

# *Causarum Investigatio* and the Two Bell's Theorems of John Bell

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# The 50<sup>th</sup> anniversary of

Physics Vol. 1, No. 3, pp. 195–200, 1964    Physics Publishing Co.    Printed in the United States

## ON THE EINSTEIN PODOLSKY ROSEN PARADOX\*

J. S. BELL†

*Department of Physics, University of Wisconsin, Madison, Wisconsin*

*(Received 4 November 1964)*

Given as “J. S. Bell, *Physics* 1, 195 (1965)” in:

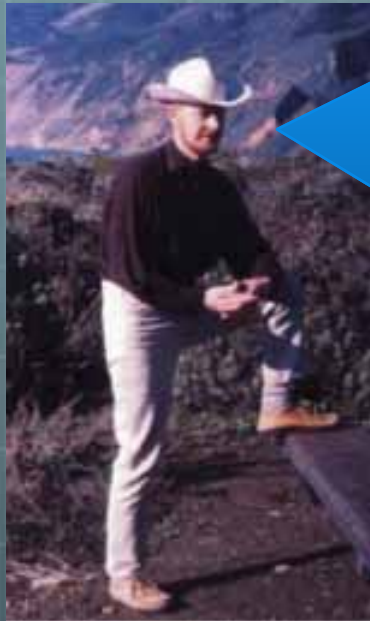
Bell (1966); Bell & Nauenberg (1966); Clauser, Horne, Shimony, & Holt (1969); Wigner (1970); Clauser & Horne (1974); Bell (1976); Shimony, Horne, & Clauser (1976); Clauser & Shimony (1978); Bell (1981); Van Fraassen (1982); Aspect, Dalibard, & Roger (1982); Aspect, Grangier, & Roger (1982); F. Rohrlich (1983); Ellis *et al.* (1983); Bell (1986) /S&US-1987/ Weihs *et al.* (1989); Greenberger, Horne, & Zeilinger (1989); Pearle (1990) ...

**AFAICT only one of Bell’s papers (1971) gives the date as 1964!**

# Outline

- **The two Bell's theorems of John Bell**
- **The on-going war over Bell's theorem(s).**
- **Back to basics: causation**
- **How causation allows a reconciliation**
- **Conclusion.**

# What *exactly* did Bell prove in 1964?



**In a theory in which parameters are added to quantum mechanics to **determine the results of individual measurements**, without changing the statistical predictions, there **must be** a mechanism whereby **the setting of one measuring device can influence the reading of another instrument, however remote.** (1964)**

i.e. quantum correlations violate the joint assumptions of:

- 🌐 “Predetermination” (of measurement results).
- 🌐 “Locality, [meaning] that the result of a measurement on one system be unaffected by operations on a distant system”.

# What *exactly* did Bell prove in 1976?

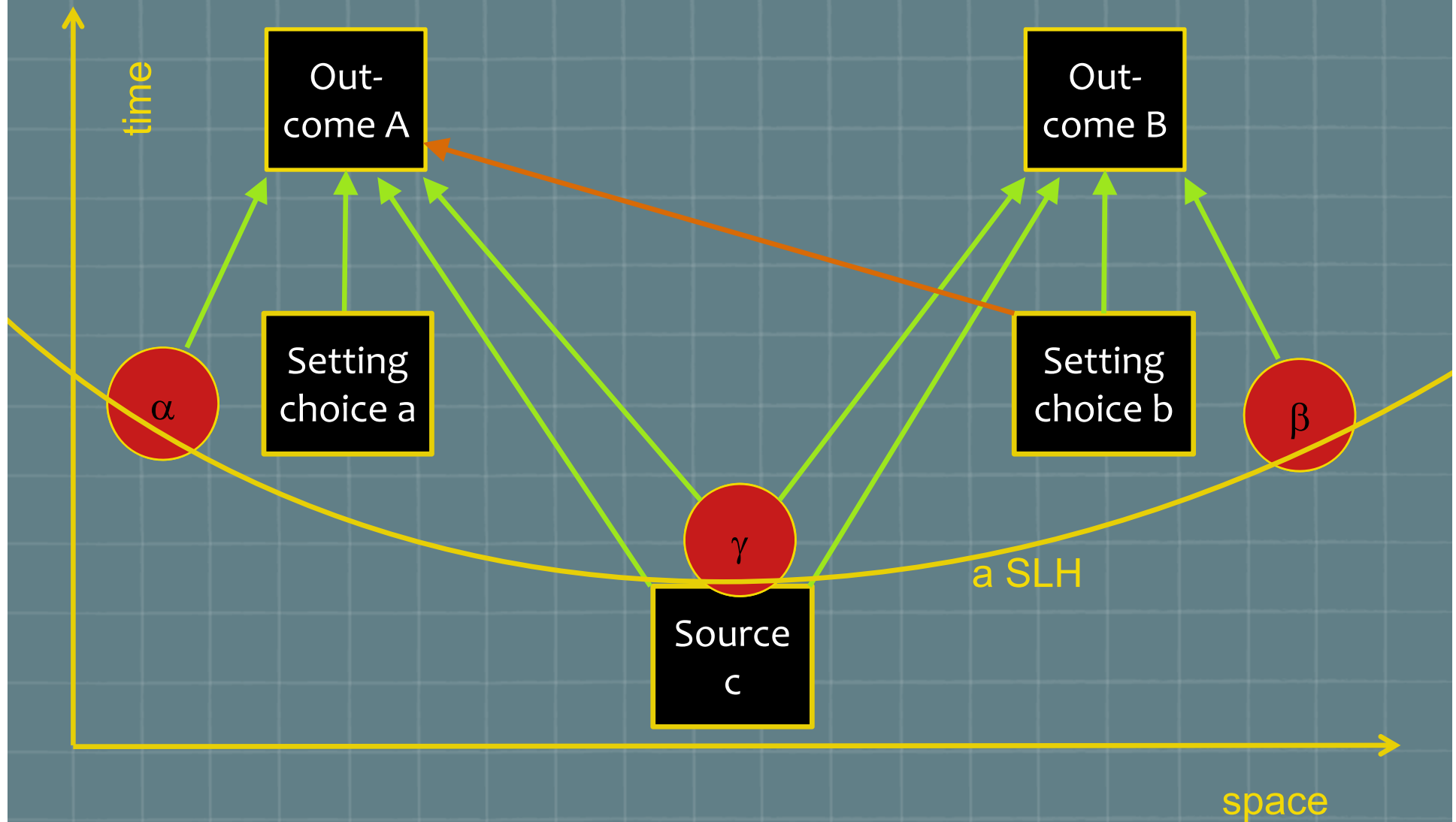


My ... notion of **local causality** is that events in [one lab] should not be causes of events in [a *space-like separated* lab], and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the *overlap in their backward light cones*. (1976)

A consequence ... of 'local causality' [is] the outcomes [in the two labs] having **no [statistical] dependence on one another nor on the settings** of the remote [measurement], but only on the local [measurement settings] and on the [common] past causes. (1990)

**Quantum** mechanics ... gives certain **correlations** which ... **cannot be [reproduced by] a locally causal theory**. (1976)

# Minkowski Diagram



# Or, if you prefer probabilities ...

- A quantum phenomena is described by a theory if

$$P_{\text{obs}}(A,B|a,b,c) = \sum_{\lambda} P(A,B|a,b,c,\lambda)P(\lambda|c), \text{ where } \lambda=(\alpha,\beta,\gamma)$$

- **Predetermination**:  $P(A,B|a,b,c,\lambda) = 0$  or  $1$ .  
≠ **Predictability**:  $P_{\text{obs}}(A,B|a,b,c) = 0$  or  $1$ .
- **Locality**:  $P(A|a,b,c,\lambda) = P(A|a,c,\lambda)$ .  
≠ **Signal locality**:  $P_{\text{obs}}(A|a,b,c) = P_{\text{obs}}(A|a,c)$ .
- **Bell 1964**: there is no theory reproducing quantum phenomena satisfying **locality** and **predetermination**.
- **Local causality**:  $P(A|a,B,b,c,\lambda) = P(A|a,\alpha,c,\gamma)$ .
- **Bell 1976**: there is no ... satisfying **local causality**.
- Both assume **no-superdeterminism (1990)**:  $P(\lambda|a,c) = P(\lambda|c)$

# Well that clears that up ...

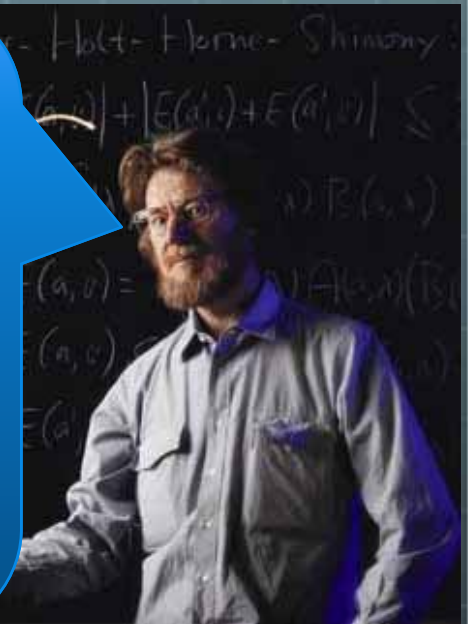
Unfortunately the confusion started even in 1976:

- 🌐 Having defined “local causality” Bell immediately started using “locality” as a synonym for it, even though it was different from “locality” as he had used in 1964.
- 🌐 Soon afterwards, Bell started claiming that by “locality” he had *always* meant local causality, and that this was the “sacred principle” which Einstein had believed in.
- 🌐 Followers of Bell therefore often state that Bell’s theorem is that quantum phenomena are nonlocal.
- 🌐 Most quantum physicists only know Bell’s 1964 theorem, and say that we can keep locality if we give up predetermination.



# Realism vs Operationalism

My theorem uses only one assumption: local causality (or 'locality' as we may call it for short). This is the only reasonable way to define the principle of relativity for statistical theories. It is essentially what EPR assumed in 1935. They showed that operational quantum mechanics is nonlocal, and I showed in 1964 that adding hidden variables cannot solve the problem. Experiments have thus proven the principle of relativity false. The world is nonlocal.



Bell's theorem uses two assumptions. The first assumption is locality. This means no signaling faster than light (which is all the principle of relativity implies) even for hidden variable theories. Operational quantum mechanics respects locality. The second assumption is realism/classicality/predetermination. Clearly it is this second assumption that we should abandon, whatever we call it. Locality is here to stay.



# Back to basics



# AXIOMS

## **AXIOM 1: MACROREALITY.**

An event observed by any observer is a real single event, and not 'relative' to anything or anyone.

## **AXIOM 2: MINKOWSKI SPACE-TIME.**

Concepts like light-cones, space-like separated (SLS), space-like hypersurfaces (SLHs) etc. can be applied unambiguously in ordinary laboratory situations.

## **AXIOM 3: (PARTIAL) TEMPORAL ORDER.**





For any event A, there is a SLH containing A that separates events in the **past** of A from events that have A in their **past**.

## **AXIOM 4: CAUSAL ARROW.**

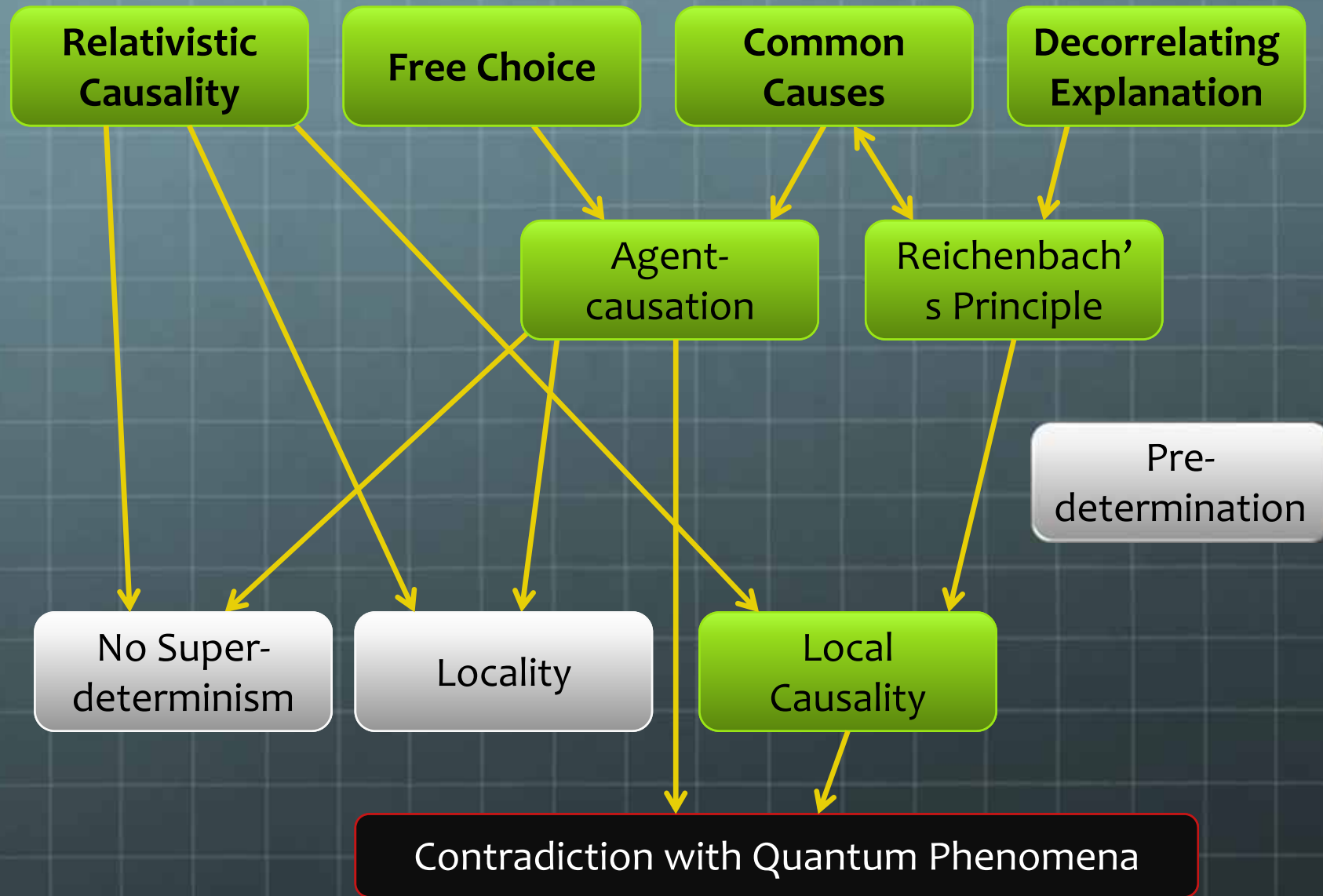
Any **cause** of an event can only be in its **past**.

(These guarantee that there are no causal loops.)

# POSTULATES

-  **POSTULATE 1: FREE CHOICE.**  
A freely chosen action has no **causes** that we need to worry about.
-  **POSTULATE 2: RELATIVISTIC CAUSALITY.**  
The **past** is the past light-cone.
-  **POSTULATE 3: COMMON CAUSES.**  
If two sets of events A and B are correlated, and no event in either is a **cause** of any event in the other, then they have a set of common **causes** C that **explains** the correlation.
-  **POSTULATE 4: DECORRELATING EXPLANATION.**  
A set of **causes** C, common to two sets of events A and B, **explains** a correlation between A and B only if conditioning on C eliminates the correlation.

# Reconciliation Version



# Operationalist Principles



## PRINCIPLE: LOCALITY

The probability of an observable event  $A$  is unchanged by conditioning on a SLS **free choice**  $b$ , even if it is already conditioned on other events as long as those events are not in the future light cone of  $b$ .



## PRINCIPLE: NO-SUPERDETERMINISM

Any set of events on a SLH are uncorrelated with any set of **freely chosen actions** subsequent to that SLH.



## PRINCIPLE: PREDETERMINATION.

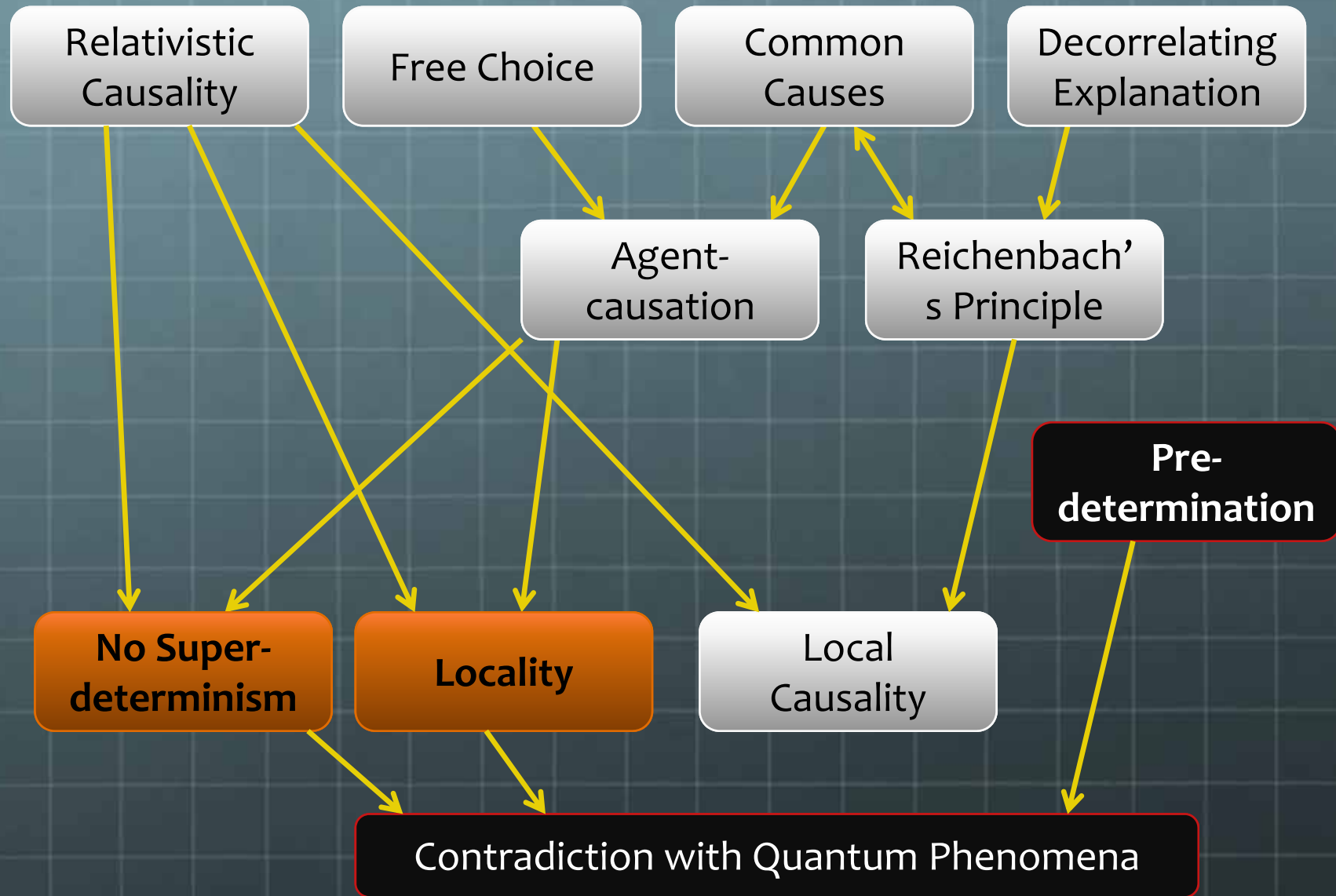
For any observable event  $A$ , and any SLH  $S$  prior to it,  $A$  has **causes** on  $S$  which, possibly in conjunction with **free choices** subsequent to  $S$ , determine  $A$ .



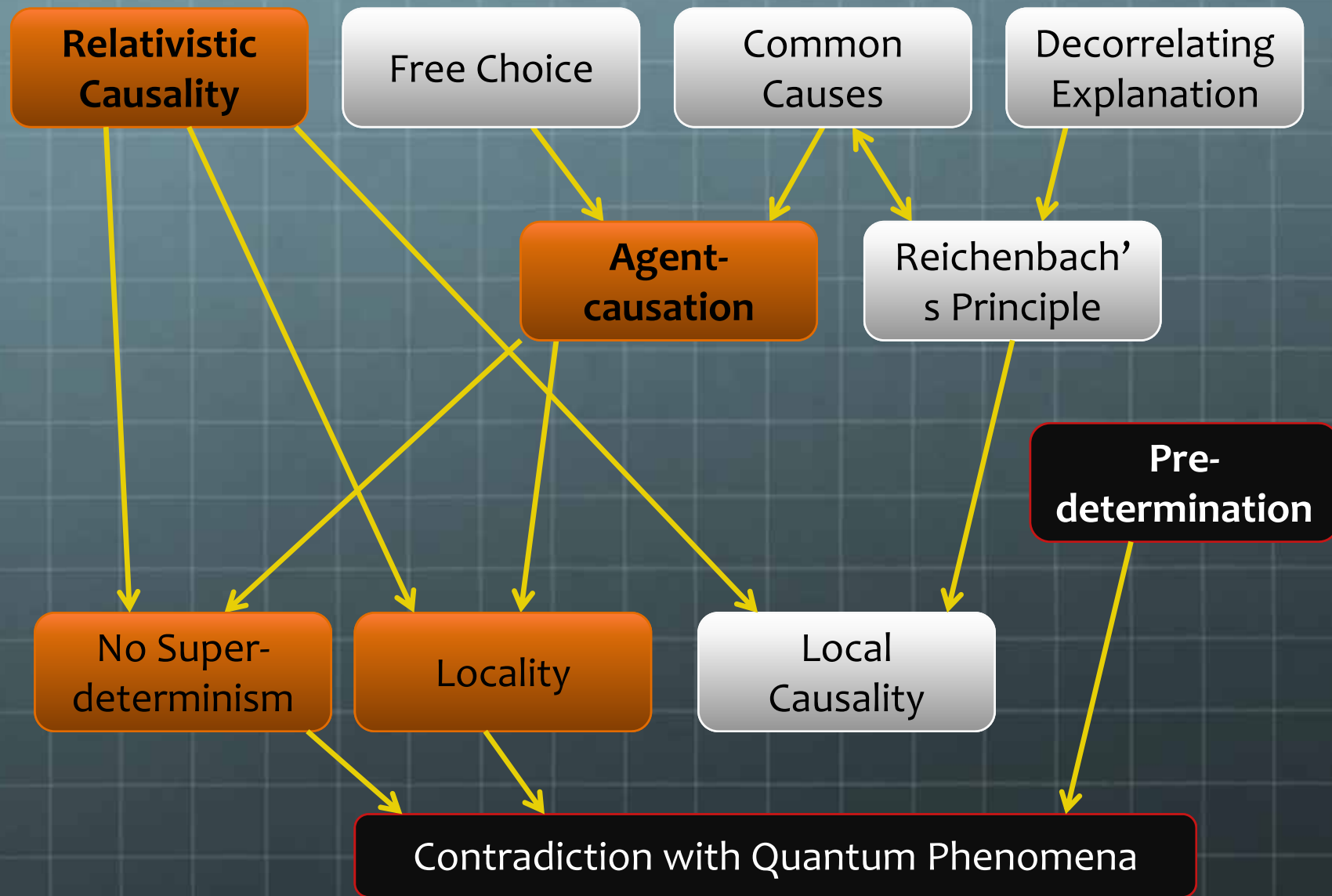
## THEOREM ~ Bell (1964)

The conjunction of these three principles contradicts QM.

# Original **Operationalist** Version (1964)



# Intermediate **Operationalist** Version



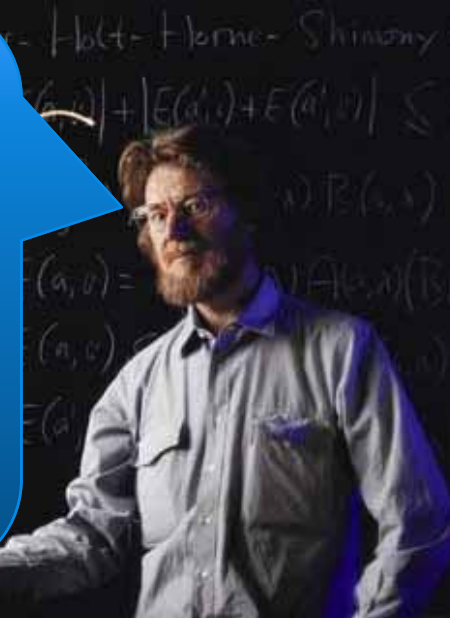


# Better **Operationalist** Principles

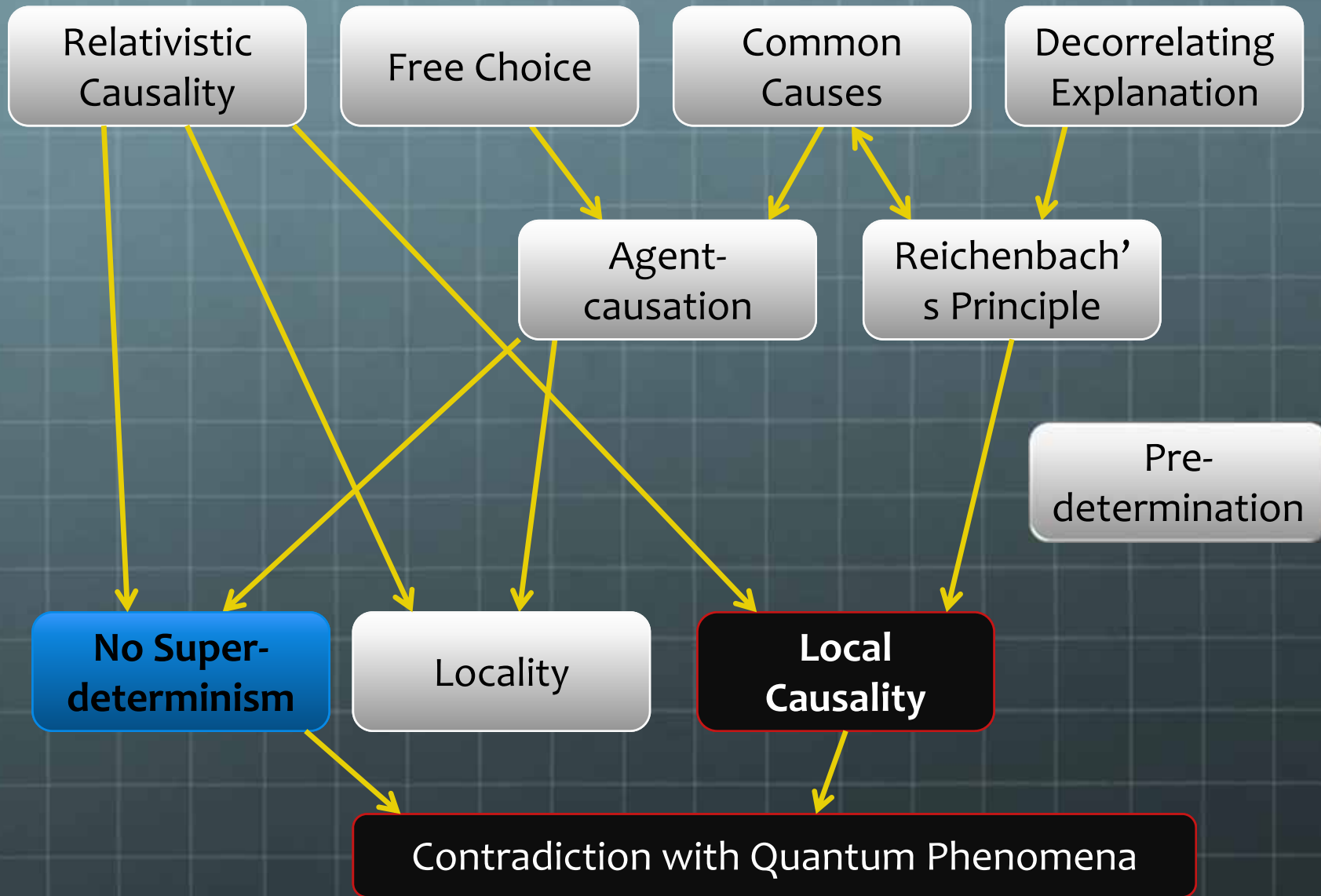
- 🌐 **PRINCIPLE: AGENT-CAUSATION**  
If a set of events  $A$  is correlated with a **freely chosen action**  $b$ , then  $b$  is a **cause** of  $A$ .
- 🌐 **LEMMA: Relativistic causality plus agent-causation imply locality, and no-superdeterminism.**

To use **agent causation** as *the* fundamental causal concept in contemporary theoretical physics is hard for me to accept. For one thing we would lose the idea that correlations must be explained. More importantly, it is desperately vague, immediately provoking the question: "What is an agent?"

Also, **predetermination** is a sacrificial lamb – it is unnecessarily strong, and so too easily given up.



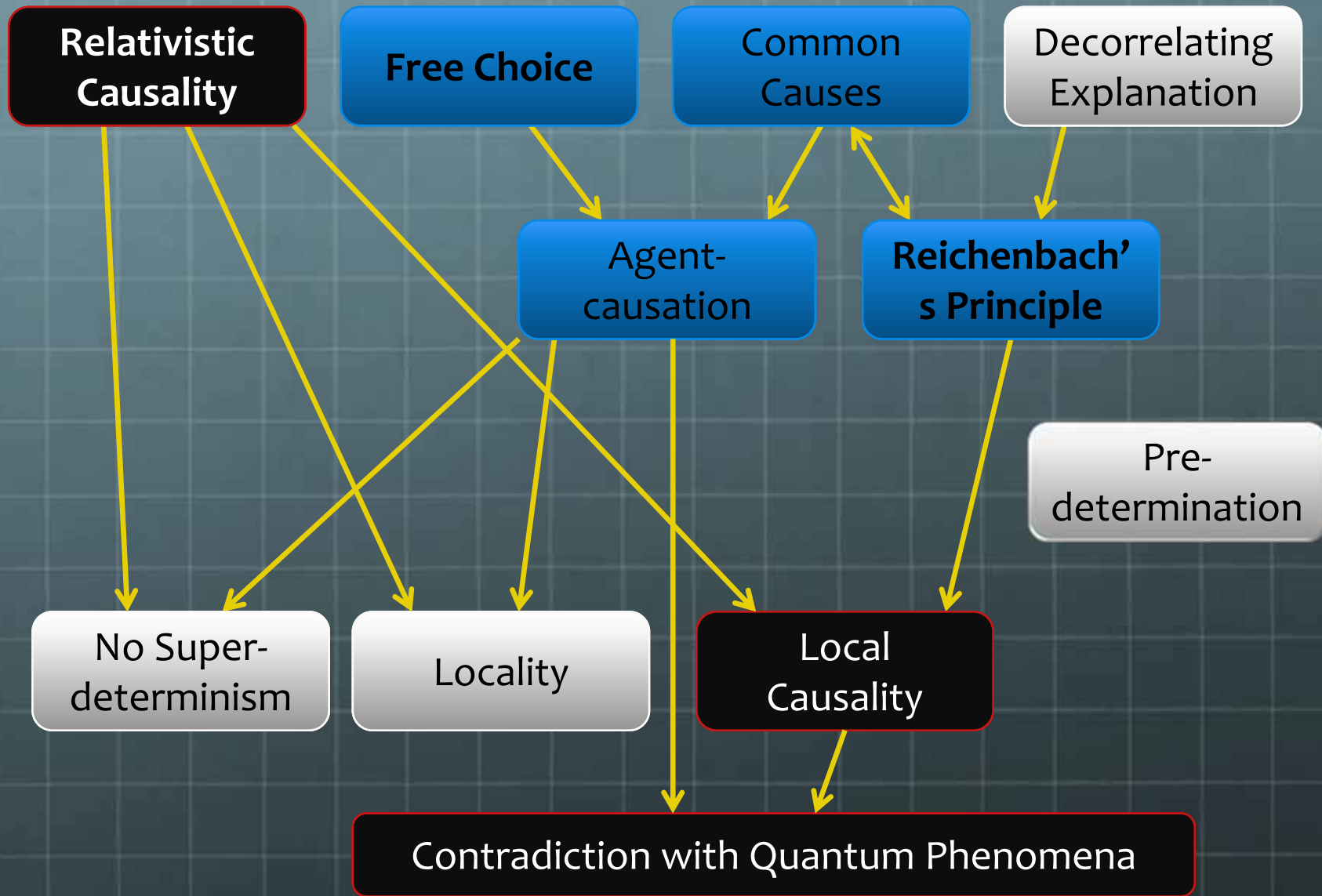
# Original **Realist** Version (1976)



# Realist Principles

- 🌐 **PRINCIPLE: LOCAL CAUSALITY**  
If two SLS sets of events  $A$  and  $B$  are correlated, then there is a set of events  $C$  in their common Minkowski past such that conditioning on  $C$  eliminates the correlation.
- 🌐 **PRINCIPLE of REICHENBACH (1956):**  
If two sets of events  $A$  and  $B$  are correlated, and neither is a **cause** of the other, then they have a set of common **causes**  $C$ , such that conditioning on  $C$  eliminates the correlation.
- 🌐 **LEMMA: Relativistic causality plus Reichenbach's Principle implies local causality.**
- 🌐 **LEMMA: Reichenbach's Principle plus Free Choice implies Agent-Causation**

# Intermediate **Realist** Version



# This **Realist** Theorem is almost there ...

- 🌍 **THEOREM ~ Bell (1976)**  
The conjunction of **Relativistic causality**, **Reichenbach's Principle** and **Free Choice** is contradicted by QM.
- 🌍 For realists, **Relativistic causality** has to be sacrificed to save **Reichenbach's Principle** (common causes that *explain* correlations).

As an operationalist, I find Reichenbach's Principle too strong as the fundamental notion of causation. Why should all correlations in nature be explicable in the way that they are in classical physics?

Also, how can you realists criticize us for having an agent-centric notion of causation when you still need to assume **Free Choice**, which is agent-centric too?



# Reconciliation

The key is to break

- 🌐 **PRINCIPLE of REICHENBACH** – If two sets of events A and B are correlated, and neither is a **cause** of the other, then they have a set of common **causes** C, such that conditioning on C eliminates the correlation.

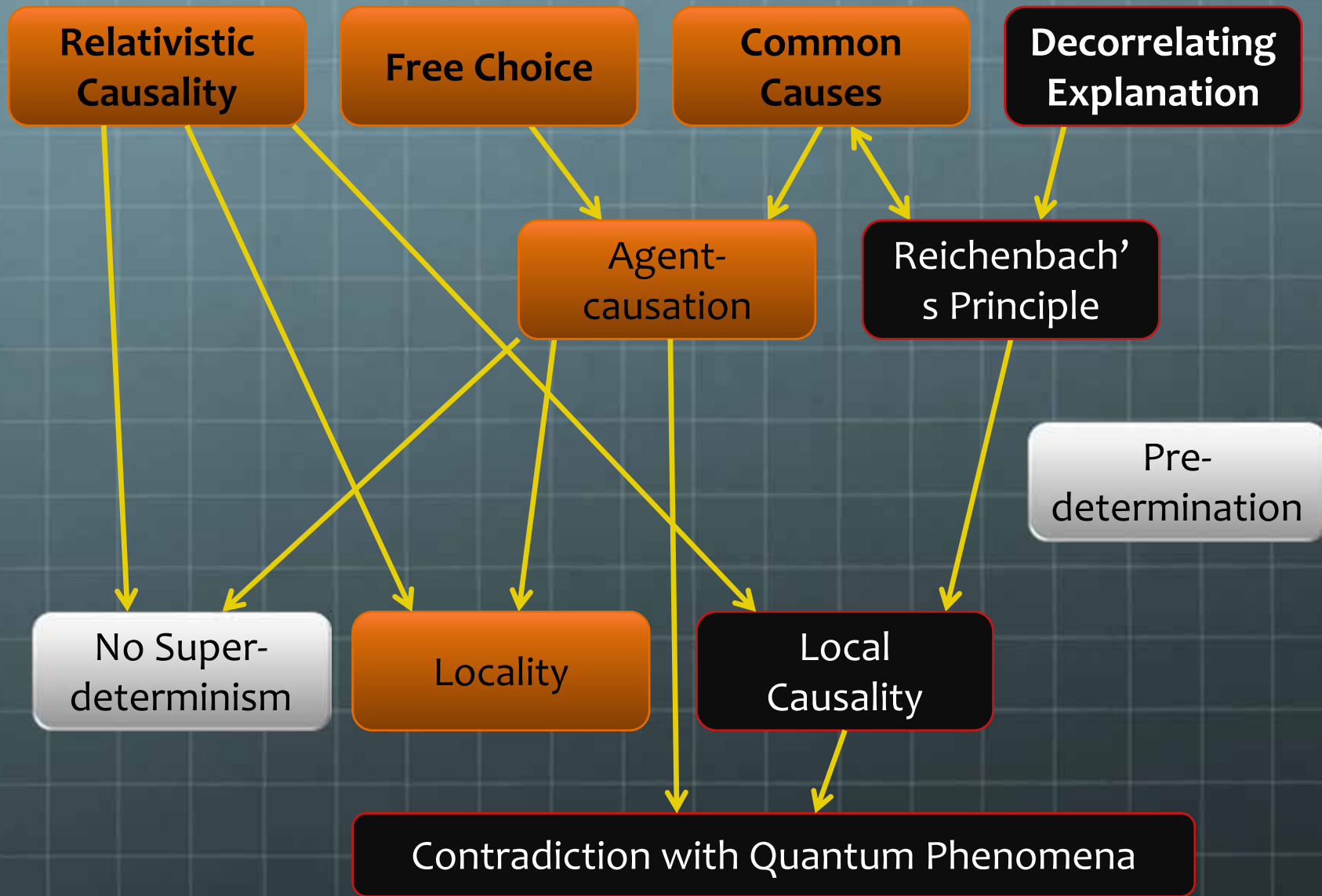
into

- 🌐 **POSTULATE 3: COMMON CAUSES** – If two sets of events A and B are correlated, and neither is a **cause** of the other, then they have a set of common **causes** C that **explains** the correlation.

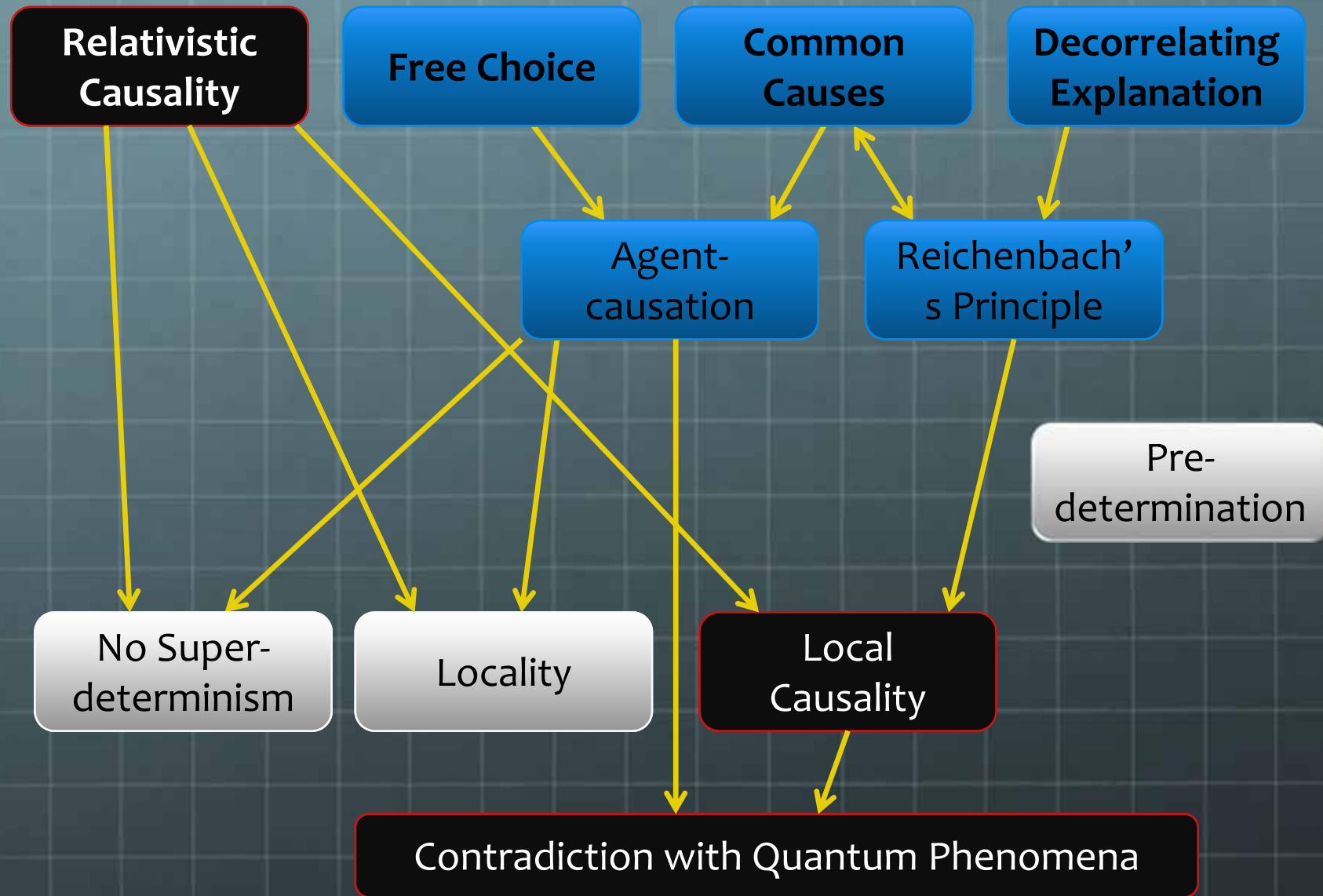
plus

- 🌐 **POSTULATE 4: DECORRELATING EXPLANATION** – A set of **causes** C, common to two sets of events A and B, **explains** a correlation between A and B only if conditioning on C eliminates the correlation.

# Final **Operationalist** Version



# Final Realist Version





# Considering causation confers ...

- A deeper understanding of why the two camps disagree, and why they prefer the different versions of Bell's theorem.
- A form of Bell's theorem that both camps might agree upon:
  - with terms not loaded with preconceptions;
  - in particular, without the problematic term “locality”;
  - with an option for each camp of rejecting a postulate (**Relativistic causality**, or **decorrelating explanation**).
- Several other conceptual advantages:  
Relativistic Causality + Reichenbach better than PI+OI; allow no-Fine-Tuning arguments; avoid absurdities of Colbeck&Renner's “Free Choice”

# Summary

- 🌐 There are actually two Bell's theorems (1964 and 1976), with essentially the same proof, but with different assumptions:
  - 🌐 Bell 1964, favoured by operationalists
  - 🌐 Bell 1976, favoured by realists
- 🌐 By considering notions of causation we can:
  - 🌐 Understand why the two camps disagree
  - 🌐 Find a form of Bell's theorem both *should* agree upon.
- 🌐 Still, we have to give up an AXIOM (macroreality; Minkowski space-time; temporal order; or causal arrow), or a POSTULATE (freedom of choice; relativistic causality; common cause; or “decorrelating explanation”).
- 🌐 **→ The universe is profoundly strange ...**