(Quantum?) Processes and Correlations with no definite causal order

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Workshop on Quantum Nonlocality, Causal Structures and Device-Independent Quantum Information

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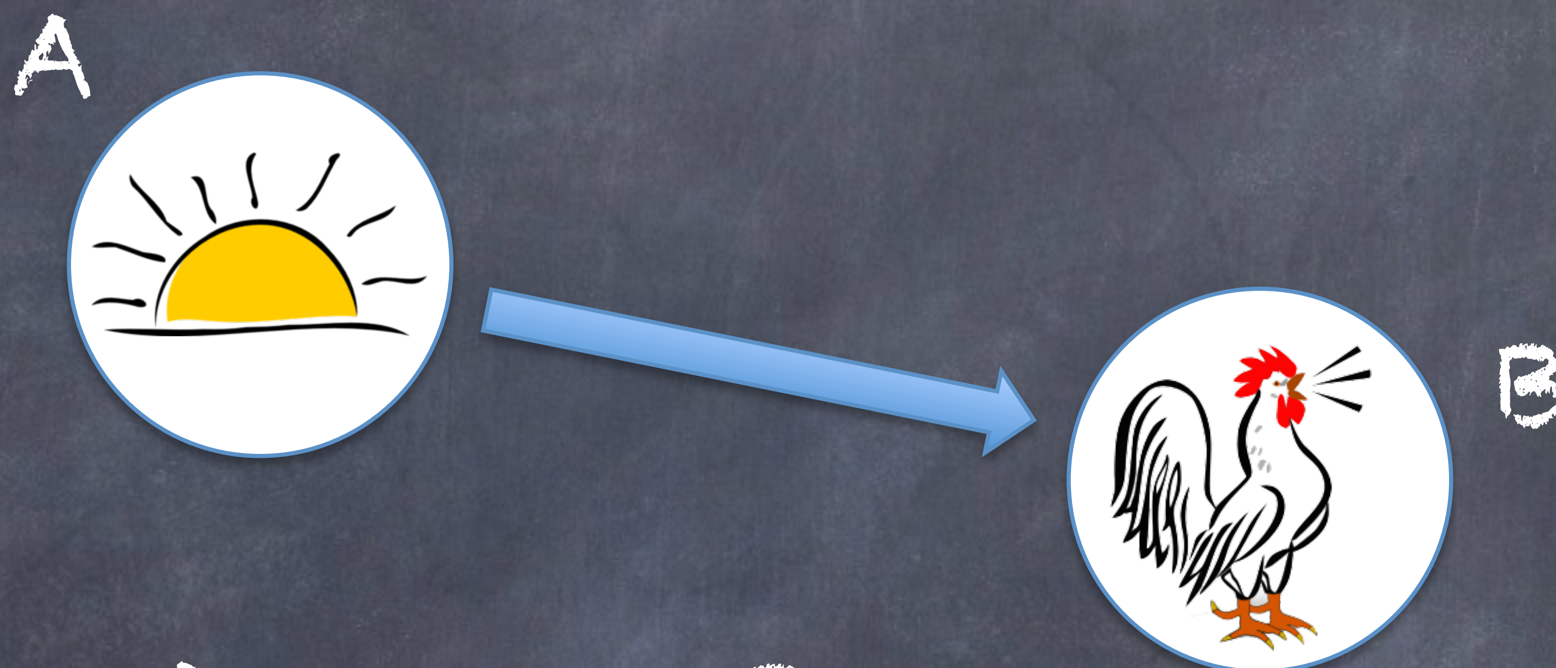
Classical causal relations



A causes B

Let's enter the quantum world...

Motivation

- In quantum mechanics, some variables may be indefinite (e.g. X, P)
- What about causal relations?
  - In "standard QM", measurements are done in space-time
    Fixed measurement positions, time evolution, tensor product structure... assume a fixed causal structure
  - Can we go beyond this?
    Remove time and causal structure from QM?
    - What new phenomenology arises?
      Experiments, applications?

Outline

- The process matrix framework
- Analogy with entanglement & Bell nonlocality
- The “Quantum switch” as a causally nonseparable process

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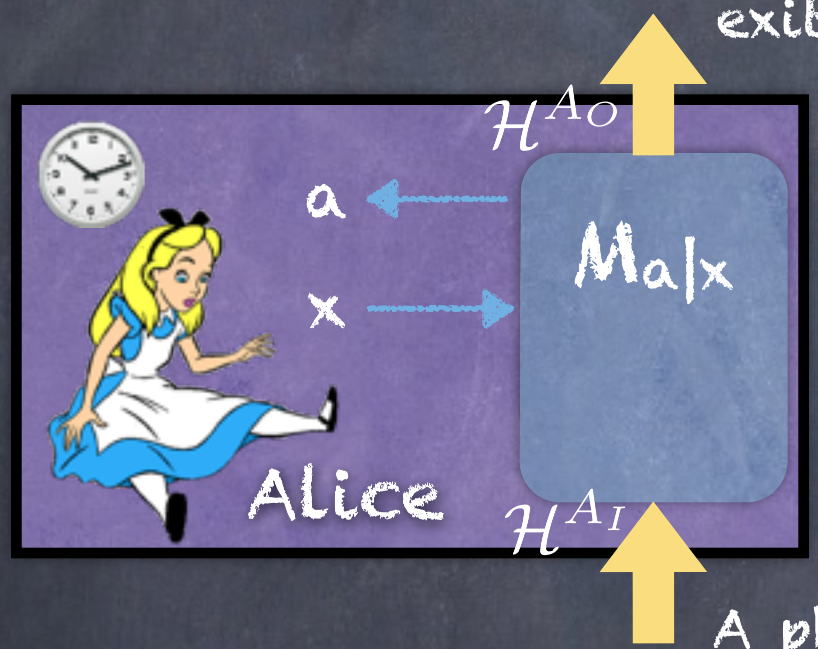
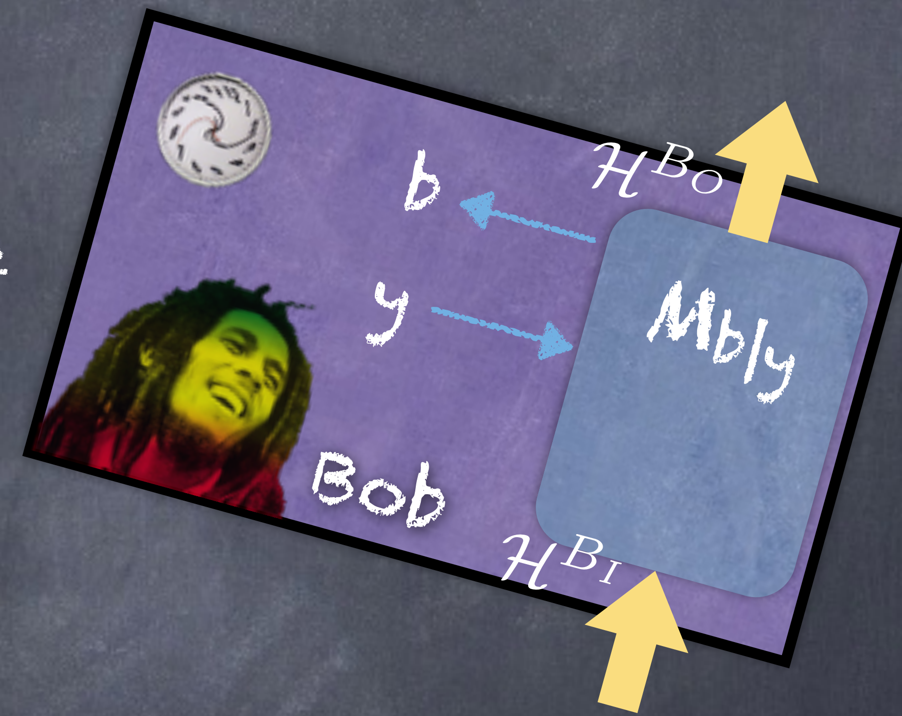
The process matrix framework

[O. Oreshkov, F. Costa, Č. Brukner, Nat. Commun. 3, 1092 (2012)]

A physical system exits the lab



Alice can choose some possible action x to perform, gets an outcome a



A physical system enters the lab

- No shared reference frame, no global time
- Assuming "local quantum mechanics": CP map $M_{a|x}$

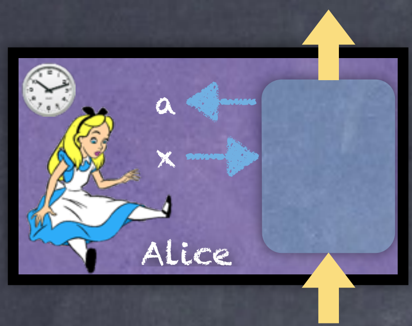
The process matrix framework

[O. Oreshkov, F. Costa, Č. Brukner, Nat. Commun. 3, 1092 (2012)]

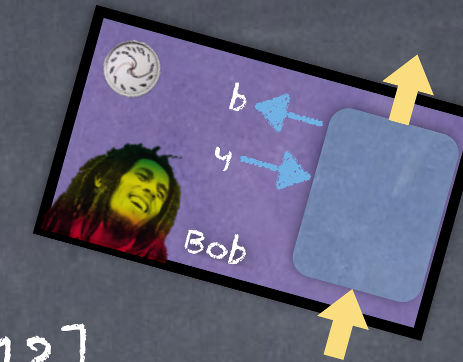
- Correlations are bilinear functions of Alice and Bob's CP maps:

$$P(a,b|x,y) = \mathrm{Tr}[M_{a|x} \otimes M_{b|y} \cdot W]$$

  - W = "Process matrix"



The process matrix framework



[OCB 2012]

- Some W matrices are compatible with a definite causal order: $W^{A\preceq B}$ or $W^{B\preceq A}$ (e.g. standard quantum circuits)
- The causal order may only be known with some probability q:

$$W_{sep} = q\, W^{A\preceq B} + (1-q)\, W^{B\preceq A}$$

  - W matrices of this form are said to be causally separable
  - Otherwise, they are causally nonseparable, and are incompatible with a definite causal order
    - Those may generate correlations with no definite causal order, which violate "causal inequalities"

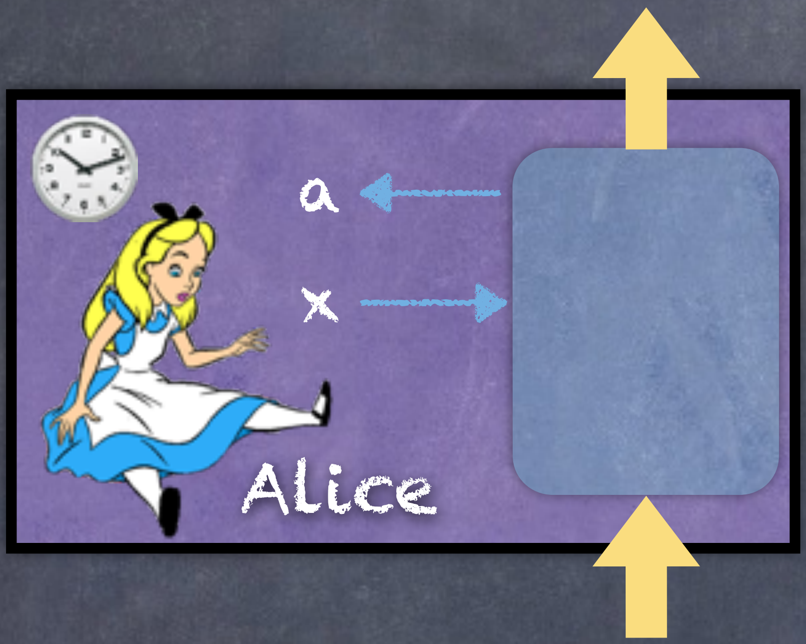
A causal game

[OCB 2012]

- Game:
  - If y'=0, Alice must guess Bob's input bit y
  - If y'=1, Bob must guess Alice's input bit x
- Success probability: $p_{succ} = 1/2\ [\ p(a=y|y'=0) + p(b=x|y'=1)\ ]$
- Assuming a definite causal order (→ no 2-way signaling):
  - $p_{succ} \leq 3/4$

A causal inequality

A causal game [OCB 2012]



- $p_{succ} = 1/2\,[\,p(a=y|y'=0) + p(b=x|y'=1)\,] \leq 3/4$
  - Can be violated in the process matrix framework:

$$W = \frac{1}{4}\left[\mathbb{1} + \frac{\mathbb{1}^{A_I} Z^{A_O} Z^{B_I} \mathbb{1}^{B_O} + Z^{A_I} \mathbb{1}^{A_O} X^{B_I} Z^{B_O}}{\sqrt{2}}\right]$$

$$M_{a|x}^{A_I A_O} = \left(\frac{\mathbb{1}+(-1)^a Z}{2}\right)^{A_I} \otimes \left(\frac{\mathbb{1}+(-1)^x Z}{2}\right)^{A_O}$$

$$M_{b|y,y'=0}^{B_I B_O} = \left(\frac{\mathbb{1}+(-1)^b X}{2}\right)^{B_I} \otimes \left(\frac{\mathbb{1}+(-1)^{y+b} Z}{2}\right)^{B_O}$$

$$M_{b|y,y'=1}^{B_I B_O} = \left(\frac{\mathbb{1}+(-1)^b Z}{2}\right)^{B_I} \otimes \frac{\mathbb{1}^{B_O}}{2}$$

$$\Rightarrow \; p_{succ} = \frac{1 + 1/\sqrt{2}}{2}$$

Process matrices vs correlations

- 2 kinds of objects which are "incompatible with any definite causal order":

  process matrices / correlations

- Do we need to violate a causal inequality to prove the causal nonseparability of a W matrix?
  - Do all causally nonseparable W matrices violate a causal inequality?
  - How to test for causal nonseparability otherwise?
- What could be observed in the lab?
  - Could we demonstrate causal nonseparability in practice, even if we don't know how to violate a causal inequality?

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Device Dependent

Entanglement

Detected by entanglement witnesses

Device Independent

Bell nonlocality

Violate Bell inequalities

A rich analogy

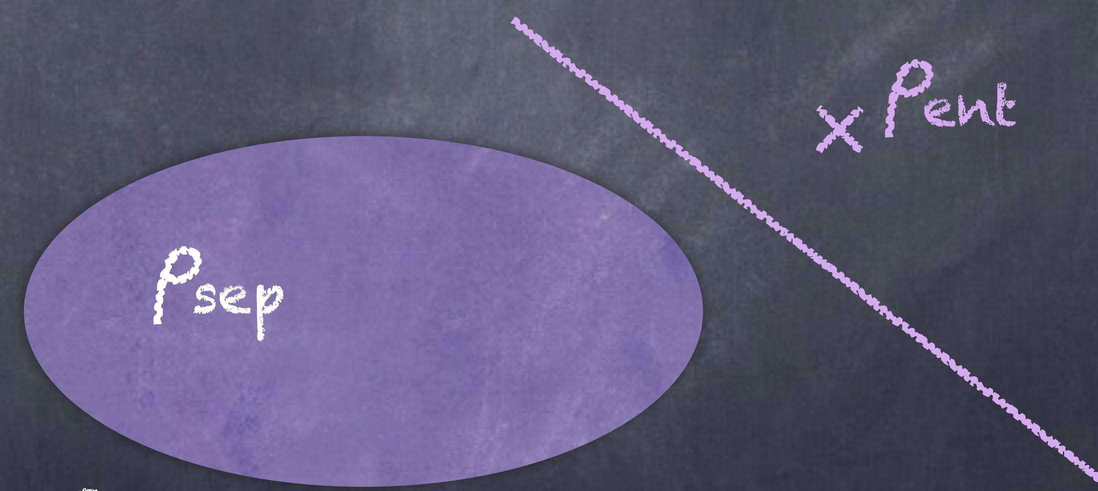
Causally nonseparable process matrices	↔	Entangled states
Causal witnesses (1)	↔	Entanglement witnesses
Correlations with no definite causal order	↔	Bell-nonlocal correlations
Causal inequalities	↔	Bell inequalities

(1) [M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]

A rich analogy

Causally nonseparable process matrices	$\Longleftrightarrow$	Entangled states

Causal witnesses [1] $\Longleftrightarrow$ Entanglement witnesses



Entanglement witness S:

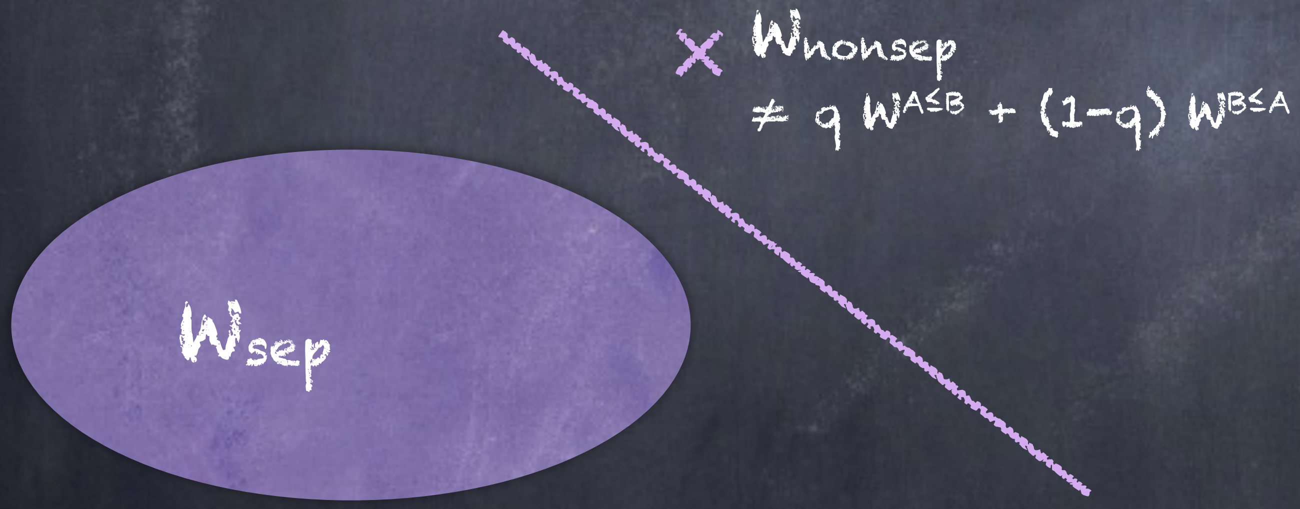
$\mathrm{Tr}[S.\rho_{ent}] < 0$ and $\mathrm{Tr}[S.\rho_{sep}] \geq 0$ for all ρ_{sep}

[1] [M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]

A rich analogy

Causally nonseparable process matrices	⟷	Entangled states

Causal witnesses [1] ⟷ Entanglement witnesses



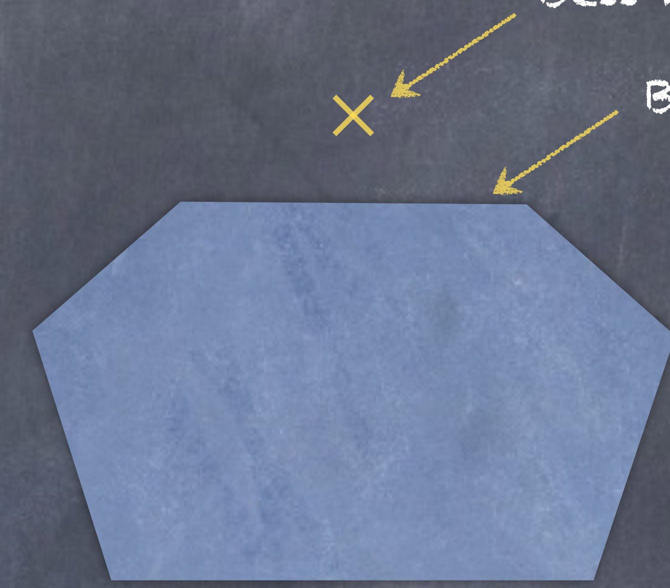
- for any W_{nonsep}, there exists a causal witness S such that

  $Tr[S.W_{nonsep}] < 0$ and
  $Tr[S.W_{sep}] \geq 0$ for all W_{sep}

- Can be constructed efficiently

[1] [M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]

A rich analogy



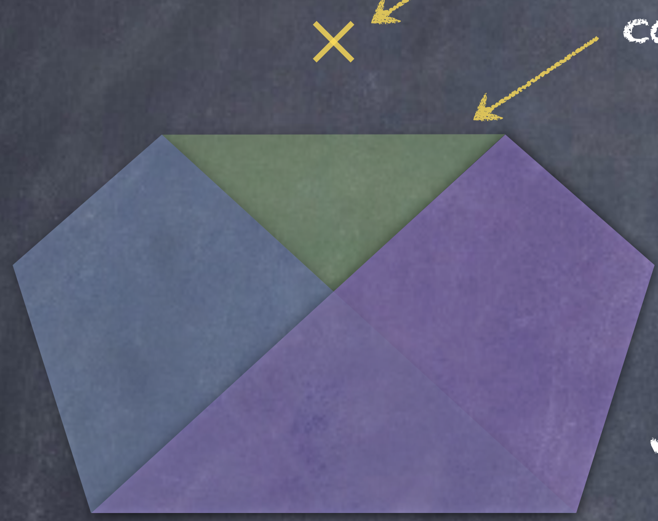
Correlations with no definite causal order	⟷	Bell-nonlocal correlations

Causal inequalities ⟷ Bell inequalities

A rich analogy

correlation with no definite causal order

causal inequality



facets of the "causal polytope" [1]

E.g. in the case of binary inputs & outputs[1]:

$p(a{=}y, b{=}x) \leq 1/2$

Correlations with no definite causal order	Bell-nonlocal correlations

Causal inequalities ⟷ Bell inequalities

[1] [CB et al., New J. Phys. (in press, 2015); arXiv:1508.01704 (quant-ph)]

A rich analogy

Device Dependent	Causally nonseparable process matrices ⟷	Entangled states
	Causal witnesses ⟷	Entanglement witnesses
Device Independent	Correlations with no definite causal order ⟷	Bell-nonlocal correlations
	Causal inequalities ⟷	Bell inequalities

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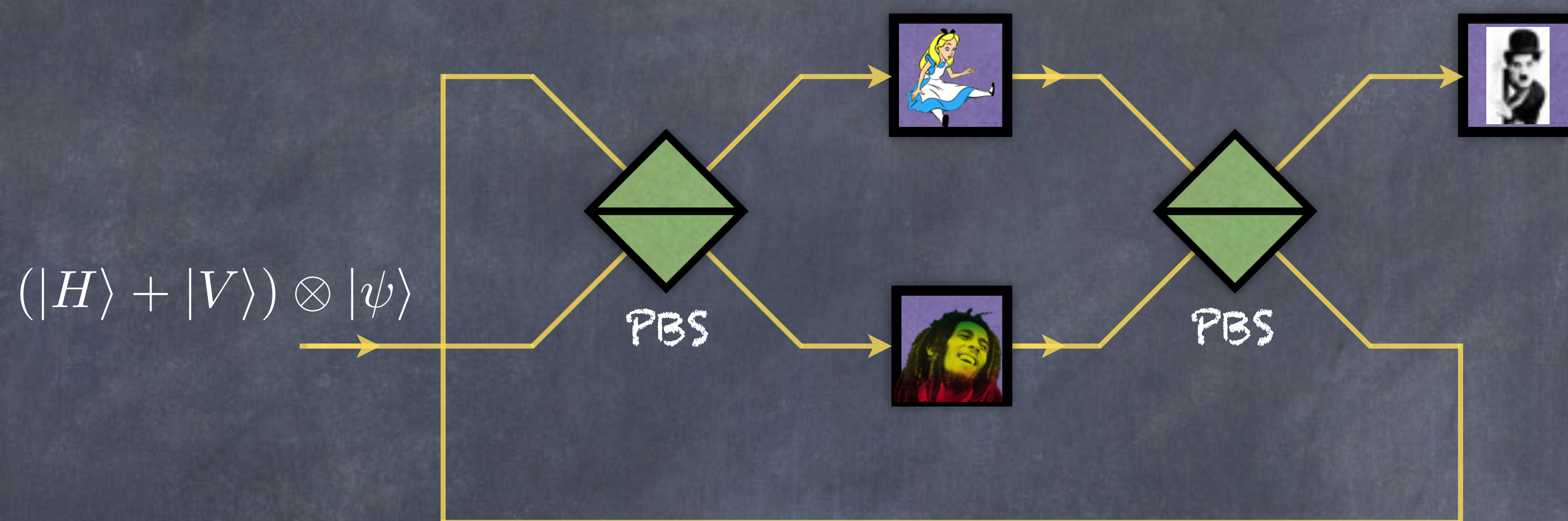
The "quantum switch"

[G. Chiribella et al., PRA 88, 022318 (2013);
Araújo et al., PRL 113, 250402 (2014);
Procopio et al., Nat. Commun. 6, 7913 (2015)]



$$\big(|H\rangle + |V\rangle\big) \otimes |\psi\rangle \to |H\rangle \otimes BA|\psi\rangle + |V\rangle \otimes AB|\psi\rangle$$

The "quantum switch"

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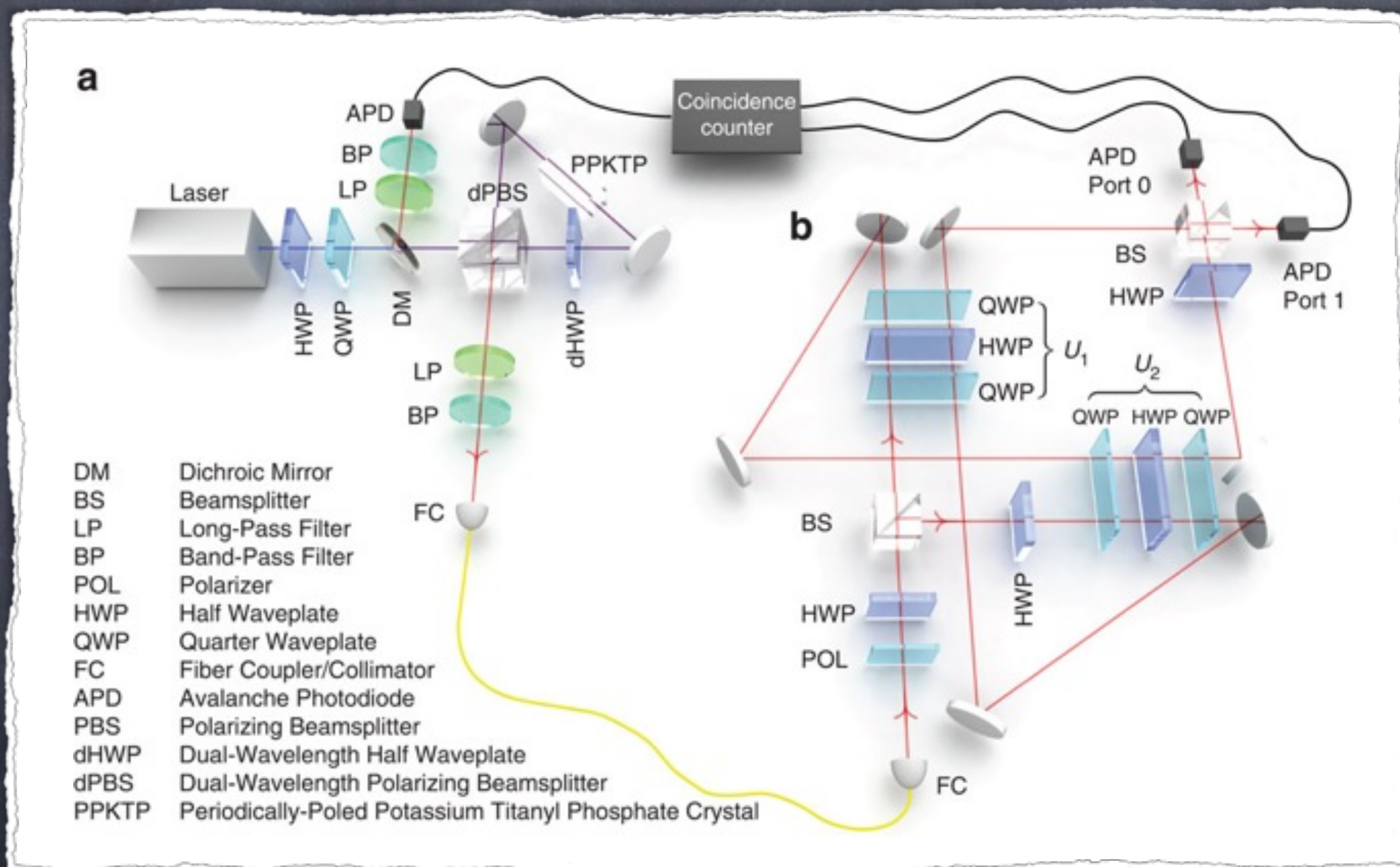

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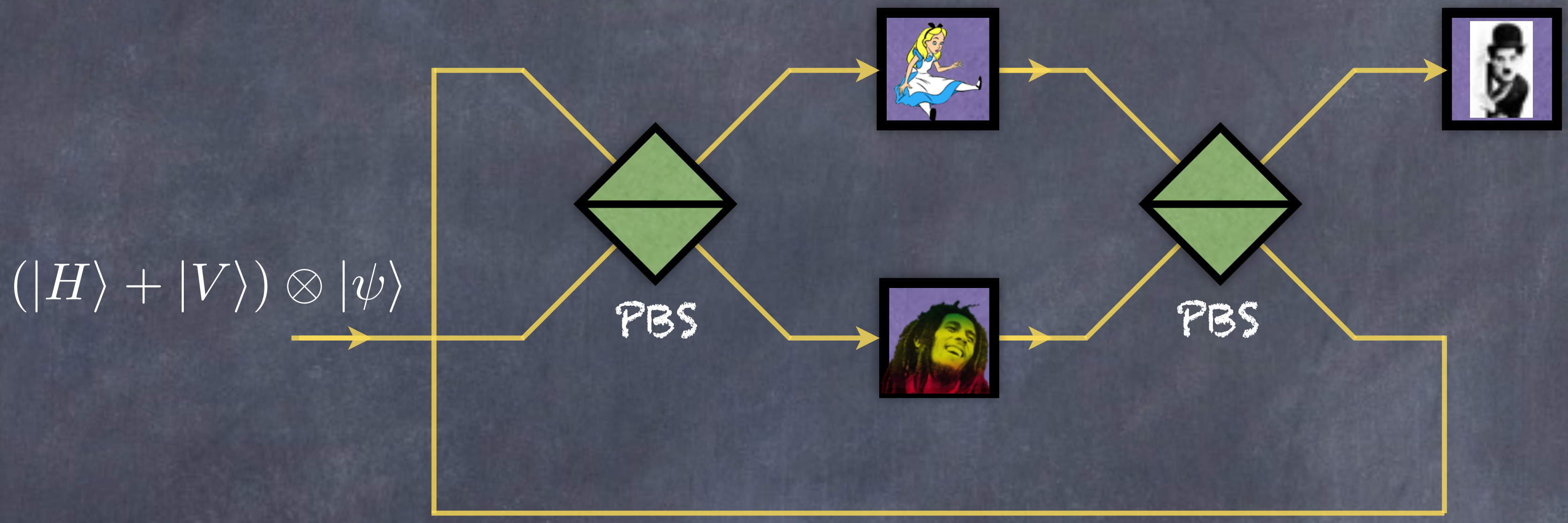

$$(|H\rangle + |V\rangle) \otimes |\psi\rangle \to |H\rangle \otimes BA|\psi\rangle + |V\rangle \otimes AB|\psi\rangle$$

- As a process matrix:

$$\begin{aligned} |w\rangle = \quad & |H\rangle^{C_I'}|\psi\rangle^{A_I}|\mathbb{1}\rangle\rangle^{A_O B_I}|\mathbb{1}\rangle\rangle^{B_O C_I} \\ & + |V\rangle^{C_I'}|\psi\rangle^{B_I}|\mathbb{1}\rangle\rangle^{B_O A_I}|\mathbb{1}\rangle\rangle^{A_O C_I} \end{aligned} \qquad W = |w\rangle\langle w|$$

- causally nonseparable!

The "quantum switch"

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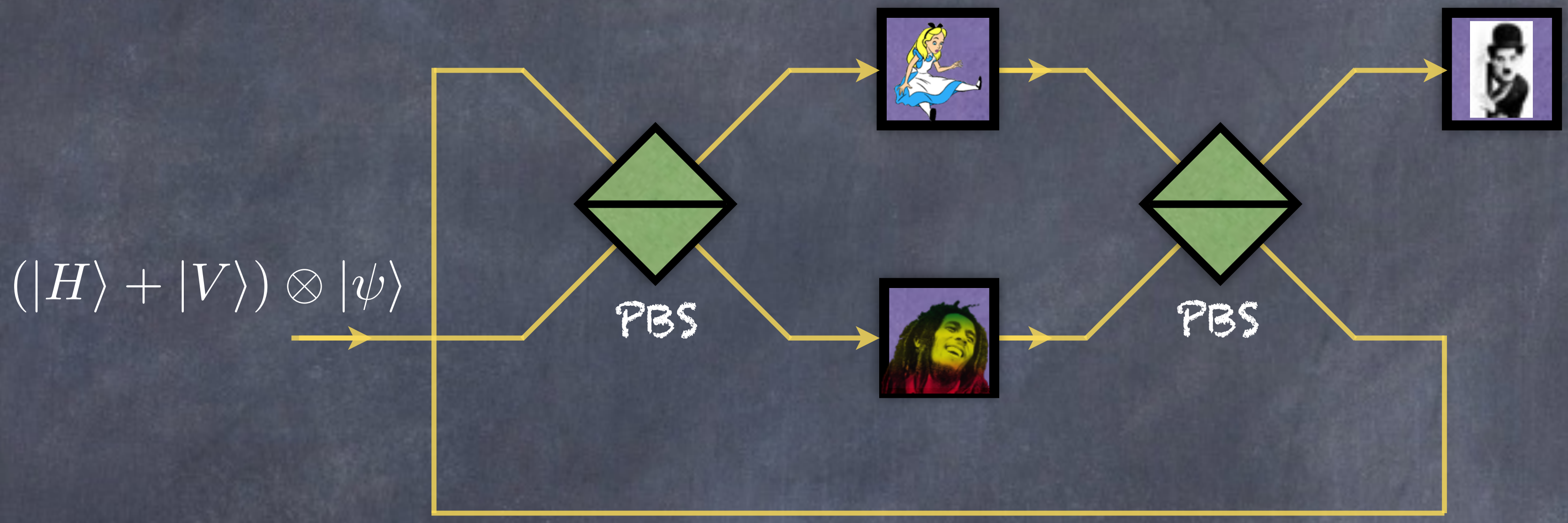

- A causal witness [1] can be constructed and measured

$$\mathrm{Tr}[S.W_{switch}] < 0 \quad \text{and} \quad \mathrm{Tr}[S.W_{sep}] \geq 0 \text{ for all } W_{sep}$$

- The quantum switch does not violate any causal inequality [1,2]

(1) [M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]
(2) [O. Oreshkov, C. Giarmatzi, arXiv:1506.05449 (2015)]

Conclusion - Outlook

- New causal relations in the quantum world: Causally non separable processes
- Gave some physical content to the process matrix formalism
- Clarified the link between causal nonseparability
  of a process and violation of a causal inequality
- Rich analogy with entanglement and Bell nonlocality:
  to be exploited further!
- Applications for Quantum Information? –> Beyond quantum computers!
- Other examples of nonseparable processes?
  Bipartite example that can be implemented?
- Violation of a causal inequality in practice???

Thank you for your attention