Device-independent quantification of Einstein-Podolsky-Rosen steerability

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Workshop on Quantum Nonlocality, Causal Structures,
and Device independent Quantum Information
10th~14th Dec. 2015
National Cheng Kung University, Taiwan

collaborators:
Costantino Budroni, Yeong-Cherng Liang and Yueh-Nan Chen

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Outline

Steering and local hidden state model

Hierarchy relation between steering, entanglement, and Bell nonlocality

Motivation

Characterizing quantum correlations and the moment matrix

The moment matrix of single party state

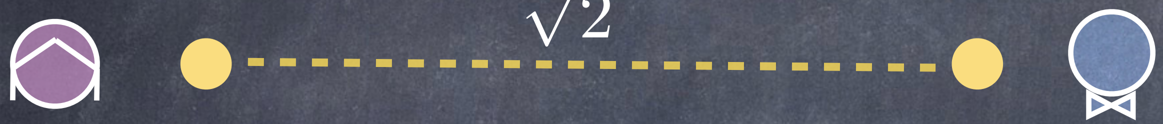
Device-independent quantification of steerability

Summary and outlook

Steering and local hidden state model

A simple demonstration of steering

$$\rho_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



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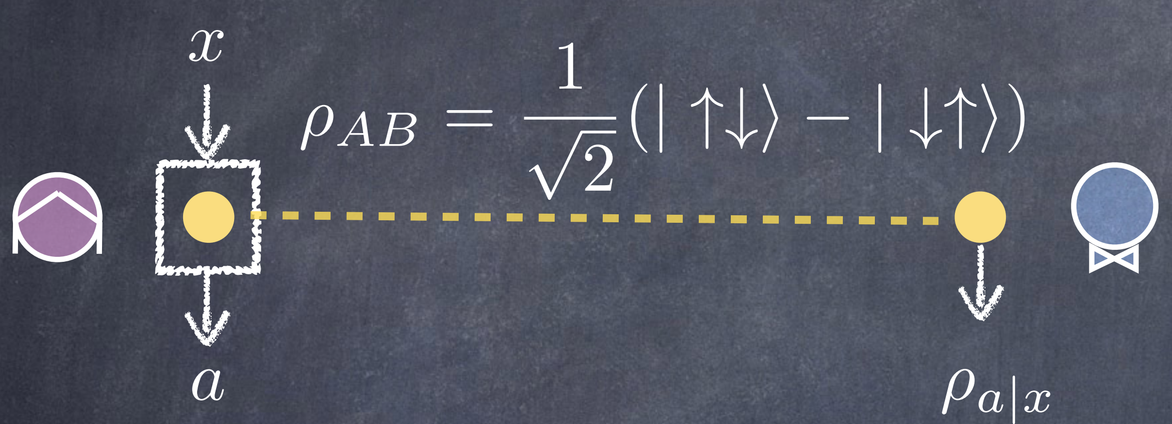

$$\rho_{a|x} = \mathrm{tr}_A[M_{a|x} \otimes \mathbb{I}\ \rho_{AB}]$$

(subnormalized state)

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$\{|0\rangle, |1\rangle\}$ eigenstates of σ_z

$\{|+\rangle, |-\rangle\}$ eigenstates of σ_x

x

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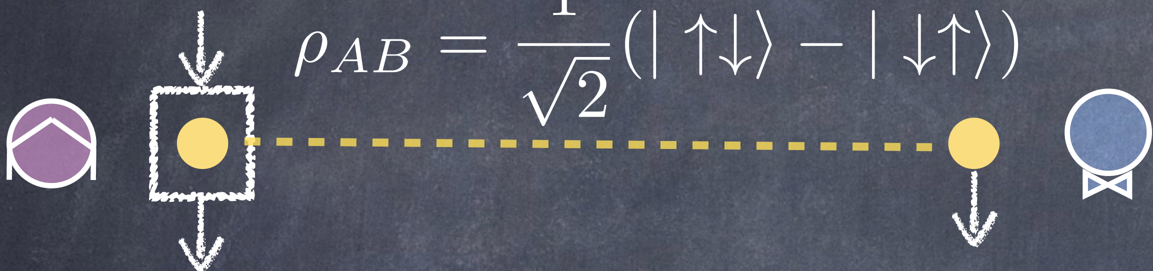

a $\rho_{a|x}$

Alice's measurement	$\hat{\rho}_{a\|x}$
σ_z	$\{\|0\rangle, \|1\rangle\}$
σ_x	$\{\|+\rangle, \|-\rangle\}$

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$\longrightarrow$ "steering"
— by Schrödinger

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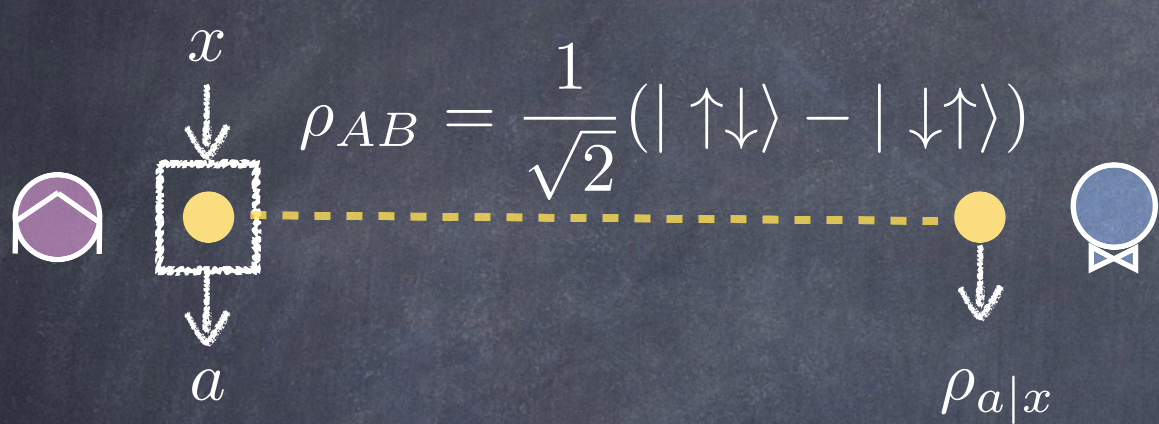

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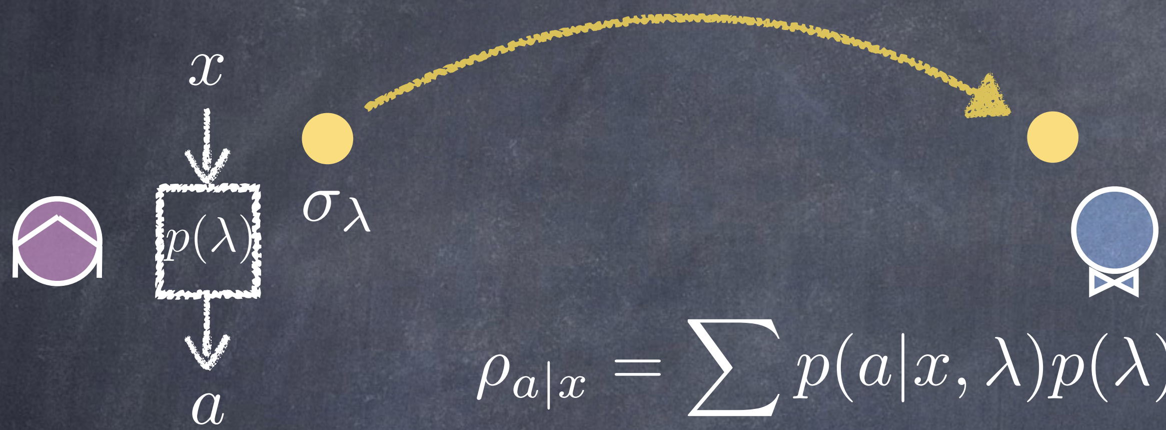

$$\rho_{a|x} = \sum_\lambda p(a|x,\lambda)p(\lambda)\sigma_\lambda$$

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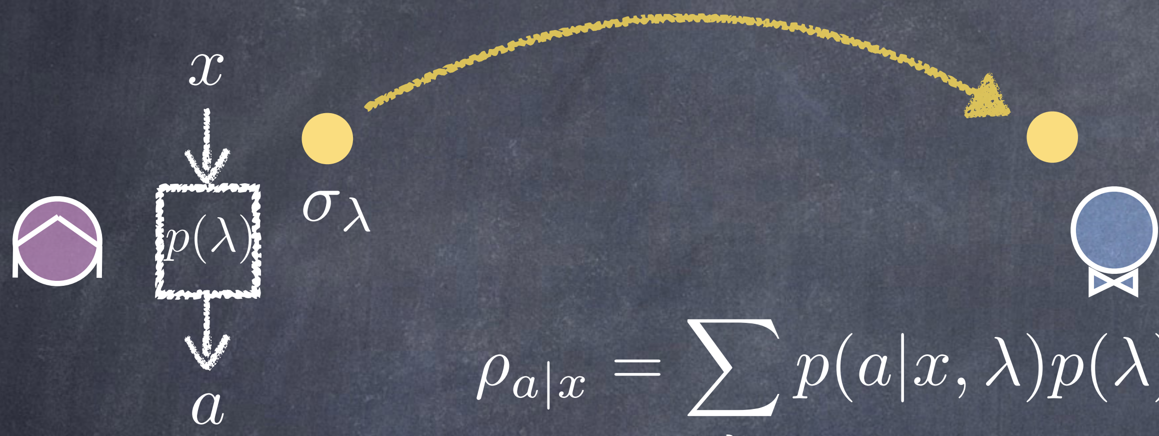

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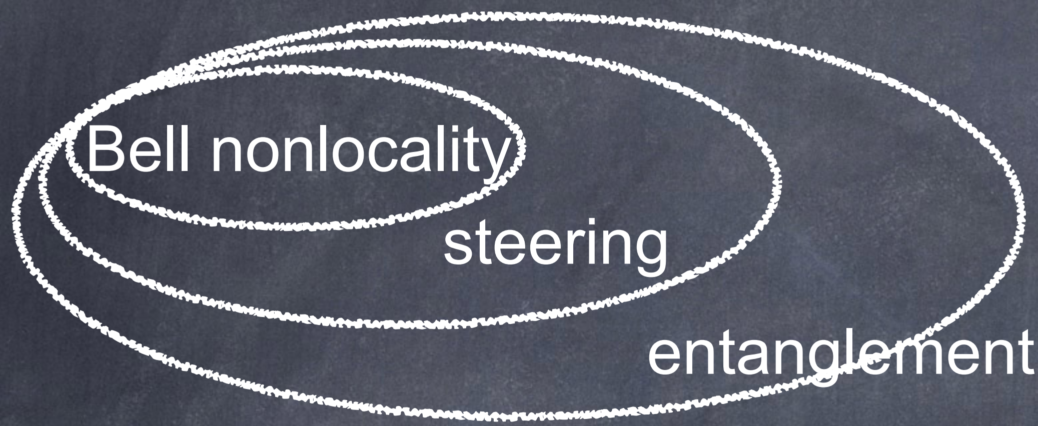
$\longleftrightarrow$ non-steerability from Alice to Bob

Hierarchy relation between steering, entanglement, and Bell nonlocality



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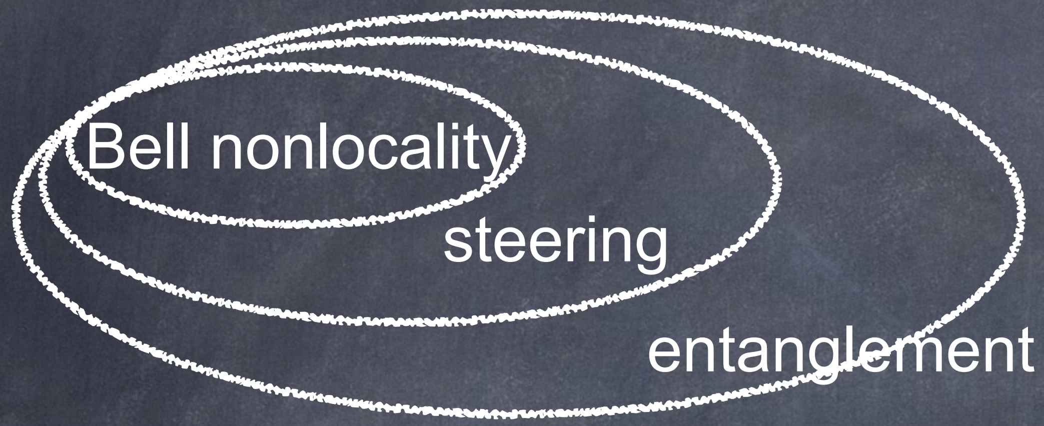

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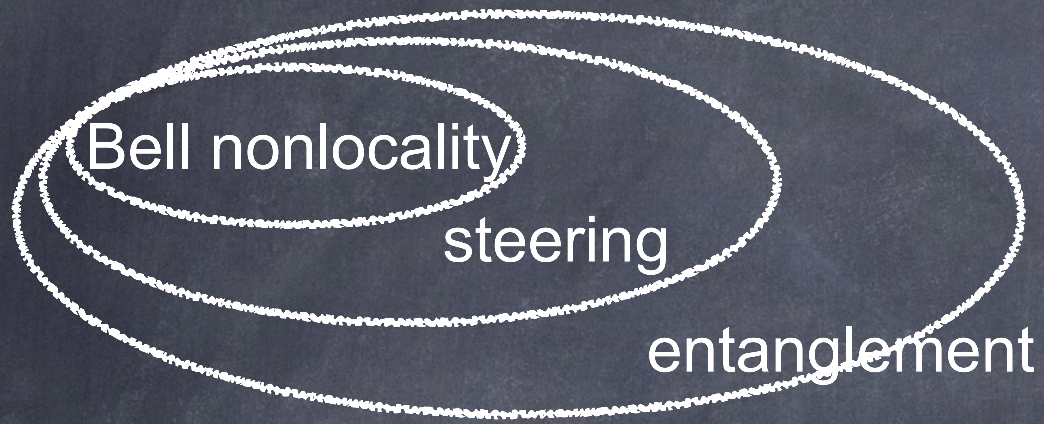

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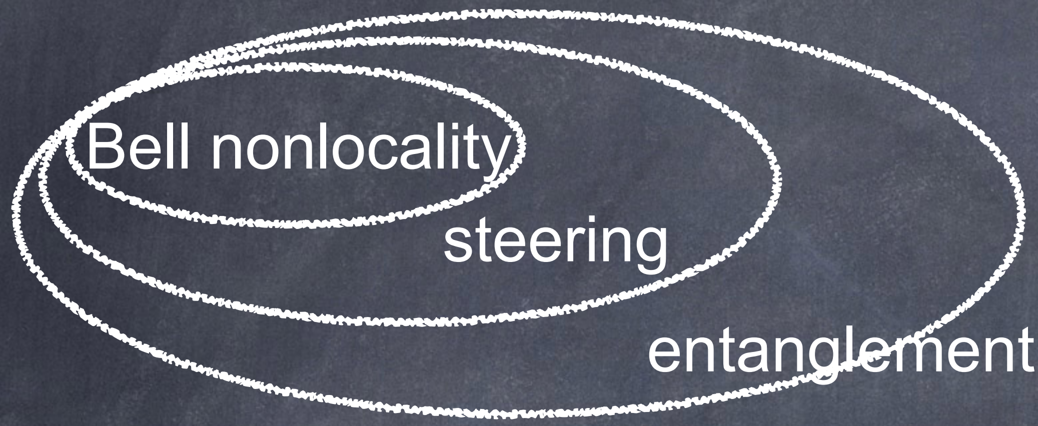

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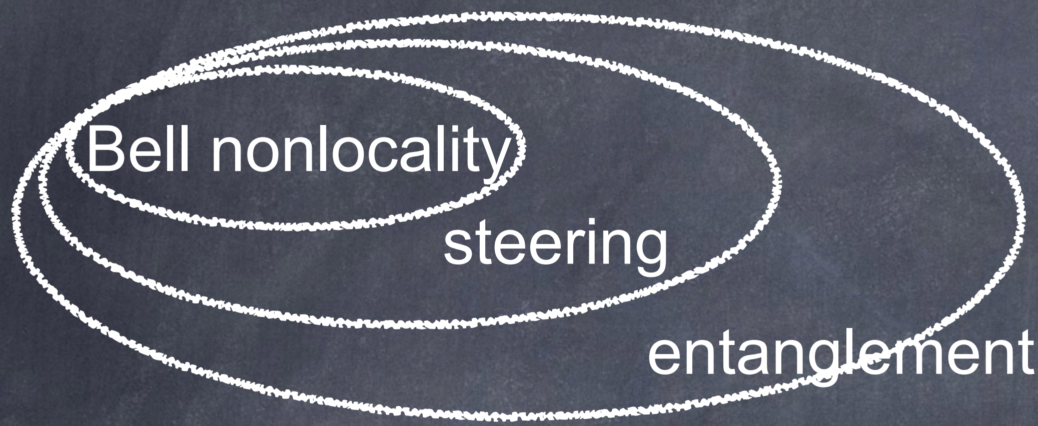

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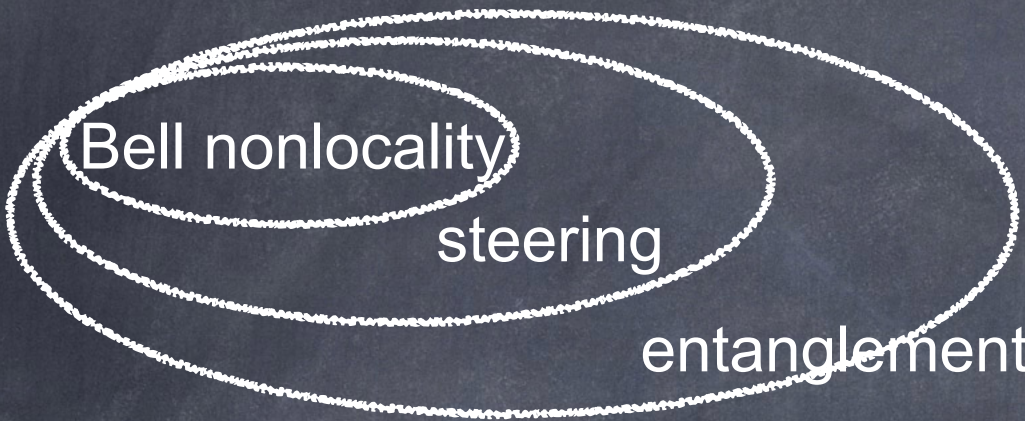

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T. Moroder
violation

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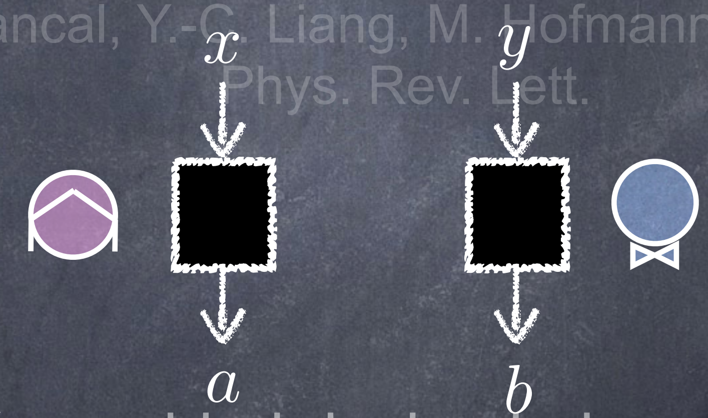
$$\boxed{\begin{array}{ll} & \min \ \mathcal{N}(\rho_{AB}) \\ \text{such that} & \sum_{abxy} I_{abxy} P(a,b|x,y) = v \end{array}}$$



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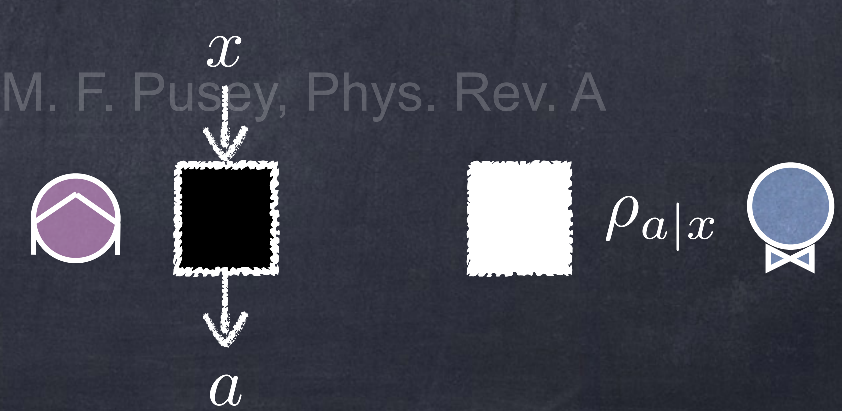
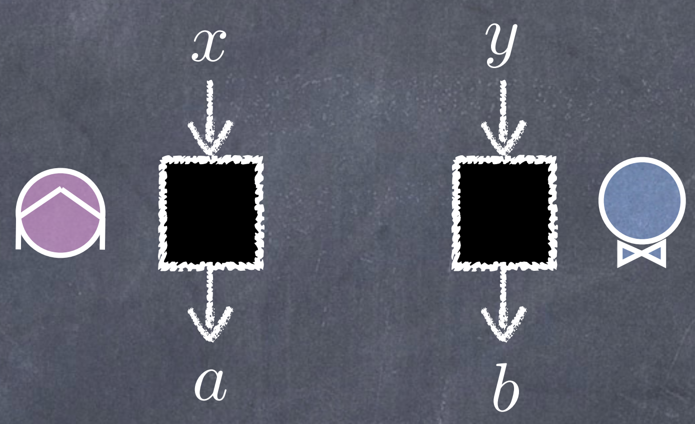
$$\boxed{\begin{array}{ll} & \min \ \mathcal{N}(\rho_{AB}) \\ \text{such that} & \sum_{ax} \mathrm{Tr}(F_{a|x}\rho_{a|x}) = u \end{array}}$$



Motivation

lower bound on the degree of steerability for given Bell inequality violation

$$\begin{aligned} &\min \quad \text{a measure of steering} \\ &\text{such that} \quad \sum_{abxy} I_{abxy} P(a,b|x,y) = v \end{aligned}$$



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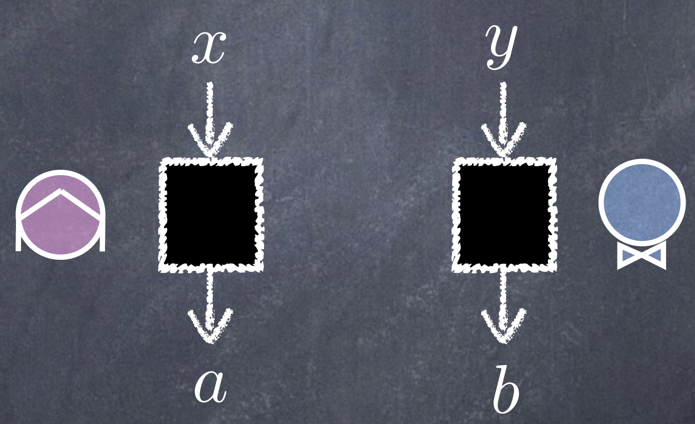

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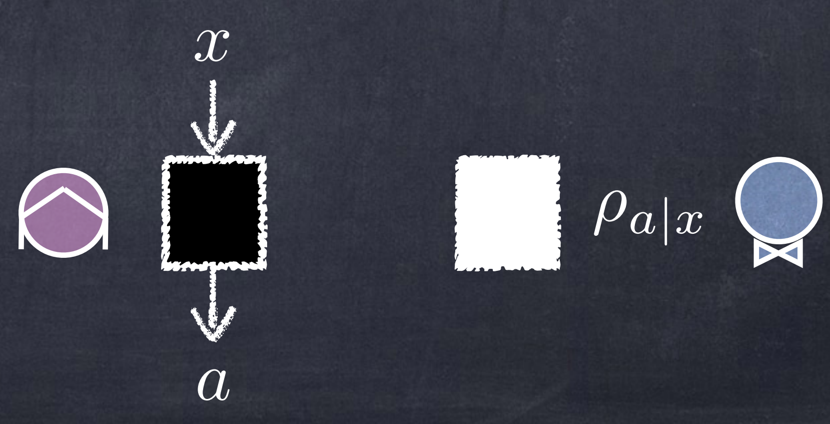

Characterizing quantum correlations and the moment matrix

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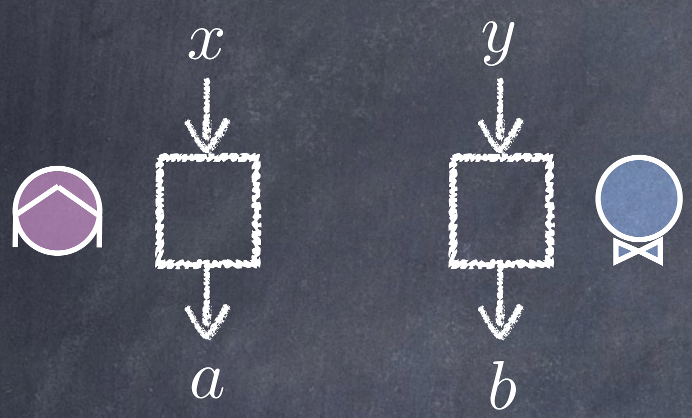

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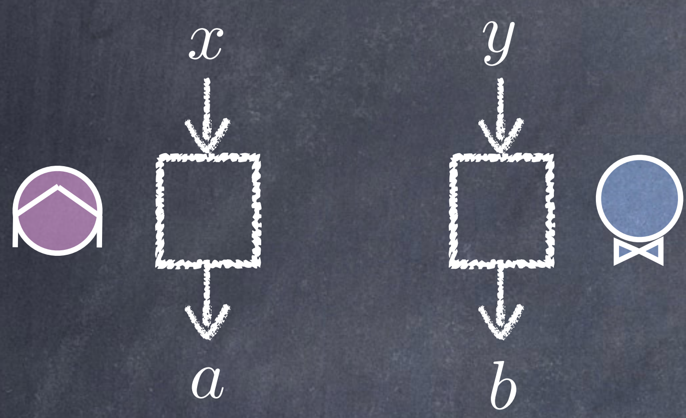

$$\{P(a,b|x,y)\} \in \mathcal{Q}$$

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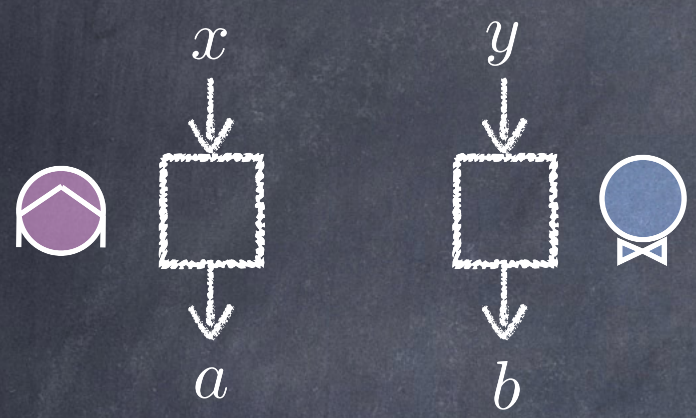

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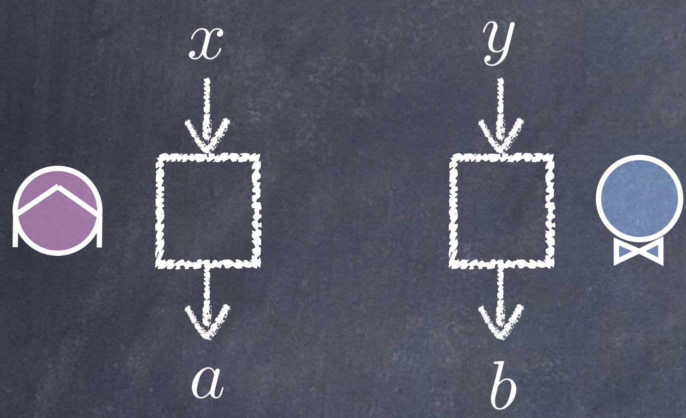

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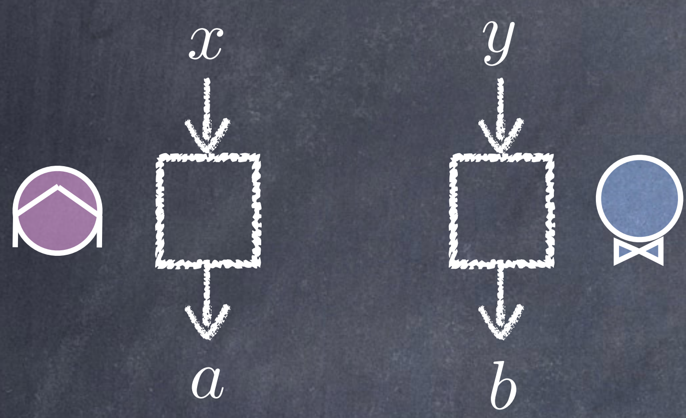

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$$\Gamma_{ij} = \mathrm{Tr}(S_i^\dagger S_j \rho_{AB})$$

$$\mathbb{S}^{(1)} = \{E_{a|x}\} \cup \{E_{b|y}\} = \{S_1, ..., S_n\}$$

$$\Gamma = \begin{pmatrix} \cdots & & \cdots \\ & \cdot & \\ \cdot & & \cdot \end{pmatrix}$$

0

$$\mathrm{Tr}(E_{a|x} \otimes E_{b|y}\ \rho_{AB}) = P(a,b|x,y)$$

unknown complex number

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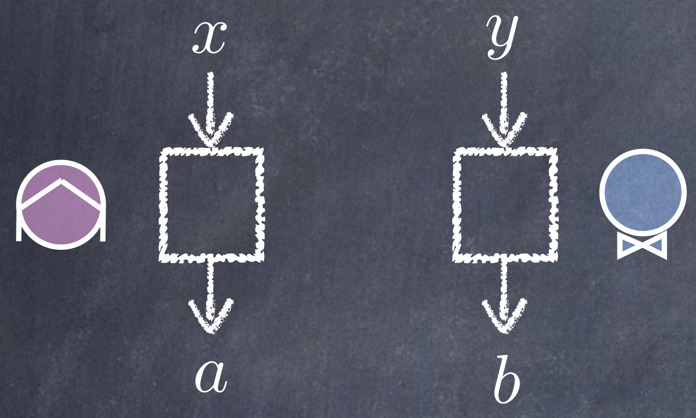

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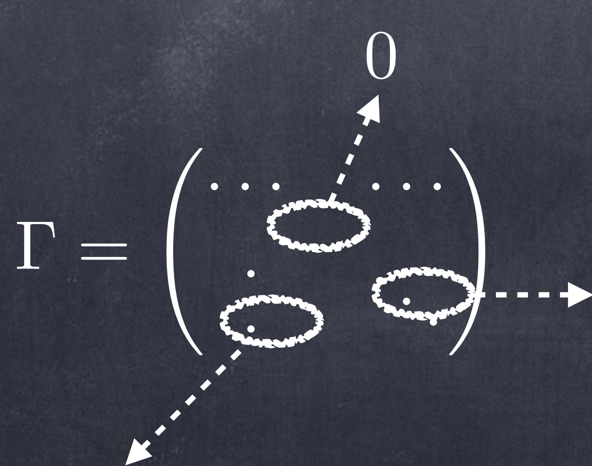

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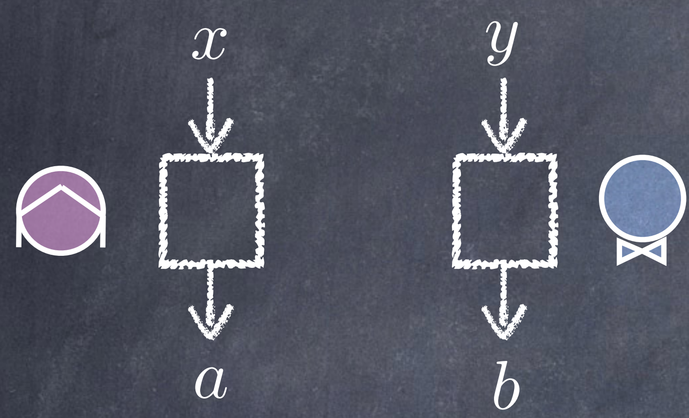

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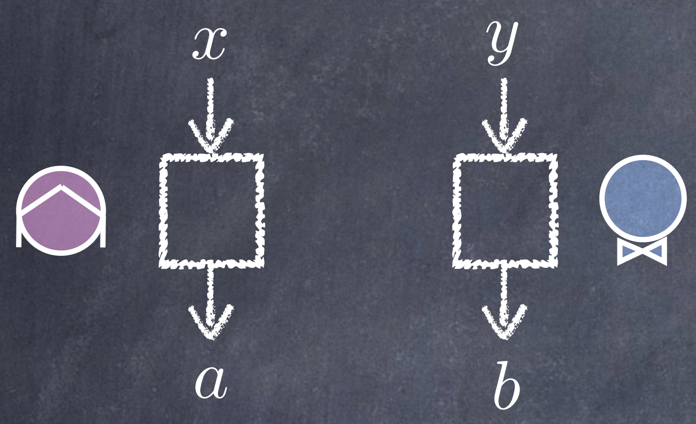

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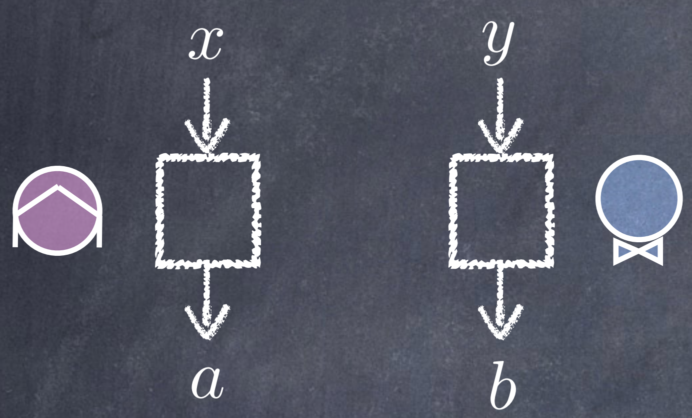

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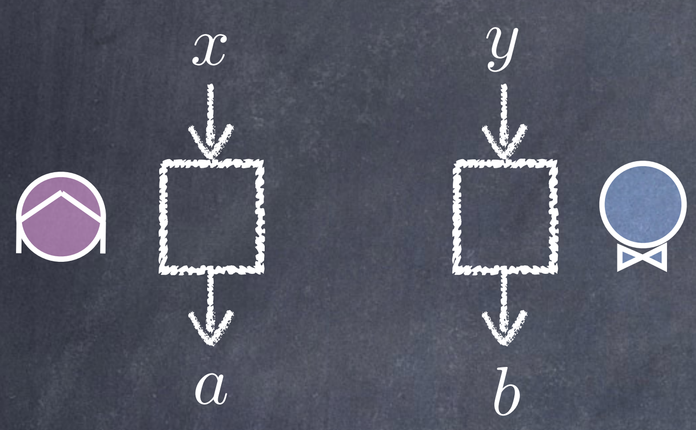

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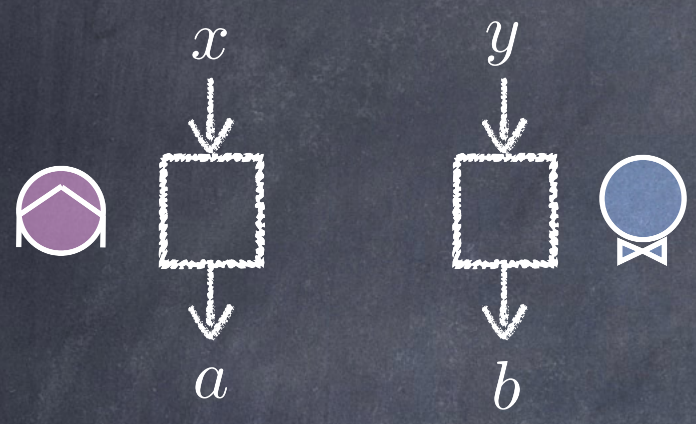

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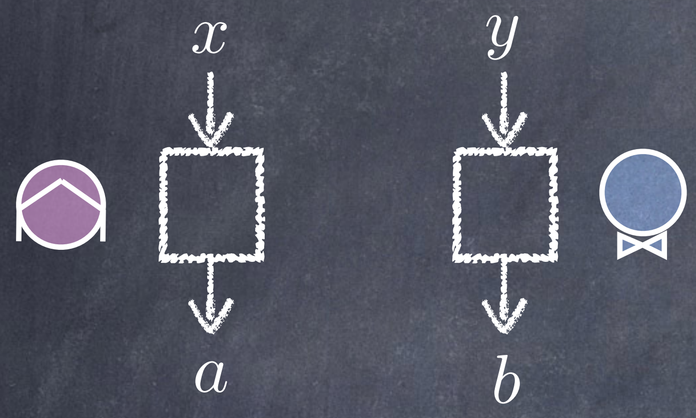

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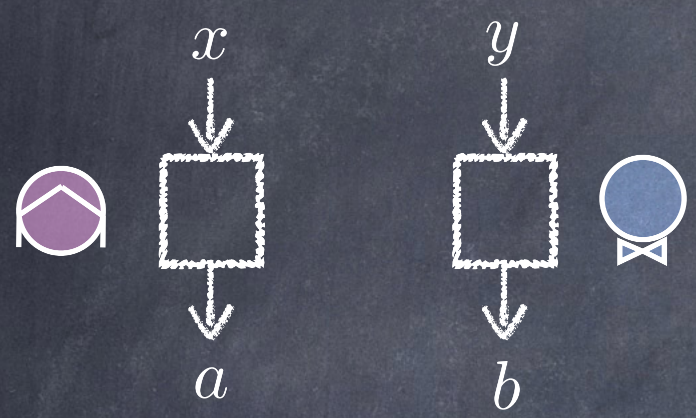

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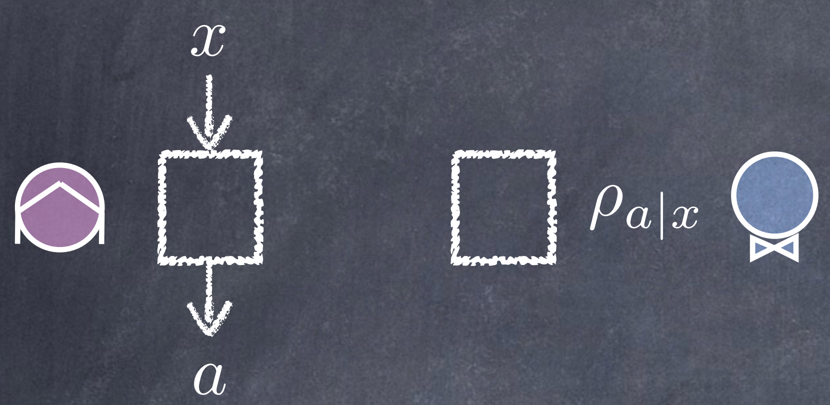
The moment matrix of assemblage



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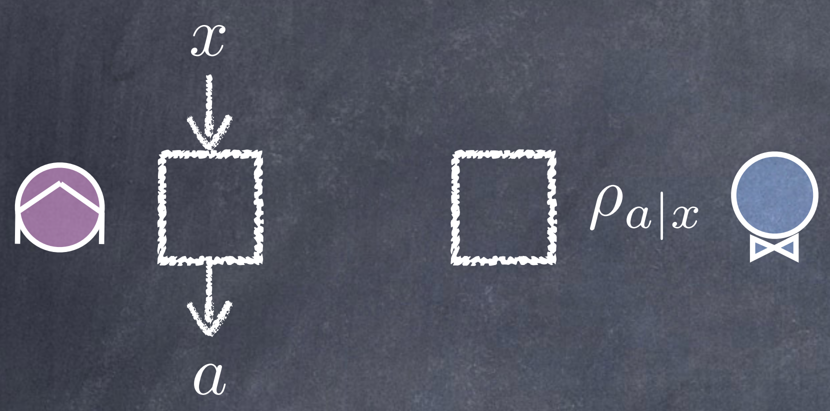

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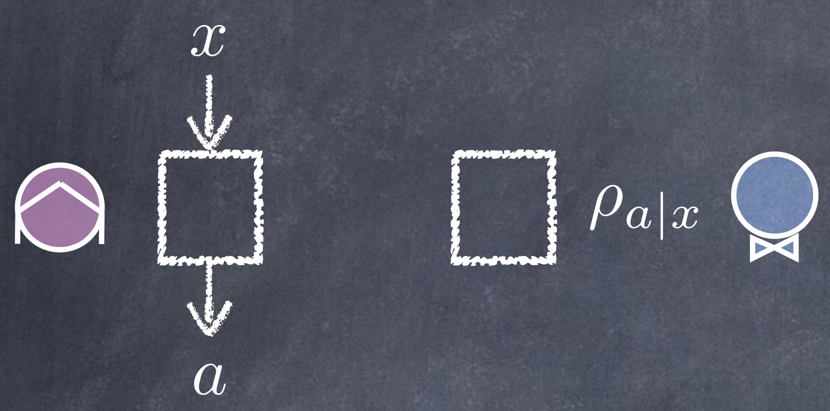

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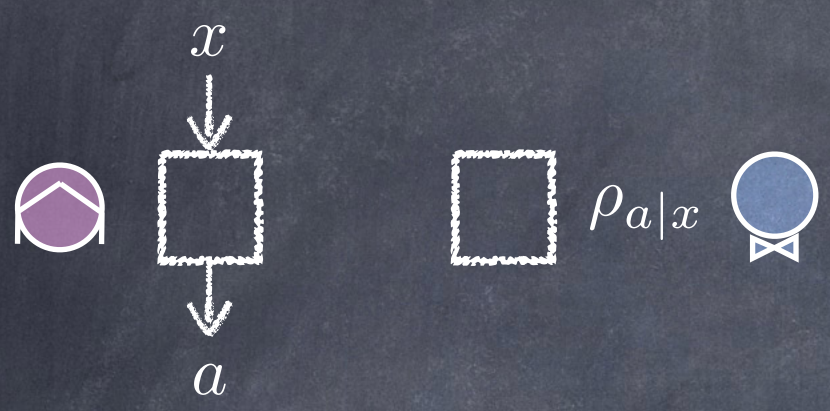

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ex. two settings $y, \ y'$, three outcomes for each setting $b \in \{0,1,2\}$ (first level)

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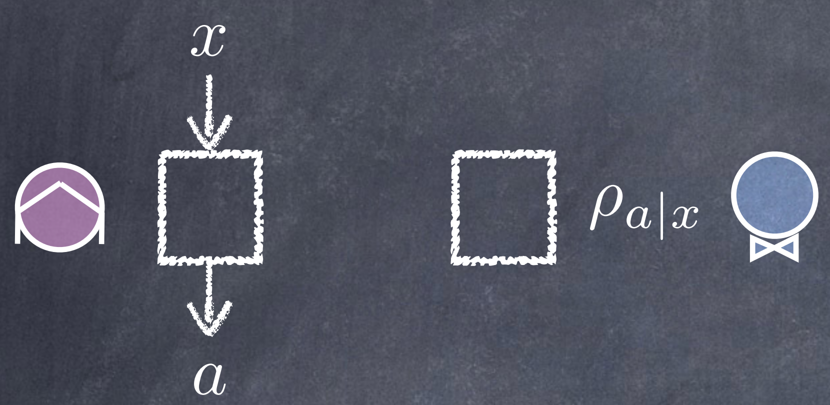

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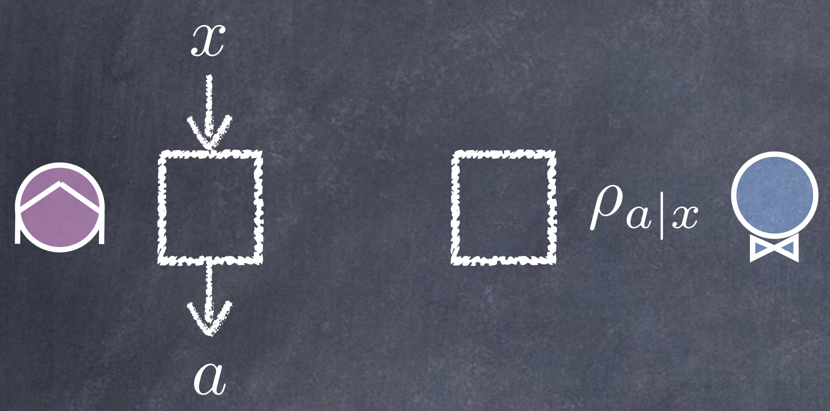

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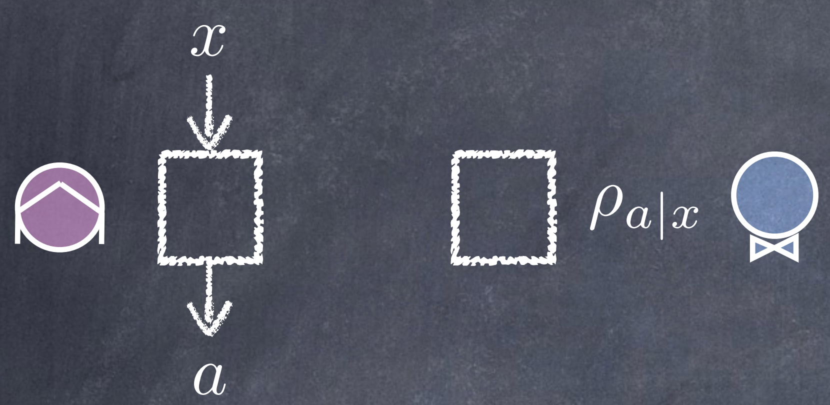

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convergence:

keep going to higher level…

The moment matrix of assemblage

Testing Tsirelson's bounds

The moment matrix of assemblage

Testing Tsirelson's bounds

$$\begin{aligned} \max \quad & I \cdot P \\ \text{subject to} \quad & \chi[\rho_{a|x}] \geq 0 & \forall a, x \\ & \sum_a \chi[\rho_{a|x}] = \sum_a \chi[\rho_{a|x'}] & \forall x \neq x' \end{aligned}$$

$$I \cdot P = \sum_{a,b,x,y} I_{a,b,x,y} P(a,b|x,y)$$

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	CHSH Ineq.	I_{3322}	I_{2233}
$l=1$	2.8284	0.3621	0.3078
$l=2$		0.2550	0.3050
$l=3$		0.2512	0.3050
$l=4$		0.2509	

	Tsirelson's bound
CHSH Ineq.	$2\sqrt{2} \approx 2.8284$
I_{3322}	≈ 0.25088
I_{2233}	$(\sqrt{11/3} - 1)/3 \approx 0.3050$

The upper bound of a given Bell inequality converges (?) to Tsirelson's bound when increasing the level of hierarchy l .

Device-independent quantification of steerability

a measure of steering - steering robustness (SR) of assemblage $\{\rho_{a|x}\}_{a,x}$

$$\min \quad t \geq 0$$

$$\text{subject to} \quad \left\{\frac{\rho_{a|x} + t\,\tau_{a|x}}{1+t}\right\}_{a,x} \quad \text{unsteerable}$$

$$\{\tau_{a|x}\}_{a,x} \quad \text{any assemblage}$$

-Marco Piani and John Watrous
Phys. Rev. Lett. **114**, 060404 (2015).

Device-independent quantification of steerability

$$\mathrm{SR} = \min \ \mathrm{tr} \sum_\lambda \sigma_\lambda - 1$$

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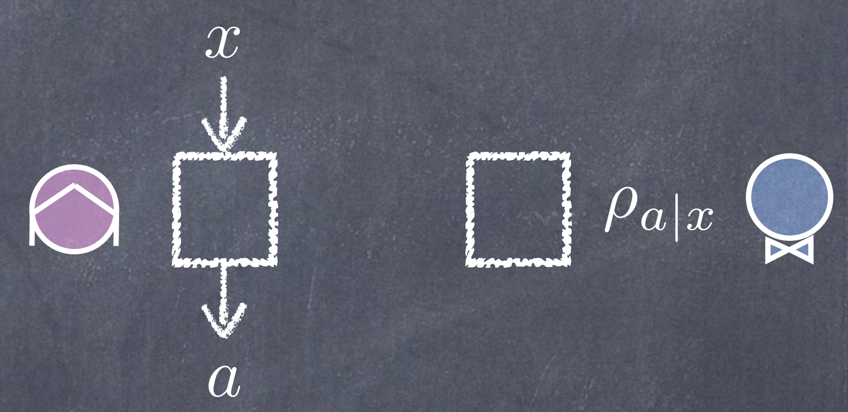
$$\sigma_\lambda \geq 0 \qquad \forall\ \lambda$$



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$$\min \quad \sum_\lambda \chi[\sigma_\lambda]_{\mathrm{tr}} - 1$$

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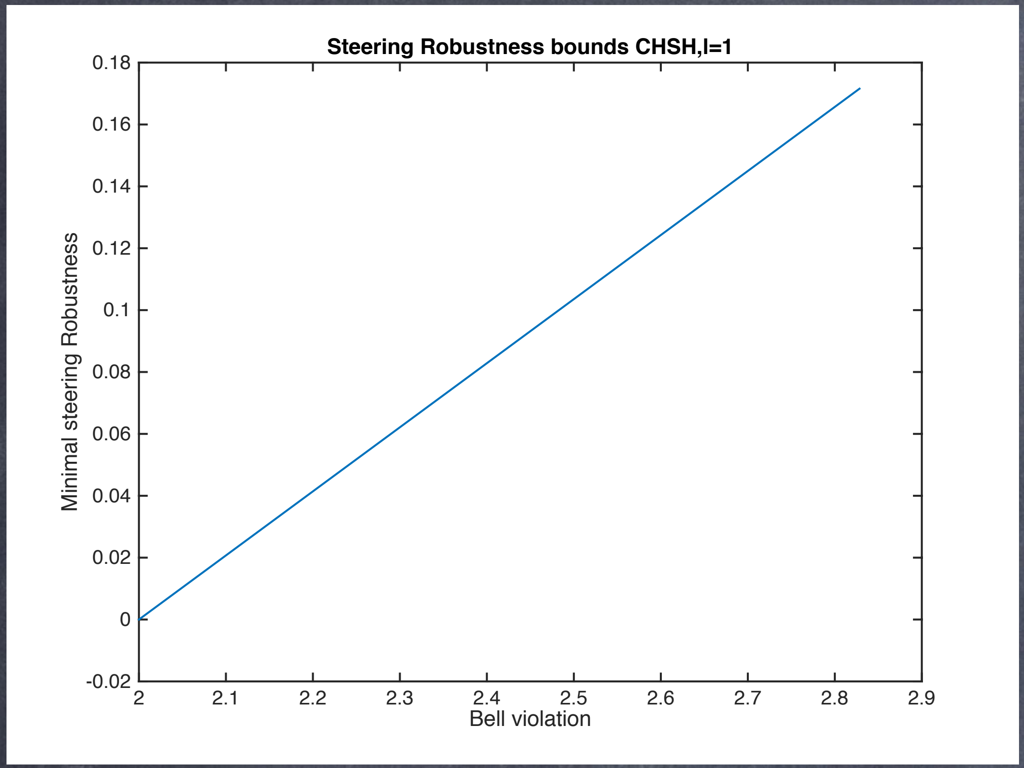
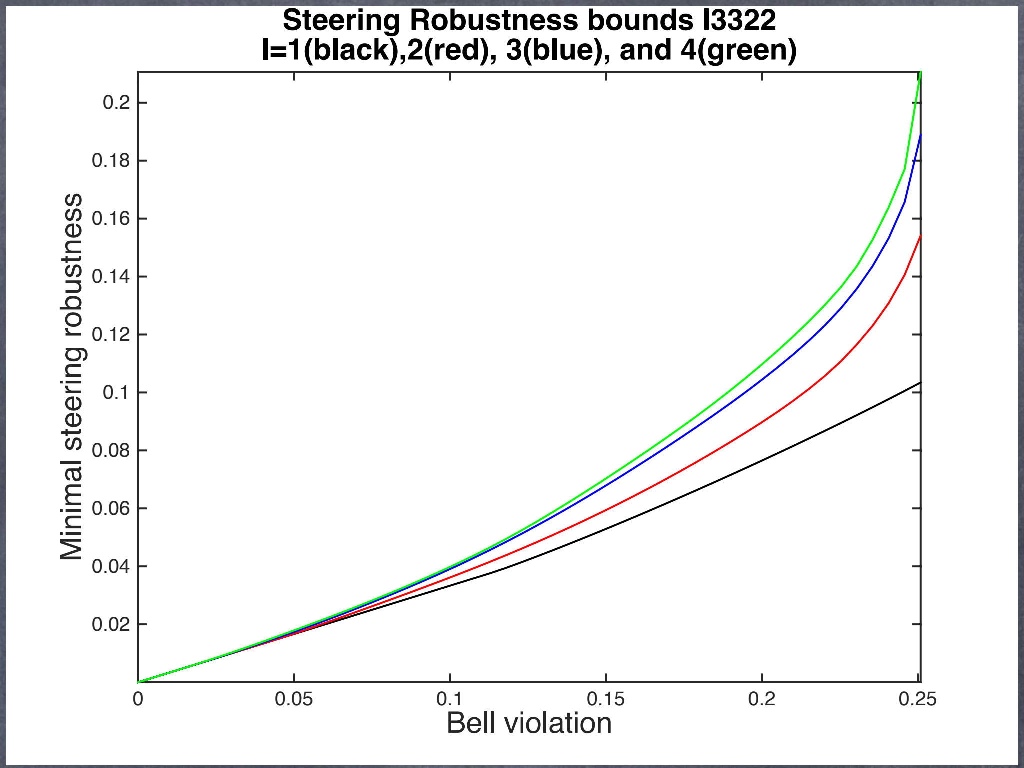
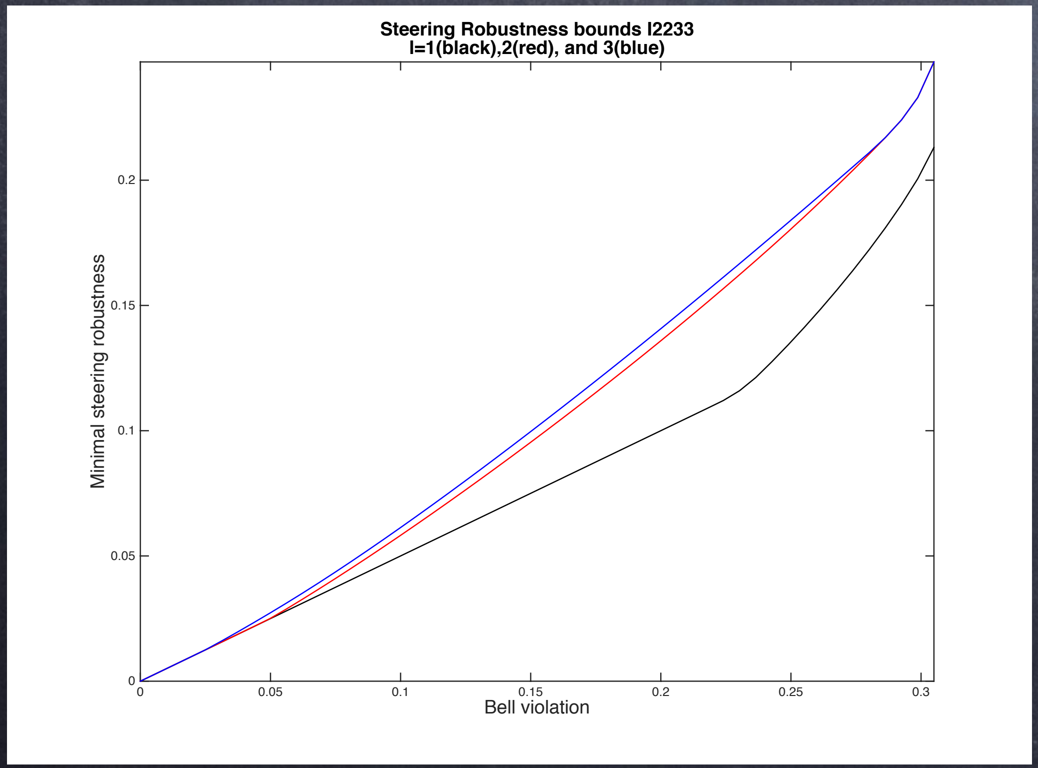
$$\sum_a \chi[\rho_{a|x}]_{\mathrm{tr}} = 1 \qquad \forall\ a, x$$

$$I \cdot P = v,$$

Device-independent quantification of steerability







Summary and Outlook

The moment matrix of assemblage

Device-independent quantification of steerability

The gap between quantum set and our approach

Device-independent quantification of measurement incompatibility

$$\boxed{\mathrm{IR}(M_{a|x}) \geq \mathrm{SR}(\sigma_{a|x})}$$

Possibility of self-testing

Thank you for your attention!