Quantum theory without predefined causal structure

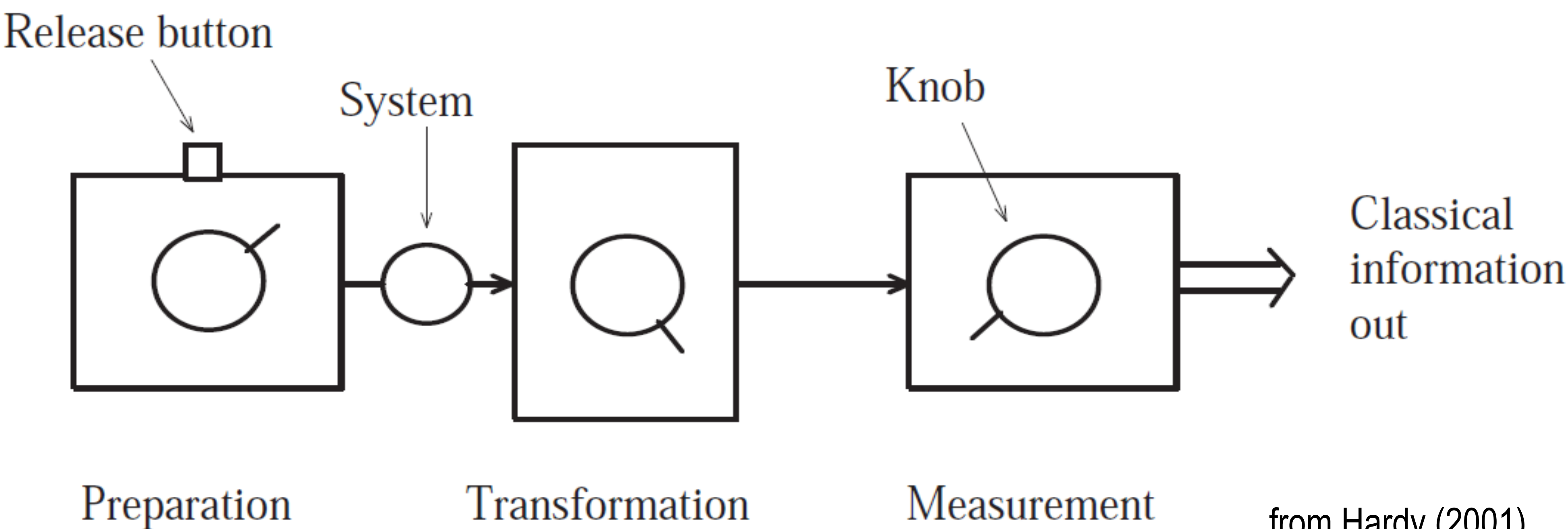
Ognyan Oreshkov

Centre for Quantum Information and Communication, Université Libre de Bruxelles

Based on work with
Caslav Brukner, Nicolas Cerf, Fabio Costa, Christina Giarmatzi

Quantum Nonlocality, Causal Structures, and Device-independent Quantum Information, Tainan, 2015

Operational Approach

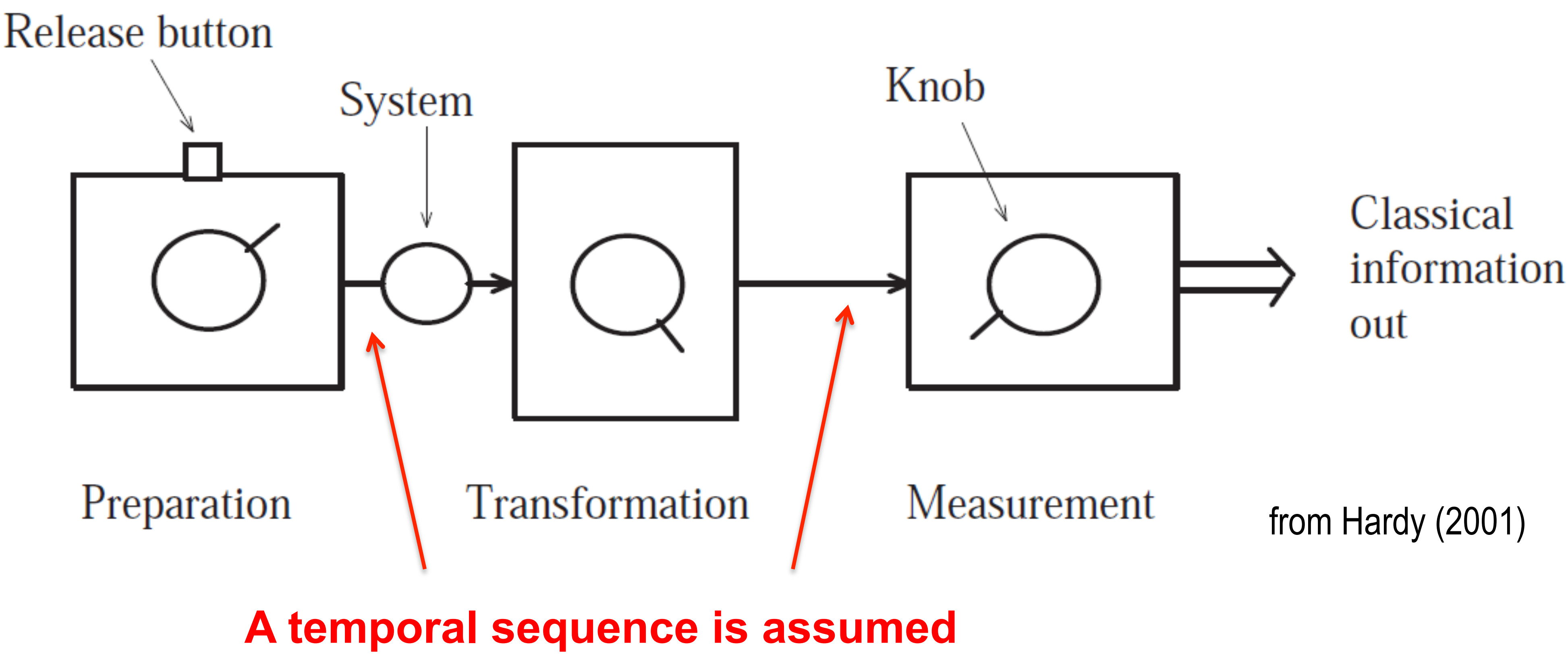


from Hardy (2001)

Significant progress in understanding QM from **operational** perspective.

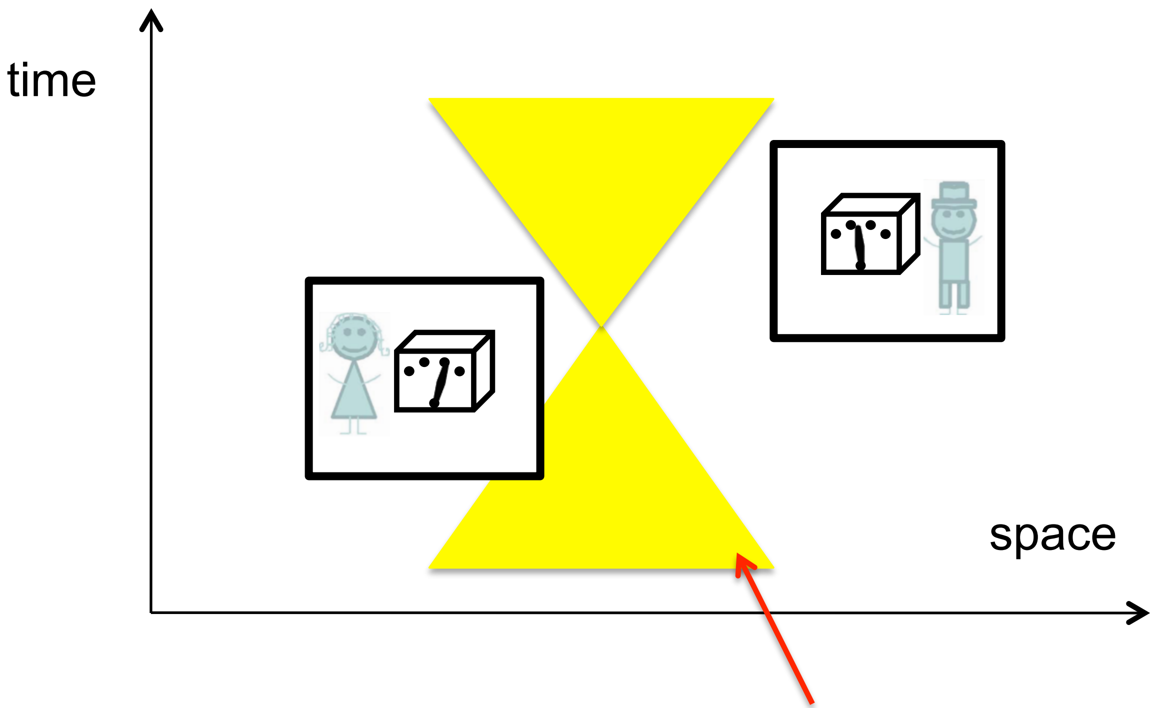
Hardy (2001), Barrett (2005), Dakic and Brukner (2009), Massanes and Mülelr (2010), Chiribella, D'Ariano, and Perinotti (2010) ….

Operational Approach



from Hardy (2001)

Correlations between experiments in space-time



A causal structure is assumed

Questions

1. Could we understand causal structure from more primitive concepts (e.g., signaling from A to B $\rightarrow$ A is in the past of B)?

2. Why does signalling always go forward in time?

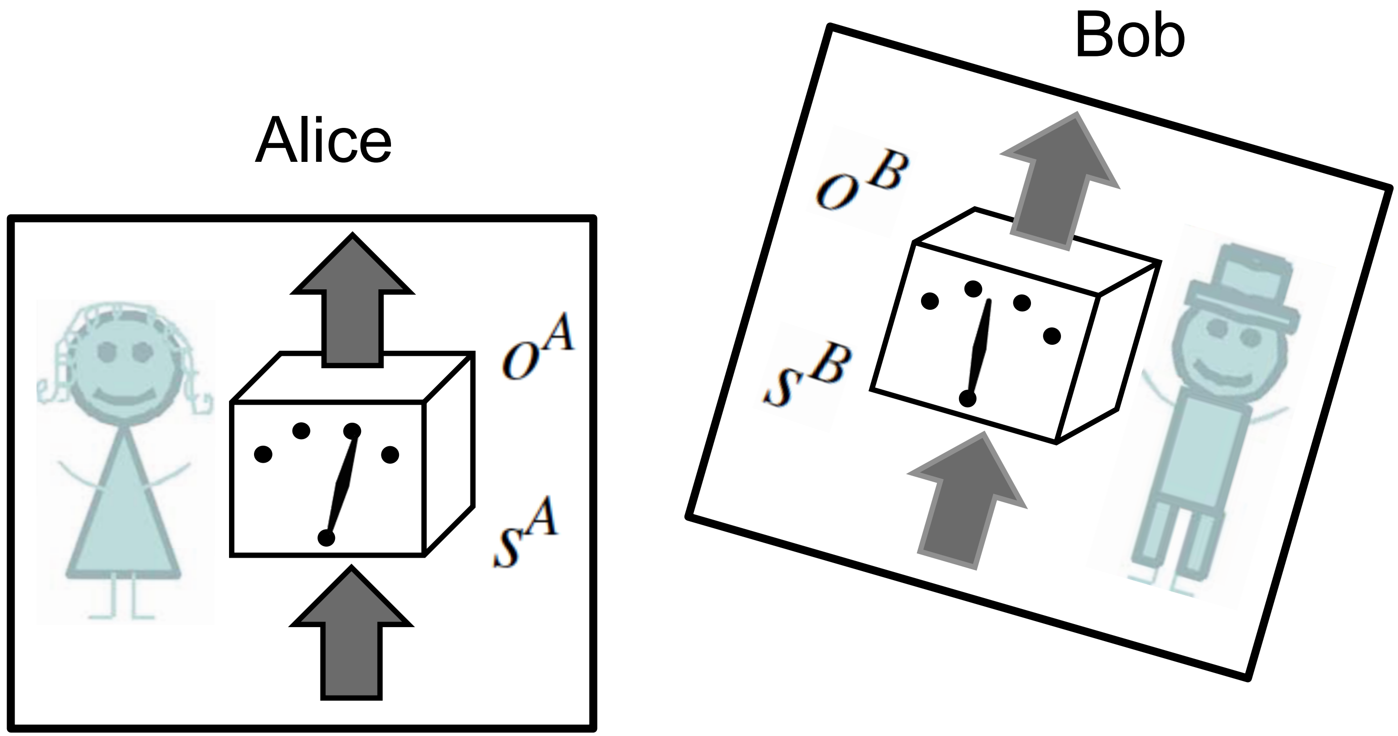
3. Can we generalize quantum theory so that time and causal structure are not predefined? (Motivation: quantum gravity)

4. What new physical possibilities would this imply?

Outline

- The process framework for operations with no causal order
- A time-symmetric operational approach to quantum theory
- Quantum theory without any prior notion of time

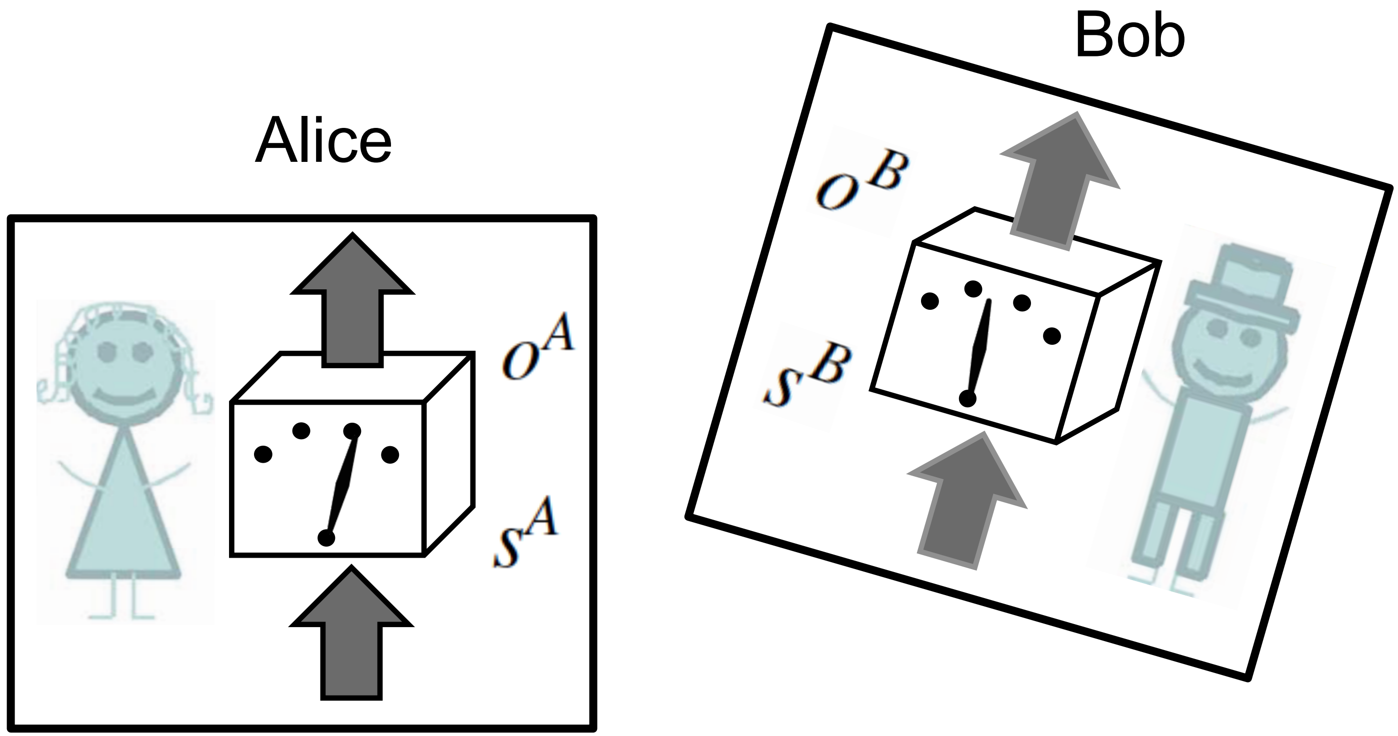
The process framework



No assumption of pre-existing causal order.

O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

The process framework



'Process'
(catalogue of probabilities)

…

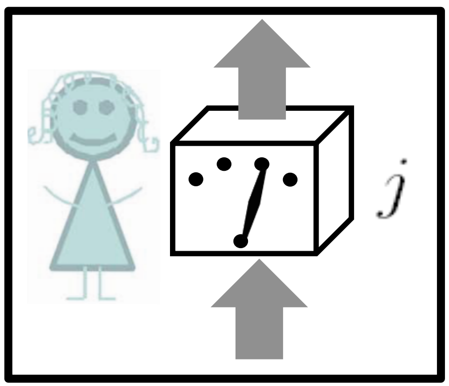
$$P(o^A, o^B | s^A, s^B)$$

…

Quantum processes

Local descriptions agree with quantum mechanics

$\mathcal{H}^2$



$\mathcal{H}^1$

Transformations = completely positive (CP) maps

$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$$

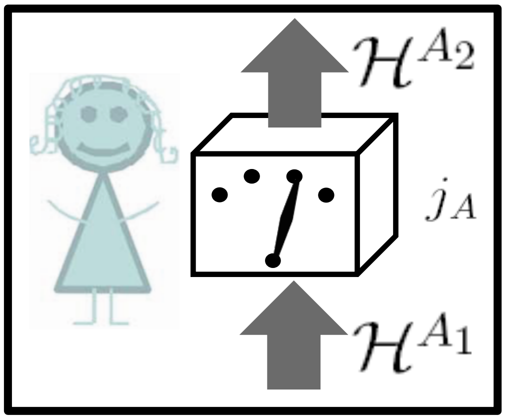
Kraus representation:

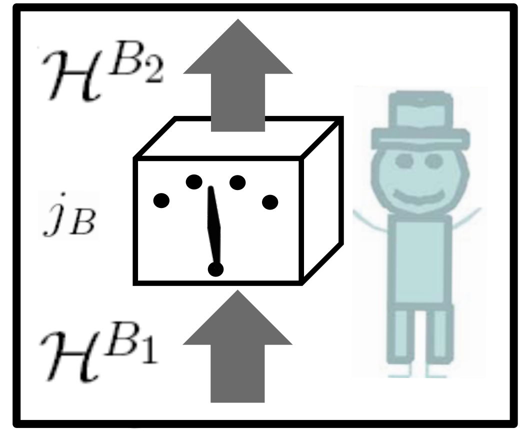
$$\mathcal{M}_j(\rho) = \sum_k E_{jk} \rho E_{jk}^\dagger$$

Completeness relation:

$$\sum_j \sum_k E_{jk}^\dagger E_{jk} = I$$

Quantum processes





$$\mathcal{M}^A_{j^A} : \mathcal{L}(\mathcal{H}^{A_1}) \to \mathcal{L}(\mathcal{H}^{A_2}) \qquad \mathcal{M}^B_{j^B} : \mathcal{L}(\mathcal{H}^{B_1}) \to \mathcal{L}(\mathcal{H}^{B_2})$$

Assumption 1: The probabilities are functions of the local CP maps,

$$P(\mathcal{M}^A_{j^A}, \mathcal{M}^B_{j^B}, \cdots)$$

Local validity of QM $\Rightarrow$ $P(\mathcal{M}^A, \mathcal{M}^B, \cdots)$ is **linear** in $\mathcal{M}^A$, $\mathcal{M}^B$, ...

Choi-Jamiołkowski isomorphism

CP maps

Positive semidefinite matrices

$$\mathcal{M}^{A} : \mathcal{L}(\mathcal{H}^{A_1}) \to \mathcal{L}(\mathcal{H}^{A_2}) \longleftrightarrow M^{A_1 A_2} \in \mathcal{L}(\mathcal{H}^{A_1}) \otimes \mathcal{L}(\mathcal{H}^{A_2})$$

The process matrix

Representation

$$P(\mathcal{M}^{A}_{j^A}, \mathcal{M}^{B}_{j^B}, \cdots) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2}_{j^A} \otimes M^{B_1 B_2}_{j^B} \otimes \cdots\right)\right]$$

Process matrix

The process matrix

Representation

$$P(\mathcal{M}^{A}_{j^A}, \mathcal{M}^{B}_{j^B}, \cdots) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2}_{j^A} \otimes M^{B_1 B_2}_{j^B} \otimes \cdots\right)\right]$$

Process matrix

Similar to Born's rule but can describe signalling!

The process matrix

Conditions on W (assuming the parties can share entanglement):

1. Non-negative probabilities: $W^{A_1 A_2 B_1 B_2 \cdots} \geq 0$

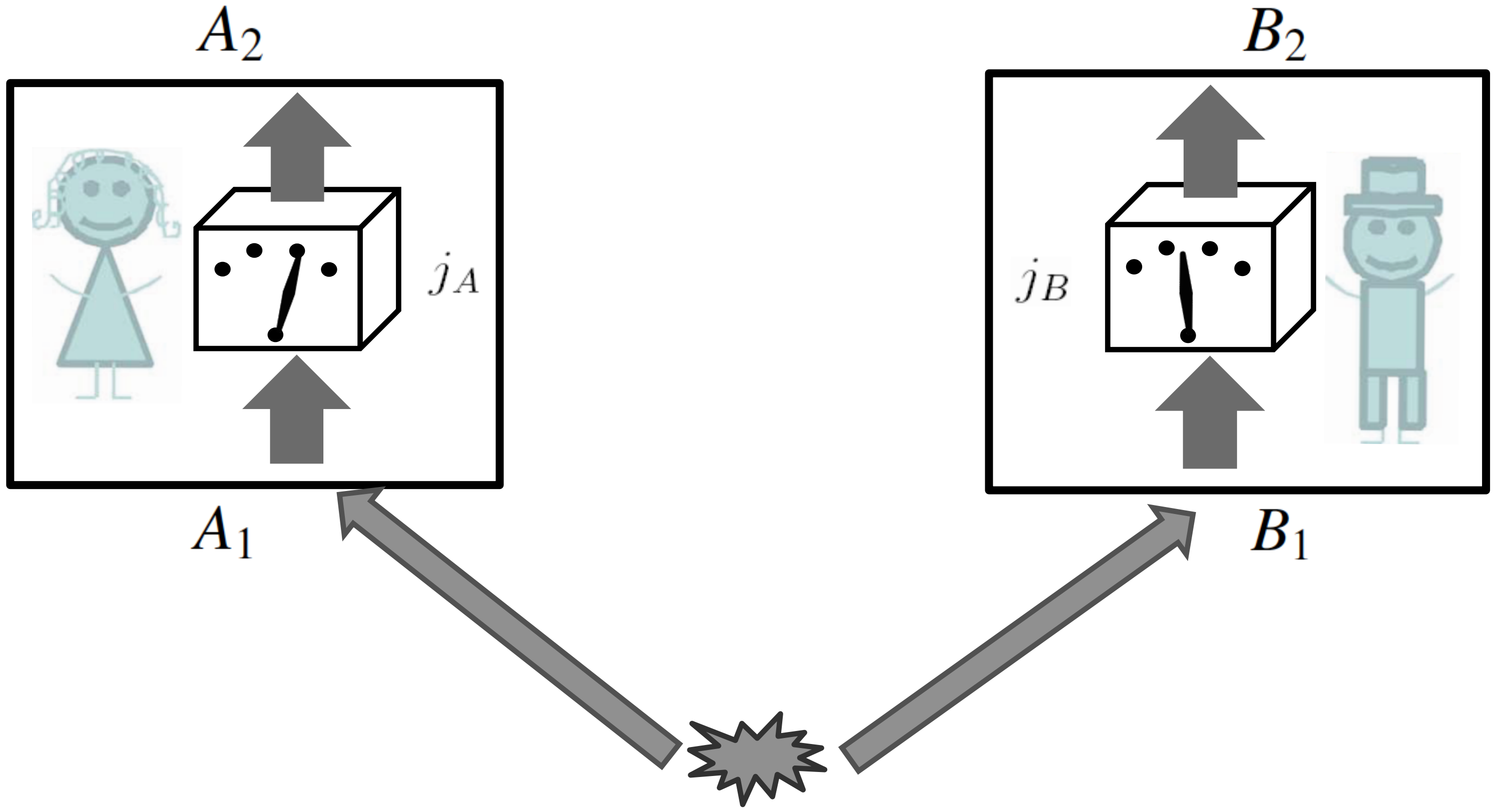
2. Probabilities sum up to 1:

$$\mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2} \otimes M^{B_1 B_2} \otimes \cdots\right)\right] = 1$$

on all CPTP maps $M^{A_1 A_2}$, $M^{B_1 B_2}$, ...

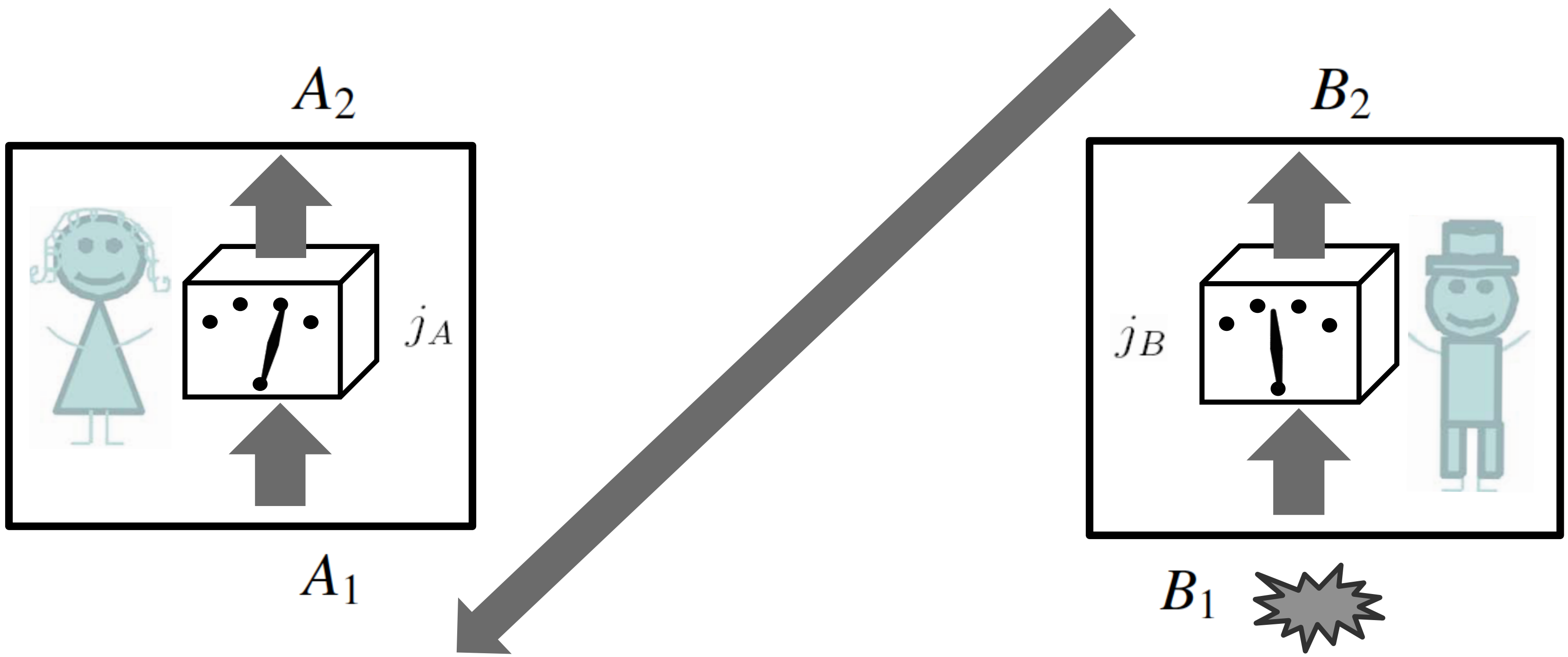
→ Simple characterization via the allowed terms in a Hilbert-Schmidt basis

Example: bipartite state



$$W^{A_1 A_2 B_1 B_2} = \rho^{A_1 B_1} \otimes \mathbb{1}^{A_2 B_2}$$

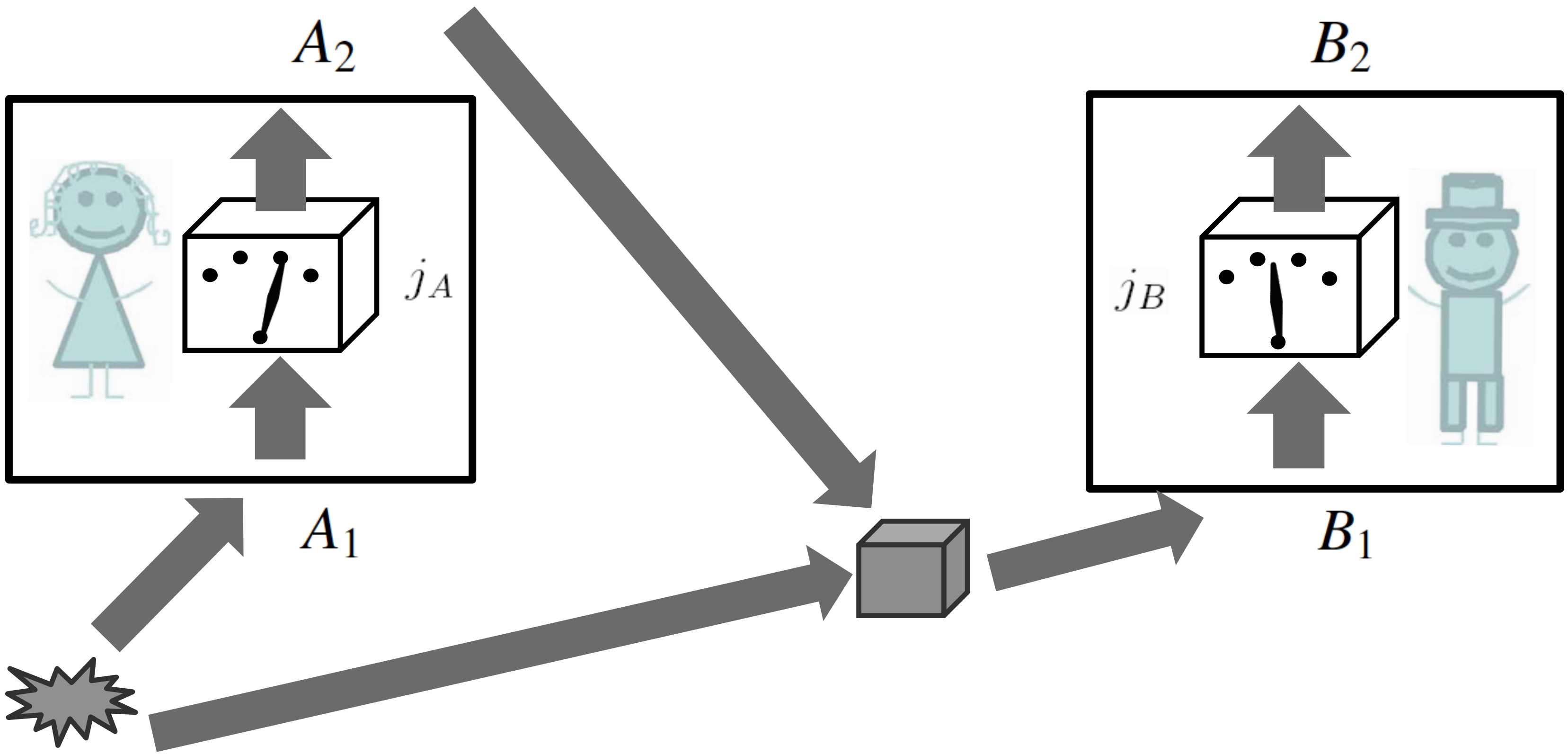
Example: channel B→A



$$W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_2} \otimes (C^{A_1 B_2})^T \otimes \rho^{B_1}$$

Example: channel with memory A→ B

(The most general possibility compatible with no signalling from B to A)



$$W^{A_1 A_2 B_1 B_2} = W^{A_1 A_2 B_1} \otimes \mathbb{1}^{B_2}$$

Bipartite processes with causal realization

$W^{A\not\leq B}$ – no signalling from A to B (ch. with memory from A to B)

$W^{B\not\leq A}$ – no signalling from B to A (ch. with memory from B to A)

Bipartite processes with causal realization

$W^{A \not\preceq B}$ – no signalling from A to B (ch. with memory from A to B)

$W^{B \not\preceq A}$ – no signalling from B to A (ch. with memory from B to A)

More generally, we may conceive **causally separable** processes (probabilistic mixtures of fixed-order processes):

$$\boxed{W^{A_1 A_2 B_1 B_2} = q W^{B \not\preceq A} + (1-q) W^{A \not\preceq B}}$$

Bipartite processes with causal realization

$W^{A \npreceq B}$ – no signalling from A to B (ch. with memory from A to B)

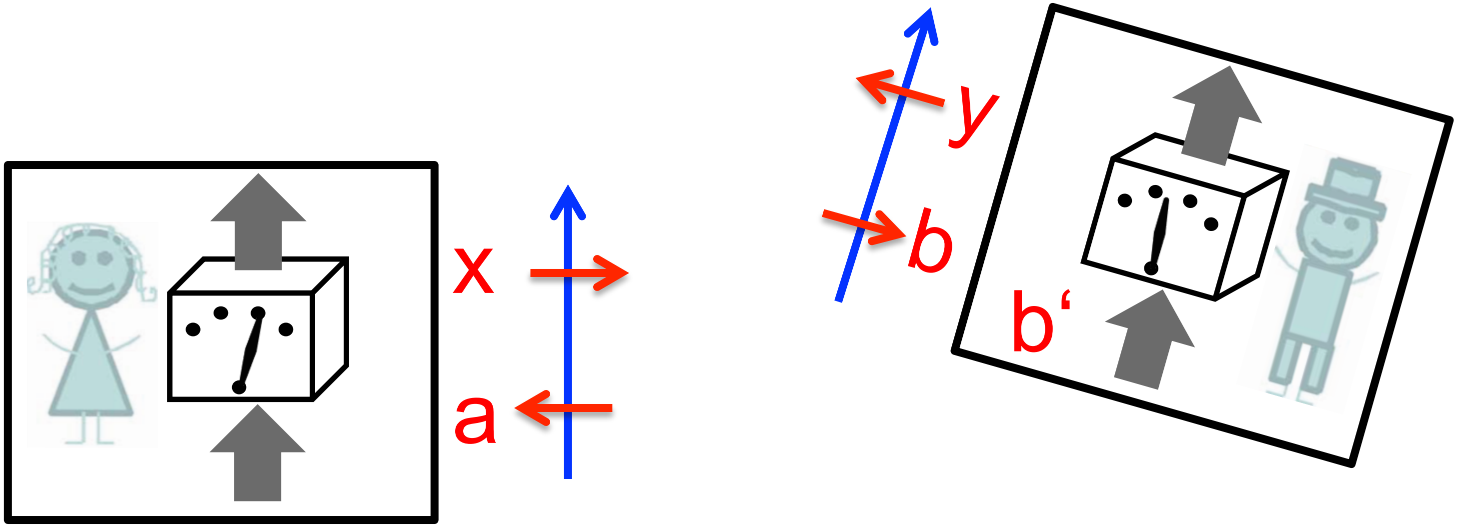
$W^{B \npreceq A}$ – no signalling from B to A (ch. with memory from B to A)

More generally, we may conceive **causally separable** processes (probabilistic mixtures of fixed-order processes):

$$\boxed{W^{A_1 A_2 B_1 B_2} = q W^{B \npreceq A} + (1-q) W^{A \npreceq B}}$$

Are all possible *W* causally separable?

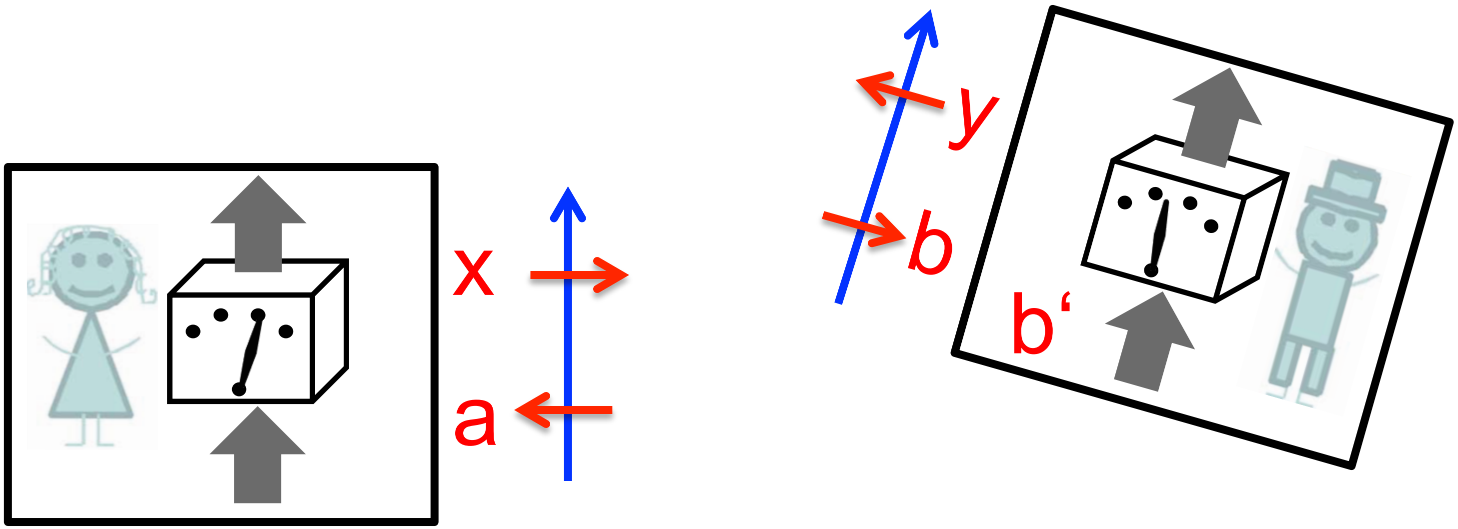
Causal game



- Alice is given bit **a** and Bob bit **b**.
- Bob is given an additional bit **b'** that tells him whether he should guess her bit (**b'=1**) or she should guess his bit (**b'=0**).
- Alice produces **x** and Bob **y**, which are their best guesses for the value of the bit given to the other.
- The goal is to maximize the probability for correct guess:

$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)]$$

Causal game



Definite causal order between the events in the experiment →

$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)] \leq \frac{3}{4}$$

A non-causal process

Can achieve probability of success $p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$

two-level systems

The operations of Alice and Bob do not occur in a definite order!

More info: O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

A non-causal process

Can achieve probability of success $p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$

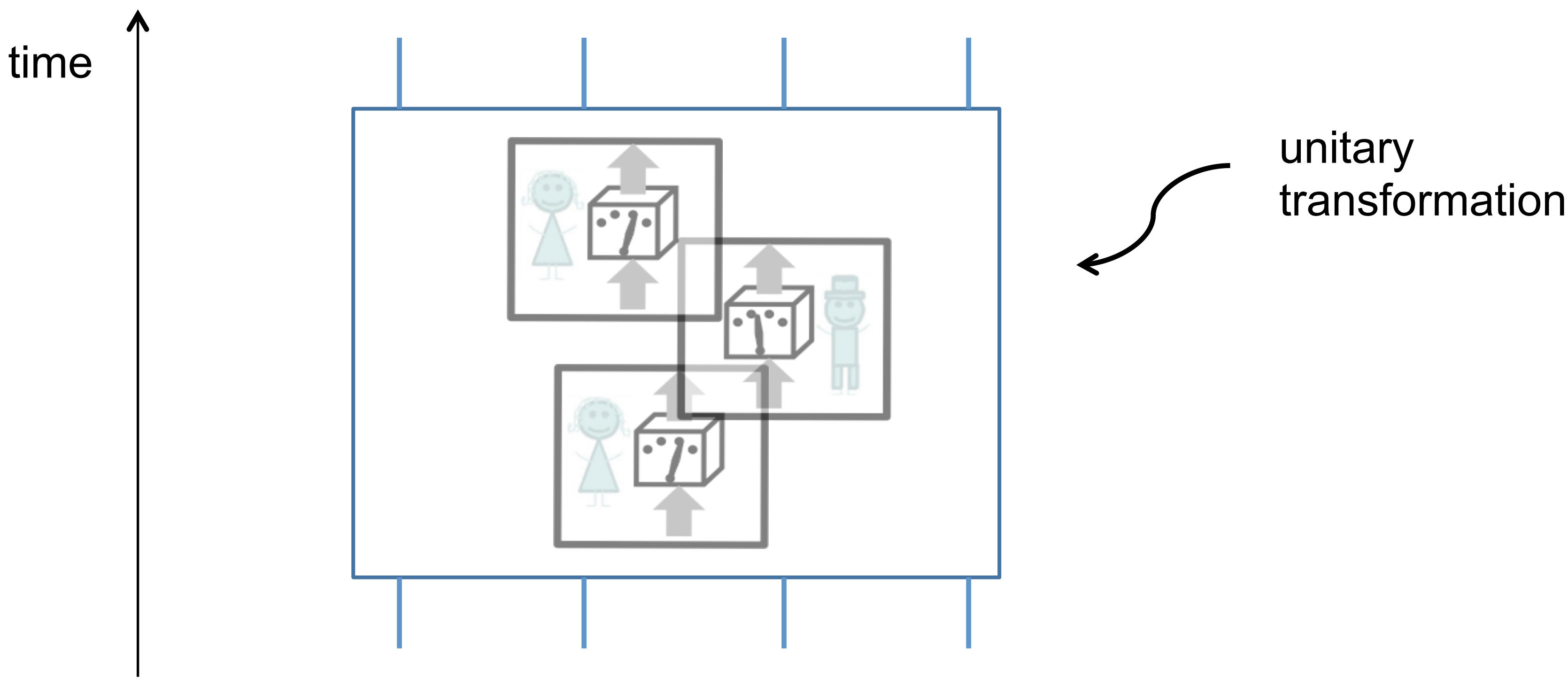
$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$

Can such process be realized in practice?

We don't know.

But it is not a priori impossible

From the outside the experiment may still agree with standard unitary evolution in time.



E.g., **quantum switch** (Chiribella, D'Ariano, Perinotti and Valiron, arXiv:0912.0195, PRA 2013)

Other causal inequalities and violations

Simplest bipartite inequalities:

Branciard, Araujo, Feix, Costa, Brukner, arXiv:1508.01704 (2015)

Multiparite inequalities:

Baumeler and Wolf

- violation with perfect signaling: Proc. ISIT 2014, 526-530 (2014)

- violation by classical local operations: Phys. Rev. A 90, 042106 (2014)
arXiv:1507.01714 (2015)

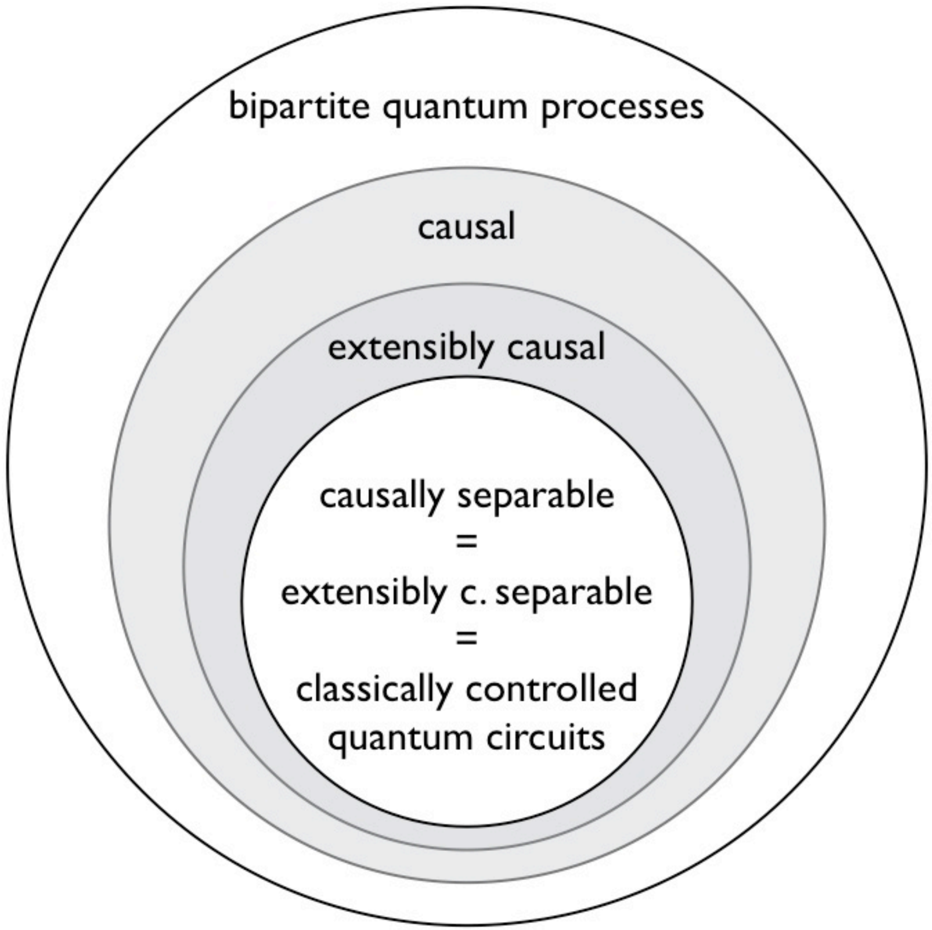
Biased version of the original inequality:

Bhattacharya and Banik, arXiv:1509.02721 (2015)

Formal theory of causality and causal separability

See O. O. and C. Giarmatzi, arXiv:1506.05449

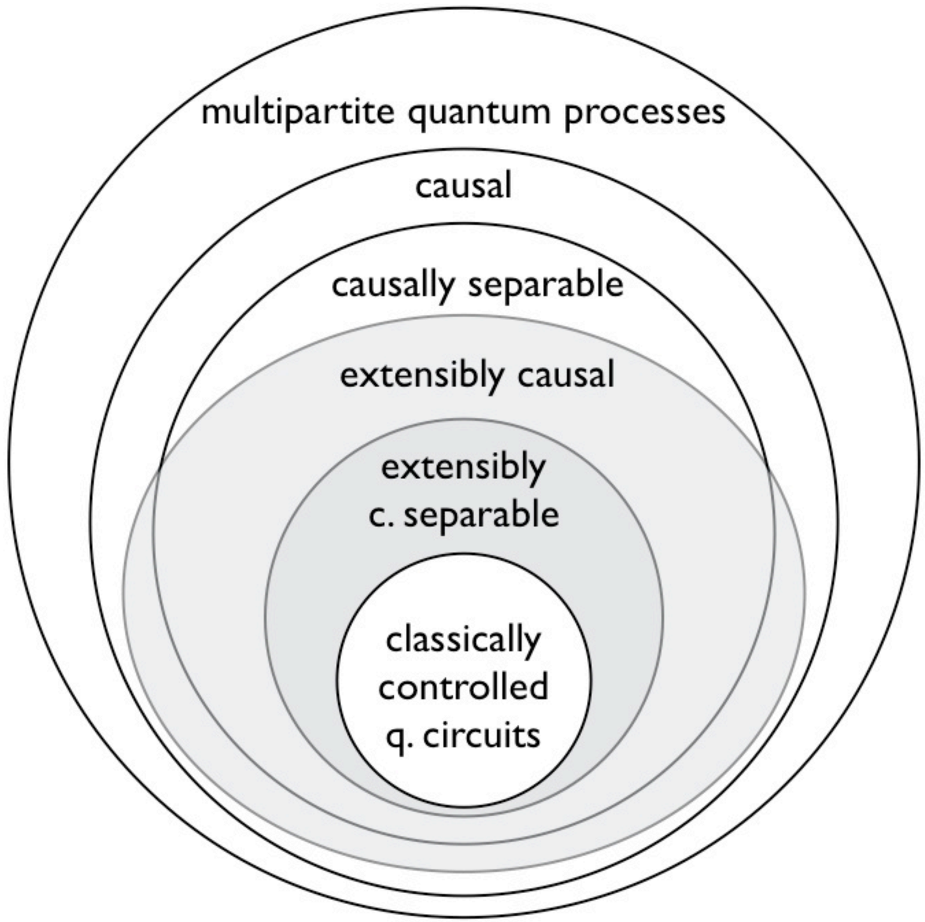
Captures the possibility for dynamical causal relations:



Formal theory of causality and causal separability

See O. O. and C. Giarmatzi, arXiv:1506.05449

Captures the possibility for dynamical causal relations:



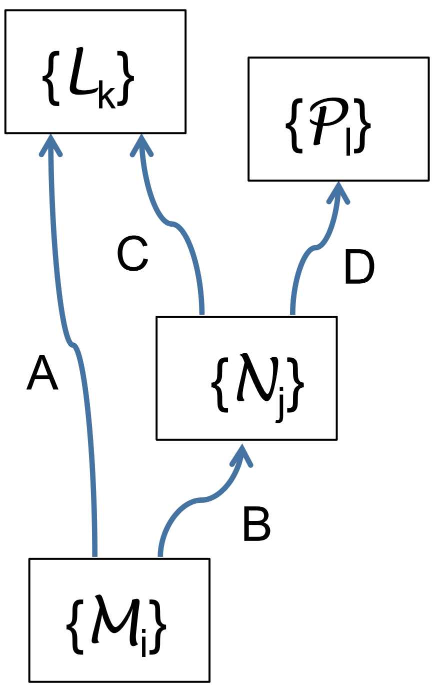
This framework still assumes time locally,
and it is time-asymmetric.

What is the origin of this time asymmetry?

Could we relax the assumption of time also locally?

The circuit framework for operational probabilistic theories

Circuit (an acyclic composition of operations with no open wires):



Probabilistic structure

Joint probabilities

$p(i, j, k, l)$

$p(i, j, k, l) \geq 0, \quad \sum_{ijkl} p(i, j, k, l) = 1$

Hardy, PIRSA:09060015; Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010) [arXiv 2009]

Time-asymmetry of standard quantum theory

Causality axiom [Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010), PRA 84, 012311 (2011)]:

Also Pegg, PLA 349, 411 (2006), ('**weak causality**').

$$\begin{tikzcd} \boxed{\{E_j\}} \\ \boxed{\{\rho_i\}} \arrow[u, "A"'] \end{tikzcd} \quad \Rightarrow \quad p(i, j) \equiv p(\rho_i, E_j)$$

In quantum theory, $p(\rho_i, E_j) = Tr(\hat{\rho}_i \hat{E}_j)$.

The marginal probabilities of the preparation events are independent of the measurement:

$$\boxed{p(\rho_i \mid \{E_j\}) = p(\rho_i)}$$

'No signalling from the future'

What do we call ‘operation’?

What do we call 'operation'?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

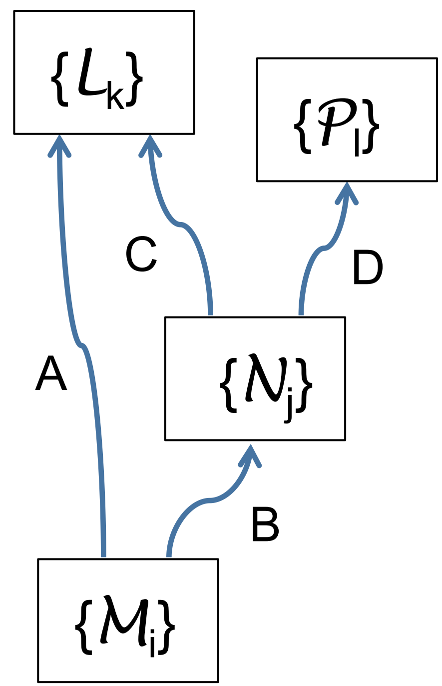
What do we call ‘operation’?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

Idea 1. The closed-box assumption

The events in a box are correlated with other events only as a result of information exchange through the wires

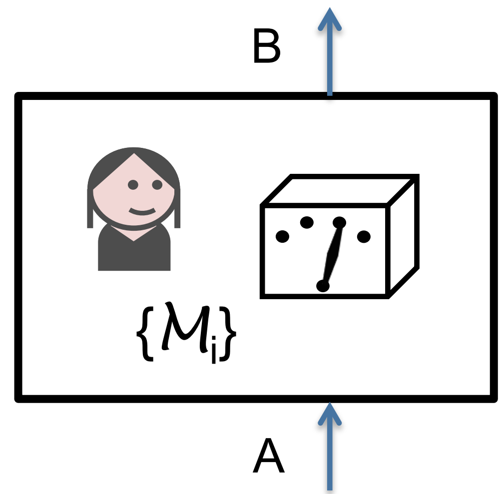


What do we call ‘operation’?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

Idea 1. The closed-box assumption



→ An operation can be realized inside an isolated box.

What do we call ‘operation’?

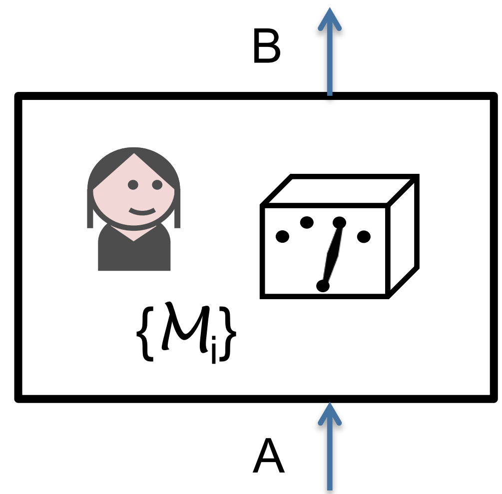
O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

Idea 2. No post-selection

The ‘choice’ of operation can be known *before* the operation is applied

(Gives the idea that an operation can be ‘chosen’.)



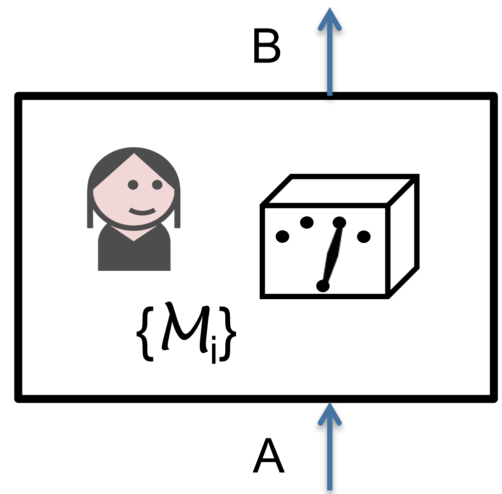
→ The causality axiom describes a constraint on pre-selected operation.

What do we call ‘operation’?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

Idea 2. No post-selection



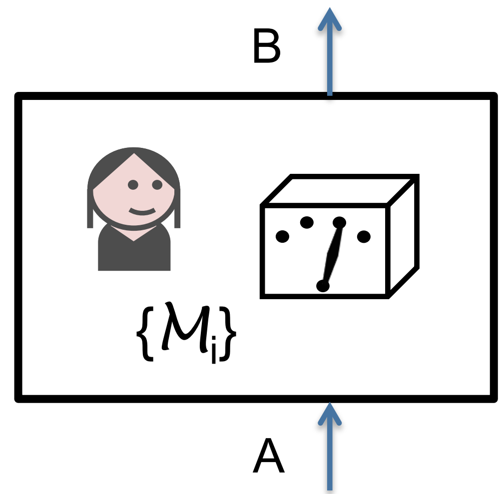
The ‘choice’ of operation can be known *before* the operation is applied

(Gives the idea that an operation can be ‘chosen’.)

The very concept of operations is time-asymmetric!

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Proposal: drop the ‘no post-selection’ criterion



Operation =

description of the possible events in a box conditional on local information

Time-symmetric quantum theory

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Joint probabilities:

The basic probability rule.

$$\begin{tikzcd} \boxed{\{E_j\}} \\ \boxed{\{\rho_i\}} \arrow[u, "A"'] \end{tikzcd} \quad \Rightarrow \quad \boxed{p(i,j) = \frac{Tr(\rho_i E_j)}{Tr(\bar{\rho}\bar{E})}}$$

where

$$\bar{\rho} = \sum_i \rho_i, \quad Tr(\bar{\rho}) = 1$$

$$\bar{E} = \sum_j E_j, \quad Tr(\bar{E}) = d$$

[Also Pegg, Barnett, Jeffers, J. Mod. Opt. 49, 913 (2002).]

New states and effects

States (equivalent preparation events): $(\rho, \bar{\rho})$, where $0 \le \rho \le \bar{\rho},\ Tr(\bar{\rho}) = 1$.

Effects (equivalent measurement events): $(E, \bar{E})$, where $0 \le E \le \bar{E},\ Tr(\bar{E}) = d$.

Joint probabilities:

$$\begin{tikzcd} \boxed{(E, \bar{E})} \\ \boxed{(\rho, \bar{\rho})} \arrow[u, "A"'] \end{tikzcd}$$

$$\Rightarrow \boxed{\begin{aligned} p[(\rho, \bar{\rho}), (E, \bar{E})] &= \frac{Tr(\rho E)}{Tr(\bar{\rho}\bar{E})}, & Tr(\bar{\rho}\bar{E}) \neq 0 \\ &= 0, & Tr(\bar{\rho}\bar{E}) = 0 \end{aligned}}$$

States can be thought of as functions on effects and vice versa.

New states and effects

States (equivalent preparation events): $(\rho,\bar{\rho})$, where $0 \le \rho \le \bar{\rho},\ Tr(\bar{\rho}) = 1$.

Effects (equivalent measurement events): $(E,\bar{E})$, where $0 \le E \le \bar{E},\ Tr(\bar{E}) = d$.

Joint probabilities:

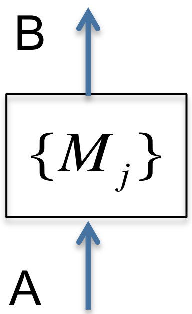
The set of states (effects) is not closed under convex combinations!

$$\begin{tikzcd} \boxed{(E,\bar{E})} \\ \boxed{(\rho,\bar{\rho})} \arrow[u, "A"] \end{tikzcd} \quad \Rightarrow \quad \boxed{\begin{aligned} p[(\rho,\bar{\rho}),(E,\bar{E})] &= \frac{Tr(\rho E)}{Tr(\bar{\rho}\bar{E})}, & Tr(\bar{\rho}\bar{E}) \neq 0 \\ &= 0, & Tr(\bar{\rho}\bar{E}) = 0 \end{aligned}}$$

States can be thought of as functions on effects and vice versa.

General operations

General operations: collections of CP maps $\{M_j\}$, s.t. $Tr(\sum_j M_j(\frac{I}{d_A})) = 1$.



Transformations: $(M, \bar{M})$, where $0 \leq M \leq \bar{M}$, $Tr(\bar{M}(\frac{I}{d_A})) = 1$.

Time reversal symmetry

Example:

$$\begin{tikzcd} \{E_k\} \\ \{M_j\} \arrow[u, "B"] \\ \{\rho_i\} \arrow[u, "A"] \end{tikzcd}$$

$$p(i,j,k) = \frac{Tr(E_k^B M_j^{A\to B}(\rho_i^A))}{Tr(\bar{E}^B \bar{M}^{A\to B}(\bar{\rho}^A))}$$

Time reversal symmetry

Example:

$$\begin{tikzcd} \{\sigma_k\} \arrow[d, "B"'] \\ \{N_j\} \arrow[d, "A"'] \\ \{F_i\} \end{tikzcd}$$

$$p(i,j,k) = \frac{Tr(F_i^A N_j^{B\to A}(\sigma_k^B))}{Tr(\overline{F}^A \overline{N}^{B\to A}(\overline{\sigma}^B))}$$

Time reversal symmetry

The exact form of time-reversal is determined by physics!

$$\boxed{\{\rho_i\}} \xrightarrow{\mathcal{T}} \boxed{\{F_i\}}$$

(play the movie backwards)

Time reversal symmetry

The exact form of time-reversal is determined by physics!

$$\begin{tikzcd} \boxed{\{\rho_i\}} \arrow[r, "\mathcal{T}"] & \boxed{\{F_i\}} \\ & \boxed{\{\sigma_m\}} \arrow[u] \end{tikzcd}$$

(play the movie backwards)

The *time-reversed image* $\{F_i\}$ *of* $\{\rho_i\}$ *is* determined relative to preparations $\{\sigma_m\}$ that have not been time-reversed.

Generalized Wigner's theorem

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

The time-symmetric theory admits more general symmetry transformations.

$$\hat{S}_{s\to e}(\rho;\overline{\rho}) = (F;\overline{F}) = \left(d\frac{S\rho^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)}; d\frac{S\overline{\rho}^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)}\right),$$

$$\hat{S}_{e\to s}(E;\overline{E}) = (\sigma;\overline{\sigma}) = \left(\frac{S^{-1\dagger}E^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})}; \frac{S^{-1\dagger}\overline{E}^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})}\right).$$

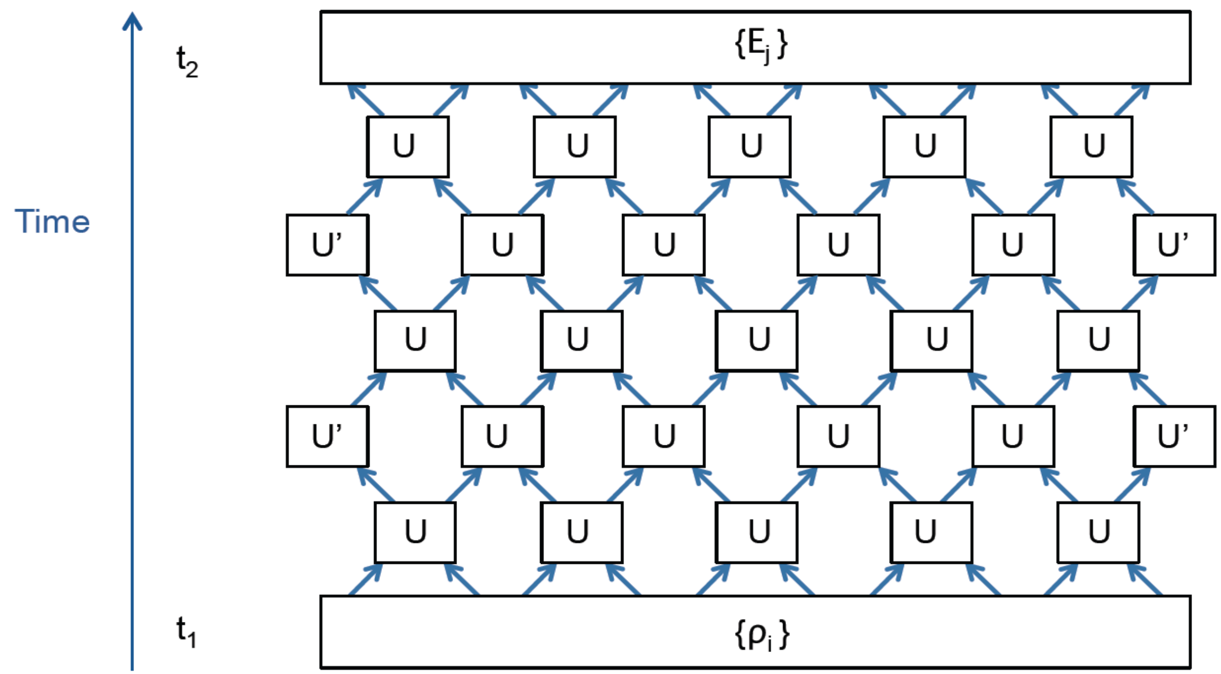
Time reversal can have this form

(Here S is an invertible operator and T denotes transposition in some basis.)

Understanding the observed asymmetry

A toy model of the universe:

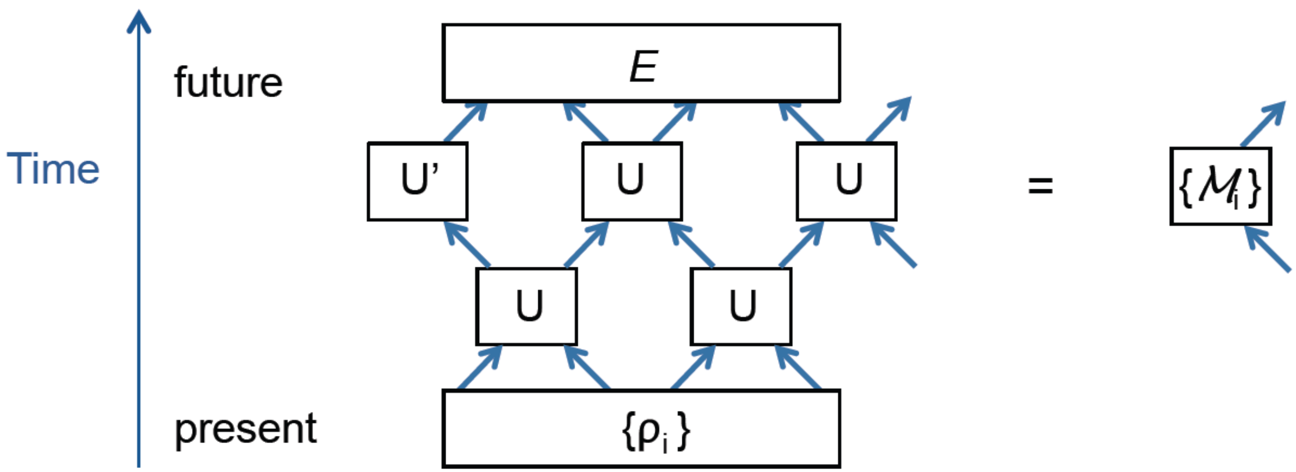
O.O. and N. Cerf, Nature Phys. 11, 853 (2015)



For an observer at t_1, all future circuits contain standard operations iff $\sum_{j\in Q} E_j = \mathbb{1}$.

(Implies that we remember the past and not the future!)

Note: it is logically possible that non-standard operations were obtainable without post-selection



A time-neutral formalism

An isomorphism dependent on time reversal

TRANSFORMATIONS EFFECTS ON PAIRS OF SYSTEMS

$$(\mathcal{M}^{A_1\to B_1};\overline{\mathcal{M}}^{A_1\to B_1}) \leftrightarrow (M^{A_1B_2};\overline{M}^{A_1B_2})$$

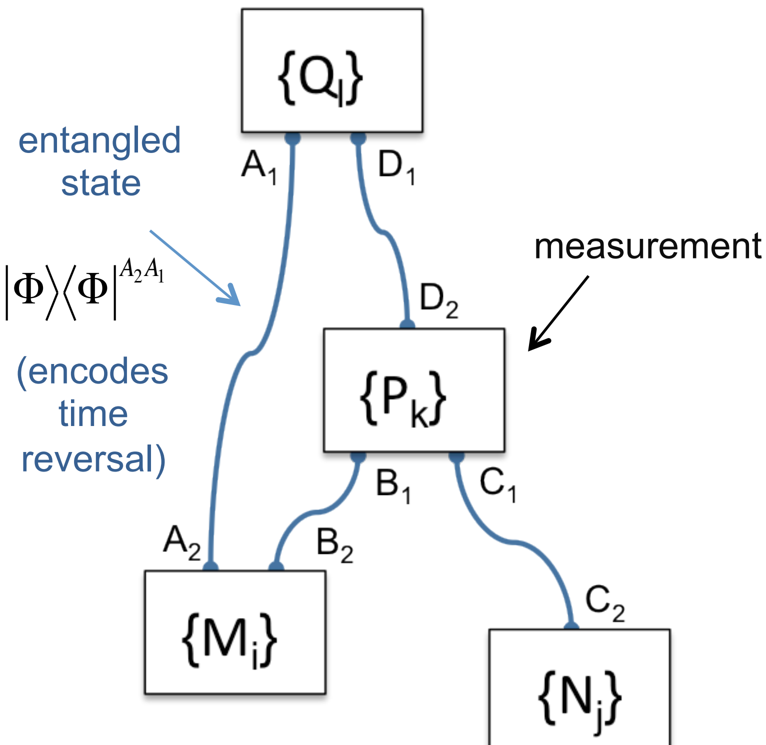
O.O. and N. Cerf, arXiv: 1406.3829

A time-neutral formalism

An isomorphism dependent on time reversal

TRANSFORMATIONS EFFECTS ON PAIRS OF SYSTEMS

$$(\mathcal{M}^{A_1\to B_1};\overline{\mathcal{M}}^{A_1\to B_1}) \leftrightarrow (M^{A_1B_2};\overline{M}^{A_1B_2})$$



Joint probabilities:

$$p(i,j,k,l\,|\,\{M_i^{A_2B_2}\},\{N_j^{C_2}\},\cdots,W) =$$

$$\frac{Tr[W^{A_1A_2B_1B_2C_1C_2D_1D_2}(M_i^{A_2B_2}\otimes N_j^{C_2}\otimes P_k^{B_1C_1D_2}\otimes Q_l^{A_1D_1})]}{\sum_{i,j,k,l} Tr[W^{A_1A_2B_1B_2C_1C_2D_1D_2}(M_i^{A_2B_2}\otimes N_j^{C_2}\otimes P_k^{B_1C_1D_2}\otimes Q_l^{A_1D_1})]}$$

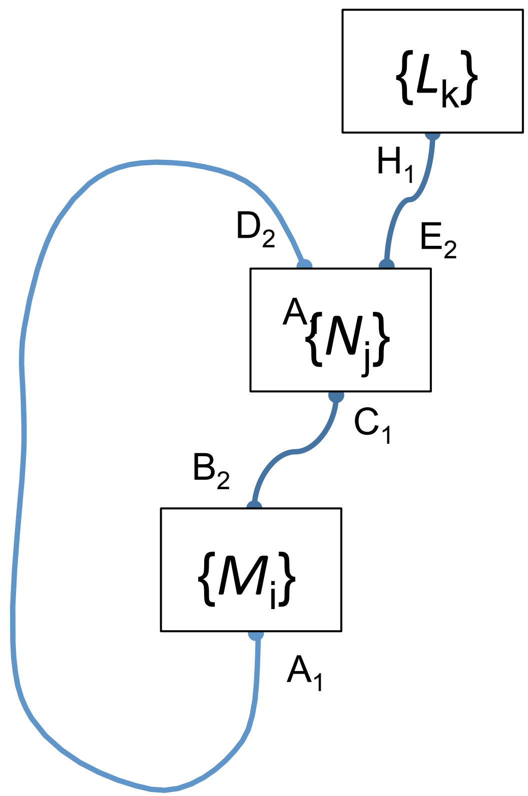
'process matrix' (encodes the connections)

$$= |\Phi\rangle\langle\Phi|^{A_1A_2}\otimes|\Phi\rangle\langle\Phi|^{B_1B_2}\otimes|\Phi\rangle\langle\Phi|^{C_1C_2}\otimes|\Phi\rangle\langle\Phi|^{D_1D_2}$$

O.O. and N. Cerf, arXiv: 1406.3829

A time-neutral formalism

Can describe circuits with *cycles*:



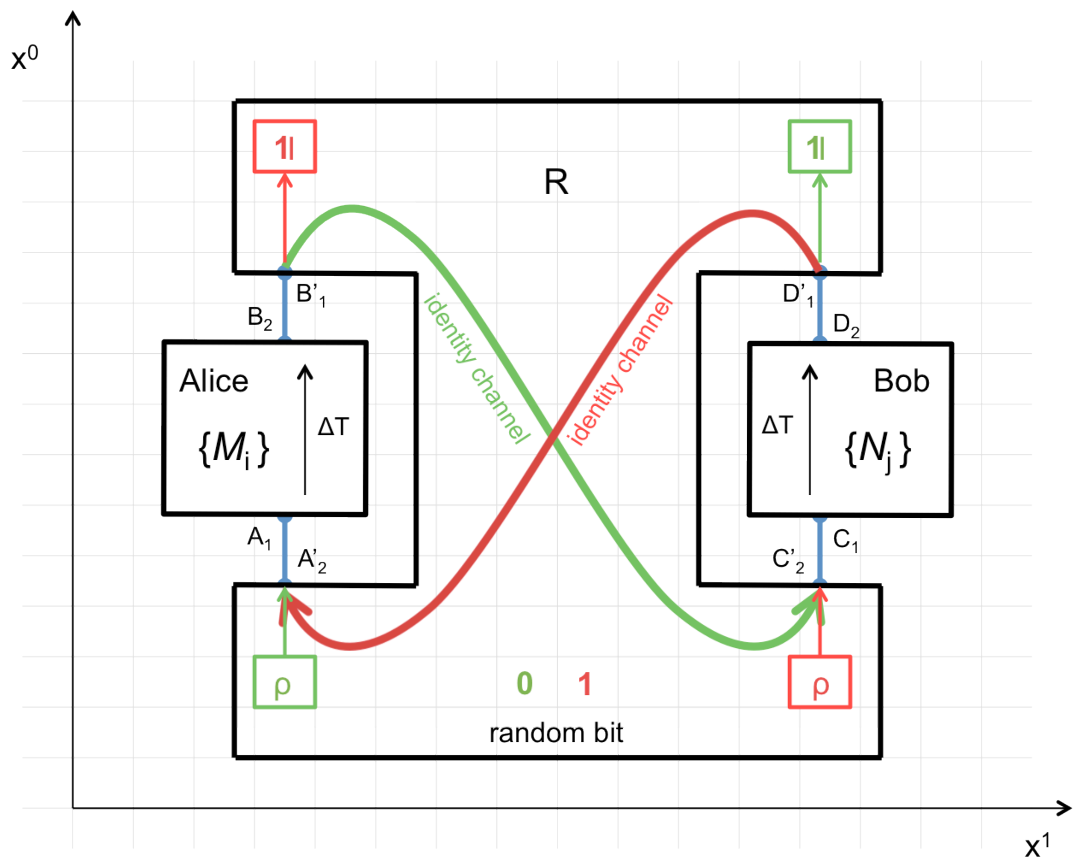
All such circuits can be realized using post-selection.

(Compatible with closed timelike curves)

O.O. and N. Cerf, arXiv: 1406.3829

A time-neutral formalism

There exist circuits with cycles that can be obtained without post-selection!

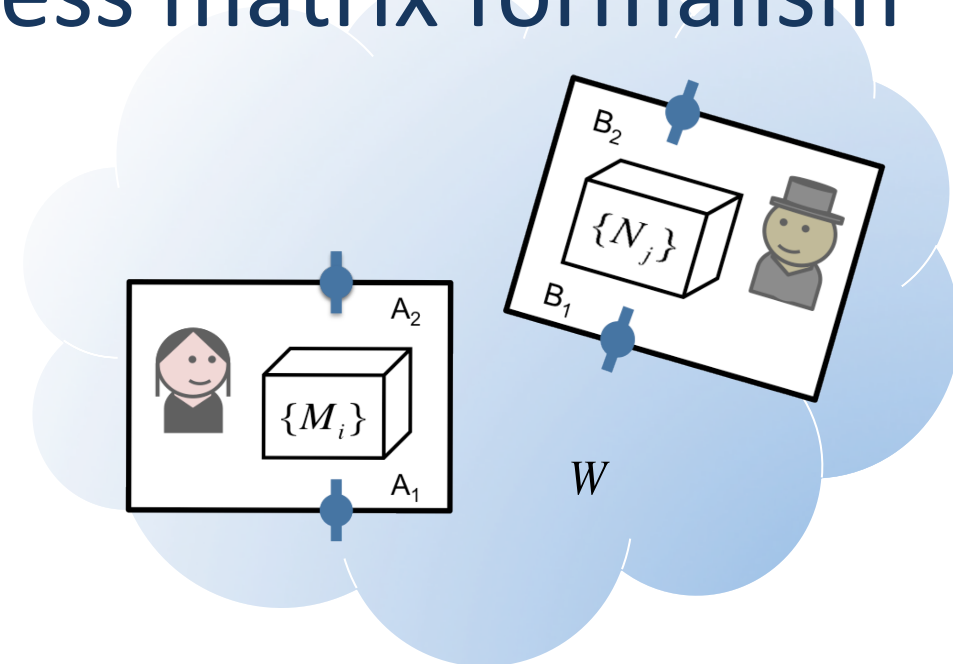


the idea of *background independence* extended to random events

(provides a framework to understand experiments realizing the quantum switch)

Time-symmetric process matrix formalism

Equivalently:



external variables

$$p(i,j,\cdots|\{M_i^{A_1A_2}\},\{M_j^{B_1B_2}\},\cdots,W)=\frac{Tr[W^{A_1A_2B_1B_2\cdots}(M_i^{A_1A_2}\otimes M_j^{B_1B_2}\otimes\cdots)]}{Tr[W^{A_1A_2B_1B_2\cdots}(\bar{M}^{A_1A_2}\otimes \bar{M}^{B_1B_2}\otimes\cdots)]}$$

The 'process matrix':

$$W^{A_1A_2B_1B_2\cdots}\geq 0,\quad Tr(W^{A_1A_2B_1B_2\cdots})=1$$

Note: Any process matrix is allowed.

O.O. and N. Cerf, arXiv: 1406.3829

Dropping the assumption of local time

Observation: The predictions are the same whether the systems are of type 1 or type 2.

Proposal: There is no a priori distinction between systems of type 1 and 2.

The concept of time should come out from properties of the dynamics!

O.O. and N. Cerf, arXiv: 1406.3829

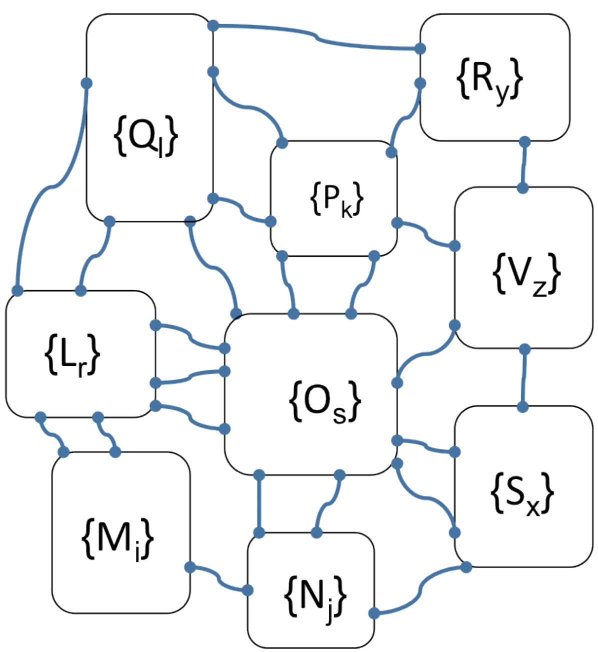
Dropping the assumption of local time

Observation: The predictions are the same whether the systems are of type 1 or type 2.

Proposal: There is no a priori distinction between systems of type 1 and 2.

The concept of time should come out from properties of the dynamics!

The general picture:



Main probability rule

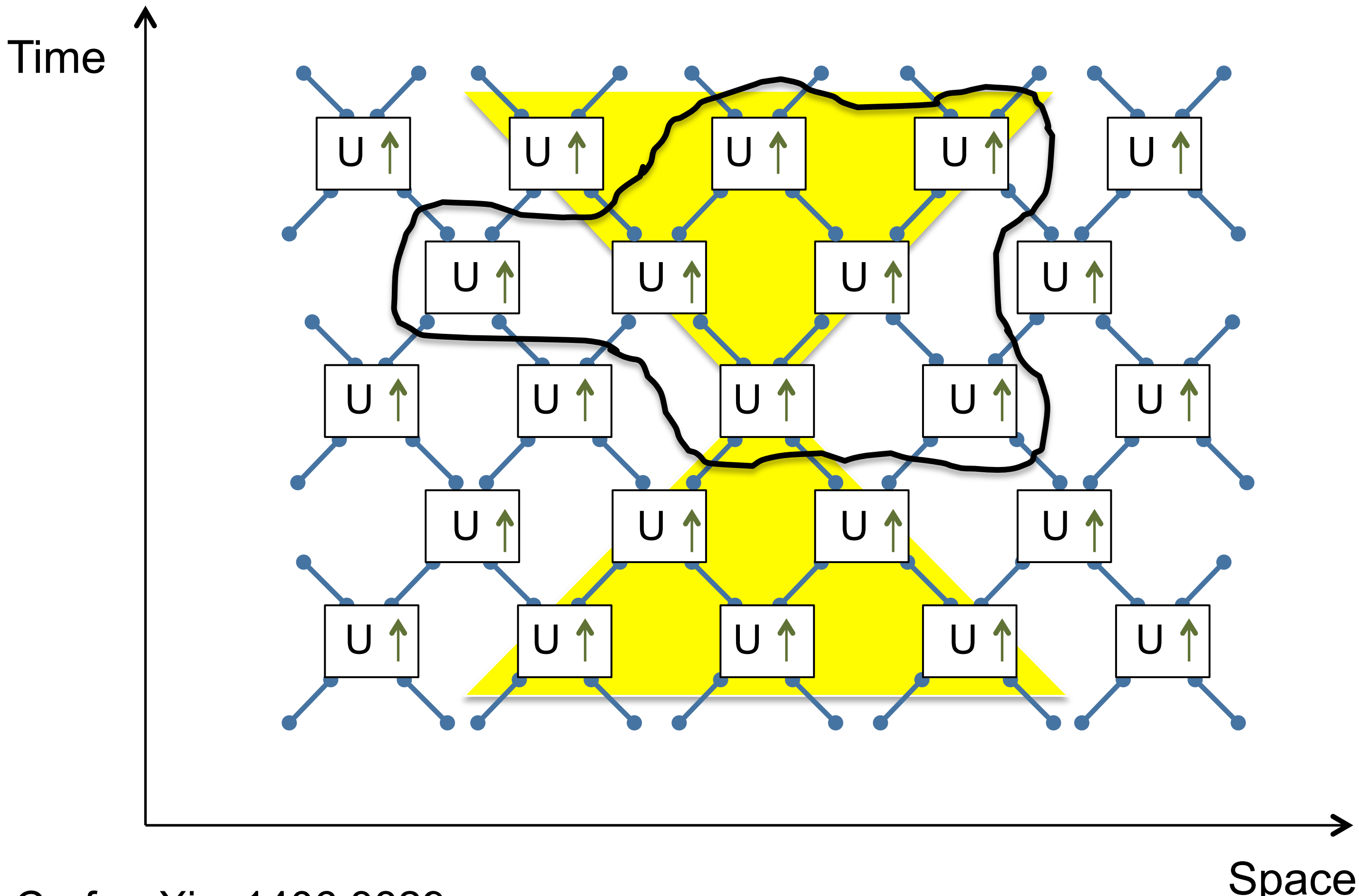
$$p(i,j,\cdots|\{M_i^{\cdots}\},\{N_j^{\cdots}\},\cdots) = \frac{Tr[W_{wires}^{\cdots\cdots}(M_i^{\cdots}\otimes N_j^{\cdots}\otimes\cdots)]}{Tr[W_{wires}^{\cdots\cdots}(\bar{M}^{\cdots}\otimes \bar{N}^{\cdots}\otimes\cdots)]}$$

O.O. and N. Cerf, arXiv: 1406.3829

Limit of quantum field theory

R. Oeckl, Phys. Lett. B 575, 318 (2003), ... , Found. Phys. 43, 1206 (2013)

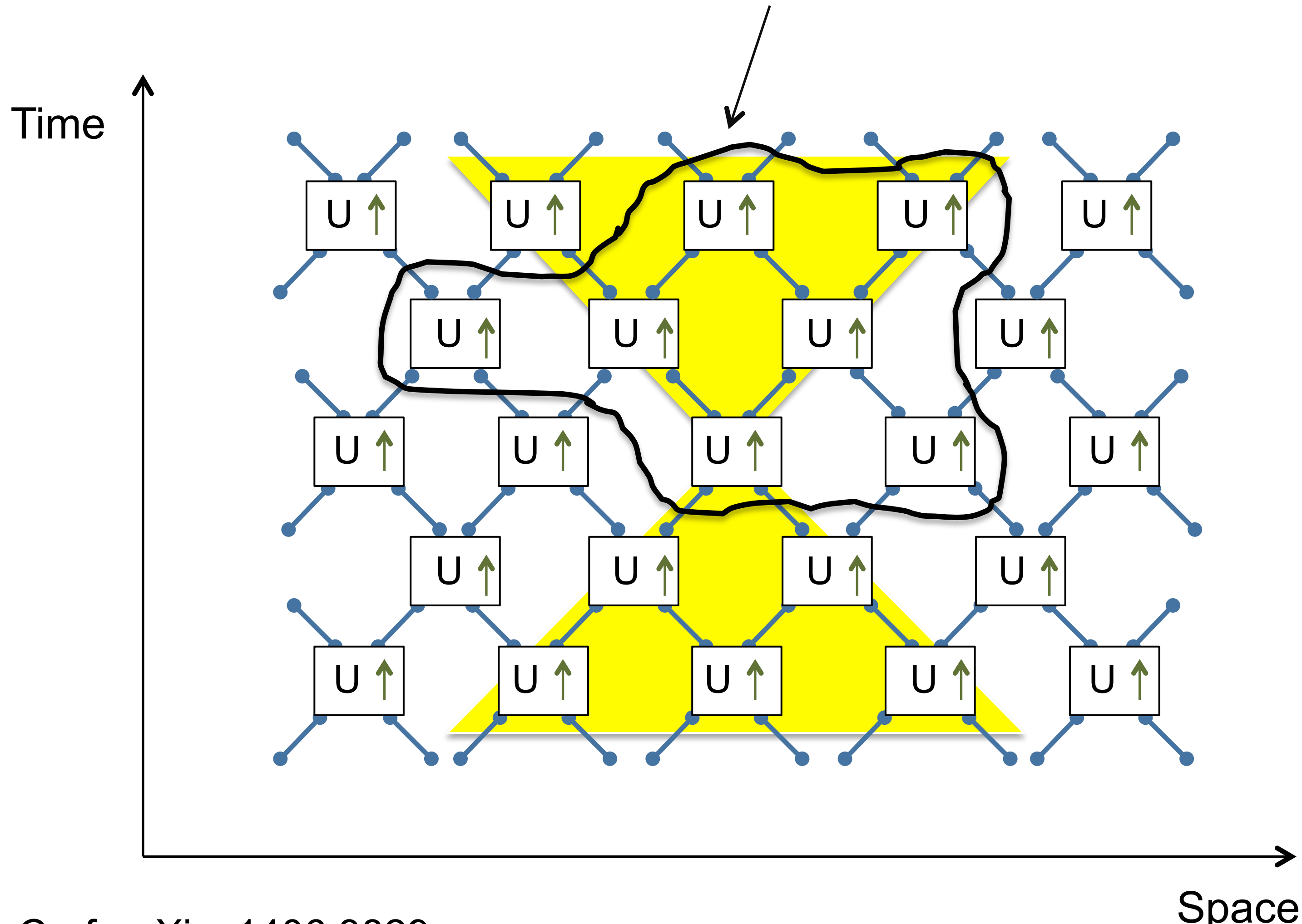
(the ‘general boundary’ approach with a few generalization)



O.O. and N. Cerf, arXiv: 1406.3829

Proposal: causal structure from correlations

The causal structure underlying the dynamics in the region is reflected in correlation properties of the state on the boundary.



O.O. and N. Cerf, arXiv: 1406.3829

Conclusion

It is possible to formulate a QT without any predefined time, which

- agrees with experiment

- has a physical and informational interpretation

- opens up the possibility to understand time and causal structure as dynamical, and explore new forms of dynamics

- Is the metric/causal structure emergent, or do we need to postulate it as another field?

- What processes/networks can be realized without post-selection (e.g., can we violate causal inequalities?)

- How to formulate general covariant laws of dynamics in this framework?

- What does it imply for the foundations of information processing?

Thank you!

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

A notion of causality should:

- **have a universal expression** (implies the multipartite case)
- **allow of *dynamical* causal order** (a given event can influence the order of other events in its future)
- **capture our intuition of causality**

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

General process: $\mathcal{W}^{A,B,\cdots} \equiv \{P(o^A, o^B, \ldots | s^A, s^B, \ldots)\}$

Intuition: The choice of setting of a given party can only influence the occurrence of events in the future and the order of such events.

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

General process: $\mathcal{W}^{A,B,\cdots} \equiv \{P(o^A, o^B, ...|s^A, s^B, ...)\}$

A process is **causal** iff there exists a random partial order $\kappa(A, B, \cdots)$ and and a probability distribution $P(\kappa(A, B, \cdots), o^A, o^B, \cdots |s^A, s^B, \cdots)$ such that for every party, e.g., *A*, and every subset *X*, *Y*, … of the other parties,

$$P(\kappa(A, X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots |s^A, s^B, \cdots)$$
$$= P(\kappa(X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots |s^B, \cdots).$$

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

General process: $\mathcal{W}^{A,B,\cdots} \equiv \{P(o^A, o^B, \ldots | s^A, s^B, \ldots)\}$

A process is **causal** iff there exists a random partial order $\kappa(A, B, \cdots)$ and and a probability distribution $P(\kappa(A, B, \cdots), o^A, o^B, \cdots | s^A, s^B, \cdots)$ such that for every party, e.g., *A*, and every subset *X*, *Y*, … of the other parties,

$$P(\kappa(A, X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots | s^A, s^B, \cdots)$$
$$= P(\kappa(X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots | s^B, \cdots).$$

(background-independent understanding of causal order)

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

Consider $$\mathcal{W}^{X^1,\cdots X^n} \equiv \mathcal{W}^{\mathcal{A},\mathcal{B}}$$

$$\mathcal{A} = \{X^1, \cdots, X^k\}$$
$$\mathcal{B} = \{X^{k+1}, \cdots, X^n\}$$

If no signaling from $\mathcal{B}$ to $\mathcal{A}$ $\longrightarrow$ exists **reduced process** $\mathcal{W}^{\mathcal{A}}$

$$\boxed{\mathcal{W}^{\mathcal{A},\mathcal{B}} \equiv \mathcal{W}^{\mathcal{B}|\mathcal{A}} \circ \mathcal{W}^{\mathcal{A}}}$$

conditional process

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

Theorem (canonical causal decomposition)**:**

$$\mathcal{W}_c^{X^1,\cdots,X^n} = \sum_{i=1}^{n} q_i \mathcal{W}^{(X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n)\npreceq X^i}, \quad q_i \geq 0$$

where

$$\mathcal{W}^{(X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n)\npreceq X^i} = \mathcal{W}_c^{X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n|X^i} \circ \mathcal{W}^{X^i}$$

(iterative formulation)

Describes causal 'unraveling' of the events in the process!

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

Theorem (canonical causal decomposition)**:**

$$\mathcal{W}_c^{X^1,\cdots,X^n} = \sum_{i=1}^{n} q_i \mathcal{W}^{(X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n)\not\preceq X^i}, \quad q_i \geq 0$$

where

$$\mathcal{W}^{(X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n)\not\preceq X^i} = \mathcal{W}_c^{X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n|X^i} \circ \mathcal{W}^{X^i}$$

(iterative formulation)

Causal correlations form polytopes! See also Branciard et al., arXiv:1508.01704.

Causal separability

O. O. and C. Giarmatzi, arXiv:1506.05449

A *quantum* process is called **causally separable** iff it can be written in a canonical causal form with every reduced and conditional process being a valid quantum process.

(analogy with Bell local and separable quantum states)

→ Agrees with the bipartite definition $$W^{A_1 A_2 B_1 B_2} = qW^{B \npreceq A} + (1-q)W^{A \npreceq B}$$

Causal separability

O. O. and C. Giarmatzi, arXiv:1506.05449

A *quantum* process is called **causally separable** iff it can be written in a canonical causal form with every reduced and conditional process being a valid quantum process.

(analogy with Bell local and separable quantum states)

→ Agrees with the bipartite definition $$W^{A_1A_2B_1B_2} = qW^{B\not\preceq A} + (1-q)W^{A\not\preceq B}$$

In the multipartite case, causality and causal separability are not equivalent!

[see also Araujo, Branciard, Costa, Feix, Giarmatzi, Brukner, NJP 17, 102001 (2015)]

Causal separability

O. O. and C. Giarmatzi, arXiv:1506.05449

A *quantum* process is called **causally separable** iff it can be written in a canonical causal form with every reduced and conditional process being a valid quantum process.

(analogy with Bell local and separable quantum states)

→ Agrees with the bipartite definition $$W^{A_1 A_2 B_1 B_2} = qW^{B \not\leq A} + (1-q)W^{A \not\leq B}$$

In the multipartite case, causality and causal separability are not equivalent!

[see also Araujo, Branciard, Costa, Feix, Giarmatzi, Brukner, NJP 17, 102001 (2015)]

Is there a simple description of multipartite causally separable processes?

Extensive causality and causal separability

O. O. and C. Giarmatzi, arXiv:1506.05449

Non-causality can be *activated* by shared entanglement!

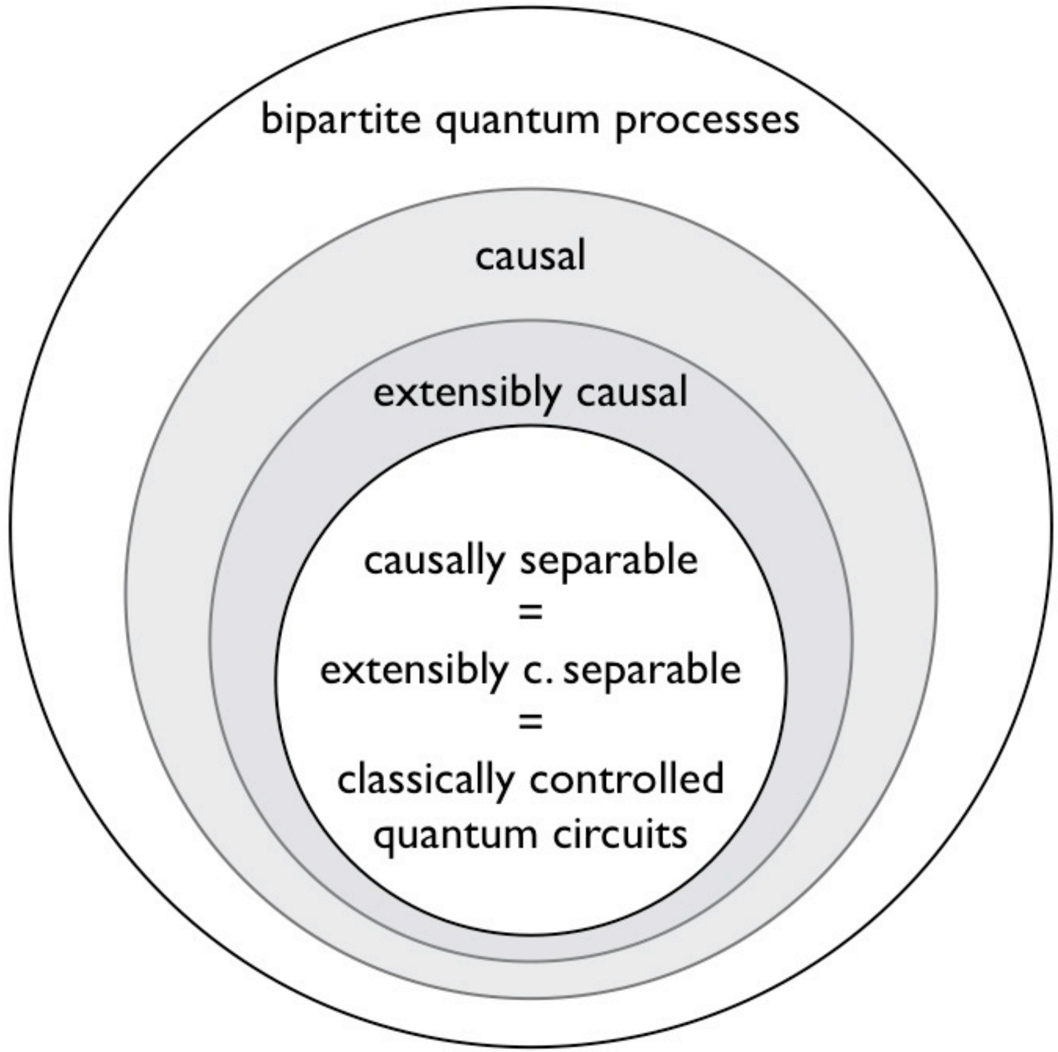
→ Define **extensively causal** / **extensively causally separable** processes as

those that remain causal / causally separable under extension with arbitrary input ancilla.

There is a simple characterization of multipartite *extensively causally separable* processes!
(see paper)

What we know about the classes of quantum processes

O. O. and C. Giarmatzi, arXiv:1506.05449



What we know about the classes of quantum processes

O. O. and C. Giarmatzi, arXiv:1506.05449