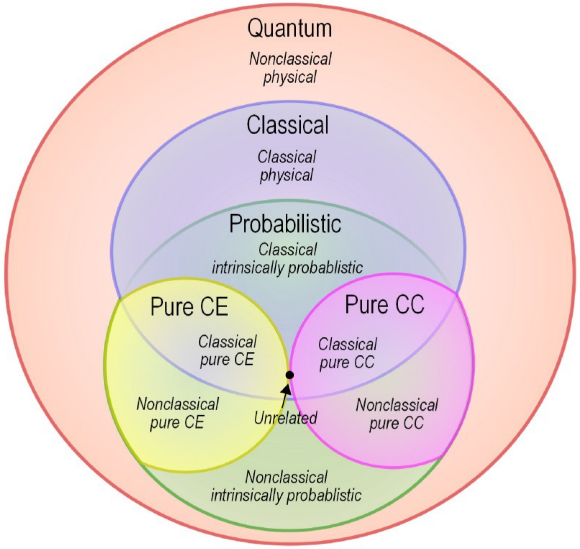
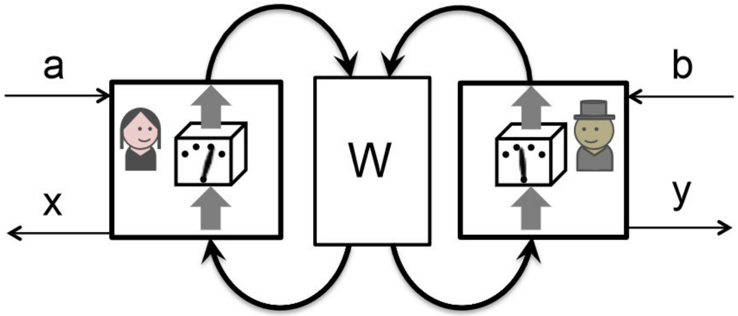




Oreshkov, Costa, Brukner, Nat Comm 3, 1092 (2012)

Non-classical causal structures in theory and experiment

Katja Ried
Perimeter Institute for Theoretical Physics

Tainan, December 10th, 2015
Quantum Non-Locality, Causal Structures and Device-independent Quantum Information

Outline

- Causality in quantum theory
- Quantum causal models
- Non-classical causal structures

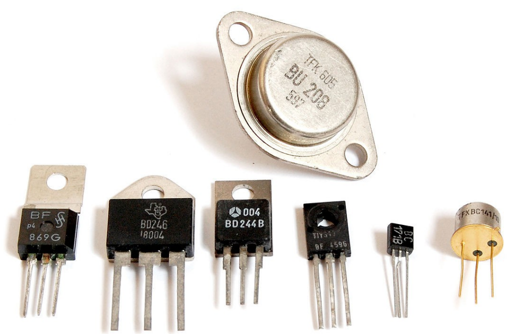
Causality in Quantum Theory

Cavity Control of a Single-Electron Quantum Cyclotron: Measuring the Electron Magnetic Moment

D. Hanneke,[*] S. Fogwell Hoogerheide, and G. Gabrielse[†]
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
(Dated: Submitted to Phys. Rev. A on 3 Sept. 2010)

Measurements with a one-electron quantum cyclotron determine the electron magnetic moment, given by $g/2 = 1.001\,159\,652\,180\,73\,(28)\,[0.28\ \text{ppt}]$, and the fine structure constant, $\alpha^{-1} = 137.035\,999\,084\,(51)\,[0.37\ \text{ppb}]$. Brief announcements of these measurements [1, 2] are supplemented here with a more complete description of the one-electron quantum cyclotron and the new measurement methods, a discussion of the cavity control of the radiation field, a summary of the analysis of the measurements, and a fuller discussion of the uncertainties.

$$\imath\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2 + V(\vec{r},t)\right]\Psi(\vec{r},t)$$



$$|\psi\rangle \in \mathcal{H}_d(\mathbb{C})$$

$$\rho(k) = \frac{E_k \rho E_k^\dagger}{Tr\left[\rho E_k^\dagger E_k\right]}$$

$$\mathcal{E}_{A\to B} \in \mathcal{CP} : \mathcal{L}(\mathcal{H}_A) \to \mathcal{L}(\mathcal{H}_B)$$

Case study: special relativity

Mathematical framework

$$\begin{bmatrix} c\,t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta n_x & -\gamma\beta n_y & -\gamma\beta n_z \\ -\gamma\beta n_x & 1+(\gamma-1)n_x^2 & (\gamma-1)n_x n_y & (\gamma-1)n_x n_z \\ -\gamma\beta n_y & (\gamma-1)n_y n_x & 1+(\gamma-1)n_y^2 & (\gamma-1)n_y n_z \\ -\gamma\beta n_z & (\gamma-1)n_z n_x & (\gamma-1)n_z n_y & 1+(\gamma-1)n_z^2 \end{bmatrix} \begin{bmatrix} c\,t \\ x \\ y \\ z \end{bmatrix}$$

Voigt, FitzGerald, Larmor, Lorentz, Poincaré and others, 1880s and 1890s

Physical principles

1. The laws according to which the nature of physical systems alter are independent of the manner in which these changes are referred to two co-ordinate systems which have a uniform translatory motion relative to each other.

2. Every ray of light moves in the "stationary co-ordinate system" with the same velocity c, the velocity being independent of the condition whether this ray of light is emitted by a body at rest or in motion.

Poincaré, Einstein, 1905 [Einstein, Ann Phys 322 (1905), translated by Megnadh Saha]

Physics of information

$$C\left[|0\rangle \otimes |0\rangle\right] = |0\rangle \otimes |0\rangle$$
$$C\left[|1\rangle \otimes |0\rangle\right] = |1\rangle \otimes |1\rangle$$
$$\Rightarrow C\left[(|0\rangle + |1\rangle) \otimes |0\rangle\right]$$
$$\neq (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)$$

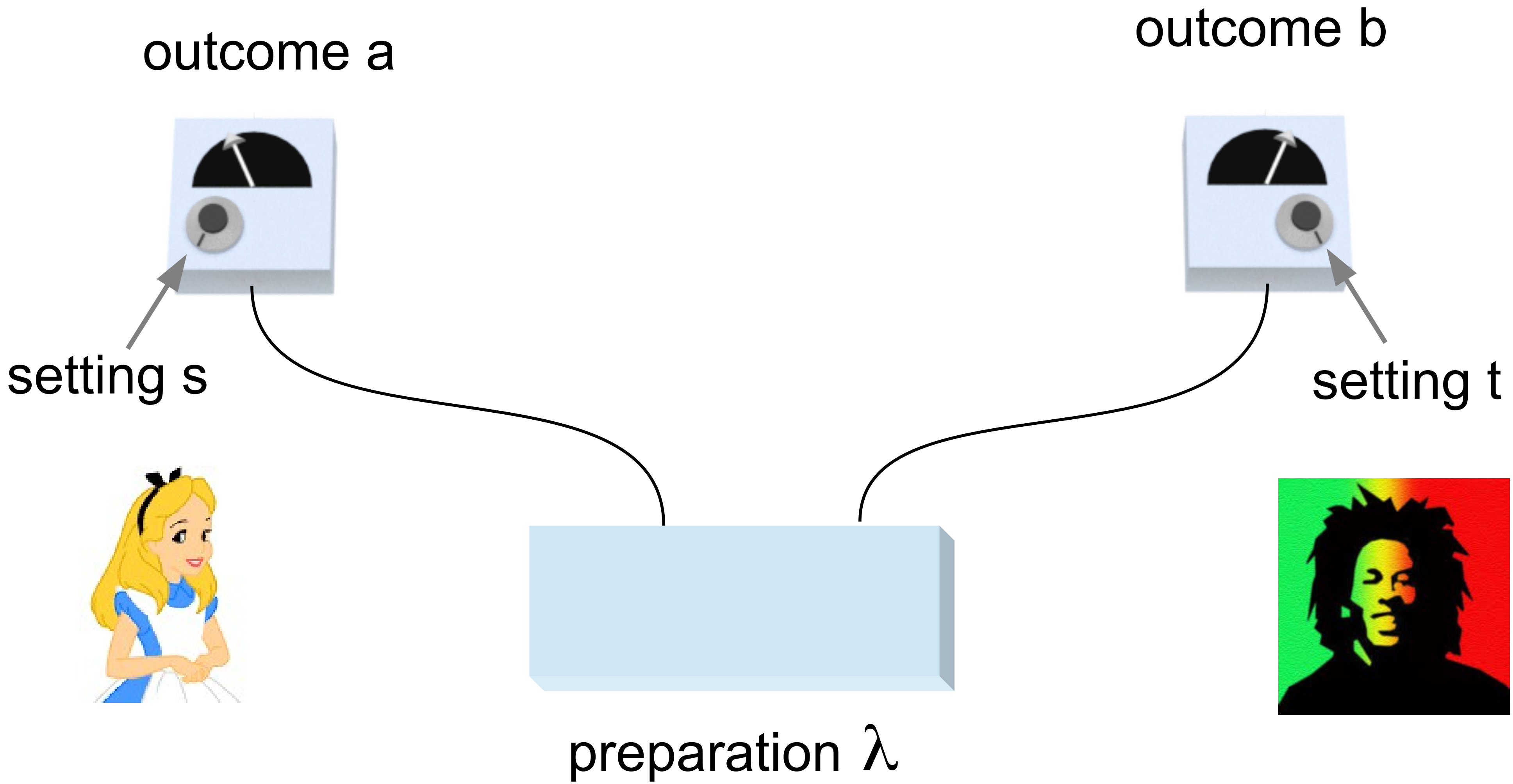
Wootters & Zurek, Nature 299, 802 (1982)

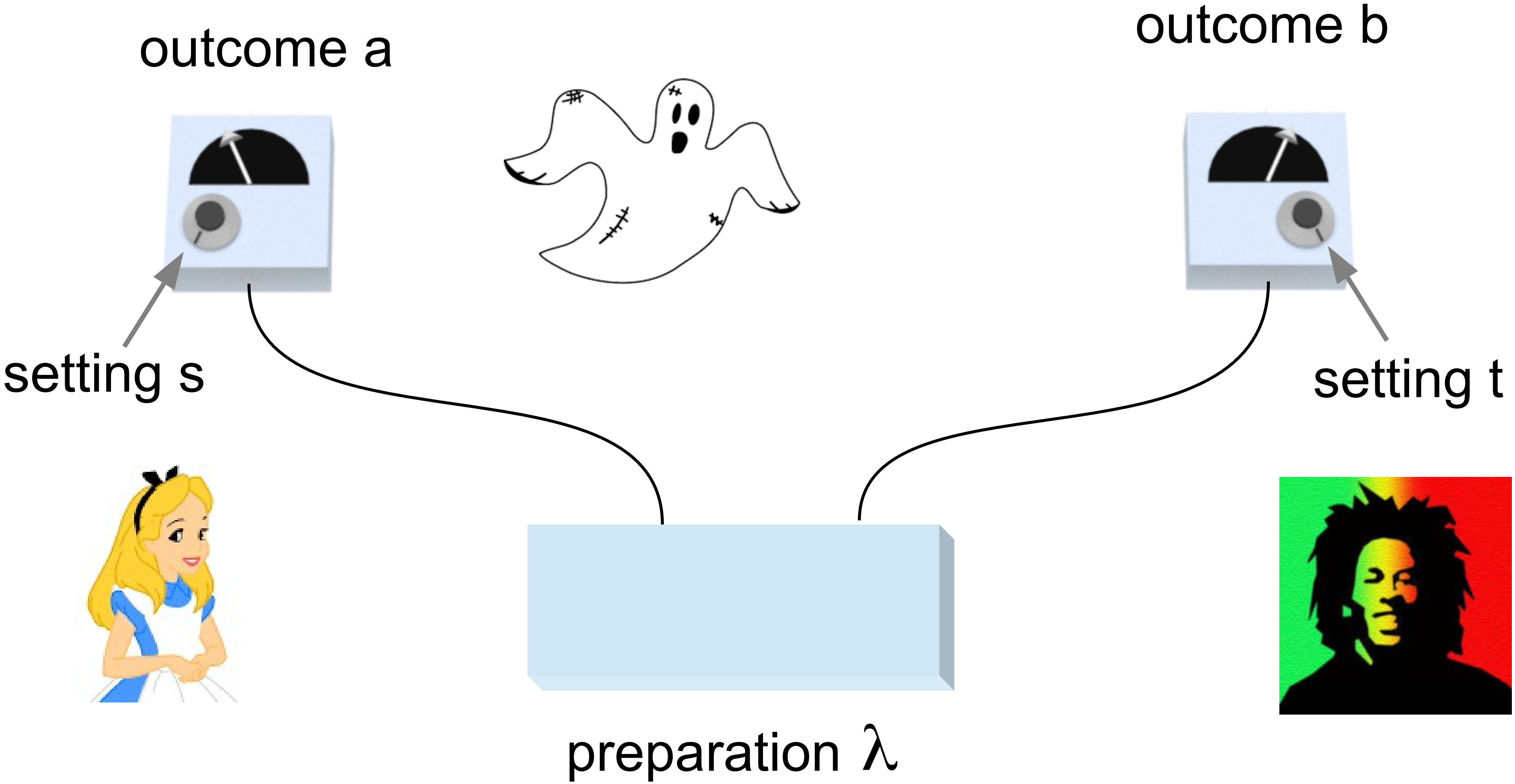
$$\Delta X \cdot \Delta P \geq \frac{\hbar}{2}$$

Heisenberg

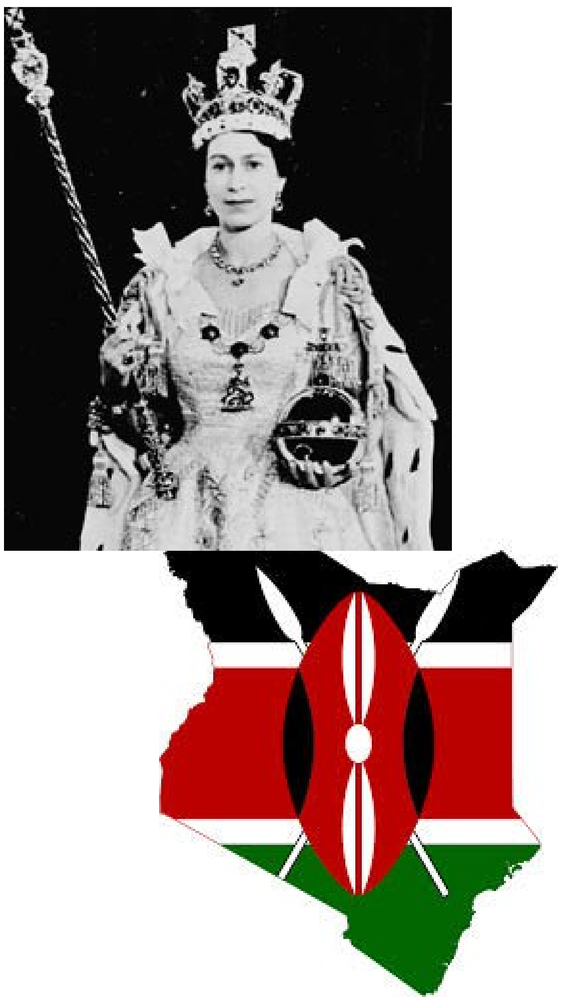
"But our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature, in part incomplete human information about Nature – all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble."

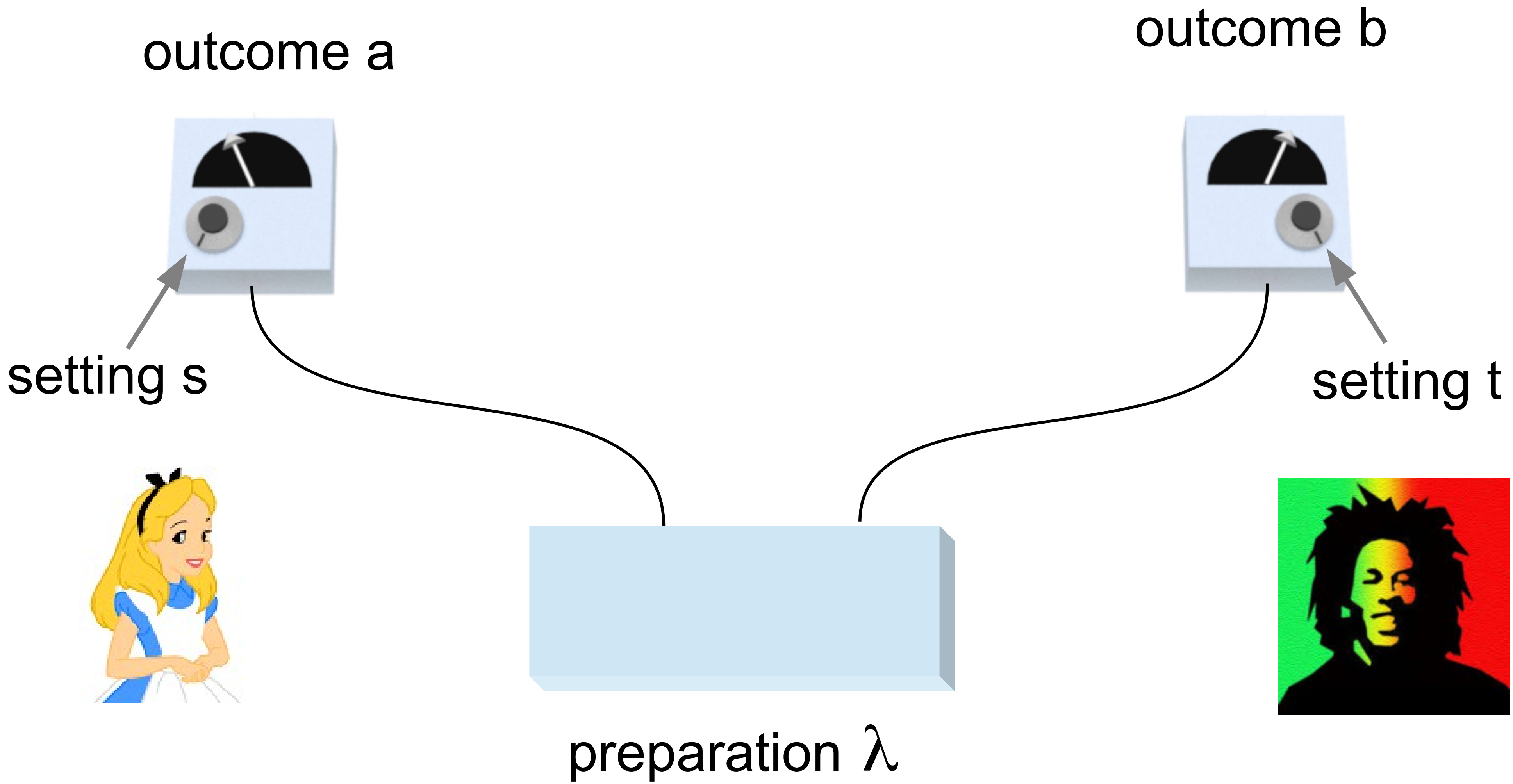
E.T. Jaynes, "Probability in quantum theory"


outcome a
outcome b
setting s
setting t
preparation λ

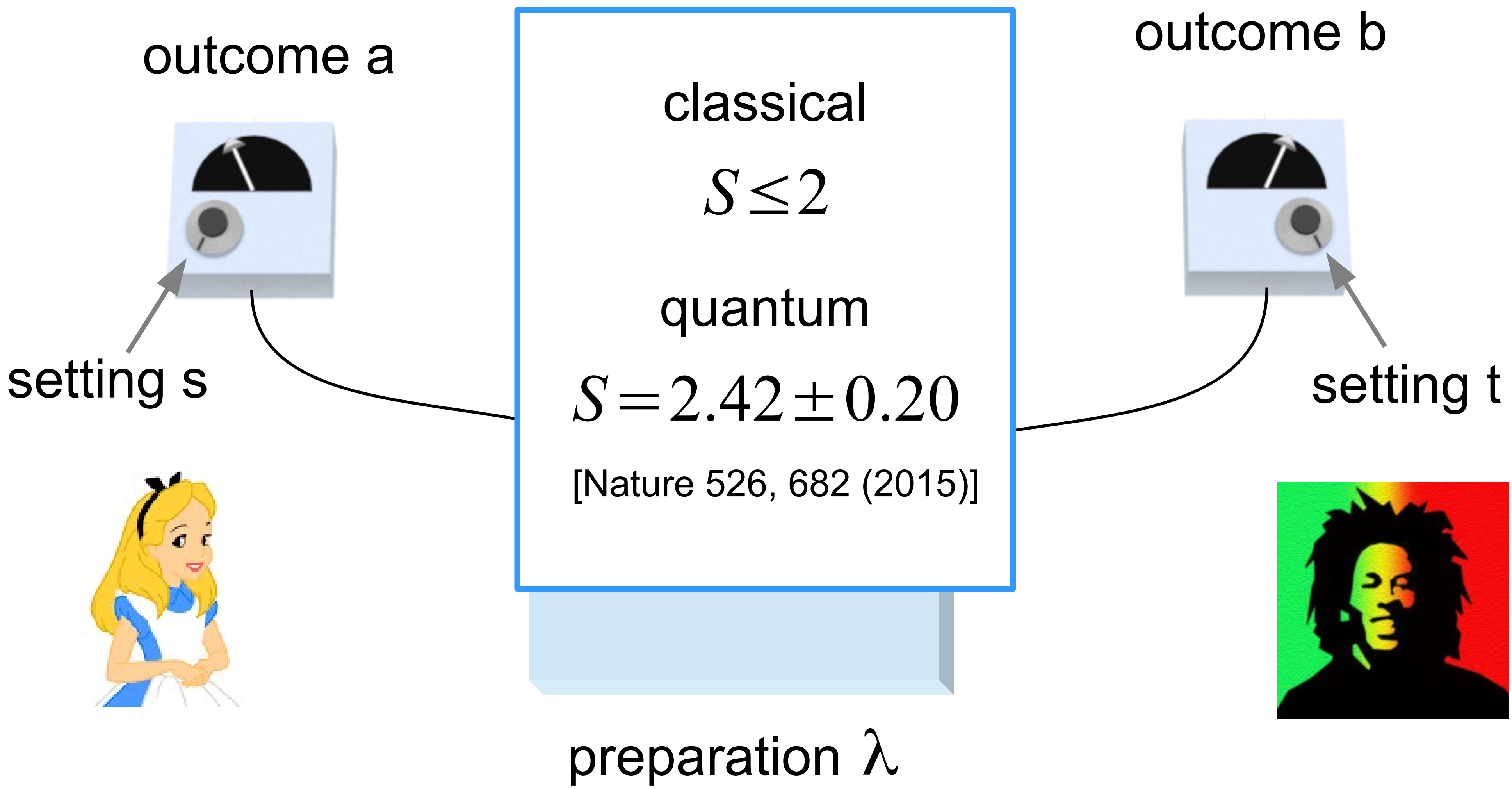

outcome a
outcome b
setting s
setting t
preparation λ

Influence vs inference





$$P(ab|st) = \Sigma_\lambda P(a|s\,\lambda)\, P(b|t\,\lambda)\, P(\lambda)$$



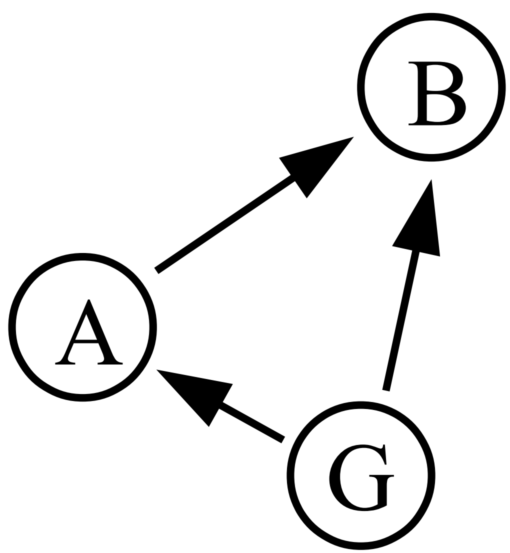
$$P(ab|st) = \Sigma_\lambda P(a|s\lambda) P(b|t\lambda) P(\lambda)$$

Quantum Causal Models

Classical causal models

Structure

pattern of influences



directed acyclic graph

+

Parameters

describe influences

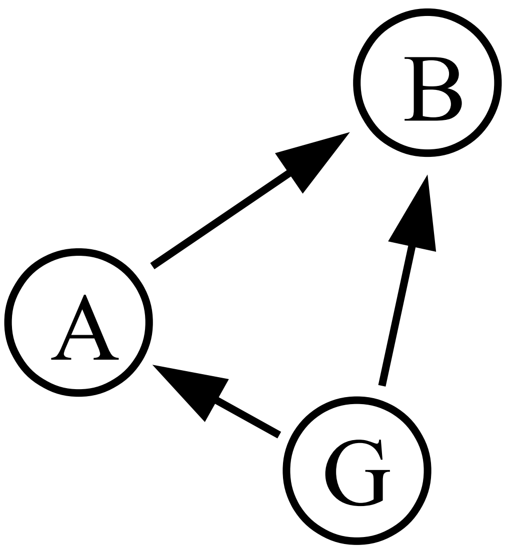
$$P(B|AG)$$
$$P(A|G)$$
$$P(G)$$

conditional probabilities

Classical causal models

Structure

pattern of influences



directed acyclic graph

+

Parameters

describe influences

$$P(B|AG)$$
$$P(A|G)$$
$$P(G)$$

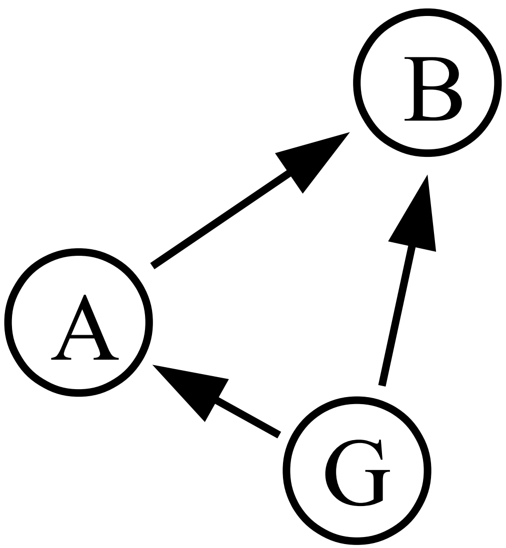
conditional probabilities

autonomous mechanisms

Classical causal models

Structure

pattern of influences



directed acyclic graph

+

Parameters

describe influences

$$P(B|AG)$$
$$P(A|G)$$
$$P(G)$$

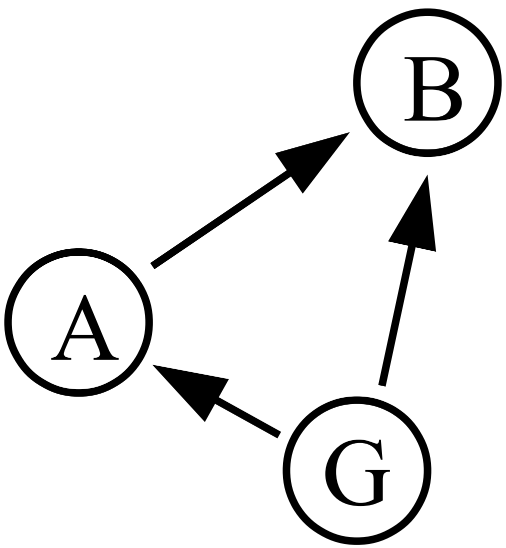
conditional probabilities

captures how information flows

Classical causal models

Structure

pattern of influences



directed acyclic graph

+

Parameters

describe influences

$$P(B|AG)$$
$$P(A|G)$$
$$P(G)$$

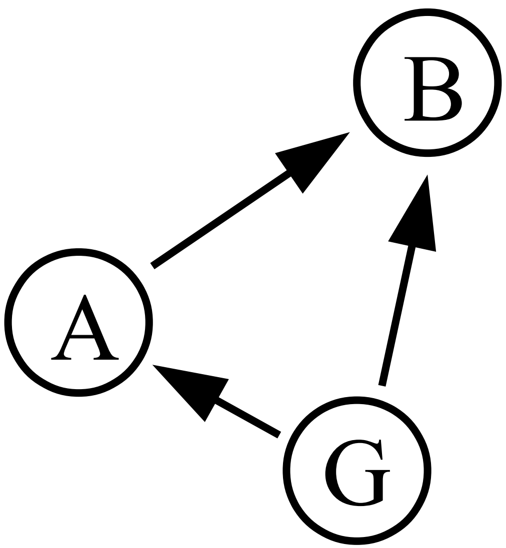
conditional probabilities

contrasts influence with inference

Quantum causal models 1.0

Structure

pattern of influences



directed acyclic graph

+

Parameters

describe influences

$$\rho\left(B|AG\right)$$
$$\rho\left(A|G\right)$$
$$\rho\left(G\right)$$

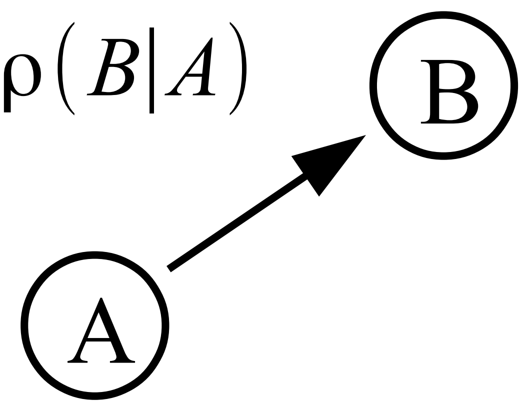
quantum states and maps

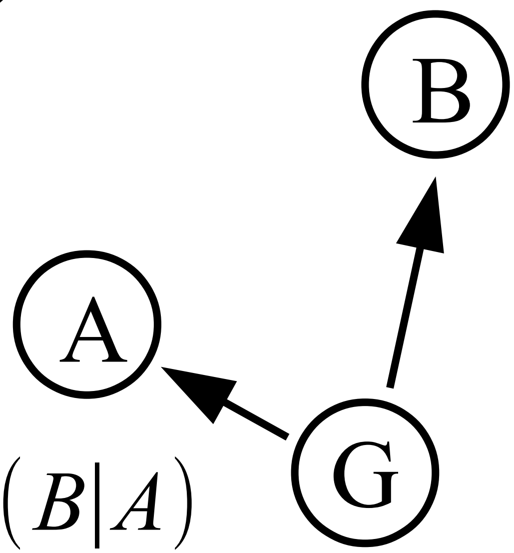
M Leifer and RW Spekkens, “Formulating quantum theory as a causally neutral theory of Bayesian inference”, arXiv:1107.5849

Quantum conditionals

quantum channels:
coherent cause-effect relations

$\rho\left(B|A\right)$

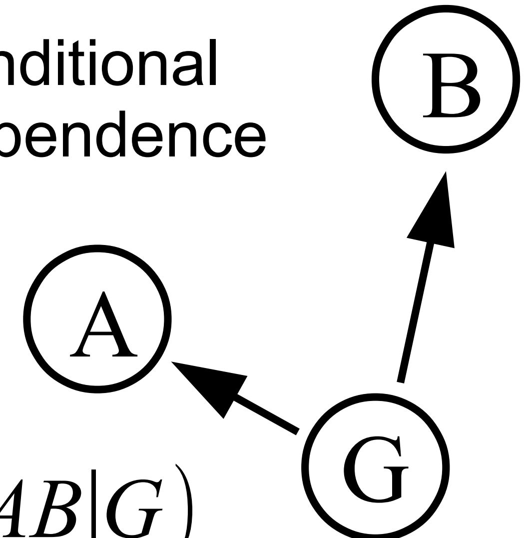




$\rho\left(B|A\right)$

influence vs inference

conditional independence



$$P\left(AB|G\right) = P\left(A|G\right)P\left(B|G\right)$$

structure and conditionals not independent

Ried et al, "A quantum advantage for inferring causal structure", Nat Phys 11, 414 (2015)

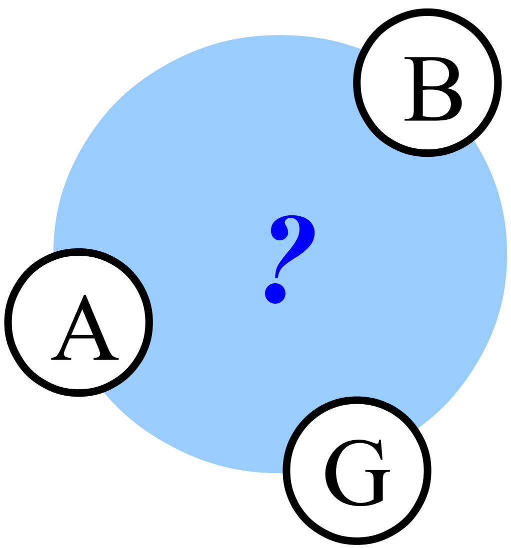
Non-Classical Causal Structures

joint work with J.-P. MacLean, R. W. Spekkens and K. J. Resch

Quantum causal models 2.0

Structure

pattern of influences



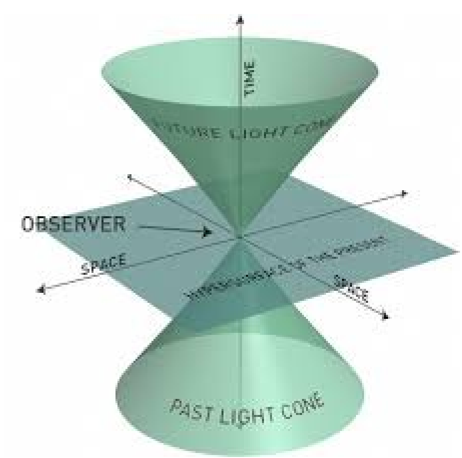
+

Parameters

describe influences

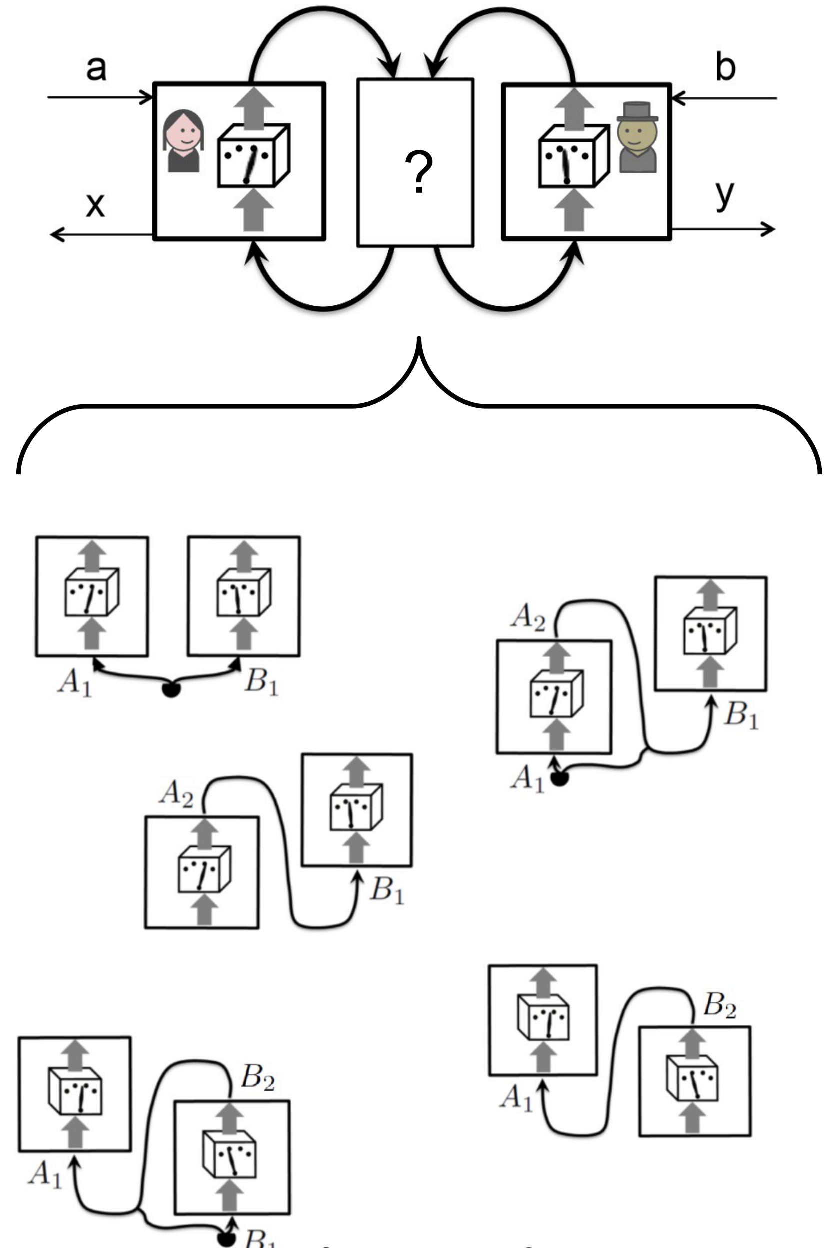
...

Quantum combinations of causal structures



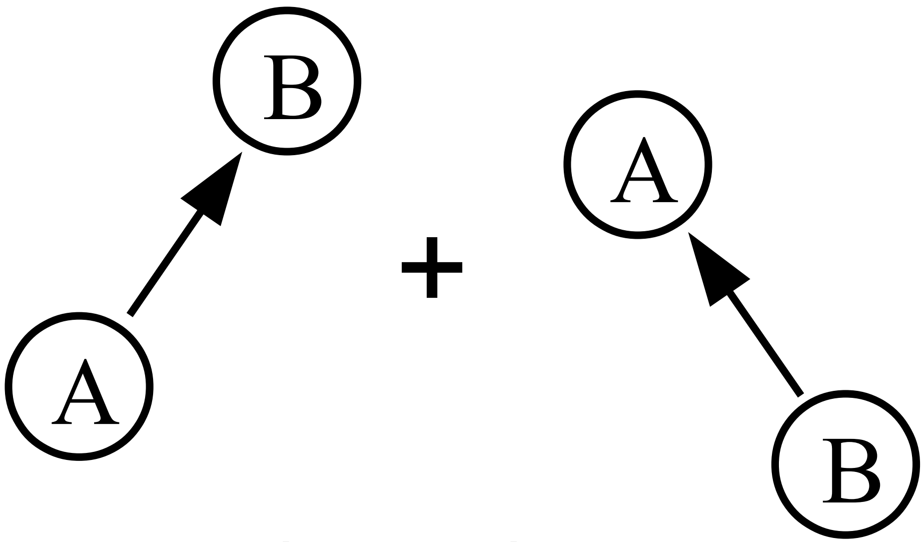
$\Rightarrow g_{\mu\nu}$

Hawking, Robb, Malamet, Sorkin, ...

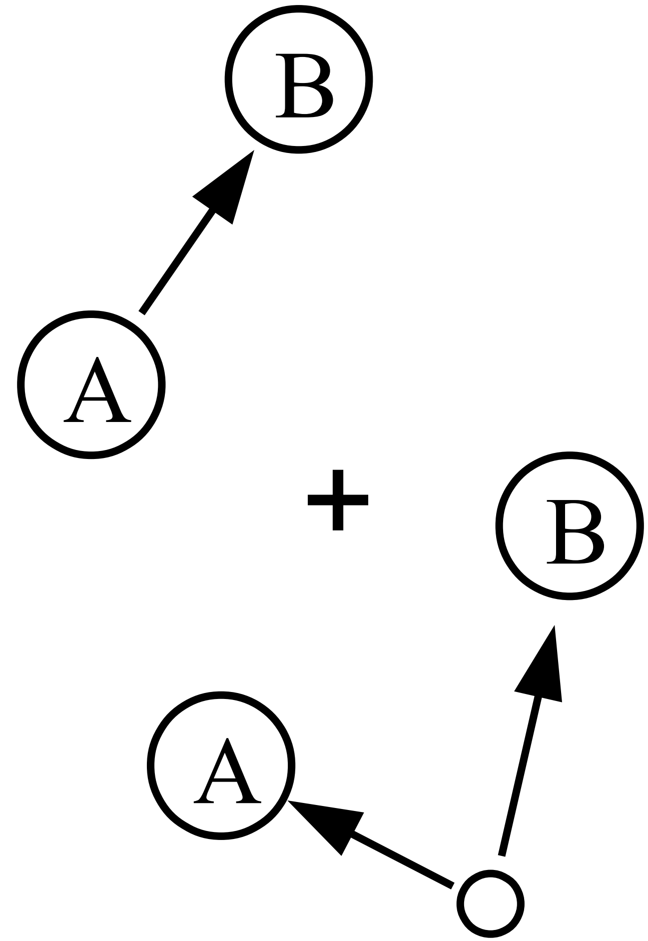


Oreshkov, Costa, Brukner, Nat Comm 3, 1092 (2012)

Quantum combinations of causal structures



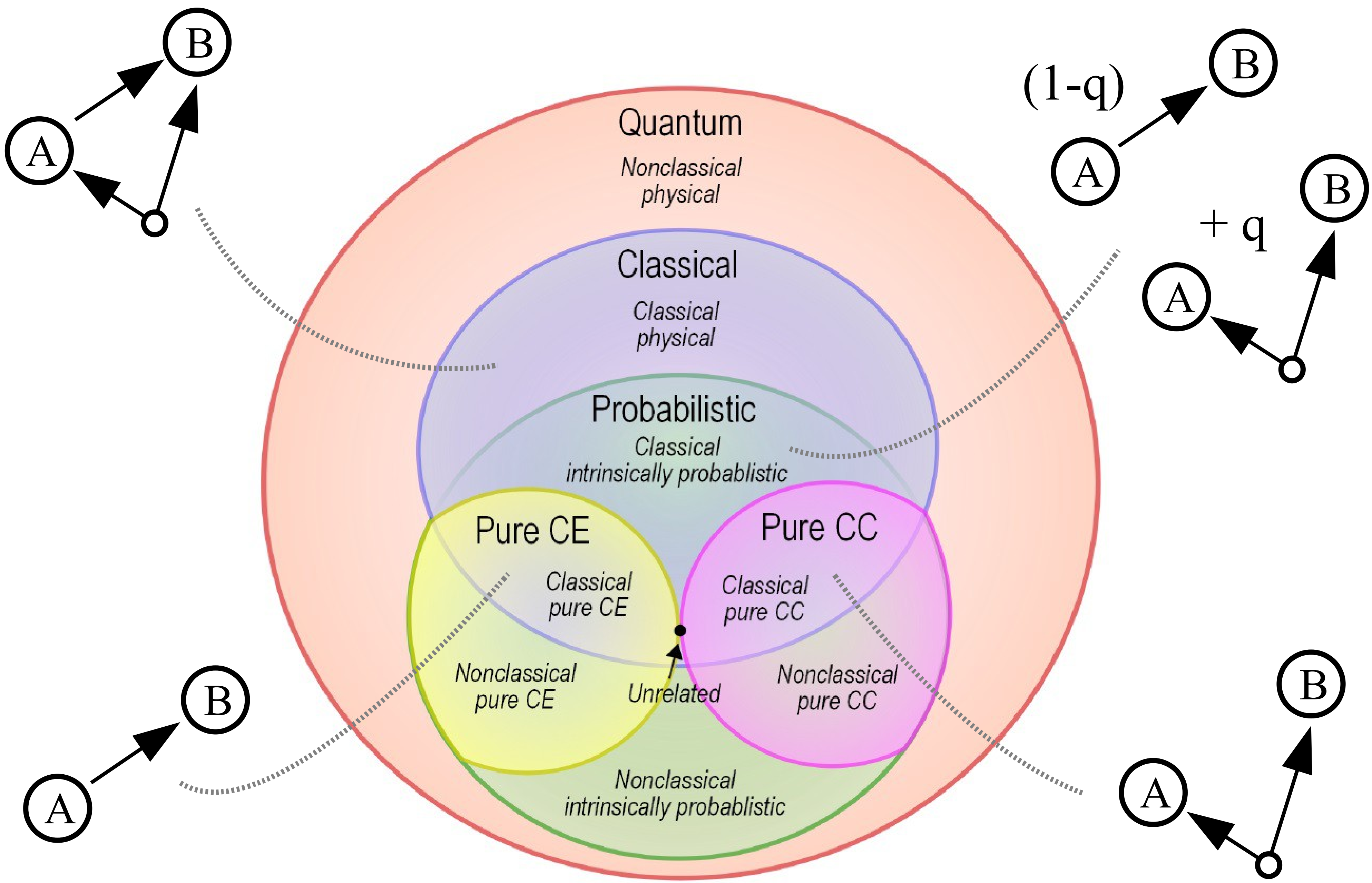
interesting
but exotic



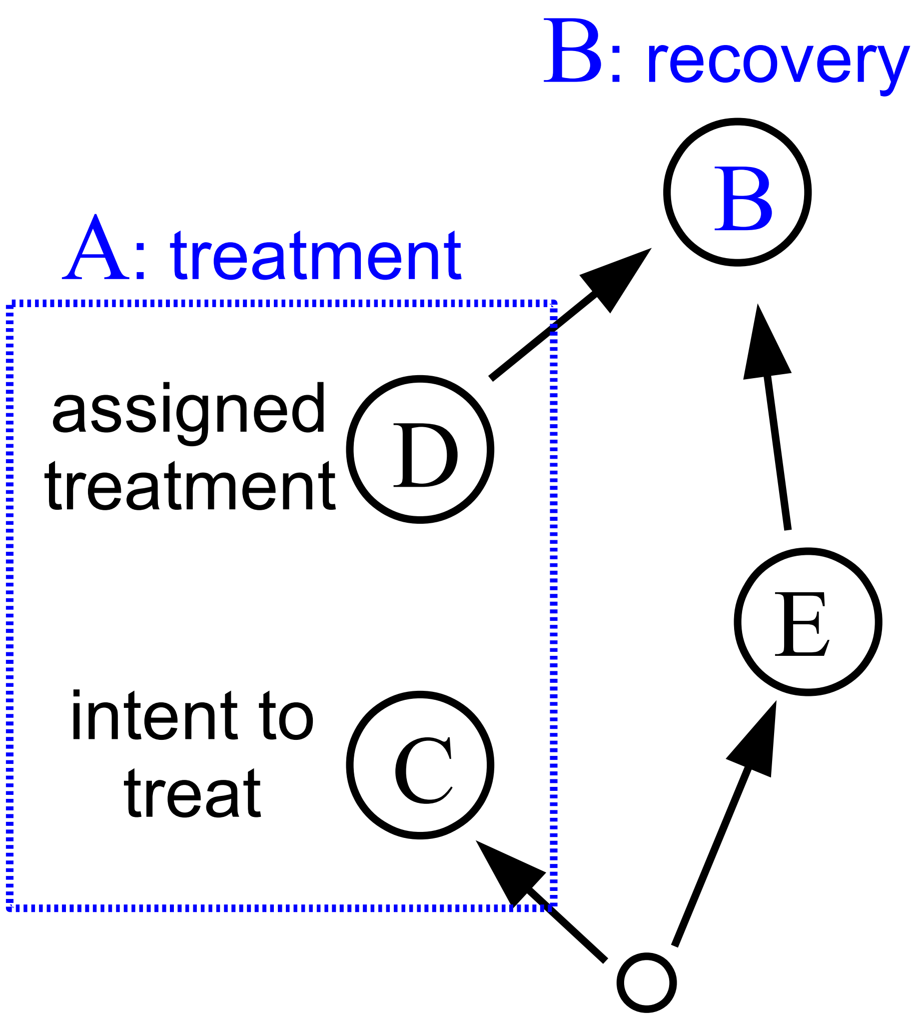
tractable prototype

Hardy, arXiv:quant-ph/0701019 (2007)
Chiribella, PR A 86, 040301 (2012)
Araújo, Costa, Brukner, PRL 113, 250402 (2014)

Combining cause-effect and common-cause relations



What determines combinations of cause-effect and common-cause?



Depends on whether B responds to...

$$P(B|DE) = P(B|D)$$

cause-effect

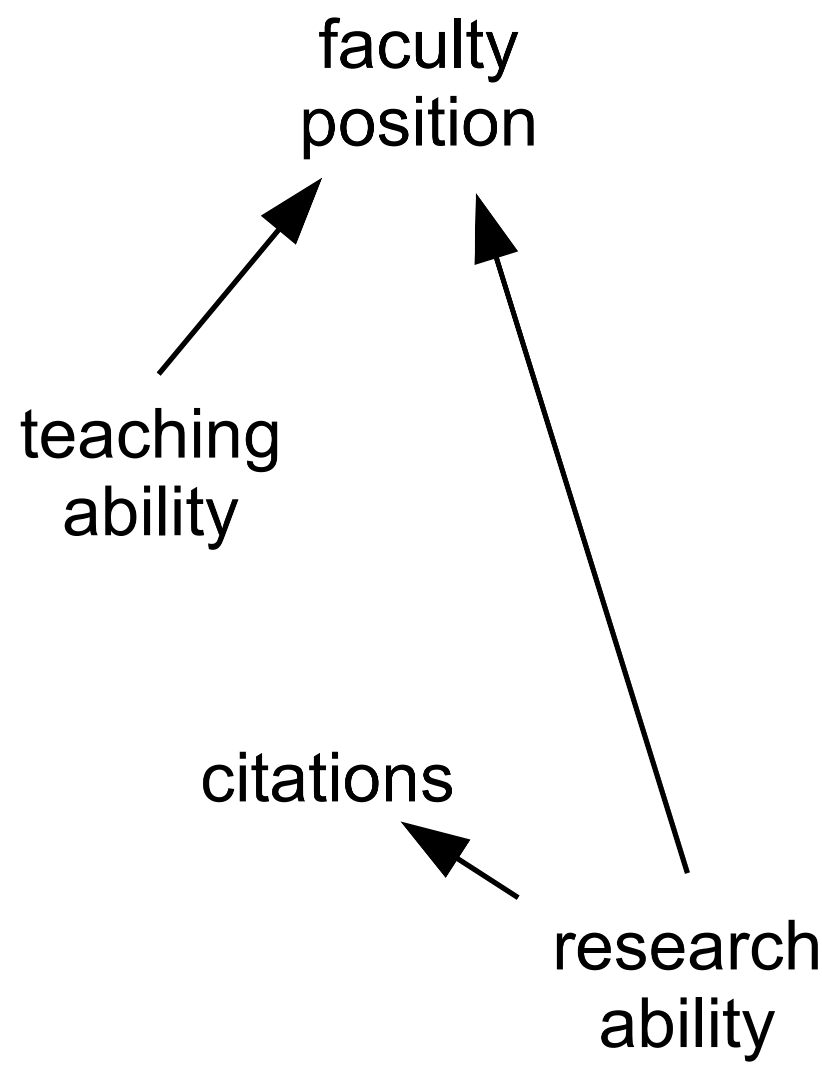
$$P(B|DE) = P(B|E)$$

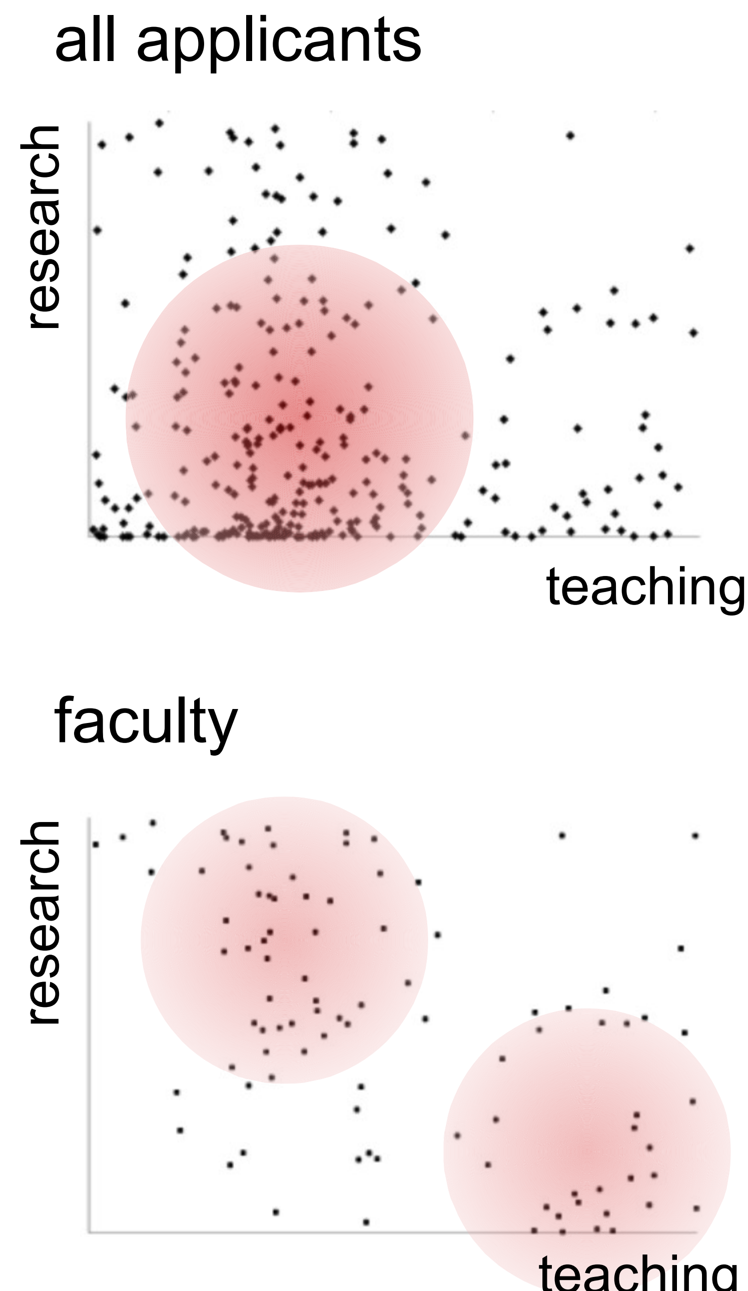
common-cause

$$P(B|DE) = q\,P(B|E) + (1-q)\,P(B|D)$$

probabilistic

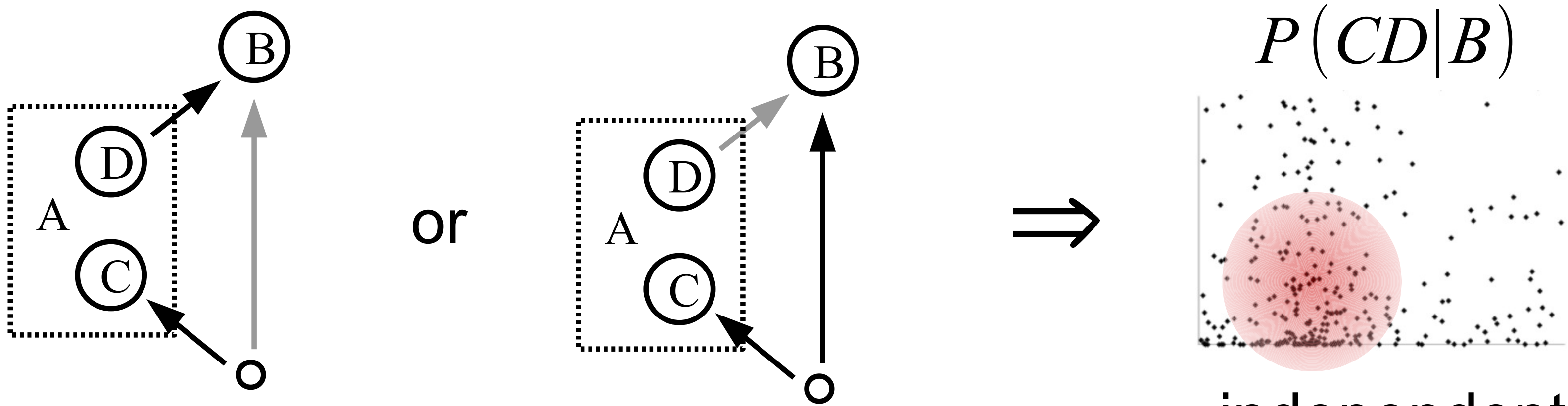
Berkson's paradox





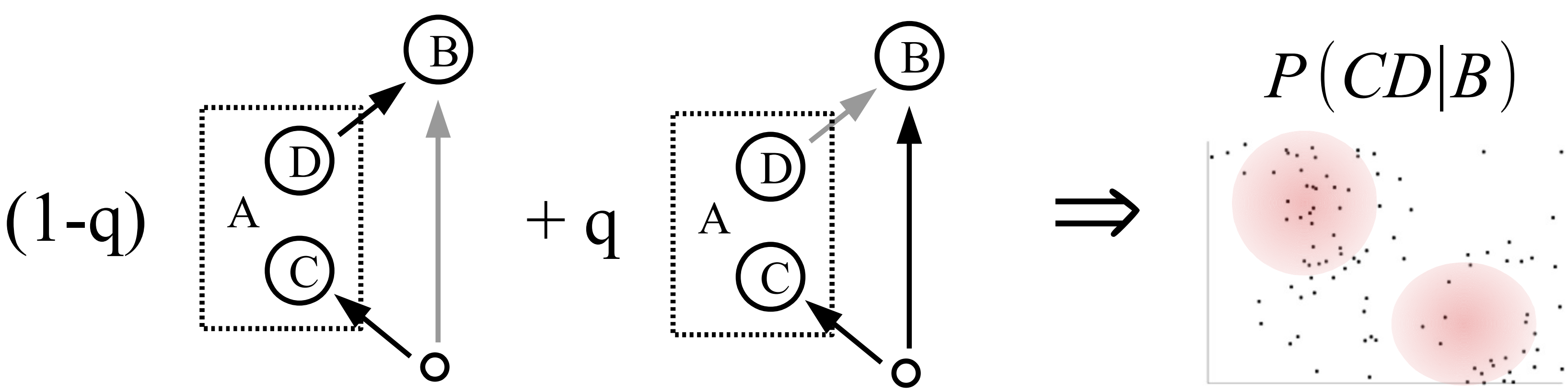
Quantifying combinations of causal structures

purely cause-effect
or purely common-cause:



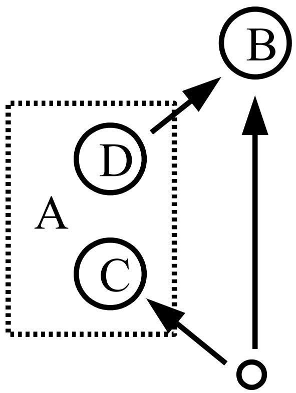
independent

probabilistic mixture:



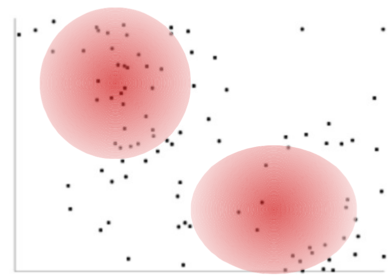
weak correlations

physical mixture (not probabilistic):



$\Rightarrow$

$$P\left(CD|B\right)$$



strong correlations

for example

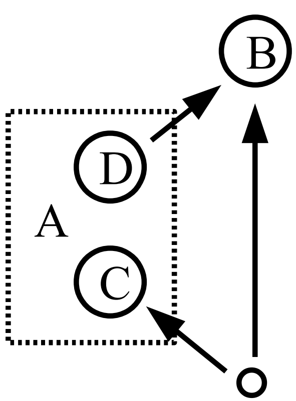
$$B = D \oplus E$$
$$C = E$$

then

$$B = 0$$
$$\Rightarrow C = D$$

perfect correlation

intrinsically quantum combination:



$\Rightarrow$

$$\rho\left(CD|B\right) \neq \Sigma_b \rho_C^{(b)} \otimes \rho_D^{(b)}$$

stronger-than-classical
correlations

Indicators

To rule out classical mixtures:

For any classical mixture, the induced state ρ_{CD} when we post-select on B is separable, ie its negativity is zero:

$$N \equiv \frac{Tr\left|T_D \rho_{CD}\right| - 1}{2} = 0$$

Conversely,

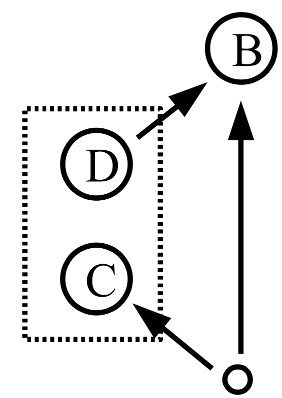
$N \neq 0 \quad \Rightarrow \quad$ intrinsically quantum mixture

Indicators

To rule out probabilistic mixtures:

For any probabilistic mixture

$$\Phi_{CB|D} = q\,\rho_{CB} \otimes Tr_D + (1-q)\tfrac{1}{2}\mathbf{1}_C^{*} \otimes \Phi_{B|D}$$



note that

$$W_0 \equiv \langle \sigma_x^C \otimes \sigma_z^B \otimes \sigma_y^D \rangle \quad \Rightarrow \quad W_0 = 0$$

Conversely

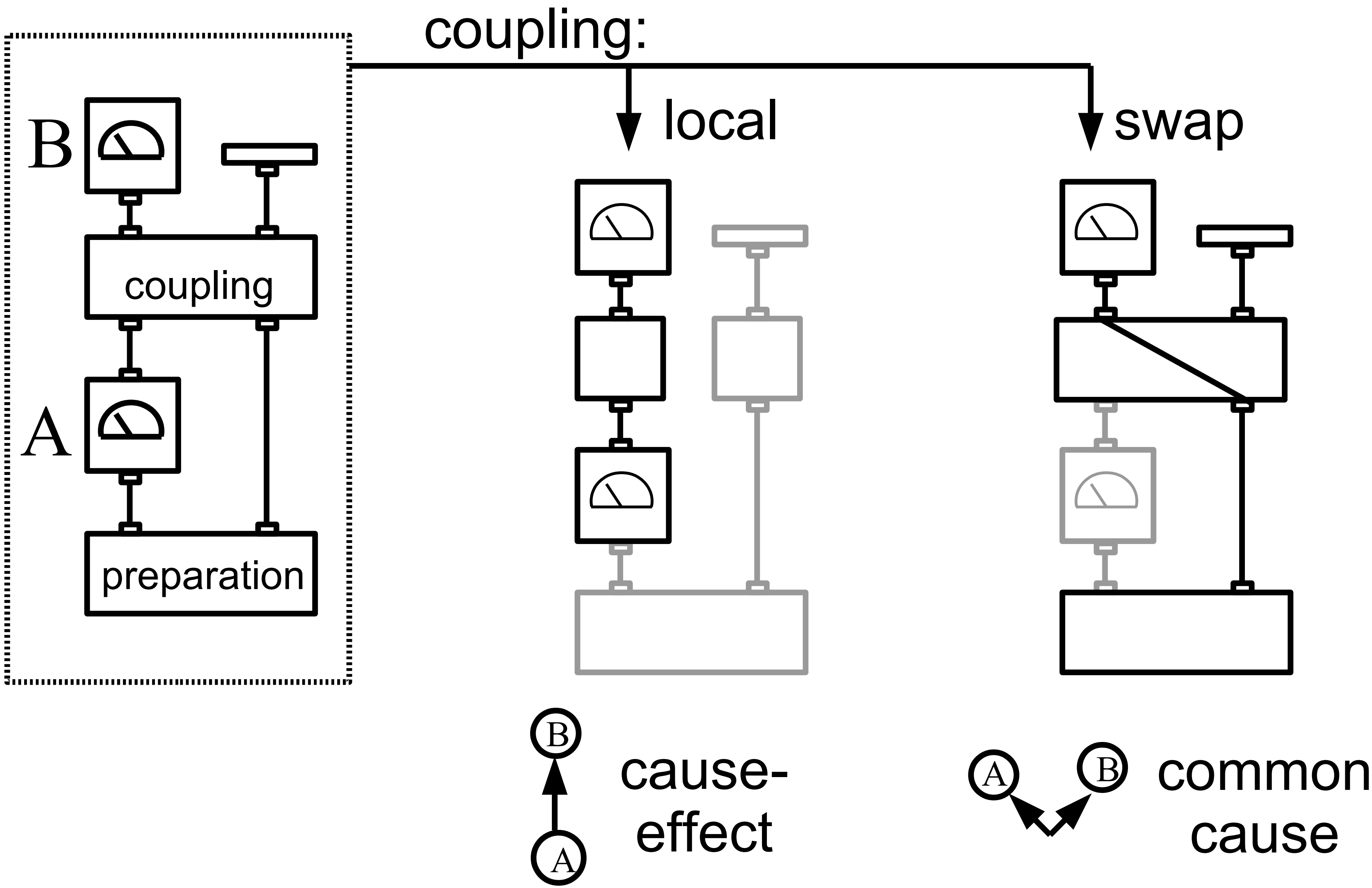
$W \neq 0 \quad \Rightarrow$ physical mixture

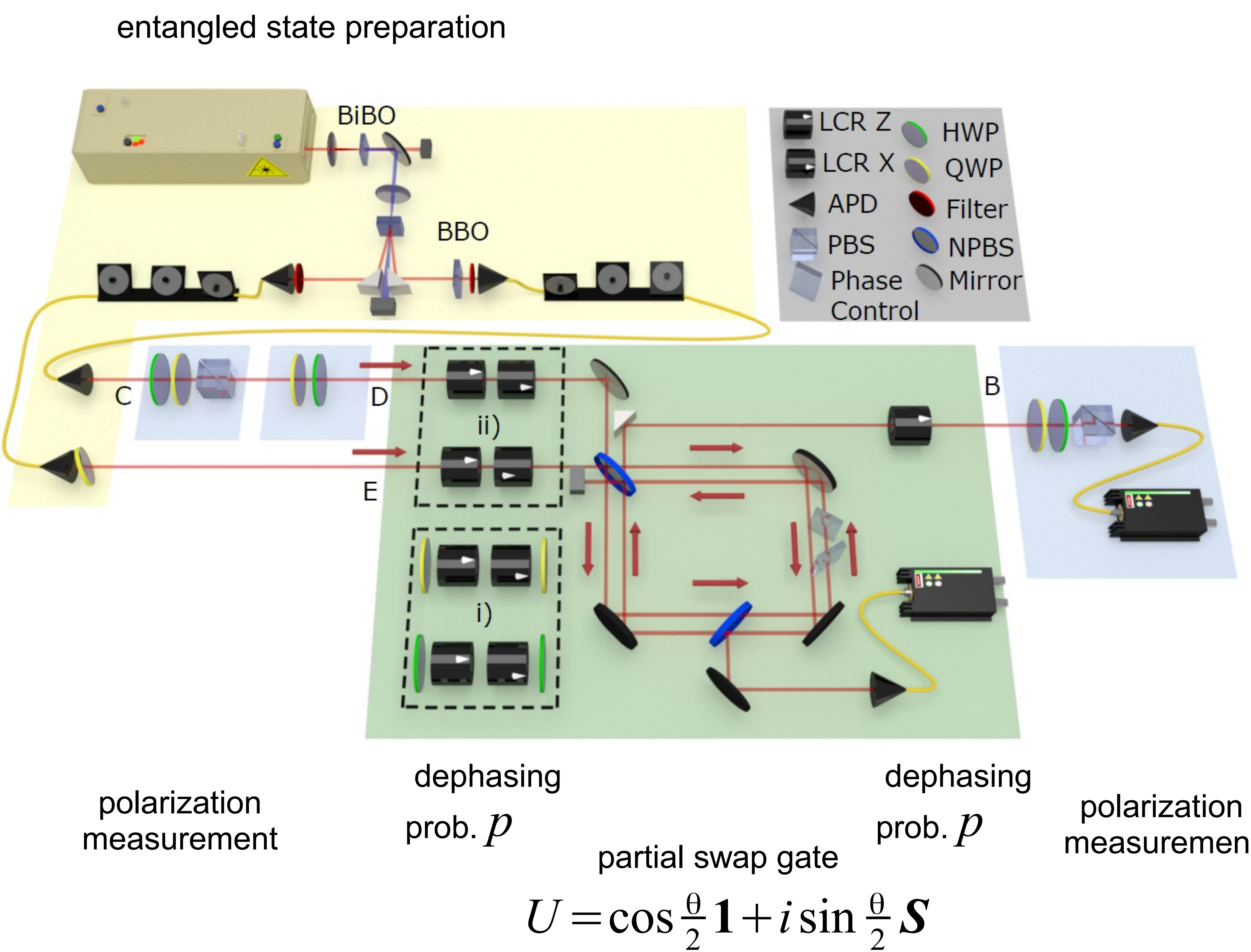
* measure σ_z on B, σ_x on C, σ_y on D, obtain P(b,c,d)

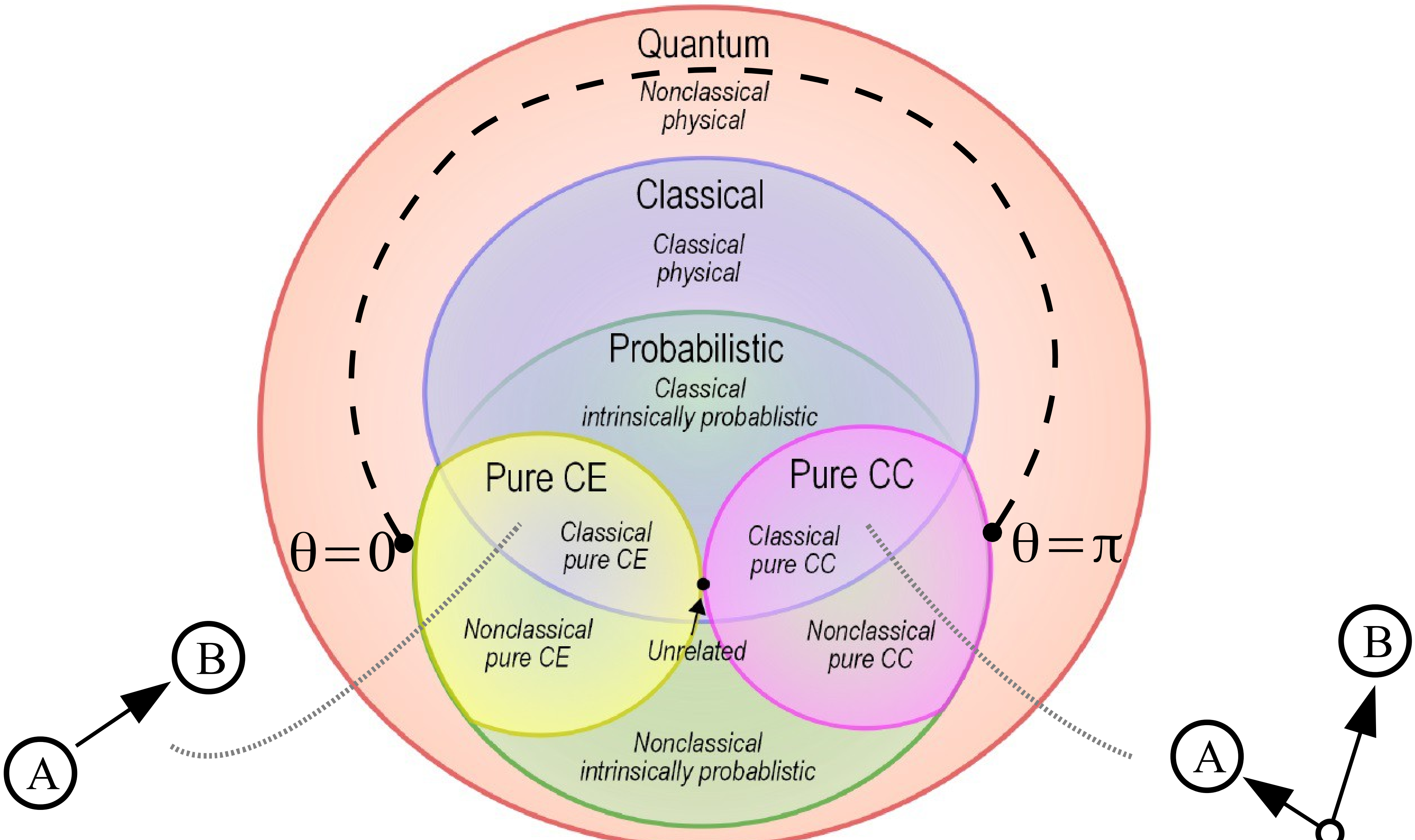
$$W \equiv P(+++)P(+--) - P(++-)P(+-+)$$
$$-P(-++)P(---) + P(-+-)P(--+)$$

Non-Classical Causal Structures in Experiment

Two quantum variables with tunable causal relation



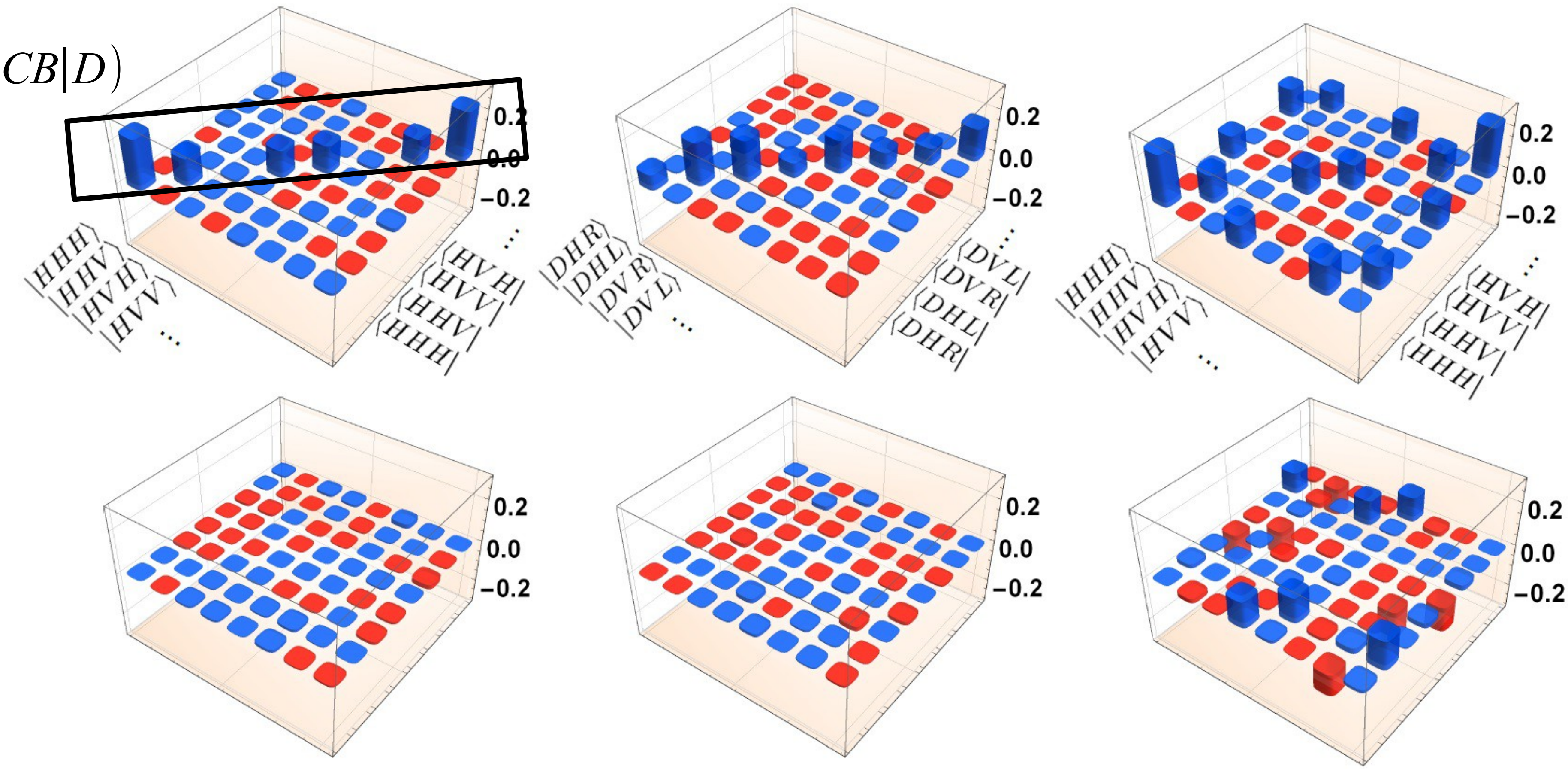

entangled state preparation
BiBO
BBO
LCR Z
LCR X
APD
PBS
Phase Control
HWP
QWP
Filter
NPBS
Mirror
C
D
E
B
ii)
i)
polarization measurement
dephasing prob. p
partial swap gate
dephasing prob. p
polarization measurement
$U = \cos\frac{\theta}{2}\mathbf{1} + i\sin\frac{\theta}{2}\boldsymbol{S}$



Quantum
Nonclassical physical
Classical
Classical physical
Probabilistic
Classical intrinsically probablistic
Pure CE
Pure CC
$\theta = 0$
$\theta = \pi$
Classical pure CE
Classical pure CC
Nonclassical pure CE
Unrelated
Nonclassical pure CC
Nonclassical intrinsically probablistic
A
B
A
B

Tomography of $\Phi_{CB|D}$

$P(CB|D)$

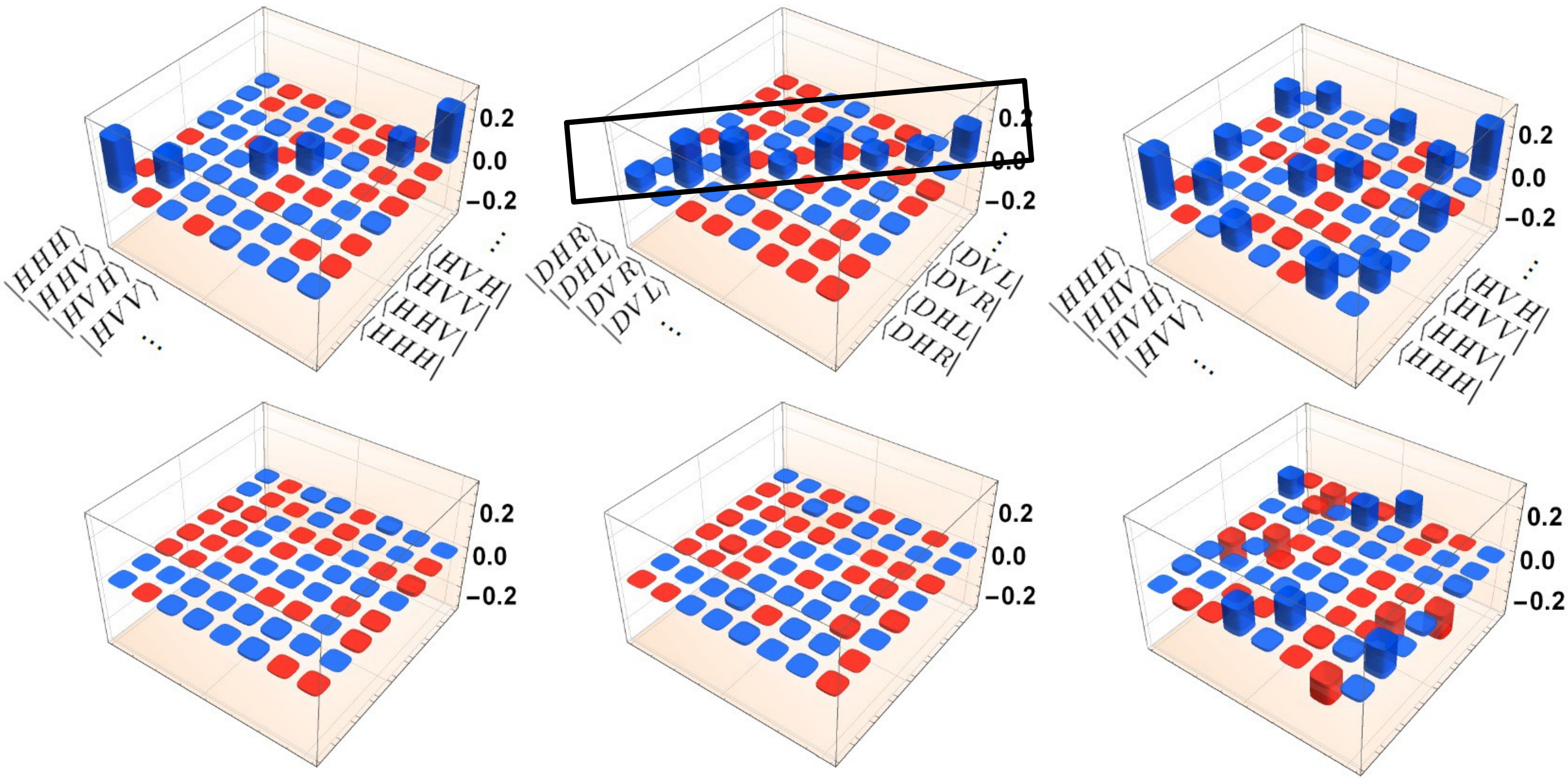


$P(CD|b=0)$

	c=0	c=1
d=0	1/2	1/4
d=1	1/4	0

$$= \frac{1}{2}\begin{array}{|c|c|}\hline 1/2 & 0 \\ \hline 1/2 & 0 \\ \hline \end{array} + \frac{1}{2}\begin{array}{|c|c|}\hline 1/2 & 1/2 \\ \hline 0 & 0 \\ \hline \end{array}$$

Tomography of $\Phi_{CB|D}$



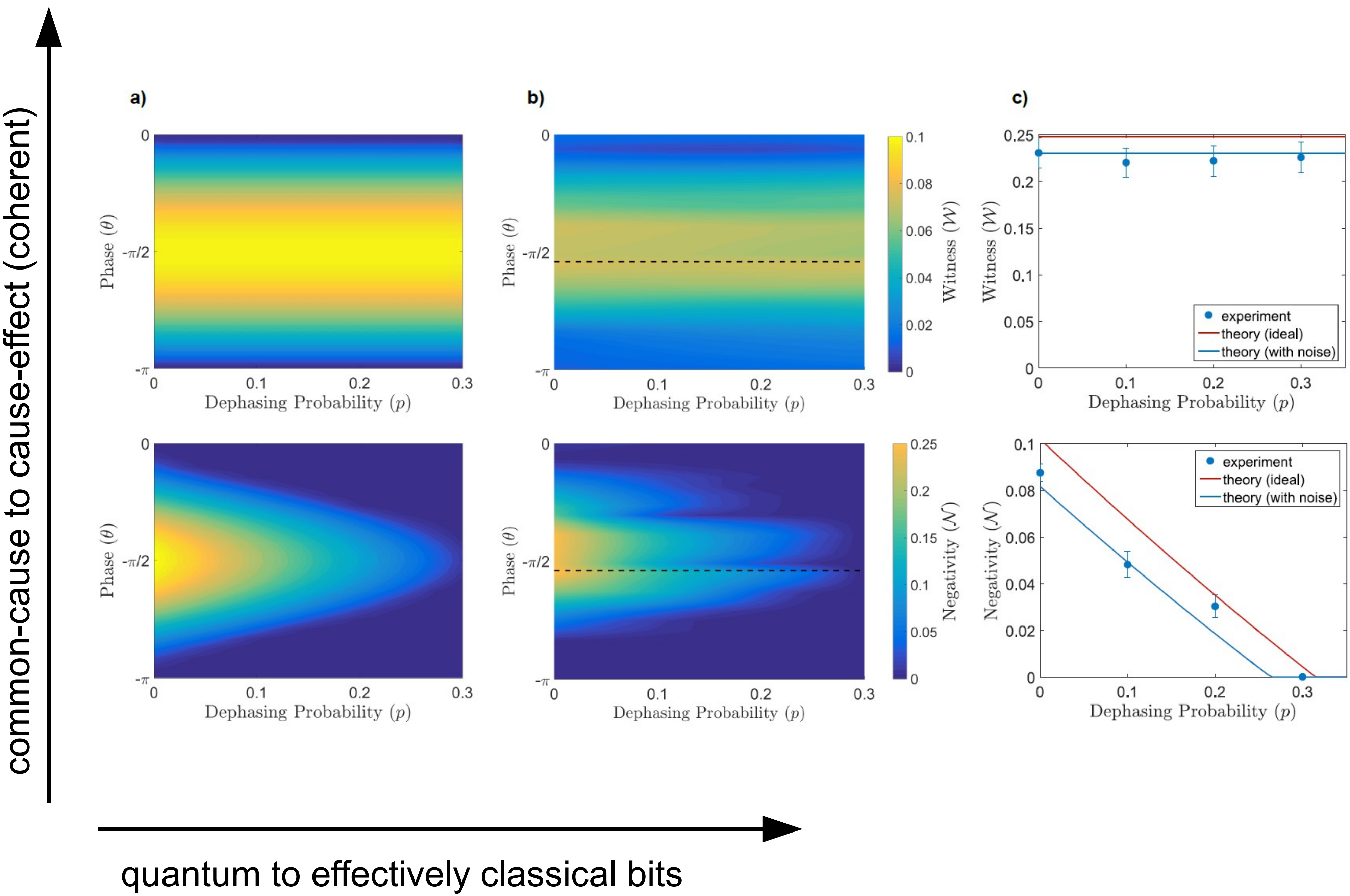
	c=0	c=1
d=0	1/8	3/8
d=1	3/8	1/8

$= \frac{1}{2}$

1/4	1/4
1/4	1/4

$+ \frac{1}{2}$

0	1/2
1/2	0



common-cause to cause-effect (coherent)
quantum to effectively classical bits
a)
b)
c)
Phase (θ)
Dephasing Probability (p)
Witness (W)
Negativity (N)
experiment
theory (ideal)
theory (with noise)
0
-π/2
-π
0.1
0.2
0.3
0.25
0.15
0.05
0.08
0.06
0.04
0.02

Highlights

- Causal models provide a new perspective on physical principles governing information in quantum theory
- Classification of combinations of common-cause and cause-effect structures, both classical and non-classical
  - operational criteria for identifying different types
  - experimental realization