Quantum Coherence Effects in Thouless Adiabatic Pump and Their Possible Observation with One Qubit

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Experiment ongoing at USTC
Outline

- Thouless topological pumping
- Remarks on quantum adiabatic theorem
- Theory: Influence of interband coherence on adiabatic pumping
- 1D model with nontrivial topological phases
- Results and experimental proposal
The operation of a (classical) pump: Transport of fluid under periodic modulations

From wiki
Thouless topological (quantum) pump

http://muellergroup.lassp.cornell.edu/

\[ V(x, t) = V(x + a, t) = V(x, t + T) \]

\[ Q = \frac{1}{2\pi} \int_0^T dt \int_{BZ} dk \Omega_{kt} \]

\[ \Omega_{kt} = i(\langle \partial_k u | \partial_t u \rangle - \langle \partial_t u | \partial_k u \rangle) \]

Pumped charge over an adiabatic cycle is a topological invariant (Chern number)
Experimental realization of Thouless pump

Centre-of-mass position of the atom cloud as a function of the pumping parameter $\varphi$:

Experimental realization of Thouless pump


Initial state is a band Wannier state, uniform population on one band
Our general results for nonequilibrium initial states:

\[
B(1) \equiv \frac{1}{\Delta k} \int_{-\Delta k/2}^{\Delta k/2} dk \int_0^1 T ds \text{Tr}[\hat{\rho}(s) \hat{A}]
\]

\[
B(1) = B^{\Pi}(1) + B^{\Delta}(1)
\]

\[
= \frac{1}{\Delta k} \int_{-\Delta k/2}^{\Delta k/2} dk \sum_{m, n, m \neq n} \text{Re} \left\{ \rho_{mn}(0) \left[ \frac{M_{mn}(s)}{\Delta_{mn}(s)} \right] \bigg|_{s=0} \right\} \int_0^1 ds A_{nn}(s) + \frac{1}{\Delta k} \int_{-\Delta k/2}^{\Delta k/2} dk \sum_{m, n, m \neq n} \rho_{nn}(0) \int_0^1 ds \frac{A_{nm}(s)M_{mn}(s) + A_{mn}(s)M_{nm}(s)}{\Delta_{mn}(s)}
\]

\[
\rho_{mn}(s) \equiv c_m(s) c^*_n(s) \quad \Delta_{mn}(s) \equiv E_m(s) - E_n(s) \quad M_{mn}(s) \equiv i\langle m(s)|\partial_s n(s)\rangle
\]
A slowly changing Hamiltonian:
\[ \hat{H}(\beta)|\psi_n(\beta)\rangle = E_n(\beta)|\psi_n(\beta)\rangle \]
with \( \beta = \beta(t) \)

Using instantaneous eigenstates \( |\psi_n(\beta)\rangle \) to analyze the time evolution

\[ |\Psi(t)\rangle = \sum_m C_m(t)|\psi_m(\beta)\rangle e^{i\Omega_m(t)} \]

where
\[ \Omega_m(t) = -\frac{1}{\hbar} \int_0^t E_n[\beta(t')] dt' \]
1\textsuperscript{st} order adiabatic perturbation

\[
\dot{C}_m = -C_m \langle \psi_m(\beta) | \frac{d\psi_m(\beta)}{dt} \rangle
\]

absence of oscillating factor

\[- \sum_{n \neq m} C_n e^{i(\Omega_n - \Omega_m)} \frac{\langle \psi_m(\beta) | \frac{d\hat{H}(\beta)}{dt} | \psi_n(\beta) \rangle}{E_n(\beta) - E_m(\beta)}\]

oscillating factor

Berry phase

\[
C_m(T) = C_m(0) e^{i\gamma_m} + \sum_{n \neq m} C_n(0) (\text{something}) \frac{1}{T}
\]
1st order Adiabatic Perturbation

Assuming initial state is exclusively at \( m \) th state

In approaching adiabatic limit:

\[
|C_m(T)|^2 = |C_m(0)|^2 + O\left(\frac{1}{T^2}\right)
\]
One “hidden” prediction from 1st order adiabatic perturbation theory

\[ C_m(T) = C_m(0)e^{i\gamma m} + \sum_{n \neq m} C_n(0) \text{(something)} \frac{1}{T} \]

Assuming initial state is a superposition state of instantaneous energy eigenstates

In approaching adiabatic limit:

\[ |C_m(T)|^2 = |C_m(0)|^2 + O \left( \frac{1}{T^2} \right) \]

\[ |C_m(T)|^2 = |C_m(0)|^2 + O \left( \frac{1}{T} \right) \]
Using Floquet topological phases as a specific context

\[ \hat{U}(t, t+T)|\psi_n(t)\rangle = e^{i\omega_n} |\psi_n(t)\rangle \]

- Naturally non-equilibrium situation
- Initial state likely a superposition state of many Floquet adiabatic eigenstates
- Floquet topological phase is a topic of considerable interest
- Will apply Thouless pumping protocol to Floquet band states
Driven quantum well
Lindner et al, Nature Physics 7 490 (2011)

Driven cold atoms
Jiang et al PRL 106, 220402 (2011)

Driven cold atoms
Ho and Gong, PRL 109, 010601 (2012)

Photonics realization
Rechtsman et al Nature 496 196 (2013)
Notation to present our theory

Periodically driven:
\[ H_\beta(x, t + \tau) = H_\beta(x, t) \]

Spatial periodicity:
\[ H_\beta(x + a, t) = H_\beta(x, t) \]

Floquet bands:
\[ U(\beta)|\psi_{n,k}(\beta)\rangle = e^{-i\omega_{n,k}(\beta)}|\psi_{n,k}(\beta)\rangle \]
\[ k \text{ is the conserved quasi-momentum label} \]

Parallel transport convention:
\[ \langle \psi_{n,k}(\beta) | d\psi_{n,k}(\beta)/d\beta \rangle = 0 \]
\[ \beta \text{ will be slowly tuned in adiabatic protocol, } H \text{ is periodic in } \beta \text{ with period } 2\pi \]
Adiabatic perturbation theory for periodically driven systems

scaled time: \[ s = \frac{t}{(T\tau)} \]
s increases from 0 to 1

adiabatic protocol lasting T periods \[ \beta = \beta(s) \]
2\pi change in one adiabatic cycle

dynamical phase: \[ \Omega_{n,k}(s) = T \int_0^s \omega_{n,k}[\beta(s)] ds \]

expressing state using instantaneous band states
\[ |\Psi(s)\rangle = \sqrt{\frac{\alpha}{2\pi}} \int dk \sum_n C_{n,k}(s)e^{-i\Omega_{n,k}(s)}|\psi_{n,k}[\beta(s)]\rangle \]
Evolution in representation of instantaneous band eigenstates

\[
\frac{dC_{n,k}}{ds} = - \sum_{m \neq n} e^{i(\Omega_{n,k} - \Omega_{m,k})} C_{m,k} \left\langle \psi_{n,k}(\beta) \left| \frac{d\psi_{m,k}(\beta)}{ds} \right. \right\rangle
\]

zeroth-order
“adiabatic theorem”

\[
\frac{dC_{n,k}}{ds} = 0
\]

1st-order
adiabatic correction

\[
C_{n,k}(1) = C_{n,k}(0) + \frac{1}{T} \sum_{m \neq n} C_{m,k}(0) \left( W_{nm,k}(s) \right|_{s=1}^{s=0}
\]

\[
W_{nm,k}(s) = \frac{\left\langle \psi_{n,k}(\beta) \left| \frac{d\psi_{m,k}(\beta)}{ds} \right. \right\rangle}{1 - e^{i(\Omega_{n,k}(s) - \Omega_{m,k}(s))}}
\]
Influence of interband coherence on population correction:

\[ C_{n,k}(1) = C_{n,k}(0) + \frac{1}{T} \sum_{m \neq n} C_{m,k}(0) \left( W_{nm,k}^{s=1} \right) \]

\[ \Delta \rho_{n,k} = |C_{n,k}(1)|^2 - \rho_{n,k}(0) \]

\[ \Delta \rho_{n,k} = \frac{2}{T} \text{Re} \left[ \sum_{m \neq n} C_{n,k}^*(0) C_{m,k}(0) \left( W_{nm,k}^{s=1} \right) \right] \]

Population correction on top of adiabatic theorem

Scales as \( \frac{1}{T^2} \) without interband coherence

Scales as \( \frac{1}{T} \) with interband coherence
Numerical examples of population contamination in adiabatic dynamics

with interband coherence

without interband coherence

linear scaling

quadratic scaling
Charge polarization change per adiabatic cycle  
(a compact derivation)

\[
|\Psi(s = 1)\rangle = \sqrt{\frac{a}{2\pi}} \int dk \sum_n C_{n,k}(1) e^{-i\Omega_{n,k}(1)} |\psi_{n,k}[\beta(s = 1)]\rangle
\]

After one-cycle

\[
|\Psi(s = 1)\rangle = \sqrt{\frac{a}{2\pi}} \int dk \sum_n C_{n,k}(1) e^{-i\Omega_{n,k}(1)} e^{-i\gamma_{n,k}(s)} |\psi_{n,k}[\beta(s = 0)]\rangle
\]

\[
\Delta \langle x \rangle = \sum_n \int dk \left[ \frac{d\gamma_{n,k}}{dk} + \frac{d\Omega_{n,k}(1)}{dk} \right] |C_{n,k}(1)|^2,
\]

\[
= \sum_n \int dk \left[ \frac{d\gamma_{n,k}}{dk} + \frac{d\Omega_{n,k}(1)}{dk} \right] [\Delta \rho_{n,k} + \rho_{n,k}(0)]
\]
Vanishes for large $T$

Weighted Berry curvature integral, survives for large $T$

Interband coherence effect survives for large $T$ and independent of $T$!

Vanishes under simple symmetry assumption
Charge polarization change over one adiabatic pumping cycle (under certain symmetry assumption)

\[
\Delta \langle x \rangle = \sum_n \int dk \int d\beta \ B_n(\beta, k) \rho_{n,k}(0)
\]

apparent contribution, reducing to a Chern number summation if occupation is uniform everywhere

\[
\sum_n \int dk \frac{d\Omega_{n,k}(1)}{dk} \Delta \rho_{n,k}
\]

novel coherence-induced correction

\[
- 2 \sum_{m \neq n} \int dk \text{Re} \left[ C^*_{n,k}(0) C_{m,k}(0) \frac{dE_{n,k}}{dk} W_{nm,k}(0) \right]
\]

Berry curvature

\[
B_n(\beta, k) = i \left[ \langle \frac{\partial \psi_{n,k}(\beta)}{\partial k} | \frac{\partial \psi_{n,k}(\beta)}{\partial \beta} \rangle - \langle \frac{\partial \psi_{n,k}(\beta)}{\partial \beta} | \frac{\partial \psi_{n,k}(\beta)}{\partial k} \rangle \right]
\]

\[
W_{nm,k}(s) = \frac{\langle \psi_{n,k}(\beta) | \frac{d\psi_{m,k}(\beta)}{ds} \rangle}{1 - e^{i[\omega_{n,k}(\beta) - \omega_{m,k}(\beta)](\Omega_{n,k}(s) - \Omega_{m,k}(s))}}
\]
A simple driven superlattice (Harper-Aubry-Andre) model

\[ \hat{H} = \sum_{m} \frac{J}{2} (a_{m}^{\dagger} a_{m+1} + h.c.) + \sum_{m} V \cos (2\pi \alpha m - \beta) \cos (\Omega t) a_{m}^{\dagger} a_{m} \]

near-neighbor hopping

modulated on-site superlattice potential

\( \alpha \) is like the magnetic flux parameter in IQHE

\( \beta \) is like quasi-momentum of 2nd dimension in IQHE

possible optics realization

warning: classical dynamics can be really complicated
Generating different topological phases in the driven superlattice model

\[ \hat{H} = \sum_m \frac{J}{2} (a_m^\dagger a_{m+1} + h.c.) + \sum_m V \cos(2\pi \alpha m - \beta) \cos(\Omega t) a_m^\dagger a_m \]

Table 1. Chern numbers of CDHM with respect to $J$ and $V$ for $\alpha = 1/3$, $T = 2$.

<table>
<thead>
<tr>
<th>Values of $J$ and $V$</th>
<th>[0.1, 3.4]</th>
<th>[3.5, 5.1]</th>
<th>[5.2, 5.4]</th>
<th>[5.5, 5.6]</th>
<th>5.7</th>
<th>[5.8, 7.5]</th>
<th>[7.6, 10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>-2</td>
<td>4</td>
<td>-8</td>
<td>-2</td>
<td>-8</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>$C_2$</td>
<td>4</td>
<td>-8</td>
<td>16</td>
<td>4</td>
<td>16</td>
<td>-8</td>
<td>4</td>
</tr>
<tr>
<td>$C_3$</td>
<td>-2</td>
<td>4</td>
<td>-8</td>
<td>-2</td>
<td>-8</td>
<td>4</td>
<td>-2</td>
</tr>
</tbody>
</table>
Adiabatic dynamics in the driven superlattice model: Initial state

\[ \hat{H} = \sum_{m} \frac{J}{2} (a_m^\dagger a_{m+1} + h.c.) + \sum_{m} V \cos(2\pi m - \beta) \cos(\Omega t) a_m^\dagger a_m \]

Floquet band structure in a 3-band example

initial state localized at site zero
Adiabatic evolution in terms of wavepacket displacement

Different solid lines are for different pumping-cycle duration

Berry curvature integral only

Our theory

Coherence effect is independent of T, survives in adiabatic limit!
Coherence is important in understanding adiabatic pumping
Detected a topological phase transition using adiabatic pumping (initial state localized at site zero, possessing interband coherence)

\[ \Delta \langle x \rangle \]

Numerical

Our theory

Berry curvature only

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<td>$-8$</td>
<td>$16$</td>
</tr>
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<td>$-2$</td>
<td>$4$</td>
<td>$-8$</td>
</tr>
</tbody>
</table>
General results due to initial state coherence:

\[ B(1) = B^{\text{II}}(1) + B^{\text{III}}(1) \]

\[ = \frac{1}{\Delta k} \int_{-\Delta k/2}^{\Delta k/2} dk \sum_{m,n,m \neq n} 2\text{Re} \left\{ \rho_{mn}(0) \left[ \frac{M_{nm}(s)}{\Delta_{mn}(s)} \right]_{s=0} \right\} \int_0^1 ds A_{nn}(s) \]

\[ + \frac{1}{\Delta k} \int_{-\Delta k/2}^{\Delta k/2} dk \sum_{m,n,m \neq n} \rho_{nn}(0) \int_0^1 ds \frac{A_{nm}(s)M_{mn}(s) + A_{mn}(s)M_{nm}(s)}{\Delta_{mn}(s)} \]

\[ \rho_{mn}(s) \equiv c_m(s)c_n^*(s) \quad \Delta_{mn}(s) \equiv E_m(s) - E_n(s) \quad M_{mn}(s) \equiv i\langle m(s)|\partial_s n(s)\rangle \]
Experimental proposal to detect coherence effects: A qubit in a rotating field

Hamiltonian:
\[ \hat{H}(\theta, \phi) = \sum_{\alpha=x,y,z} h_{\alpha}(\theta, \phi) \hat{\sigma}_\alpha \]

Adiabatic protocol:
\[ \phi = \phi(s) \quad (s = t / \tau) \]

Generalized force:
\[ \hat{A}_{\theta} = \partial_\theta \hat{H}(\phi) = \sum_{\alpha=x,y,z} \partial_\theta h_{\alpha}(\phi) \hat{\sigma}_\alpha \]

In adiabatic limit:
\[ B(1) = \frac{1}{\Delta \theta} \int_{-\Delta \theta/2}^{\Delta \theta/2} d\theta \sum_{m,n,m \neq n} 2 \text{Re} \left\{ \rho_{mn}(0) \left[ \frac{M_{nm}(s)}{\Delta_{mn}(s)} \right] \bigg|_{s=0} \right\} \int_0^1 ds \partial_\theta E_n(\phi) \]
\[ + \frac{i}{\Delta \theta} \int_{-\Delta \theta/2}^{\Delta \theta/2} d\theta \sum_n \rho_{nn}(0) \int_0^1 ds \left[ \langle \partial_s n(\phi) | \partial_\theta n(\phi) \rangle - \langle \partial_\theta n(\phi) | \partial_s n(\phi) \rangle \right] \]

Weighted integral of Berry curvature

Quasimomentum in a synthetic dimension

Pure interband coherence effect
A qubit in a rotating field

Hamiltonians:

\[ H = \omega_1 \sin \theta \left( S_x \cos \phi + S_y \sin \phi \right) + \left( \delta_1 \cos \theta + \delta_2 \right) S_z \]

Generalized force:

\[ A = \partial_\theta H = \omega_1 \cos \theta \left( S_x \cos \phi + S_y \sin \phi \right) - \delta_1 \sin \theta \cdot S_z \]

Different adiabatic protocols:

\[ \phi(s) = 2\pi s, \quad \pi(s + s^2), \quad 2\pi \sin \left( \frac{\pi}{2} s \right), \quad 2\pi s^2 \quad (s = t / \tau) \]

Parameter settings:

\[ \omega_1 = 20\text{MHz}, \quad \delta_1 = 10\text{MHz}, \quad \delta_2 = 0 \sim 20\text{MHz} \]
Different adiabatic protocols

Initial state with interband coherence: $\rho_{+-}(0) = 1/2$

- $2\pi \sin \left( \frac{\pi}{2} s \right)$
- $2\pi s$
- $\pi(s + s^2)$

Signal independent of protocol duration $T$

Pumping sensitively depends on the turn-on rate of the protocol

Signal captures phase transition points
Concluding Remarks

Correction to adiabatic theorem: initial-state coherence between different instantaneous eigenstates can be important.

The found correction naturally emerges in considering adiabatic pumping for nonequilibrium initial states, which often occurs, for example, in considering Floquet topological phases.

The found coherence effect survives well in the presence of dephasing (results not shown)

The found effect can be used to detect topological phase transitions, and its simulation using single-qubit system is ongoing.
Thank you!