A hierarchy of steering criteria (NBA-style)

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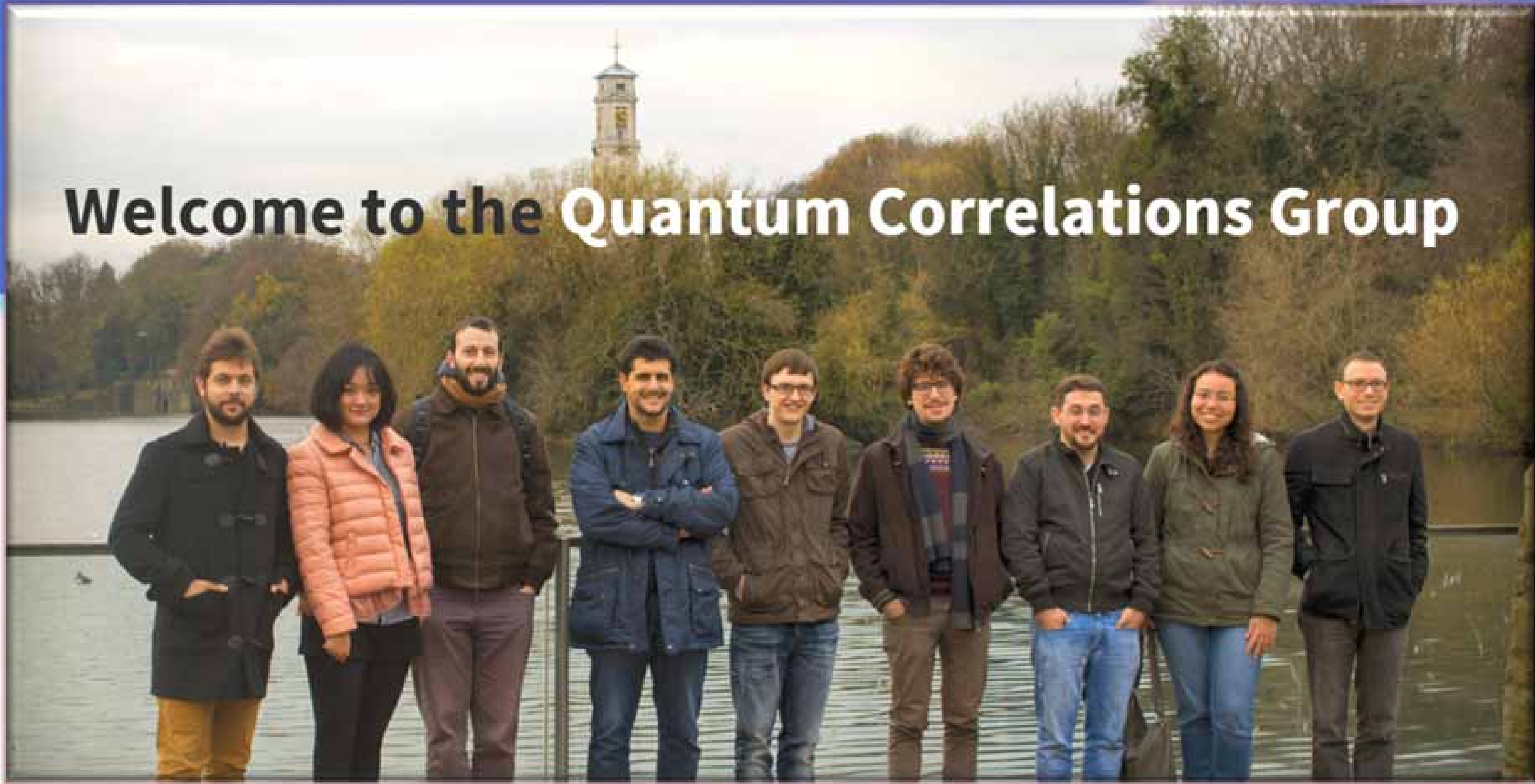
Welcome to the Quantum Correlations Group

Steering, Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox

H. M. Wiseman, S. J. Jones, and A. C. Doherty
Phys. Rev. Lett. **98**, 140402 – Published 6 April 2007

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Hierarchy of Steering Criteria Based on Moments for All Bipartite Quantum Systems

Ioannis Kogias, Paul Skrzypczyk, Daniel Cavalcanti, Antonio Acín, and Gerardo Adesso
Phys. Rev. Lett. **115**, 210401 – Published 17 November 2015

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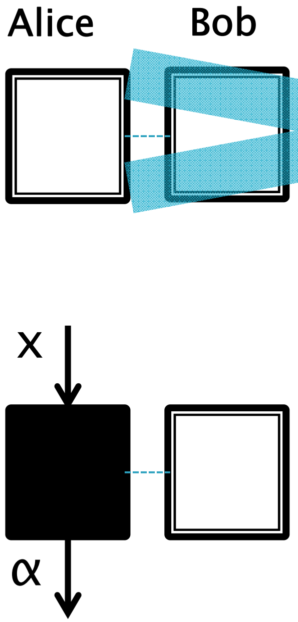


$$\rho_{AB} = \sum_{\lambda} p_{\lambda}\, \rho_{A,\lambda} \otimes \rho_{B,\lambda}$$

Independent of Alice's Hilbert space.

$$\sigma^{B}_{a|x} = Tr_{A}\left[\left(M_{a|x} \otimes 1_{B}\right)\rho_{AB}\right] \rightarrow \sum_{\lambda} q_{\lambda}\, p\left(a \mid x,\lambda\right)\rho_{B,\lambda}$$

unsteerable

$\left\{\sigma^{B}_{a|x}\right\}$: **assemblage**

1) All separable states lead to unsteerable assemblages

2) Some entangled states also lead to unsteerable assemblages (in the sense that Alice and Bob cannot detect the existing entanglement if they make no assumptions on Alice's side)

$\{\sigma^B_{a|x}\}$: assemblage

$\sigma^{US}_{a|x} = \sum_\lambda q_\lambda \, p(a|x,\lambda)\, \rho_{B,\lambda}$

Experimental criteria for steering and the Einstein-Podolsky-Rosen paradox

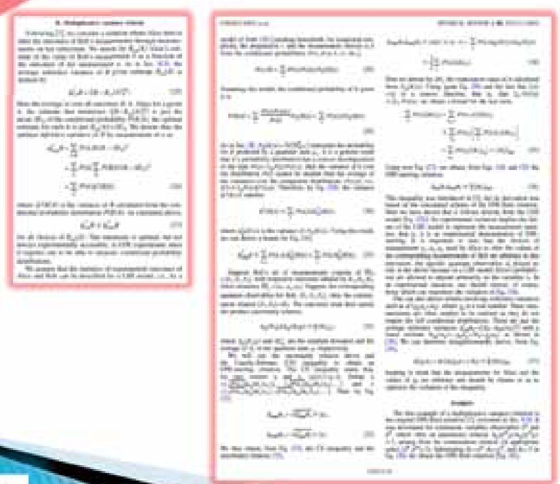
E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid
Phys. Rev. A **80**, 032112 – Published 18 September 2009

- Take advantage of the convexity of the set, by using convex/concave functions to form steering criteria (*cumbersome*)

- E.g.,

$$\Delta_{\inf}B_1\Delta_{\inf}B_2 \geq \tfrac{1}{2}|\langle B_3\rangle|_{\inf}$$

Long proof!



...or, ask Howard!

- x Tailored to specific measurem enarios
- x If it doesn't work, *pray to come up h a better one*!

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Matthew F. Pusey
Phys. Rev. A **88**, 032313 – Published 13 September 2013

Paul Skrzypczyk, Miguel Navascués, and Daniel Cavalcanti
Phys. Rev. Lett. **112**, 180404 – Published 8 May 2014

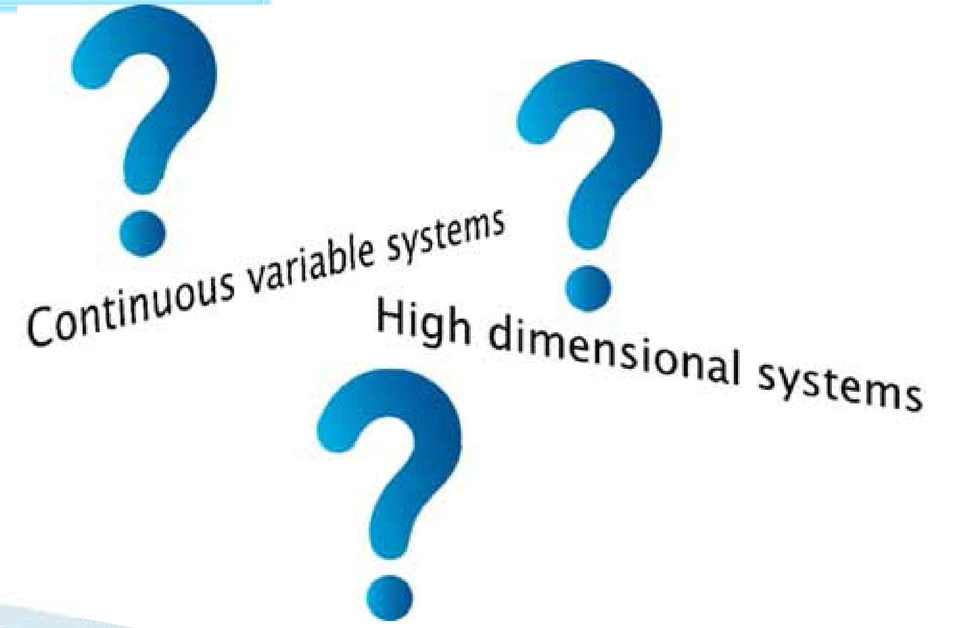
Marco Piani and John Watrous
Phys. Rev. Lett. **114**, 060404 – Published 12 February 2015

Steering maps and their application to dimension-bounded steering, arXiv:1412:2623 (2014)
Tobias Moroder, (et al.- 1) , Otfried Gühne

- Powerful SDP methods
- Freely choose the measurements performed
- Can increase number of measurements to improve steering detection

x Works only for small dimensional systems (small assemblage)

x Purely numerical methods



Our proposal: Moment matrix approach

$$\Gamma_{ij} = \langle S_i^\dagger S_j \rangle_{\rho_{AB}} \qquad \Gamma \geq 0 \qquad \Gamma^\dagger = \Gamma$$

- Entanglement

Inseparability Criteria for Continuous Bipartite Quantum States

E. Shchukin and W. Vogel
Phys. Rev. Lett. **95**, 230502 – Published 28 November 2005; Errata Phys. Rev. Lett. **96**, 129902 (2006); Phys. Rev. Lett. **95**, 249904 (2005)

$$S = \begin{pmatrix} q_A & p_A & q_B & p_B \end{pmatrix}$$

$$\Gamma[\rho_{AB}] = \begin{pmatrix} \langle q_A^2 \rangle & \langle q_A p_A \rangle & \langle q_A q_B \rangle & \langle q_A p_B \rangle \\ & \langle p_A^2 \rangle & \langle p_A q_B \rangle & \langle p_A p_B \rangle \\ & & \langle q_B^2 \rangle & \langle q_B p_B \rangle \\ & & & \langle p_B^2 \rangle \end{pmatrix} \geq 0 \quad \Rightarrow \quad \Gamma\left[\rho_{AB}^{T_B}\right] = \begin{pmatrix} \langle q_A^2 \rangle & \langle q_A p_A \rangle & \langle q_A q_B \rangle & -\langle q_A p_B \rangle \\ & \langle p_A^2 \rangle & \langle p_A q_B \rangle & -\langle p_A p_B \rangle \\ & & \langle q_B^2 \rangle & -\langle q_B p_B \rangle \\ & & & \langle p_B^2 \rangle \end{pmatrix} \geq 0$$

- If no violation found, add more observables to the set: HIERARCHY

e.g., $S' = \begin{pmatrix} q_A & p_A & q_B & p_B & p_B^2 & \ldots \end{pmatrix}$

Something similar for steering?

- We don't have bipartite state in our hands (neither do we have a PPT-like technique)
- We only have the set of Bob's conditional states
- But…

If Bob's assemblage $\{\sigma_{a|x}\}$ is *unsteerable,* there always exists a separable bipartite state,

$$\overline{\rho}_{AB} = \sum_{\lambda} p_{\lambda}\, \overline{\rho}_{A,\lambda} \otimes \rho_{B,\lambda},$$

and *commuting observables* for Alice,

$$x \leftrightarrow A_x, \qquad [A_x, A_{x'}] = 0,$$

that can reproduce Bob's assemblage,

$$\sigma_{a|x}^{US} = \mathrm{Tr}_A\left[\left(M_{a|x} \otimes 1_B\right) \overline{\rho}_{AB}\right].$$

- For any unsteerable assemblage, we can always define a moment matrix –with commuting observables on Alice– that is positive semi-definite,

$$\rho_{AB}:\mathrm{US} \quad \rightarrow \quad \bar{\rho}_{AB}: \ \Gamma[\bar{\rho}_{AB}] \geq 0$$

Similar to PPT for entanglement

- **Example I: All continuous variable states up to second-order moments**

$$S = \begin{pmatrix} A_q & A_p & q_B & p_B \end{pmatrix}, \qquad \text{with} \ \left[A_q, A_p\right] = 0.$$

$<A_qA_p>$: unobservable

$$\Gamma[\bar{\rho}_{AB}] = \begin{pmatrix} \langle A_q^2 \rangle & \langle A_q A_p \rangle & \langle A_q q_B \rangle & \langle A_q p_B \rangle \\ \langle A_p A_q \rangle & \langle A_p^2 \rangle & \langle A_p q_B \rangle & \langle A_p p_B \rangle \\ & & \langle q_B^2 \rangle & \langle q_B p_B \rangle \\ & & & \langle p_B^2 \rangle \end{pmatrix} \geq 0 \quad \Rightarrow \quad \Gamma_R[\bar{\rho}_{AB}] = \begin{pmatrix} a & R & c_1 & 0 \\ R & a & 0 & c_2 \\ c_1 & 0 & b & i \\ 0 & c_2 & i & b \end{pmatrix} \geq 0, \ \forall R$$

- We can show *analytically,*

$$\det \Gamma_R \geq 0 \quad \Rightarrow \quad \boxed{\mathrm{V}_{AB} + i\,0_A \oplus \Omega_B \geq 0}$$

Wiseman et al.'s necessary and sufficient steering criterion for Gaussian states and Gaussian measurements.

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Example II: d=2 Werner states

$$\rho_{AB} = w\left|\psi^-\right\rangle_{AB}\left\langle\psi^-\right| + \frac{1-w}{4}1_{AB}$$

- We choose: $S = \left(1\otimes 1 \quad A_X\otimes X \quad A_Y\otimes Y \quad A_Z\otimes Z\right), \quad X,Y,Z : \text{Pauli}$

$$\Gamma_{\Re} = \begin{pmatrix} 1 & \langle A_X X\rangle & \langle A_Y Y\rangle & \langle A_Z Z\rangle \\ \langle A_X X\rangle & \langle A_X^2 X^2\rangle & \langle A_X A_Y XY\rangle & \langle A_X A_Z XZ\rangle \\ \langle A_Y Y\rangle & \langle A_Y A_X YX\rangle & \langle A_Y^2 Y^2\rangle & \langle A_Y A_Z YZ\rangle \\ \langle A_Z Z\rangle & \langle A_Z A_X ZX\rangle & \langle A_Z A_Y ZY\rangle & \langle A_Z^2 Z^2\rangle \end{pmatrix} = \begin{pmatrix} 1 & \langle A_X X\rangle & \langle A_Y Y\rangle & \langle A_Z Z\rangle \\ & 1 & i\,R_1 & i\,R_2 \\ & & 1 & i\,R_3 \\ & & & 1 \end{pmatrix} \geq 0, \quad \forall \Re$$

A] We can proceed *analytically,*

$$\det\Gamma_{\Re} \geq 0 \;\Rightarrow\; \boxed{\langle A_X\otimes X\rangle^2 + \langle A_Y\otimes Y\rangle^2 + \langle A_Z\otimes Z\rangle^2 \leq 1}$$

PAPER

Loophole-free Einstein–Podolsky–Rosen experiment via quantum steering

Bernhard Wittmann[1,2,6,7], Sven Ramelow[1,2,6,7], Fabian Steinlechner[2], Nathan K Langford[2], Nicolas Brunner[3], Howard M Wiseman[4], Rupert Ursin[2] and Anton Zeilinger[1,2,5]

Published 24 May 2012 • IOP Publishing and Deutsche Physikalische Gesellschaft • New Journal of Physics

Violated for all $w \geq \frac{1}{\sqrt{3}}$

B] We can proceed *numerically.*

- Checking whether Γ is positive semi-definite for all free parameters is an instance of a semi-definite program.

(just like in NPA)

$$
\begin{aligned}
\lambda_\star^{\max} = \max_{\lambda, \{\Gamma_{ij}\}} \quad & \lambda \\
\text{subject to} \quad & \Gamma - \lambda \mathbb{1} \geq 0 \\
& \mathrm{Tr}[\Gamma A_i] = b_i, \quad i = 1, \ldots, k \\
& \mathrm{Tr}\left[\Gamma C_j\right] = 0, \quad j = 1, \ldots, l
\end{aligned}
$$

Maximizes smallest eigenvalue of Γ

Observable values

Linear constraints between among free parameters

We find numerically a violation for all $w > \frac{1}{\sqrt{3}}$

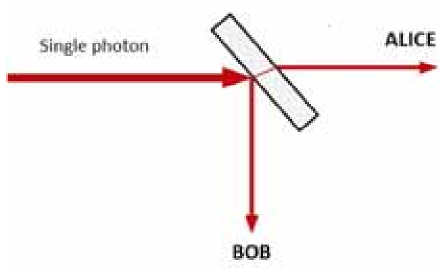
- The dual of the SDP gives the optimal steering witness for this class of states and measurements,

$$\boxed{\langle A_X \otimes X \rangle + \langle A_Y \otimes Y \rangle + \langle A_Z \otimes Z \rangle \geq -\sqrt{3}}$$

satisfied by all unsteerable assemblages, and violated for all $w > \frac{1}{\sqrt{3}}$.

Experimental criteria for steering and the Einstein-Podolsky-Rosen paradox
E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid
Phys. Rev. A **80**, 032112 – Published 18 September 2009

Beating the state of the art: *steering a single photon*



$$\rho_{AB}(w) = w\left|\psi^-\right\rangle_{AB}\left\langle\psi^-\right| + (1-w)\left|00\right\rangle_{AB}\left\langle 00\right|,$$

where, $\left|\psi^-\right\rangle_{AB} = \frac{1}{\sqrt{2}}\left(\left|10\right\rangle_{AB} - \left|01\right\rangle_{AB}\right).$

- We would like to check the steerability of this state with experimentally-friendly measurements.
- We consider only quadrature measurements for Alice and Bob.
- Best current detection is due to Jones and Wiseman:

Nonlocality of a single photon: Paths to an Einstein-Podolsky-Rosen-steering experiment
S. J. Jones and H. M. Wiseman
Phys. Rev. A **84**, 012110 – Published 12 July 2011

They consider *infinitely many* binned quadrature measurements for both Alice and Bob, and find steering for all,

$$\boldsymbol{w \geq 0.77}$$

- Entropic steering criteria, employing two (unbinned) quadrature measurements for both Alice and Bob, get violated for a much weaker $\boldsymbol{w \geq 0.94}$.

- Criteria using only second-order moments do not get violated for any $\boldsymbol{w}$.

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$$\rho_{AB}(w) = w|\psi^-\rangle_{AB}\langle\psi^-| + (1-w)|00\rangle_{AB}\langle 00|,$$

- To make a fair comparison, we also choose only quadrature measurements for both Alice and Bob, with Alice having only two inputs,

$$S = \{\mathbb{1}\otimes\mathbb{1}, A_0\otimes q_B, A_0\otimes p_B, A_1\otimes q_B, A_1\otimes p_B, A_0^2\otimes\mathbb{1}, A_1^2\otimes\mathbb{1}, \mathbb{1}\otimes q_B^2, \mathbb{1}\otimes q_Bp_B, \mathbb{1}\otimes p_Bq_B, \mathbb{1}\otimes p_B^2\}$$

- 11x11 moment matrix : tractable only numerically (but very efficiently ~0.3 sec).
- To see what measurements are involved, let's see the optimal steering witness given by the dual of the SDP,

$$(A_0 \leftrightarrow q_A, \quad A_1 \leftrightarrow p_A)$$

$$\begin{aligned}\beta = 8.1657 &- (\langle A_0\otimes q_B\rangle + \langle A_1\otimes p_B\rangle) + 0.2508\,(\langle A_0\otimes q_B^3\rangle + \langle A_1\otimes p_B^3\rangle) - 0.3110\,(\langle A_0^2\rangle + \langle A_1^2\rangle)\\ &+ 0.3205\,(\langle A_0^2\otimes q_B^2\rangle + \langle A_1^2\otimes p_B^2\rangle) + 0.3020\,(\langle A_0^2\otimes p_B^2\rangle + \langle A_1^2\otimes q_B^2\rangle) - 0.0001\,(\langle A_0^3\otimes q_B\rangle + \langle A_1^3\otimes p_B\rangle)\\ &+ 7.7217\,(\langle q_B^4\rangle + \langle p_B^4\rangle) + 15.5451\,\langle q_B^2p_B^2\rangle - 31.0941\,(\langle q_B^2\rangle + \langle p_B^2\rangle) - 31.0903i\,\langle q_Bp_B\rangle \geq 0\end{aligned}$$

- The witness get's a negative value for all $w \geq \frac{2}{3} \approx \mathbf{0.6667}$.

Nonclassical moments and their measurement

E. V. Shchukin and W. Vogel

Phys. Rev. A **72**, 043808 – Published 12 October 2005

Summary

We introduced a new method for steering detection:

- Valid for any dimension (discrete or continuous)
- Other steering criteria are derived as special cases (just like with Shchukin and Vogel for CV entanglement)
- Able to beat current best steering criteria
- Provides a systematic framework to analytically derive new non-linear criteria
- Provides optimal steering witnesses
- Allows you to add/remove any measurement you want to include/exclude in the detection

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Thank you!