Causality, Consistency, Complexity

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Based on collaboration and discussion with:
Mateus Araújo, Veronika Baumann, Ämin Baumeler, Harvey Brown, Caslav Brukner, Sandro Coretti, Bora Dakic, Paul Erker, Adrien Feix, Jürg Fröhlich, Nicolas Gisin, Esther Hänggi, Arne Hansen, Marcus Huber, Alberto Montina, Benno Salwey, and Andreas Winter

Tainan, Taiwan, December 2015

Overview

Causality

Overview

Causality

Correlations

Overview

Causality

Reichenbach's principle

Correlations

Overview

Causality

Reichenbach's principle

Correlations

The law of causality [...] is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm

Bertrand Russell

1913

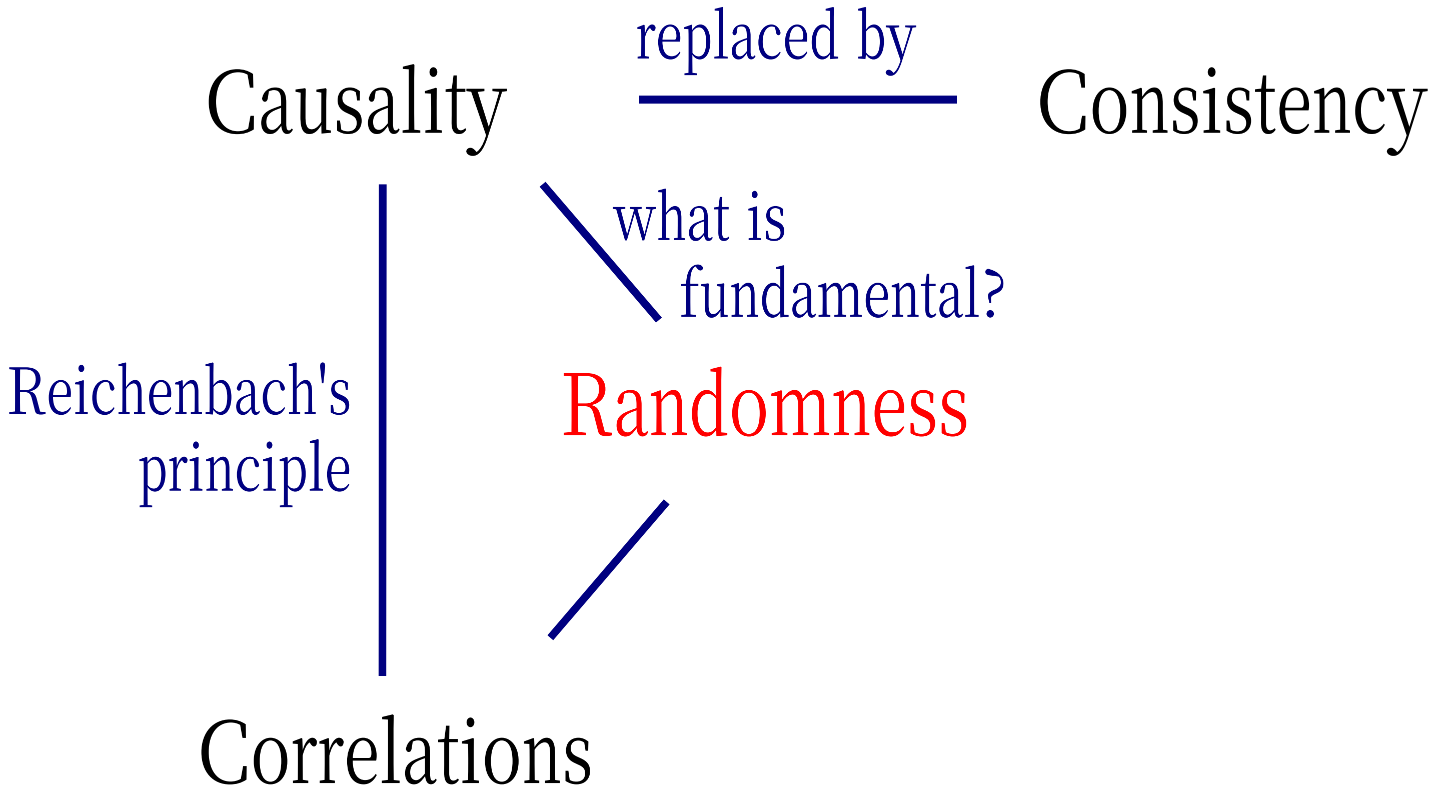
Overview

Causality —replaced by— Consistency

Reichenbach's principle

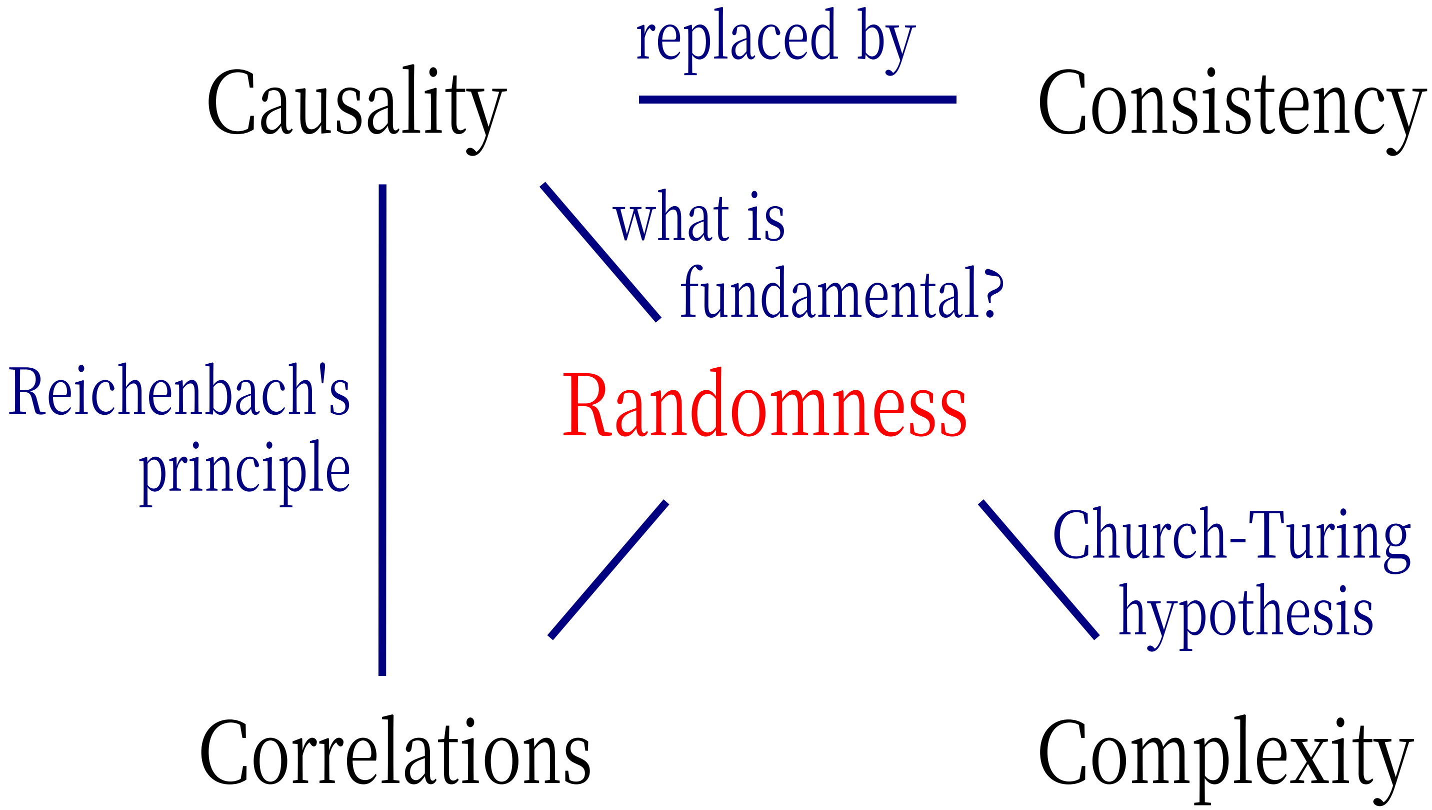
Correlations

Overview



Causality
replaced by
Consistency
what is
fundamental?
Reichenbach's
principle
Randomness
Correlations

Overview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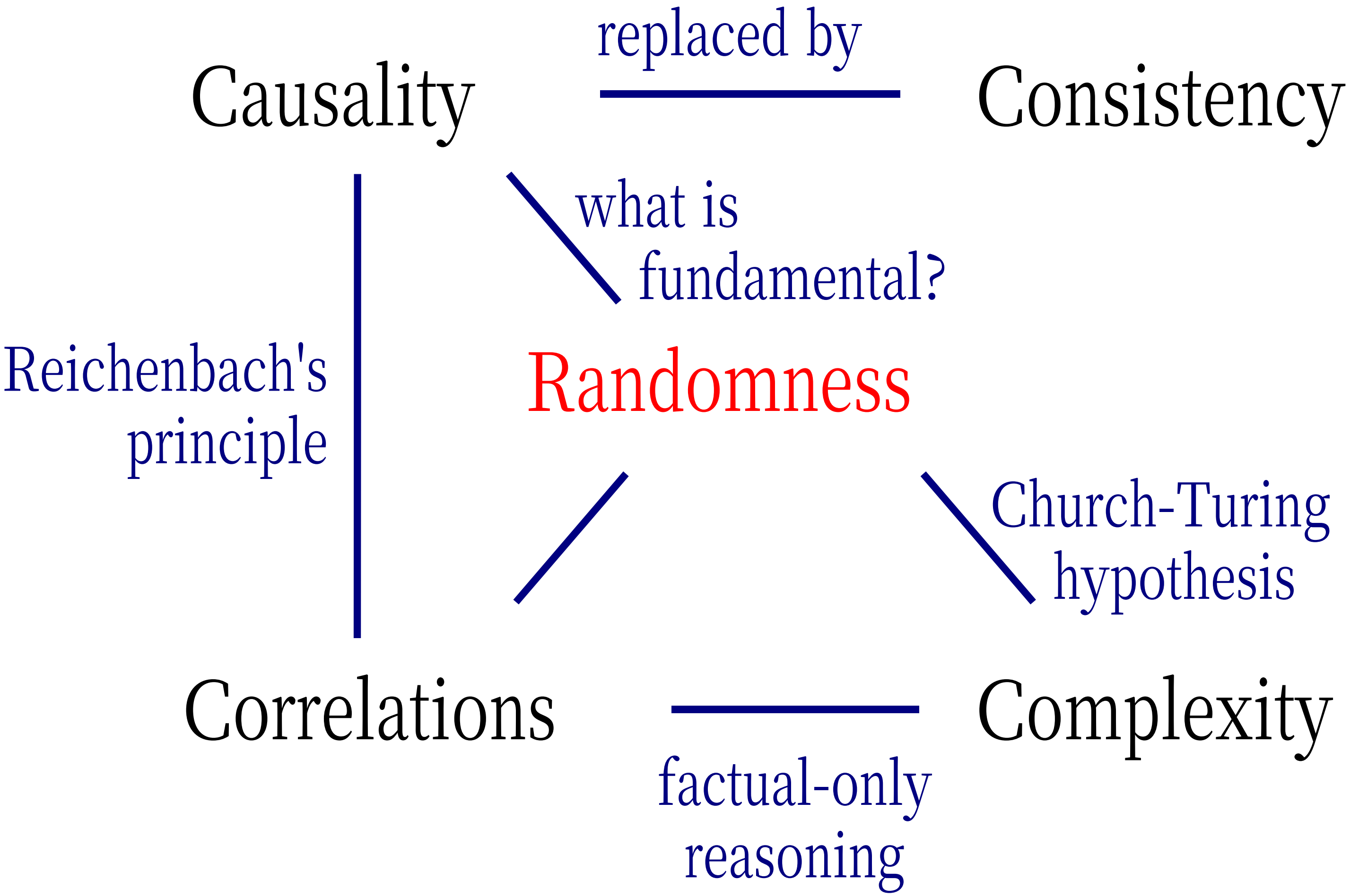
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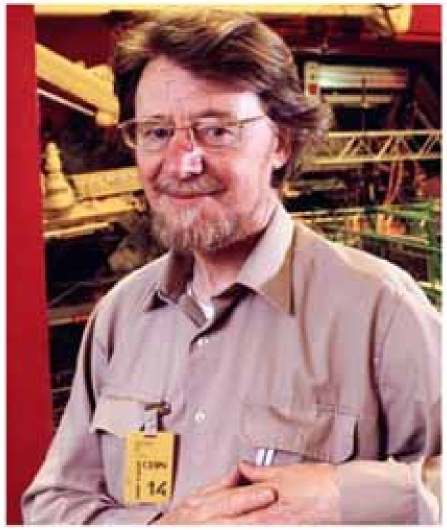
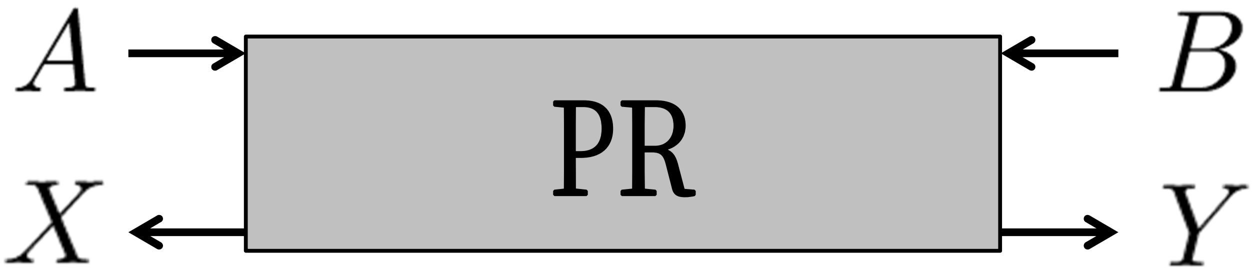
factual-only reasoning

Non-Local Correlations



John Bell
1964



$$X \oplus Y = A \odot B$$

1994
Sandu Popescu

Daniel Rohrlich

Explaining Correlations in a Causal Structure

1) Common Cause

$$A \leftarrow C \rightarrow B$$

2) Influence

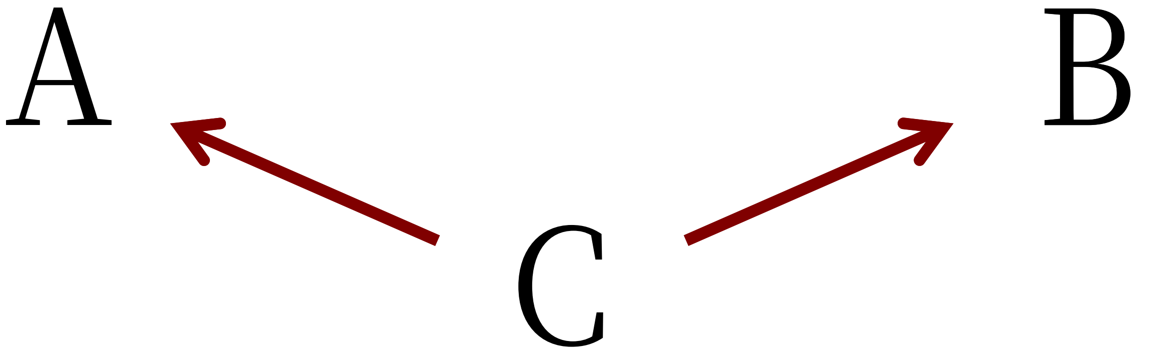
$$A \longrightarrow B$$

1956

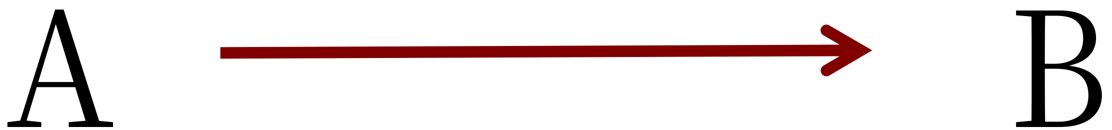
Hans Reichenbach

Explaining Correlations in a Causal Structure

1) Common Cause



2) Influence

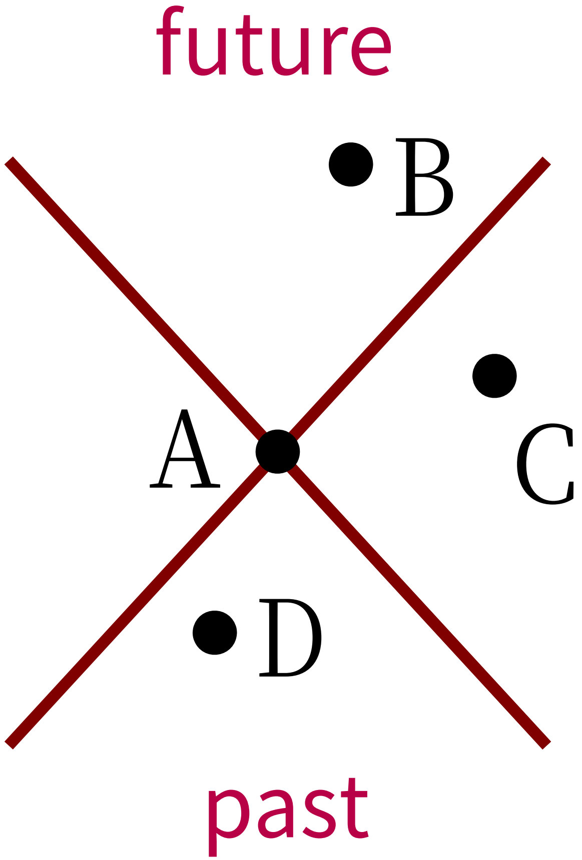


3) Drop the Causal Structure?

1956

Hans Reichenbach

Causality and Randomness



A is free if it is
independent of C anc
(of all except its futu

Renner

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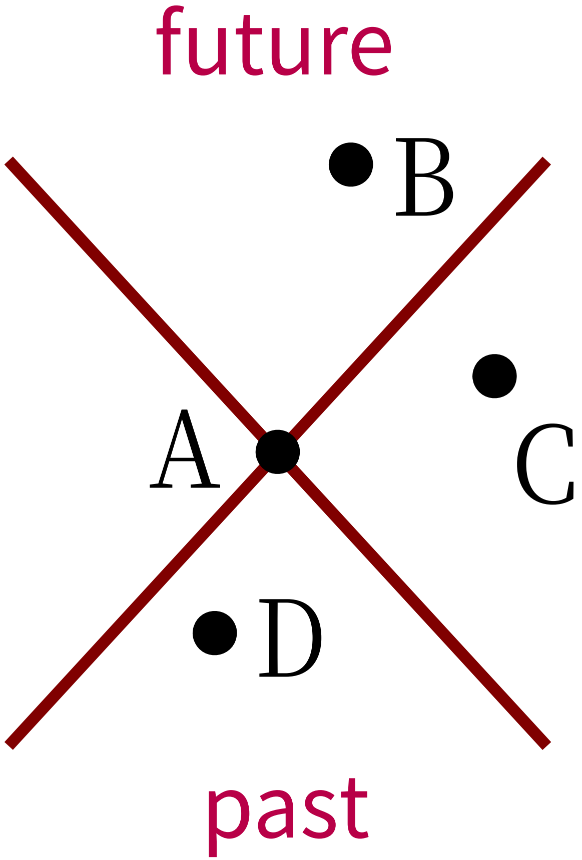
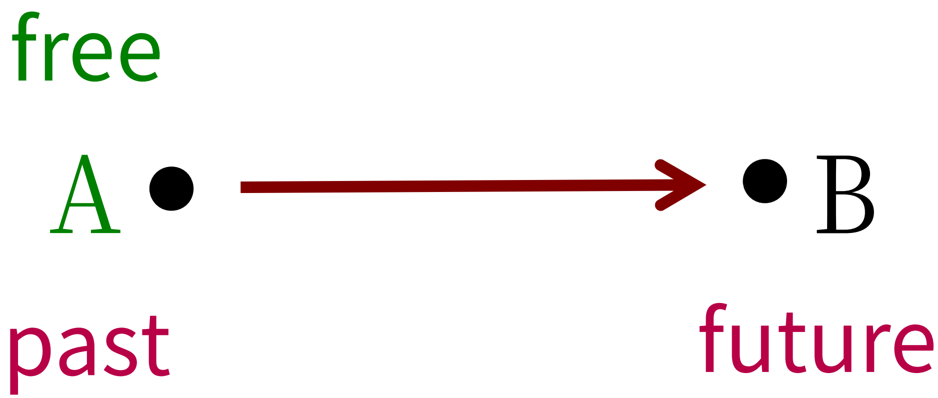
Causality and Randomness

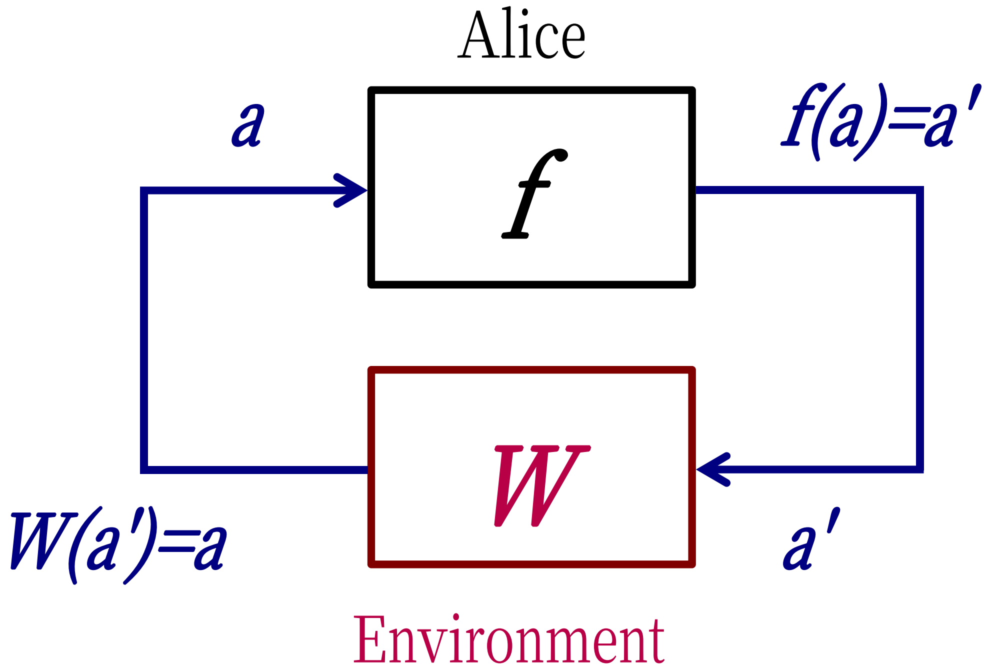


If A is free and B is
correlated with A,
then B is in A's futur



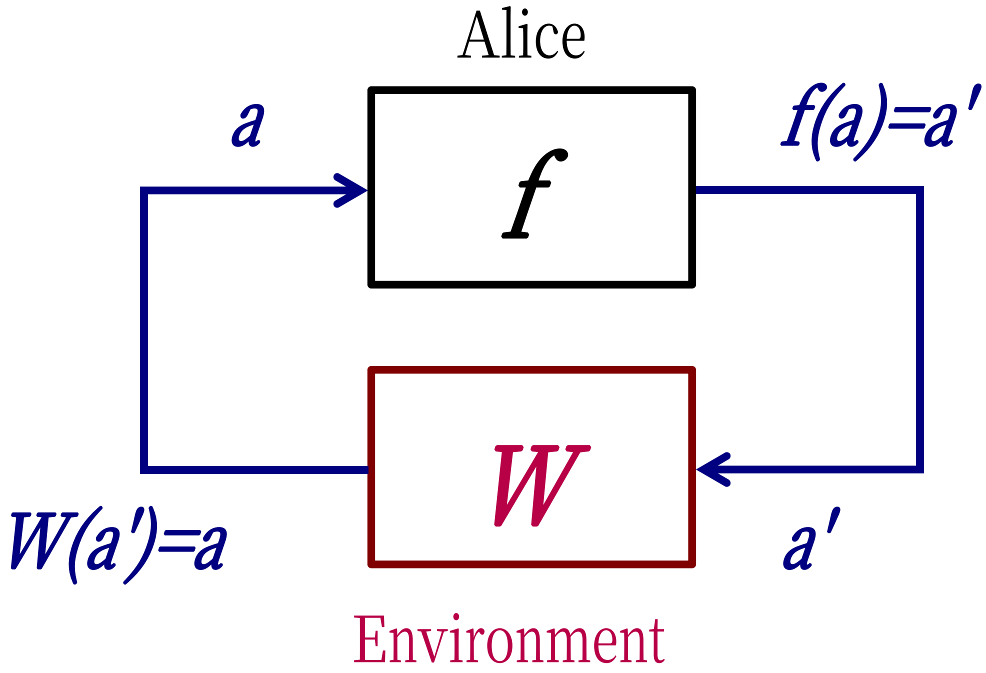
A is free if it is
independent of C and D
(of all except its future)

Dropping the Causal Structure



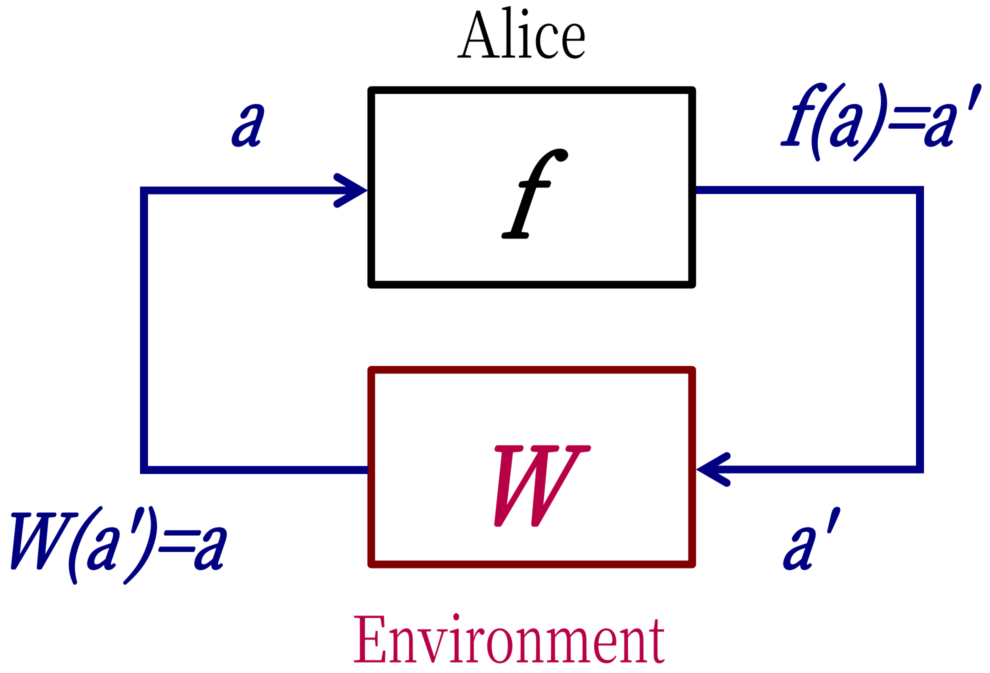
Oreshkov Costa Brukner Baumeler Feix

Dropping the Causal Structure



W is *consistent* if no logical contradiction arises, whatever Alice does

Dropping the Causal Structure

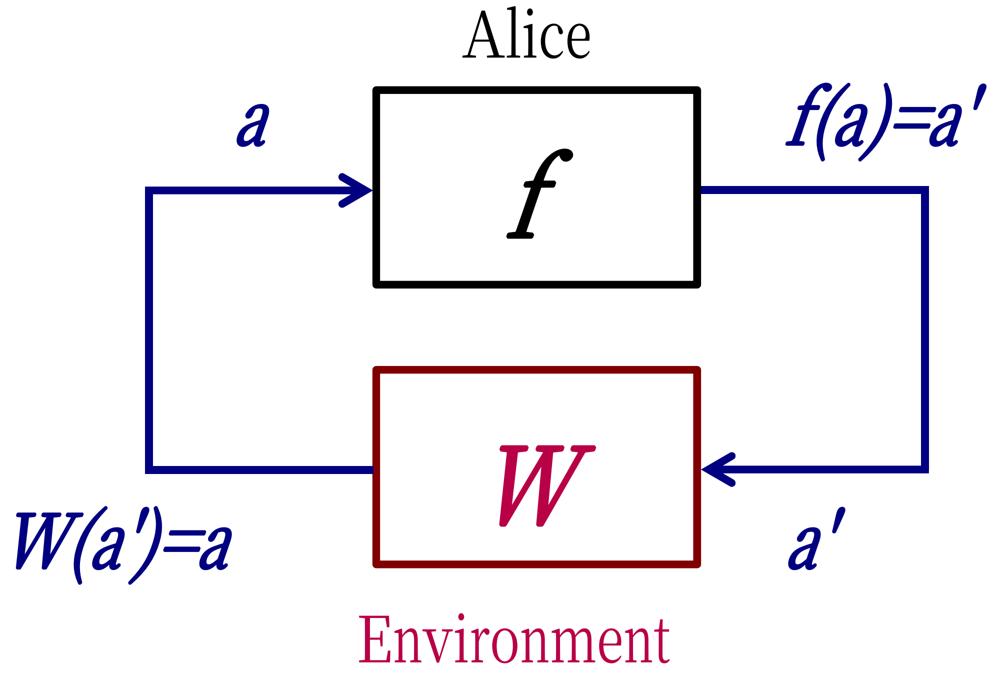


Example 1: $W(a') = a \oplus 1$

Contradiction for $f(a) = a$

(”Grandfather paradox”)

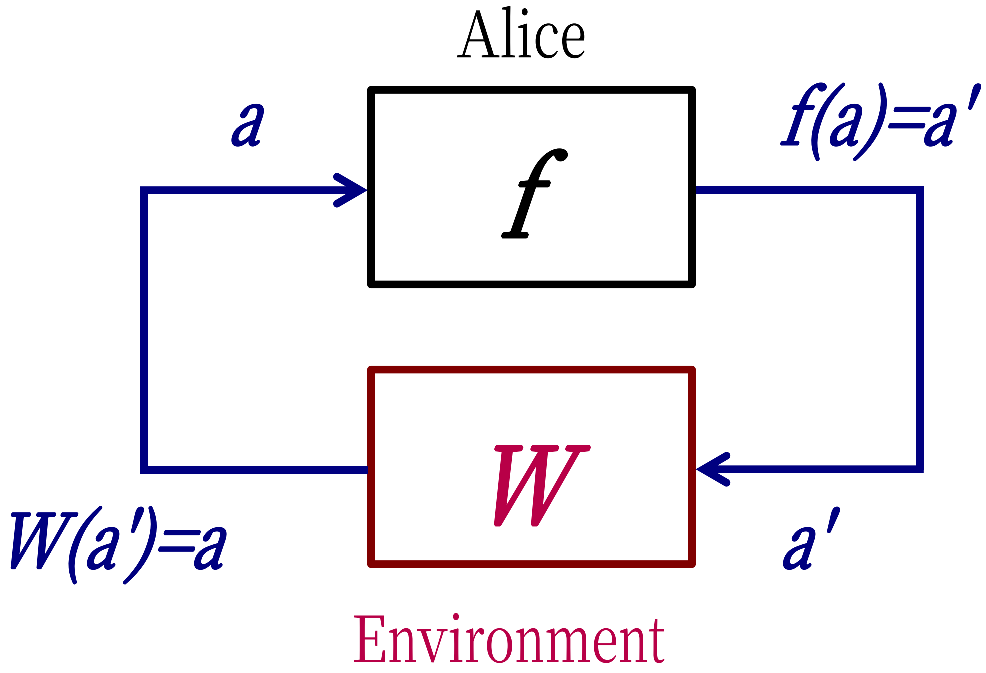
Dropping the Causal Structure



Example 2: $W(a') = a$

Contradiction for $f(a) = a \oplus 1$

Dropping the Causal Structure

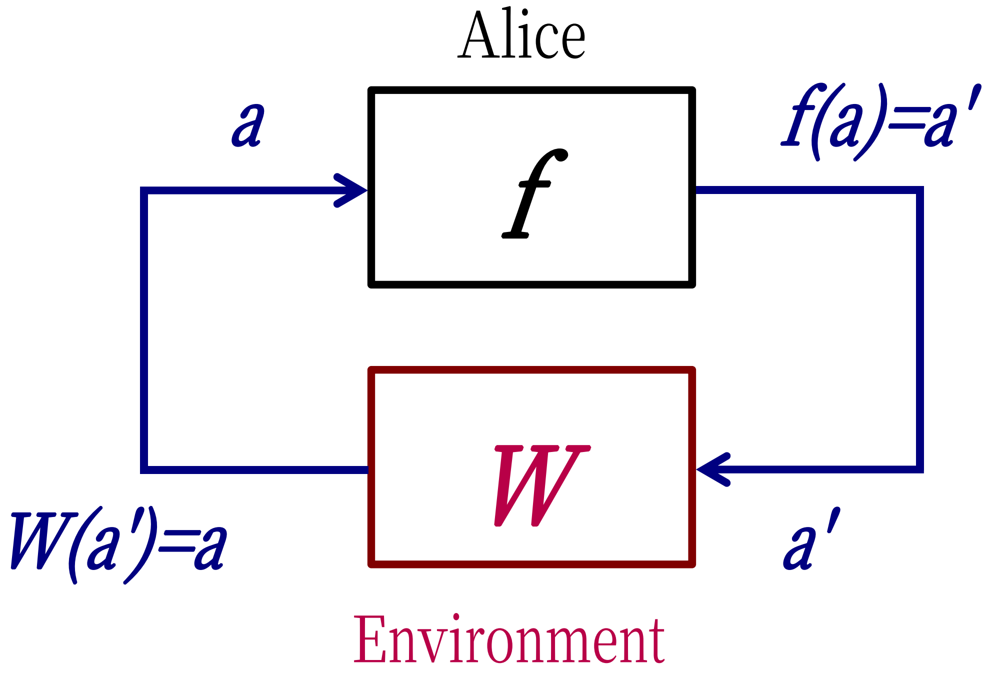


Example 3: $W(a') = 0$

No contradiction.

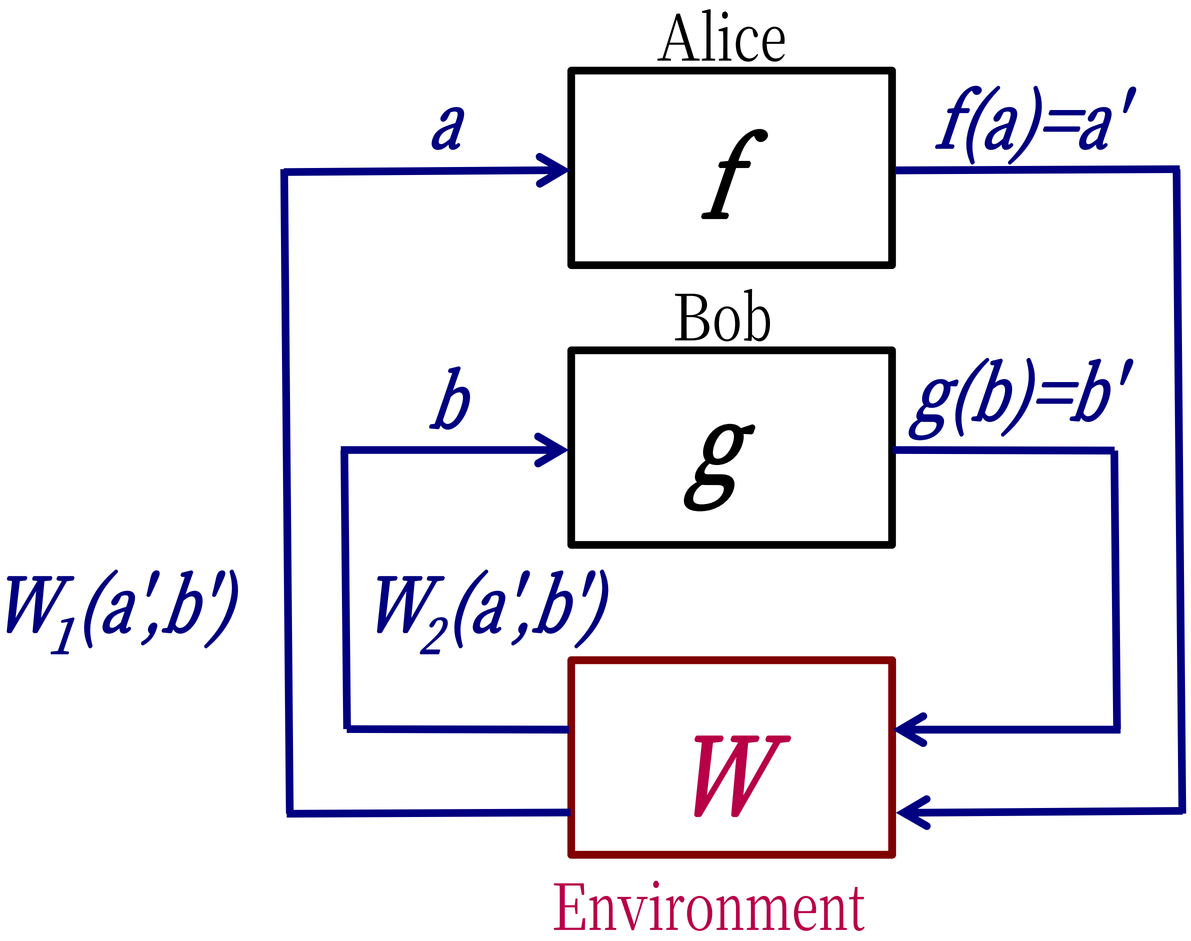
But: can be modeled *causally*: $W \longrightarrow^{0} A \longrightarrow f(0)$

Dropping the Causal Structure



Do there exist *logically consistent* scenarios that are *not causal*?

Dropping the Causal Structure



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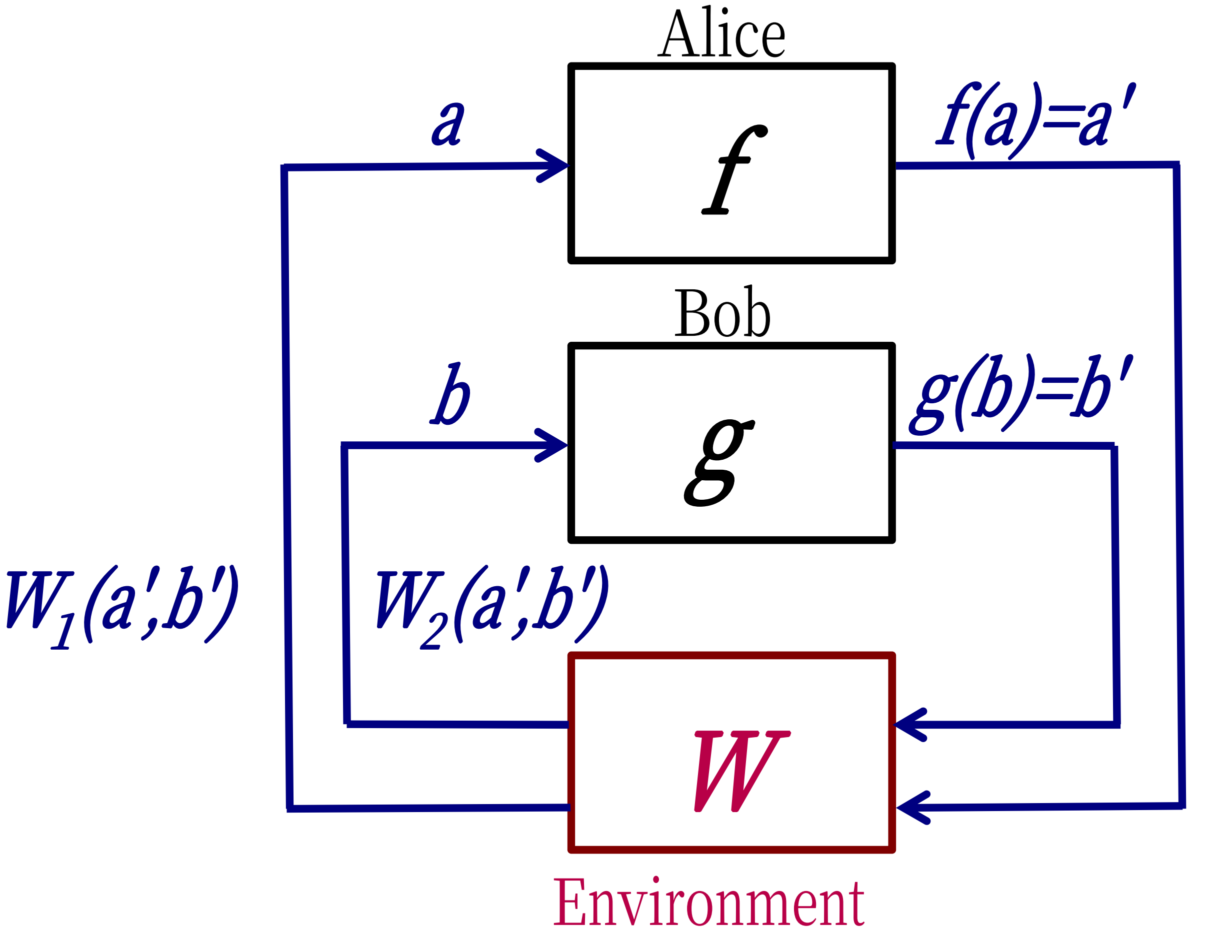
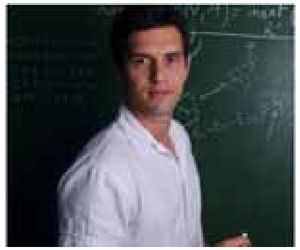
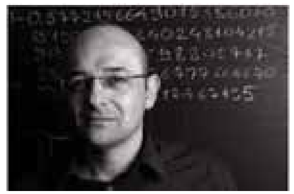




Oreshkov

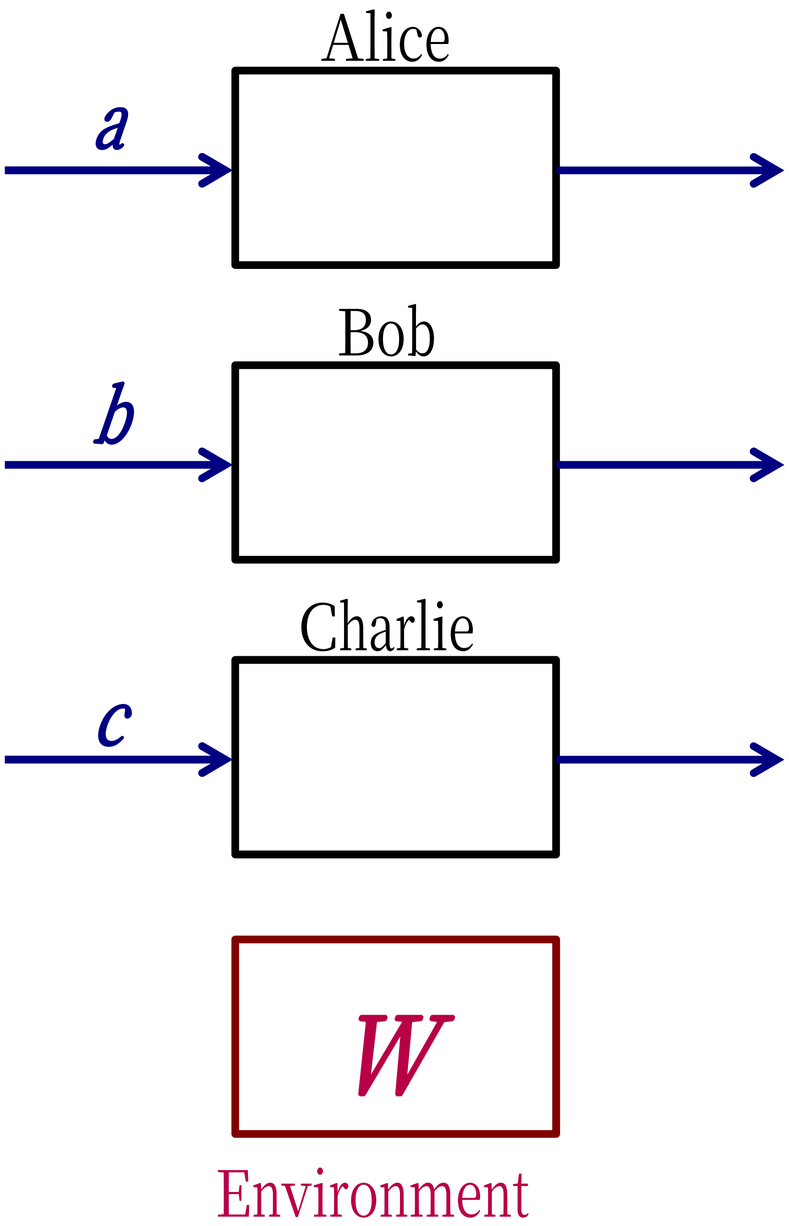
Costa



Brukner

Two parties: No. *Consistency implies causality*

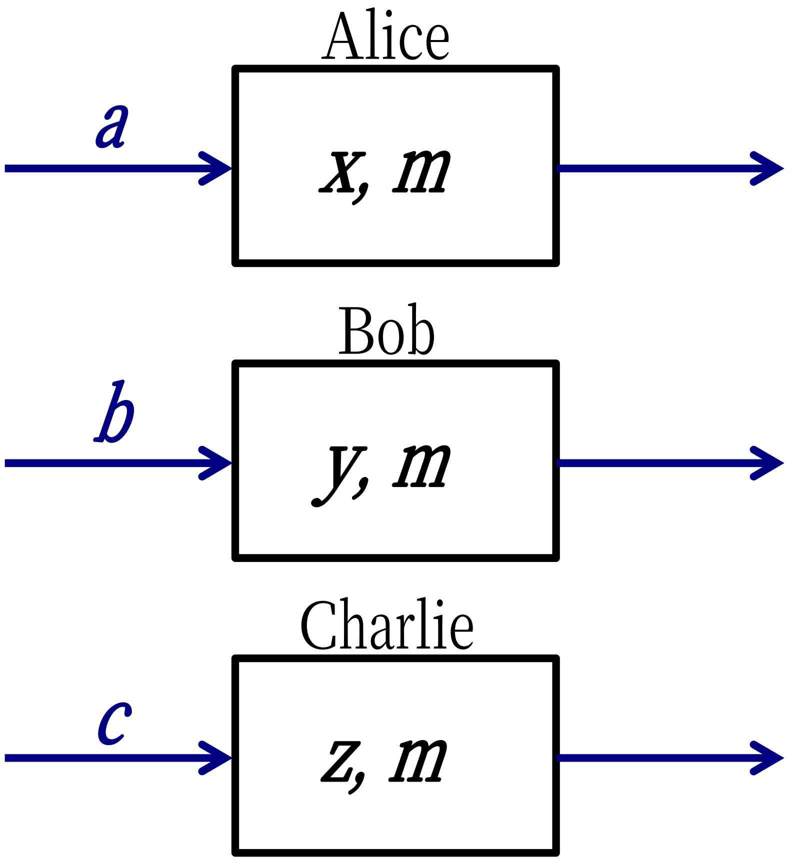
Consistency *without* Causality is Possible



Baumeler

Feix

The Three-Party Game



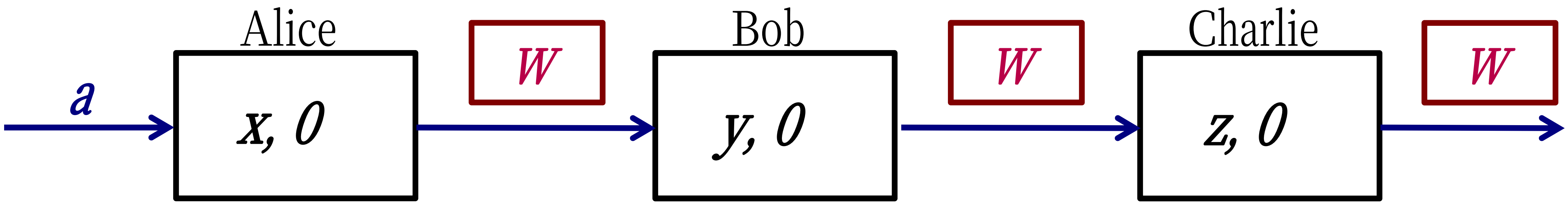
Game won if

$$m = 0 \implies a = y \oplus z$$

$$m = 1 \implies b = x \oplus z$$

$$m = 2 \implies c = x \oplus y$$

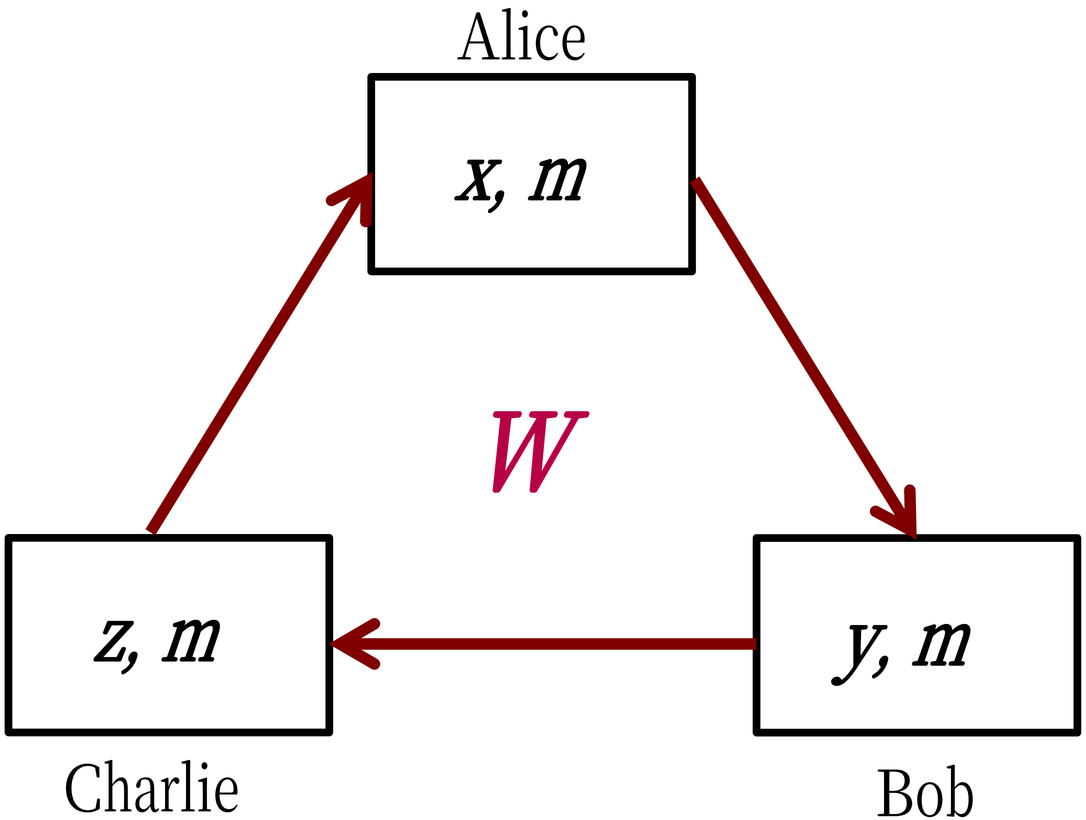
Game Cannot Always Be Won with Fixed Order



$$\mathrm{Prob}[a = y \oplus z] = \frac{1}{2}$$

Otherwise: Game lost

Always Winning the Game with a Consistent W



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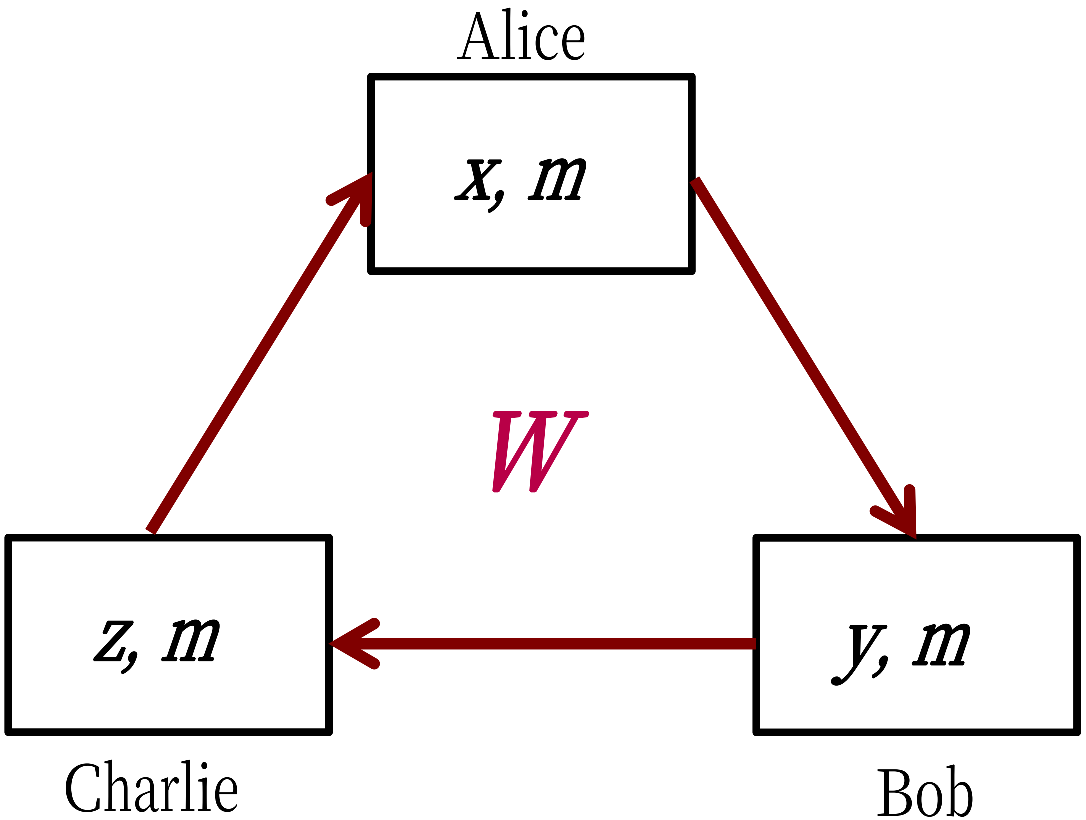
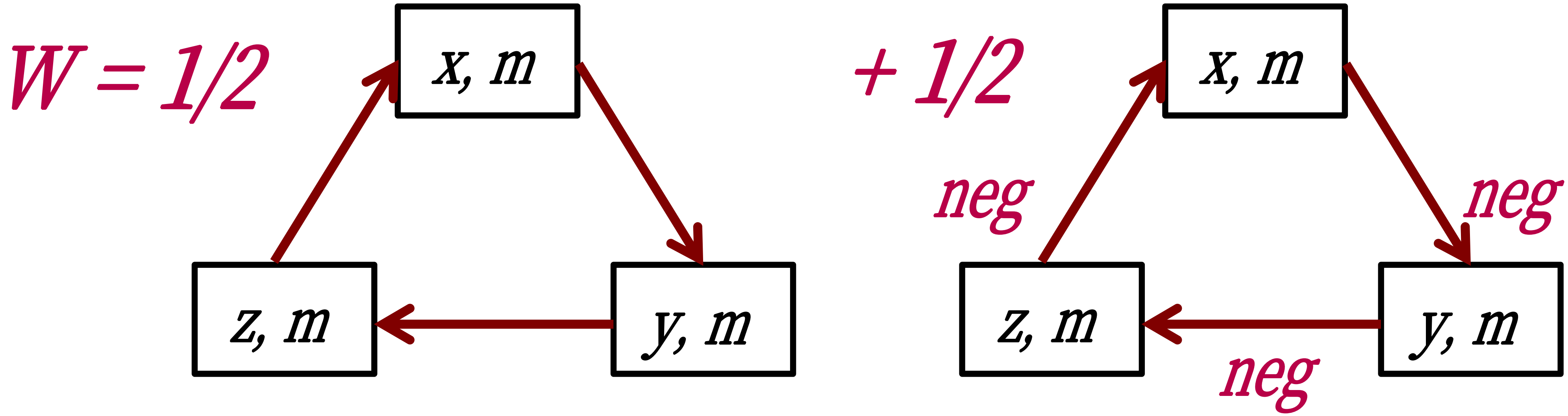


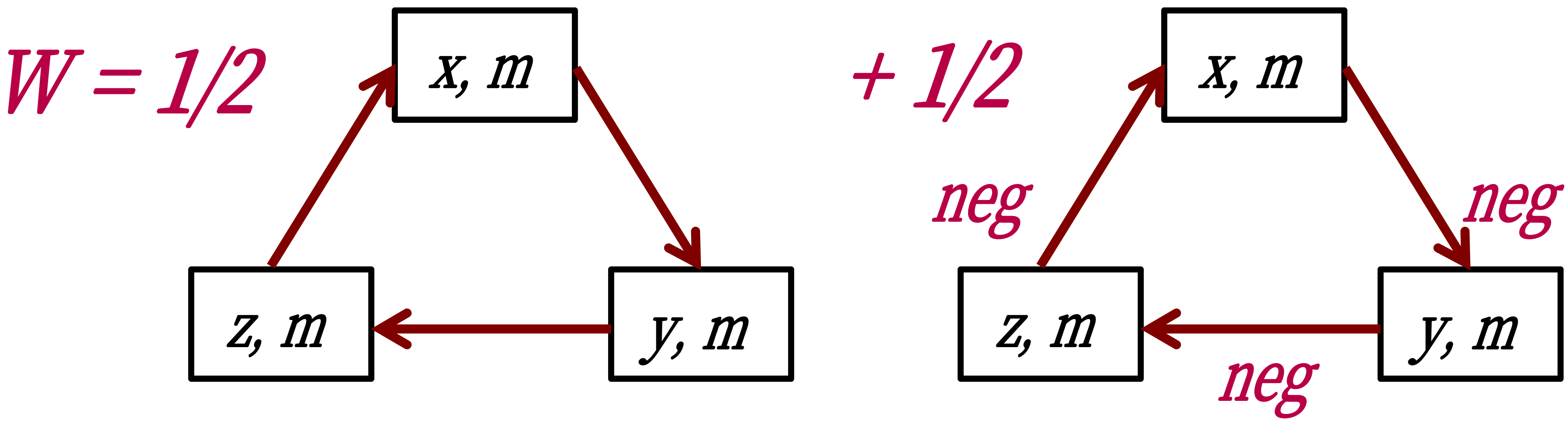
W inconsistent

Always Winning the Game with a Consistent *W*



W is *consistent*:
The value a party receives is *independent* of what she sent

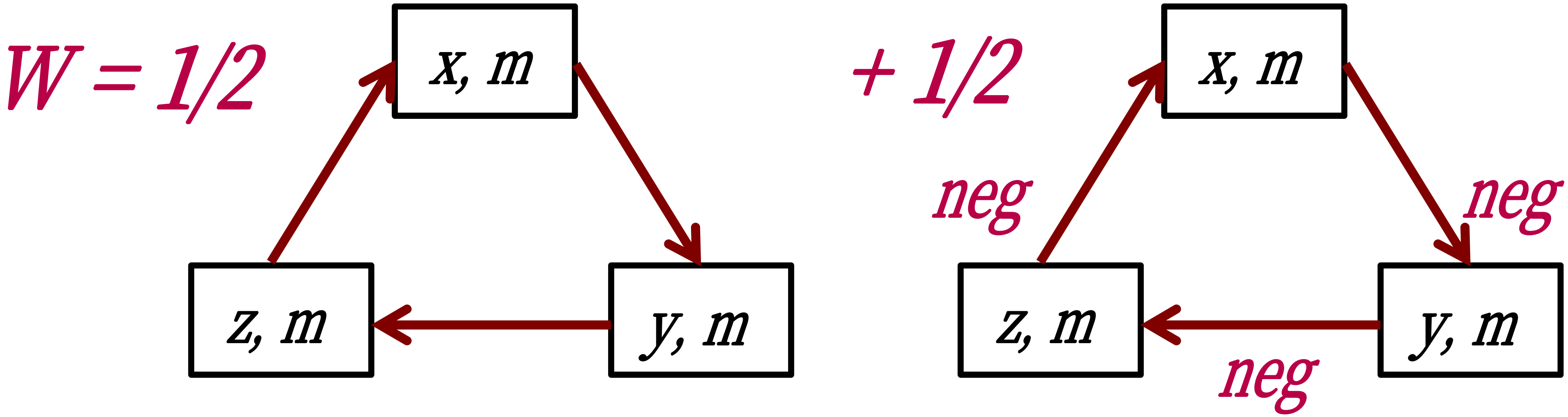
Always Winning the Game with a Consistent *W*



W allows for winning the game:
Each party sends the value she receives XORed with her own

Always Winning the Game with a Consistent *W*

Example: m=0



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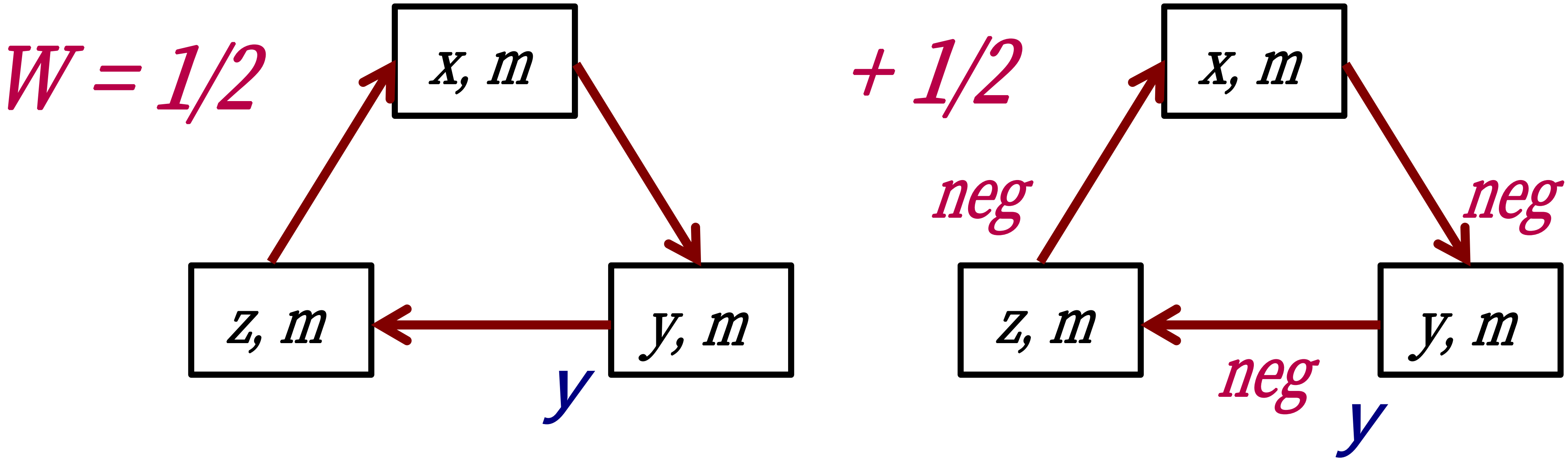

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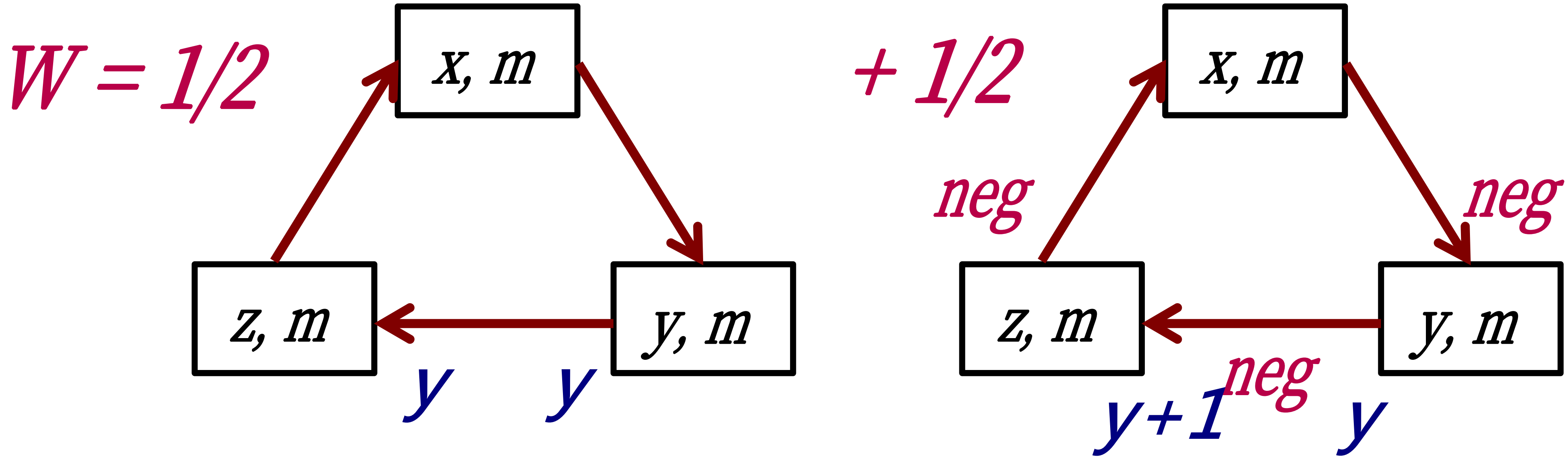

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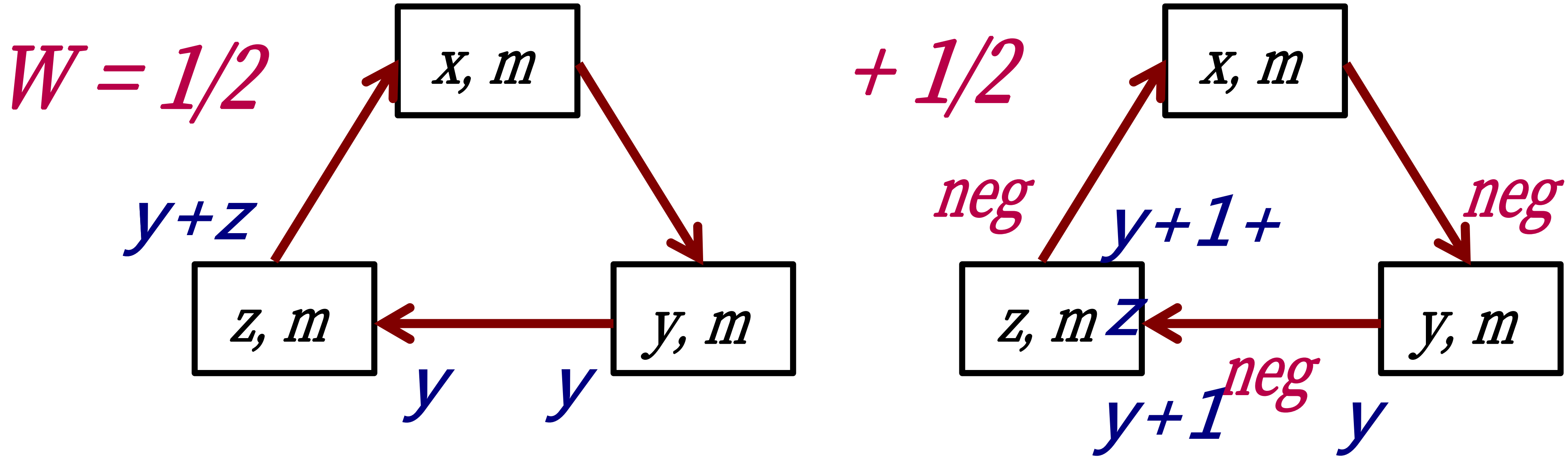

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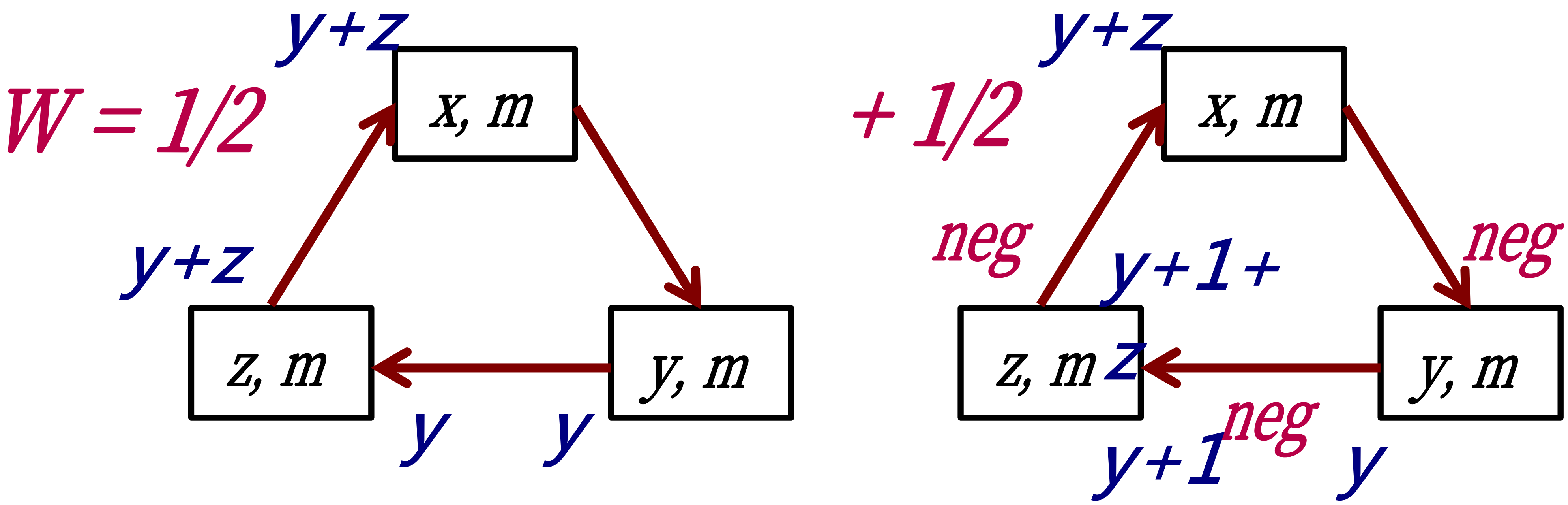
Each party sends the value she receives XORed with her own

Always Winning the Game with a Consistent *W*

Example: m=0. The game is won.



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Each party sends the value she receives XORed with her own

Back to Randomness

How to define it intrinsically?

Back to Randomness

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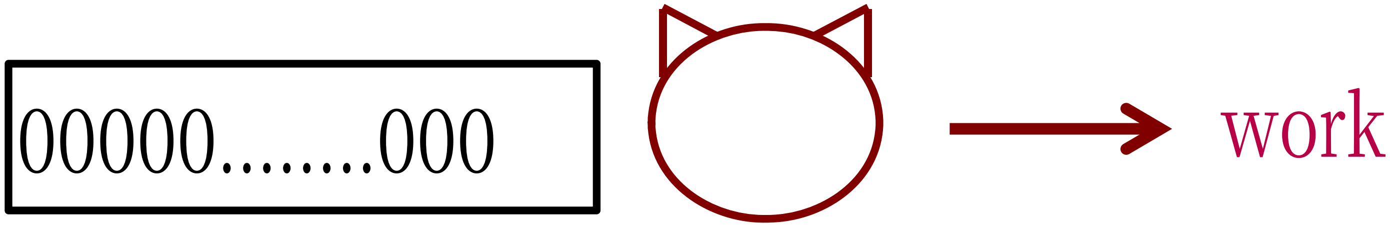
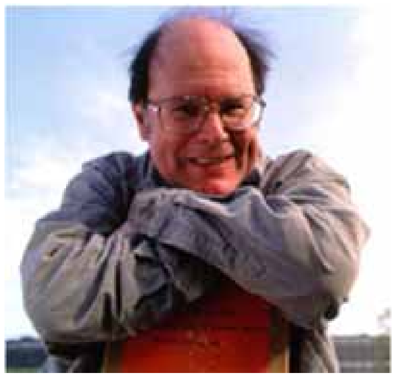
$\boxed{00000........000}$ is *not* random

Back to Randomness

How to define it intrinsically?





Charles H. Bennett

Rolf Landauer

Back to Randomness

How to define it intrinsically?

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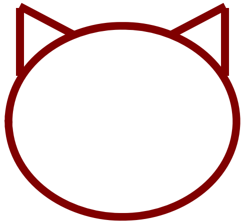
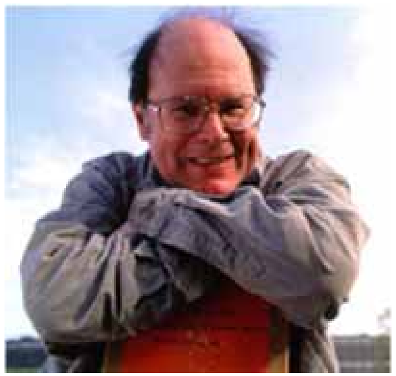
N



work $kTN \ln 2$



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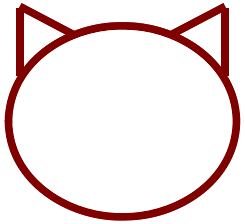
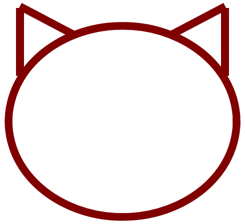
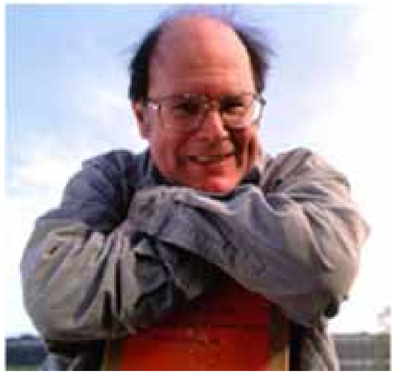
00000........000  → work kTN ln 2

N

31415926535..  → work kTN ln 10

N



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Back to Randomness

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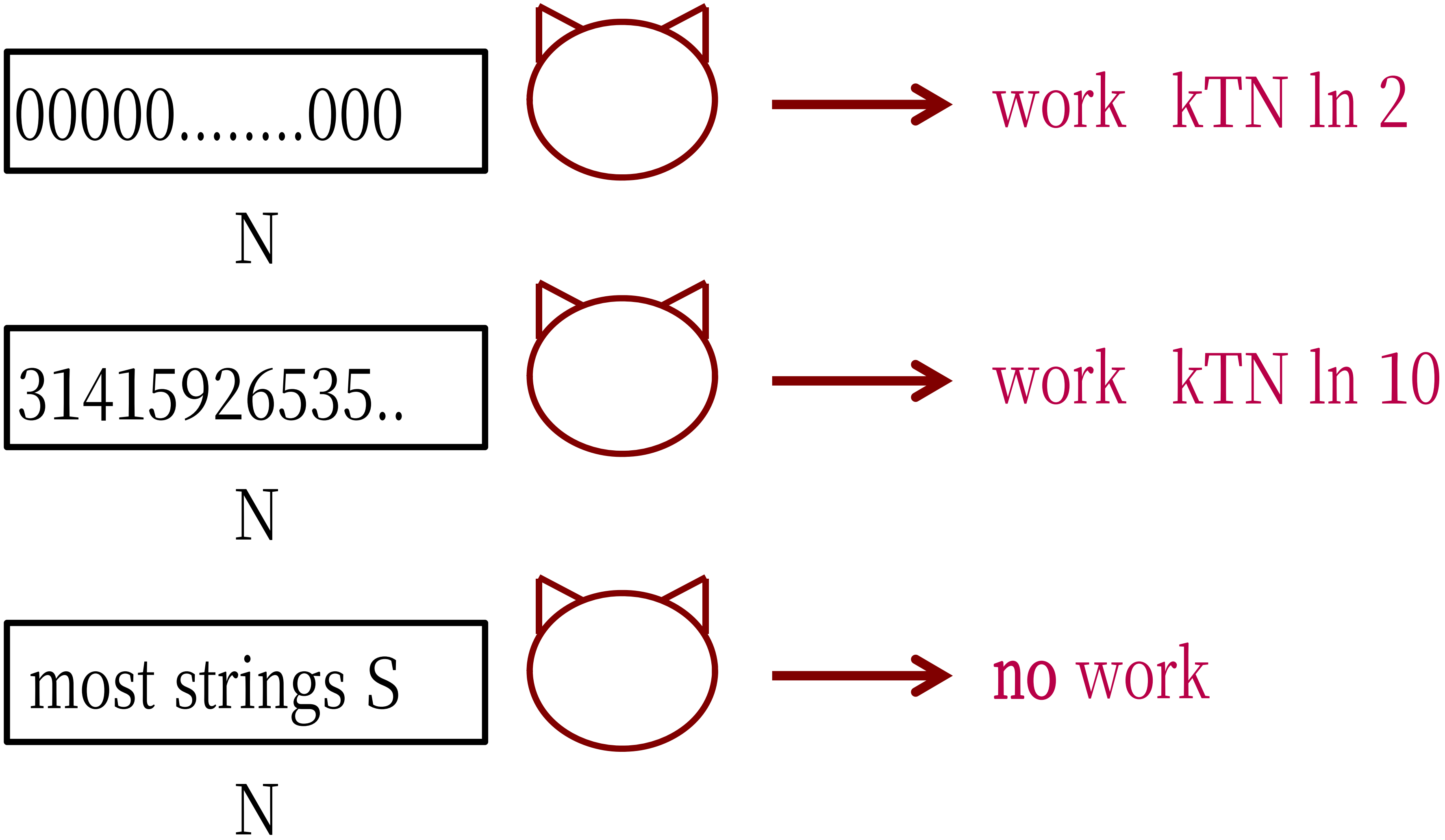

Back to Randomness

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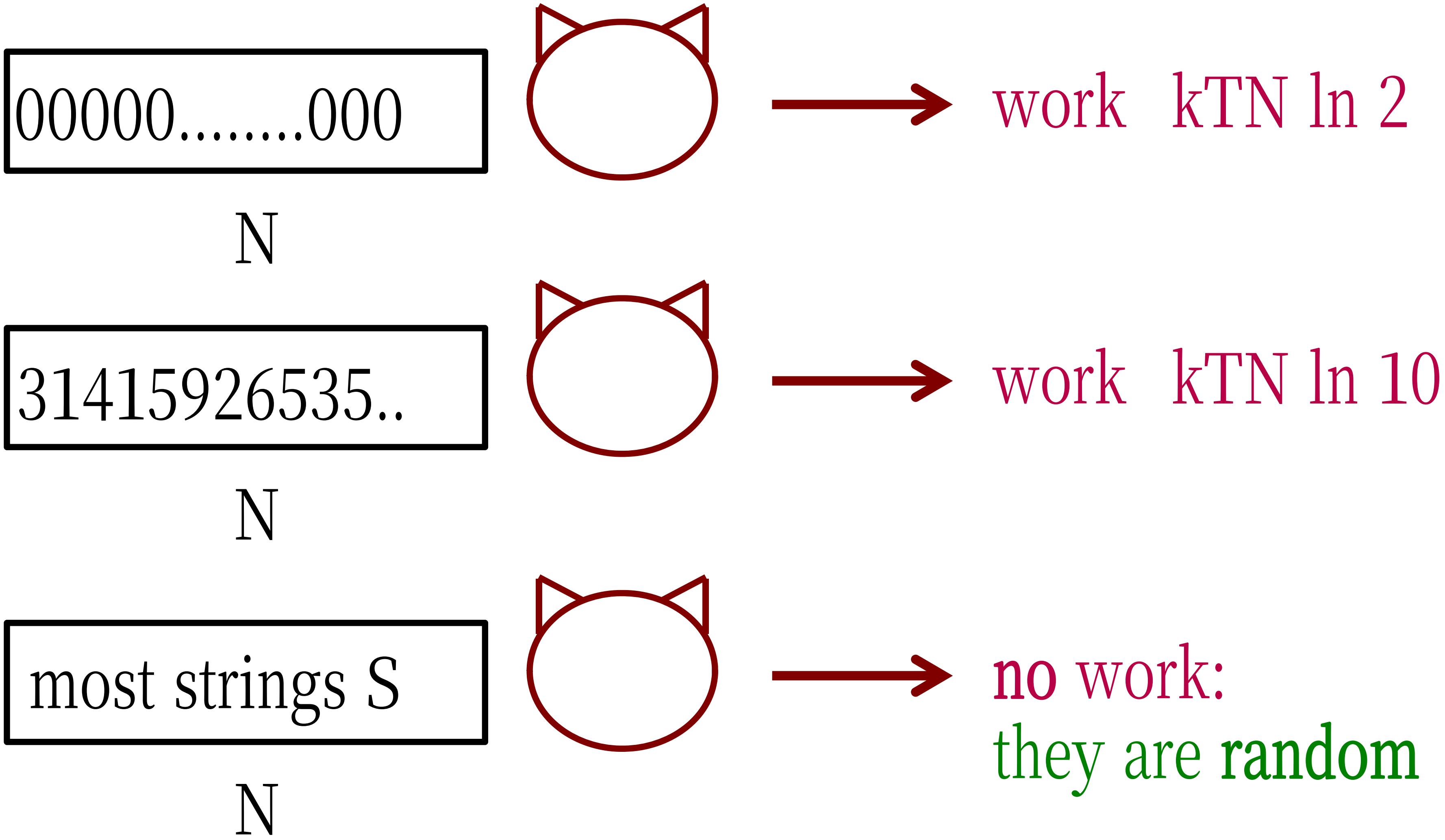

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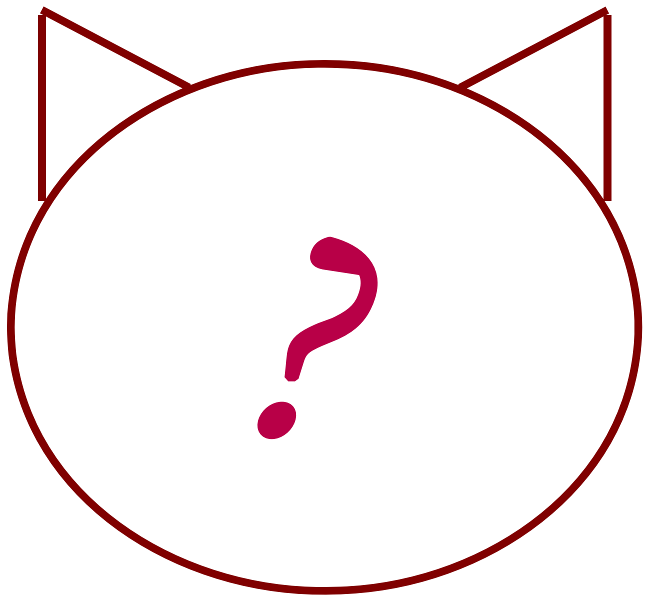
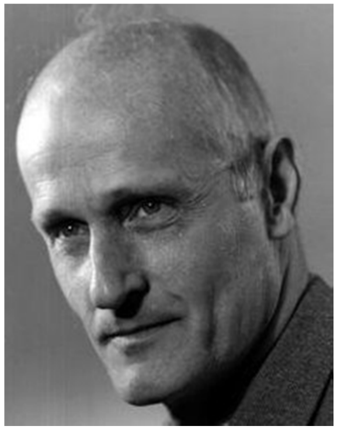


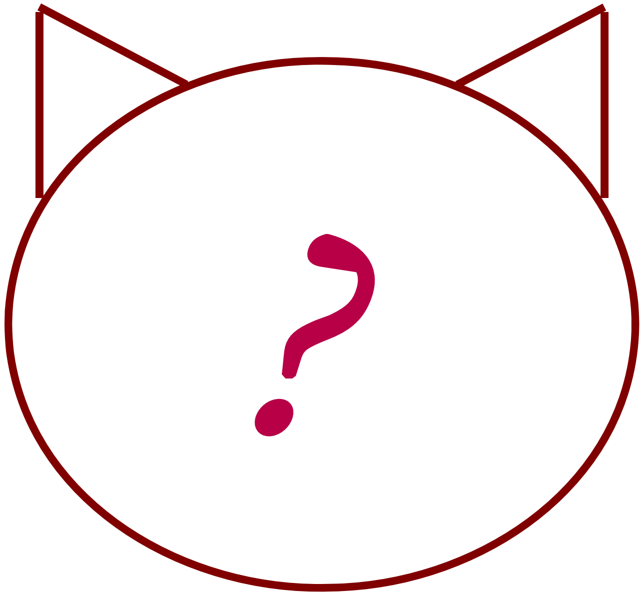
Back to Randomness

Alan Turing
1936

Alonzo Church



1943
Stephen Kleene



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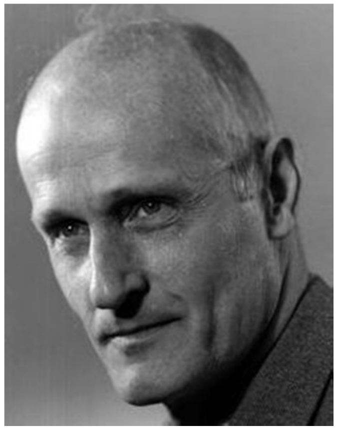

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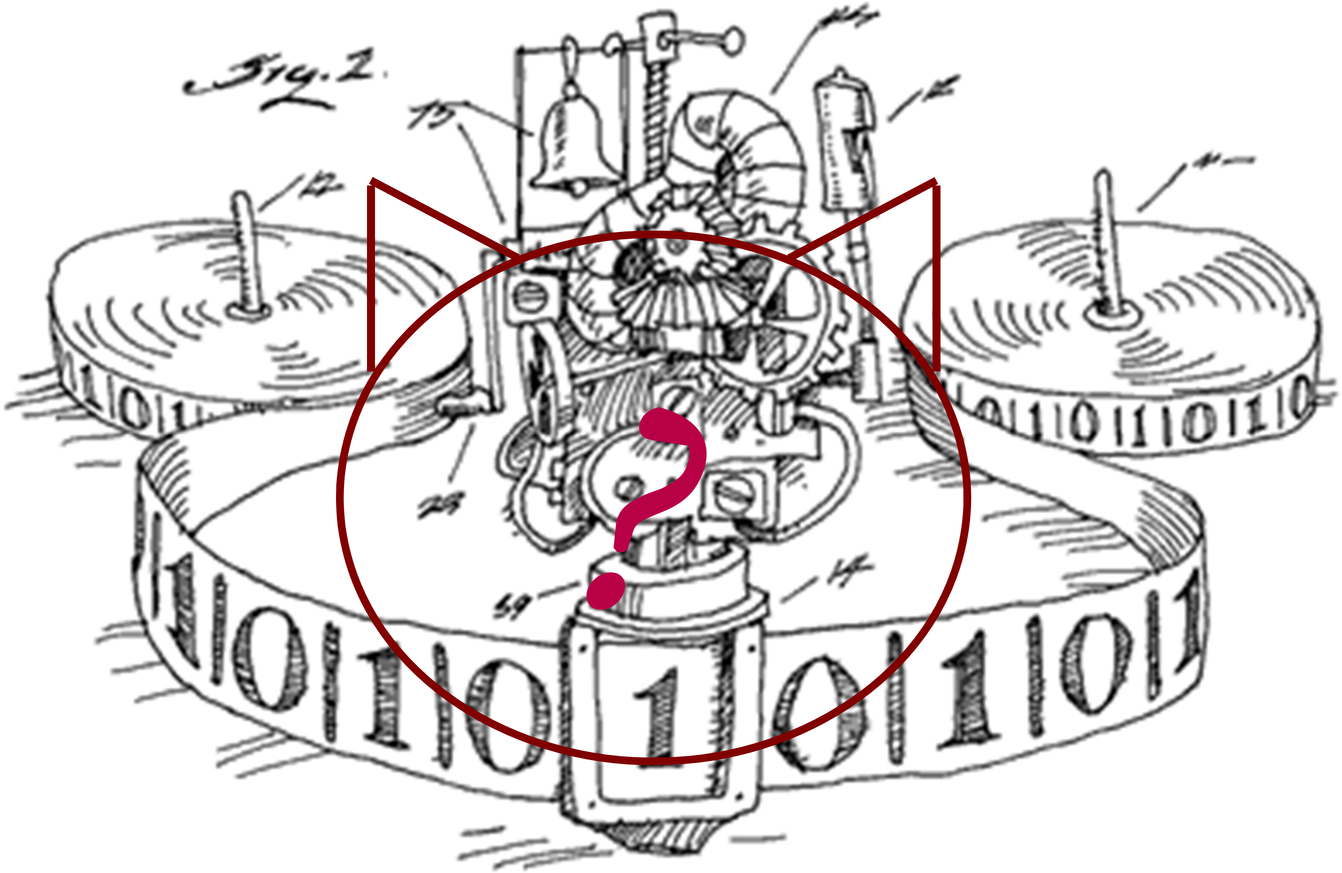

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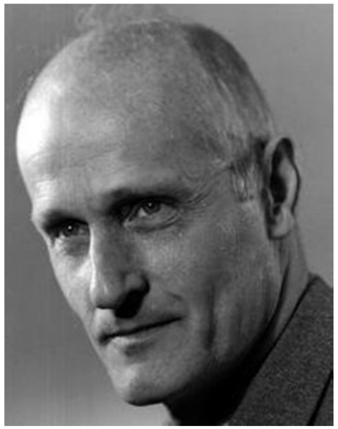
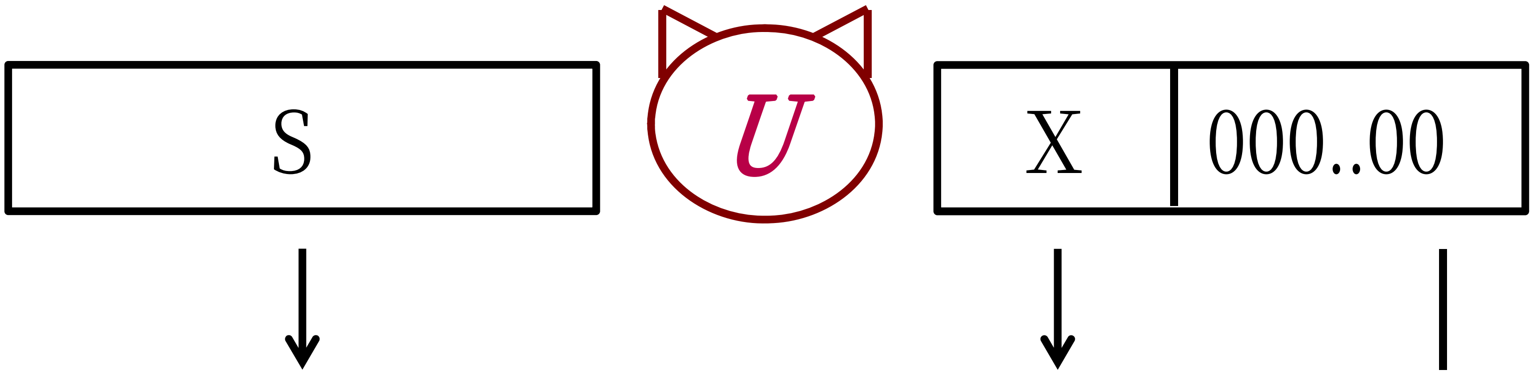


1943
Stephen Kleene

Work Extraction

The model



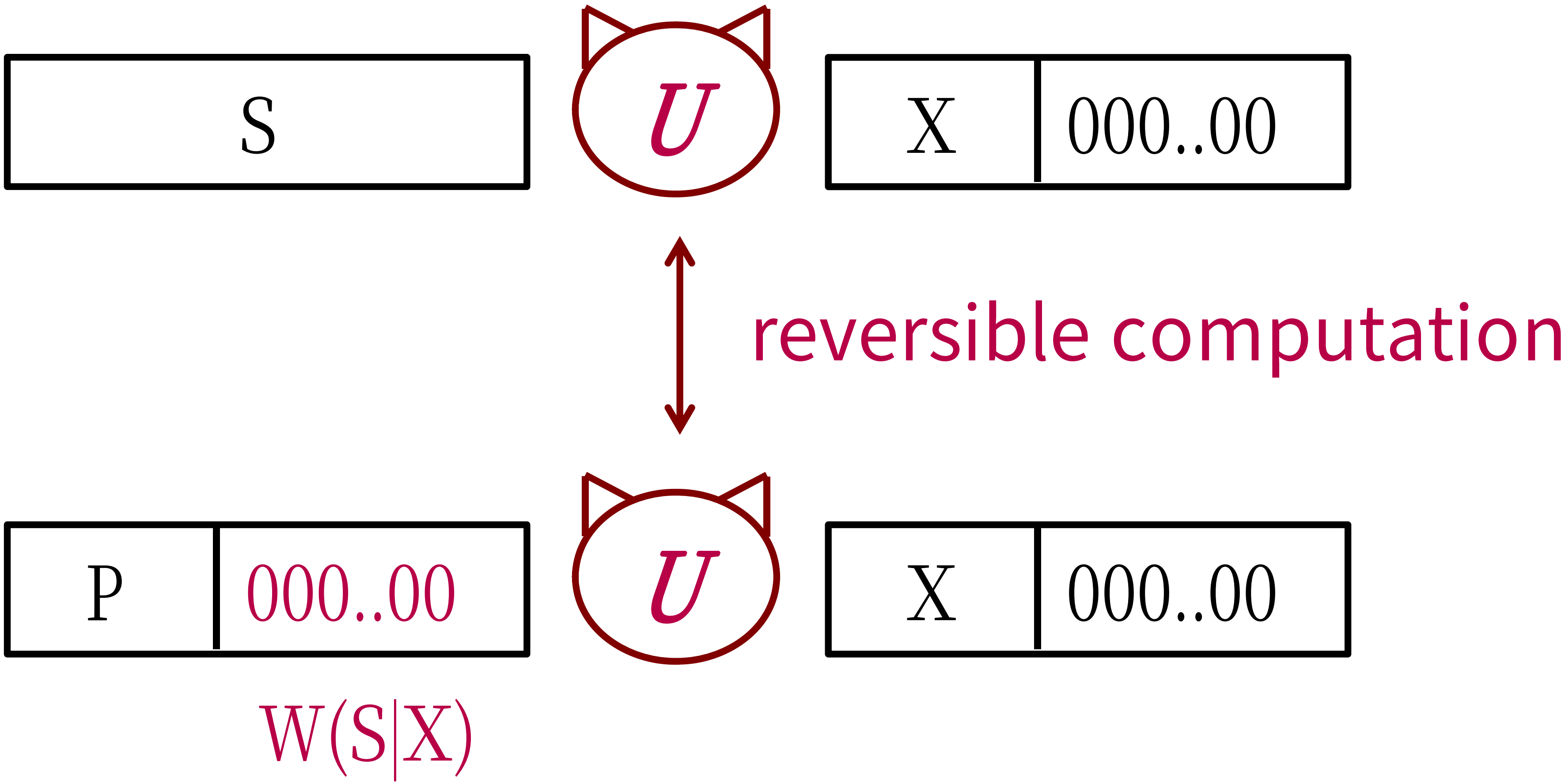
finite string from which work is to be extracted

knowledge about S (initial state)

finite, sufficiently long tape

Work Extraction

The model



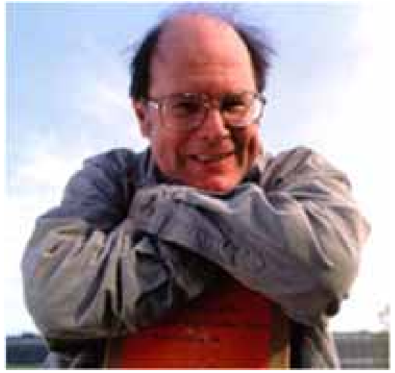
→ work: W(S|X) kT ln2

Bounds on the Work Value

Upper bound:

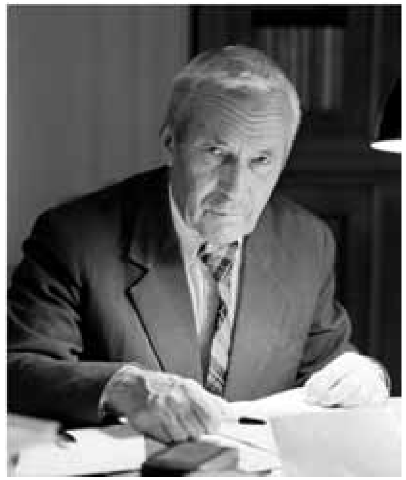
$$W(S|X) \leq \text{len}(S) - K_{\mathcal{U}}(S|X)$$

length of the shortest program for U to compute S given X



Charles H. Bennett

Andrei Kolmogorov



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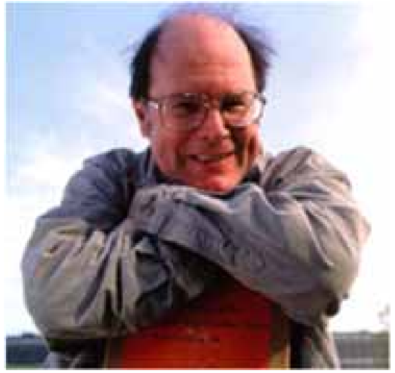


Charles H. Bennett

Lower bound:

C *compression algorithm* with helper, *i.e.*,

$C : S||X \mapsto C(S,X)||X$ reversible:

$$W(S|X) \geq \mathrm{len}(S) - \mathrm{len}(C(S,X))$$

Wojciech Zurek

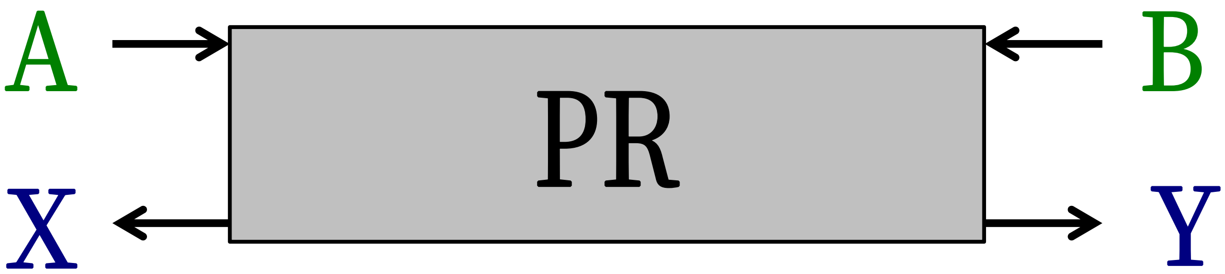
Back to Non-Local Correlations

Back to Non-Local Correlations

Counterfactual reasoning

Consequence of non-signaling:

If all (A,B) combinations are possible…



… then X and Y must be perfectly random

Back to Non-Local Correlations

Factual-only reasoning

$$x_i \oplus y_i = a_i \odot b_i$$

Back to Non-Local Correlations

Factual-only reasoning

(a,b) incompressible:

$$\frac{K(a^n, b^n)}{2n} \longrightarrow 1$$

for $n \longrightarrow \infty$ and $a^n := (a_1, \ldots, a_n)$

$$x_i \oplus y_i = a_i \odot b_i$$

Back to Non-Local Correlations

Factual-only reasoning

non-signaling

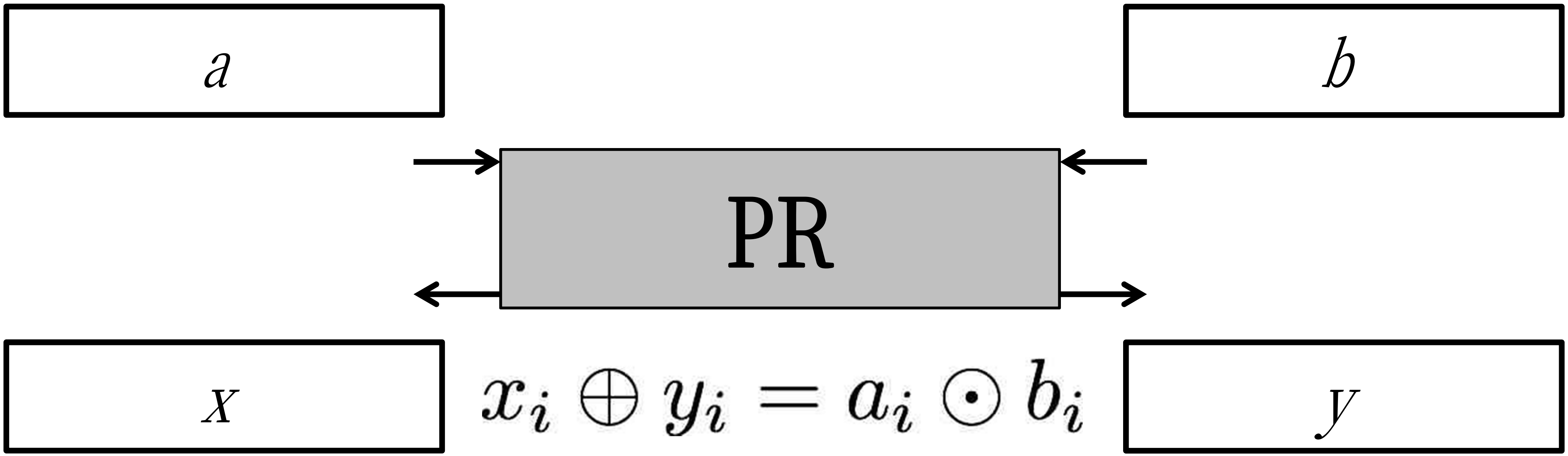
$$\frac{K(x^n|a^n)}{K(x^n|a^n,b^n)} \longrightarrow 1$$

for $n \longrightarrow \infty$, and symmetric

$$x_i \oplus y_i = a_i \odot b_i$$

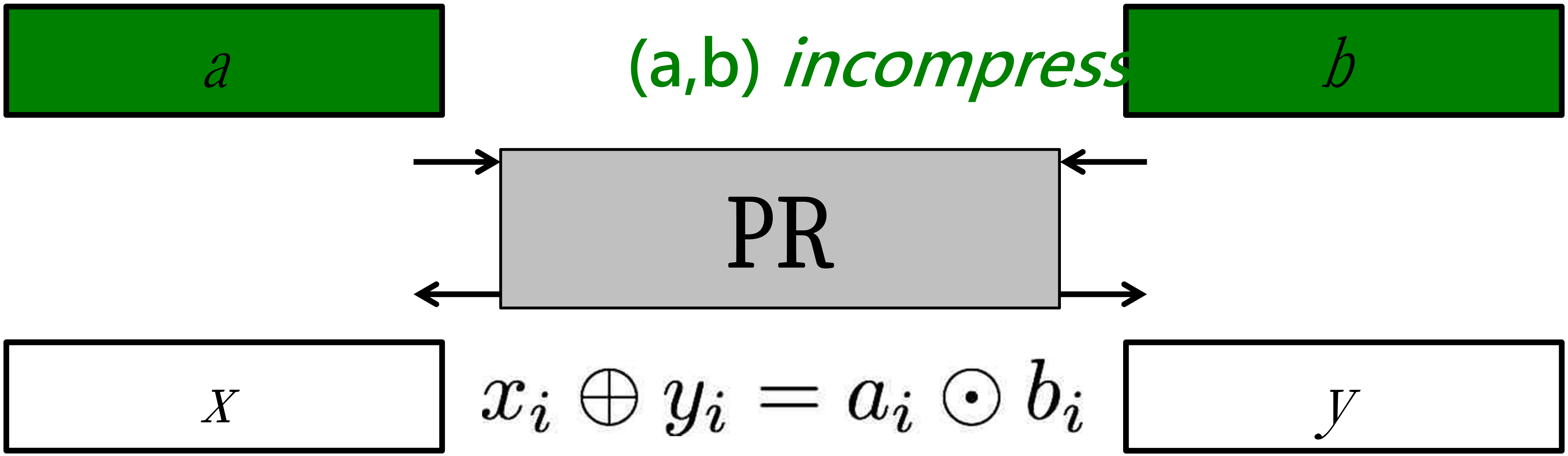
Back to Non-Local Correlations

Factual-only reasoning



Back to Non-Local Correlations

Factual-only reasoning



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Back to Non-Local Correlations

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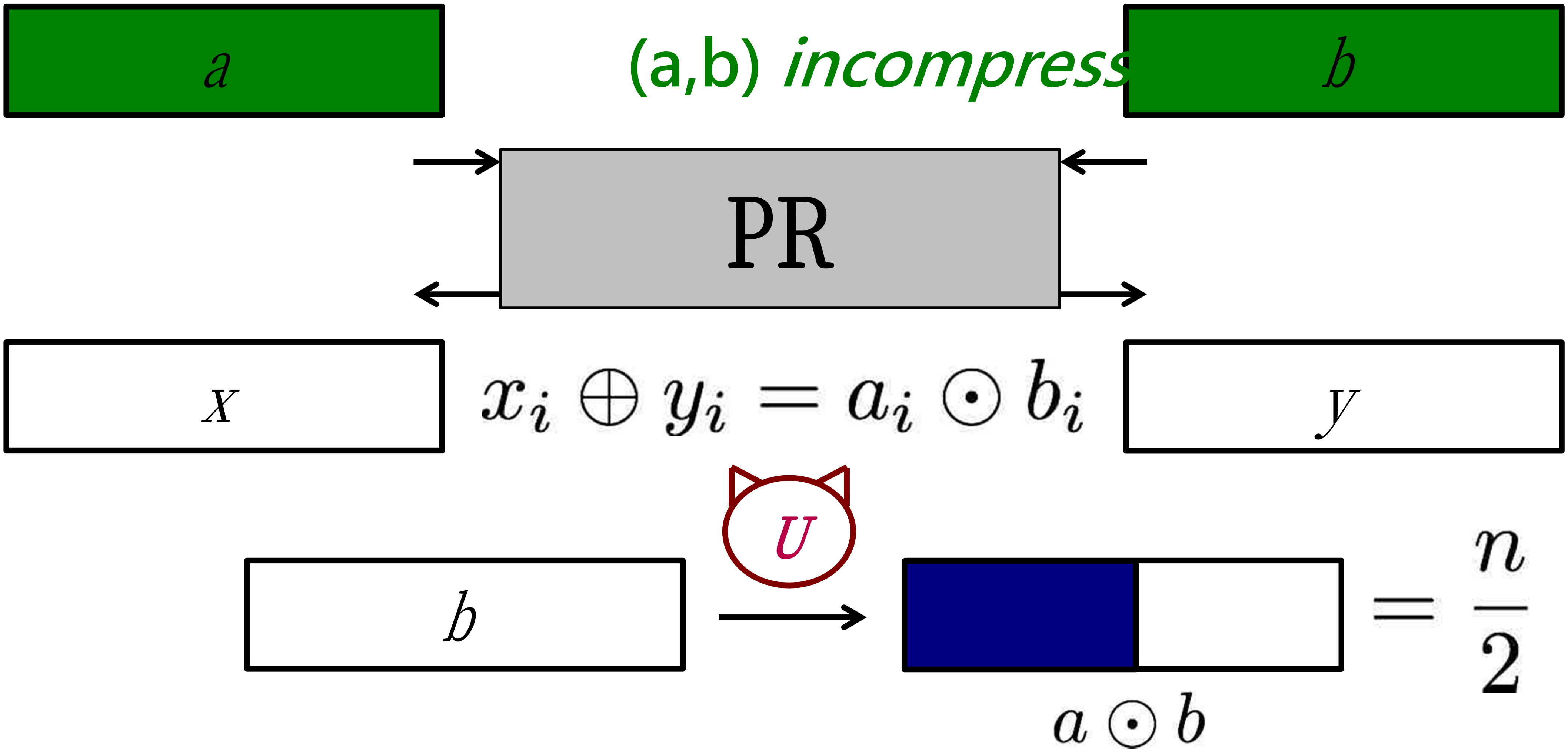

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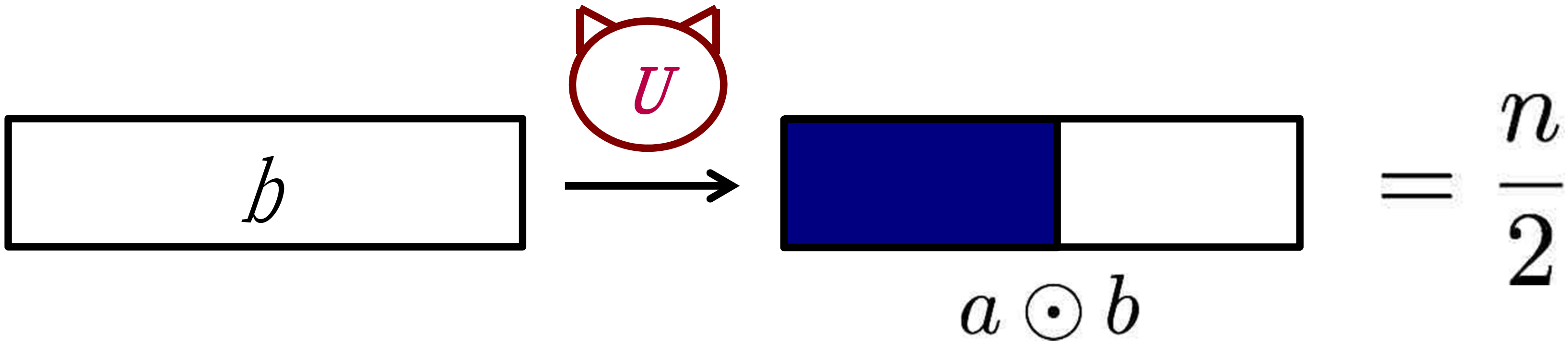

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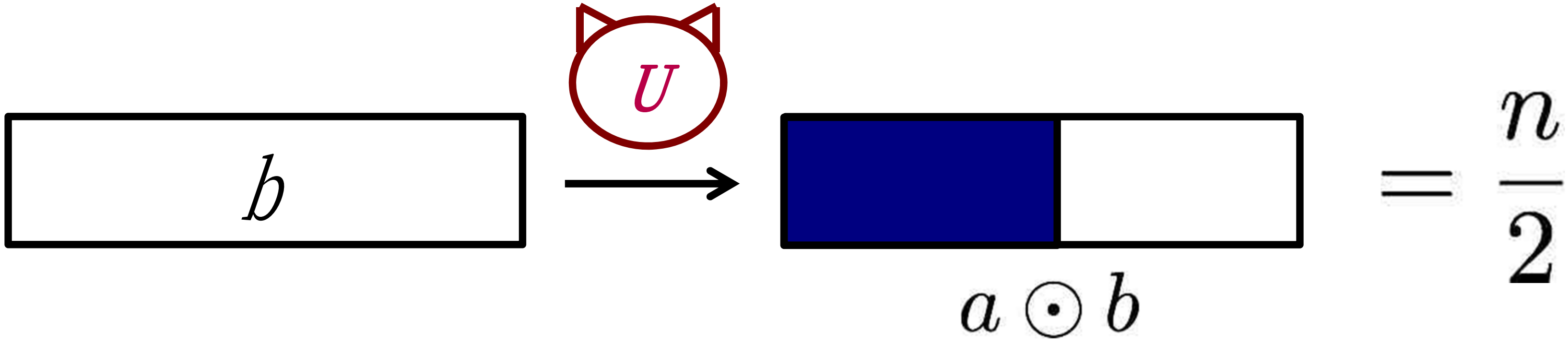

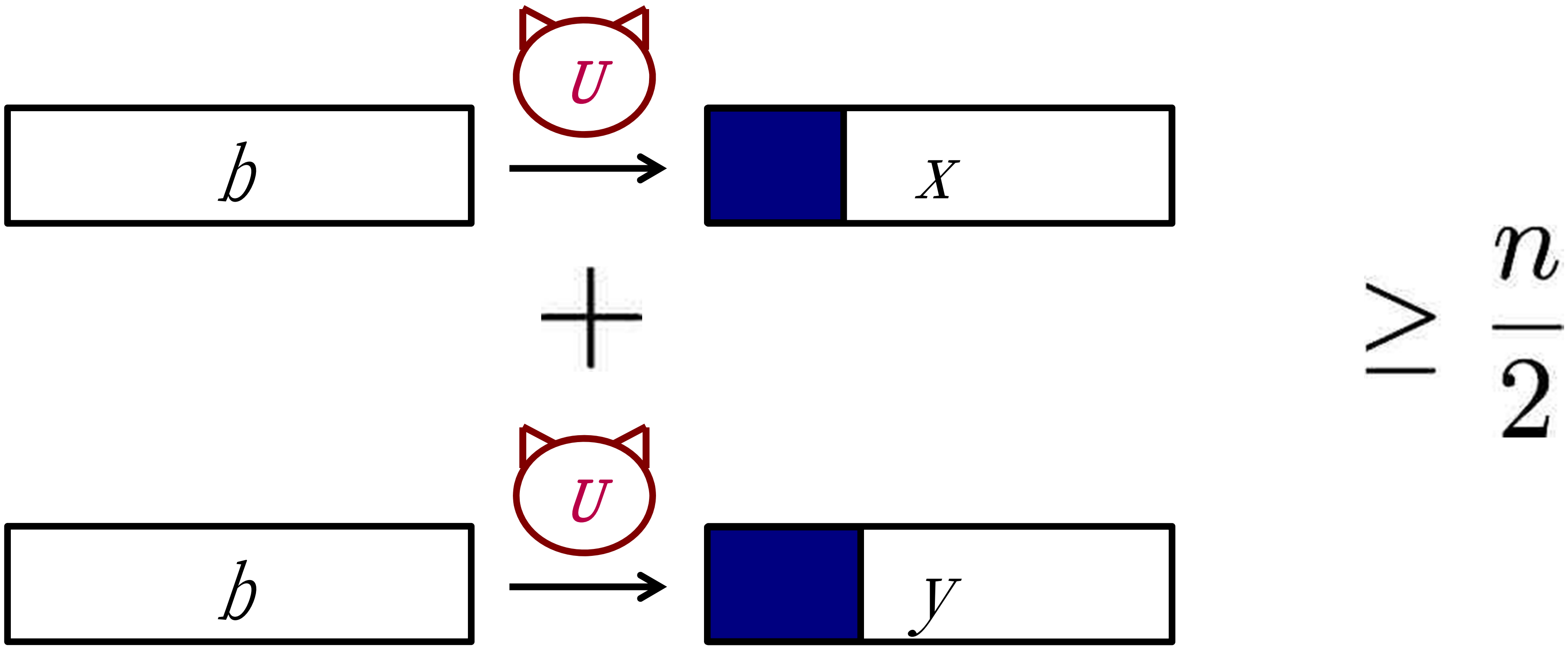

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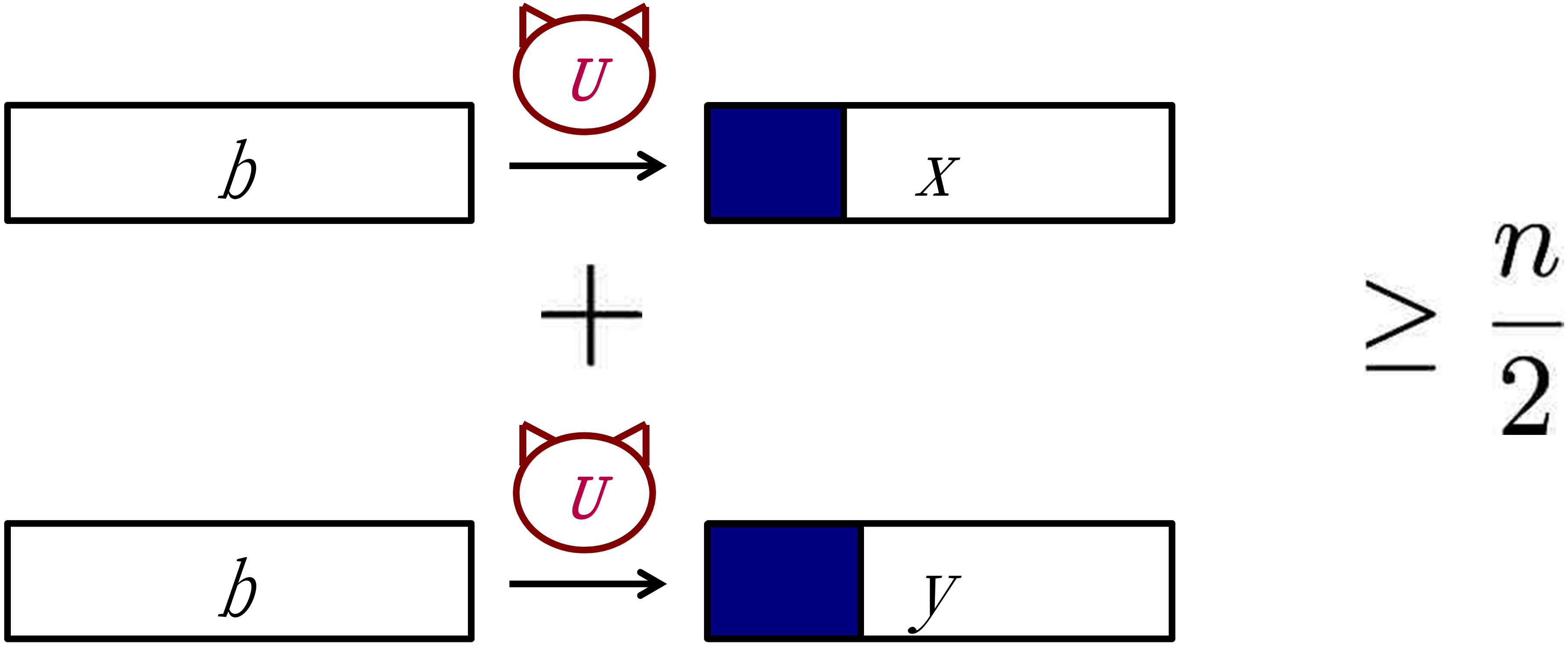

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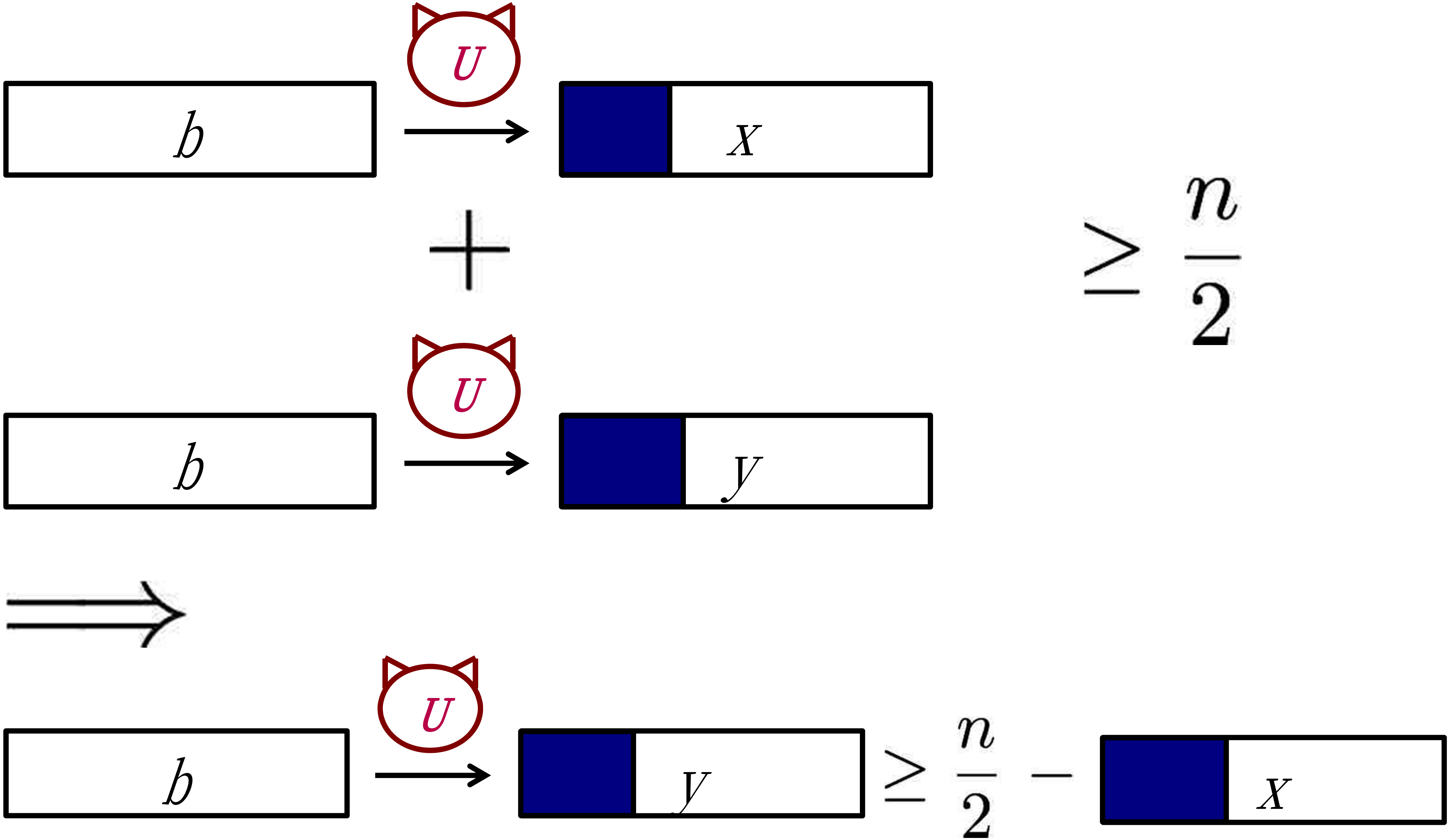
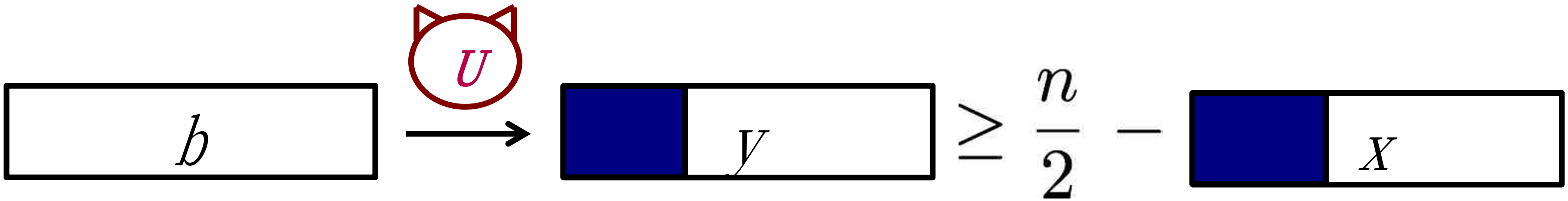
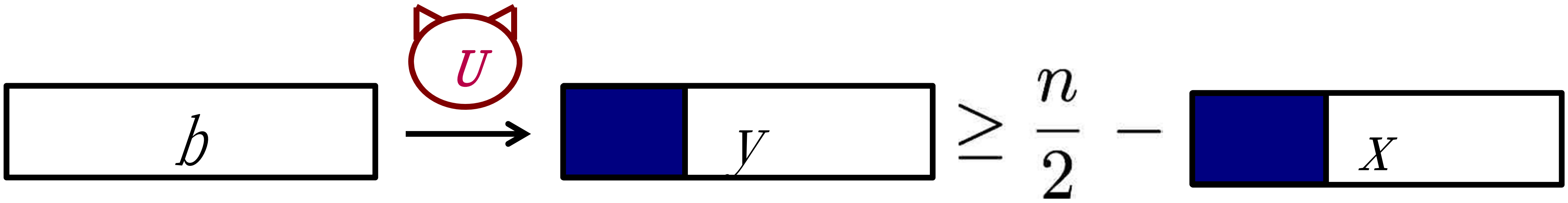
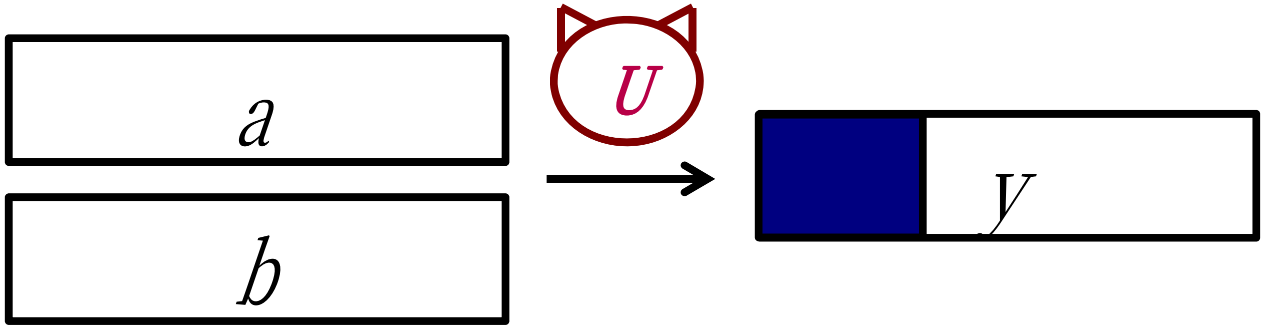
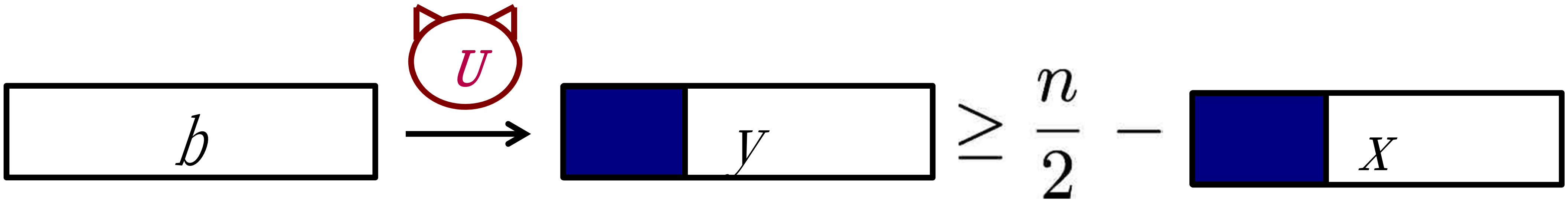



b
U
y
≥ n/2 −
x



On the other hand:





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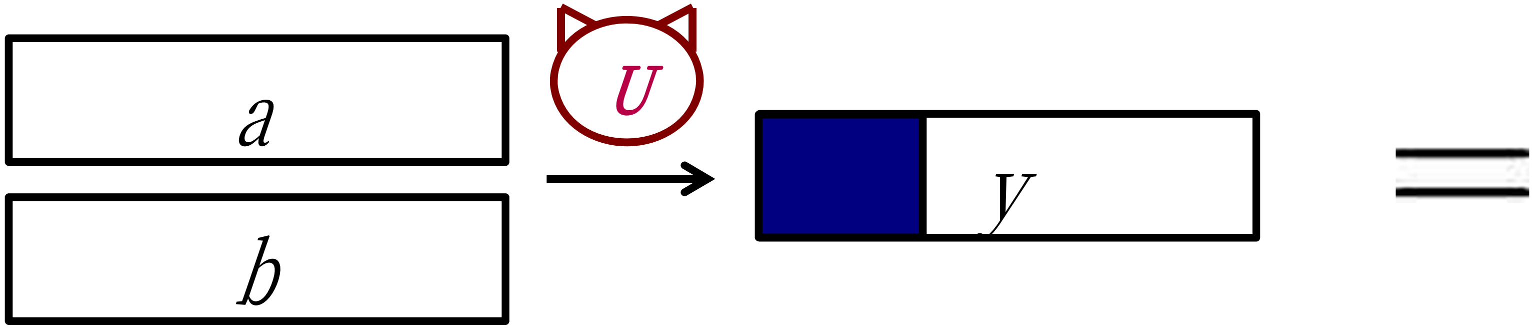
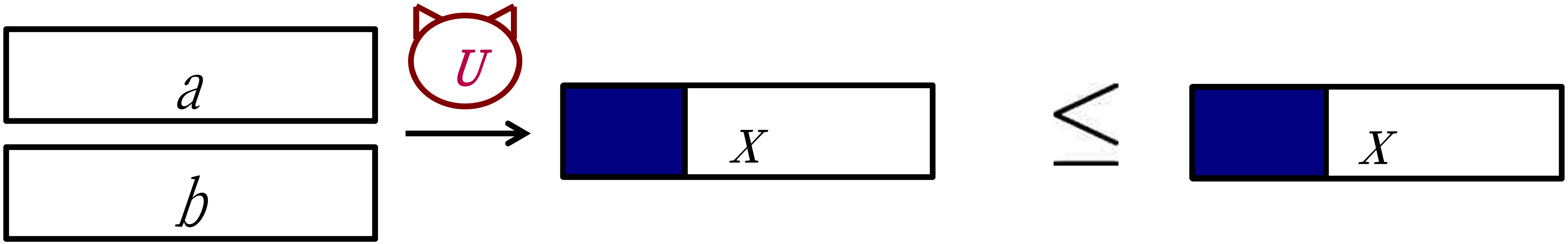
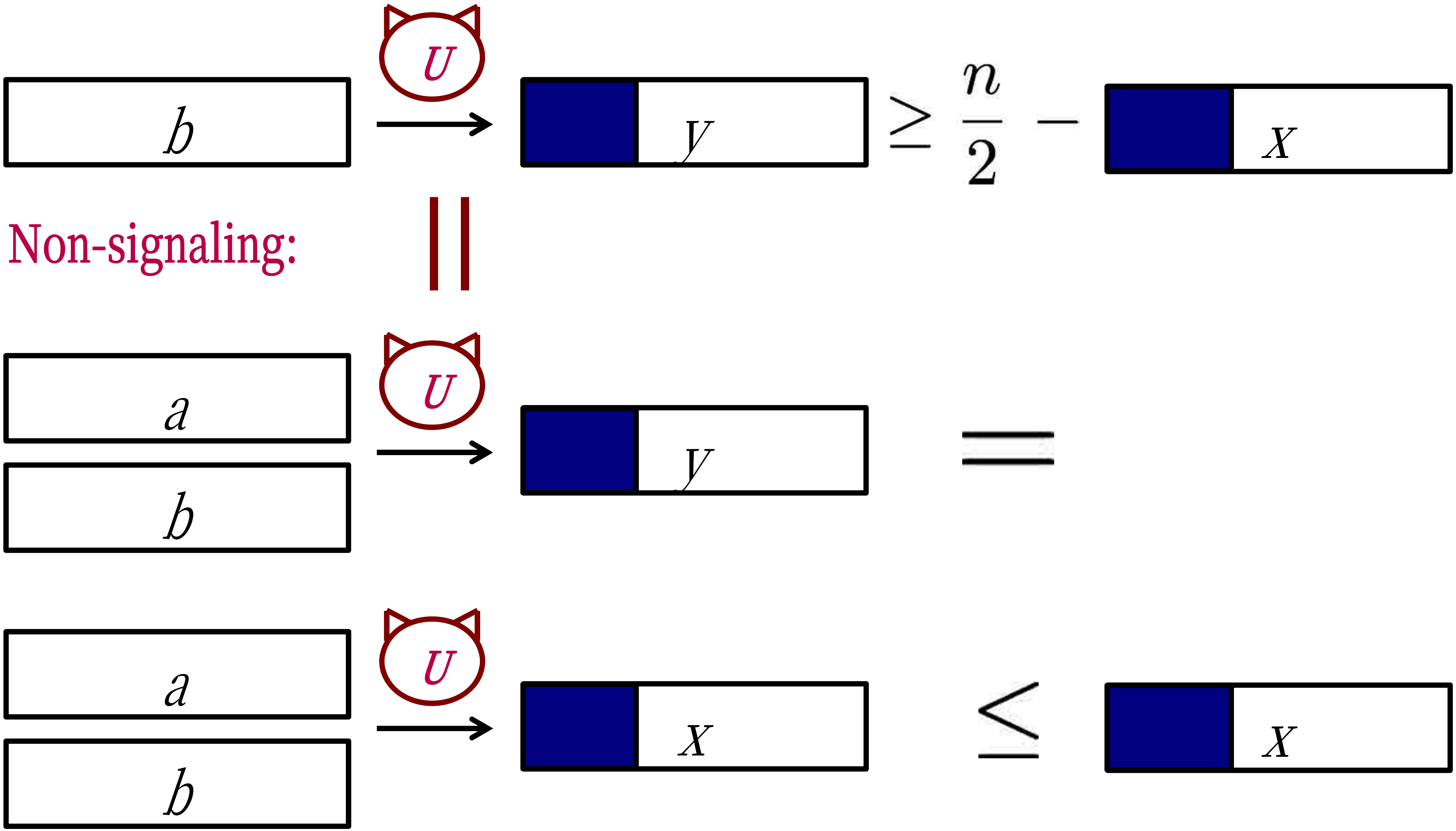
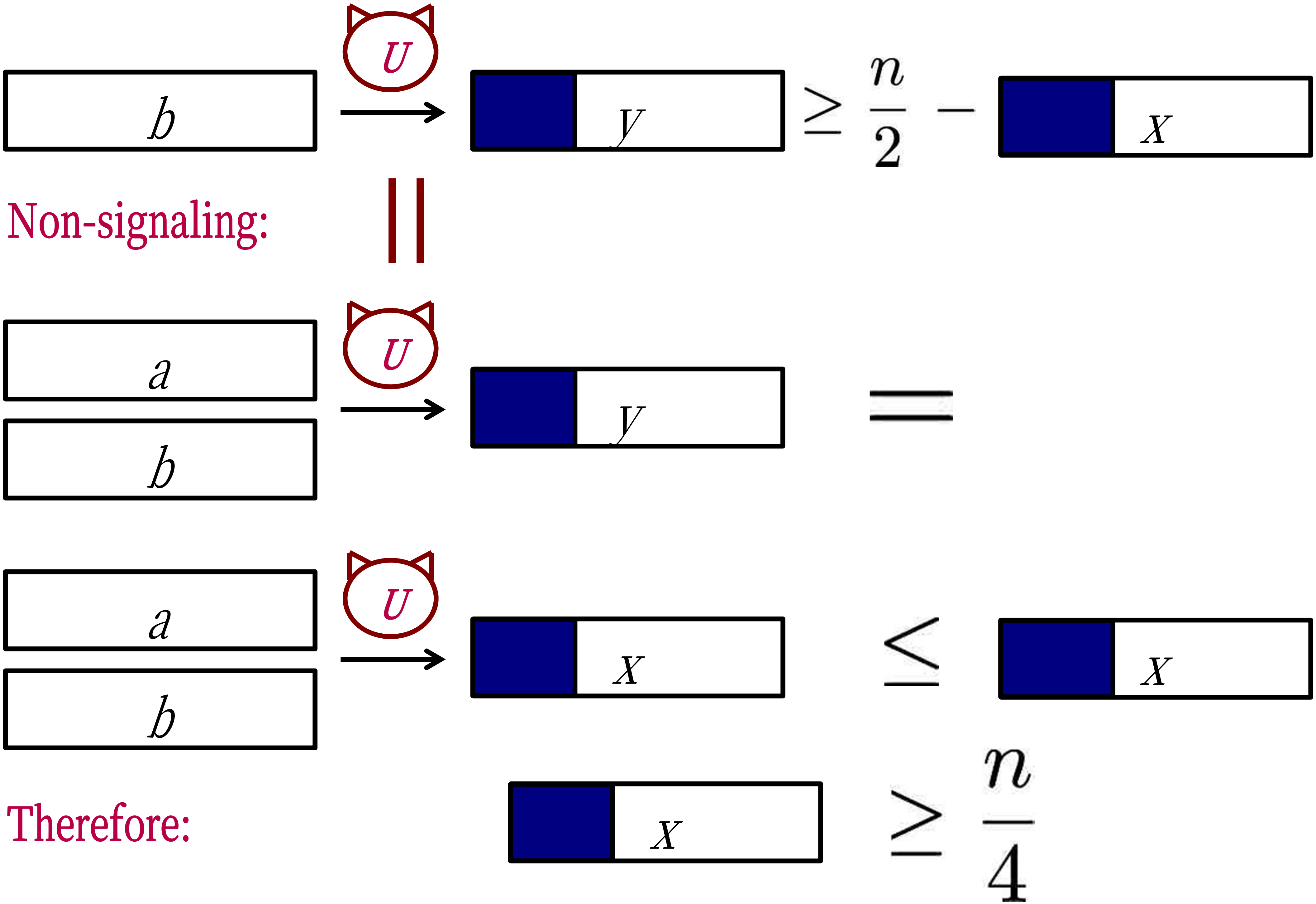






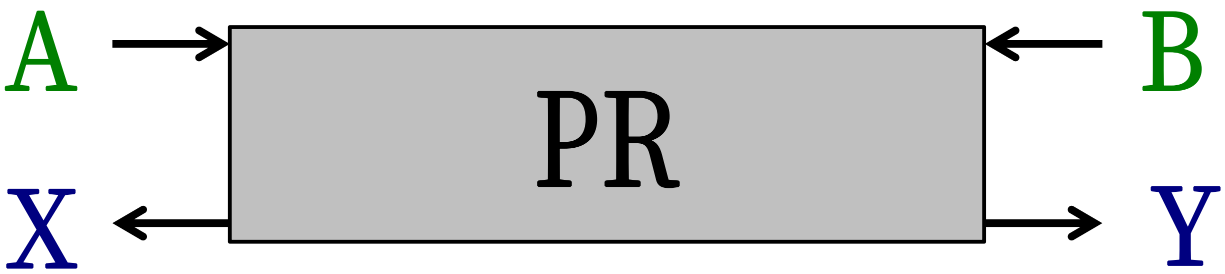
Non-signaling:
U
b
y
$\geq \frac{n}{2} -$
x
a
b
y
=
a
b
x
$\leq$
x


b
U
y
$\geq \frac{n}{2} -$
x
Non-signaling:
a
b
U
y
=
a
b
U
x
$\leq$
x
Therefore:
x
$\geq \frac{n}{4}$

Non-Local Correlations

Factual-only reasoning

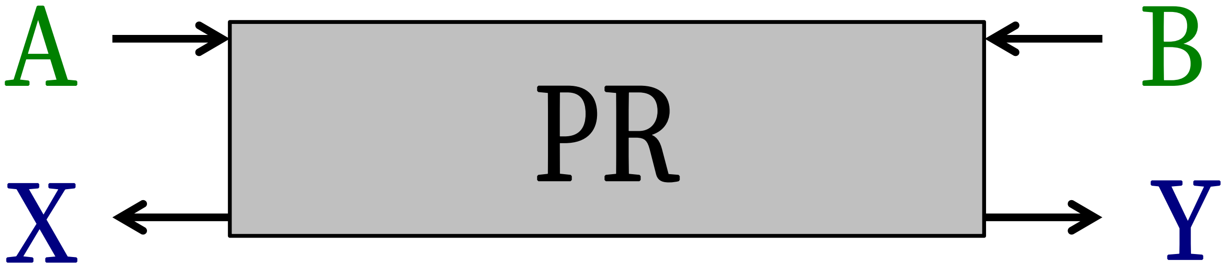
If (A,B) is incompressible…



Non-Local Correlations

Factual-only reasoning

If (A,B) is incompressible...



... then X and Y cannot be computable

Non-Local Correlations

Factual-only reasoning

If (A,B) is incompressible...

... then X and Y cannot be computable

Non-Local Correlations

Factual-only reasoning

If (A,B) is incompressible…

… then X and Y cannot be computable

even given the respective inputs

from parallel-repetition theorem

Ran Raz 1998

Non-Local Correlations

All-or-Nothing Feature of *Church-Turing Hypothesis*

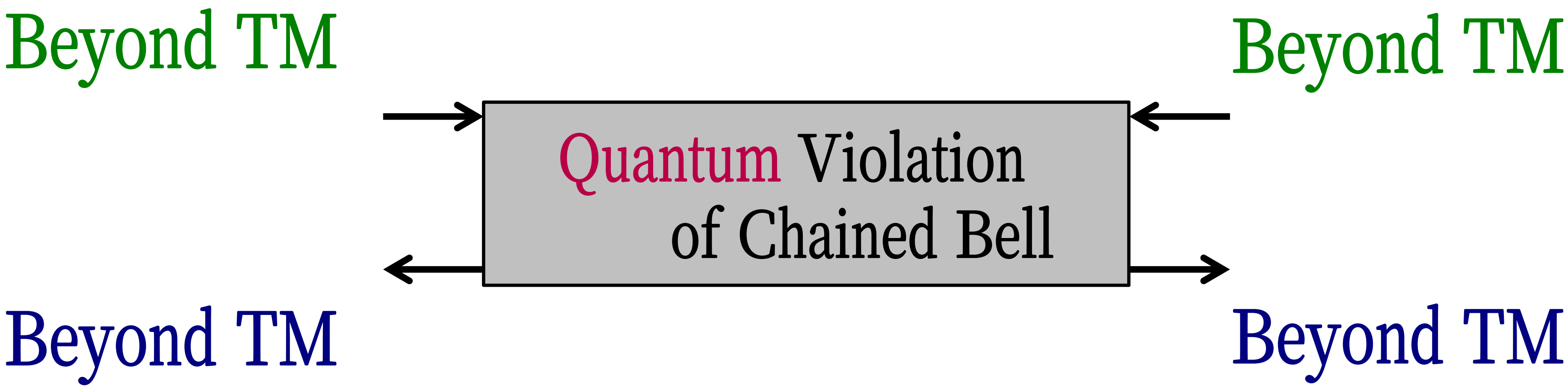
Beyond TM

Beyond TM

Quantum Violation of Chained Bell

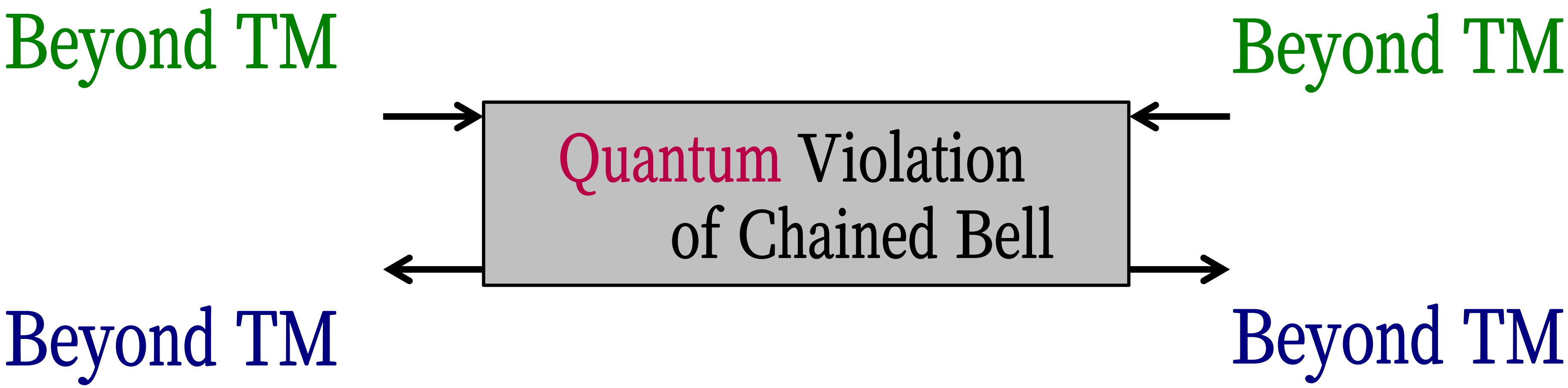
Non-Local Correlations

All-or-Nothing Feature of Church-Turing Hypothesis



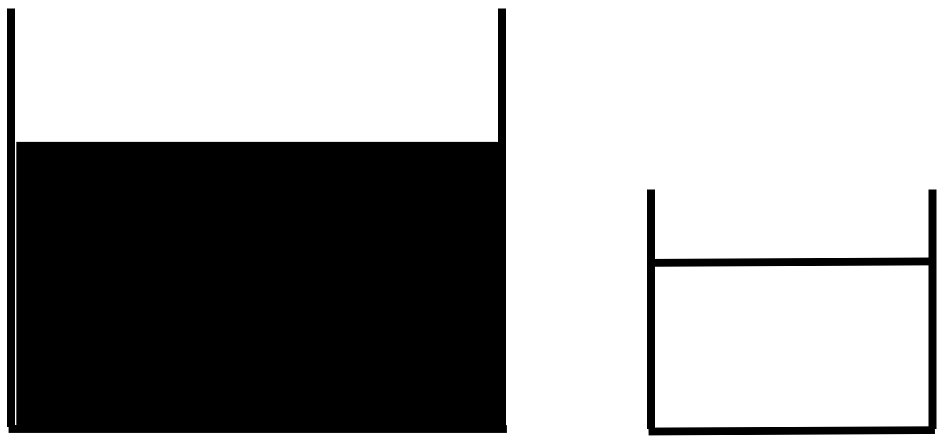
Non-Local Correlations

All-or-Nothing Feature of *Church-Turing Hypothesis*



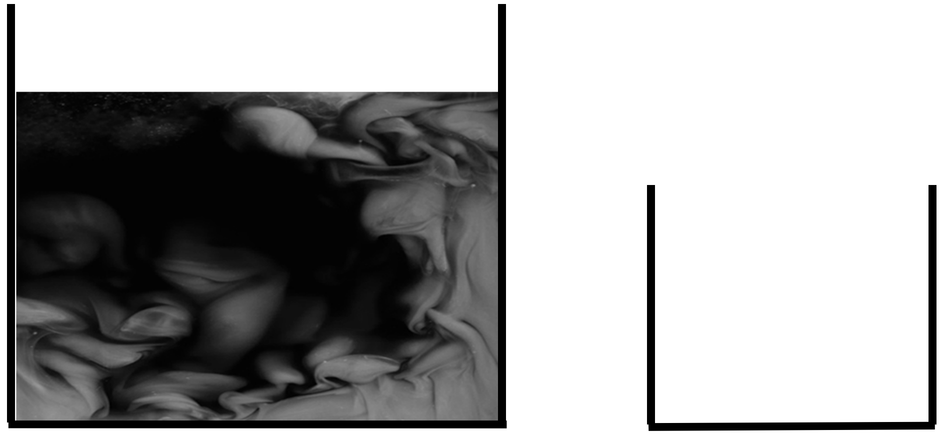
If the experimenter can generate (maximally) uncomputable data, then so can the measured photons

Epilogue: Complexity and Time Asymmetry



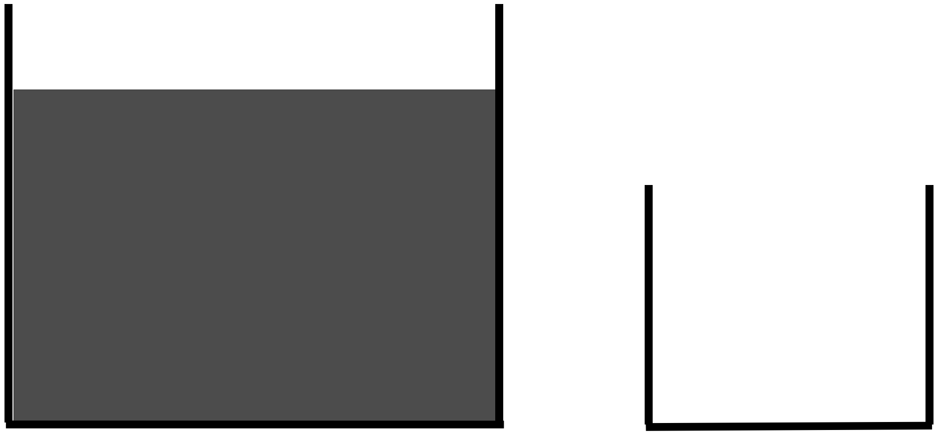
Carroll
Aaronson

Epilogue: Complexity and Time Asymmetry



Carroll
Aaronson

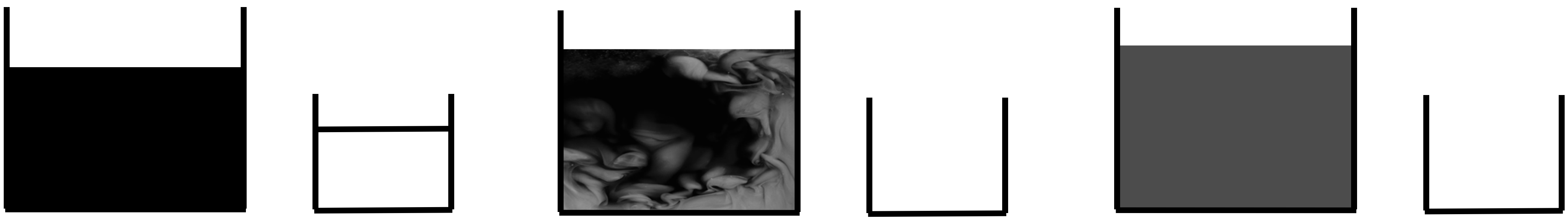
Epilogue: Complexity and Time Asymmetry



Carroll
Aaronson

Epilogue: Complexity and Time Asymmetry

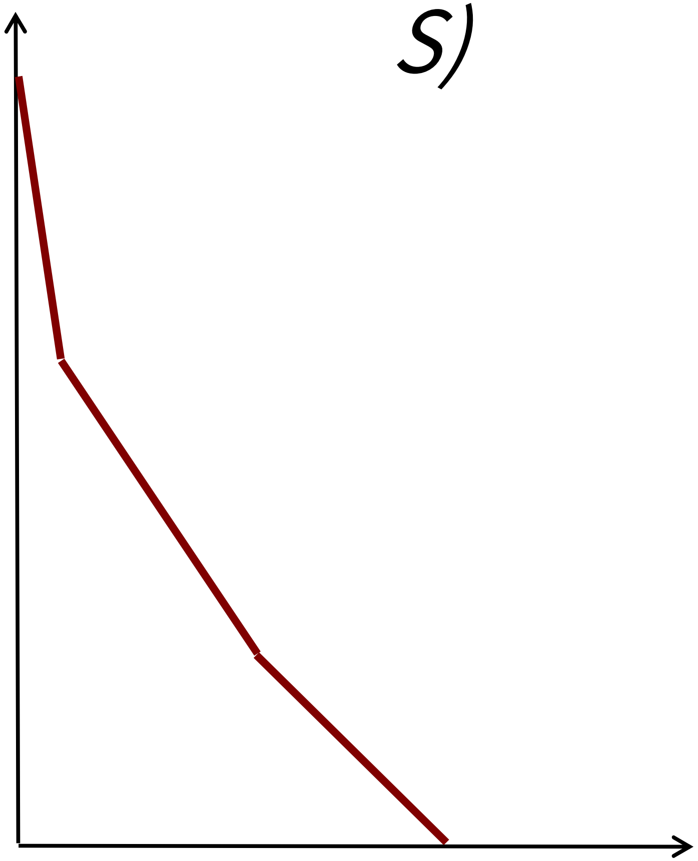
Which quantity is monotonic in time? Macrostate size?



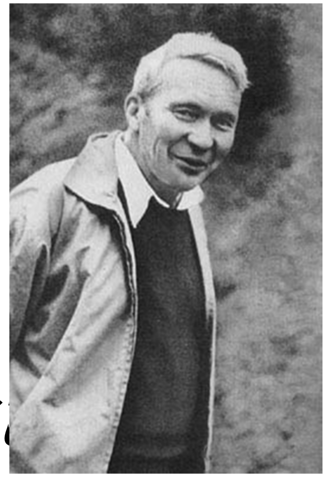
Which is not? Structure?

Epilogue: Complexity and Time Asymmetry

Context-free macrostate (of a finite string S)



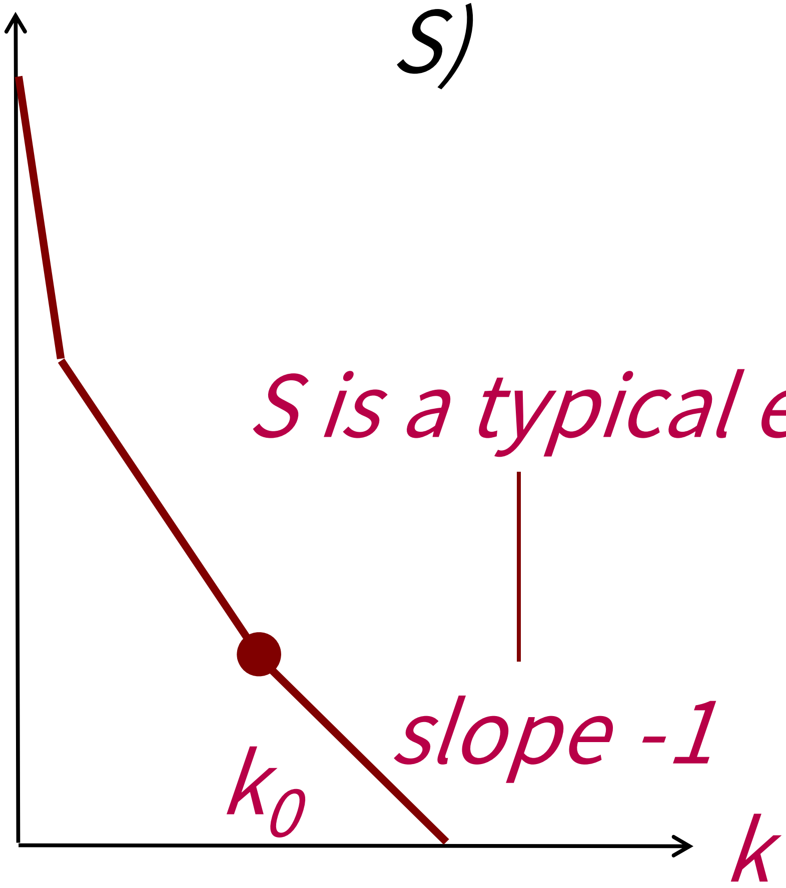
Kolmogorov sufficient statis



Epilogue: Complexity and Time Asymmetry

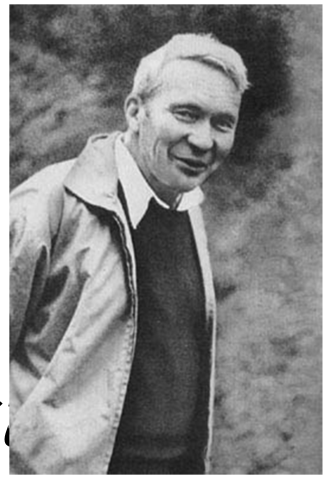
Context-free macrostate (of a finite string S)

log-size of smallest set $M(k)$ with complexity k containing S



S is a typical element of $M(k_0)$

Kolmogorov sufficient statis



Epilogue: Complexity and Time Asymmetry

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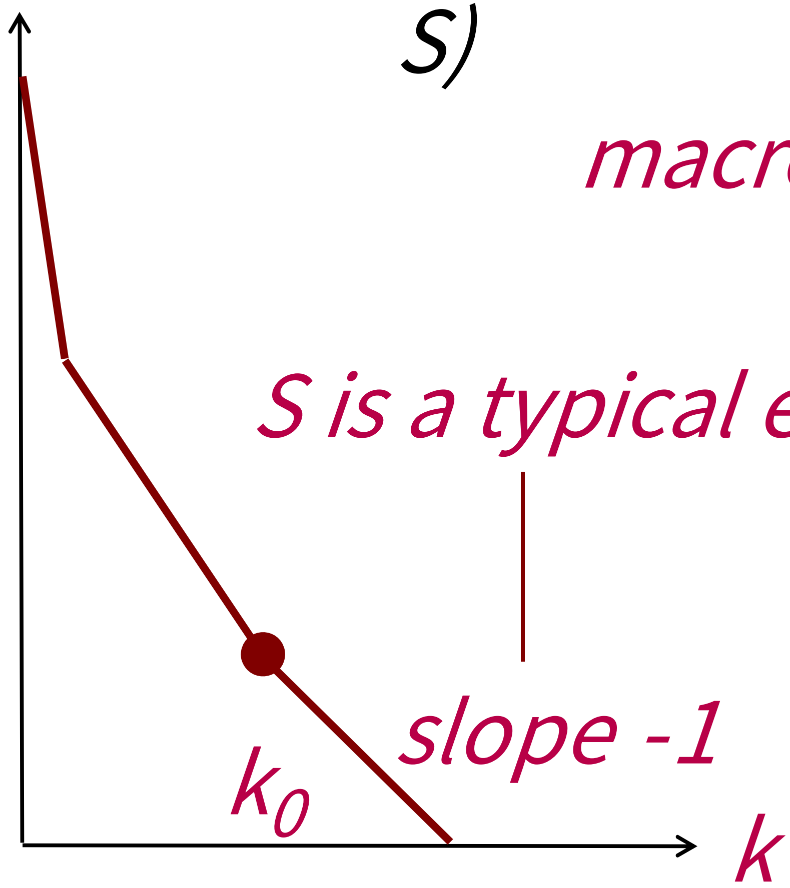
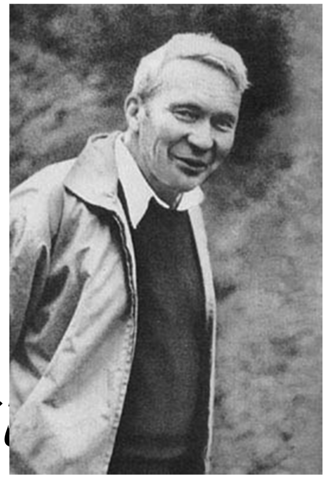


macrostate: $M(k_0)$

S is a typical element of $M(k_0)$



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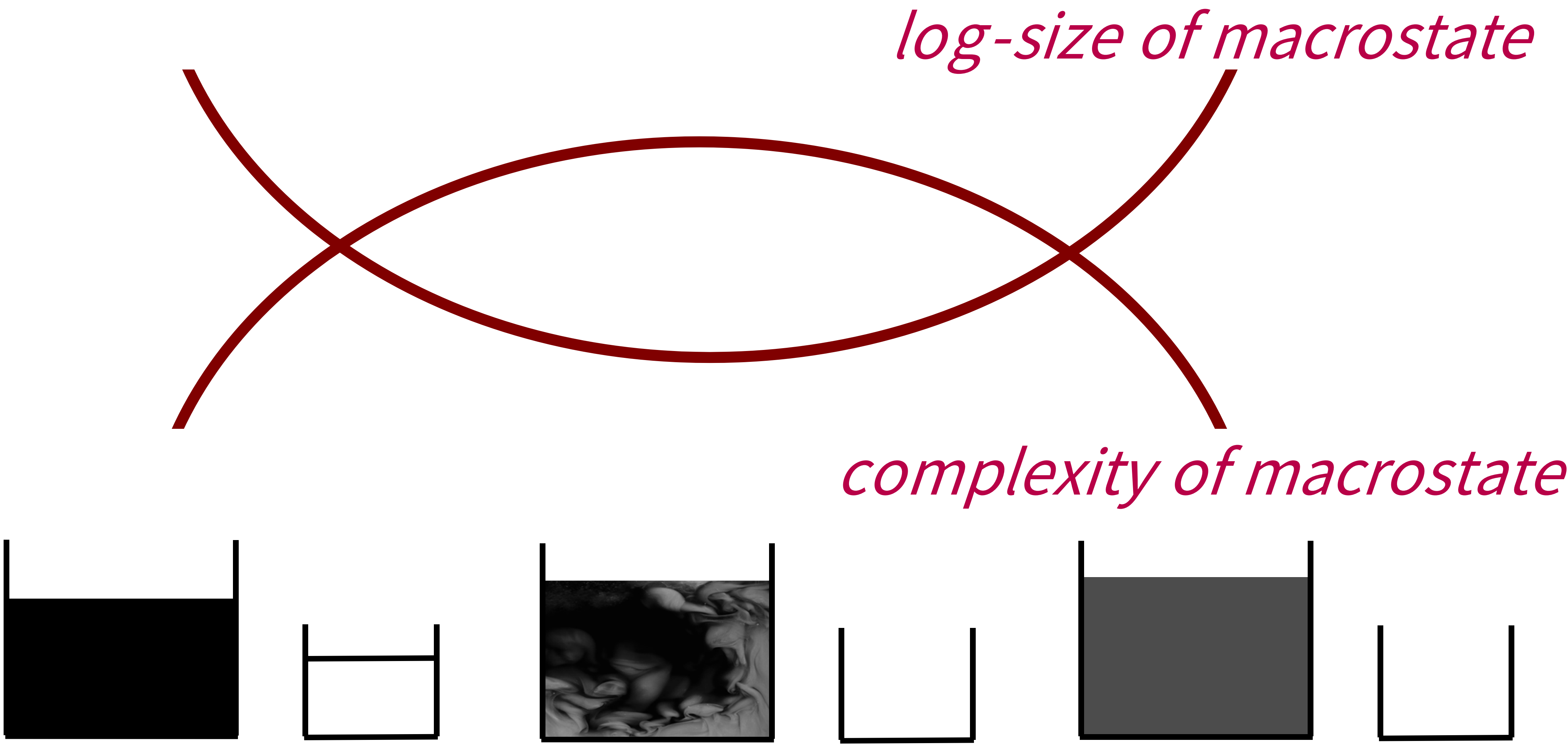
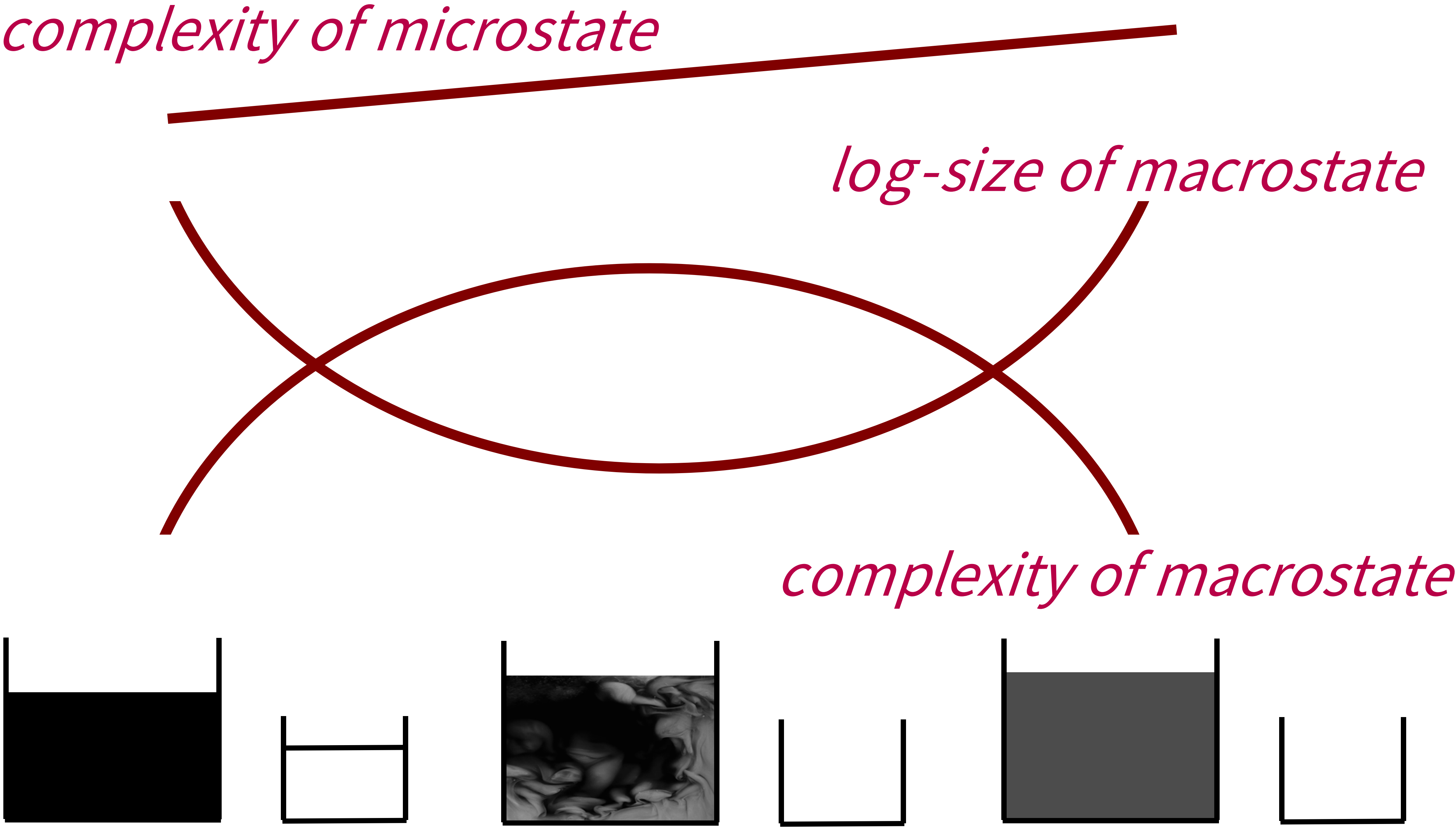
Epilogue: Complexity and Time Asymmetry



Epilogue: Complexity and Time Asymmetry



Epilogue: Complexity and Time Asymmetry

Observations

logical reversibility $\Longrightarrow$ K(tape/microstate) non-decreasing

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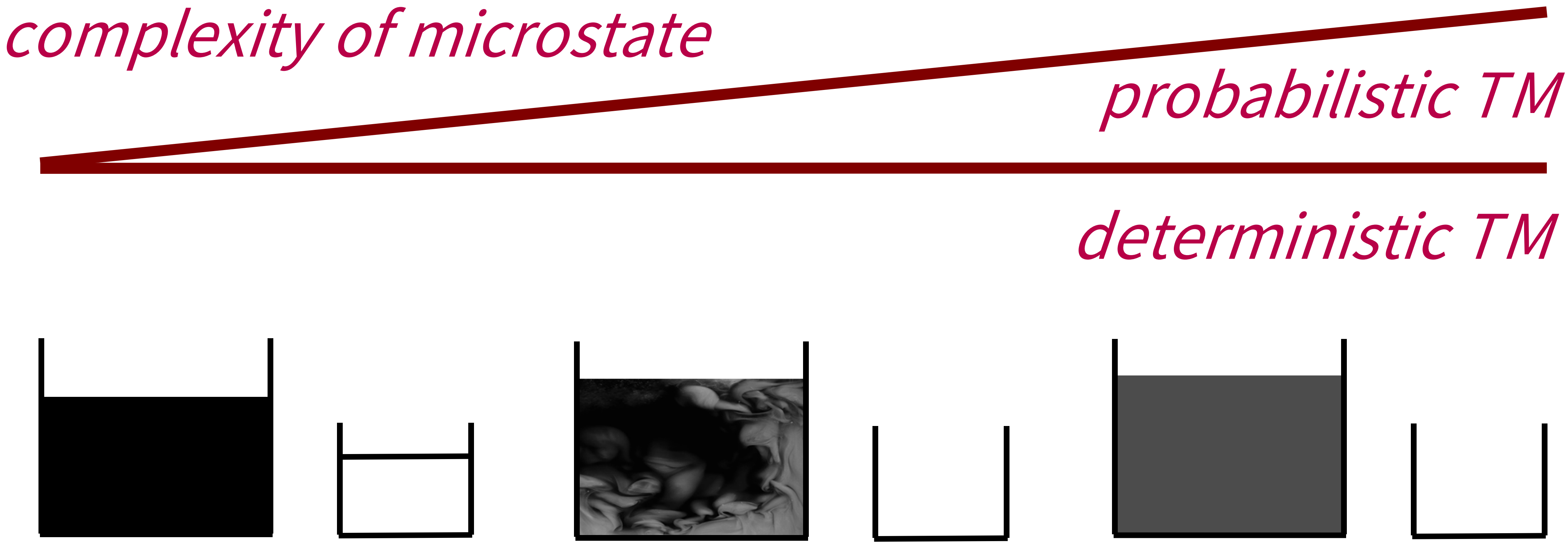

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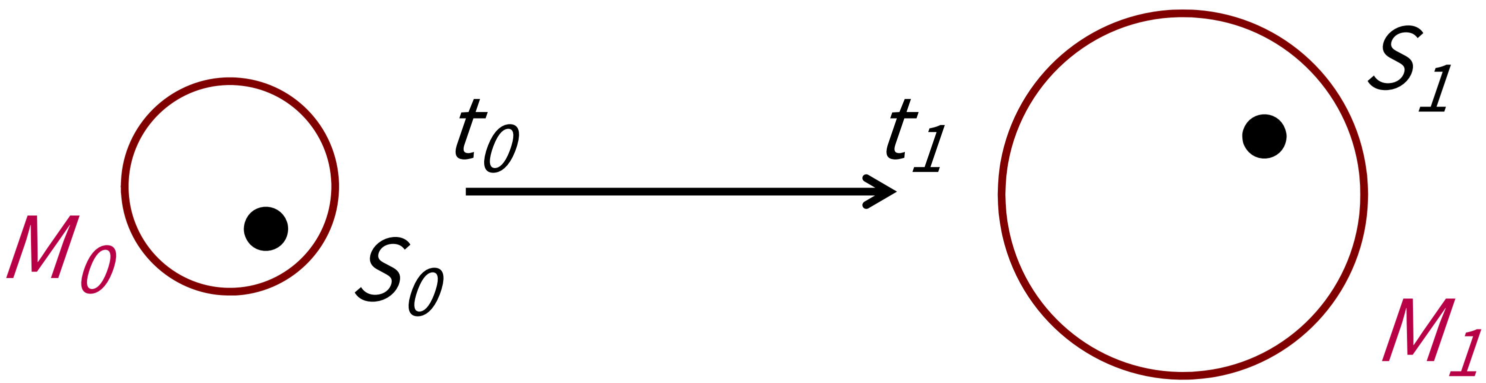
$\Longrightarrow$ simple macrostate not shrinking



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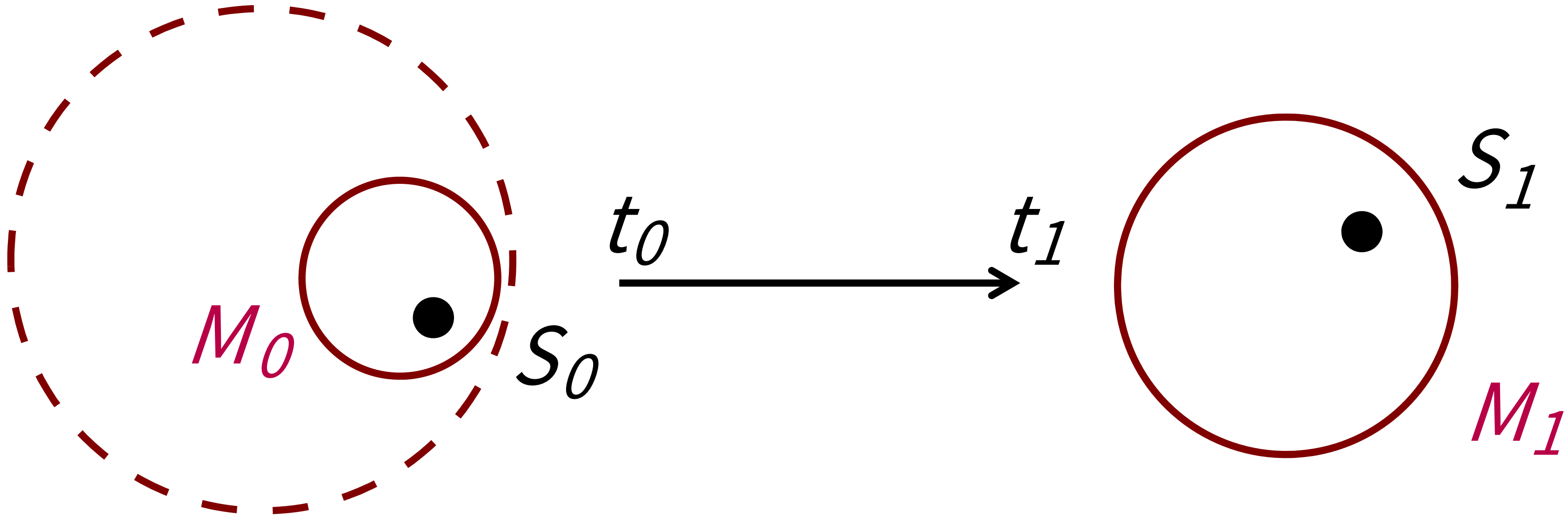

Based on collaboration and discussion with:

Mateus Araújo, Veronika Baumann, Ämin Baumeler, Harvey Brown, Caslav Brukner, Sandro Coretti, Bora Dakic, Paul Erker, Adrien Feix, Jürg Fröhlich, Nicolas Gisin, Esther Hänggi, Arne Hansen, Marcus Huber, Alberto Montina, Benno Salwey, and Andreas Winter