# A guided tour from locality to noncontexuality (and back again)

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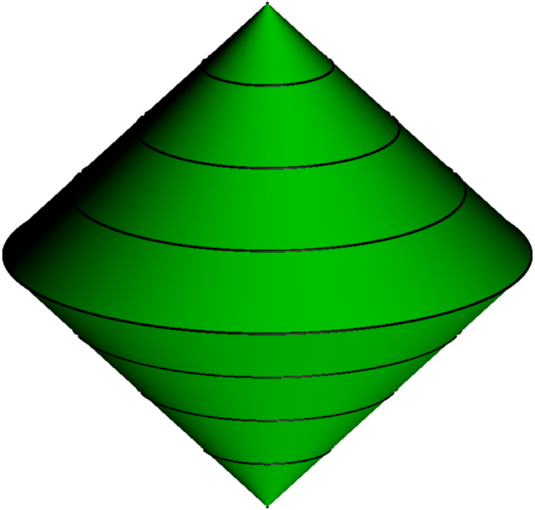
with Rob Spekkens

# Entanglement witnesses

$$\mathrm{tr}\,((X \otimes X)\rho) + \mathrm{tr}\,((Z \otimes Z)\rho) \leq 1$$

# Trusted measurements



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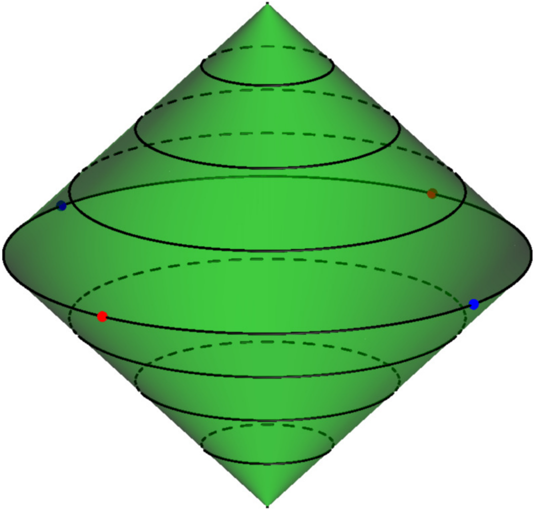
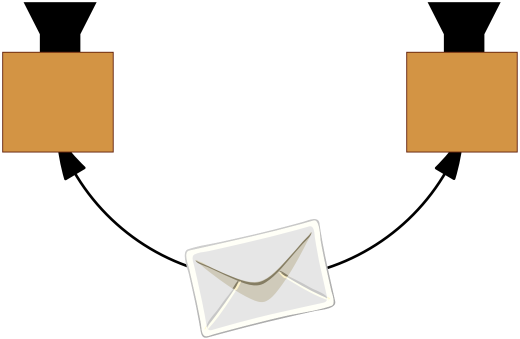


# Untrusted measurements

# Classical correlations

| Alice | Bob | Name |
|---|---|---|
| Trusted | Trusted | Separability |
| Trusted | Untrusted | Unsteerability |
| Untrusted | Untrusted | Bell locality |

# Time-like case



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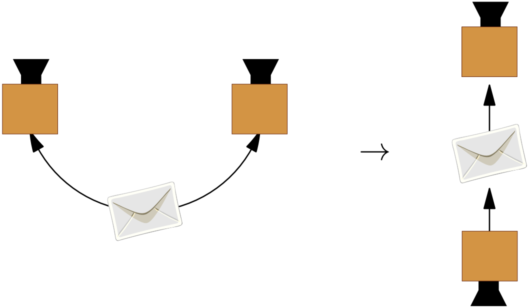
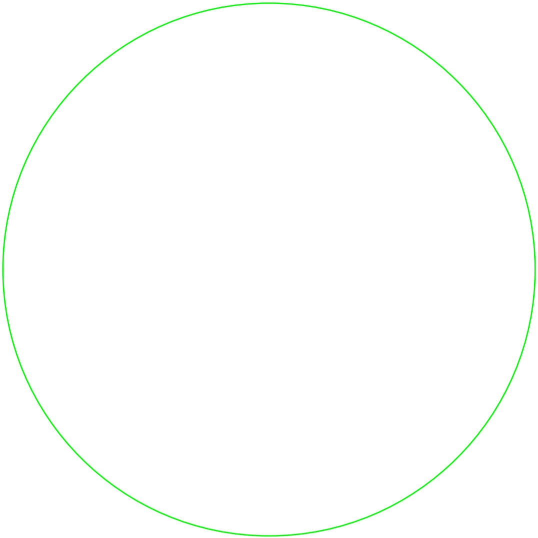


# Trusted preparation



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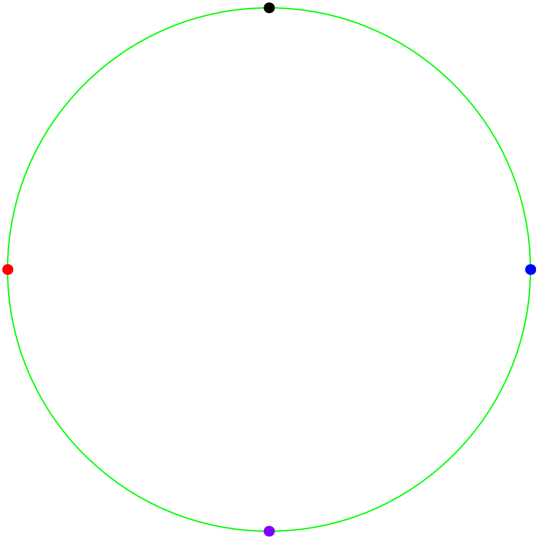

# Classical channels

| Input | Output | Name |
|---|---|---|
| Trusted | Trusted | Entanglement-breaking |
| Trusted | Untrusted | Jointly measurable[1] |
| Untrusted | Trusted | Anything! |
| Untrusted | Untrusted | Anything! |

[1] arXiv:1502.03010

# Generalising trust

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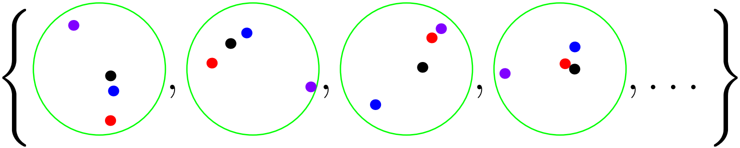

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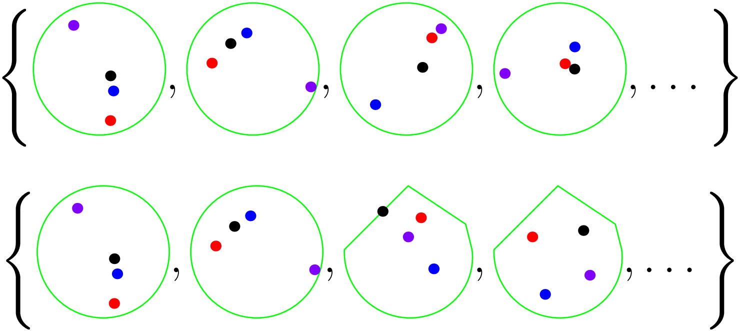

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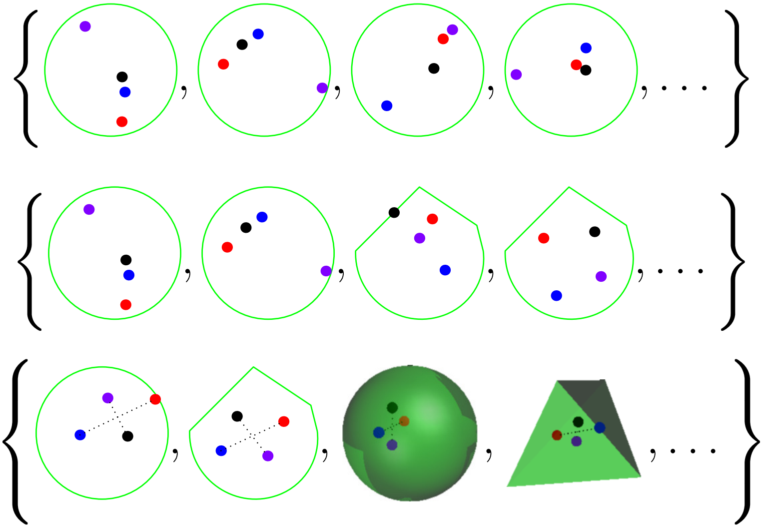

# Classical channels

| Input | Output | Name |
| --- | --- | --- |
| Trusted | Trusted | Entanglement-breaking |
| Trusted | Untrusted | Jointly measurable |
| Untrusted | Trusted | Anything! |
| Untrusted | Untrusted | Anything! |

# Classical channels

| Input | Output | Name |
|---|---|---|
| G-trusted | G-trusted | G-entanglement-breaking |
| G-trusted | Untrusted | G-jointly-measurable |
| Untrusted | G-trusted | Anything! |
| Untrusted | Untrusted | Anything! |

# Classical channels

| Input | Output | Name |
|---|---|---|
| G-trusted | G-trusted | Noncontextual |
| G-trusted | Untrusted | Preparation noncontextual |
| Untrusted | G-trusted | Anything![2] |
| Untrusted | Untrusted | Anything! |

[2]Measurement noncontextual

# Ontological models?

$$p(k|\mathcal{P},\mathcal{M}) = \int p(\lambda|\mathcal{P})p(k|\lambda,\mathcal{M})d\lambda$$

[3] cf Busch quant-ph/9909073

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$p(k|\lambda,\mathcal{M})$ are probabilities that respect coarse-graining and mixtures of $\mathcal{M}$.

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A noncontextual $p(k|\lambda, \mathcal{M})$ only depends on the operational equivalence class of $\mathcal{M}$.

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A noncontextual $p(k|\lambda,\mathcal{M})$ only depends on the operational equivalence class of $\mathcal{M}$.
$\implies p(k|\lambda,\cdot)$ is a state for each $\lambda$.[3]

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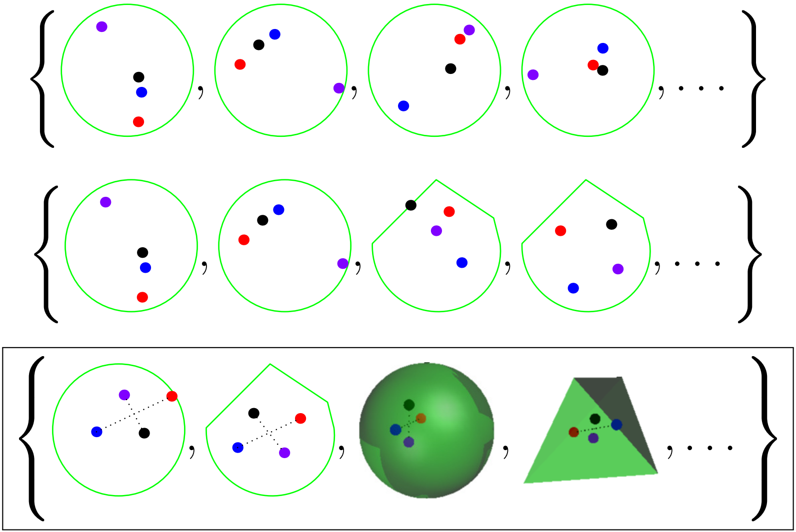
$p(\lambda|\mathcal{P})$ are probabilities that respect mixtures of $\mathcal{P}$.

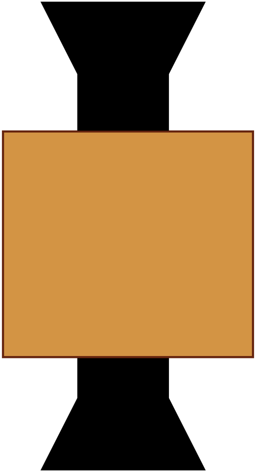
A noncontextual $p(\lambda|\mathcal{P})$ only depends on the operational equivalence class of $\mathcal{P}$.

$\implies p(\lambda|\cdot)$ is a measurement with outcome $\lambda$.

# Traditional g-trust



# What about transformations?



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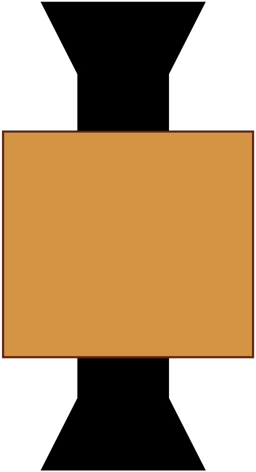

$\to p(\lambda'|\lambda)$

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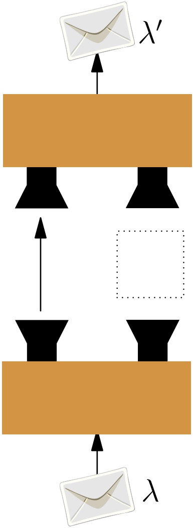


Chiribella *et. al.*, arXiv:0804:0180

# A closer analogy: $NS$-trust

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Label preparations $(a, x)$

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Define conditional probabilities $p(a|x)$

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Label preparations $(a, x)$
Define conditional probabilities $p(a|x)$
Trust that $\sum_a p(a|x)\mathcal{P}_{a,x}$ independent of $x$

# A closer analogy: $NS$-trust

| Input | Output | Name |
|---|---|---|
| Trusted | Trusted | Entanglement-breaking |
| Trusted | Untrusted | Jointly measurable |
| NS-trusted | Trusted | Steering-like[4] |
| NS-trusted | Untrusted | Bell-like |

[4] Chen *et. al.* arXiv:1310.4970

# Summary

Summary: Preparation and measurement noncontextual model $\iff$ measure-and-prepare channel between g-trusted devices.

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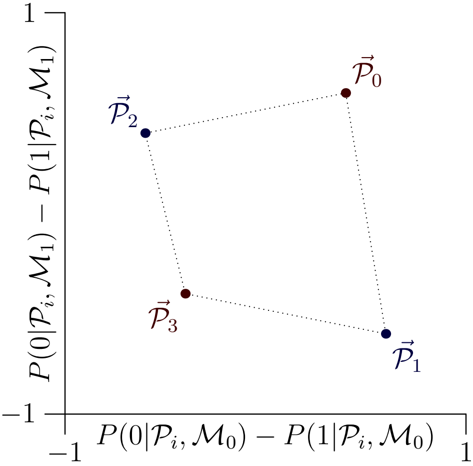
# Two binary measurements on four preparations, arXiv:1506.04178



# Two binary measurements on four preparations, arXiv:1506.04178

$$P(b|\mathcal{P}_i, \mathcal{M}_y) \text{ noncontextual}$$
$$\Updownarrow$$
$$P(a, b|x, y) \text{ Bell-local}$$

# Two binary measurements on four preparations, arXiv:1506.04178

$$P(b|\mathcal{P}_i, \mathcal{M}_y) \text{ noncontextual}$$
$$\Updownarrow$$
$$P(a, b|x, y) \text{ Bell-local}$$

1. Calculate $p, q$
2. Convert $P(b|\mathcal{P}_i, \mathcal{M}_y)$ to $P(a, b|x, y)$
3. Plug into CHSH

# Two binary measurements on four preparations, arXiv:1506.04178

$$\begin{vmatrix} x_0 & y_0 & x_0 + y_0 - 1 & 1 \\ x_1 & y_1 & -x_1 + y_1 + 1 & 1 \\ x_2 & y_2 & x_2 - y_2 + 1 & 1 \\ x_3 & y_3 & -x_3 - y_3 - 1 & 1 \end{vmatrix} \leq 0.$$

Where

$$x_i = P(0|\mathcal{P}_i, \mathcal{M}_0) - P(1|\mathcal{P}_i, \mathcal{M}_0)$$

$$y_i = P(0|\mathcal{P}_i, \mathcal{M}_1) - P(1|\mathcal{P}_i, \mathcal{M}_1)$$