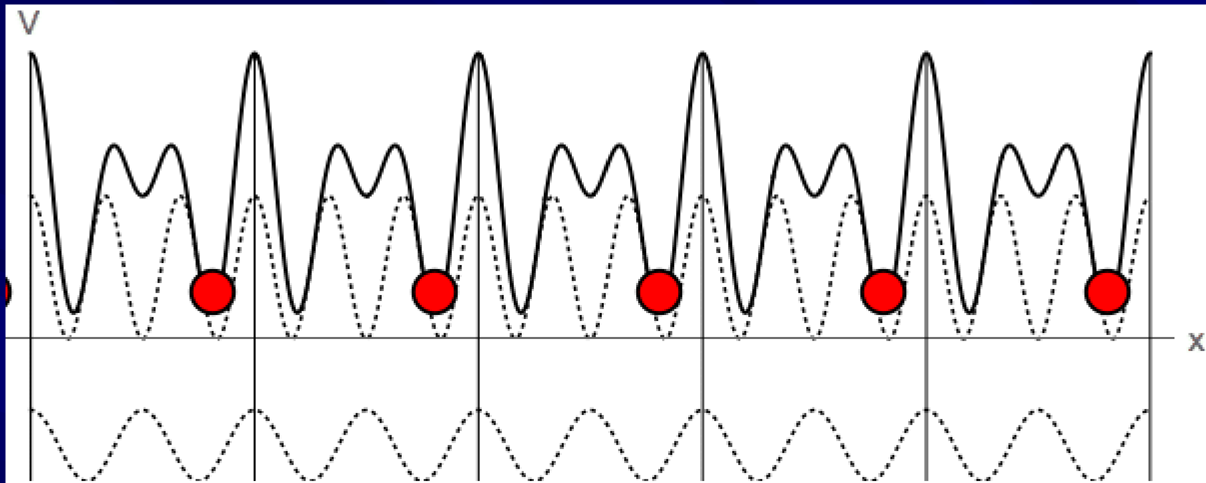




From wiki

Thouless topological (quantum) pump



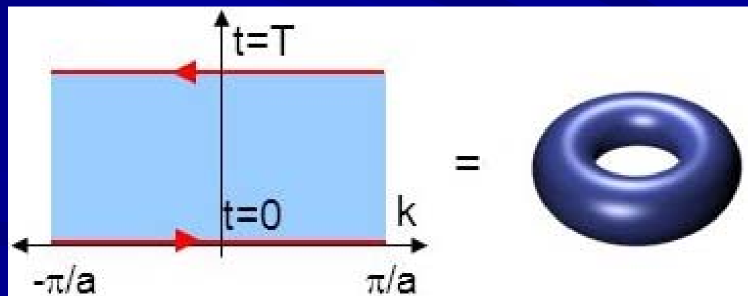


http://muellergroup.lassp.cornell.edu/

$$V(x,t) = V(x+a,t) = V(x,t+T)$$

$$Q = \frac{1}{2\pi}\int_0^T dt \int_{BZ} dk\, \Omega_{kt}$$

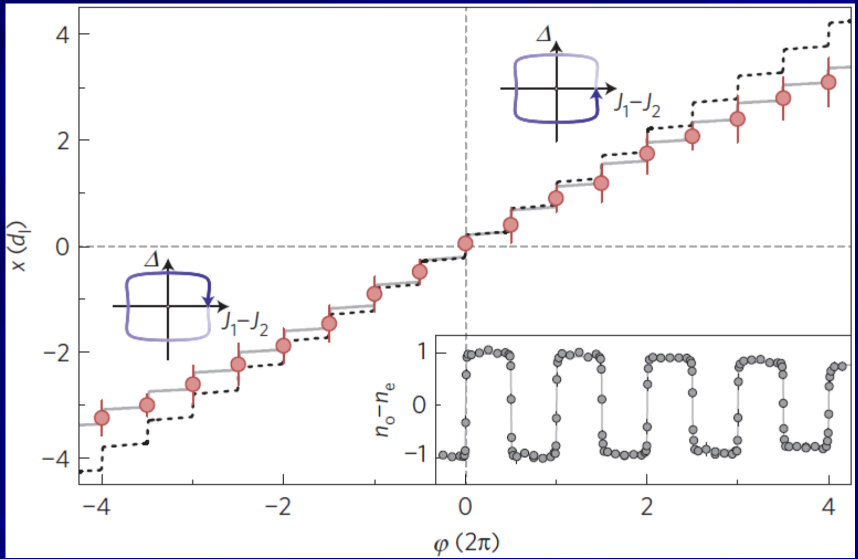
$$\Omega_{kt} = i(\langle \partial_k u | \partial_t u \rangle - \langle \partial_t u | \partial_k u \rangle)$$



Pumped charge over an adiabatic cycle is a topological invariant (Chern number)

Experimental realization of Thouless pump

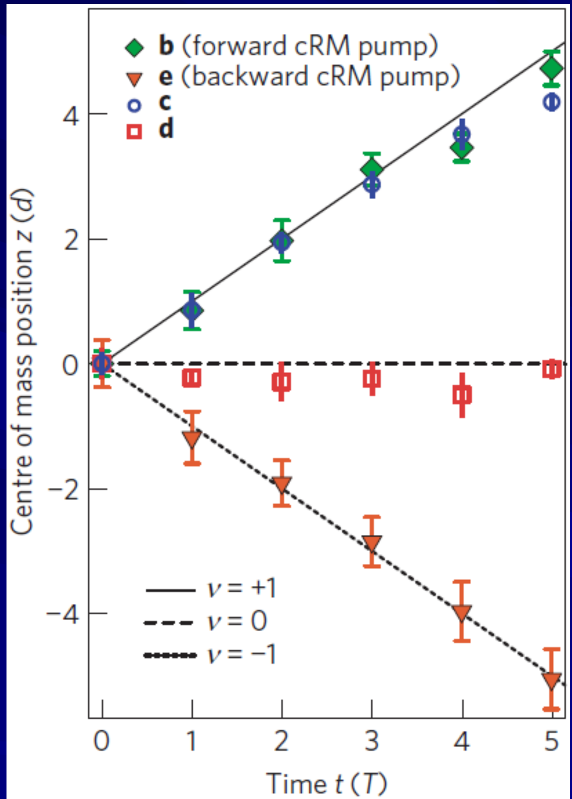
- Centre-of-mass position of the atom cloud as a function of the pumping parameter φ:



Initial state is a band Wannier state, uniform population on one band

M. Lohse, *et al*. A Thouless quantum pump with ultracold bosonic atoms in an optical superlattice. Nature Physics, **12**, 350 (2016).

Experimental realization of Thouless pump



Initial state is a band Wannier state, uniform population on one band

S. Nakajima, *et al*. Topological Thouless pumping of ultracold fermions. Nature Physics, **12**, 296 (2016).

Our general results for nonequilibrium initial states :

$$B(1) \equiv \frac{1}{\Delta k}\int_{-\frac{\Delta k}{2}}^{\frac{\Delta k}{2}} dk \int_0^1 \mathbf{T} ds \mathrm{Tr}[\hat{\rho}(s)\hat{A}]$$

$$B(1) = B^{\mathrm{II}}(1) + B^{\mathrm{III}}(1)$$

initial state coherence effect

$$= \frac{1}{\Delta k}\int_{-\frac{\Delta k}{2}}^{\frac{\Delta k}{2}} dk \sum_{m,n,m\neq n} 2\mathrm{Re}\left\{\rho_{mn}(0)\left[\frac{M_{nm}(s)}{\Delta_{mn}(s)}\right]\Big|_{s=0}\right\}\int_0^1 ds A_{nn}(s)$$

$$+ \frac{1}{\Delta k}\int_{-\frac{\Delta k}{2}}^{\frac{\Delta k}{2}} dk \sum_{m,n,m\neq n} \rho_{nn}(0)\int_0^1 ds \frac{A_{nm}(s)M_{mn}(s) + A_{mn}(s)M_{nm}(s)}{\Delta_{mn}(s)}$$

$$\rho_{mn}(s) \equiv c_m(s)c_n^*(s)$$

$$\Delta_{mn}(s) \equiv E_m(s) - E_n(s)$$

$$M_{mn}(s) \equiv i\langle m(s)|\partial_s n(s)\rangle$$

Adiabatic Theorem:

(state population does not change in adiabatic processes)

A slowly changing Hamiltonian: $\hat{H}(\beta)|\psi_n(\beta)\rangle = E_n(\beta)|\psi_n(\beta)\rangle$ with $\beta = \beta(t)$

Using intantaneous eigenstates $|\psi_n(\beta)\rangle$ to analyze the time evolution

$$|\Psi(t) = \sum_m C_m(t)|\psi_m(\beta)\rangle e^{i\Omega_m(t)}$$

where $\Omega_m(t) = -\frac{1}{\hbar}\int_0^t E_n[\beta(t')]dt'$

1st order adiabatic perturbation

$$\dot{C}_m = -C_m \langle \psi_m(\beta) | \frac{d\psi_m(\beta)}{dt} \rangle$$

absence of oscillating factor

$$- \sum_{n \neq m} C_n e^{i(\Omega_n - \Omega_m)} \frac{\langle \psi_m(\beta) | \frac{d\hat{H}(\beta)}{dt} | \psi_n(\beta) \rangle}{E_n(\beta) - E_m(\beta)}$$

oscillating factor

Berry phase

$$\Rightarrow \quad C_m(T) = C_m(0) e^{i\gamma_m} + \sum_{n \neq m} C_n(0) (\text{something}) \frac{1}{T}$$

1^{st} order Adiabatic Perturbation

Berry phase

$$C_m(T) = C_m(0)e^{i\gamma_m} + \sum_{n \neq m} C_n(0)(\text{something})\frac{1}{T}$$

Assuming initial state is exclusively at m th state

In approaching adiabatic limit:

$$|C_m(T)|^2 = |C_m(0)|^2 + O\left(\frac{1}{T^2}\right)$$

One "hidden" prediction from 1st order adiabatic perturbation theory

$$C_m(T) = C_m(0)e^{i\gamma_m} + \sum_{n\neq m} C_n(0)(\text{something})\frac{1}{T}$$

Assuming initial state is a superposition state of instantaneous energy eigenstates

In approaching adiabatic limit:

~~$$|C_m(T)|^2 = |C_m(0)|^2 + O\left(\frac{1}{T^2}\right)$$~~

$$|C_m(T)|^2 = |C_m(0)|^2 + O\left(\frac{1}{T}\right)$$

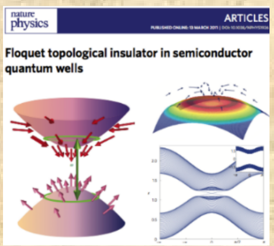
Using Floquet topological phases as a specific context

$$\hat{U}(t, t+T)\,|\psi_n(t)\rangle = e^{i\omega_n}\,|\psi_n(t)\rangle$$

- Naturally non-equilibrium situation
- Initial state likely a superposition state of many Floquet adiabatic eigenstates
- Floquet topological phase is a topic of considerable interest
- Will apply Thouless pumping protocol to Floquet band states

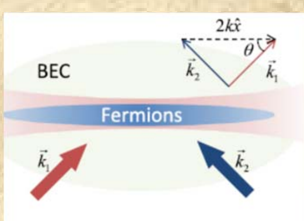
Concept of “Floquet topological phase” emerging in 2011-2013

Driven quantum well



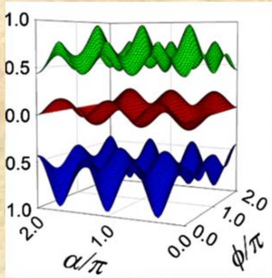
Lindner et al,
Nature Physics 7
490 (2011)

Driven cold atoms



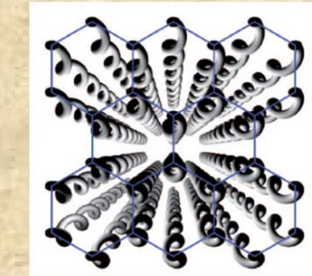
Jiang et al PRL
106, 220402 (2011)

Driven cold atoms



Ho and Gong,
PRL 109, 010601 (2012)

Photonics realization



Rechtsman et al
Nature 496
196 (2013)

Notation to present our theory

Periodically driven: $H_\beta(x, t+\tau) = H_\beta(x,t)$

Spatial periodicity: $H_\beta(x+a, t) = H_\beta(x,t)$

Floquet bands: $U(\beta)|\psi_{n,k}(\beta)\rangle = e^{-i\omega_{n,k}(\beta)}|\psi_{n,k}(\beta)\rangle$

k is the conserved quasi-momentum label

Parallel transport convention: $\langle\psi_{n,k}(\beta)|\mathrm{d}\psi_{n,k}(\beta)/\mathrm{d}\beta\rangle = 0$

β will be slowly tuned in adiabatic protocol,
H is periodic in β with period 2π

Adiabatic perturbation theory for periodically driven systems

scaled time: $s = t/(T\tau)$ — s increases from 0 to 1

adiabatic protocol lasting T periods: $\beta = \beta(s)$ — 2π change in one adiabatic cycle

dynamical phase: $\Omega_{n,k}(s) = T \int_0^s \omega_{n,k}[\beta(s)]\mathrm{d}s$

expressing state using instantaneous band states:

$$|\Psi(s)\rangle = \sqrt{\frac{a}{2\pi}} \int \mathrm{d}k \sum_n C_{n,k}(s) e^{-i\Omega_{n,k}(s)} |\psi_{n,k}[\beta(s)]\rangle$$

Evolution in representation of instantaneous band eigenstates

$$\frac{\mathrm{d}C_{n,k}}{\mathrm{d}s} = -\sum_{m\neq n} e^{i(\Omega_{n,k}-\Omega_{m,k})} C_{m,k} \left\langle \psi_{n,k}(\beta) \middle| \frac{\mathrm{d}\psi_{m,k}(\beta)}{\mathrm{d}s} \right\rangle$$

zeroth-order
"adiabatic theorem"

$$\frac{\mathrm{d}C_{n,k}}{\mathrm{d}s} = 0$$

1st-order
adiabatic correction

$$C_{n,k}(1) = C_{n,k}(0) + \frac{1}{T}\sum_{m\neq n} C_{m,k}(0) \left(W_{nm,k}(s) \Big|_{s=0}^{s=1} \right)$$

$$W_{nm,k}(s) = \frac{\left\langle \psi_{n,k}(\beta) \middle| \frac{\mathrm{d}\psi_{m,k}(\beta)}{\mathrm{d}s} \right\rangle}{1 - e^{i[\omega_{n,k}(\beta)-\omega_{m,k}(\beta)]}} e^{i[\Omega_{n,k}(s)-\Omega_{m,k}(s)]}$$

Influence of interband coherence on population correction:

$$C_{n,k}(1) = C_{n,k}(0) + \frac{1}{T}\sum_{m\neq n} C_{m,k}(0)\left(W_{nm,k}(s)\Big|_{s=0}^{s=1}\right)$$

$$\Delta\rho_{n,k} = |C_{n,k}(1)|^2 - \rho_{n,k}(0)$$

$$\Delta\rho_{n,k} = \frac{2}{T}\mathrm{Re}\left[\sum_{m\neq n} C^*_{n,k}(0)C_{m,k}(0)\left(W_{nm,k}\Big|_{s=0}^{s=1}\right)\right]$$

population correction
on top of adiabatic theorem

scales as $1/T^2$

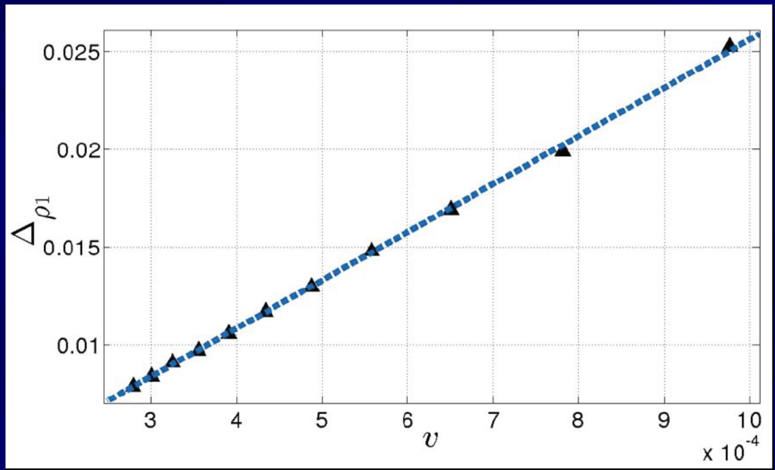
without interband
coherence

scales as $1/T$

with interband
coherence

Numerical examples of population contamination in adiabatic dynamics

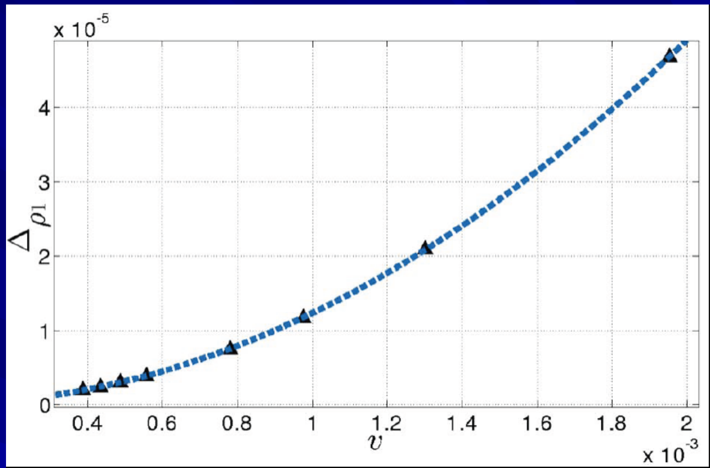
with interband coherence



$1/T$

linear scaling

without interband coherence



$1/T$

quadratic scaling

Charge polarization change per adiabatic cycle
(a compact derivation)

$$|\Psi(s=1)\rangle = \sqrt{\frac{a}{2\pi}} \int \mathrm{d}k \sum_n C_{n,k}(1) e^{-i\Omega_{n,k}(1)} |\psi_{n,k}[\beta(s=1)]\rangle$$

After one-cycle

$$|\Psi(s=1)\rangle = \sqrt{\frac{a}{2\pi}} \int \mathrm{d}k \sum_n C_{n,k}(1) e^{-i\Omega_{n,k}(1)} e^{-i\gamma_{n,k}(s)} |\psi_{n,k}[\beta(s=0)]\rangle$$

$$x \sim i\frac{d}{dk}$$

$$\begin{aligned}\Delta\langle x\rangle &= \sum_n \int \mathrm{d}k \left[\frac{\mathrm{d}\gamma_{n,k}}{\mathrm{d}k} + \frac{\mathrm{d}\Omega_{n,k}(1)}{\mathrm{d}k}\right] |C_{n,k}(1)|^2, \\ &= \sum_n \int \mathrm{d}k \left[\frac{\mathrm{d}\gamma_{n,k}}{\mathrm{d}k} + \frac{\mathrm{d}\Omega_{n,k}(1)}{\mathrm{d}k}\right] [\Delta\rho_{n,k} + \rho_{n,k}(0)]\end{aligned}$$

$$\sum_n \int \mathrm{d}k \frac{\mathrm{d}\gamma_{n,k}}{\mathrm{d}k} \Delta\rho_{n,k} \sim 1/T$$

vanishes for large T

$$\sum_n \int \mathrm{d}k \frac{\mathrm{d}\gamma_{n,k}}{\mathrm{d}k} \rho_{n,k}(0)$$

weighted Berry curvature integral, survives for large T

$$\sum_n \int \mathrm{d}k \frac{\mathrm{d}\Omega_{n,k}(1)}{\mathrm{d}k} \Delta\rho_{n,k} \sim T \times 1/T$$

Interband coherence effect survives for large T and independent of T !

$$\sum_n \int \mathrm{d}k \frac{\mathrm{d}\Omega_{n,k}(1)}{\mathrm{d}k} \rho_{n,k}(0)$$

vanishes under simple symmetry assumption

Charge polarization change over one adiabatic pumping cycle
(under certain symmetry assumption)

$$\Delta\langle x\rangle = \sum_n \int \mathrm{d}k \int d\beta\; B_n(\beta,k)\rho_{n,k}(0) \quad + \quad \sum_n \int \mathrm{d}k\, \frac{\mathrm{d}\Omega_{n,k}(1)}{\mathrm{d}k}\Delta\rho_{n,k}$$

apparent contribution, reducing
to a Chern number summation
if occupation is uniform everywhere

novel coherence-induced correction

$$-\,2\sum_{m\neq n}\int \mathrm{d}k\ \mathrm{Re}\left[C^*_{n,k}(0)C_{m,k}(0)\frac{\mathrm{d}E_{n,k}}{\mathrm{d}k}W_{nm,k}(0)\right]$$

Berry curvature

$$B_n(\beta,k) \equiv i\left[\left\langle\frac{\partial\psi_{n,k}(\beta)}{\partial k}\middle|\frac{\partial\psi_{n,k}(\beta)}{\partial\beta}\right\rangle - \left\langle\frac{\partial\psi_{n,k}(\beta)}{\partial\beta}\middle|\frac{\partial\psi_{n,k}(\beta)}{\partial k}\right\rangle\right]$$

$$W_{nm,k}(s) = \frac{\left\langle\psi_{n,k}(\beta)\middle|\frac{\mathrm{d}\psi_{m,k}(\beta)}{\mathrm{d}s}\right\rangle}{1-e^{i[\omega_{n,k}(\beta)-\omega_{m,k}(\beta)]}}e^{i[\Omega_{n,k}(s)-\Omega_{m,k}(s)]}$$

A simple driven superlattice (Harper-Aubry-Andre) model

$$\hat{H} = \sum_m \frac{J}{2}\left(a_m^\dagger a_{m+1} + h.c.\right) + \sum_m V\cos\left(2\pi\alpha m - \beta\right)\cos(\Omega t) a_m^\dagger a_m$$

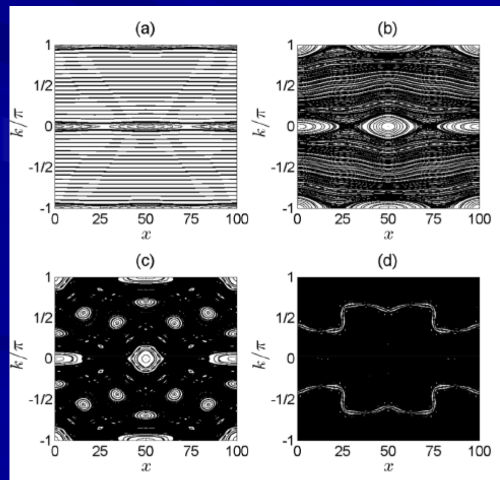
neareast-neighbor hopping

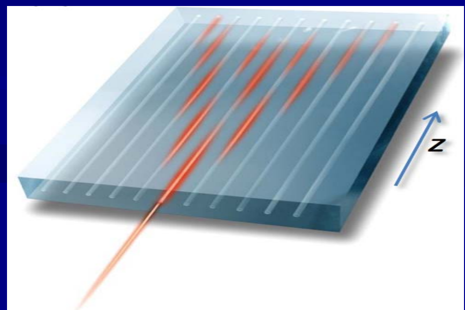
modulated on-site superlattice potential

α is like the magnetic flux parameter in IQHE

β is like quasi-momentum of 2nd dimension in IQHE

warning: classical dynamics can be really complicated





possible optics realization

Generating different topological phases in the driven superlattice model

$$\hat{H} = \sum_m \frac{J}{2}\left(a_m^\dagger a_{m+1} + h.c.\right) + \sum_m V\cos\left(2\pi\alpha m - \beta\right)\cos(\Omega t) a_m^\dagger a_m$$

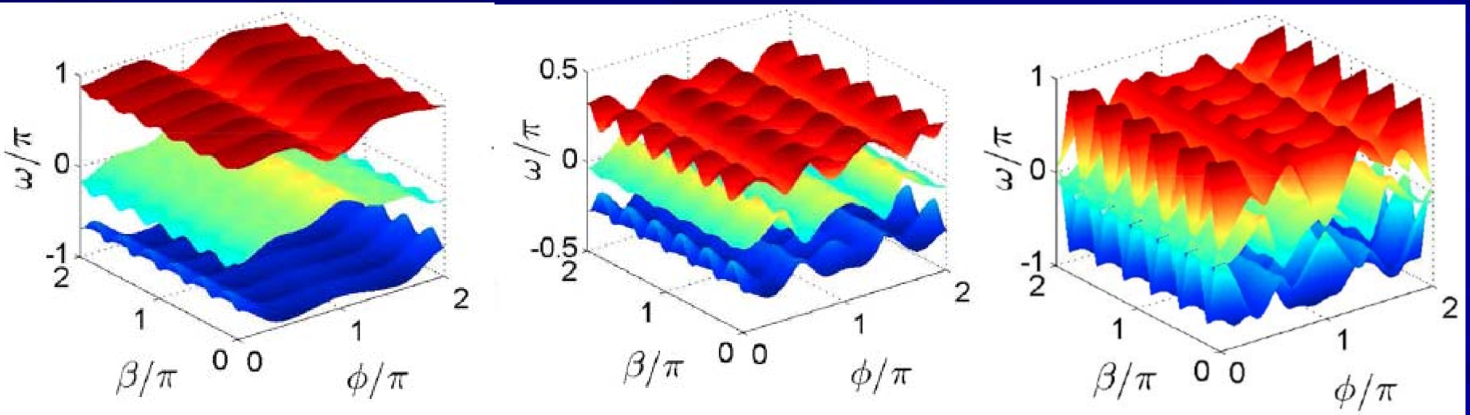
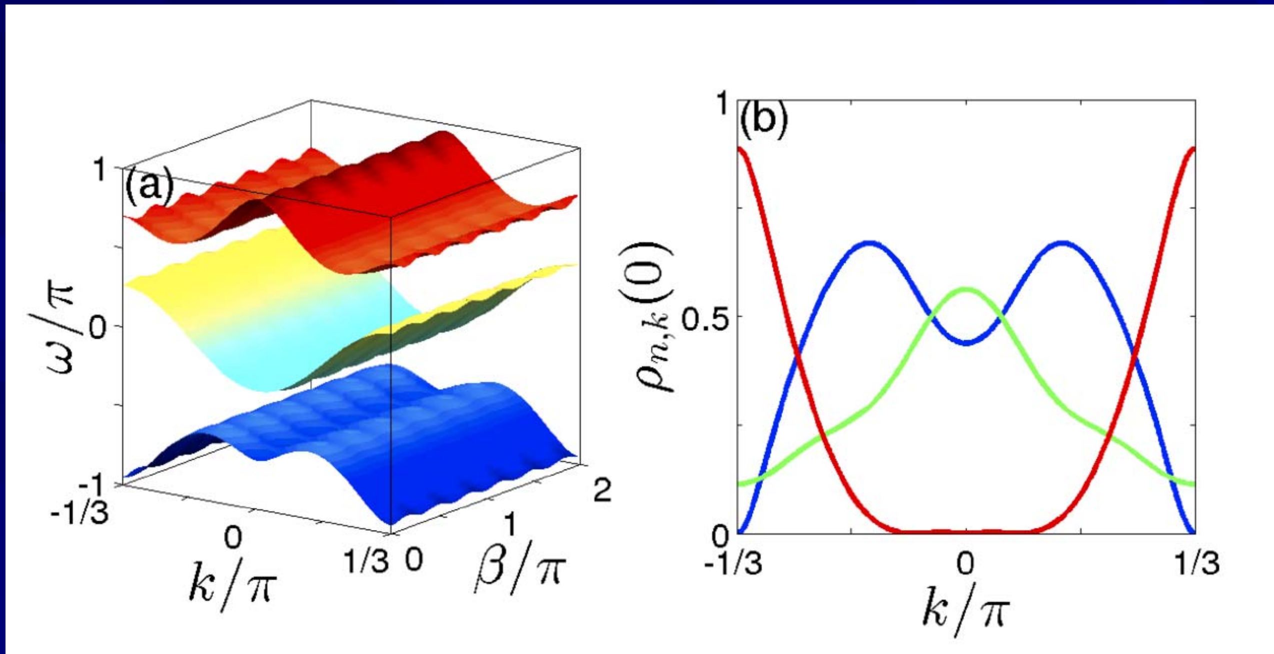


Table 1. Chern numbers of CDHM with respect to J and V for $\alpha = 1/3, T = 2$.

Values of J and V	[0.1, 3.4]	[3.5, 5.1]	[5.2, 5.4]	[5.5, 5.6]	5.7	[5.8, 7.5]	[7.6, 10]
C_1	−2	4	−8	−2	−8	4	−2
C_2	4	−8	16	4	16	−8	4
C_3	−2	4	−8	−2	−8	4	−2

Adiabatic dynamics in the driven superlattice model: Initial state

$$\hat{H} = \sum_m \frac{J}{2}\left(a_m^\dagger a_{m+1} + h.c.\right) + \sum_m V\cos\left(2\pi\alpha m - \beta\right)\cos(\Omega t) a_m^\dagger a_m$$



Floquet band structure in a 3-band example

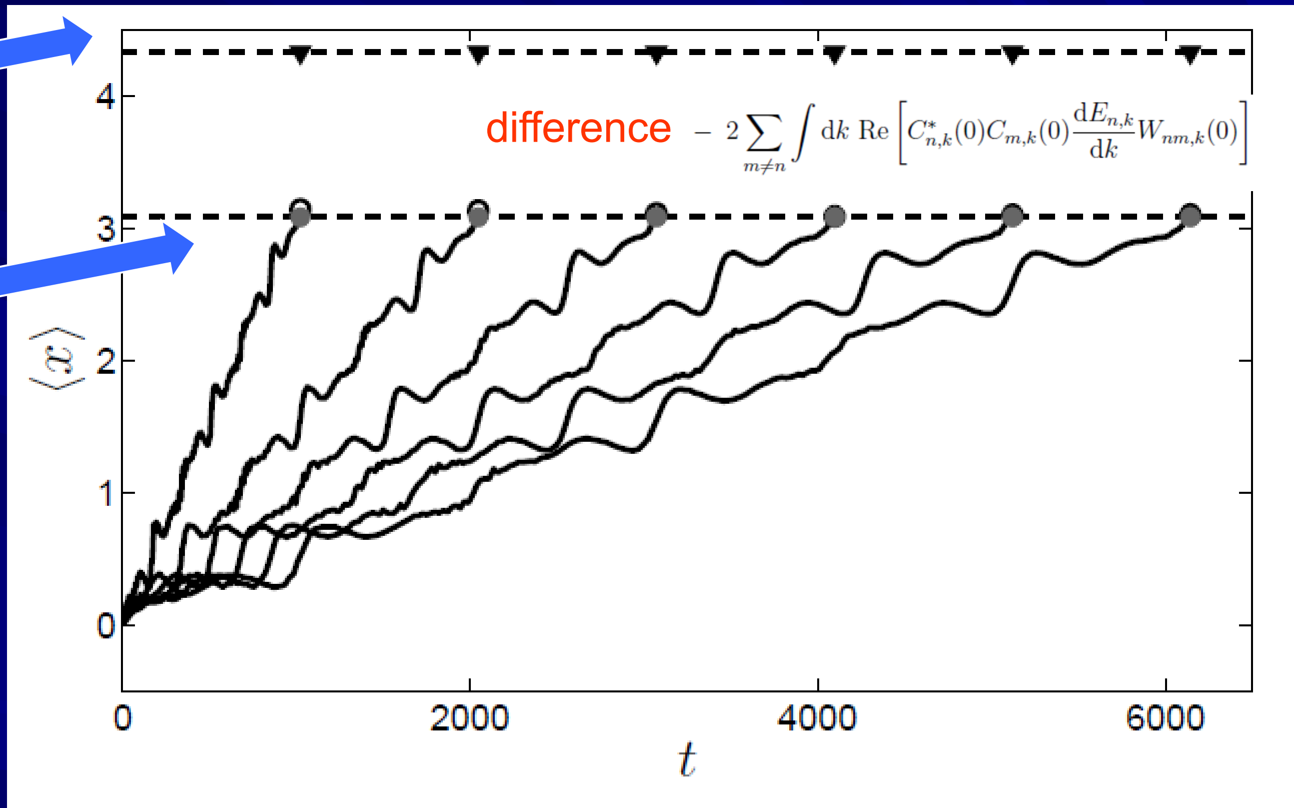
initial state localized at site zero

Adiabatic evolution in terms of wavepacket displacement

different solid lines are for different pumping-cycle duration

Berry curvature integral only

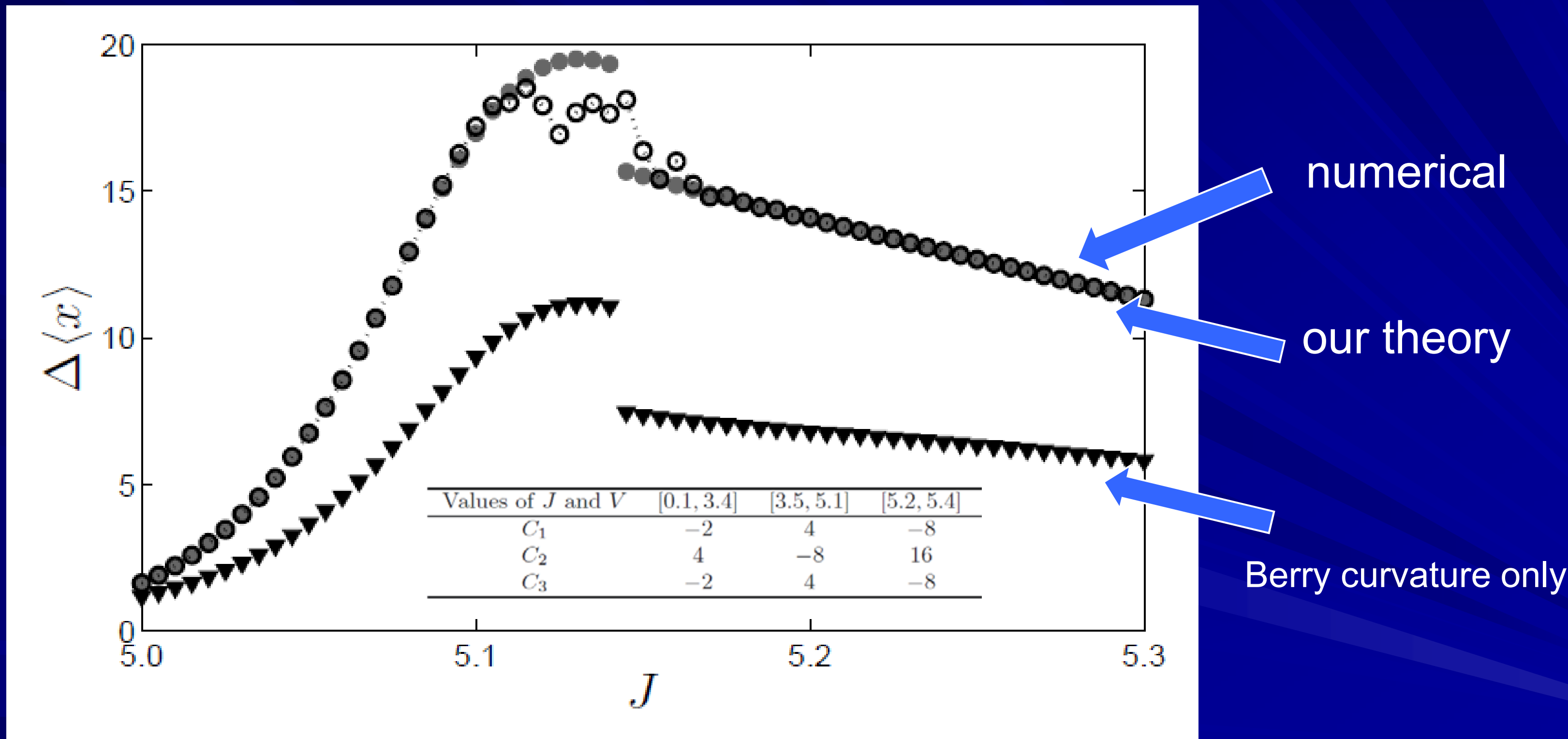
Our theory



Coherence effect is independent of T, survives in adiabatic limit!

Coherence is important in understanding adiabatic pumping

Detecting a topological phase transition using adiabatic pumping (initial state localized at site zero, possessing interband coherence)



Values of J and V	$[0.1, 3.4]$	$[3.5, 5.1]$	$[5.2, 5.4]$
C_1	-2	4	-8
C_2	4	-8	16
C_3	-2	4	-8

General results due to initial state coherence :

$$B(1) \equiv \frac{1}{\Delta k}\int_{-\frac{\Delta k}{2}}^{\frac{\Delta k}{2}} dk \int_0^1 \mathbf{T} ds \mathrm{Tr}[\hat{\rho}(s)\hat{A}]$$

$$B(1) = B^{\mathrm{II}}(1) + B^{\mathrm{III}}(1)$$

initial state coherence effect

$$= \frac{1}{\Delta k}\int_{-\frac{\Delta k}{2}}^{\frac{\Delta k}{2}} dk \sum_{m,n,m\neq n} 2\mathrm{Re}\left\{\rho_{mn}(0)\left[\frac{M_{nm}(s)}{\Delta_{mn}(s)}\right]\Big|_{s=0}\right\}\int_0^1 ds A_{nn}(s)$$

$$+ \frac{1}{\Delta k}\int_{-\frac{\Delta k}{2}}^{\frac{\Delta k}{2}} dk \sum_{m,n,m\neq n} \rho_{nn}(0)\int_0^1 ds \frac{A_{nm}(s)M_{mn}(s) + A_{mn}(s)M_{nm}(s)}{\Delta_{mn}(s)}$$

$$\rho_{mn}(s) \equiv c_m(s)c_n^*(s) \qquad \Delta_{mn}(s) \equiv E_m(s) - E_n(s) \qquad M_{mn}(s) \equiv i\langle m(s)|\partial_s n(s)\rangle$$

Experimental proposal to detect coherence effects: A qubit in a rotating field

Hamiltonian: $\hat{H}(\theta,\phi) = \sum_{\alpha=x,y,z} h_\alpha(\theta,\phi)\hat{\sigma}_\alpha$

Quasimomentum in a synthetic dimension

Adiabatic protocol: $\phi = \phi(s) \quad (s = t/\tau)$

Generalized force: $\hat{A}_\theta = \partial_\theta \hat{H}(\phi) = \sum_{\alpha=x,y,z} \partial_\theta h_\alpha(\phi)\hat{\sigma}_\alpha$

In adiabatic limit:

Pure *interband coherence* effect

$$B(1) = \frac{1}{\Delta\theta}\int_{-\frac{\Delta\theta}{2}}^{\frac{\Delta\theta}{2}} d\theta \sum_{m,n,m\neq n} 2\mathrm{Re}\left\{\rho_{mn}(0)\left[\frac{M_{nm}(s)}{\Delta_{mn}(s)}\right]\Big|_{s=0}\right\}\int_0^1 ds\partial_\theta E_n(\phi)$$

$$+\frac{i}{\Delta\theta}\int_{-\frac{\Delta\theta}{2}}^{\frac{\Delta\theta}{2}} d\theta \sum_n \rho_{nn}(0)\int_0^1 ds[\langle\partial_s n(\phi)|\partial_\theta n(\phi)\rangle - \langle\partial_\theta n(\phi)|\partial_s n(\phi)\rangle]$$

Weighted integral of *Berry curvature*

A qubit in a rotating field

Hamiltonians:

$$H = \omega_1 \sin\theta \left(S_x \cos\phi + S_y \sin\phi \right) + \left(\delta_1 \cos\theta + \delta_2 \right) S_z$$

Generalized force:

$$A = \partial_\theta H = \omega_1 \cos\theta \left(S_x \cos\phi + S_y \sin\phi \right) - \delta_1 \sin\theta \cdot S_z$$

Different adiabatic protocols:

$$\phi(s) = 2\pi s, \quad \pi(s + s^2), \quad 2\pi \sin\left(\frac{\pi}{2} s \right), \quad 2\pi s^2 \quad \left(s = t / \tau \right)$$

Parameter settings:

$$\omega_1 = 20\text{MHz}, \quad \delta_1 = 10\text{MHz}, \quad \delta_2 = 0 \sim 20\text{MHz}$$

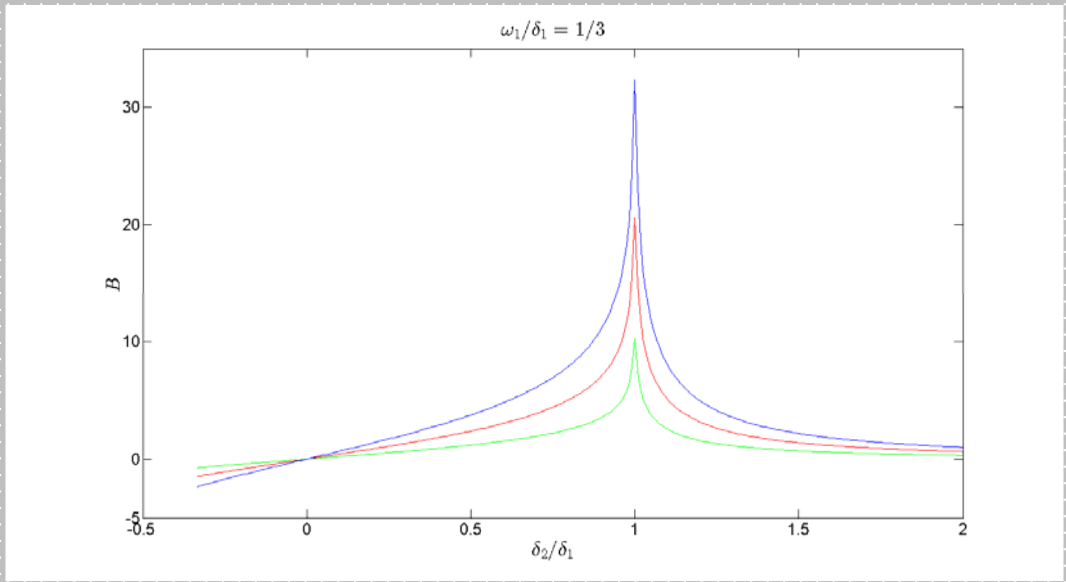
Different adiabatic protocols

Initial state with interband coherence: $\rho_{+-}(0) = 1/2$

— : $2\pi \sin\left(\frac{\pi}{2}s\right)$

— : $2\pi s$

— : $\pi(s + s^2)$



Signal independent of protocol duration T

Pumping sensitively depends on the turn-on rate of the protocol

Signal captures phase transition points

Concluding Remarks

Correction to adiabatic theorem: initial-state coherence between different instantaneous eigenstates can be important.

The found correction naturally emerges in considering adiabatic pumping for nonequilbrium initial states, which often occurs, for example, in considering Floquet topological phases.

The found coherence effect survives well in the presence of dephasing (results not shown)

The found effect can be used to detect topological phase transitions, and its simulation using single-qubit system is ongoing.

Thank you !