Generation of continuous-variable quadripartite cluster states in a circuit QED system

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Outline

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II. Four-step Generation of CV quadripartite cluster states

III. One-step Generation of CV quadripartite cluster states

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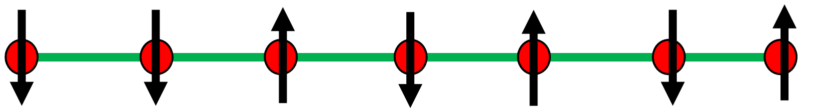
I. Introduction

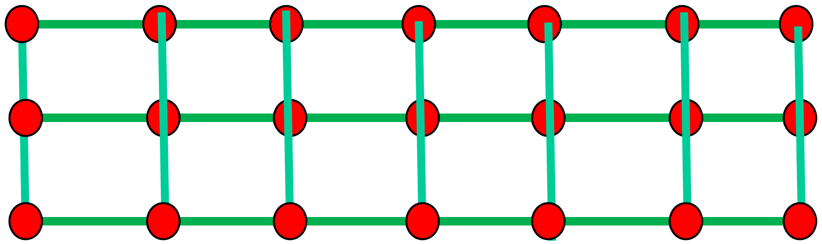
Qubit Cluster

assemble of qubits: spin, ion or atom, a single bosonic mode

d-dimensional lattice:



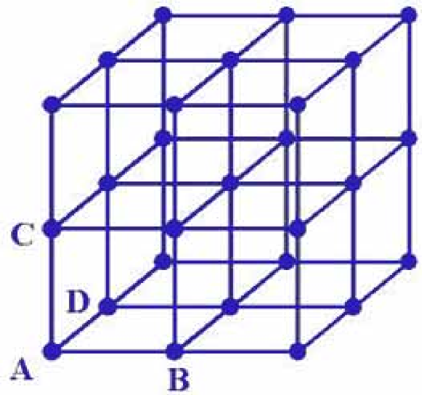
d=1



d=2

short range interaction:

$$H_{\text{int}} = \hbar g(t) \sum_{i,i'} f(i-i') \frac{1-\sigma_z^{(i)}}{2} \frac{1-\sigma_z^{(i')}}{2}$$



空间格子

d=3

For the nearest neighbor interaction

$$f(i-i') = \delta_{i+1,i'}$$

Each of qubits is initially prepared in the state

$$|\varphi_{0i}> = [|0>_i + |1>_i]/\sqrt{2}$$

where $\frac{1-\sigma_z^{(i)}}{2}|0>_i = 0|0>_i, \ \frac{1-\sigma_z^{(i)}}{2}|1>_i = 1|0>_i$

At time t $|\phi_N> = U(\theta)\otimes_{\mathrm{i=1}}^{\mathrm{N}}|\varphi_{0i}>$

$$U(\theta) = \exp(-i\hbar^{-1}\int H_{\mathrm{int}}(t)dt) = \exp(-i\theta\sum_i \frac{1+\sigma_z^{(i)}}{2}\frac{1-\sigma_z^{(i+1)}}{2})$$

Hans J. Briegel and Robert Raussendorf, Phys. Rev. Lett. 86, 5188(2001)

when $\theta = \int g(t)dt = (2n+1)\pi = \pi, 3\pi, 5\pi, \ldots$

$$|\phi_N\rangle = \frac{1}{2^{N/2}} \otimes_{i=1}^{N} \left(|0\rangle_i \, \sigma_z^{(i+1)} + |1\rangle_i\right) \text{ with } \sigma_z^{(N+1)} = 1$$

For example

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_1 |0\rangle_2 + |1\rangle_1 |1\rangle_2\right)$$

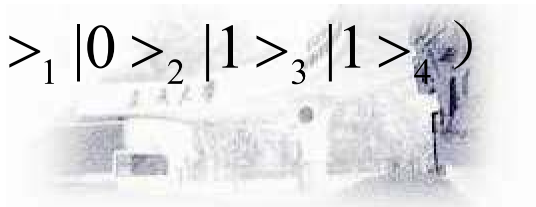
$$|\phi_3\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_1 |0\rangle_2 |0\rangle_3 + |1\rangle_1 |1\rangle_2 |1\rangle_3\right) = |GHZ_3\rangle$$

$$|\phi_4\rangle = \frac{1}{2}\left(|0\rangle_1 |0\rangle_2 |0\rangle_3 |0\rangle_4 + |0\rangle_1 |0\rangle_2 |1\rangle_3 |1\rangle_4 + |1\rangle_1 |1\rangle_2 |0\rangle_3 |0\rangle_4 - |1\rangle_1 |1\rangle_2 |1\rangle_3 |1\rangle_4\right)$$

N>3

not equivalent to

$$|GHZ_4\rangle = \frac{1}{2}\left(|0\rangle_1 |0\rangle_2 |0\rangle_3 |0\rangle_4 + |0\rangle_1 |0\rangle_2 |1\rangle_3 |1\rangle_4\right)$$



Features: (1) any two of the four qubits can be projected into a Bell state by measuring the other two qubits

$$|\phi_4\rangle = \frac{1}{2}|0\rangle_1|0\rangle_2(|0\rangle_3|0\rangle_4 + |1\rangle_3|1\rangle_4) + \frac{1}{2}|1\rangle_1|1\rangle_2(|0\rangle_3|0\rangle_4 - |1\rangle_3|1\rangle_4)$$

(2) it is harder to destroy the entanglement by local operations

$$|\phi_4\rangle = \frac{1}{2}|0\rangle_1(|0\rangle_2|0\rangle_3|0\rangle_4 + |0\rangle_2|1\rangle_3|1\rangle_4) + \frac{1}{2}|1\rangle_1(|1\rangle_2|0\rangle_3|0\rangle_4 - |1\rangle_1|1\rangle_2|1\rangle_3|1\rangle_4)$$

$$|GHZ_4\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4 + |1\rangle_1|1\rangle_2|1\rangle_3|1\rangle_4)$$

Generalize to dimensions d=2,3 cases

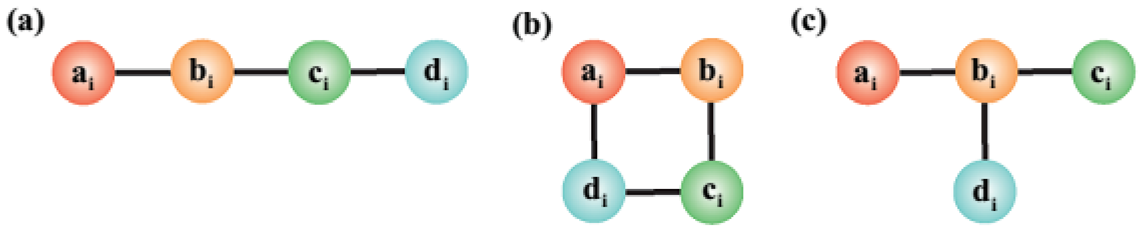
$$|\Phi_C\rangle = \otimes_{c\in C}\left(|0\rangle_c \otimes_{\gamma\in\Gamma}\sigma_z^{(c+\gamma)} + |1\rangle_c\right)$$

$$d=2: \quad \Gamma=\{(1,0),(0,1)\};$$

$$d=3: \quad \Gamma=\{(1,0,0),(0,1,0),(0,0,1)\}$$

$$\sigma_z^{(c+\gamma)} \equiv 1, \; when \; c+\gamma \notin C$$

For d=2 and N=4



Quantum Computation: A unitary transformation

The two-qubit CNOT and arbitrary one-qubit rotations form a universal set for any quantum computation tasks.

All unitary operations on arbitrarily many qubits can be expressed as compositions of these gates.

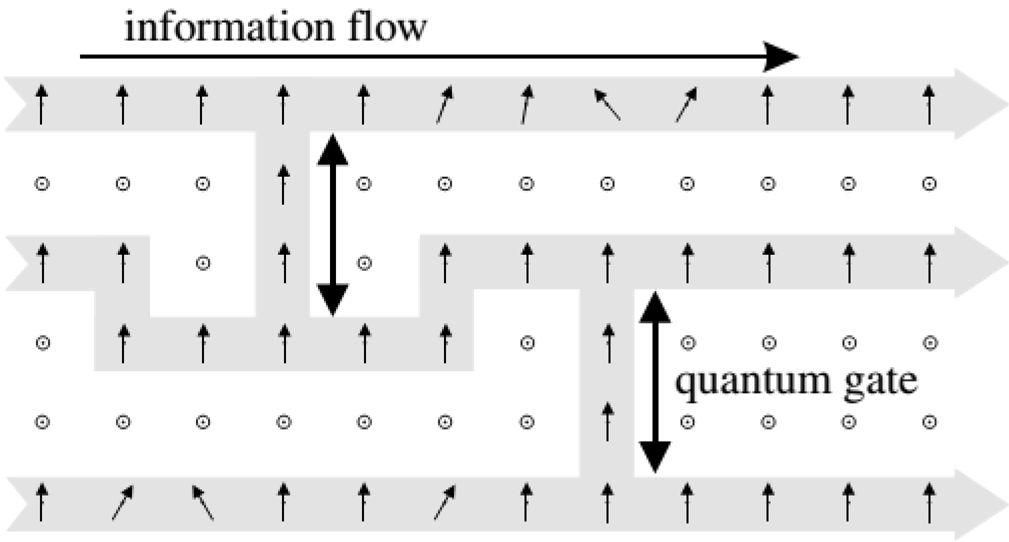
A. Barencoet al., Phys. Rev. A52, 3457 (1995)

A One-Way Quantum Computer

R. Raussendorf and H.-J. Briegel, Phys. Rev. Lett. 86, 5188(2001)

A qubit cluster is initially prepared in a cluster state

$$\sigma_x^{(a)} \bigotimes_{a'\in ngbh(a)} \sigma_z^{(a')} \,|\,\Phi >_C = \pm \,|\,\Phi >_C$$



By only performing one-qubit measurements in a certain order and in a certain basis, the following operations can realized:

(a) Information propagation on a horizontal line $|\varphi_{in}>_1 \rightarrow |\varphi_{in}>_n$

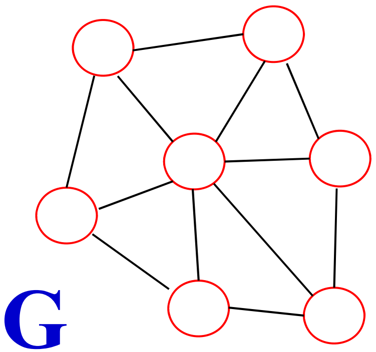
(b) Arbitrary one-qubit rotation $U_R \in SU(2)$

(c) CNOT gate along a vertical line

$$CNOT(c, t_{in} \rightarrow t_{out}) = |0>_{cc}<0| \otimes 1^{(t_{in} \rightarrow t_{out})} + |1>_{cc}<1| \otimes \sigma_x^{(t_{in} \rightarrow t_{out})}$$

In continuous-variable case, one has a fully connected vertex graph G. Each vertex represents a single-mode optical field ($\hat{x}$, $\hat{p}$; $\hat{a}=\hat{x}+i\,\hat{p}$) **. If the correlation conditions**

$$\mathrm{Var.}\left(\hat{p}_a - \sum_{b\in N_a} \hat{x}_b\right) = 0, \qquad a \in G$$



G

the graph G is in a cluster state.

A one-way quantum computation can be implemented on a continuous-variable cluster state.

Shuhong Hao etal., PHYSICAL REVIEW A89, 032311 (2014)
Xiaolong Su, NATURE COMMUNICATIONS3828, (2013)

One-way quantum computation requires:

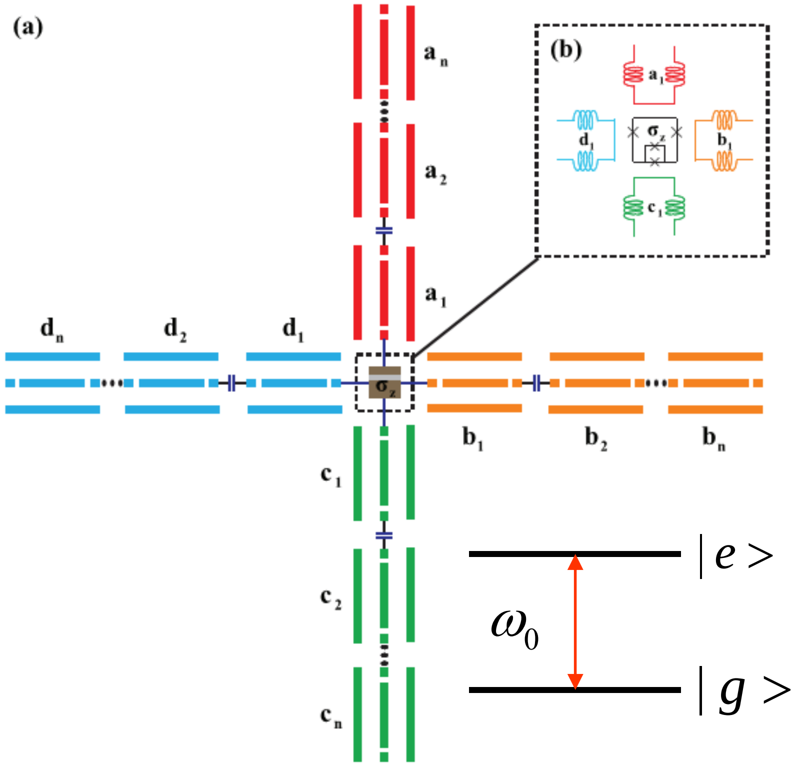
(1) a number of one-qubit measurements
(2) a qubit-cluster consisting of many qubits
one-qubit rotation: five qubits
two-qubit CNOT gate: four qubits

A system under consideration for one-way quantum computation must be scalable.

Question: how to prepare a circuit QED system in a cluster state?

II. Four-step generation of CV quadripartite cluster states

Zhen Li et al., PHYSICAL REVIEW A93, 042305 (2016)



$$H = H_0 + H_1 + H_2 + H_{\text{drive}}$$

$$H_0 = \frac{\omega_0}{2}\sigma_z + \sum_{n=1}^{N}(\omega_a a_n^\dagger a_n + \omega_b b_n^\dagger b_n + \omega_c c_n^\dagger c_n + \omega_d d_n^\dagger d_n)$$

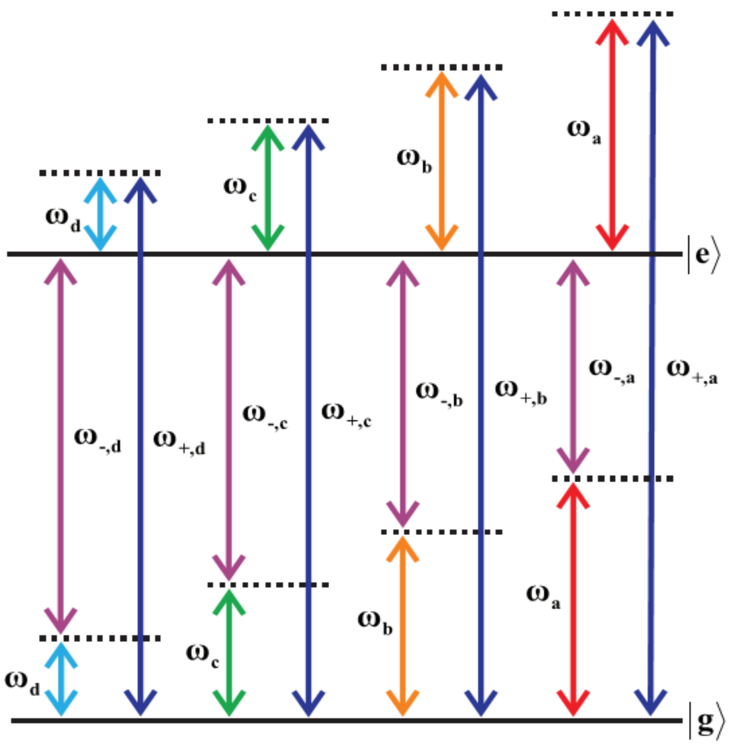
$$H_1 = (\sigma_- + \sigma_+)[g_a(a_1 + a_1^\dagger) + g_b(b_1 + b_1^\dagger) + g_c(c_1 + c_1^\dagger) + g_d(d_1 + d_1^\dagger)]$$

$$\sigma_+ = |e><g|, \quad \sigma_- = |g><e|$$

$$\sigma_z = |e><e| - |g><g|$$

$$H_2 = \sum_{n=1}^{N-1} J(a_n^\dagger a_{n+1} + b_n^\dagger b_{n+1} + c_n^\dagger c_{n+1} + d_n^\dagger d_{n+1} + H.c.)$$

$$H_{driv} = -\sigma_z \sum_{k=a,b,c,d} [\xi_{+,k}\omega_{+,k}\cos(\omega_{+,k}t+\phi_{+,k}) + \xi_{-,k}\omega_{-,k}\cos(\omega_{-,k}t+\phi_{-,k})]$$



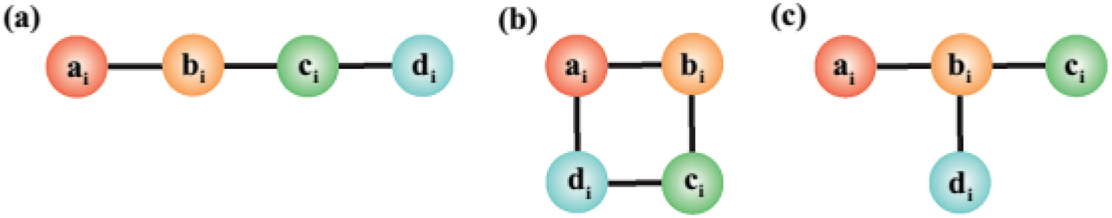
$$\frac{d\rho}{dt} = -i[H,\rho] + \frac{\Gamma}{2}D[\sigma_-]\rho$$

$$D[\sigma_-]\rho = (2\sigma_-\rho\sigma_+ - \rho\sigma_+\sigma_- - \sigma_+\sigma_-\rho)$$

$$a_j = (X_j - iP_j)/\sqrt{2}$$

$$V\left(P_j - \sum_{i\in N_j} X_i\right) \to 0, \quad j = 1,2,3,4,$$

Four possible cluster graphs



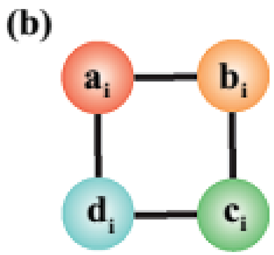
(a) Linear cluster

$$\begin{pmatrix} \bar{A}_n \\ \bar{B}_n \\ \bar{C}_n \\ \bar{D}_n \end{pmatrix} = \begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{i}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & -i\sqrt{\frac{2}{5}} & -\sqrt{\frac{2}{5}} \\ \frac{1}{\sqrt{15}} & \frac{i}{\sqrt{15}} & -\frac{2}{\sqrt{15}} & -i\sqrt{\frac{3}{5}} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \\ c_n \\ d_n \end{pmatrix}$$

$$V\left(P_j - \sum_{i \in N_j} X_i\right) \Longrightarrow$$

$$\begin{aligned} V(P_{a_n} - X_{b_n}) &= V(\bar{A}_n + \bar{A}_n^\dagger), \\ V(P_{b_n} - X_{a_n} - X_{c_n}) &= \tfrac{3}{2} V(\bar{B}_n + \bar{B}_n^\dagger), \\ V(P_{c_n} - X_{b_n} - X_{d_n}) &= \tfrac{1}{4} V(\bar{A}_n + \bar{A}_n^\dagger) + \tfrac{5}{4} V(\bar{C}_n + \bar{C}_n^\dagger) \\ V(P_{d_n} - X_{c_n}) &= \tfrac{1}{6} V(\bar{B}_n + \bar{B}_n^\dagger) + \tfrac{5}{6} V(\bar{D}_n + \bar{D}_n^\dagger) \end{aligned}$$

(b) Square cluster



$$\begin{pmatrix} \bar{A}_n \\ \bar{B}_n \\ \bar{C}_n \\ \bar{D}_n \end{pmatrix} = \begin{pmatrix} \frac{i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{2i}{\sqrt{15}} & \frac{1}{\sqrt{15}} & i\sqrt{\frac{3}{5}} & \frac{1}{\sqrt{15}} \\ \frac{1}{\sqrt{15}} & -\frac{2i}{\sqrt{15}} & \frac{1}{\sqrt{15}} & i\sqrt{\frac{3}{5}} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \\ c_n \\ d_n \end{pmatrix}$$

$$P_{a_n} - X_{b_n} - X_{d_n} = -\frac{\sqrt{3}}{\sqrt{2}}(\bar{A}_n + \bar{A}_n^\dagger),$$

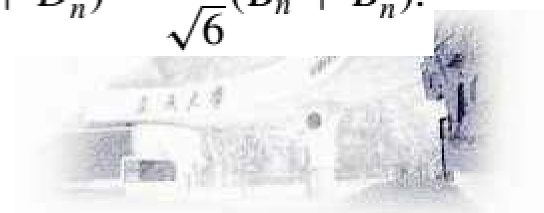
$$P_{b_n} - X_{a_n} - X_{c_n} = -\frac{\sqrt{3}}{\sqrt{2}}(\bar{B}_n + \bar{B}_n^\dagger),$$

$$P_j - \sum_{i \in N_j} X_i \quad \Rightarrow$$

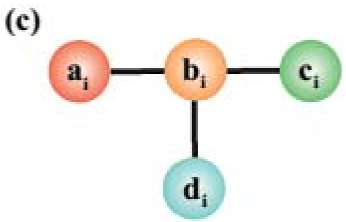
$$P_{c_n} - X_{b_n} - X_{d_n} = -\frac{\sqrt{5}}{\sqrt{6}}(\bar{C}_n + \bar{C}_n^\dagger) - \frac{2}{\sqrt{6}}(\bar{A}_n + \bar{A}_n^\dagger),$$

$$P_{d_n} - X_{a_n} - X_{c_n} = -\frac{\sqrt{5}}{\sqrt{6}}(\bar{D}_n + \bar{D}_n^\dagger) - \frac{2}{\sqrt{6}}(\bar{B}_n + \bar{B}_n^\dagger).$$

$$V(P_j - \sum_{i \in N_j} X_i) \to 0$$



(c) T-type square cluster



$$\begin{pmatrix} \bar{A}_n \\ \bar{B}_n \\ \bar{C}_n \\ \bar{D}_n \end{pmatrix} = \begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{2} & \frac{i}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{i}{\sqrt{6}} & \frac{1}{\sqrt{6}} & i\sqrt{\frac{2}{3}} & 0 \\ -\frac{i}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{i}{2\sqrt{3}} & \frac{i\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \\ c_n \\ d_n \end{pmatrix}$$

$$P_j - \sum_{i \in N_j} X_i \Rightarrow$$

$$P_{a_n} - X_{b_n} = -(\bar{A}_n + \bar{A}_n^{\dagger}),$$

$$P_{b_n} - X_{a_n} - X_{c_n} - X_{d_n} = -\sqrt{2}(\bar{B}_n + \bar{B}_n^{\dagger}),$$

$$P_{c_n} - X_{b_n} = -\frac{1}{2}(\bar{A}_n + \bar{A}_n^{\dagger}) - \frac{\sqrt{3}}{2}(\bar{C}_n + \bar{C}_n^{\dagger}),$$

$$P_{d_n} - X_{b_n} = -\frac{\sqrt{3}}{\sqrt{12}}(\bar{A}_n + \bar{A}_n^{\dagger}) - \frac{1}{\sqrt{12}}(\bar{C}_n + \bar{C}_n^{\dagger}) - \frac{2\sqrt{2}}{\sqrt{12}}(\bar{D}_n + \bar{D}_n^{\dagger})$$

$$V(P_j - \sum_{i \in N_j} X_i) \to 0$$

The problem is equivalent how to prepare the four independent combined modes into single-mode amplitude squeezed states.

Engineer the effective Hamiltonian for generation of the single-mode squeezed states.

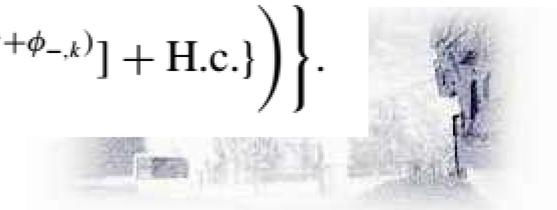
Under the unitary transformations

$$U_1 = \exp(-iH_0 t) \;\; and \;\; U_2 = T\exp[-i\int dt' H_{drive}(t')]$$

H_1

$$\begin{aligned} H = \Big\{ & \sigma_+ e^{i\omega_0 t}[g_a(a_1 e^{-i\omega_a t} + a_1^\dagger e^{i\omega_a t}) + g_b(b_1 e^{-i\omega_b t} + b_1^\dagger e^{i\omega_b t}) \\ & + g_c(c_1 e^{-i\omega_c t} + c_1^\dagger e^{i\omega_c t}) + g_d(d_1 e^{-i\omega_d t} + d_1^\dagger e^{i\omega_d t})] \\ & \times \Big(1 - \sum_j \{\xi_{+,k}[e^{i(\omega_{+,k}t+\phi_{+,k})} - e^{-i(\omega_{+,k}t+\phi_{+,k})}] \\ & + \xi_{-,k}[e^{i(\omega_{-,k}t+\phi_{-,k})} - e^{-i(\omega_{-,k}t+\phi_{-,k})}] + \text{H.c.}\}\Big)\Big\}. \end{aligned}$$

keep the terms up to the first order of the parameter $\xi_{\pm,k}$

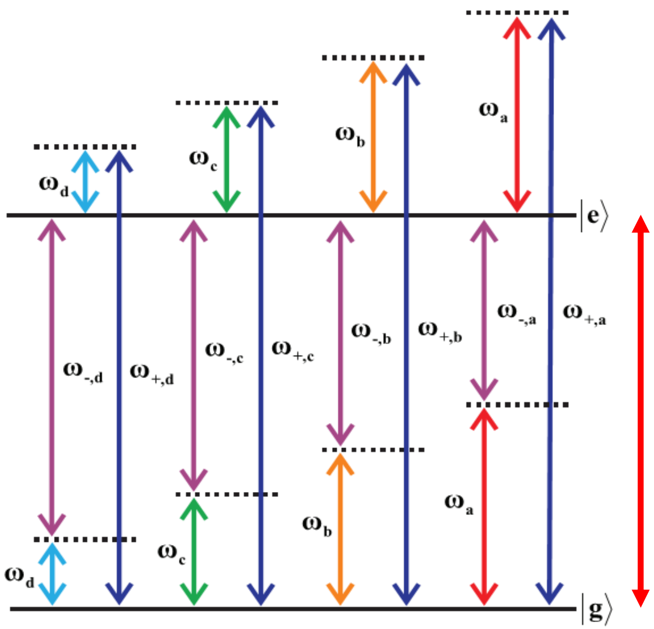


In the rotating wave approximation with the condition

$$\{\omega_k, \omega_0, \omega_{\pm,k}, \omega_k - \omega_{k'}\} >> g_j$$

and choosing $\omega_{+,k} = \omega_0 + \omega_k$ *and* $\omega_{-,k} = \omega_0 - \omega_k$

one can select out the resonant interaction



ω_0

$$\begin{aligned}
H^{\text{eff}} = \{&\sigma_+[(G_{+,a}e^{-i\phi_{+,a}}a_1^\dagger + G_{-,a}e^{-i\phi_{-,a}}a_1) \\
&+(G_{+,b}e^{-i\phi_{+,b}}b_1^\dagger + G_{-,b}e^{-i\phi_{-,b}}b_1) \\
&+(G_{+,c}e^{-i\phi_{+,c}}c_1^\dagger + G_{-,c}e^{-i\phi_{-,c}}c_1) \\
&+(G_{+,d}e^{-i\phi_{+,d}}d_1^\dagger + G_{-,d}e^{-i\phi_{-,d}}d_1)] + \text{H.c.}\},
\end{aligned}$$

with $G_{+,k} = \xi_{+,k} g_k,\ G_{-,k} = \xi_{-,k} g_k$

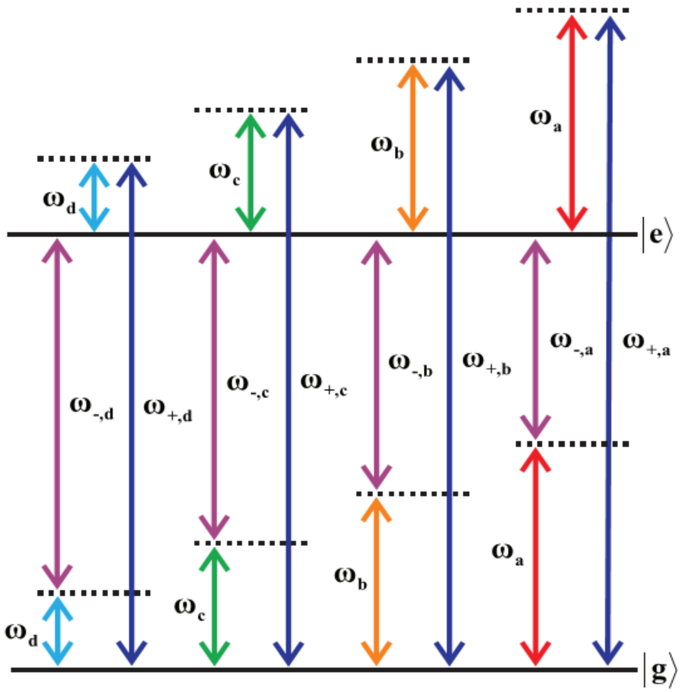
(1) The first step

choosing

$$\xi_{\pm,a}=\xi_{\pm,b}=\xi_{\pm},$$

$$\xi_{+,c}=\xi_{-,c}=\xi_{+,d}=\xi_{-,d}=0$$

$$\phi_{+,a}=\frac{3}{2}\pi,\quad \phi_{-,a}=\frac{1}{2}\pi,\quad \phi_{+,b}=\phi_{-,b}=\pi.$$



$$\bar{H}^{\mathrm{eff}}_{1,1\mathrm{st}}=\sqrt{2(G_-^2-G_+^2)}\sigma_+\underline{[\bar{A}_1\cosh r+\bar{A}_1^\dagger\sinh r]}+\mathrm{H.c.},$$

$$with\ r=\tanh^{-1}(G_+/G_-)$$

$$\bar{H}_2=\sum_{n=1}^{N-1}J(\bar{A}_n^\dagger\bar{A}_{n+1}+\bar{B}_n^\dagger\bar{B}_{n+1}+\bar{C}_n^\dagger\bar{C}_{n+1}+\bar{D}_n^\dagger\bar{D}_{n+1}+\mathrm{H.c.}).$$

$$\frac{d}{dt}\bar{\rho}=-i\left[\bar{H}^{\mathrm{eff}}_{1,1\mathrm{st}}+\bar{H}_2,\bar{\rho}\right]+\frac{\Gamma}{2}D[\sigma_-]\bar{\rho}.$$

squeezing representation by

$$S(\bar{A}_n)=\exp[(-)^n \frac{r}{2}(\bar{A}_n^{+2}-\bar{A}_n^2)]$$

$$U=\otimes_{n=1}^{N} S(\bar{A}_n)$$

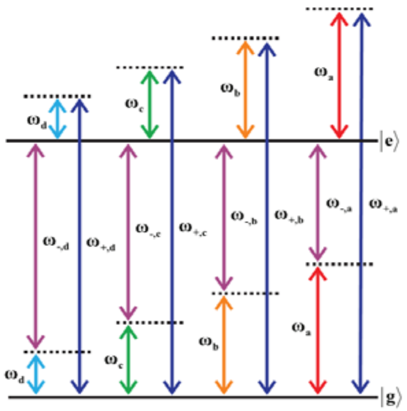
$$\frac{d}{dt}\bar{\rho}' = -i[\sqrt{2(G_-^2 - G_+^2)}(\sigma_+\bar{A}_1 + \sigma_-\bar{A}_1^\dagger) + \bar{H}_2, \bar{\rho}'] + \kappa D[\sigma_-]\bar{\rho}'.$$

$$\bar{H}_2 = \sum_{n=1}^{N-1} J(\bar{A}_n^\dagger \bar{A}_{n+1} + \bar{B}_n^\dagger \bar{B}_{n+1} + \bar{C}_n^\dagger \bar{C}_{n+1} + \bar{D}_n^\dagger \bar{D}_{n+1} + \text{H.c.}).$$

steady state:

$$\bar{\rho} = |g\rangle\langle g| \bigotimes_{n=1}^{N} S(\bar{A}_n)|0_{(\bar{A}_n)}\rangle\langle 0_{(\bar{A}_n)}|S^\dagger(\bar{A}_n) \otimes \rho_{\bar{B}_n\bar{C}_n\bar{D}_n}.$$

(2) The second step

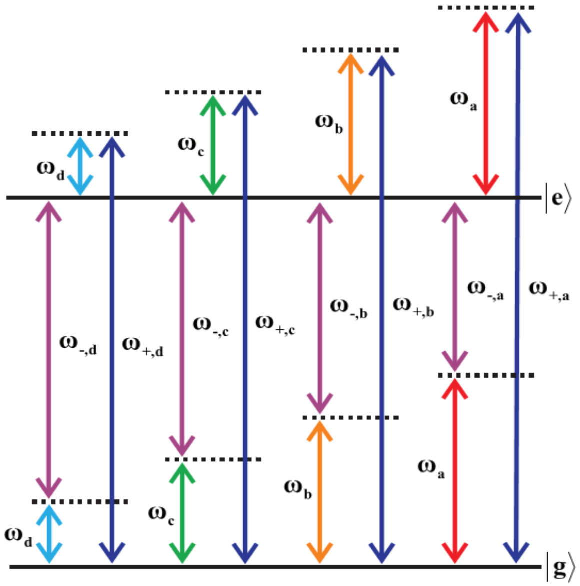


$$\xi_{\pm,a} = \xi_{\pm,b} = \xi_{\pm,c} = \xi_\pm, \quad \xi_{\pm,d} = 0$$

$$\phi_{\pm,a} = \phi_{\pm,c} = \pi, \quad \phi_{+,b} = \tfrac{3}{2}\pi, \quad \phi_{-,b} = \tfrac{1}{2}\pi, \quad \phi_{\pm,d} = 0.$$

$$\bar{H}_{1,2\text{nd}} = \sqrt{3(G_-^2 - G_+^2)}\sigma_+[\bar{B}_1 \cosh r + \bar{B}_1^\dagger \sinh r] + \text{H.c.}$$

(3) The third step



$$\xi_{\pm,a} = \quad \xi_{\pm,b} = \xi_{\pm}, \qquad \phi_{-,a} = \phi_{+c} = \tfrac{3}{2}\pi, \quad \phi_{+,a} = \phi_{-,c} = \tfrac{1}{2}\pi,$$

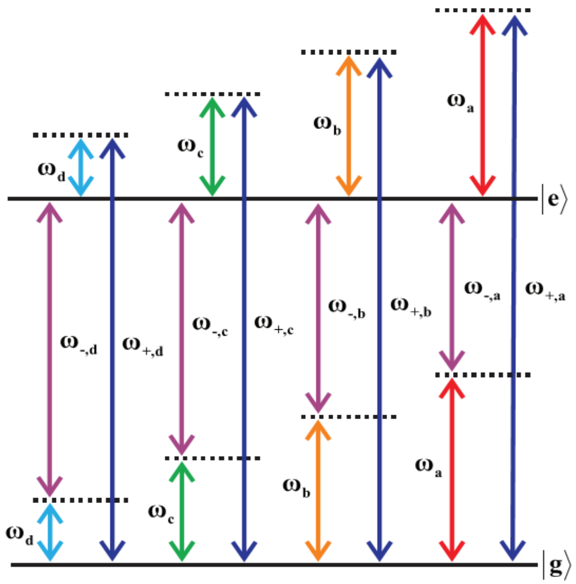
$$\xi_{\pm,c} = \quad \xi_{\pm,d} = 2\xi_{\pm} \qquad \phi_{\pm,b} = \phi_{\pm,d} = \pi.$$

$$\bar{H}_{1,3\mathrm{rd}} = \sqrt{10(G_{-}^{2} - G_{+}^{2})}\sigma_{+}[\underline{\bar{C}_{1}\cosh r + \bar{C}_{1}^{\dagger}\sinh r}] + \mathrm{H.c.}$$

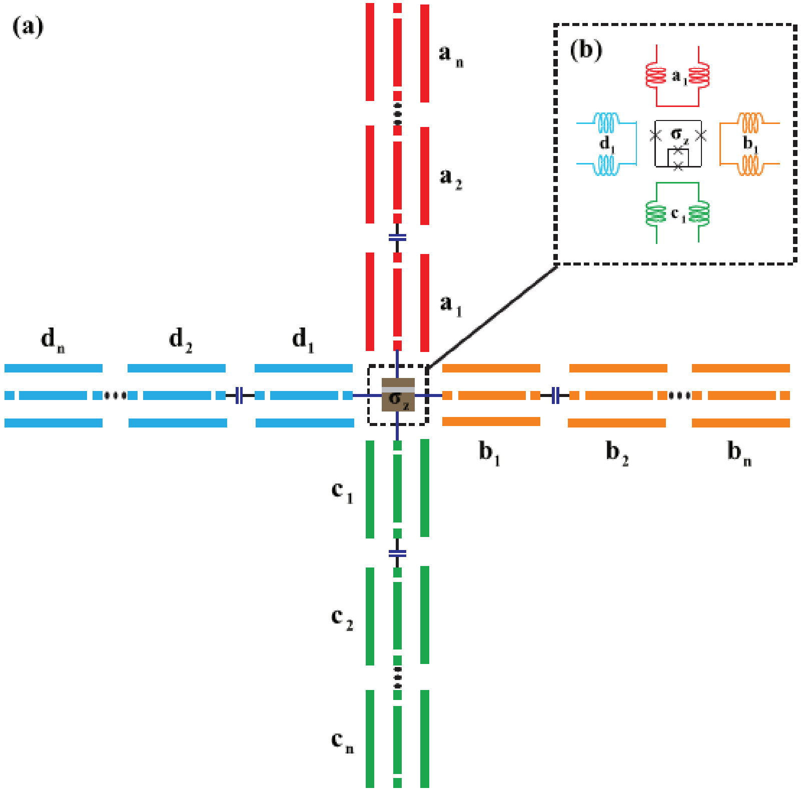
(3) The fourth step

$$\xi_{\pm,a} = \xi_{\pm,b} = \xi_{\pm},$$
$$\xi_{\pm,c} = 2\xi_{\pm}, \quad \xi_{\pm,d} = 3\xi_{\pm}$$

$$\phi_{\pm,a} = 0,$$
$$\phi_{+,b} = \phi_{-,d} = \tfrac{1}{2}\pi, \quad \phi_{-,b} = \phi_{+,d} = \tfrac{3}{2}\pi,$$
$$\phi_{+,c} = \phi_{-,c} = \pi.$$



$$\bar{H}_{1,4\text{th}} = \sqrt{15(G_-^2 - G_+^2)}\sigma_+[\bar{D}_1 \cosh r + \bar{D}_1^\dagger \sinh r] + \text{H.c.}$$



$$\bar{H}_2 = \sum_{n=1}^{N-1} J(\bar{A}_n^\dagger \bar{A}_{n+1} + \bar{B}_n^\dagger \bar{B}_{n+1} + \bar{C}_n^\dagger \bar{C}_{n+1} + \bar{D}_n^\dagger \bar{D}_{n+1} + \mathrm{H.c.}).$$

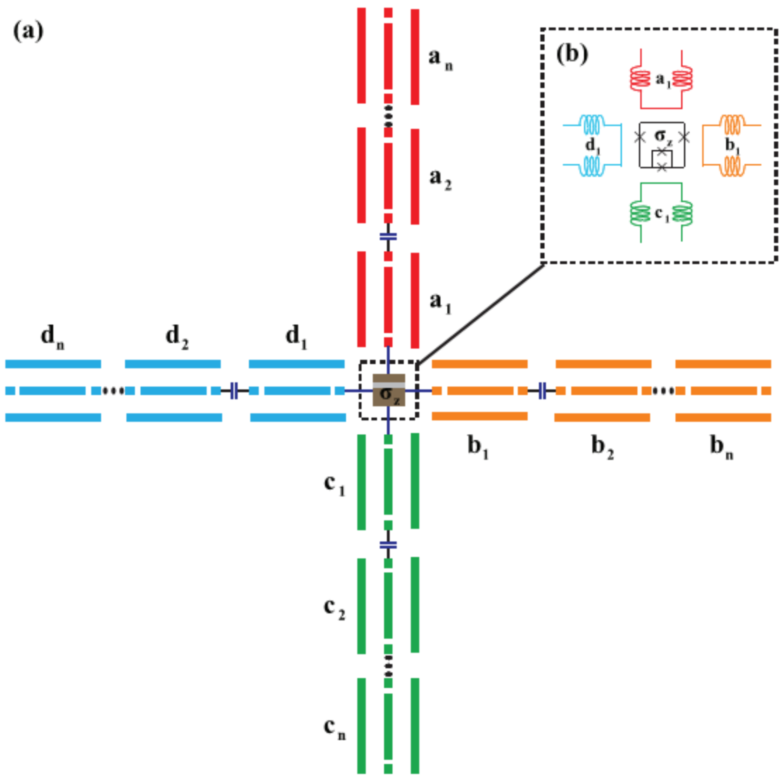
$$Vx1 = V\left(P_{a_n} - X_{b_n}\right) = e^{(-1)^n 2r},$$
$$Vx2 = V\left(P_{b_n} - X_{a_n} - X_{c_n}\right) = \tfrac{3}{2} e^{(-1)^n 2r},$$
$$Vx3 = V\left(P_{c_n} - X_{b_n} - X_{d_n}\right) = \tfrac{3}{2} e^{(-1)^n 2r},$$
$$Vx4 = V\left(P_{d_n} - X_{c_n}\right) = e^{(-1)^n 2r}.$$

$$n = 1, 3, 5, 7, \ldots$$

$$Vx1,\ Vx2,\ Vx3,\ Vx4 \rightarrow 0$$

For resonators with $n = 2, 4, 6, 8, \ldots$

Consider the local canonical transformation: $P_n \to X_n, \; X_n \to -P_n$

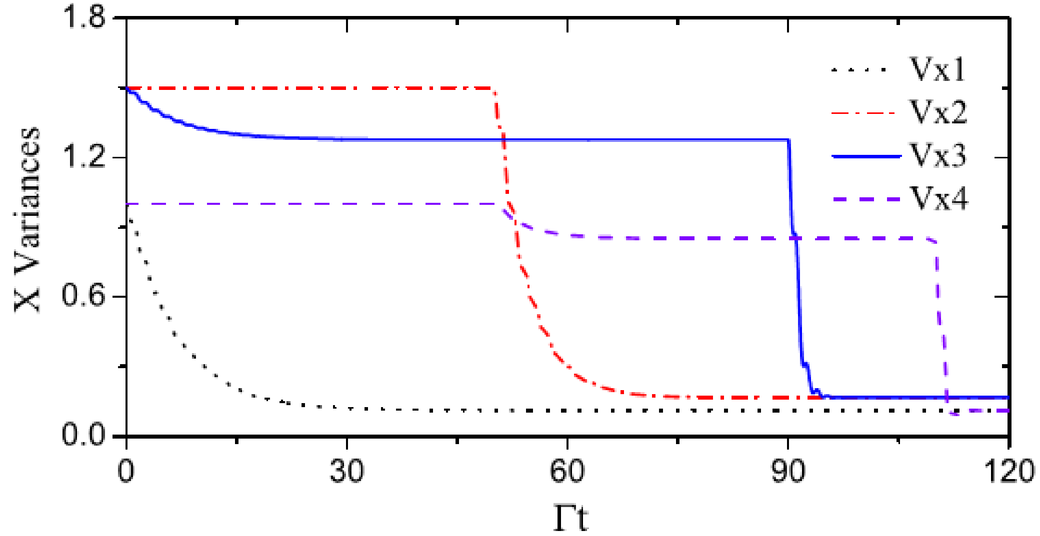


$$
\begin{aligned}
X_{a_n} + P_{b_n} &= i(\bar{A}_n - \bar{A}_n^\dagger), \\
X_{b_n} + P_{a_n} + P_{c_n} &= \sqrt{\tfrac{3}{2}}\, i(\bar{B}_n - \bar{B}_n^\dagger), \\
X_{c_n} + P_{b_n} + P_{d_n} &= \frac{1}{2} i(\bar{A}_n - \bar{A}_n^\dagger) + \frac{\sqrt{5}}{2} i(\bar{C}_n - \bar{C}_n^\dagger), \\
X_{d_n} + P_{c_n} &= \sqrt{\tfrac{1}{6}}\, i(\bar{B}_n - \bar{B}_n^\dagger) + \sqrt{\tfrac{5}{6}}\, i(\bar{D}_n - \bar{D}_n^\dagger).
\end{aligned}
$$

$$
\begin{aligned}
Vp1 &= V\left(X_{a_n} + P_{b_n}\right) = e^{-2r}, \\
Vp2 &= V\left(X_{b_n} + P_{a_n} + P_{c_n}\right) = \tfrac{3}{2} e^{-2r}, \\
Vp3 &= V\left(X_{c_n} + P_{b_n} + P_{d_n}\right) = \tfrac{3}{2} e^{-2r}, \\
Vp4 &= V\left(X_{d_n} + P_{c_n}\right) = e^{-2r}.
\end{aligned}
$$

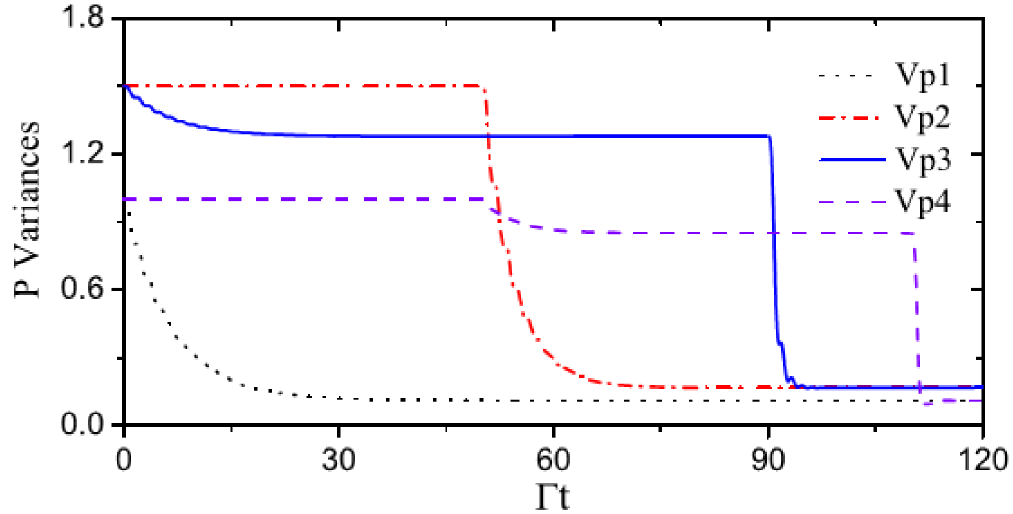
$$\frac{d}{dt}\bar{\rho} = -i\left[\bar{H}^{\mathrm{eff}}_{1,1\mathrm{st}} + \bar{H}_2, \bar{\rho}\right] + \frac{\Gamma}{2}D[\sigma_-]\bar{\rho}.$$

$$|\psi_i> = |g> \otimes |0>$$

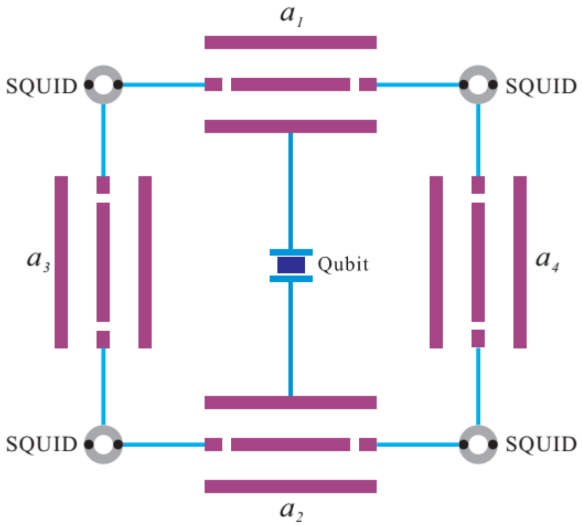


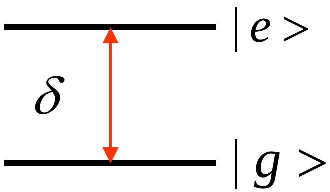
$$g/2\pi = 0.025GHz,\ \Gamma/2\pi = 0.02GHz$$

$$J/2\pi = 0.01GHz,\ \xi_- = 0.2,\ \xi_+ = 0.16$$



III. One-step generation of CV quadripartite cluster states





$$\sigma_+ = |e><g|, \quad \sigma_- = |g><e|$$

$$\sigma_z = |e><e| - |g><g|$$

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$$H_0 = \frac{\delta}{2}\sigma_z + \sum_{j=1}^{4}\omega_j a_j^\dagger a_j$$

$$H_{RR} = \sum_{m=1}^{2}\sum_{n=3}^{4}\alpha_{mn}(t)(a_m^\dagger + a_m)(a_n^\dagger + a_n)$$

$$\boxed{\alpha_{mn}(t) = g_{mn}\cos(\omega_{dmn}t + \phi_{mn})}$$

$$H_{QR} = \sum_{j=1}^{2} g(\sigma_+ + \sigma_-)(a_j^\dagger + a_j)$$

$$H_{driv} = -\sigma_z \sum_{l=1}^{2}[\xi_{l1}\omega_{dl1}\cos(\omega_{dl1}t + \phi_{l1}) + \xi_{l2}\omega_{dl2}\cos(\omega_{dl2}t + \phi_{l2})]$$

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \Gamma D[\sigma_-]\rho$$

$$H = H_0 + H_{RR} + H_{QR} + H_{drive}(t)$$

$$D[\sigma_-]\rho = 2\sigma_-\rho\sigma_+ - \rho\sigma_+\sigma_- - \sigma_+\sigma_-\rho$$

Under the unitary transformation

$$U_1 = \exp(-iH't) \text{ with } H' = \frac{\delta}{2}\sigma_z + \sum_{j=1}^{4}\omega_{0j}a_j^\dagger a_j$$

$$H'_0 = 2\Omega(-a_1^\dagger a_1 + a_2^\dagger a_2 - a_3^\dagger a_3 + a_4^\dagger a_4),$$

$$H'_{QR} = \sum_{j=1}^{2} g_j e^{i\delta t}\sigma_+(a_j^\dagger e^{-i\omega_{0j}t} + a_j e^{i\omega_{0j}t}) + H.c.,$$

$$H'_{RR} = \sum_{m=1}^{2}\sum_{n=3}^{4} g_{mn}\cos(\omega_{dmn}t + \phi_{mn}) \times$$

$$(a_m^\dagger e^{-i\omega_{0m}t} + a_m e^{i\omega_{0m}t})(a_n^\dagger e^{-i\omega_{0n}t} + a_n e^{i\omega_{0n}t})$$

$$\omega_{01} - \omega_1 = \omega_{03} - \omega_3 = 2\Omega$$

$$\omega_{02} - \omega_2 = \omega_{04} - \omega_4 = -2\Omega$$

Under the unitary transformation: $U_2 = T\exp[-i\int dt' H_{drive}(t')]$

$$H'_{QR} = \sum_{j=1}^{2} g_j e^{i\delta t}\sigma_+ (a_j^\dagger e^{-i\omega_{0i}t} + a_j e^{i\omega_{0i}t}) \times$$
$$[1 - \sum_{k=1}^{2}\sum_{l=1}^{2}\xi_{kl}(e^{i(\omega_{dkl}t+\phi_{kl})} - e^{-i(\omega_{dkl}t+\phi_{kl})})] + H.c.$$

choosing

$$\omega_{d11} = \delta - \omega_{01} = \delta - \omega_1 - 2\Omega,$$
$$\omega_{d12} = \delta + \omega_{01} = \delta + \omega_1 + 2\Omega,$$
$$\omega_{d21} = \delta - \omega_{02} = \delta - \omega_2 + 2\Omega,$$
$$\omega_{d22} = \delta + \omega_{02} = \delta + \omega_2 - 2\Omega,$$

$$\phi_{11} = -\frac{\pi}{2}, \phi_{12} = \frac{\pi}{2}, \phi_{21} = \phi_{22} = \pi$$

In the rotating wave approximation with the condition

$$\{\omega_j, \delta, \omega_{d1l}, \omega_{d1l}\} >> g_j,\ \Omega$$

one can select out the resonant interaction

$$H''_{QR} = \sigma_+ (iG_+ a_1^\dagger - iG_- a_1 - G_+ a_2^\dagger - G_- a_2) + H.c.$$

$$G_+ = g\xi_{11} = g\xi_{21}, \ \ and \ G_- = g\xi_{12} = g\xi_{22}$$

choosing

$$\begin{aligned}
\omega_{d13} &= \omega_{01} - \omega_{03} = \omega_1 - \omega_3, \\
\omega_{d23} &= \omega_{02} - \omega_{03} = \omega_2 - \omega_3 - 4\Omega, \\
\omega_{d14} &= \omega_{01} - \omega_{04} = \omega_1 - \omega_4 + 4\Omega, \\
\omega_{d24} &= \omega_{02} - \omega_{04} = \omega_2 - \omega_4,
\end{aligned}
\qquad
\phi_{13} = \pi, \phi_{23} = -\frac{\pi}{2}, \phi_{14} = \frac{\pi}{2}, \phi_{24} = 0,$$

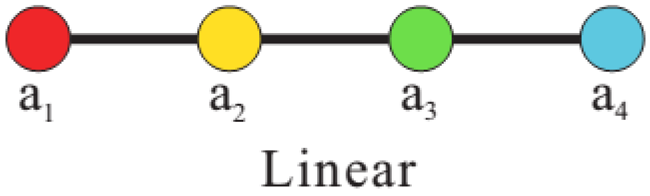
and $$g_{13} = g_{14} = g_{24} = \Omega, \ \ g_{23} = 3\Omega$$

one can select out the resonant interaction

$$H''_{RR} = -\frac{\Omega}{2} a_1^\dagger a_3 - i\frac{3\Omega}{2} a_2^\dagger a_3 + i\frac{\Omega}{2} a_1^\dagger a_4 + \frac{\Omega}{2} a_2^\dagger a_4 + H.c.,$$

$$
\begin{aligned}
H_{eff} &= H_0' + H_{QR}'' + H_{RR}'' \\
&= 2\Omega(-a_1^\dagger a_1 + a_2^\dagger a_2 - a_3^\dagger a_3 + a_4^\dagger a_4) \\
&\quad +[-\frac{\Omega}{2} a_1^\dagger a_3 - i\frac{3\Omega}{2} a_2^\dagger a_3 + i\frac{\Omega}{2} a_1^\dagger a_4 + \frac{\Omega}{2} a_2^\dagger a_4 \\
&\quad +\sigma_+(iG_+ a_1^\dagger - iG_- a_1 - G_+ a_2^\dagger - G_- a_2) + H.c.].
\end{aligned}
$$

Consider a linear cluster



$$
\begin{pmatrix} \bar{A}_1 \\ \bar{A}_2 \\ \bar{A}_3 \\ \bar{A}_4 \end{pmatrix} = \begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{i}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & -\frac{2i}{\sqrt{10}} & -\frac{2}{\sqrt{10}} \\ \frac{1}{\sqrt{15}} & \frac{i}{\sqrt{15}} & -\frac{2}{\sqrt{15}} & -\frac{3i}{\sqrt{15}} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}
$$

$$H_{eff}=\sigma^{\dagger}(G_{-}\bar{A}_1+G_{+}\bar{A}_1^{\dagger})-i\sqrt{6}\Omega\bar{A}_1^{\dagger}\bar{A}_2+i\sqrt{\frac{5}{6}}\Omega\bar{A}_2^{\dagger}\bar{A}_3-i\frac{5}{\sqrt{6}}\Omega\bar{A}_3^{\dagger}\bar{A}_4+H.c..$$

$$\frac{d\tilde{\rho}}{dt}=-i\left[H_{eff},\tilde{\rho}\right]+\Gamma D[\sigma_{-}]\tilde{\rho},$$

Steady state:

$$|\psi_S\rangle=\otimes_{j=1}^{4}\exp[-\frac{r}{2}(\bar{A}_j^{\dagger^2}-\bar{A}_j^2)]\left|0_{\bar{A}_j}\right\rangle\otimes|g\rangle$$

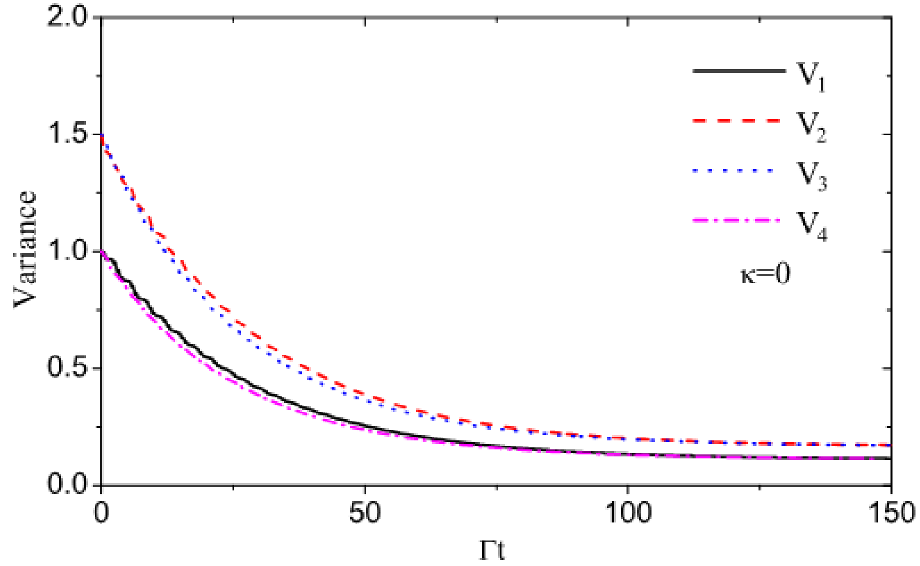
$$r=\tanh^{-1}(G_{+}/G_{-})$$

$$V\left(P_j-\sum_{i\in N_j}X_i\right)\Longrightarrow$$

$$\begin{aligned}
V_1&=V(\bar{A}_1+\bar{A}_1^{\dagger})=e^{-2r},\\
V_2&=\frac{3}{2}V(\bar{A}_2+\bar{A}_2^{\dagger})=\frac{3}{2}e^{-2r},\\
V_3&=\frac{1}{4}V(\bar{A}_1+\bar{A}_1^{\dagger})+\frac{5}{4}V(\bar{A}_3+\bar{A}_3^{\dagger})=\frac{3}{2}e^{-2r},\\
V_4&=\frac{1}{6}V(\bar{A}_2+\bar{A}_2^{\dagger})+\frac{5}{6}V(\bar{A}_4+\bar{A}_4^{\dagger})=e^{-2r}.
\end{aligned}$$

$$\frac{d\tilde{\rho}}{dt} = -i\left[H_{eff}, \tilde{\rho}\right] + \Gamma D[\sigma_-]\tilde{\rho},$$

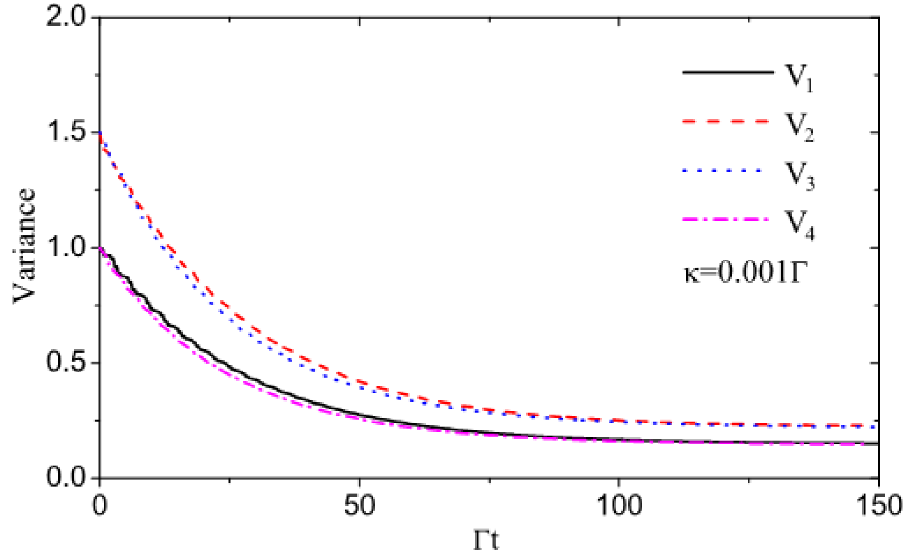
$$|\psi_i\rangle = |g\rangle \otimes |0\rangle$$



$$g/2\pi = 50MHz,\ \ G_-/2\pi = 10MHz$$

$$G_+/2\pi = 8MHz,\ \ \Gamma/2\pi = 10MHz$$

$$\Omega/2\pi = 5MHz$$



IV. Summary

Circuit QED system: four resonators and a single gap tunable superconducting qubit.

(1) Nearest neighbor interaction between two resonators with constant coupling strengths; energy levels of the qubit are periodically modulated. By properly choosing modulation frequencies and phases, CV cluster states of the resonators can be generated via four steps by the decay of the qubit.

(2) The interaction between two neighbor resonators and energy levels of the qubit are periodically modulated. By properly choosing modulation frequencies and phases, we find one-step process for generation of CV quadripartite cluster states of the resonators via the decay of the qubit.

THANK YOU !