(Quantum?) Processes and Correlations with no definite causal order

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Classical causal relations

A causes B

Let's enter the quantum world...
Motivation

- In quantum mechanics, some variables may be indefinite (e.g. X, P)

- What about causal relations?
  - In “standard QM”, measurements are done in space-time
    Fixed measurement positions, time evolution, tensor product structure... assume a fixed causal structure
  - Can we go beyond this?
    Remove time and causal structure from QM?

- What new phenomenology arises?
  Experiments, applications?
Outline

- The process matrix framework
- Analogy with entanglement & Bell nonlocality
- The “Quantum switch” as a causally nonseparable process
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The process matrix framework

[O. Oreshkov, F. Costa, Č. Brukner, Nat. Commun. 3, 1092 (2012)]

A physical system enters the lab

A physical system exits the lab

Alice can choose some possible action $x$ to perform, gets an outcome $a$

- No shared reference frame, no global time
- Assuming “local quantum mechanics”: CP map $M_{a|x}$
The process matrix framework

[O. Oreshkov, F. Costa, Č. Brukner, Nat. Commun. 3, 1092 (2012)]

Correlations are bilinear functions of Alice and Bob’s CP maps:

\[ P(a, b|x, y) = \text{Tr}[M_a|x \otimes M_b|y \cdot W] \]

\[ W = \text{“Process matrix”} \]
Some $W$ matrices are compatible with a definite causal order: $W^{A\rightarrow B}$ or $W^{B\rightarrow A}$ (e.g. standard quantum circuits)

The causal order may only be known with some probability $q$:

$$W_{sep} = q W^{A\rightarrow B} + (1-q) W^{B\rightarrow A}$$

- $W$ matrices of this form are said to be causally separable
- Otherwise, they are causally nonseparable, and are incompatible with a definite causal order
- Those may generate correlations with no definite causal order, which violate "causal inequalities"
A causal game

Game:
- If $y' = 0$, Alice must guess Bob's input bit $y$
- If $y' = 1$, Bob must guess Alice's input bit $x$

Success probability: $p_{\text{succ}} = \frac{1}{2} \left[ p(a = y | y' = 0) + p(b = x | y' = 1) \right]$

Assuming a definite causal order (→ no 2-way signaling):
- $p_{\text{succ}} \leq \frac{3}{4}$

A causal inequality [OCB 2012]
A causal game

\[ W = \frac{1}{4} \left[ 1 + \frac{1^{A_I} Z^{A_O} Z^{B_I} 1^{B_O} + Z^{A_I} 1^{A_O} X^{B_I} Z^{B_O}}{\sqrt{2}} \right] \]

\[ \Rightarrow \ p_{\text{succ}} = \frac{1 + 1/\sqrt{2}}{2} \]

\[ M_{a|x} = \frac{1+(-1)^{a}Z}{2} \otimes \frac{1+(-1)^{x}Z}{2} \]
\[ M_{b|y, y'} = \frac{1+(-1)^{b}X}{2} \otimes \frac{1+(-1)^{y+b}Z}{2} \]

\[ M_{b|y, y'=0} = \frac{1+(-1)^{b}X}{2} \otimes 1^{B_O} \]
\[ M_{b|y, y'=1} = 1^{B_I} \otimes \frac{1^{B_O}}{2} \]

\( p_{\text{succ}} = 1/2 \left[ p(a=y|y'=0) + p(b=x|y'=1) \right] \leq 3/4 \)

\( \triangleright \) Can be violated in the process matrix framework:

\( [\text{OCB 2012}] \)
Process matrices vs correlations

- 2 kinds of objects which are "incompatible with any definite causal order":
  - process matrices / correlations

- Do we need to violate a causal inequality to prove the causal nonseparability of a W matrix?
  - Do all causally nonseparable W matrices violate a causal inequality?
  - How to test for causal nonseparability otherwise?

- What could be observed in the lab?
  - Could we demonstrate causal nonseparability in practice, even if we don't know how to violate a causal inequality?
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Entanglement

Detected by entanglement witnesses

Bell nonlocality

$P(a,b|x,y)$

Violate Bell inequalities
A rich analogy

<table>
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<tr>
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<th>Entangled states</th>
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\(^{(1)}\) [M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]
A rich analogy

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Entanglement witness $S$:

$$\text{Tr}[S, \rho_{\text{ent}}] < 0 \quad \text{and} \quad \text{Tr}[S, \rho_{\text{sep}}] \geq 0 \quad \text{for all} \quad \rho_{\text{sep}}$$

(1) [M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]
A rich analogy

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\[
W_{\text{nonsep}} \neq q W^{A \rightarrow B} + (1-q) W^{B \rightarrow A}
\]

- for any \(W_{\text{nonsep}}\), there exists a **causal witness** \(S\) such that
  \[
  \text{Tr}[S.W_{\text{nonsep}}] < 0 \quad \text{and} \quad \text{Tr}[S.W_{\text{sep}}] \geq 0 \quad \text{for all } W_{\text{sep}}
  \]
- Can be constructed efficiently

\(^{(1)}\) [M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]
A rich analogy

Correlations with no definite causal order ↔ Bell-nonlocal correlations

Causal inequalities ↔ Bell inequalities

Bell nonlocal correlation
Bell inequality

facets of the “local polytope”
A rich analogy

Correlations with no definite causal order

Bell-nonlocal correlations

Causal inequalities

Bell inequalities

E.g. in the case of binary inputs $a$ and outputs $b$:

$$p(a=y, b=x) \leq 1/2$$

A rich analogy

Device Dependent

Causally nonseparable process matrices ↔ Entangled states

Causal witnesses ↔ Entanglement witnesses

Device Independent

Correlations with no definite causal order ↔ Bell-nonlocal correlations

Causal inequalities ↔ Bell inequalities
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The “quantum switch”


\[ \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) \otimes |\psi\rangle \]

\[ \mapsto |H\rangle \otimes BA |\psi\rangle + |V\rangle \otimes AB |\psi\rangle \]
The "quantum switch"

[G. Chiribella et al., PRA 88, 022318 (2013); Araújo et al., PRL 113, 250402 (2014); Procopio et al., Nat. Commun. 6, 7913 (2015)]
The "quantum switch"

[G. Chiribella et al., PRA 88, 022318 (2013);
Araújo et al., PRL 113, 250402 (2014);
Procopio et al., Nat. Commun. 6, 7913 (2015)]

\[ (|H\rangle + |V\rangle) \otimes |\psi\rangle \rightarrow |H\rangle \otimes BA|\psi\rangle + |V\rangle \otimes AB|\psi\rangle \]

\[ |w\rangle = |H\rangle^{C^I} |\psi\rangle^{A_I} |1\rangle^{AOB_I} |1\rangle^{BOC_I} + |V\rangle^{C^I} |\psi\rangle^{B_I} |1\rangle^{BOA_I} |1\rangle^{AOC_I} \]

\[ W = |w\rangle\langle w| \]

\[ \text{causally nonseparable!} \]
The “quantum switch”

| G. Chiribella et al., PRA 88, 022318 (2013); Araújo et al., PRL 113, 250402 (2014); Procopio et al., Nat. Commun. 6, 7913 (2015) |

\[
\begin{align*}
&\text{PBS} & \quad (|H\rangle + |V\rangle) \otimes |\psi\rangle \\
&\quad \text{PBS} & \\
&\quad \text{PBS} & \\
\end{align*}
\]

- A causal witness\(^{(1)}\) can be constructed and measured
  \[\text{Tr}[S.W_{\text{switch}}] < 0 \quad \text{and} \quad \text{Tr}[S.W_{\text{sep}}] \geq 0\] for all \(W_{\text{sep}}\)
- The quantum switch does not violate any causal inequality\(^{(1,2)}\)

\(^{(1)}\) M. Araújo, CB et al., New J. Phys. 17, 102001 (2015)]
Conclusion - Outlook

- New causal relations in the quantum world: Causally nonseparable processes
- Gave some physical content to the process matrix formalism
- Clarified the link between causal nonseparability of a process and violation of a causal inequality
- Rich analogy with entanglement and Bell nonlocality: to be exploited further!
- Applications for Quantum Information? → Beyond quantum computers!
- Other examples of nonseparable processes? Bipartite example that can be implemented?
- Violation of a causal inequality in practice???
Thank you for your attention