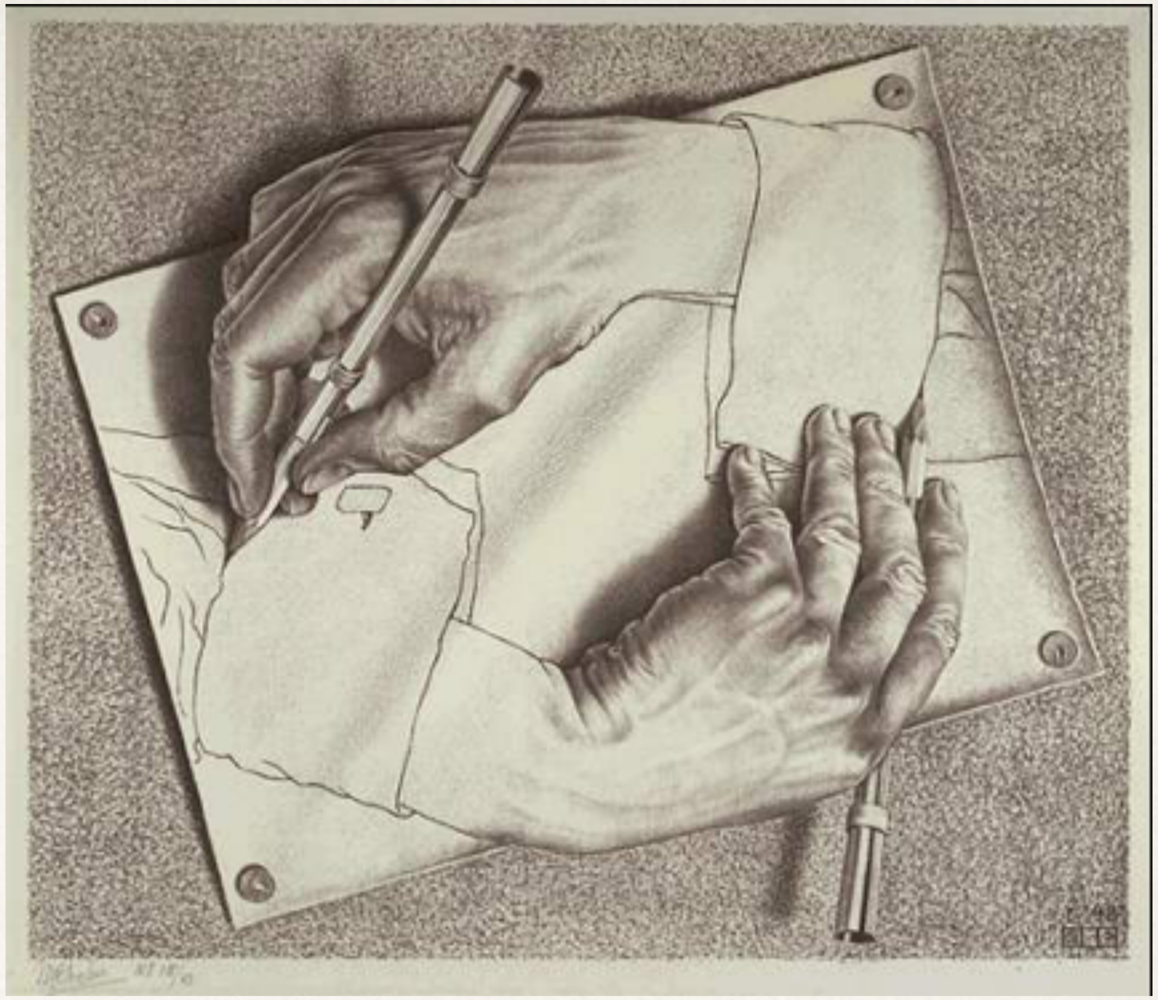

Processes without causal order as a model of computation

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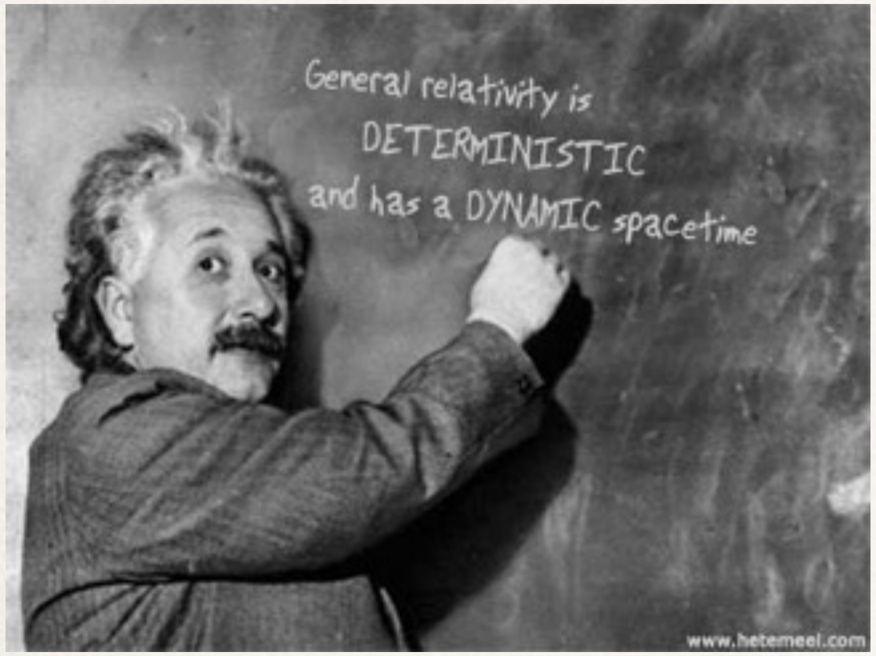
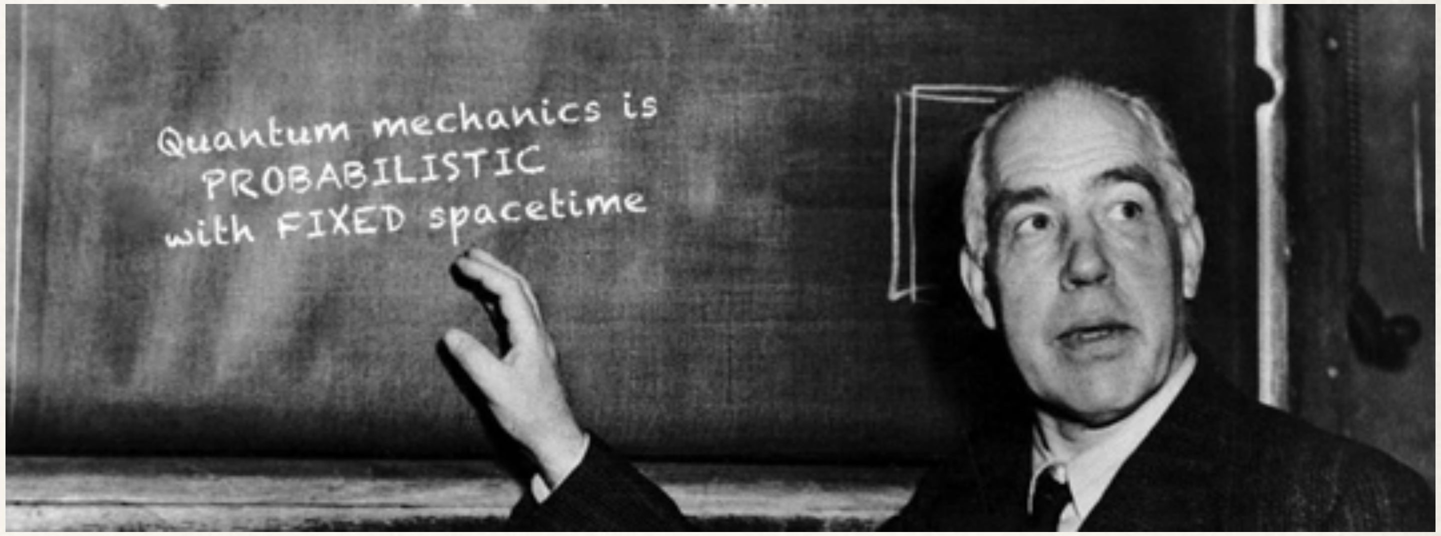
December 11, 2015 — Workshop on Quantum Nonlocality, Causal Structures and Device-Independent Quantum Information, National Cheng Kung University, Tainan, Taiwan

Outline

- **Motivation**
- Classical correlations without causal order
- Circuit model without causal order
- Examples
- Conclusion

Motivation





General relativity

Quantum physics

$\underbrace{\hspace{6em}}$

Quantum gravity

Motivation

General relativity: *dynamic* spacetime and *deterministic*

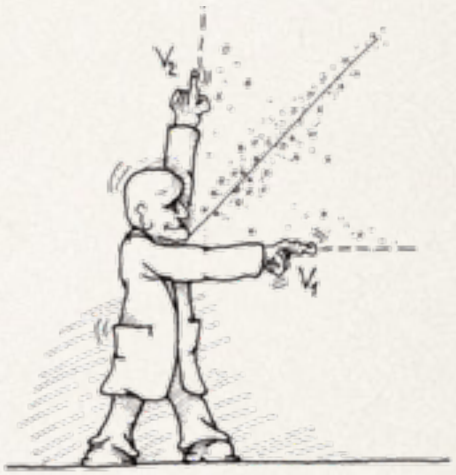
Quantum physics: *fixed* spacetime and *probabilistic*

- Quantum gravity: *probabilistic* theory with a *dynamic* spacetime?[1]

[1] *L. Hardy, arXiv:0509120 [gr-qc] (2005).*

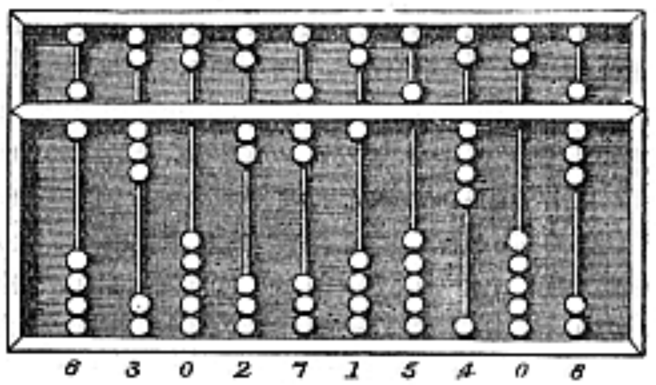
Motivation

- Replace *background time* with weaker assumption of *logical consistency*.



- What <u>correlations</u> are possible in a world *without background time?*

- What <u>computations</u> are possible in world *without background time*?



Correlations without predefined causal order

- Work is mainly based on the 

framework by Oreshkov, Costa, and Brukner.[1]

[1] *O. Oreshkov, F. Costa, Č. Brukner, Nat. Commun.* ***3****, 1092 (2012).*

Correlations without predefined causal order

- Correlations that arise when we drop

  background time

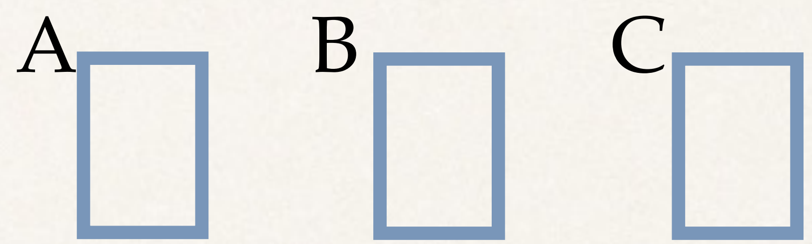
  and assume

  local validity of a theory,
  local time, and
  logical consistency only.

unique fixed point

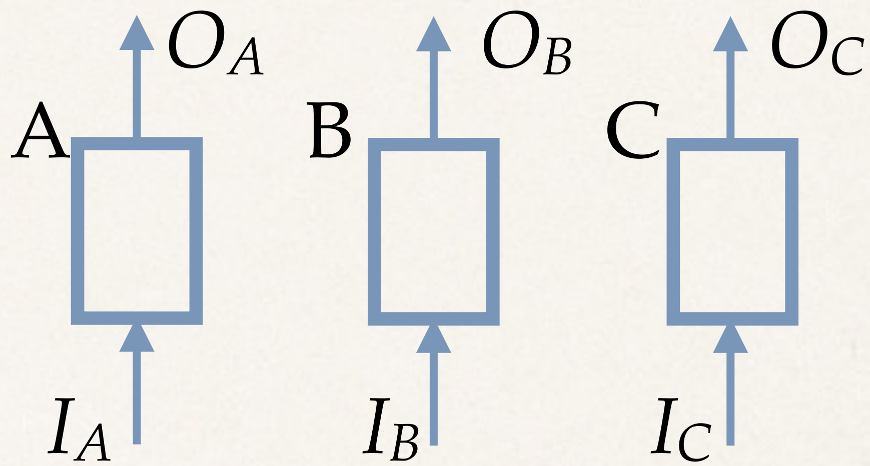
Classical correlations without causal order

- Operational approach:
  Parties: A, B, C (isolated)



Classical correlations without causal order

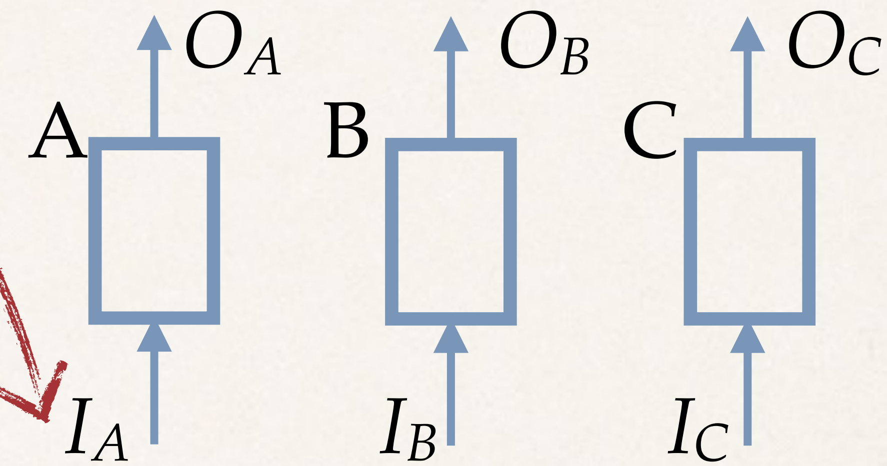
- Operational approach:
  Parties: A, B, C (isolated)



- Local time:
  A first receives random variable I_A, then sends random variable O_A (equivalently for B and C).

Classical correlations without causal order

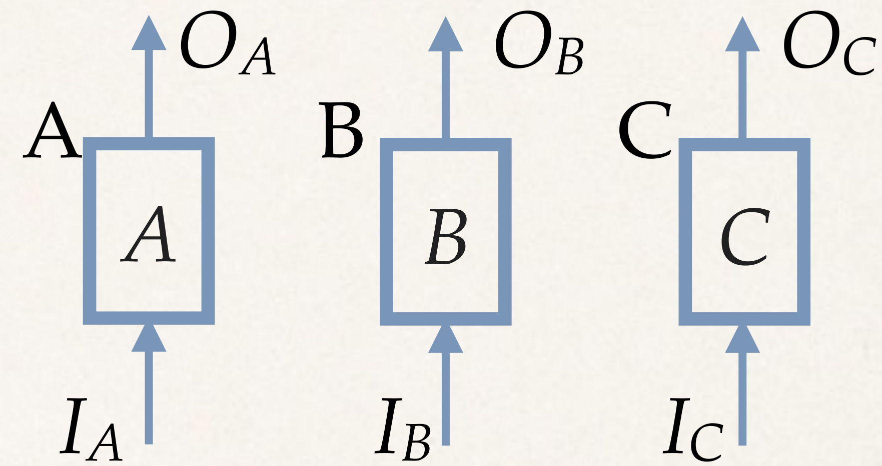
random variable



- Operational approach:
  Parties: A, B, C (isolated)
- Local time:
  A first receives random variable I_A, then sends random variable O_A (equivalently for B and C).

Classical correlations without causal order

- Operational approach:
  Parties: A, B, C (isolated)

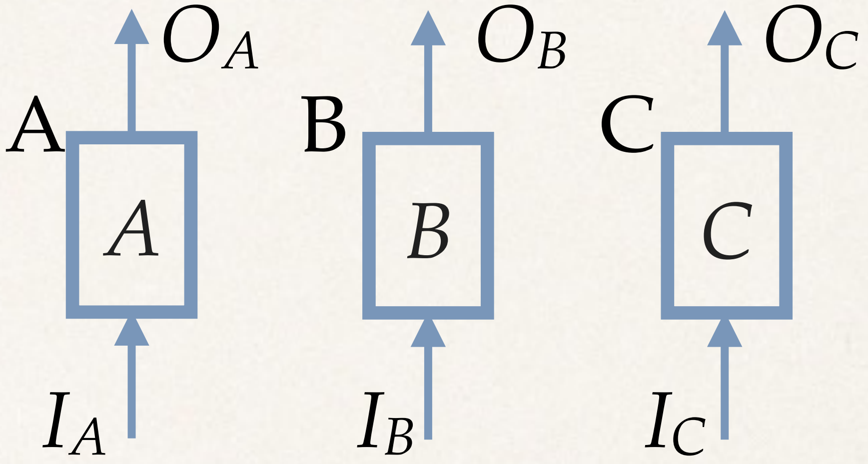


- Local time:
  A first receives random variable I_A, then sends random variable O_A (equivalently for B and C).

- Local validity of probability theory:
  A's operation is a conditional probability distribution
  $A = P_{O_A|I_A}$ (equivalently for B and C).

Classical correlations without causal order

- Operational approach: Probability $P_{\boldsymbol{O},\boldsymbol{I}}$ is *linear* in the choice of operations of A, B, and C.



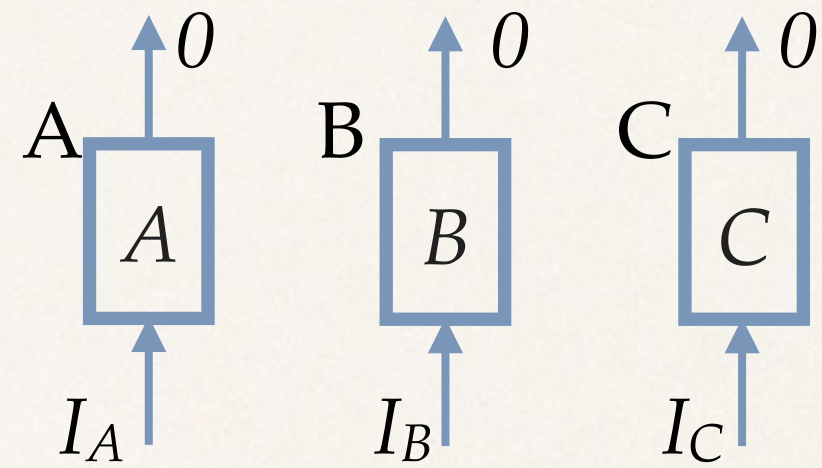
- $\boldsymbol{o} = (o_A, o_B, o_C), \quad \boldsymbol{i} = (i_A, i_B, i_C)$

- $P(\boldsymbol{o}, \boldsymbol{i}) = e(\boldsymbol{o}, \boldsymbol{i}) P(o_A|i_A) P(o_B|i_B) P(o_C|i_C)$

- Condition: $\forall \boldsymbol{o}, \boldsymbol{i} : e(\boldsymbol{o}, \boldsymbol{i}) \geq 0$

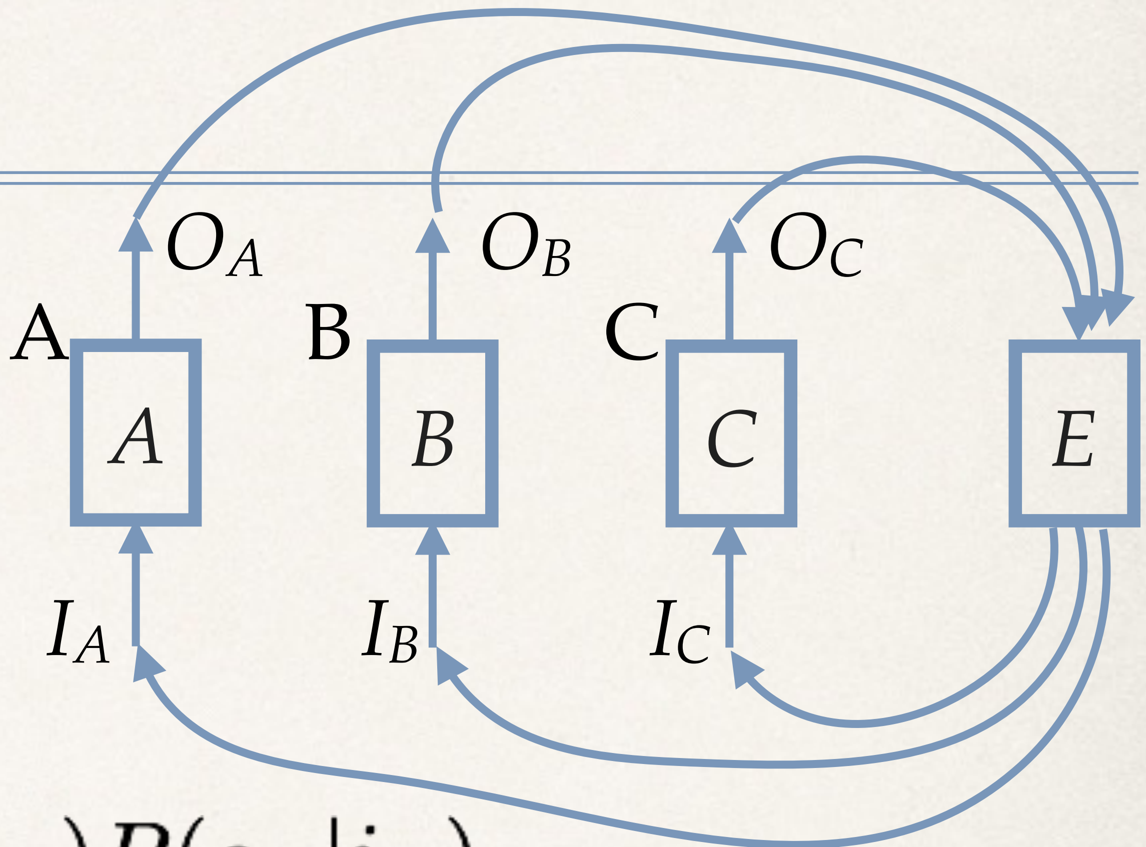
Classical correlations without causal order

- Let $P(o_A|i_A) = \delta_{0,o_A}$,
  $P(o_B|i_B) = \delta_{0,o_B}$,
  $P(o_C|i_C) = \delta_{0,o_C}$



- $P(\boldsymbol{o},\boldsymbol{i}) = e(\boldsymbol{o},\boldsymbol{i})P(o_A|i_A)P(o_B|i_B)P(o_C|i_C)$
- $\sum_{\boldsymbol{o},\boldsymbol{i}} P(\boldsymbol{o},\boldsymbol{i}) = \sum_{\boldsymbol{o},\boldsymbol{i}} e(\boldsymbol{o},\boldsymbol{i})\delta_{0,o_A}\delta_{0,o_B}\delta_{0,o_c} = \sum_{\boldsymbol{i}} e(\mathbf{0},\boldsymbol{i}) = 1$
- $\forall \boldsymbol{o} : \sum_{\boldsymbol{i}} e(\boldsymbol{o},\boldsymbol{i}) = 1$: Conditional probability distribution E=$P(\boldsymbol{i}|\boldsymbol{o}) = e(\boldsymbol{o},\boldsymbol{i})$

Classical correlations without causal order

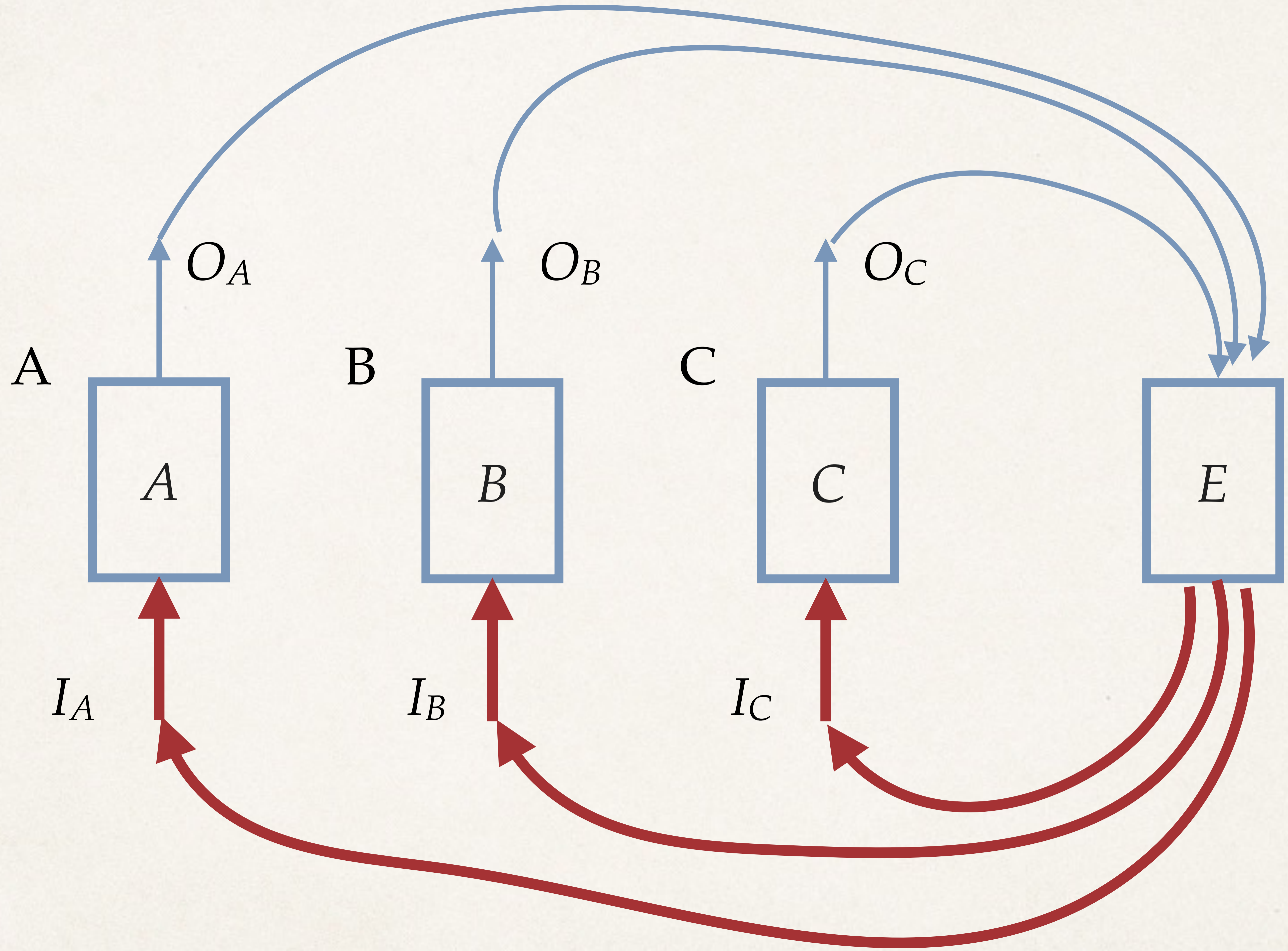


- Let $P(o_A|i_A) = \delta_{0,o_A}$,

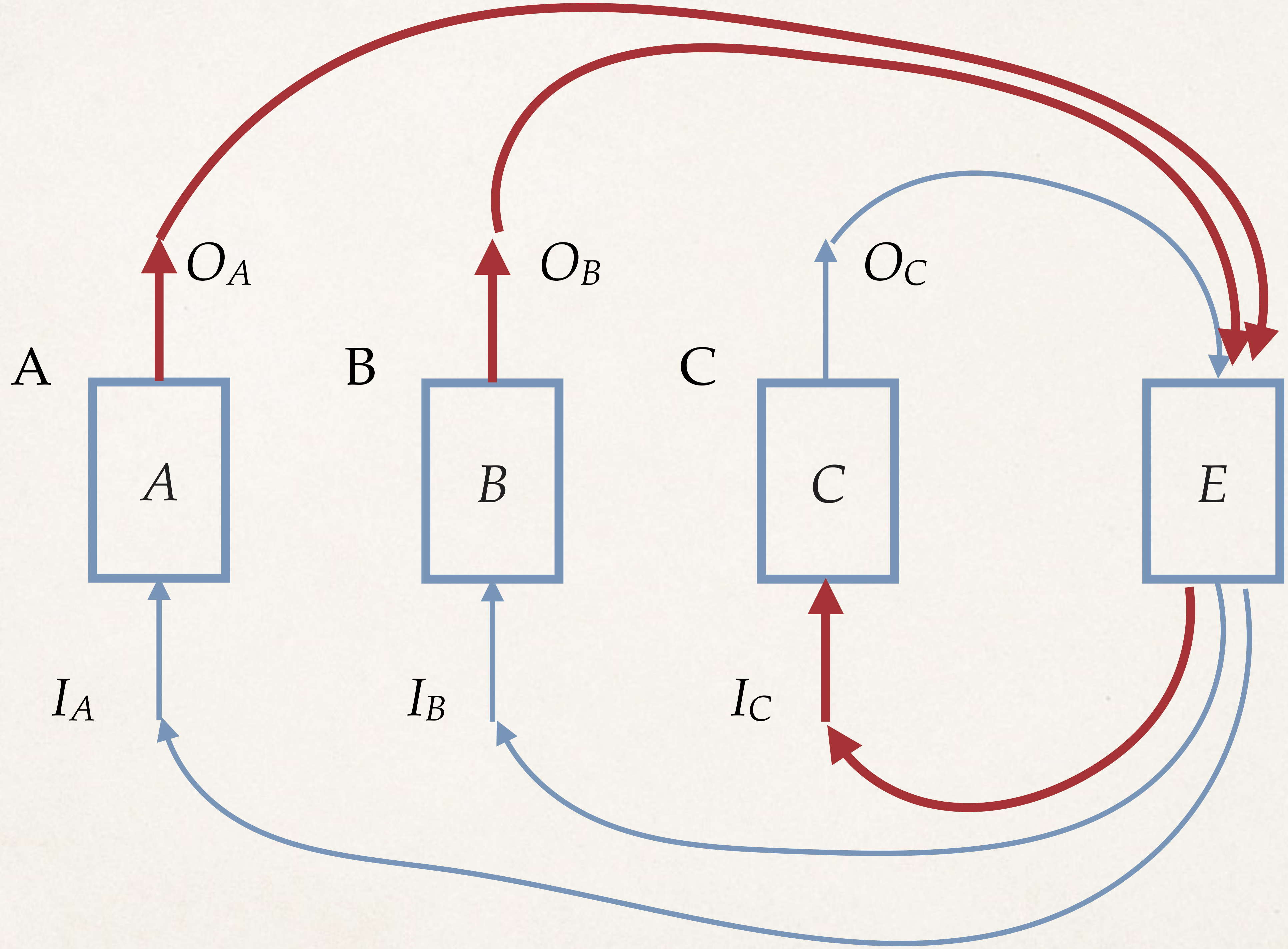
  $P(o_B|i_B) = \delta_{0,o_B}$,

  $P(o_C|i_C) = \delta_{0,o_C}$
- $P(\boldsymbol{o},\boldsymbol{i}) = e(\boldsymbol{o},\boldsymbol{i})P(o_A|i_A)P(o_B|i_B)P(o_C|i_C)$
- $\sum_{\boldsymbol{o},\boldsymbol{i}} P(\boldsymbol{o},\boldsymbol{i}) = \sum_{\boldsymbol{o},\boldsymbol{i}} e(\boldsymbol{o},\boldsymbol{i})\delta_{0,o_A}\delta_{0,o_B}\delta_{0,o_c} = \sum_{\boldsymbol{i}} e(\boldsymbol{0},\boldsymbol{i}) = 1$
- $\forall \boldsymbol{o} : \sum_{\boldsymbol{i}} e(\boldsymbol{o},\boldsymbol{i}) = 1$: Conditional probability distribution E=$P(\boldsymbol{i}|\boldsymbol{o}) = e(\boldsymbol{o},\boldsymbol{i})$

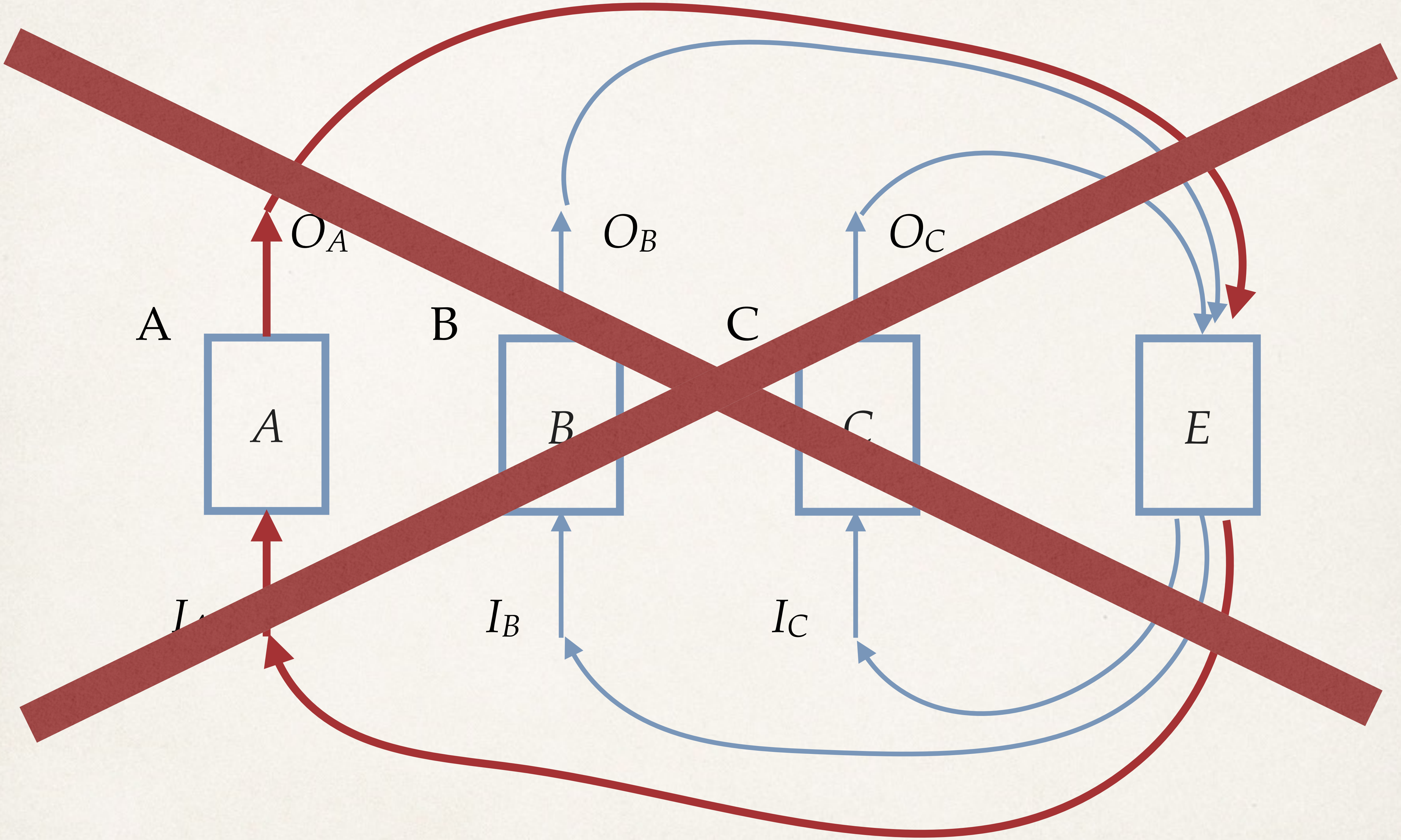
- State



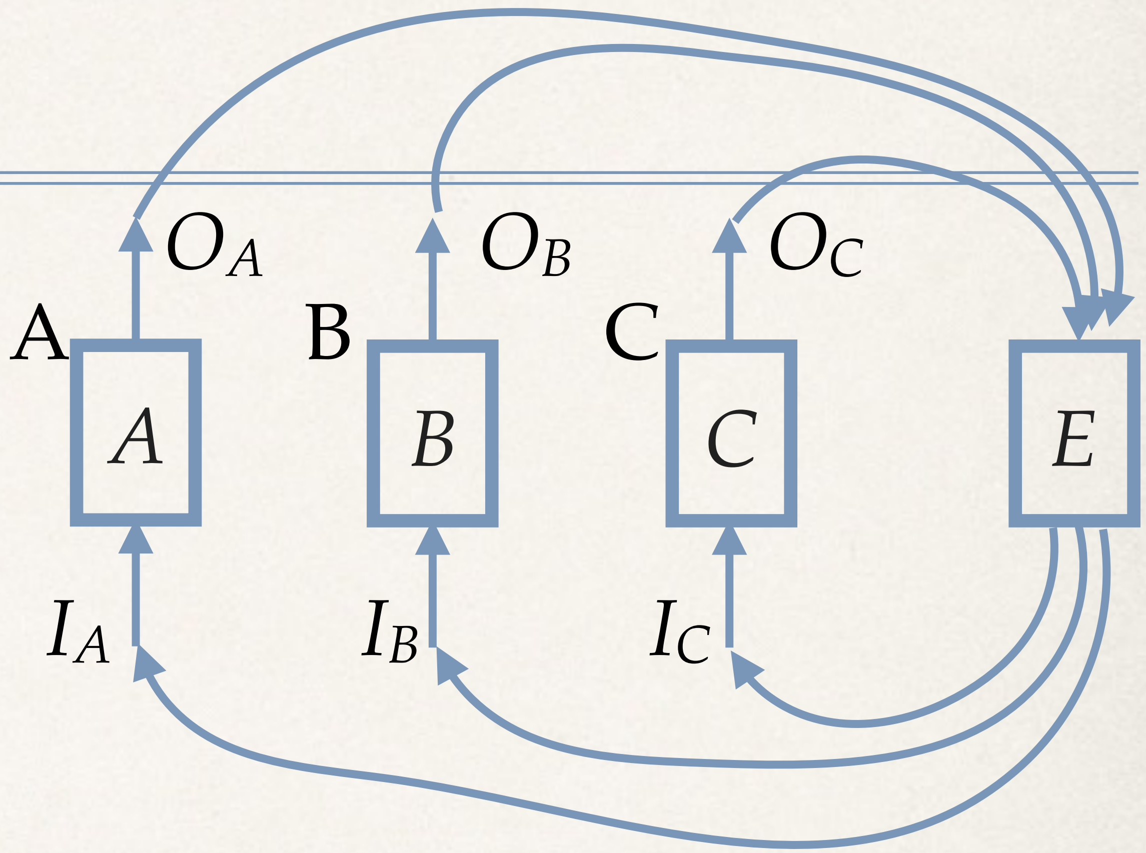
- Channel from A and B to C



- Channel from A to A. Not allowed!



Logical consistency



- Logical consistency:
  For every choice of
  $P(o_A|i_A), P(o_B|i_B), P(o_C|i_C)$:

$$\sum_{\boldsymbol{o},\boldsymbol{i}} P(\boldsymbol{i}|\boldsymbol{o}) P(o_A|i_A) P(o_B|i_B) P(o_C|i_C) = 1$$

Logical consistency

- Formulation of $\sum_{\boldsymbol{o},\boldsymbol{i}} P(\boldsymbol{i}|\boldsymbol{o})P(o_A|i_A)P(o_B|i_B)P(o_C|i_C) = 1$

  with stochastic matrices.

- Define the stochastic matrices $\hat{E}, \hat{A}, \hat{B}, \hat{C}$ as

  $\vec{\boldsymbol{i}}^T \hat{E} \vec{\boldsymbol{o}} = P(\boldsymbol{i}|\boldsymbol{o})$

  $\vec{o}_A^T \hat{A} \vec{i}_A = P(o_A|i_A)$ with $\vec{\boldsymbol{o}} = \vec{o}_A \otimes \vec{o}_B \otimes \vec{o}_C$

  $\vec{o}_B^T \hat{C} \vec{i}_B = P(o_B|i_B)$ $\quad \vec{\boldsymbol{i}} = \vec{i}_A \otimes \vec{i}_B \otimes \vec{i}_C$

  $\vec{o}_C^T \hat{B} \vec{i}_C = P(o_C|i_C)$

Logical consistency

- $$\sum_{\boldsymbol{o},\boldsymbol{i}} P(\boldsymbol{i}|\boldsymbol{o})P(o_A|i_A)P(o_B|i_B)P(o_C|i_C)$$
  $$= \sum_{\vec{\boldsymbol{o}},\vec{\boldsymbol{i}}} \left(\vec{\boldsymbol{i}}^T \hat{E} \vec{\boldsymbol{o}}\right) \left(\vec{o}_A^T \hat{A} \vec{i}_A\right) \left(\vec{o}_B^T \hat{C} \vec{i}_B\right) \left(\vec{o}_C^T \hat{B} \vec{i}_C\right)$$
  $$= \sum_{\vec{\boldsymbol{o}},\vec{\boldsymbol{i}}} \left(\vec{\boldsymbol{i}}^T \hat{E} \vec{\boldsymbol{o}}\right) \left(\vec{\boldsymbol{o}}^T (\hat{A} \otimes \hat{B} \otimes \hat{C}) \vec{\boldsymbol{i}}\right)$$
  $$= \mathrm{Tr}\left(\hat{E}(\hat{A} \otimes \hat{B} \otimes \hat{C})\right) = 1$$

Unique fixed point

Set of deterministic local operations

- Logical consistency:

$$\forall \hat{A}, \hat{B}, \hat{C} \in \mathcal{D} : \mathrm{Tr}\left(\underbrace{\hat{E}(\hat{A} \otimes \hat{B} \otimes \hat{C})}_{M}\right) = 1$$

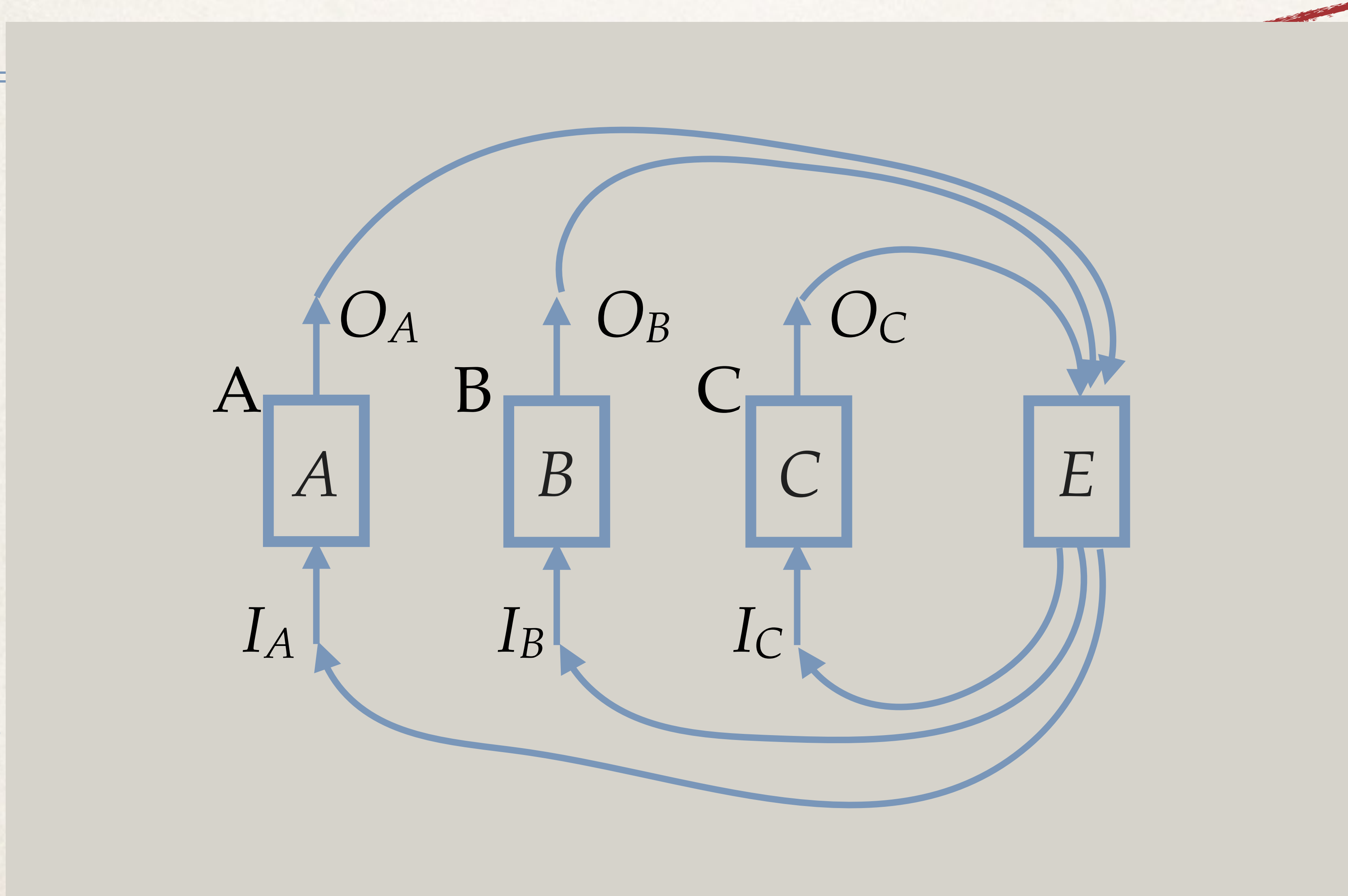
- Assume $\hat{E}$ is deterministic (only 0′s and 1′s). Diagonal of M has a single 1.

- For every choice of operations: unique fixed point

$$\begin{array}{cc} & \boldsymbol{i'} \\ \boldsymbol{i'} & \begin{bmatrix} 0 & & & & & & \\ & \ddots & & & & & \\ & & 0 & & & & \\ & & & \mathbf{1} & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{bmatrix} \end{array}$$

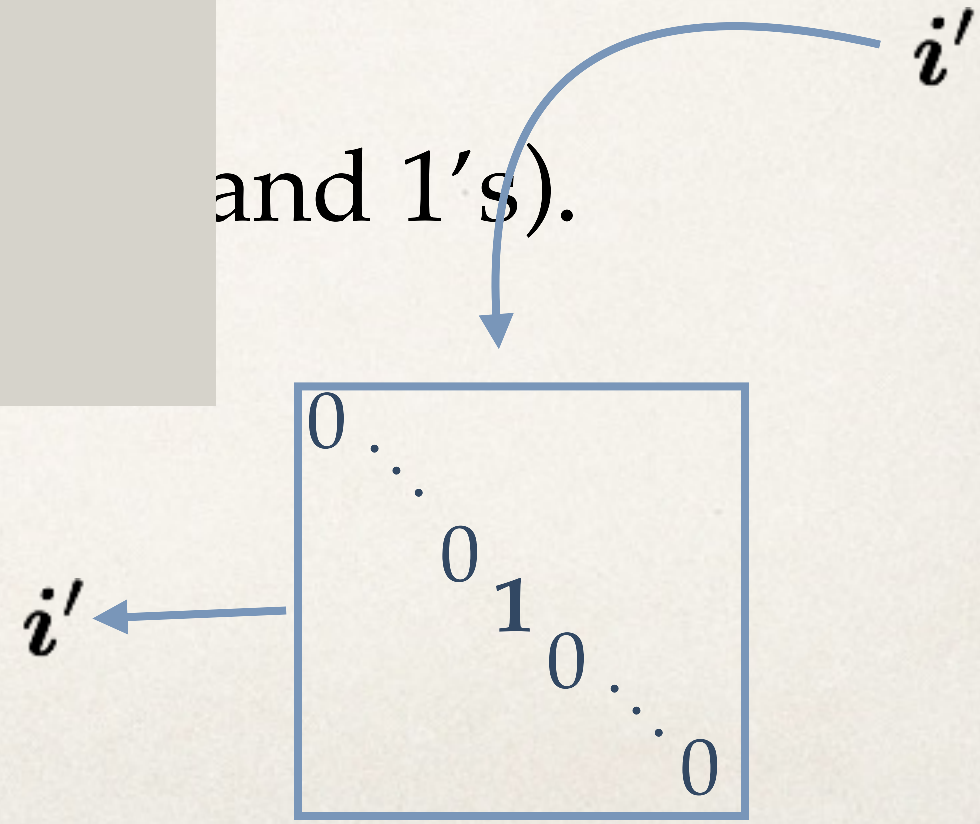
Unique fixed point

Set of deterministic
local operations



$\hat{C})\big) = 1$

and 1′s).



- For every choice of operations: unique fixed point

Unique fixed point

Set of deterministic local operations

- Logical consistency:

$$\forall \hat{A}, \hat{B}, \hat{C} \in \mathcal{D} : \mathrm{Tr}\left(\hat{E}(\hat{A} \otimes \hat{B} \otimes \hat{C})\right) = 1$$

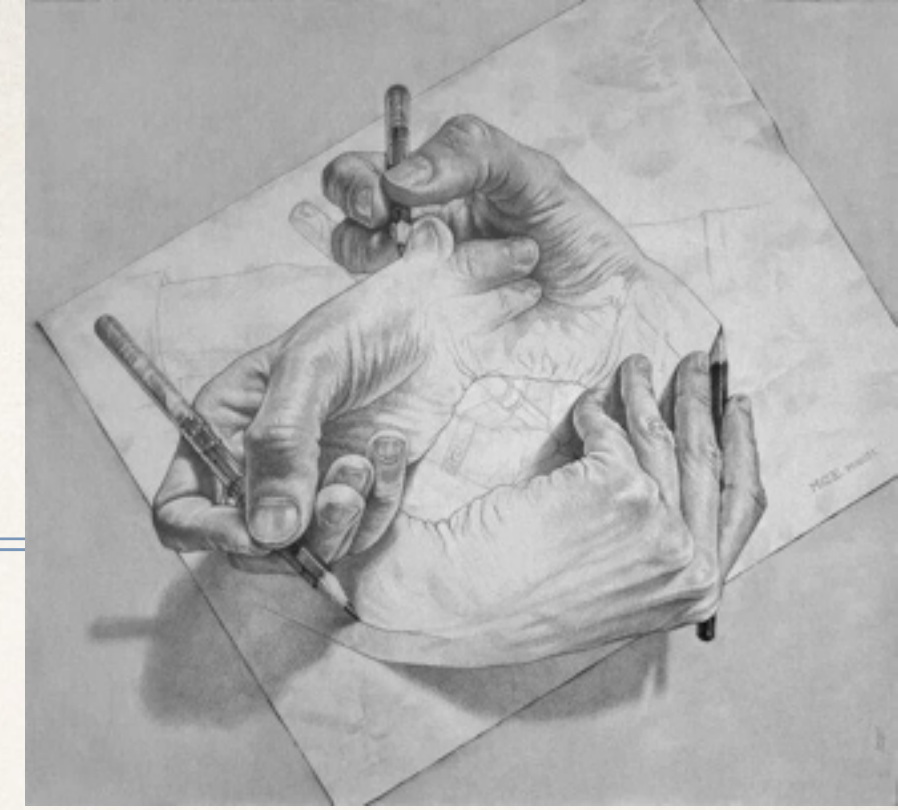
- Assume $\hat{E}$ is not deterministic.

$$\hat{E} = \sum_i p(i)\hat{E}_i \quad \text{with } \hat{E}_i \in \mathcal{D}$$

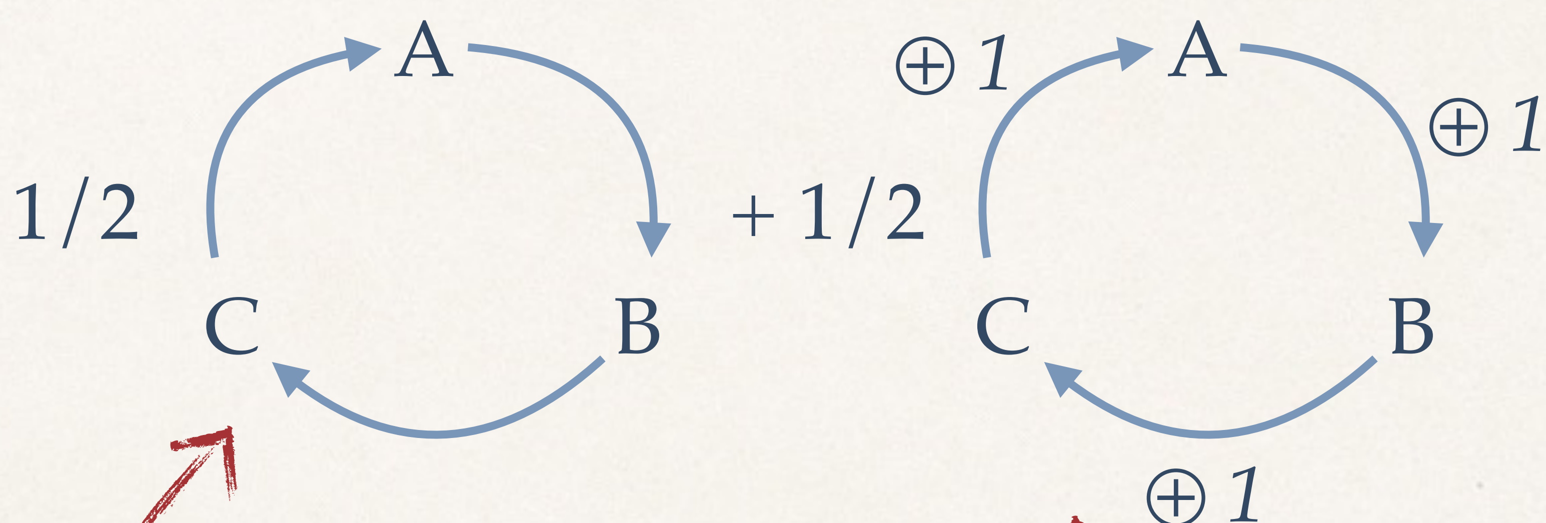
For every choice of operations:

$$\sum_i p(i)(\#\text{fixed points with } \hat{E}_i) = 1$$

Unique fixed point



- Example (binary channels):



Local operation: identity
2 fixed points

Local operation: identity
0 fixed points

Outline

- Motivation
- Classical correlations without causal order
- **Circuit model without causal order** (arXiv:1511.05444)
- Examples
- Conclusion

Model of computation

- Can we use this property to *find* fixed points?

Before:

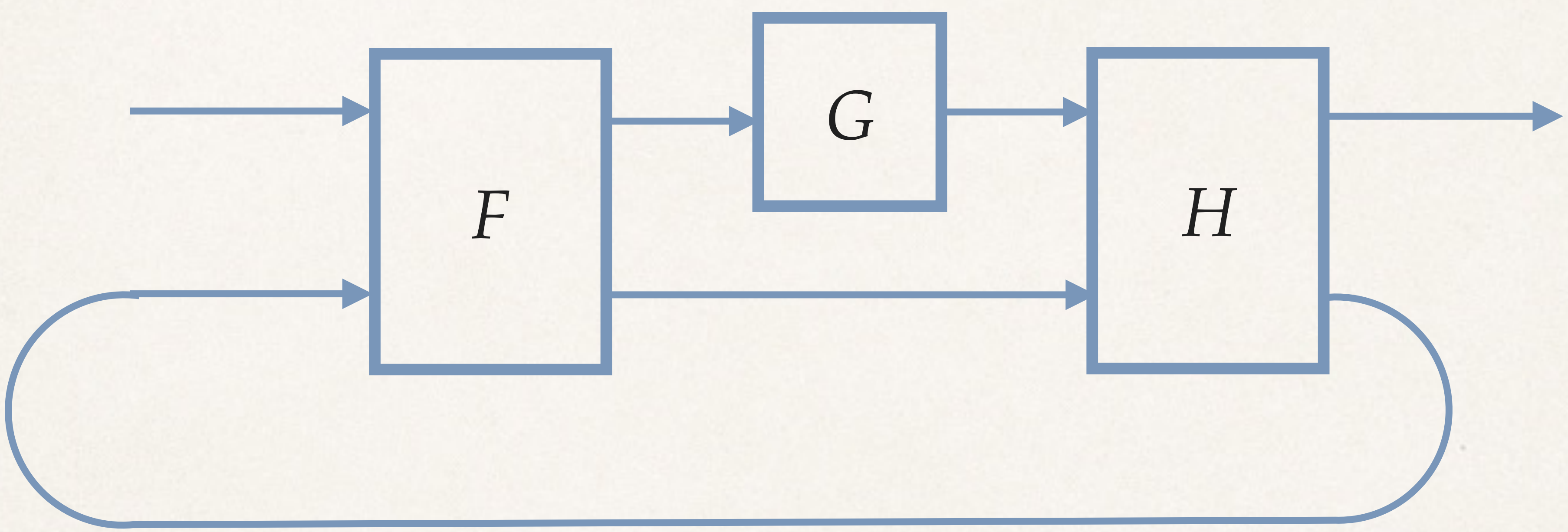
- Parties
- Order not fixed
- Logical consistency: $\forall \hat{A}, \hat{B}, \hat{C}$ unique F.P.

Model of computation:

- Gates
- Arbitrary wiring
- Logical consistency: for every input: loops in circuit have unique F.P.

Model of computation

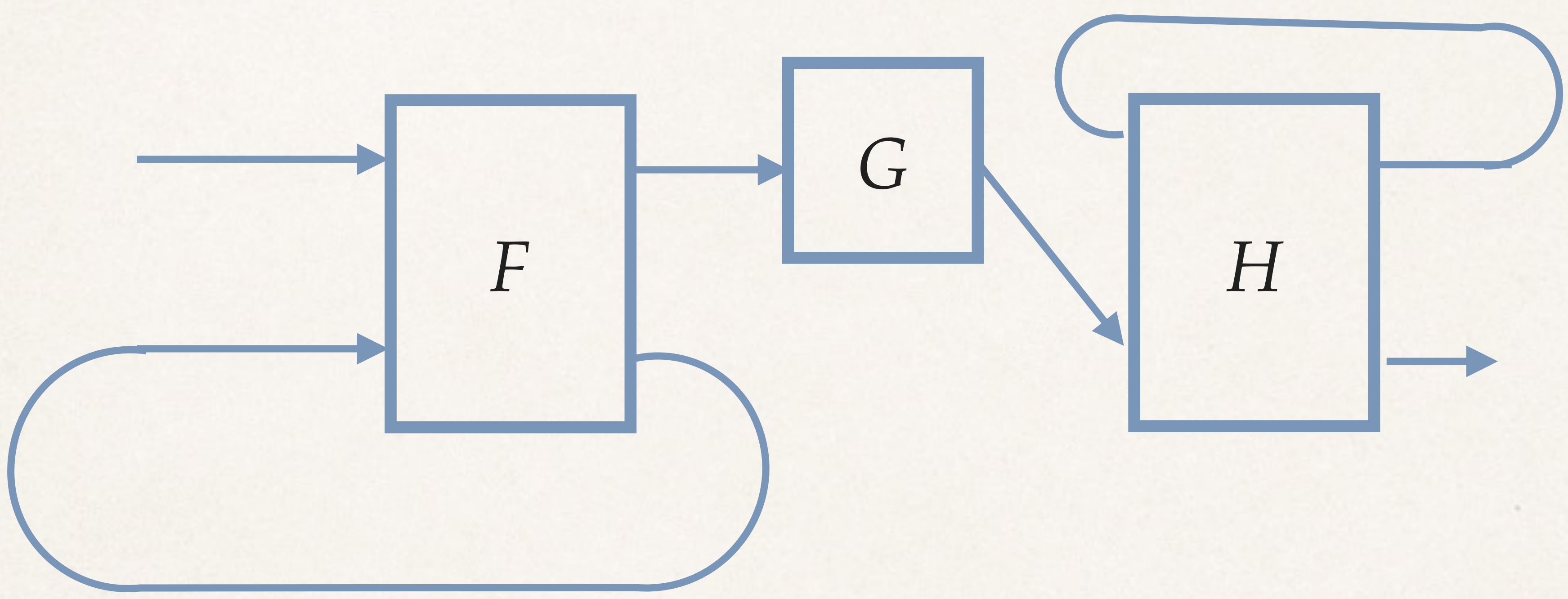
- Arbitrary wiring of gates



- Logical consistency:
  unique fixed point on loops for every input

Model of computation

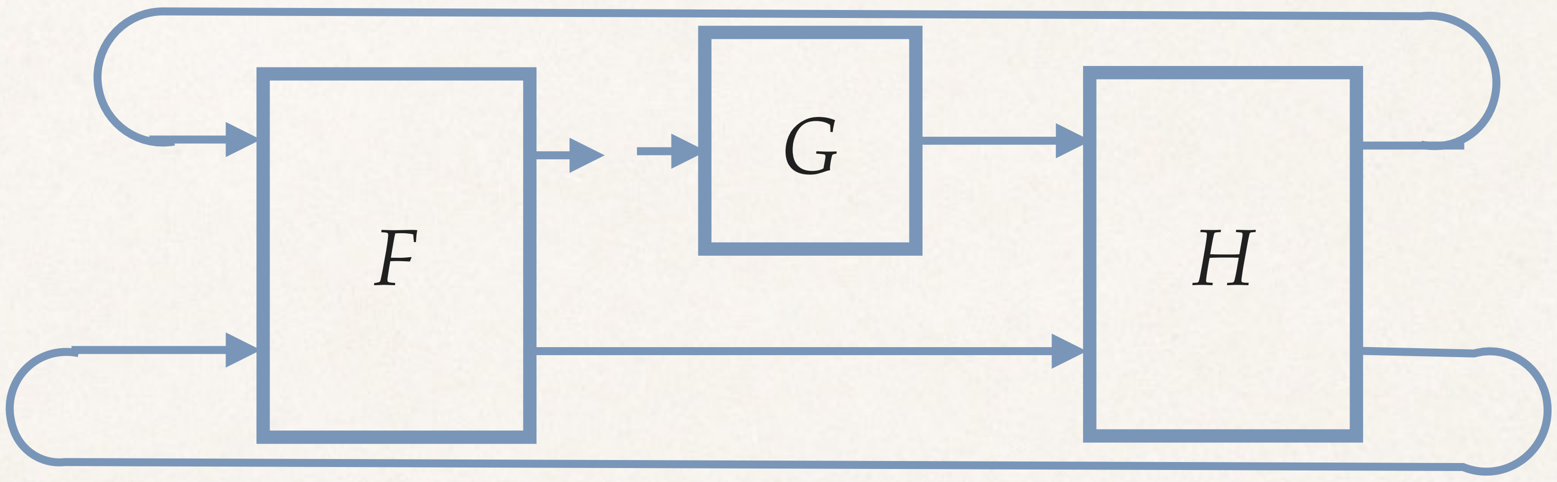
- Arbitrary wiring of gates



- Logical consistency:
  unique fixed point on loops for every input

Model of computation

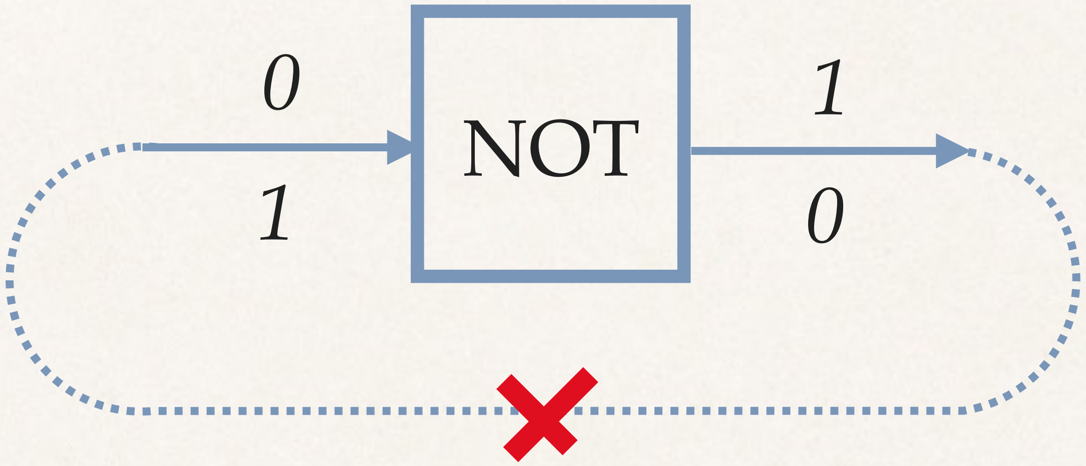
- Arbitrary wiring of gates



- Logical consistency:
  unique fixed point on loops for every input

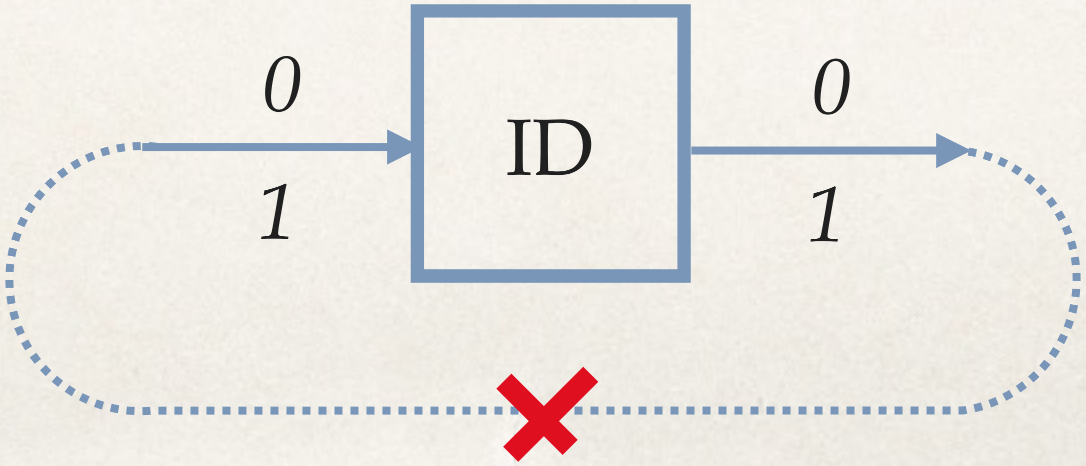
Model of computation

- Not all wirings are logically consistent
  Example: Grandfathers paradox



#fixed-points: 0

Example: Causal paradox



#fixed-points: 2

Outline

- Motivation
- Classical correlations without causal order
- Circuit model without causal order
- **Examples**
- Conclusion

Example: Fixed point search

- Given a black box **B**.
  Promise: **B** has *exactly one* fixed point.

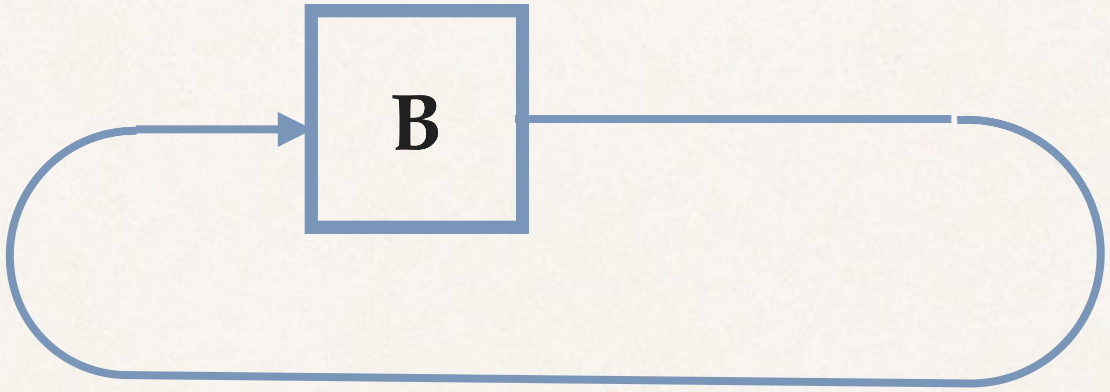
$$\hat{B} = \sum_{i=1}^{N} \vec{e}_i \vec{i}^T, \quad \text{with } |\{\vec{i} | \vec{i} = \vec{e}_i\}| = 1$$

  What is the query complexity to find the fixed point $(\vec{i} = \vec{e}_i)$?

- Worst case: *N-1*

Example: Fixed point search

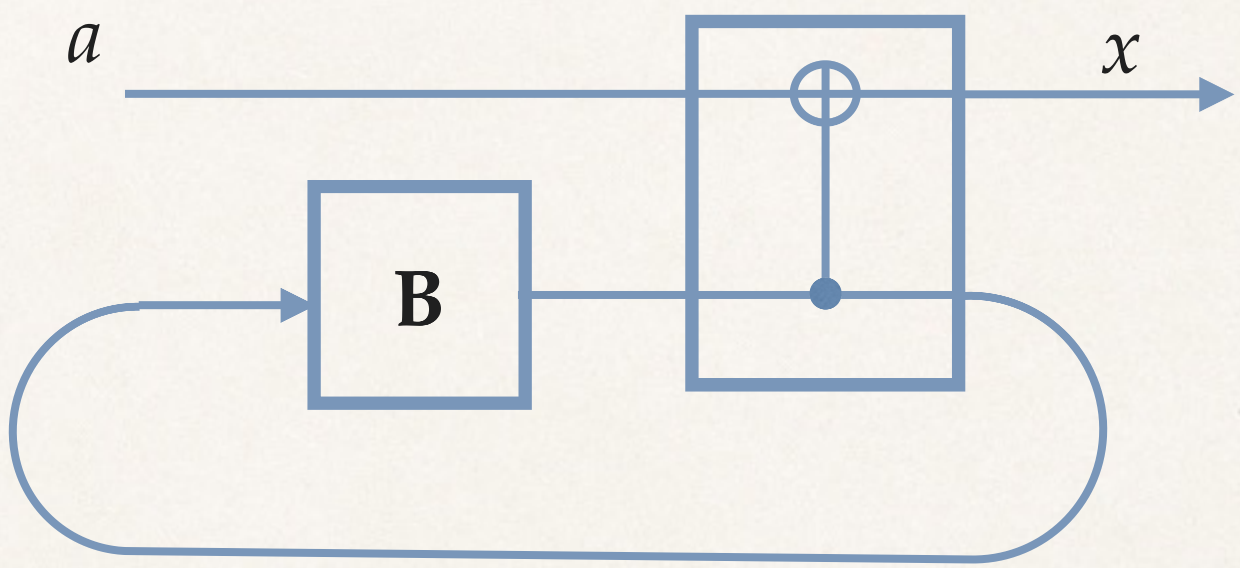
- Circuit without causal order:



- Logically consistent: fixed point is unique

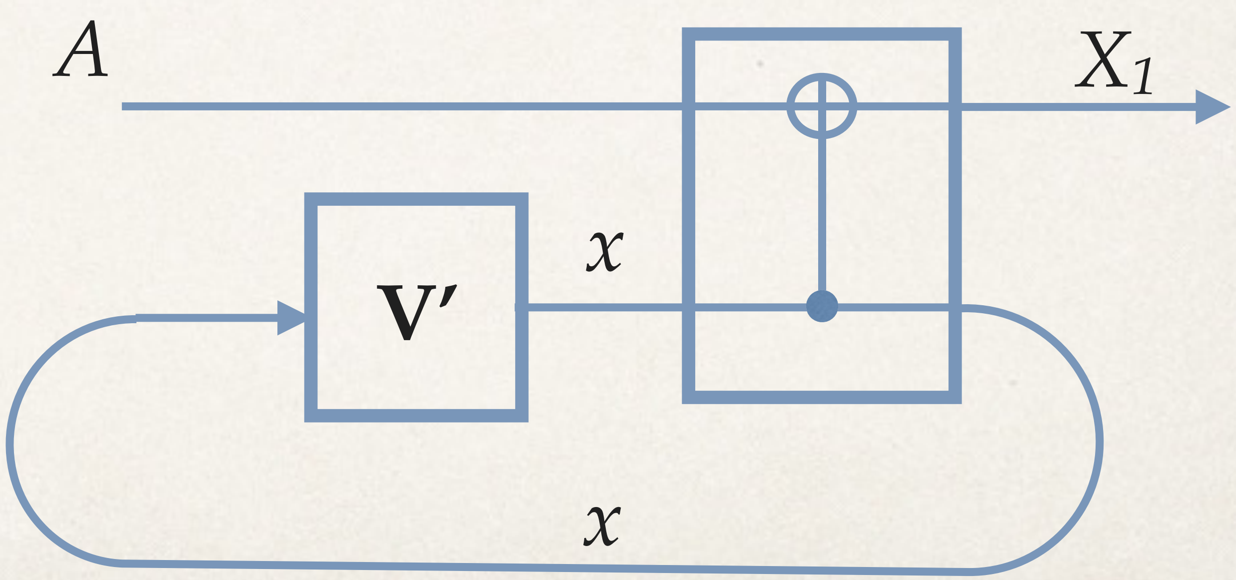
Example: Fixed point search

- Read out fixed point



Example: Finding solution to problem in NP with a unique solution

- Let V be a poly-time verifier for a problem Q and we are guaranteed that Q has a unique solution x.
- We can find x in a polynomial number of steps.
- V checks x,
  if x is the solution,
  then output x
  else output x+1



Conclusion

- Logical consistency is a weaker assumption than global time.
- More general correlations (also classically).
- More powerful circuit model.

Thank you.