EPR STEERING AND THE STEERING ELLIPSOID

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Joint work with

- Michael Hall
- Malcolm Anderson
- Marcin Zwierz
- Howard Wiseman

- Based on: JOSA B, Vol. 32, Issue 4, pp. A40-A49 (2015)

Quantum steering ellipsoid

- Way of visualising two-qubit state

$$\rho = \frac{1}{4}(I + \boldsymbol{a}.\boldsymbol{\sigma} \otimes I + I \otimes \boldsymbol{b}.\boldsymbol{\sigma} + \sum_{ij} T_{ij}\sigma_i \otimes \sigma_j)$$

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- Bob performs POVM on his qubit $\{F_f\}$
- Upon getting outcome "f", Alice is steered to

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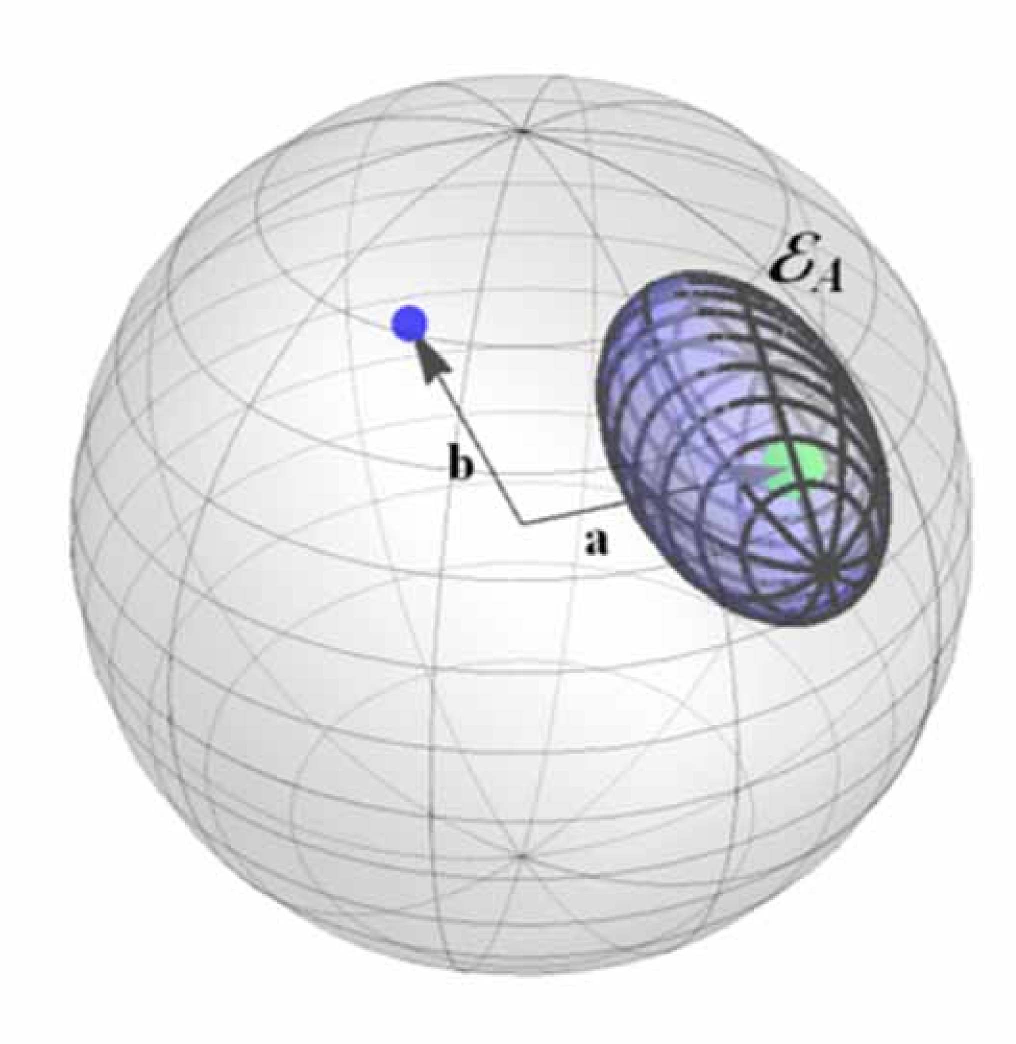
- The steering ellipsoid (for Alice) is generated by varying over all $\{F_f\}$

Quantum steering ellipsoid



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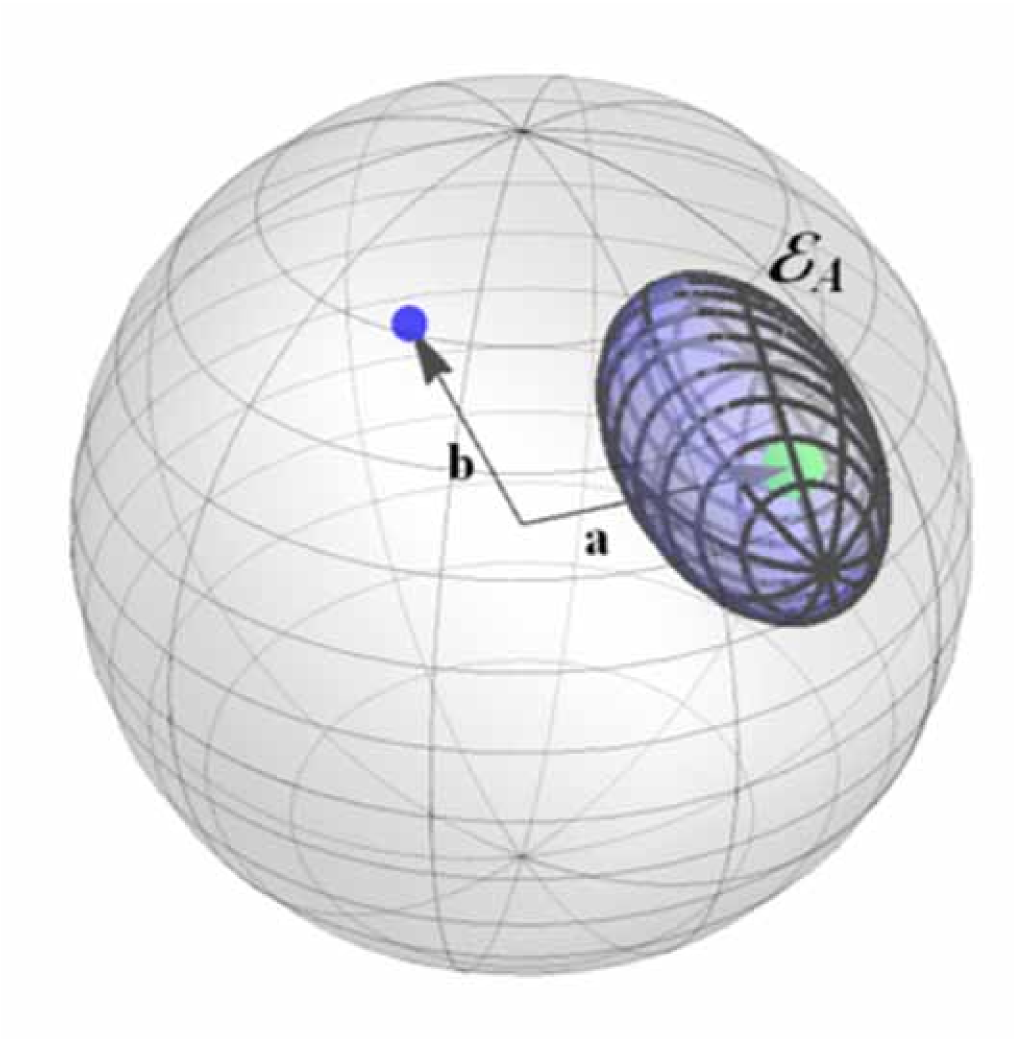
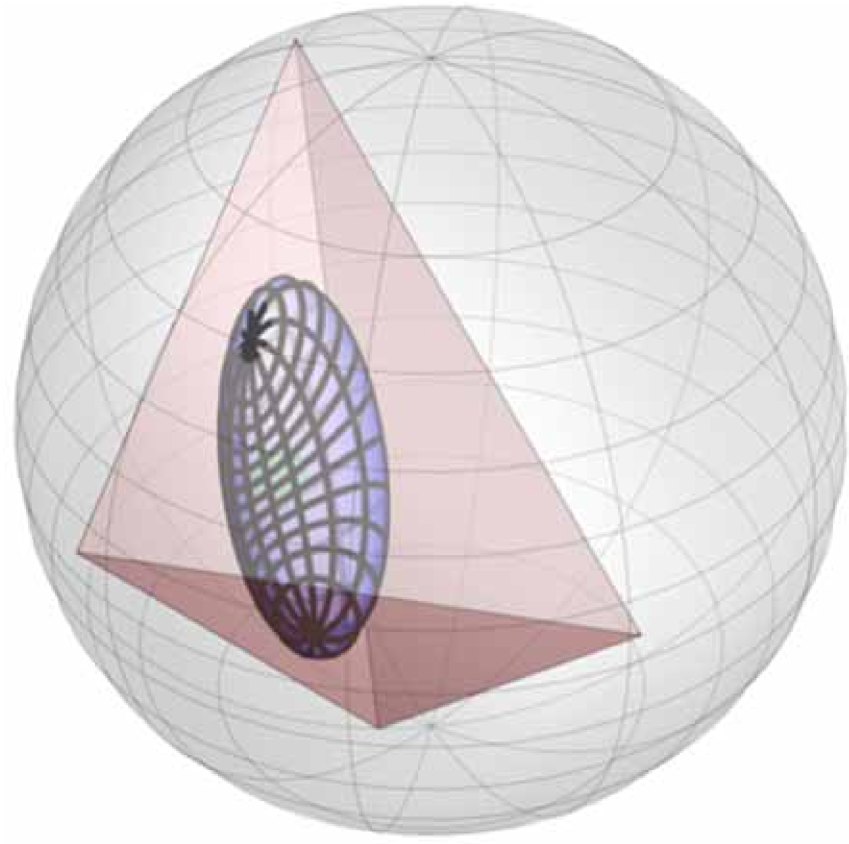


- Reconstruct state (up to local Us)
- Rank of ellipsoid
- Volume V > V* > 0
- Geometric condition for entanglement

Steering ellipsoid and non-locality

- The quantum steering ellipsoid gives us an iff condition for a non-separable state



Steering ellipsoid and non-locality

- The quantum steering ellipsoid gives us an iff condition for entanglement

- Can it say anything about other forms of non-locality?

Steering ellipsoid and non-locality

- Werner state $$\rho = (1-p)\frac{I}{4} + p|\psi^-\rangle\langle\psi^-|$$

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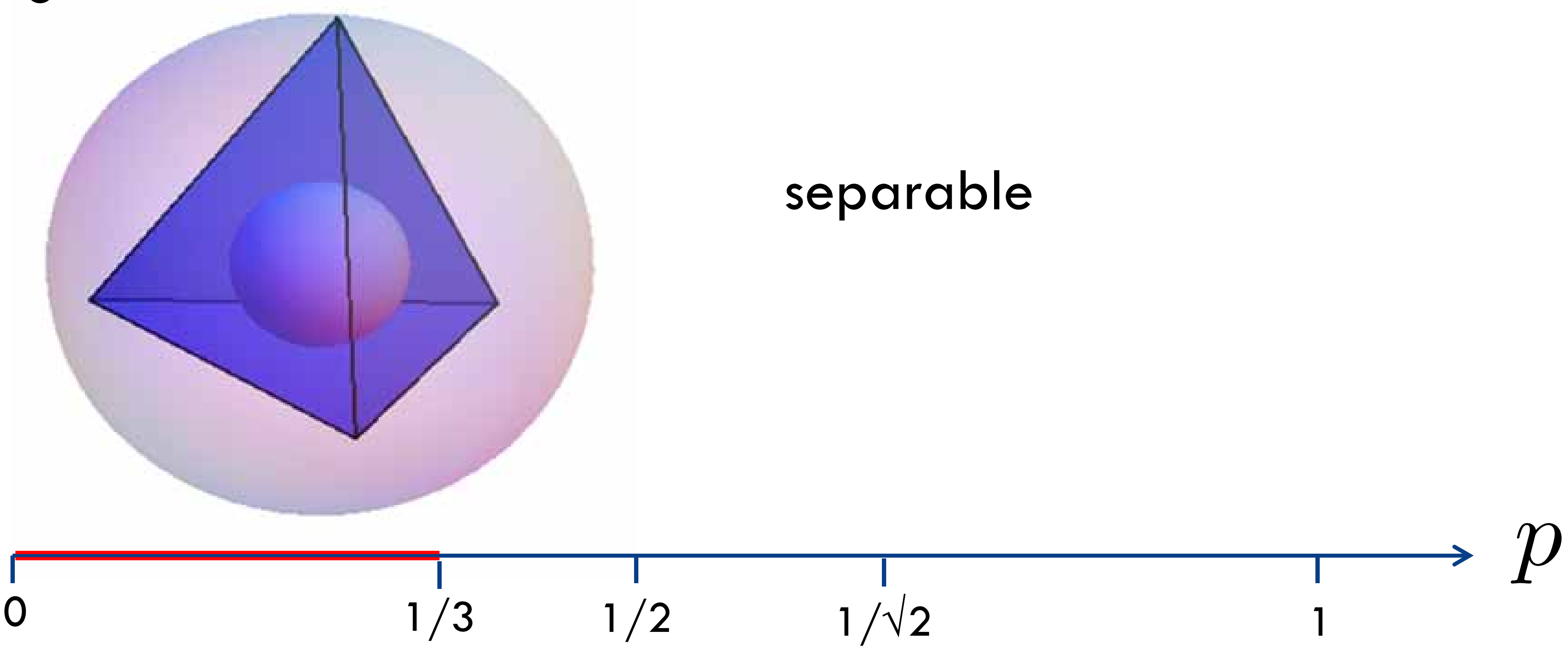

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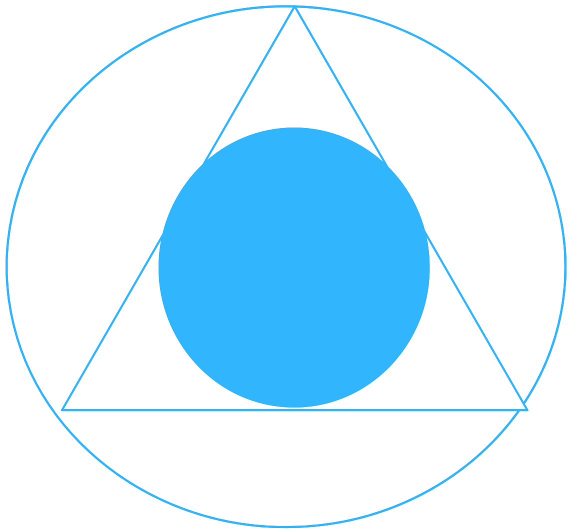

non-(EPR)-steerable

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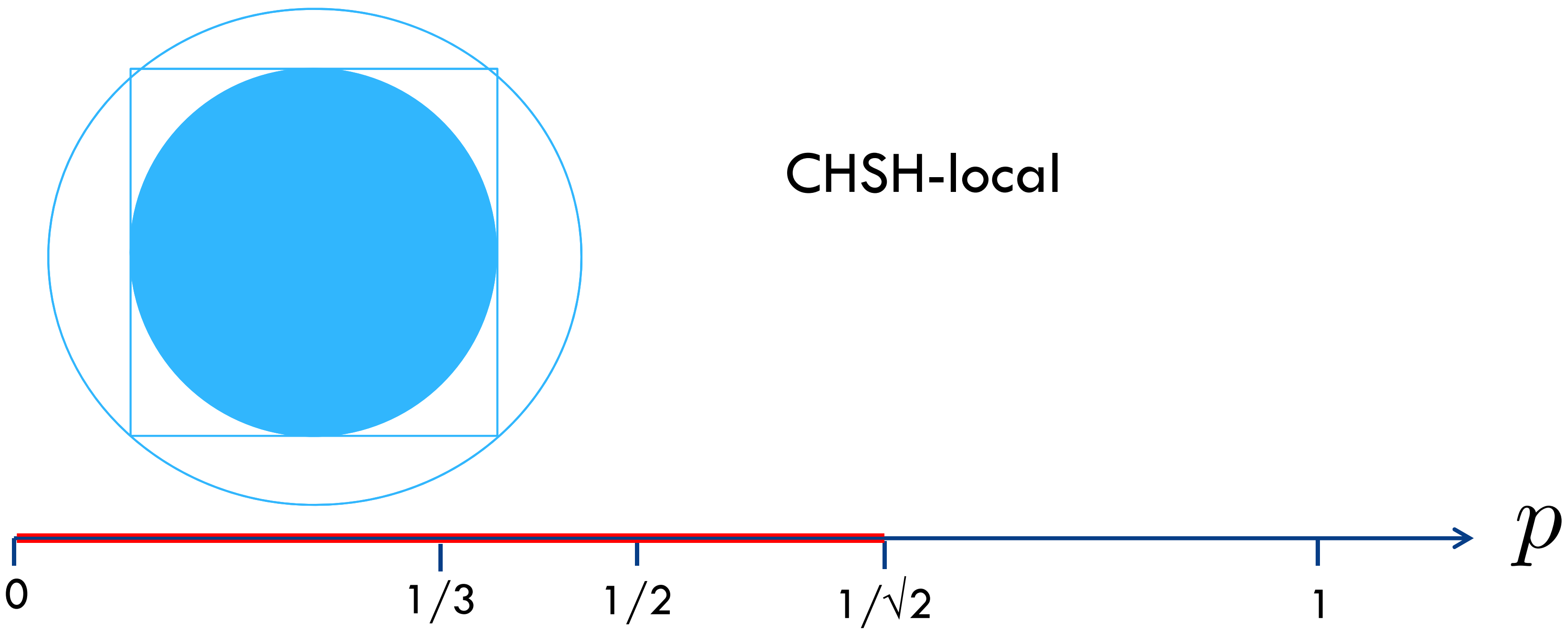

“T-states”

- Recall

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"T-states"

- Set a = b = 0

$$\rho = \frac{1}{4}(I + \sum_{ij} T_{ij}\sigma_i \otimes \sigma_j)$$

“T-states”

- Mixture of Bell states

$$\rho = \frac{1}{4}\left(I + \sum_{i=1}^{3} t_i \sigma_i \otimes \sigma_i\right)$$

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- Quantum steering ellipsoid
  - Semiaxes $|t_i|$
  - Aligned along x,y,z – axes
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- Quantum steering ellipsoid
  - Semiaxes $|t_i|$
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  - Centred at 0
  - Does not possess nice geometric conditions for LHS or LHV models (but if separable still obeys nested tet cond)

“T-states” QSE for LHS models

- Can still use QSE of T-states to obtain deterministic LHS model for all projective measurements
- We conjecture the model is *optimal*

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- Each outcome $f = \pm 1$ occurs with $p_f = \frac{1}{2}$

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- What should (p_f, ρ_f) be given by in LHS model?

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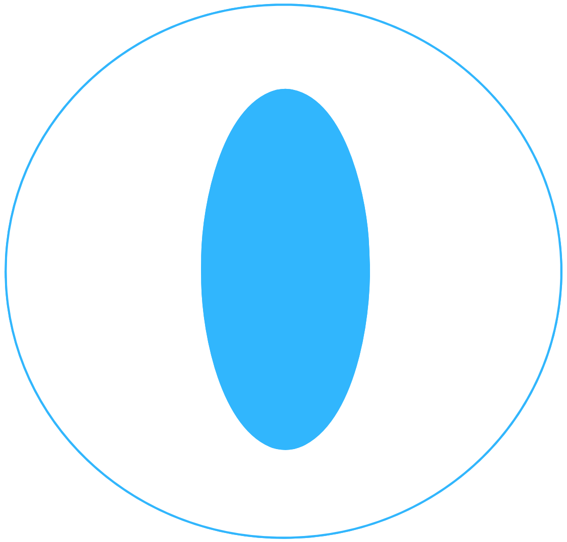
$$\{T\boldsymbol{f} : |\boldsymbol{f}| = 1\}$$



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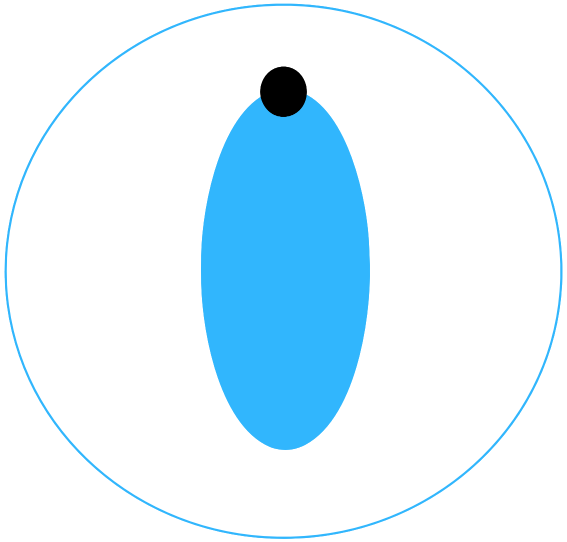

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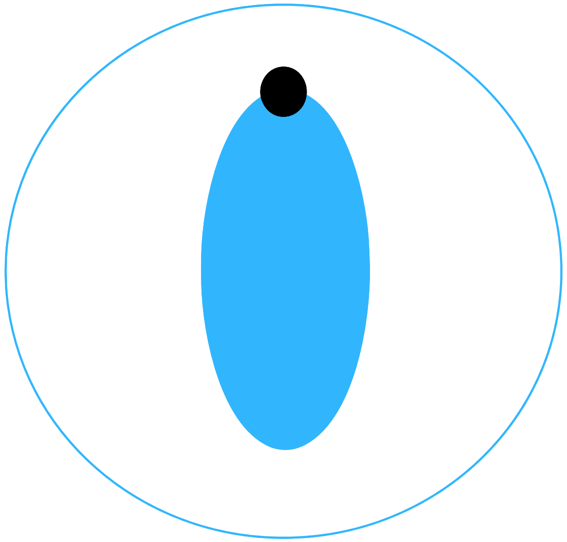

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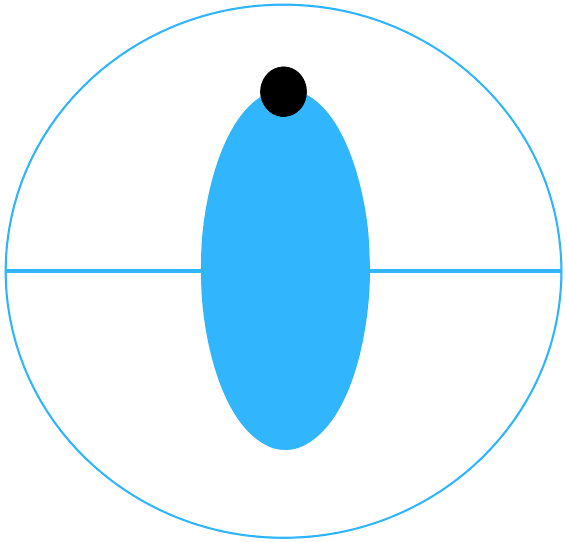

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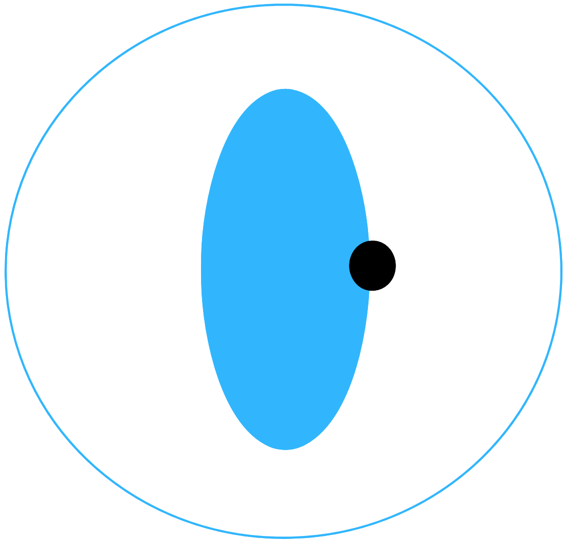

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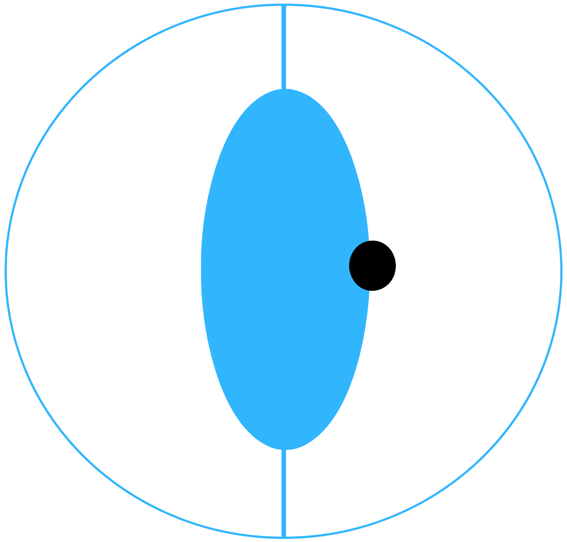

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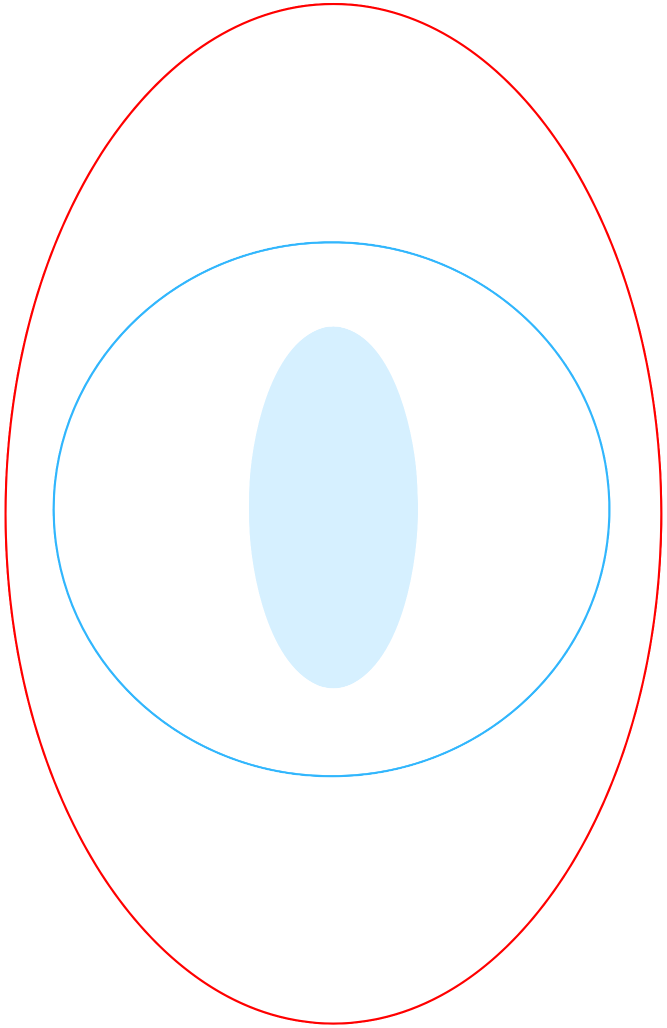

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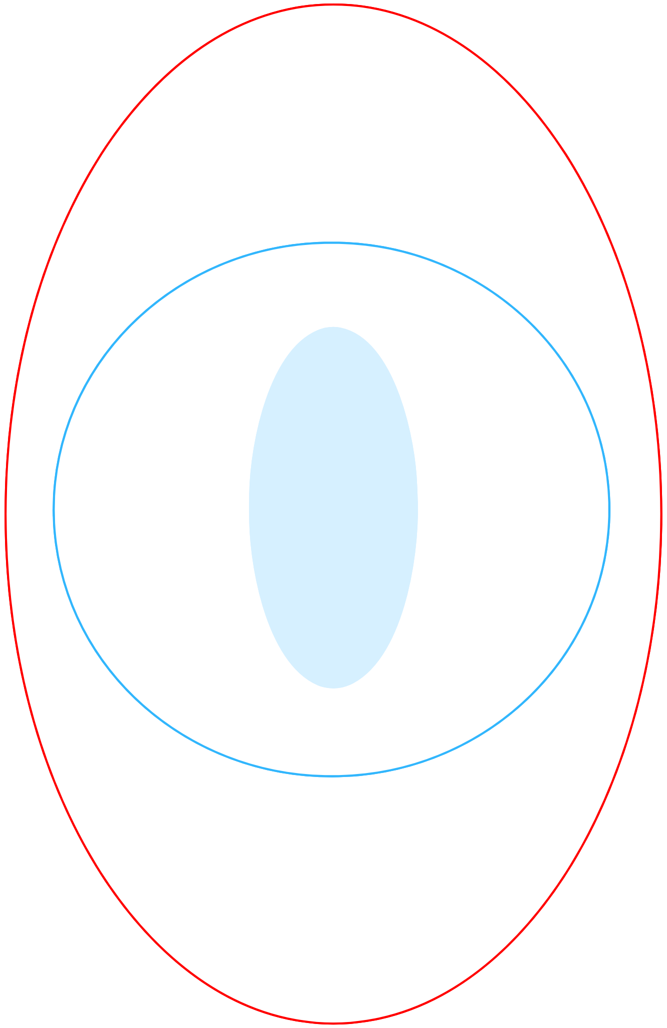

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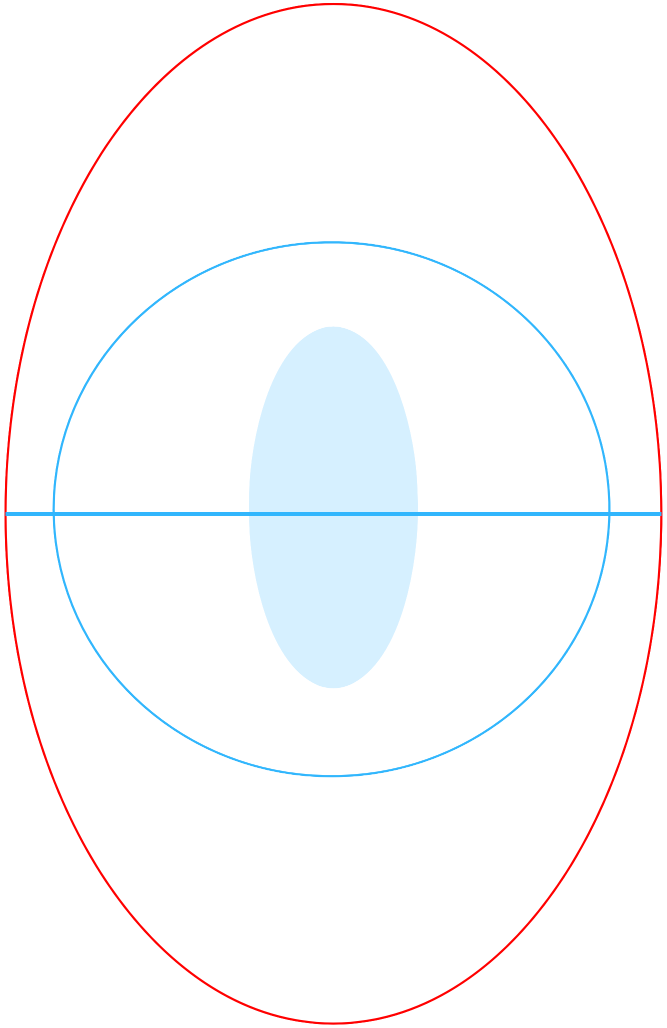

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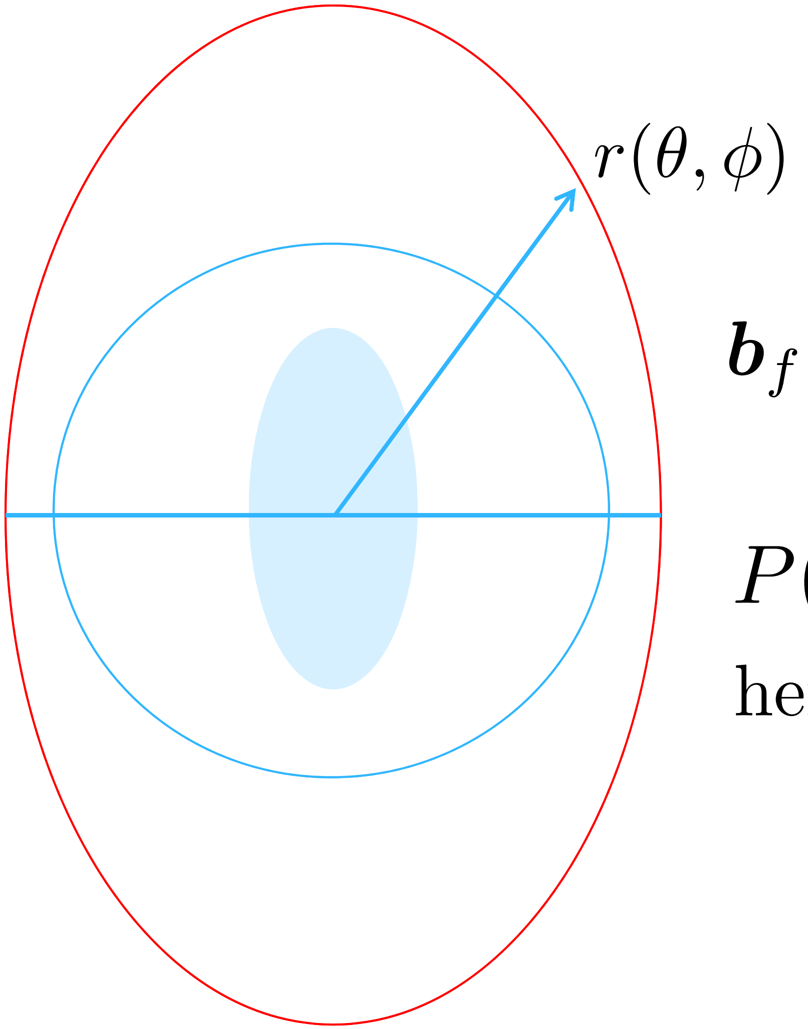

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$$P(\boldsymbol{n}) \propto r(\theta,\phi)^4$$

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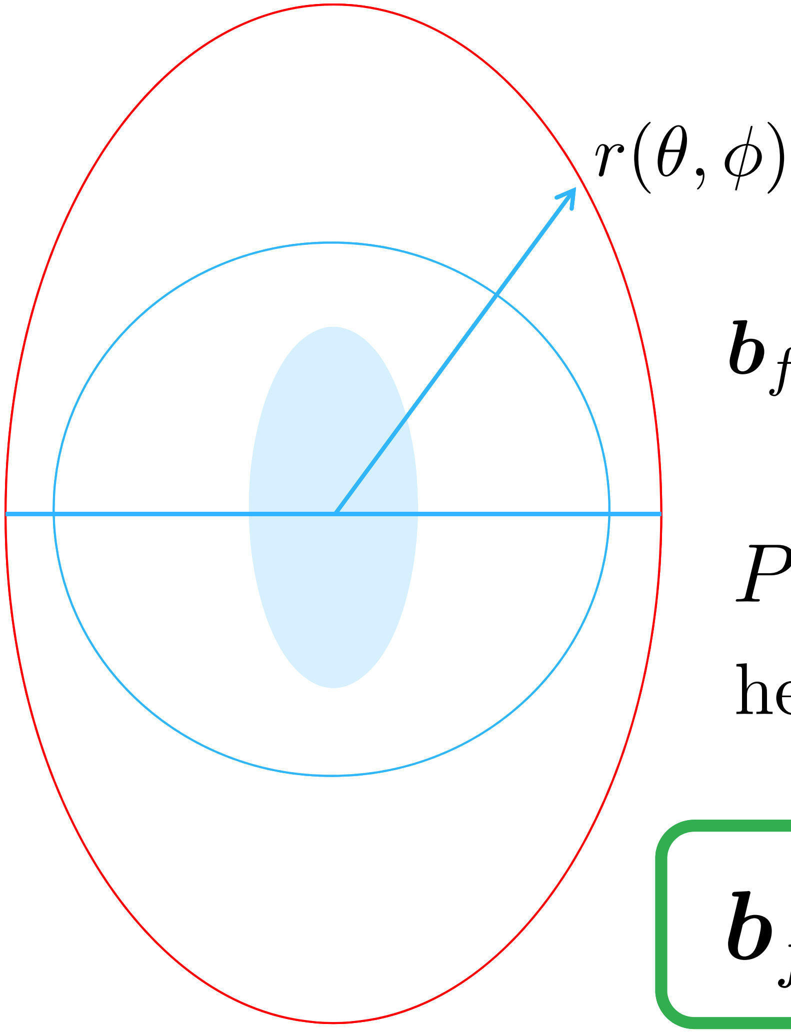

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Surface of EPR-steerable T-states

$$(s_1, s_2, s_3) = (|t_1|, |t_2|, |t_3|)$$

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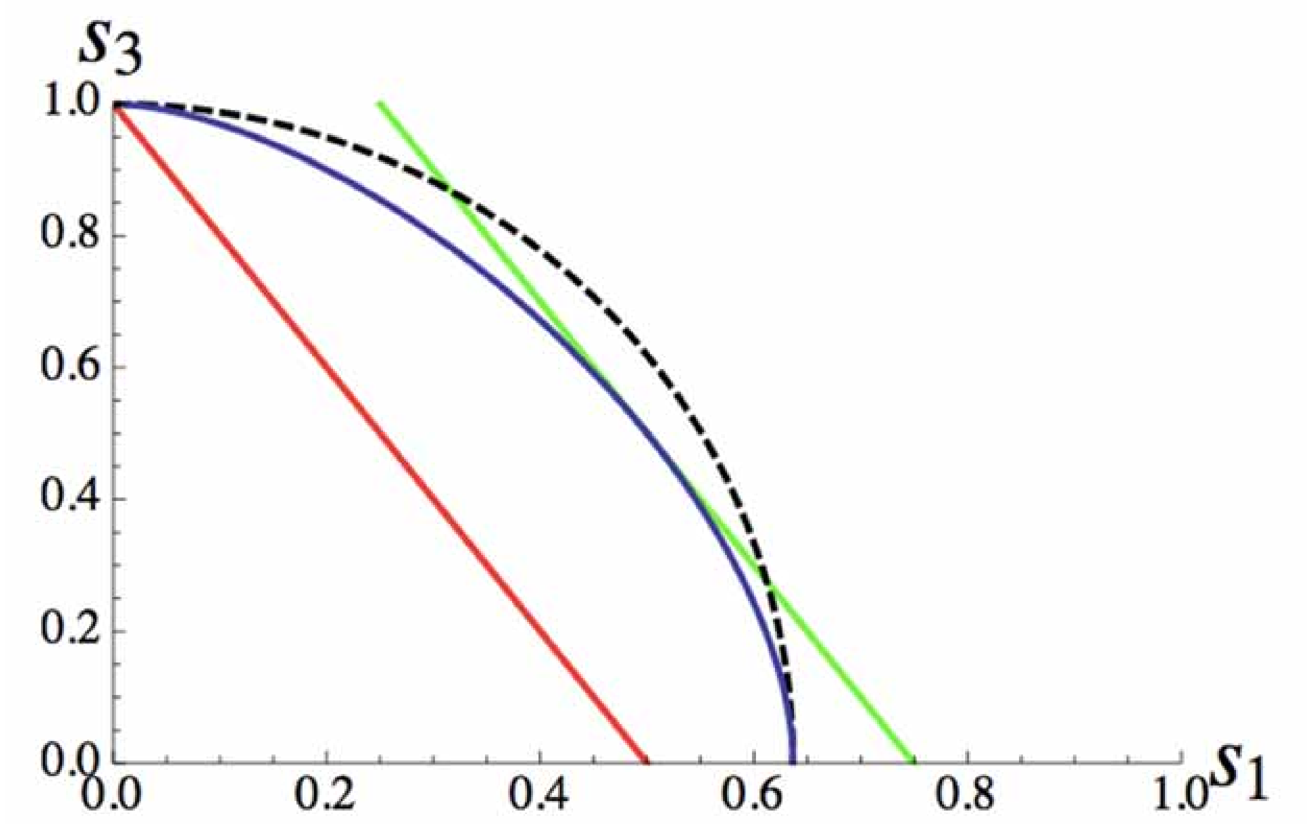

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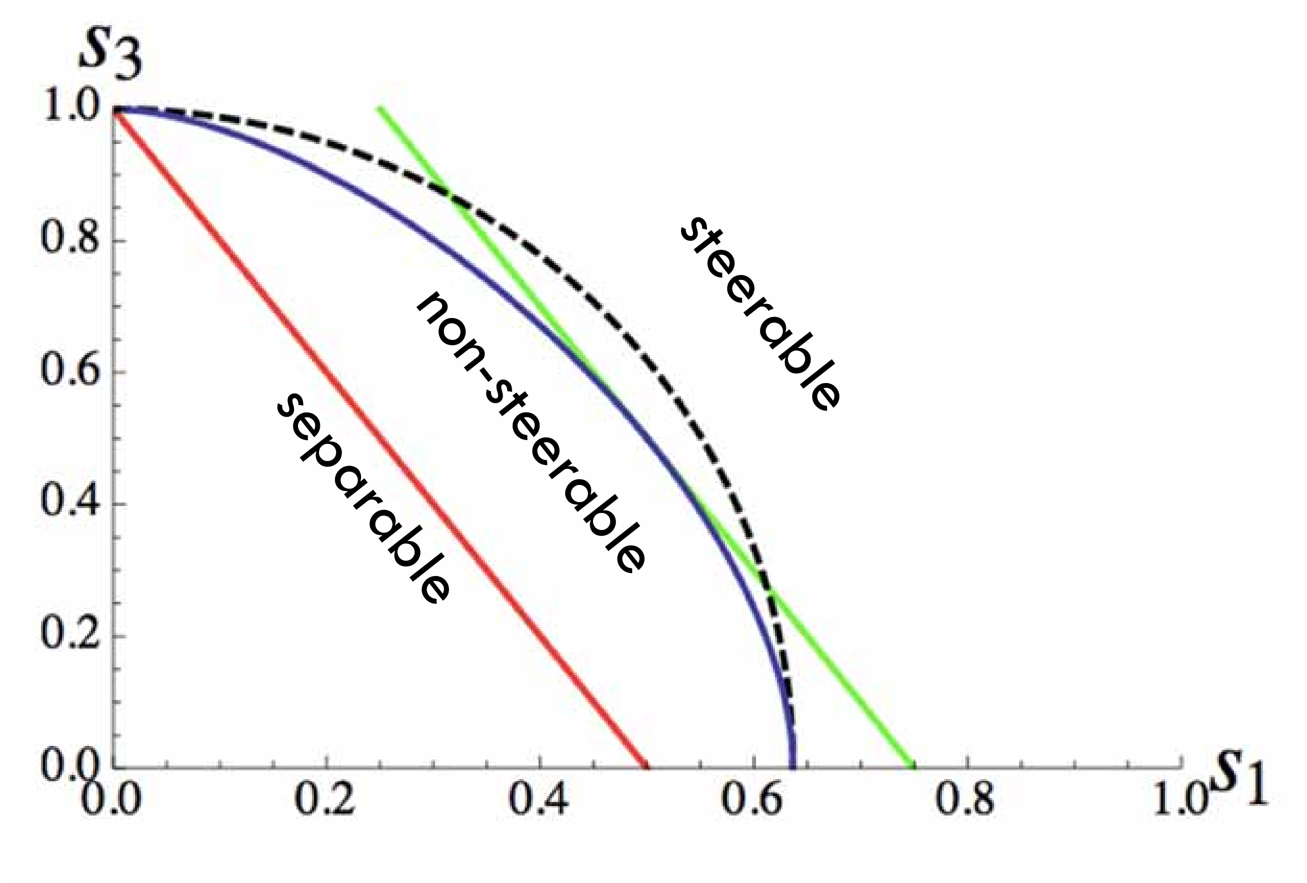

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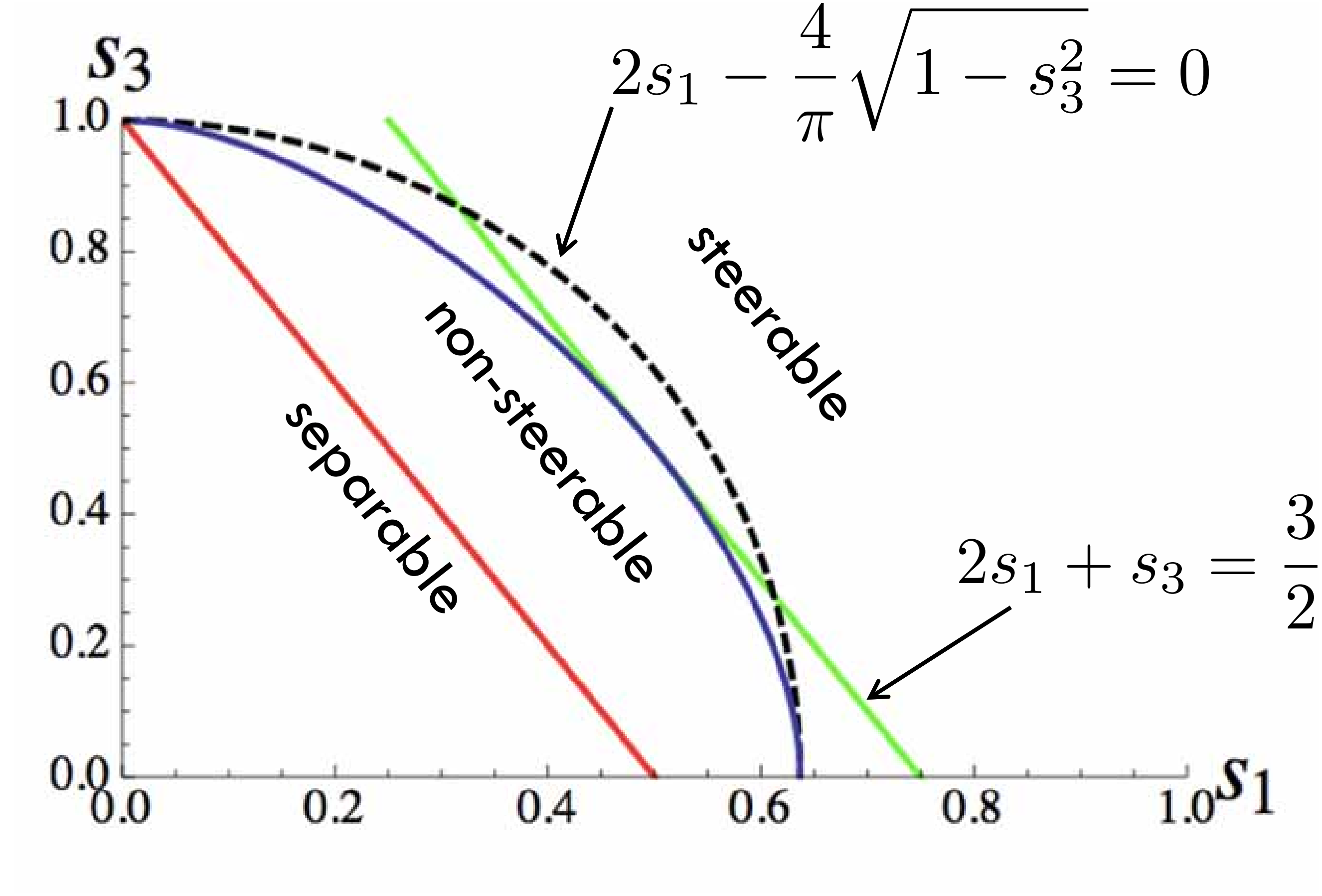
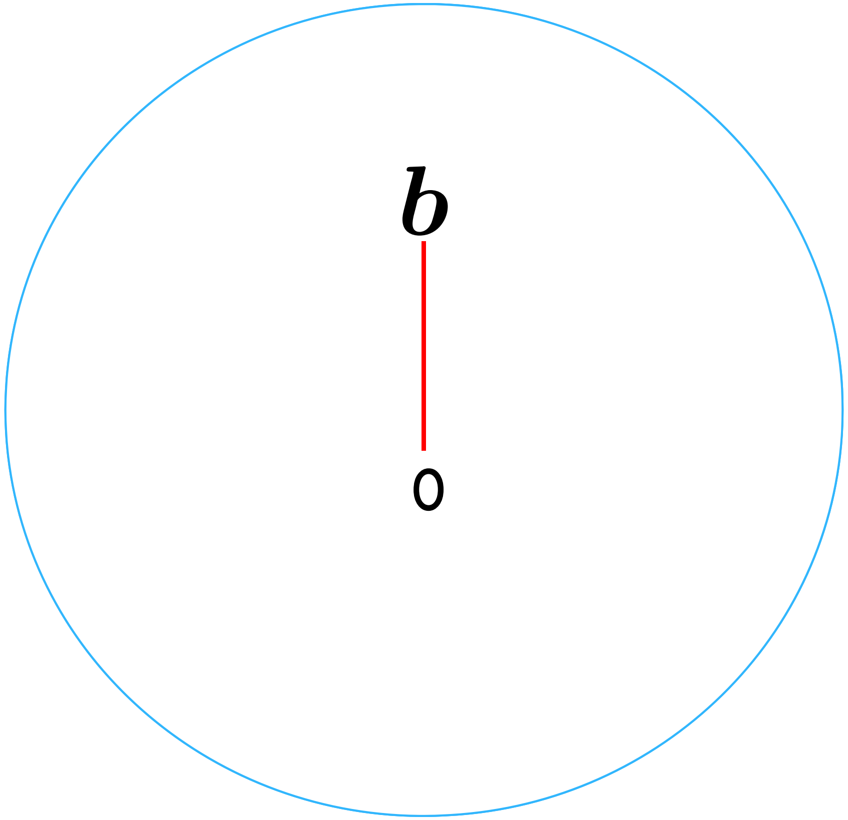


LHS models for general states



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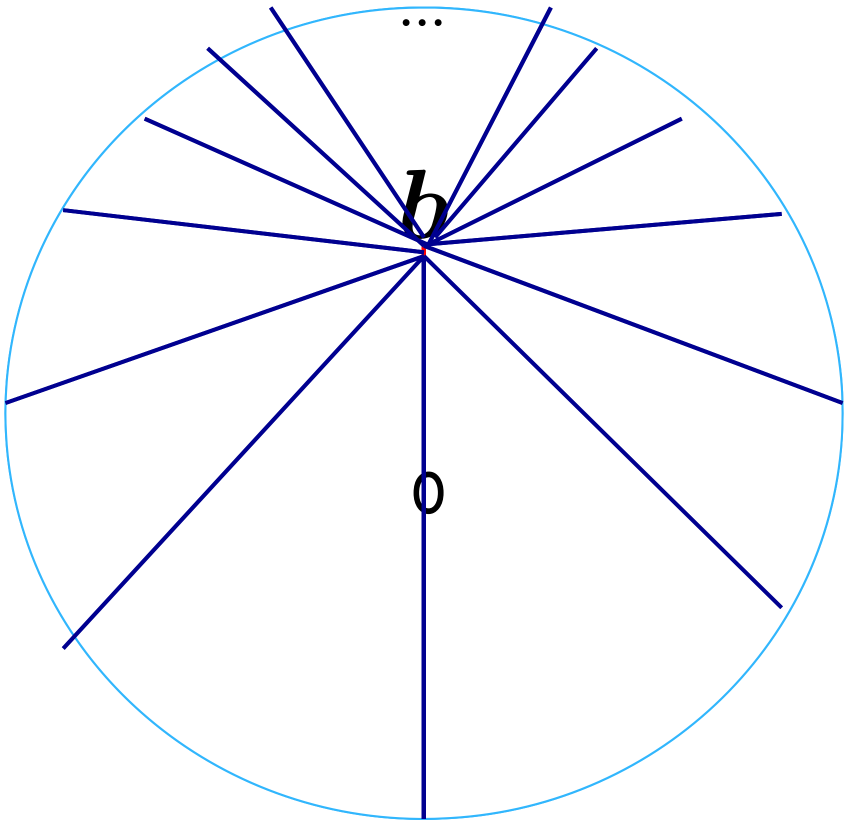

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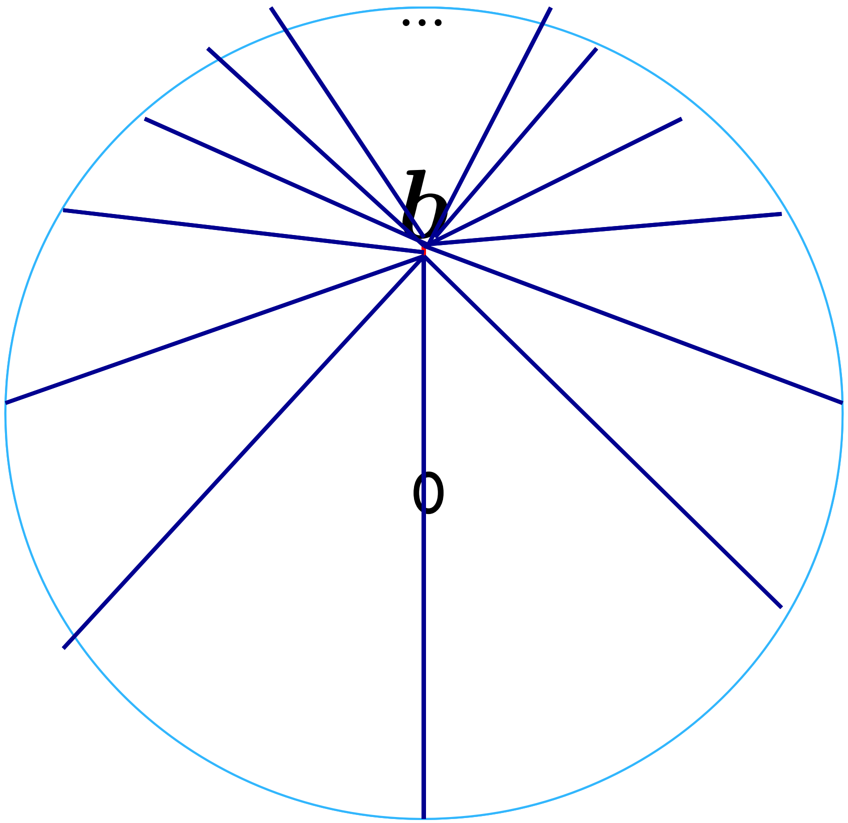

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$$\gamma(\theta)$$

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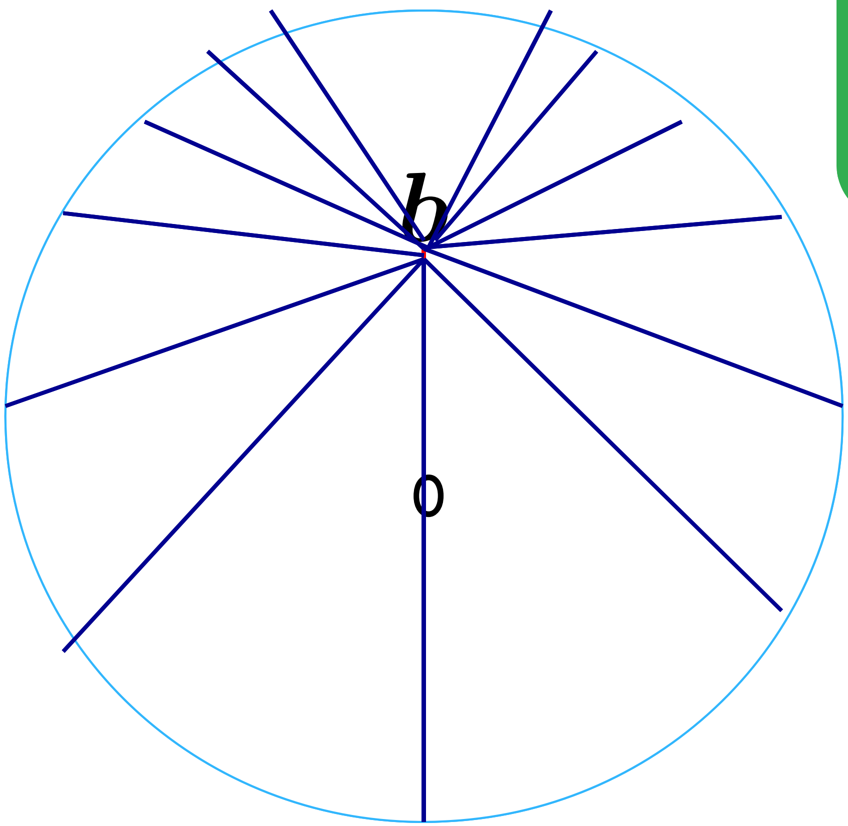

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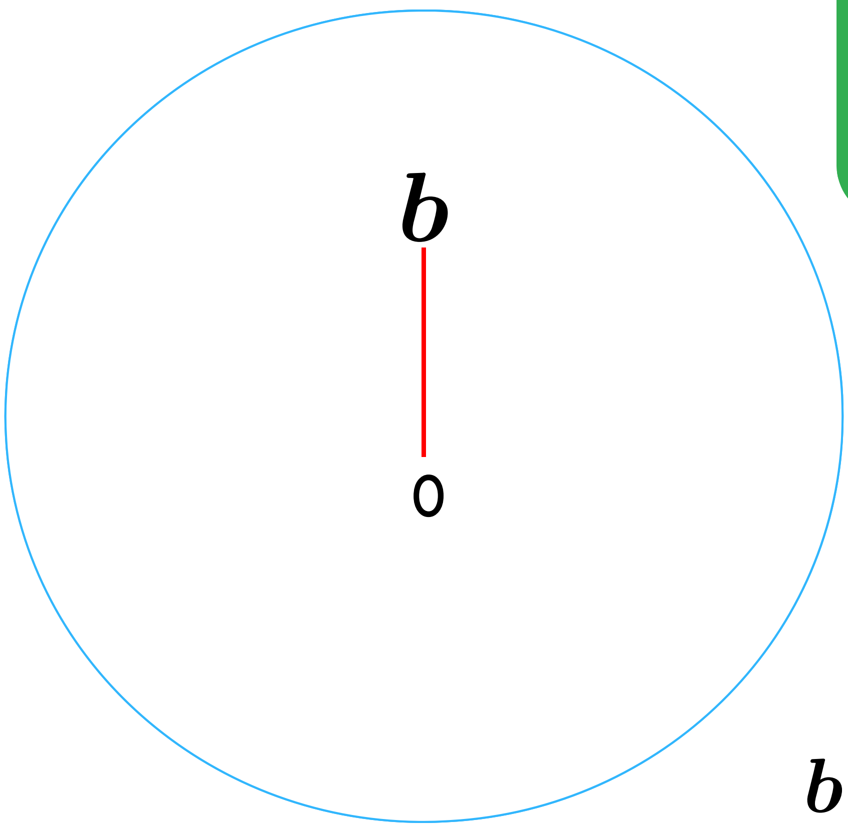

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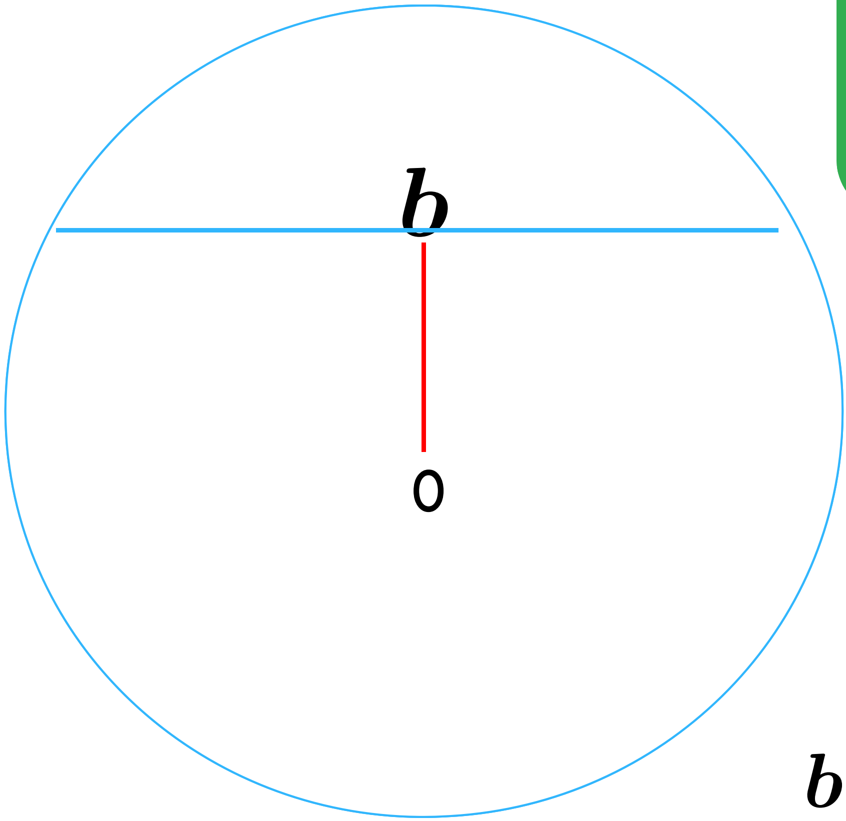

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Further work

- For two-qubit states with maximally mixed reduced states, prove our LHS model based on the QSE is *optimal*
- Can QSE be useful for developing LHS models for general two-qubit states?
- What about LHS models for POVMs? Difficult already for Werner states, but QSE is depiction of all steered states, including POVM

Thanks!