LHV models for quantum states and measurements

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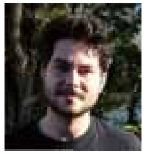
FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

NCKU Tainan, Dec 2015

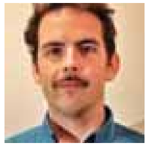
Joint work with


Joe Bowles


Flavien Hirsch

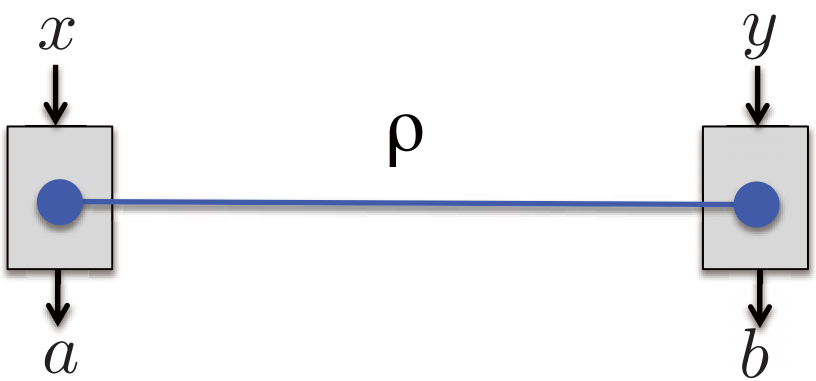

Marco Túlio Quintino


Tamás Vértesi (Debrecen)

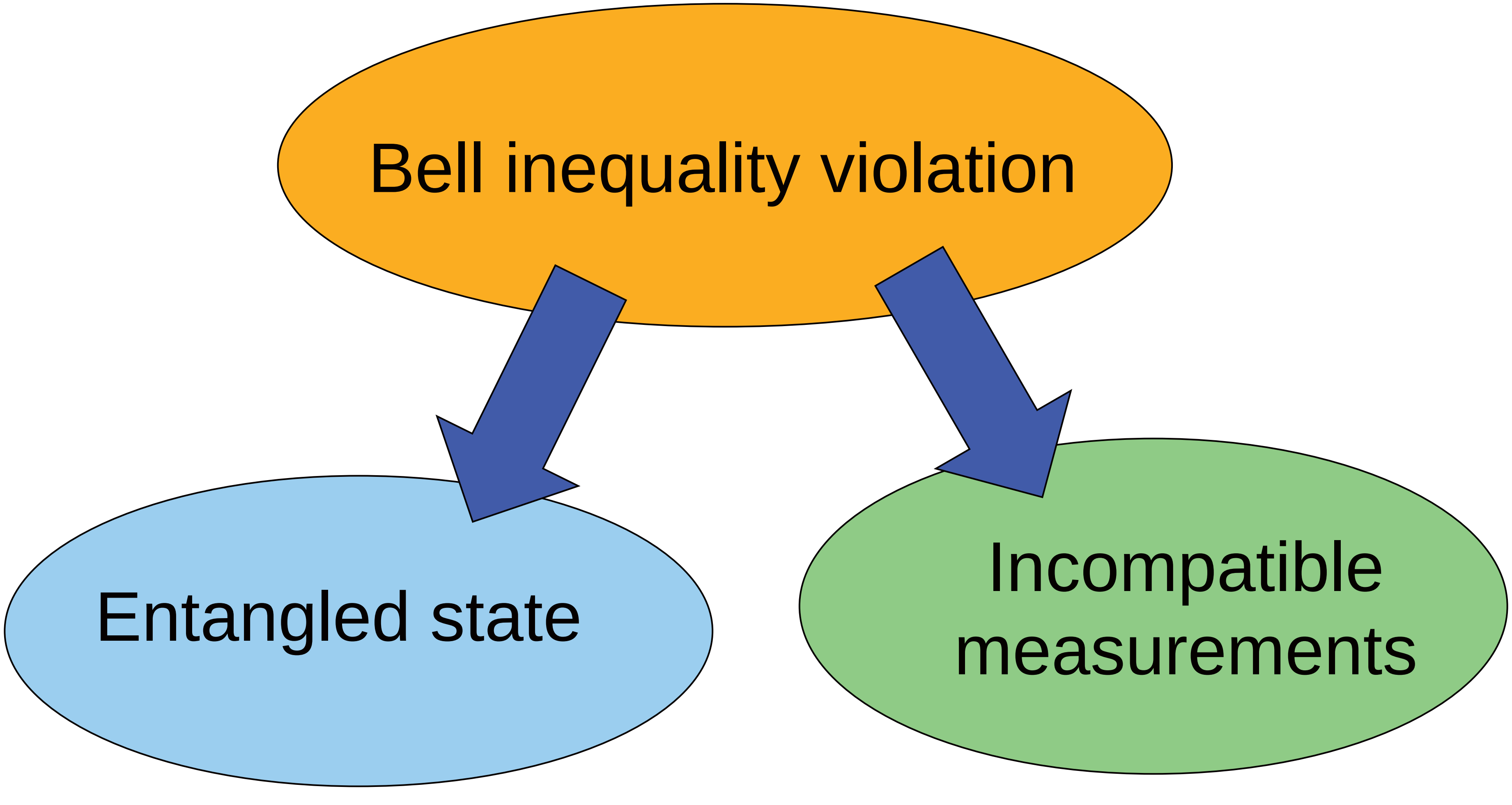

Miguel Navascués (Vienna)


Matthew Pusey (PI)

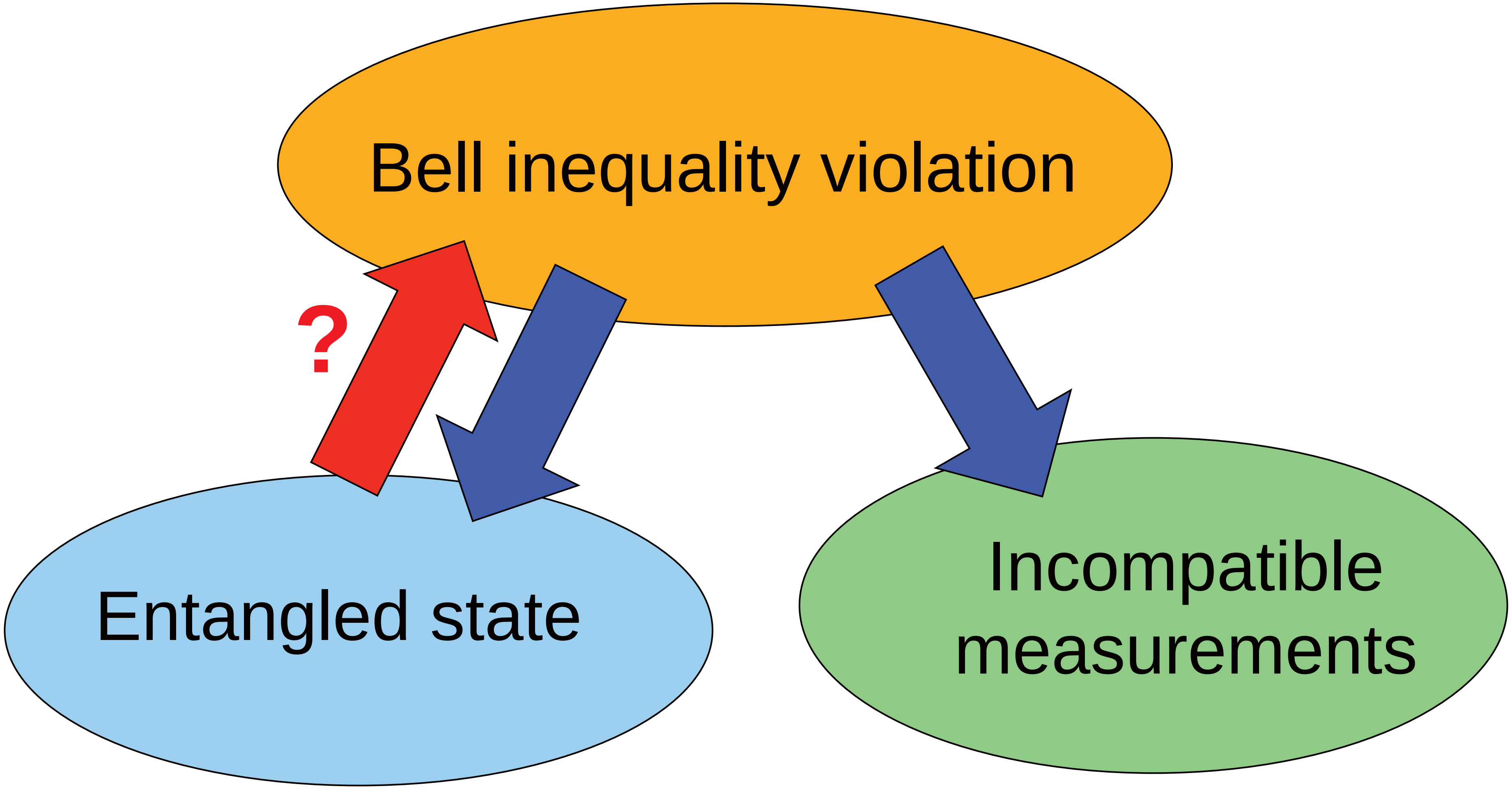
Quantum Nonlocality



Data: $p(ab|xy) = \mathrm{Tr}(\varrho M_{a|x} \otimes M_{b|y})$


Bell inequality violation
Entangled state
Incompatible measurements

Question 1


Bell inequality violation
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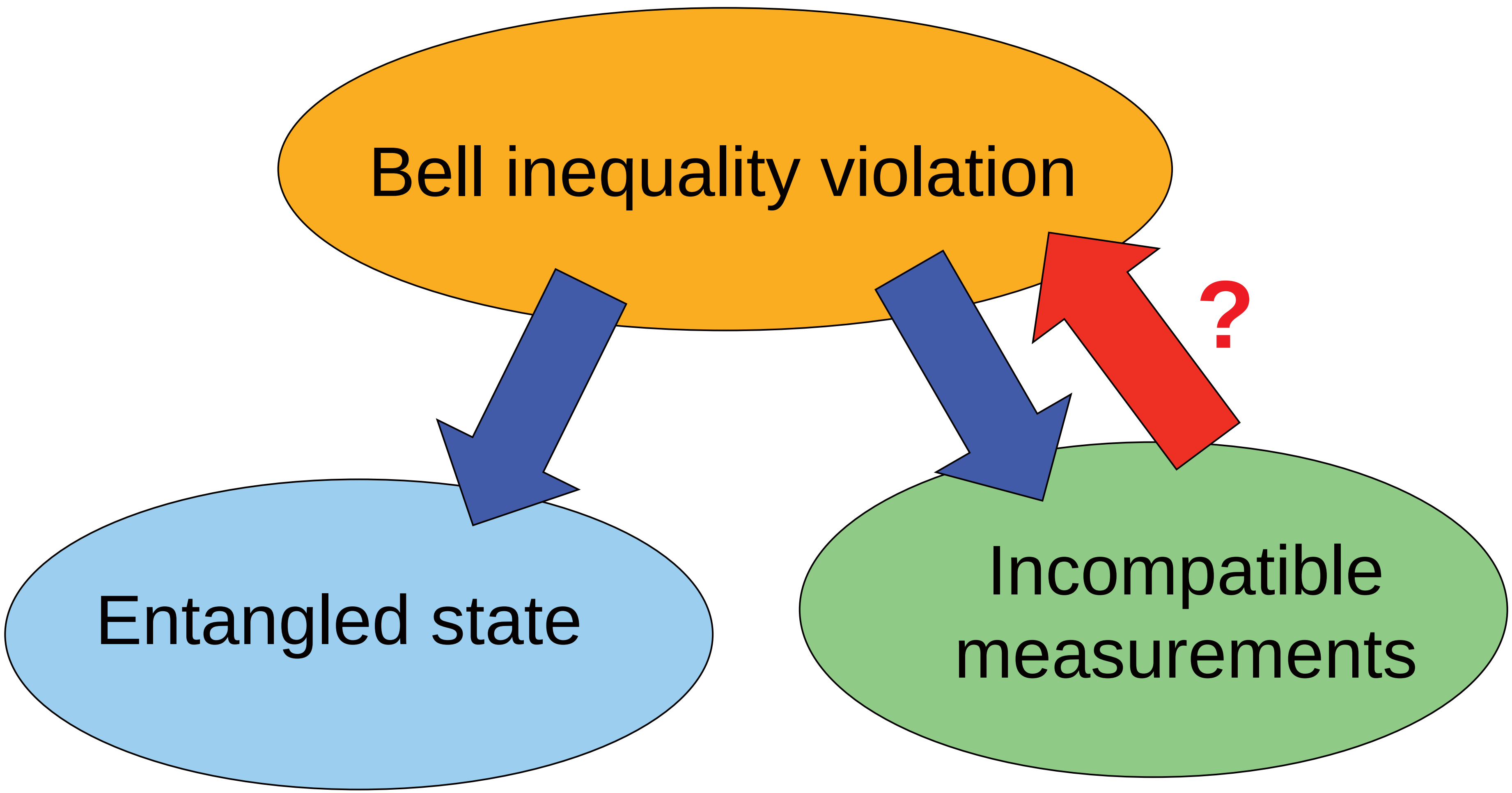
Question 2


Bell inequality violation
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Q1: LHV model for an entangled quantum state

Q1: LHV model for an entangled quantum state

Q2: LHV model for a set of incompatible quantum measurements

Question 1

General method for constructing LHV models for entangled quantum states

Hirsch et al. arxiv 2015 also: Cavalcanti et al. arxiv 2015

Question 1

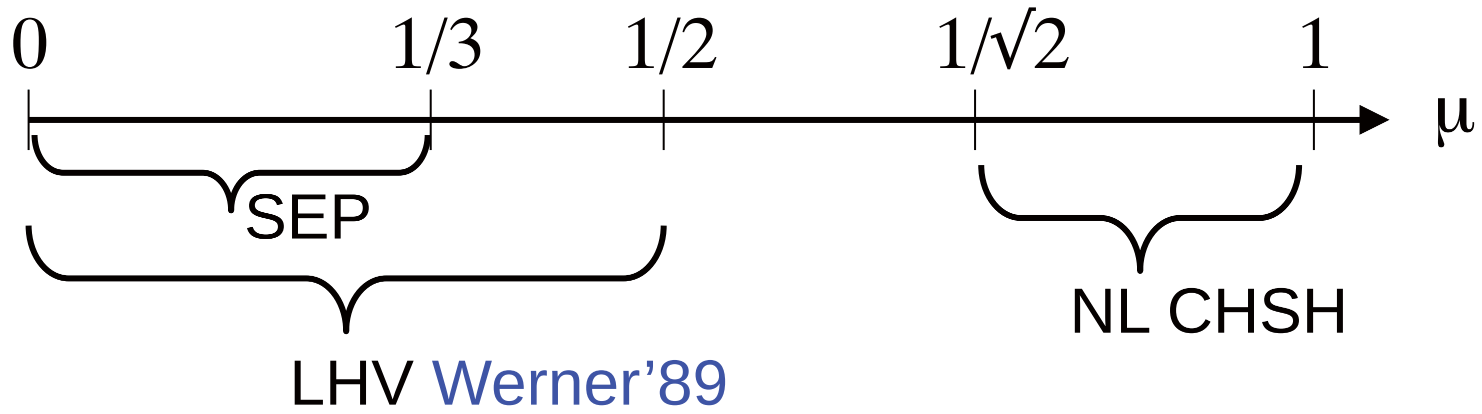
General method for constructing LHV models for entangled quantum states

- Applicable to any entangled state
- Can be implemented on a standard computer
- Converging sequence of tests

Hirsch et al. arxiv 2015 also: Cavalcanti et al. arxiv 2015

Example: Werner states

$$\rho_W^\mu = \mu \left|\phi_+\right\rangle \left\langle\phi_+\right| + (1-\mu)\frac{\mathbb{1}}{4}$$



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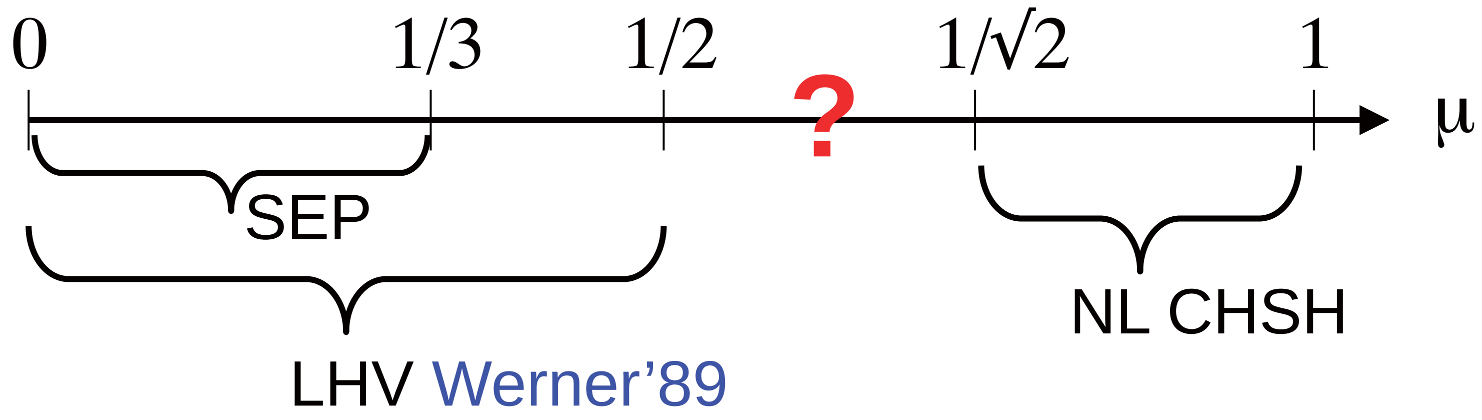

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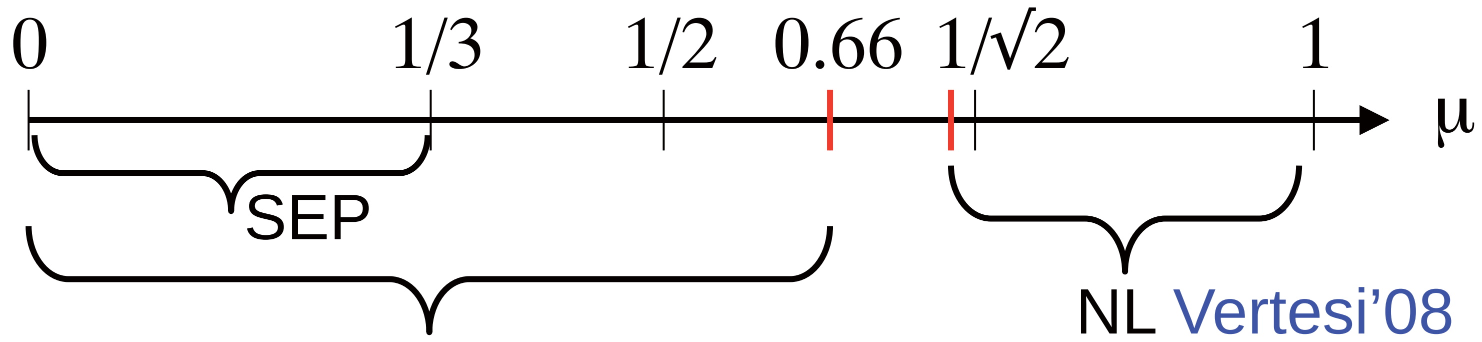

LHV Acin Gisin Toner'06

Based on Grothendieck's constant (Krivine)

Example: Werner states

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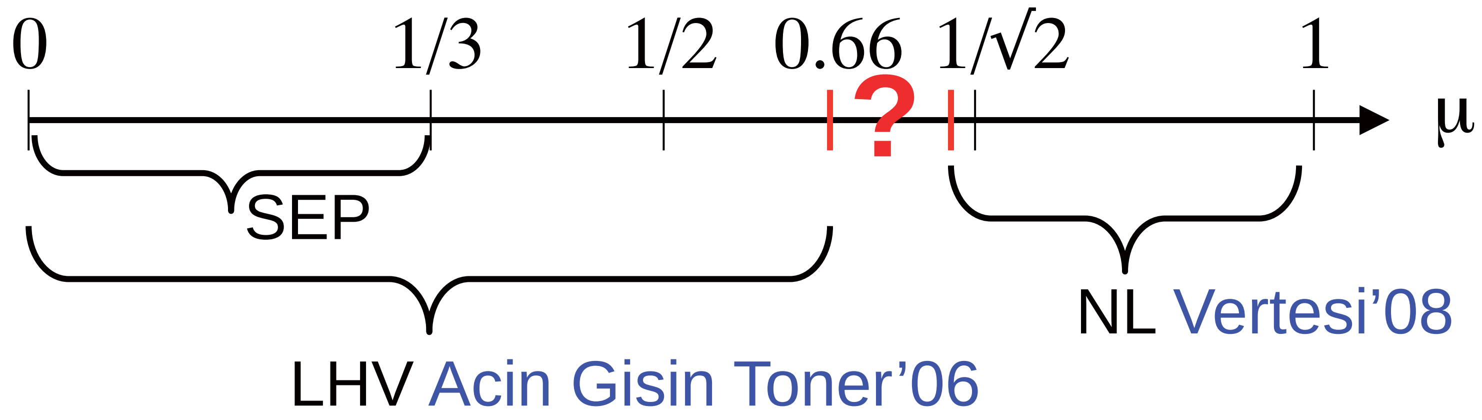


Based on Grothendieck's constant (Krivine)

What about a generic state?

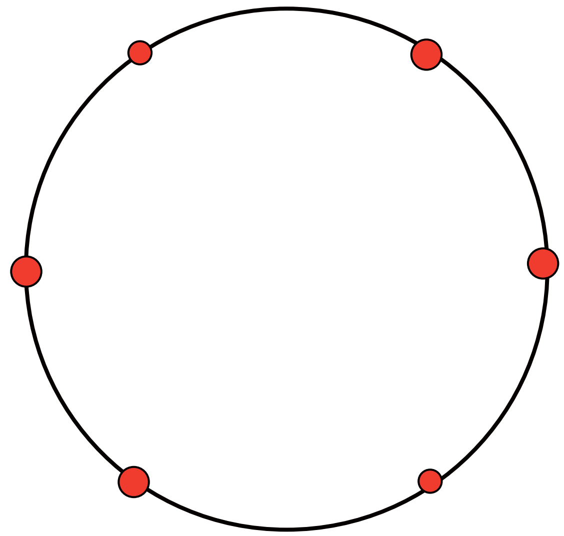
Main idea

Map the problem to a simpler one

LHV model for **ALL** measurements on ρ

LHV model for a **FINITE** set of measurements on ρ' (close to ρ)

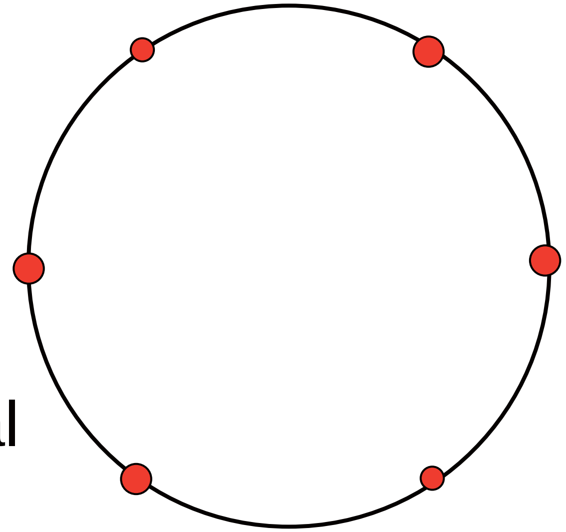
Main idea



Initial state ρ'

Take finite sets of meas $\{M_{a|x}\}$ and $\{M_{b|y}\}$

Main idea



Initial state ρ'

Take finite sets of meas $\{M_{a|x}\}$ and $\{M_{b|y}\}$

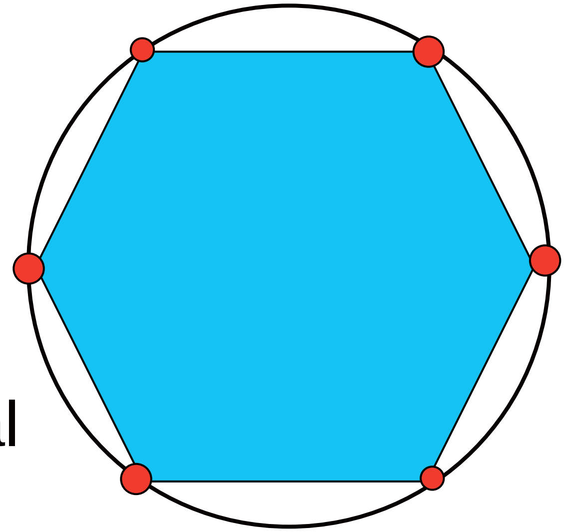
Check $p(a,b|xy) = tr(\rho' M_{a|x} M_{b|y})$ is local

Main idea

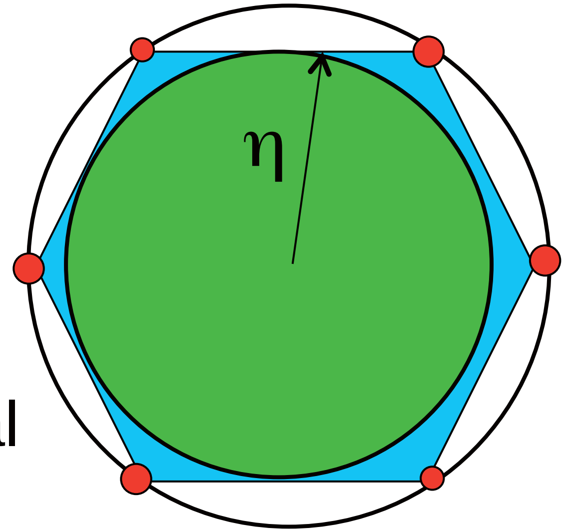
<u>Initial state</u> ρ'

Take finite sets of meas $\{M_{a|x}\}$ and $\{M_{b|y}\}$

Check $p(a,b|xy) = tr(\rho' \, M_{a|x} \, M_{b|y})$ is local



Main idea



Initial state ρ'

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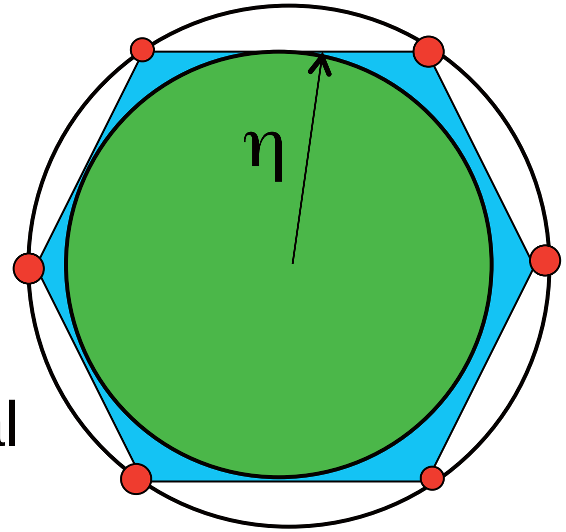
Check $p(a,b|xy) = tr(\rho' M_{a|x} M_{b|y})$ is local

→ ρ' is local for all **noisy measurements**

$$M^{\eta}_{\pm|\vec{x}} = \frac{1}{2}(\mathbb{1} \pm \eta\, \hat{x} \cdot \vec{\sigma})$$

Main idea



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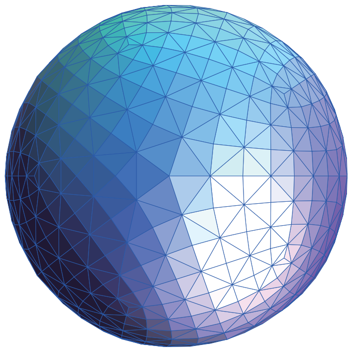
⇒ ρ' is local for all **noisy measurements**

$$M^{\eta}_{\pm|\vec{x}} = \frac{1}{2}(\mathbb{1} \pm \eta\, \hat{x} \cdot \vec{\sigma})$$

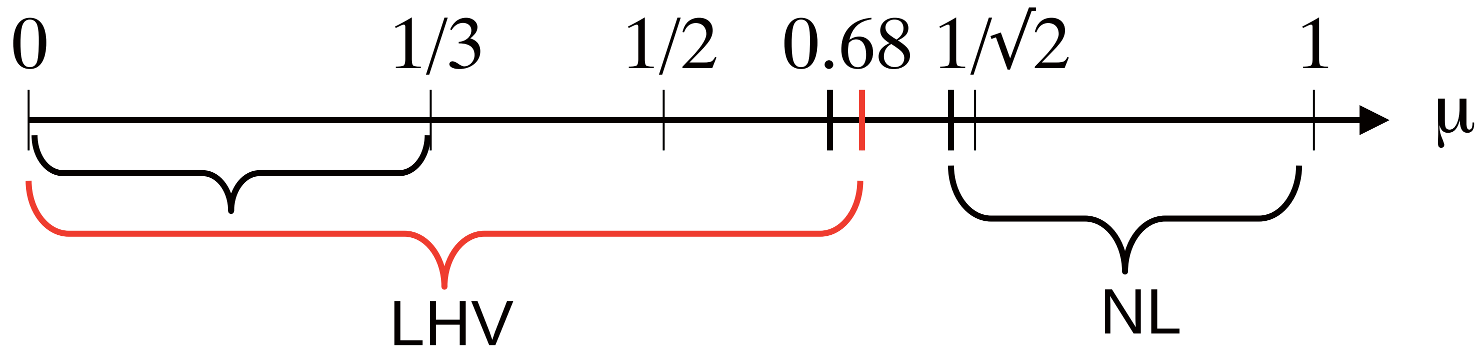
⇒ Noisy version of ρ' is local for all pure meas

Target state $\rho = \eta^2 \rho + (1-\eta^2)\, \sigma_{sep}$

Application: Werner states

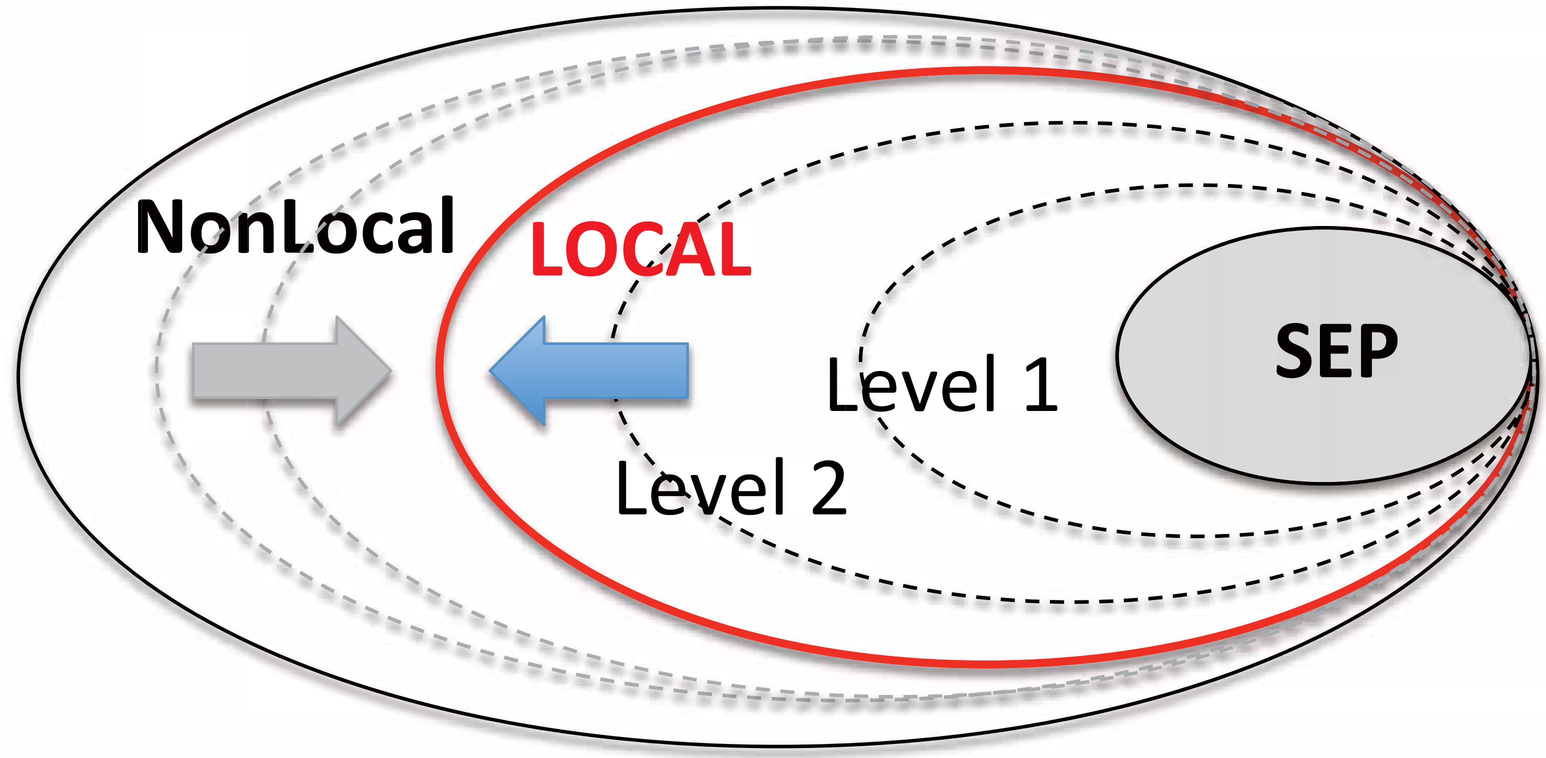


$$\rho_W^\mu = \mu \left|\phi_+\right\rangle \left\langle\phi_+\right| + (1-\mu)\frac{\mathbb{1}}{4}$$



➡ Better bound on Grothendieck constant K_3

Iterative procedure



To be explored

- POVMs
- Higher dimensions
- Multipartite systems

Question 2

Can all incompatible sets of measurements lead to Bell violation?

The problem

Incompatibility → Bell violation ??

The problem

Incompatibility → Bell violation ??

Projective measurements

Incompatibility (commutativity) → CHSH violation

Khalfin Tsirelson'85

The problem

Incompatibility → Bell violation ??

<u>Projective measurements</u>

Incompatibility (commutativity) → CHSH violation

Khalfin Tsirelson'85

What about POVMs?

Joint measurability

POVM $\{M_a\}$ & $\{M_b\}$ are JM if there is joint POVM $\{C_{ab}\}$
s.t. $M_a = \Sigma_b\, C_{ab}$ & $M_b = \Sigma_a\, C_{ab}$

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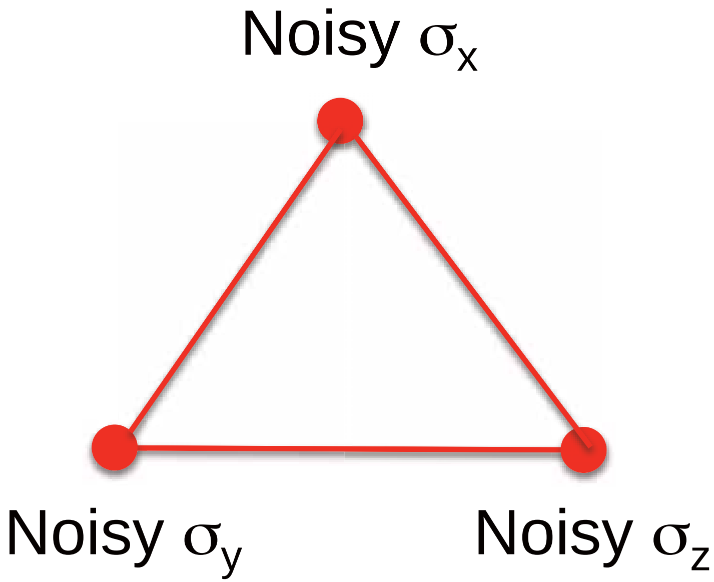
POVM $\{M_a\}$ & $\{M_b\}$ are JM if there is joint POVM $\{C_{ab}\}$ s.t. $M_a = \Sigma_b C_{ab}$ & $M_b = \Sigma_a C_{ab}$

Example: Noisy Pauli meas

$$M^{\eta}_{\pm|\vec{x}} = \frac{1}{2}(\mathbb{1} \pm \eta\, \hat{x} \cdot \vec{\sigma})$$

Partial JM does not imply fully JM



Heinossari'08, Liang et al'11

JM vs nonlocality

- 2 binary POVMs Wolf et al. 2007

  Incompatibility → CHSH violation

JM vs nonlocality

- 2 binary POVMs Wolf et al. 2007

  Incompatibility → CHSH violation

- Incompatibility ⟺ steering

  Quintino et al & Uola et al ’14

Here

⇒ Construct a LHV model for some $\mathcal{M}$

Our candidate

Continuous set of noisy qubit measurements

$\mathcal{M} = \{M^{\eta}_{\pm|\hat{x}}\}$ for all Bloch vectors $\vec{x}$

where $M^{\eta}_{\pm|\vec{x}} = \frac{1}{2}(\mathbb{1} \pm \eta\, \hat{x} \cdot \vec{\sigma})$

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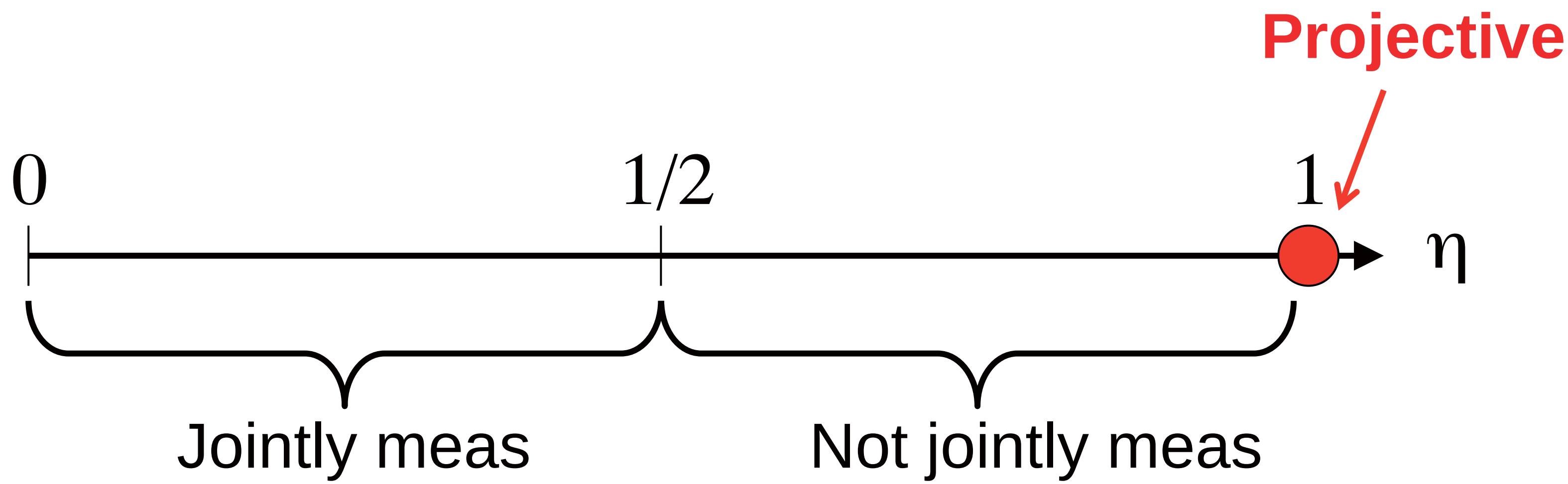

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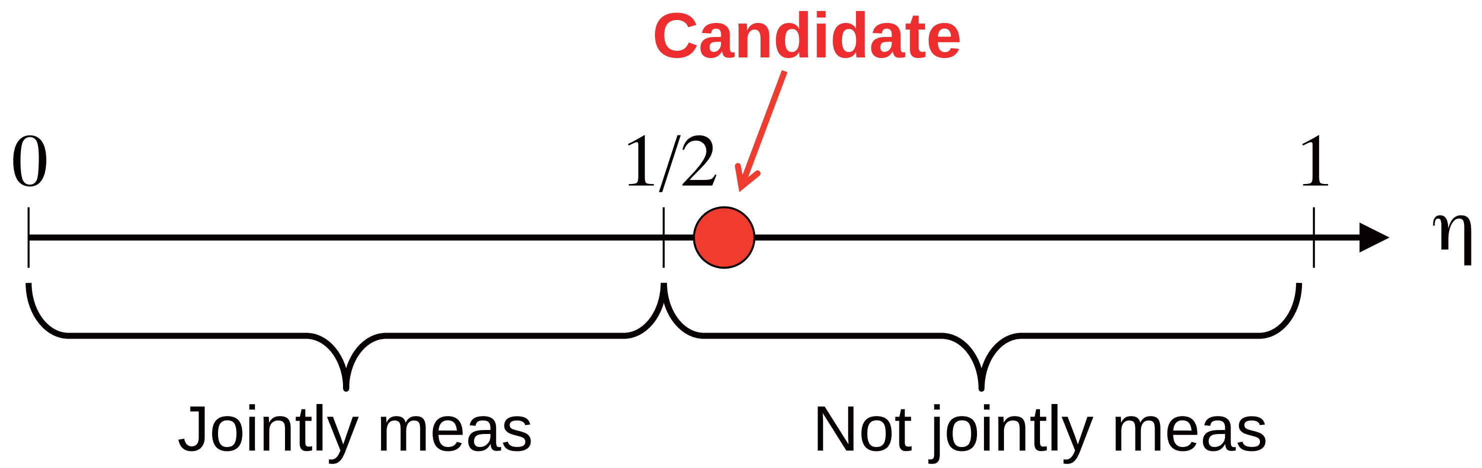
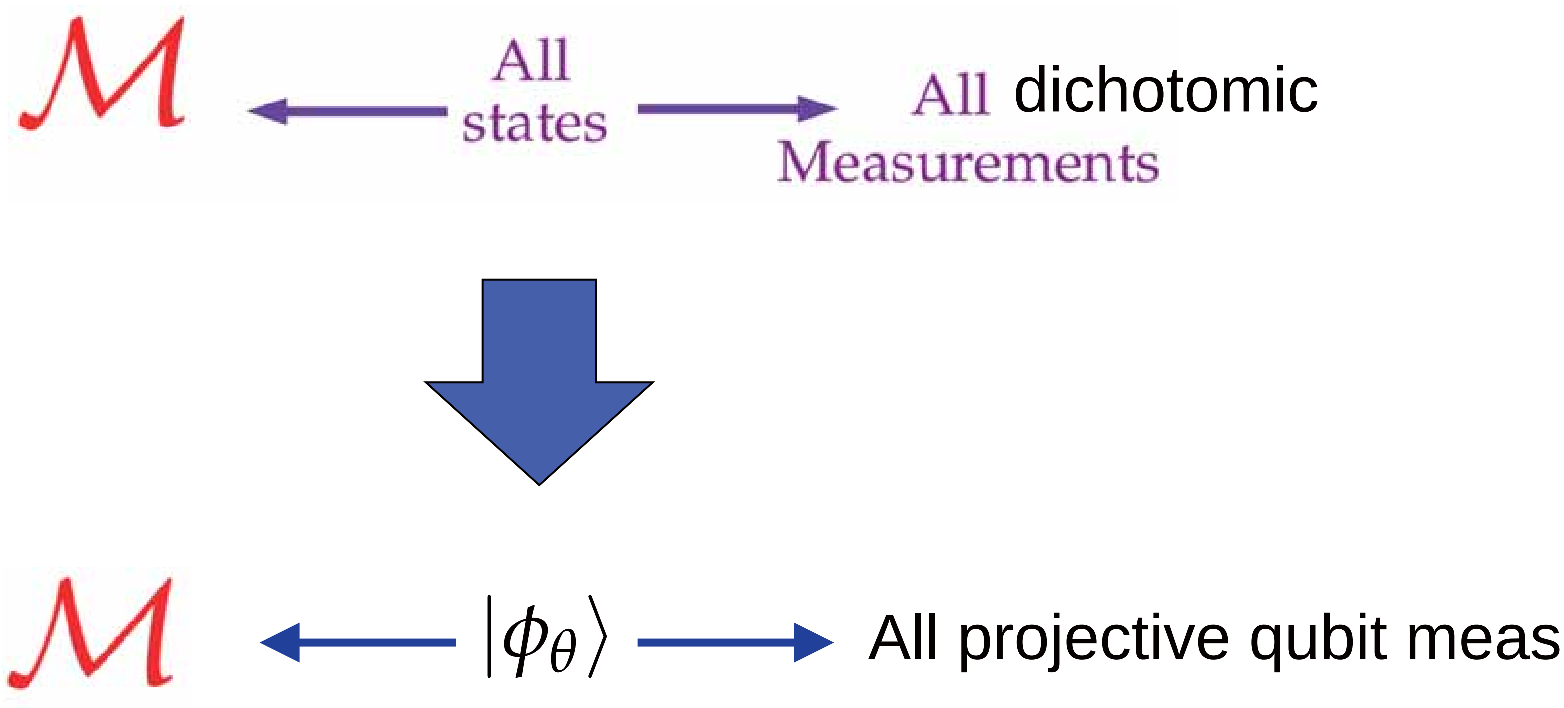


$\mathcal{M}$
All states
All dichotomic Measurements

Step 1



Schmidt form $|\phi_\theta\rangle = \cos\theta\,|00\rangle + \sin\theta\,|11\rangle$

Step 2

$\mathcal{M}$ ⟵ $|\phi_\theta\rangle$ ⟶ All projective meas $\Pi_{b|y}$

To show: $p(ab|xy) = \mathrm{tr}(|\phi_\theta\rangle\langle\phi_\theta| M^{\eta}_{a|\hat{x}} \otimes \Pi_{b|\hat{y}})$ is local

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$$\mathrm{tr}(|\phi_\theta\rangle\langle\phi_\theta| M^\eta_{a|\hat{x}} \otimes \Pi_{b|\hat{y}}) = \mathrm{tr}(\rho^\eta_\theta \Pi_{a|\hat{x}} \otimes \Pi_{b|\hat{y}})$$

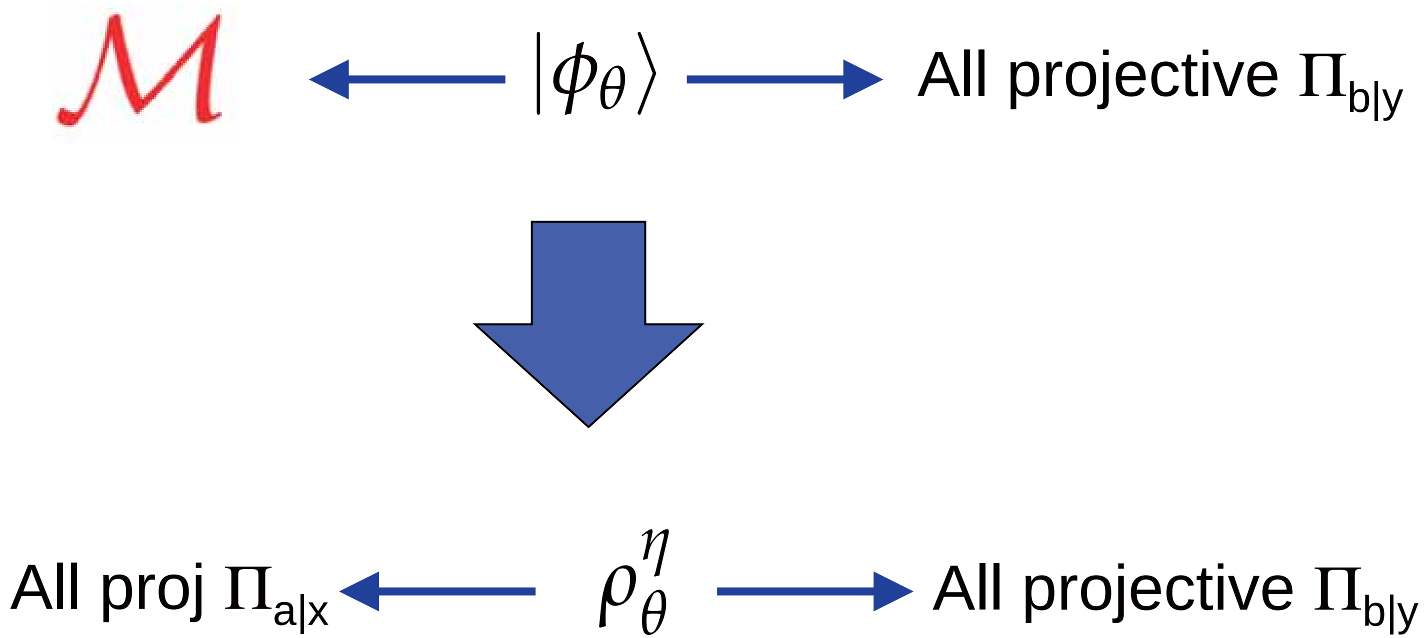
noisy meas
pure state

⟺

pure meas
noisy state

$$\rho^\eta_\theta = \eta\,|\phi_\theta\rangle\langle\phi_\theta| + (1-\eta)\frac{\mathbb{1}}{2} \otimes \rho_B$$

Step 2



$\mathcal{M}$
$|\phi_\theta\rangle$
All projective $\Pi_{b|y}$
All proj $\Pi_{a|x}$
ρ_θ^η
All projective $\Pi_{b|y}$

Step 2

$\mathcal{M} \longleftarrow |\phi_\theta\rangle \longrightarrow$ All projective $\Pi_{b|y}$

$\Downarrow$

All proj $\Pi_{a|x} \longleftarrow \rho_\theta^\eta \longrightarrow$ All projective $\Pi_{b|y}$

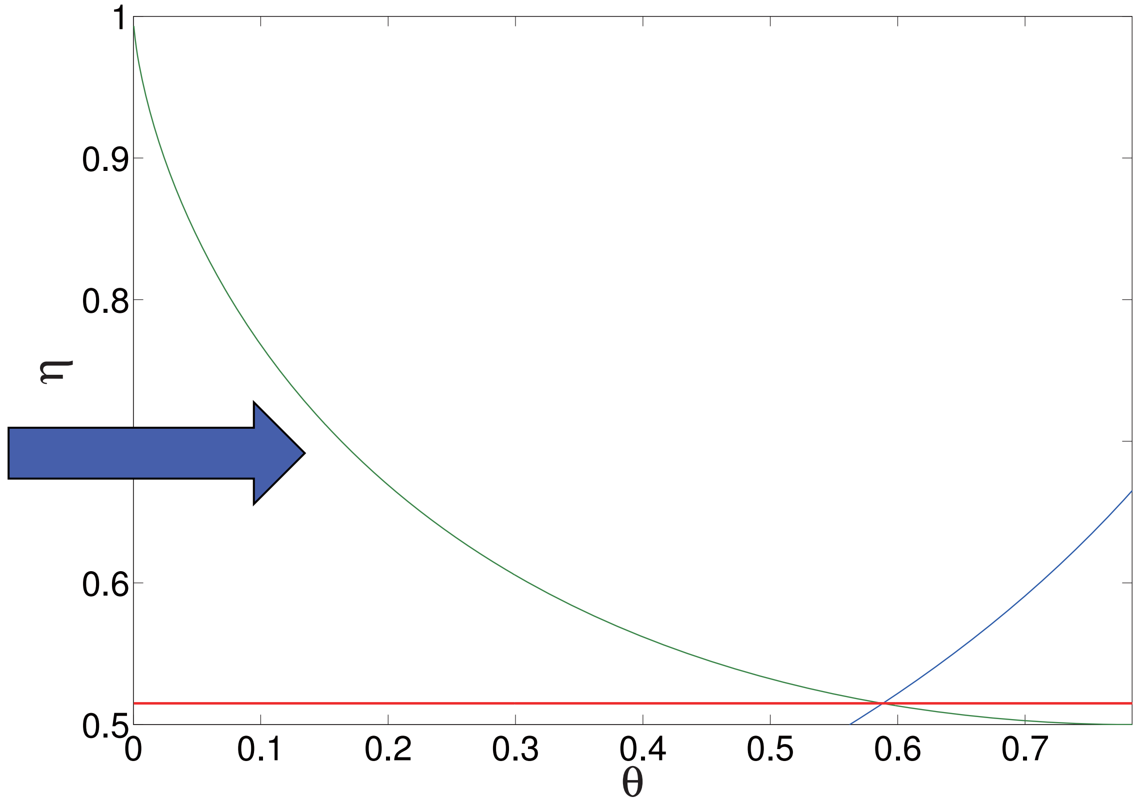
$\mathcal{M}$ is local $\Longleftrightarrow$ ρ_θ^η is local

Step 3

Show that $\rho_\theta^\eta = \eta\,|\phi_\theta\rangle\langle\phi_\theta| + (1-\eta)\frac{\mathbb{1}}{2}\otimes\rho_B$

is local for η > ½ and for all $\theta \in [0, \pi/4]$.

J. Bowles' talk



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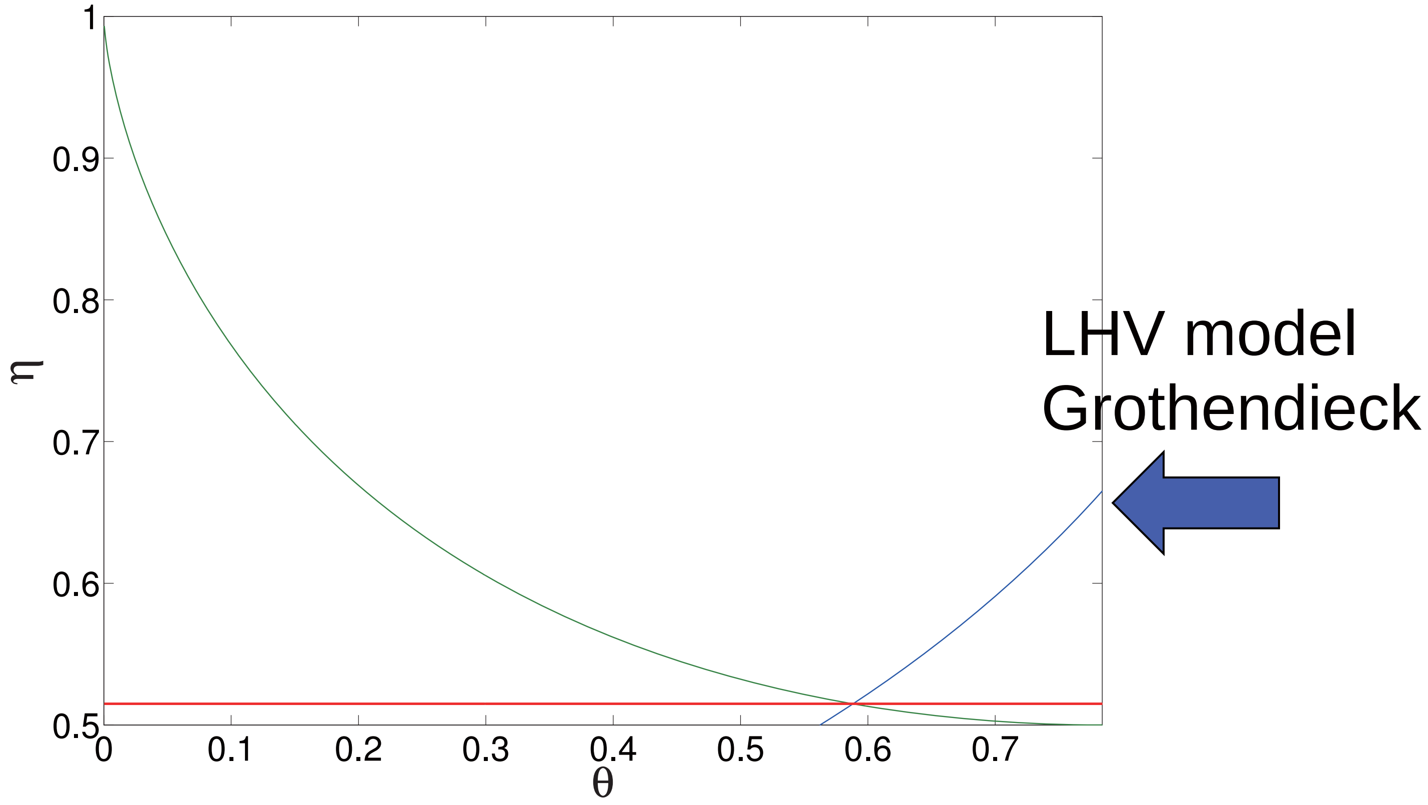

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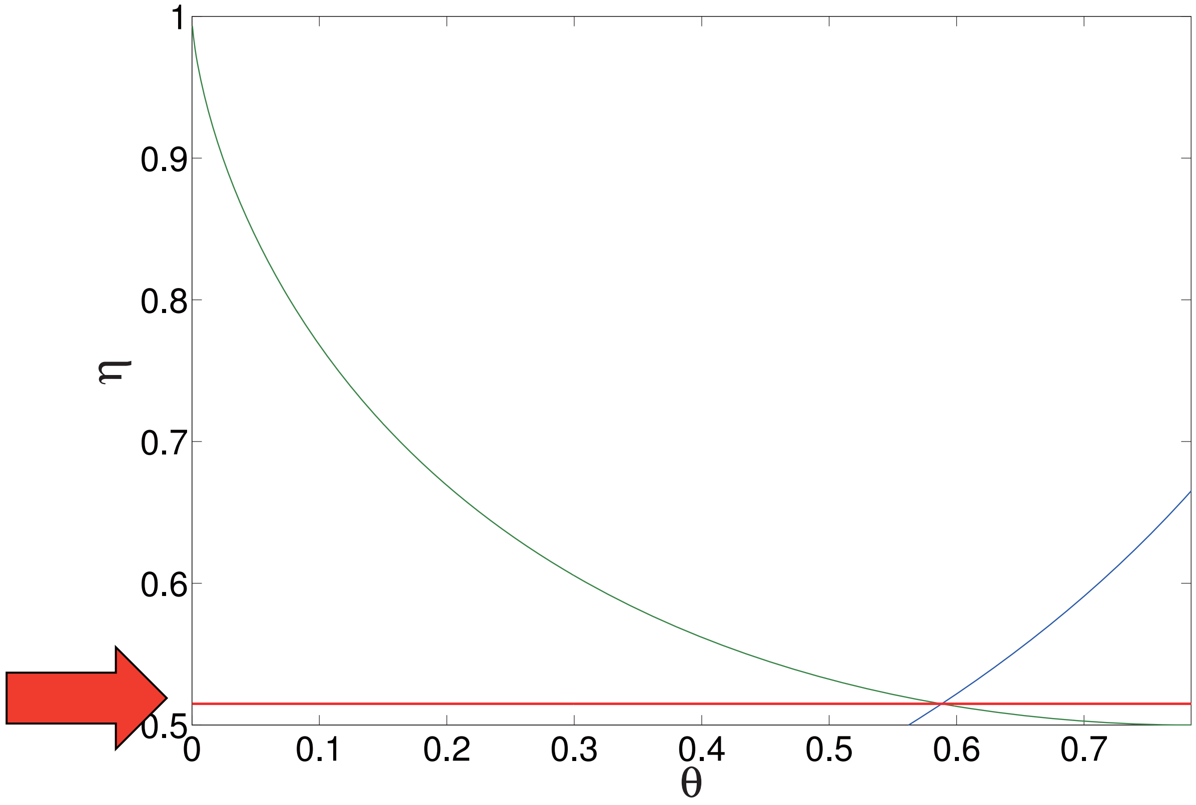
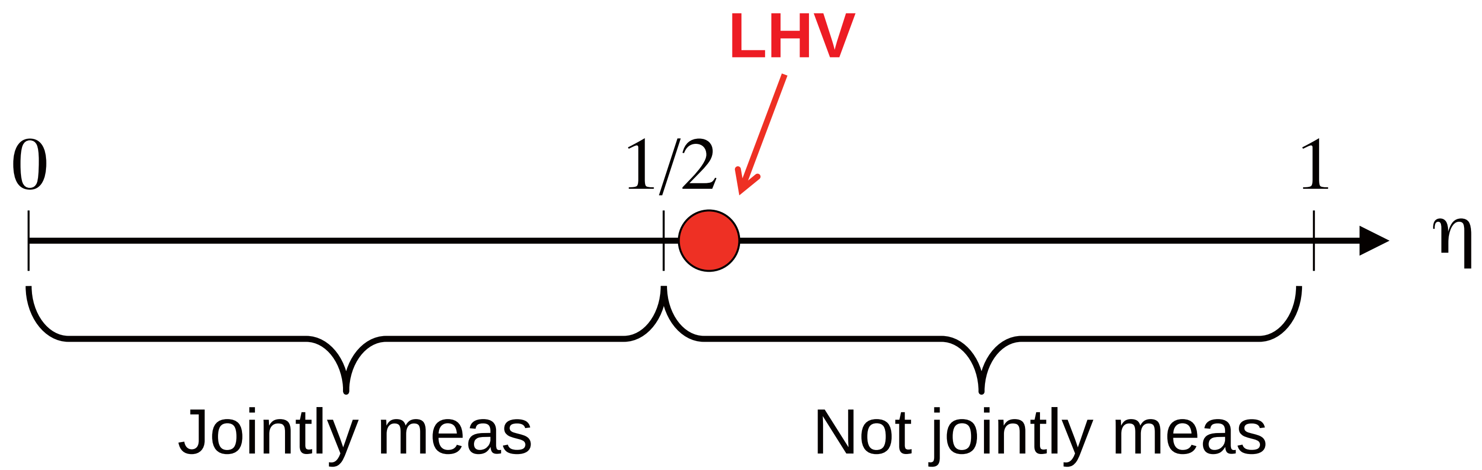
η ≈ 0.51



Summary

LHV model for a set of incompatible POVMs

$$\mathcal{M}_A^\eta = \{M^\eta_{\pm|\hat{x}}\}$$ for all Bloch vectors $\vec{x}$



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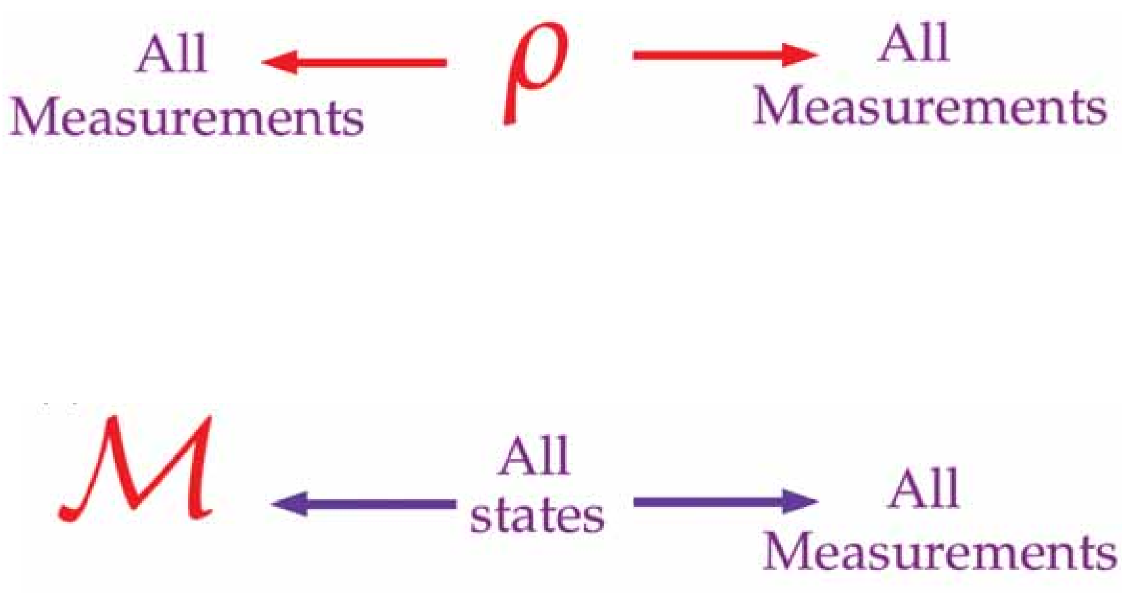
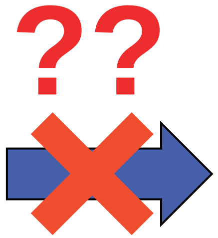


Open questions

- LHV model for set of few measurements
- Incompatibility (JM) 

Bell nonlocality
- Notion of incompatibility corresponding to Bell nonlocality
- Activation