Non-equilibrium quantum phase transition induced by periodic driving

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8th Asia-Pacific Conference & Workshop on Quantum Information Science
Dec. 12-15, 2014, Tainan

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Outline

1. Introduction
   - Quantum phase transition (QPT) & periodic driving
   - Non-equilibrium QPT induced by periodic driving
2. Floquet control to decoherence
   - Introduction to decoherence control and motivation
   - System and mechanism
   - Non-equilibrium phase diagram
   - Conclusions
3. Generating multiple Majorana fermions (MFs)
   - Introduction to MF & motivation for multiple MFs
   - Our results
   - Conclusions
4. Summary

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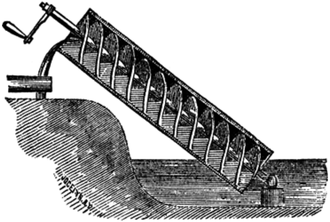
Phase transition

- Phase transition (PT): System driven by control parameters changes from one state of matter to another
- The classification of PT:
  1. CPT: Abrupt change of $\mathcal{F}$ by changing $T-$relevant parameters
  2. QPT: Abrupt change of E_g of a zero-T Q. many-body system, e.g. abrupt opening of a bandgap
- Difficulty: The parameters are hard to change once the material sample of the system is fabricated

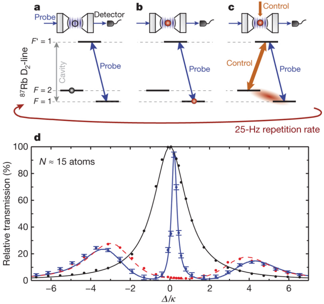
Do we have more efficient way to manipulate energy spectrum than changing parameters?

Periodic driving: A useful way

- to control system



Archimedes' screw



EIT: periodic interaction between the control laser and atoms. Mücke, et al., Nat. 465, 755 (2010)

offers high controllability of Q. system because time, as an extra dimension, is added to the system

Can we manipulate the energy spectrum of Q. many-body system by periodic driving?

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What governs the physics of a periodic system?

Static sys.	Periodic sys.
$\hat{H}\|\varphi_n\rangle = E_n\|\varphi_n\rangle$	$[\hat{H}(t) - i\hbar\partial_t]\|\phi_n(t)\rangle = \varepsilon_n\|\phi_n(t)\rangle$ $\|\phi_n(t)\rangle = \|\phi_n(t+T)\rangle$
$\|\Psi(t)\rangle = \sum_n c_n e^{\frac{-iE_n t}{\hbar}}\|\varphi_n\rangle$ $c_n = \langle\varphi_n\|\Psi(0)\rangle$	$\|\Psi(t)\rangle = \sum_n c_n e^{\frac{-i\varepsilon_n t}{\hbar}}\|\phi_n(t)\rangle$ $c_n = \langle\phi_n(0)\|\Psi(0)\rangle$

1. ε_n plays the same role as the eigenenergy in the static system. Quasienergy
2. $|\phi_n(t)\rangle$ plays the same role as the $\hat{H}$-eigenstate in the static system. Quasi-stationary state

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1. ε_n plays the same role as the eigenenergy in the static system. Quasienergy
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$$\left[\hat{H}(t) - i\hbar\partial_t\right]|\phi_n(t)\rangle = \varepsilon_n|\phi_n(t)\rangle \Leftrightarrow \hat{U}_T|\phi_n(0)\rangle = e^{\frac{-i}{\hbar}\varepsilon_n T}|\phi_n(0)\rangle$$

$$\hat{U}_T = \hat{\mathcal{T}}e^{-\frac{i}{\hbar}\int_0^T \hat{H}(t)dt} \equiv e^{\frac{-i}{\hbar}\hat{H}_{\text{eff}}T}, \quad \hat{H}_{\text{eff}}|\phi_n(0)\rangle = \varepsilon_n|\phi_n(0)\rangle$$

The quasi-stationary-state properties of $\hat{H}(t)$ are carried by $\hat{H}_{\text{eff}}$

is characterized by the abrupt opening of a bandgap in quasi-energy spectrum by the driving parameters

- The energy of the system is in non-conserving: Non-equilibrium
- A time-dependent analogy of QPT in periodically driven system

Non-equilibrium QPT induced by periodic driving

is characterized by the abrupt opening of a bandgap in quasi-energy spectrum by the driving parameters

- The energy of the system is in non-conserving: Non-equilibrium
- A time-dependent analogy of QPT in periodically driven system

The versatility of driving scheme enable us to

1. realize Q. phases not accessible for the static system in the same setting
2. explore novel Q. phases with no analogy with its static system

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References

1. Qing-Jun Tong, Jun-Hong An, L. C. Kwek, Hong-Gang Luo, and C. H. Oh, *Simulating Zeno physics by quantum quench with superconducting circuits*, **Phys. Rev. A 89, 060101(R) (2014)**.
2. Chong Chen, Jun-Hong An, Hong-Gang Luo, C. P. Sun, and C. H. Oh, *Floquet control on quantum dissipation in spin chain*, **arXiv:1408.0105**.

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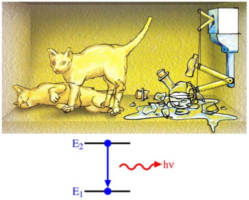
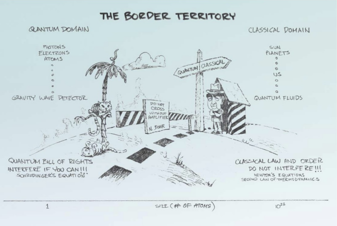
Decoherence: A basic issue of Q. physics

- The loss of the phase ordering between the components of a Q. superposition
- The reason why Q. behaviors are different from classical one



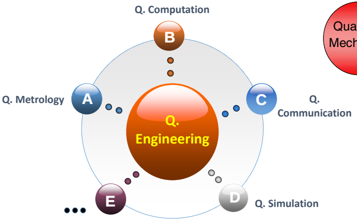
$|\Psi\rangle \to |00\cdots 0\rangle$

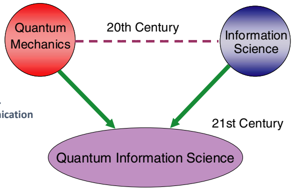


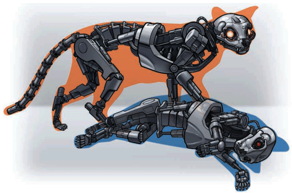
From W. H. Zurek (1991)

1. The border between Q. and classical worlds
2. The quantum-classical transition of a physical systems
3. The paradoxes when Q. laws are applied to macroscopic systems

Decoherence: Renewed interest in Q. engineering





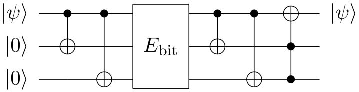


Mechanical analog to Schrödinger's cat. From Cho, Science 327, 516 (2010).

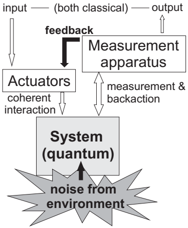
- Great technique innovations from basic physical principle
- The realization of QE is bounded by decoherence

  QE relies on the possibility to manipulate the state of a great number of qubits in a controlled way and to maintain coherence over a long time
- How to control decoherence is a crucial issue in QE

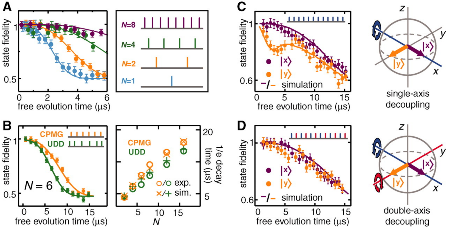
Decoherence control



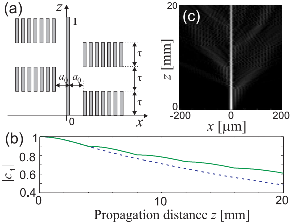
Q. error correction for one qubit-flip error



Feedback control. Tóth, arXiv:1210.0360



Dynamical decoupling. de Lange, Wang, Ristè, Dobrovitski, Hanson, Science 330, 60 (2010)



Zeno effect. Longhi, PRL 97, 110402 (2006)

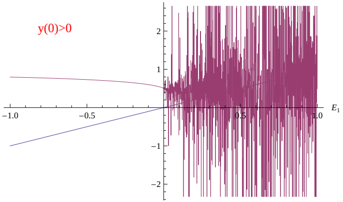
Bound-state induced decoherence suppression

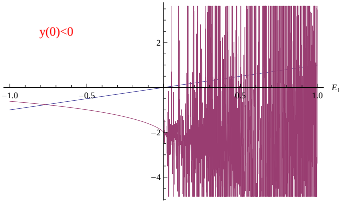
- Bound state: A mid-gap eigen state of the system-reservoir system: $|\varphi_1\rangle = c_0\,|+,\{0_k\}\rangle + \sum_k c_k\,|-,1_k\rangle$,

$$\left.\begin{array}{c}\omega_0 c_0 + \sum_k g_k c_k = E_1 c_0 \\ g_k^* c_0 + \omega_k c_k = E_1 c_k\end{array}\right\} \Rightarrow y(E_1) \equiv \omega_0 - \int_0^\infty \frac{J(\omega)}{\omega - E_1} d\omega = E_1$$

Tong, An, Luo, Oh, PRA 81, 052330 (2010); PRB 84, 174301 (2011)

- When is bound state formed?



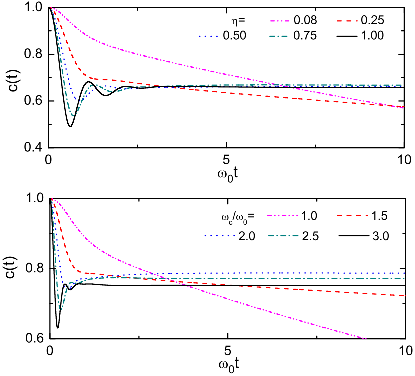


For Ohmic-like spectral density, the bound state is formed when: $y(0) < 0 \Rightarrow \omega_0 - \eta\omega_c\gamma(s) < 0$

Consequence (I): Anomalous decoherence

For $J(\omega) = \eta\frac{\omega^3}{\omega_0^2}e^{-\omega/\omega_c}$, the bound state is formed

when: $\omega_0 - 2\eta\frac{\omega_c^3}{\omega_0^2} < 0$

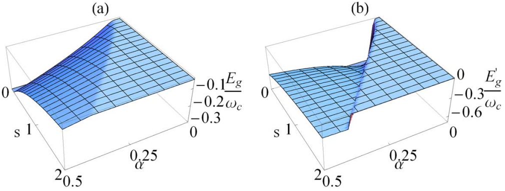


The evolution of quantum coherence $c(t) = |c_0(t)|$. $\frac{\omega_c}{\omega_0} = 1.0$ in upper panel (the bound state is formed when $\eta > 0.5$) and $\eta = 0.08$ in the lower panel (the bound state is formed when $\omega_c > 1.84\omega_0$)

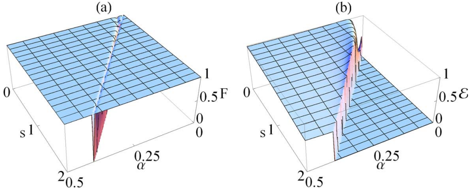
Tong, An, Luo, Oh, J. Phys. B: At. Mol. Opt. Phys. 43, 095505 (2010)

Consequence (II): Quantum phase transition

$J(\omega) = 2\pi\alpha\omega_c^{1-s}\omega^s\Theta(\omega_c - \omega)$, the bound state is formed： $\alpha > \frac{2s\Delta}{\omega_c}$.



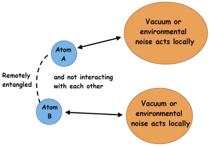
Ground-state energy and its first-derivative



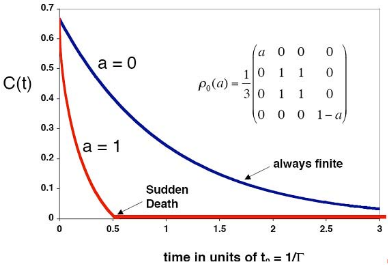
Ground-state fidelity and entanglement

From Liu, An, Chen, Tong, Luo, Oh, Phy. Rev. A 87, 052139 (2013)

Consequence (III): Entanglement preservation

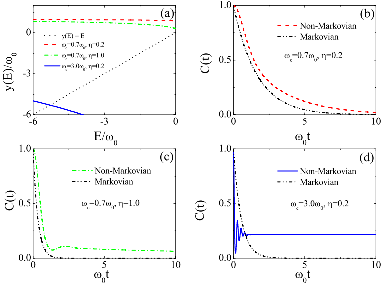


BM result



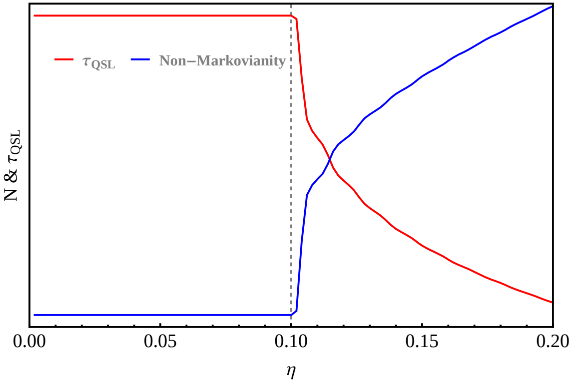
T. Yu & J. Eberly, PRL 93, 140404 (2004)

NM result



Tong, An, Luo, Oh, Phys. Rev. A 81, 052330 (2010)

Consequence (IV): Quantum speed limit & non-Markovianity



Ohmic spectrum. The bound state is formed when $\eta > 0.1$. Liu, Yang, An, Luo (2014)

- Non-Markovianity: Breuer, Laine, and Piilo, Phys. Rev. Lett. 103, 210401 (2009)
- Quantum speed limit time: Deffner and Lutz, Phys. Rev. Lett. 111, 010402 (2013)

Motivation

The decoherence of the system connects tightly with the properties of the energy spectrum of the total system

- Can we manipulate the energy spectrum of the total system forming the bound state via periodic driving so that the decoherence of the open system can be suppressed?

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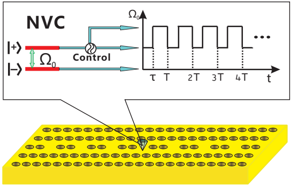
System and scheme



A periodically driven two-level system interacting with a coupled cavity array constructed by photonic crystal

$$\omega_0(t) = \begin{cases} \omega_c, & t \in [mT, mT+\tau] \\ \omega_c + \Delta, & t \in [mT+\tau, (m+1)T] \end{cases}$$

$$\begin{aligned} \hat{H}(t) &= \omega_0(t)\hat{\sigma}_+\hat{\sigma}_- + \omega_c \sum_{j=0}^{N-1} \hat{b}_j^\dagger \hat{b}_j + (g\hat{\sigma}_+\hat{b}_0 + \xi \sum_{j=0}^{N-2} \hat{b}_{j+1}^\dagger \hat{b}_j + \text{h.c.}) \\ &= \omega_0(t)\hat{\sigma}_+\hat{\sigma}_- + \sum_k [\omega_k \hat{a}_k^\dagger \hat{a}_k + \frac{g}{\sqrt{N}}(\hat{\sigma}_+\hat{a}_k + \text{h.c.})] \end{aligned}$$

- An environment with finite band width $\omega_k = \omega_c + \xi \cos k x_0$

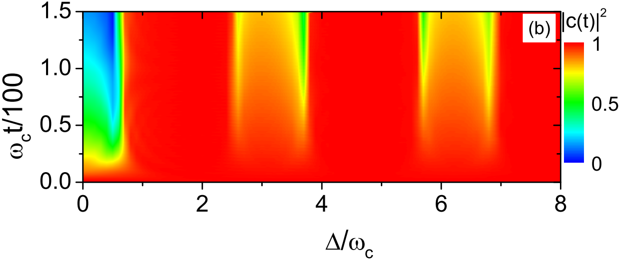
Decoherence dynamics

- Objective: Prevent the TLS from spontaneous emission
- Initial state: $|\Psi(0)\rangle = |+,\{0_k\}\rangle$

$|\Psi(t)\rangle = c(t)|+,\{0_k\}\rangle + \sum_k d_k(t)|-,1_k\rangle$, where

$$\dot{c}(t) + i\omega_0(t)c(t) + \int_0^t f(t-\tau)c(\tau)d\tau = 0, \tag{1}$$

with $f(t) = \sum_k g^2 e^{-i\omega_k t}/N$.

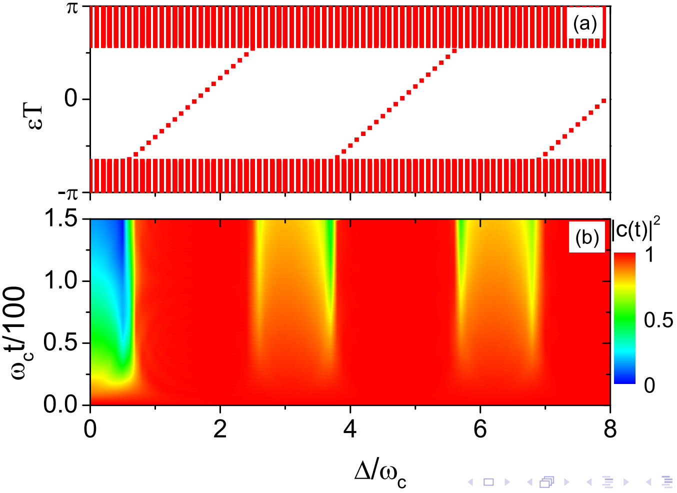


The modulated dynamics of the TLS in different driving amplitudes. The parameters are chosen as $g = 0.05\omega_c$, $\xi = 0.4\omega_c$, $N = 1001$, $T = 3.0\omega_c^{-1}$, and $\tau = 1.0\omega_c^{-1}$.

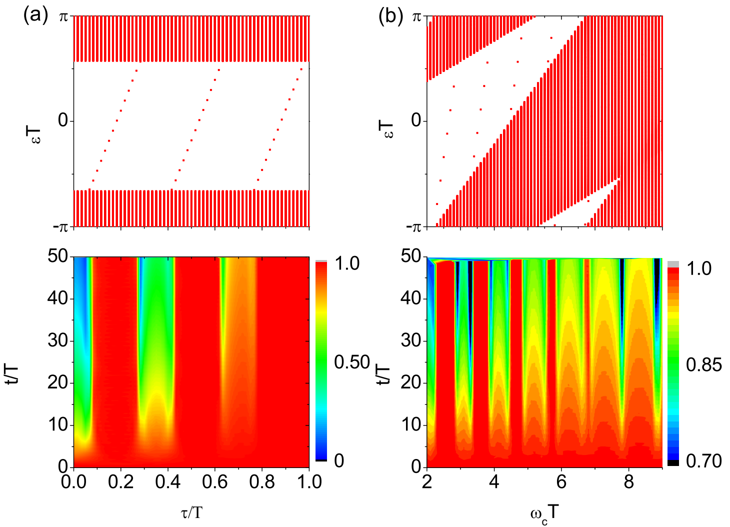
Mechanism

The Floquet quasi-energy spectrum plays essential role

$$e^{-i\hat{H}_2(T-\tau)}e^{-i\hat{H}_1\tau}|\Phi_n(0)\rangle = e^{-i\hat{H}_{\text{eff}}T}|\Phi_n(0)\rangle = e^{\frac{-i}{\hbar}\varepsilon_n T}|\Phi_n(0)\rangle$$



Other driving conditions



The Floquet spectrum (up) and the corresponding dynamics of the TLS (down) in different τ [column (a)] and in different T [column (b)]. In column (a), $T = 3\omega_c^{-1}$. In column (b), $\tau = 1.0\omega_c^{-1}$. $\Delta = 6.0\omega_c$ in both cases. Other parameters are the same as before.

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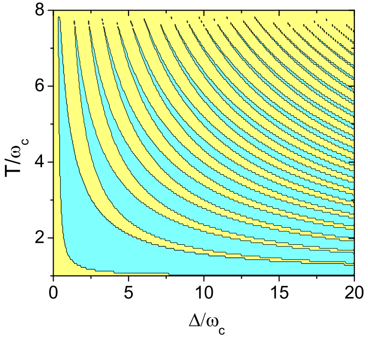
Non-equilibrium phase diagram

- The phases of a static system are characterized by its eigen energies
- The phases of a periodically driven system are characterized by its Floquet quasi-energies



The non-equilibrium phase diagram of the driven system. The light yellow and cyan areas means, respectively the phase that without and with the Floquet bound state. $\tau = 1.0\omega_c^{-1}$

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Conclusions

- The decoherence of the TLS can be suppressed by the periodic driving
- A one-to-one correspondence between decoherence suppression and the formation of a Floquet bound state is established
- It suggests a mechanism to beat decoherence by periodic driving

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Reference

1. Qing-Jun Tong, Jun-Hong An, Jiangbin Gong, Hong-Gang Luo, C. H. Oh, *Generating Many Majorana Modes via Periodic Driving: A Superconductor Model*, **Phys. Rev. B 87, 201109(R) (2013)**.

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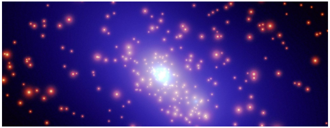
MF: Elementary particle

In 1937, Majorana: *Neutral spin-$\frac{1}{2}$ particle can be described by a real Dirac's equation, and would thus be identical to its antiparticle* E. Majorana, 14, 171 (1937)

- The particle at the border between matter and antimatter
  - A particle of its own antiparticle: $\hat{\gamma}_A = (\hat{c}^\dagger + \hat{c})/2,\ \hat{\gamma}_B = (\hat{c}^\dagger - \hat{c})/2i \Rightarrow \hat{\gamma}_j = \hat{\gamma}_j^\dagger$

  - Non-abelian statistics
- Where is MF: No experimental evidence. People suspect

Neutrino



Dark matter

F. Wilczek, Nature Phys. 5, 614 (2009)

MF: Quasi-particle excitation in CMP

A superconductor system:

- Particle-hole symmetry: Its eigenmode is formed by the quasi-particle excitation:

$$\hat{\gamma}_E = \sum_{j=1}^{N} (u_{j,E}\hat{c}_j^\dagger + v_{j,E}\hat{c}_j)$$

  which may have

$$\hat{\gamma}_{E=0} = \hat{\gamma}_{E=0}^\dagger$$

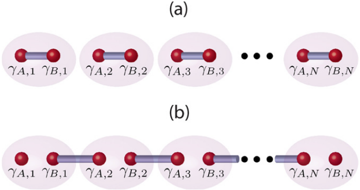
- MF can be simulated as a zero-energy quasi-excitation inside the vortex of a p-wave superconductor

  Ivanov, PRL 86, 268 (2001)

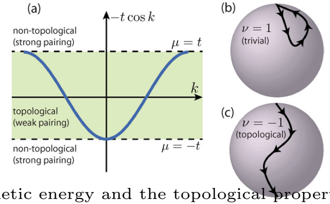
1D Kitaev model

$$
\begin{aligned}
\hat{H} &= -\mu \sum_{j}^{N} \hat{c}_j^\dagger \hat{c}_j - \frac{1}{2} \sum_{j}^{N-1} (t\hat{c}_j^\dagger \hat{c}_{j+1} + \Delta \hat{c}_j \hat{c}_{j+1} + \text{H.c.}) \\
&= \sum_k (\hat{c}_k^\dagger, \ \hat{c}_{-k}) \mathbf{h}_k \cdot \boldsymbol{\sigma} \begin{pmatrix} \hat{c}_k \\ \hat{c}_{-k}^\dagger \end{pmatrix} \qquad \text{(Bogoliubov-de Gennes Ham.)} \\
&= \frac{-\mu}{2} \sum_{j}^{N} (1 + i\hat{\gamma}_{B,j}\hat{\gamma}_{A,j}) - \frac{i}{4} \sum_{j}^{N-1} [(\Delta + t)\hat{\gamma}_{B,j}\hat{\gamma}_{A,j+1} \\
&\quad +(\Delta - t)\hat{\gamma}_{A,j}\hat{\gamma}_{B,j+1}]
\end{aligned}
$$

J. Alicea, Rep. Prog. Phys. 75, 076501 (2012)

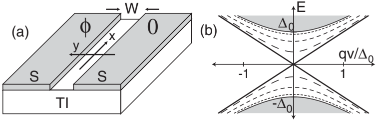


MFs in 1D Kitaev model. (a): $\mu \neq 0$ and $t = \Delta = 0$. (b): $\mu = 0$ and $t = \Delta \neq 0$

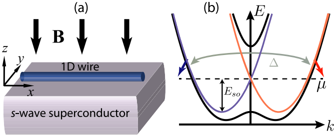


Kinetic energy and the topological properties. $\nu = \prod_{k=0,\pi} \text{sgn}(\mathbf{h}_k)$

Schemes on realization



2D topological insulator. Fu and Kane, PRL 100, 096407 (2008)



1D nanowire. (b): Time-reversal symmetry is present (red & blue) and broken by **B** (black). Lutchyn, Sau, and Das Sarma, PRL 105, 077001 (2010)

Conditions

1. Superconductor-topological insulator-superconductor structure
2. Majorana bound states at vortices

Conditions

1. Spin-orbit interactions in the nanowire
2. In the proximity to an s-wave superconductor
3. Moderate magnetic field

MF in QIS: Topological Q. computating

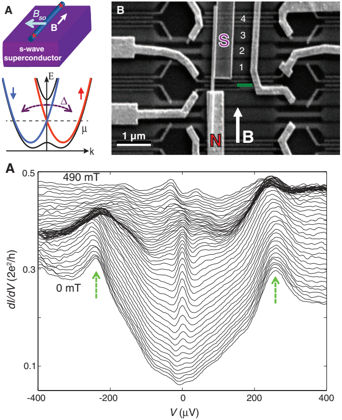
MFs define a topologically protected Q. memory

- 2 Majorana separated bound states = 1 fermion
  - 2 degenerate states (full/empty) = 1 qubit
- 2N separated Majoranas = N qubits
- Q. information is stored non locally
  - Immune from local decoherence
- Braiding performs unitary operations: Non-Abelian statistics

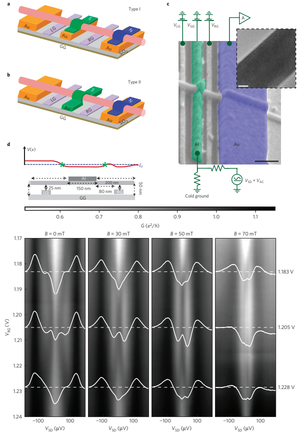
Ivanov, PRL 86, 268 (2001); Kitaev (2003)

Detection of MFs: The Q. transport idea

Zero-bias peak of differential conductance as signature of MFs



InSb nanowires contacted with one normal (gold) and one superconducting (NbTiN) electrode. Mourik *et al.*, Science 336, 1003 (2012)



Al-InAs nanowire on gold pedestals above p-type silicon. Das, Ronen, Most, Oreg, Heiblum and Shtrikman, Nat. Phys. 8, 887 (2012)

Zero-bias peak of differential conductance

Subsequent studies show that, the zero-bias peak can also be generated in topologically trivial system due to

- the strong disorder in the nanowire

  Liu, Potter, Law, Lee, PRL 109, 267002 (2012)

  Pikulin, Dahlhaus, Wimmer, Schomerus, Beenakker, NJP 14, 125011 (2012)

- the smooth confinement potential at the wire end

  Kells, Meidan, Brouwer, PRB 86, 100503(R) (2012)

"... implies that the mere observation of a zero-bias peak in the tunneling conductance is not an exclusive signature of a topological superconducting phase, even in the ideal clean single channel limit"

More ways to double-confirm the formation of MF in the relevant systems are desired

Motivation

1. Can periodic driving induce more MFs to enhance the experimental signature generated by MFs?
2. Can periodic driving supply a novel way to identify the experimental signal generated by MFs from other mechanism?

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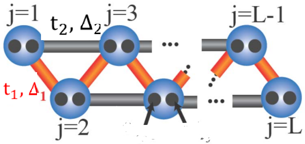
Model

$$\hat{H} = -\mu \sum_{l=1}^{N} \hat{c}_l^\dagger \hat{c}_l - \sum_{a=1}^{2} \sum_{l=1}^{N-a} (t_a \hat{c}_l^\dagger \hat{c}_{l+a} + \Delta_a \hat{c}_l^\dagger \hat{c}_{l+a}^\dagger + \text{H.c.})$$



- Chemical potential: μ
- Hopping amplitude: t_a
- Pairing potential: $\Delta_a = |\Delta_a| e^{i\phi_a}$

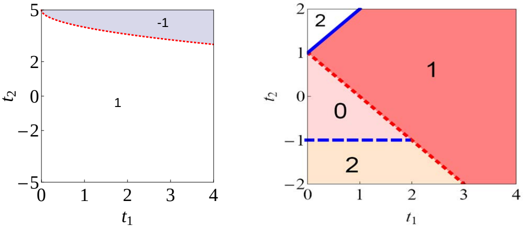
$\phi = \phi_1 - \phi_2$ determines the topological class [1]

	Symmetry	T. class	T. invariant	MMs[2]
$\phi = 0, \pi$	T,PH,C	BDI	$\mathbb{Z}$	2
ϕ =other	PH	D	$\mathbb{Z}_2$	1

[1] Ryu, Schnyder, Furusaki, and Ludwig, NJP **12**, 065010 (2010)

[2] Schnyder, Ryu, Furusaki, and Ludwig, PRB **78**, 195125 (2008)

How to generate more MFs



Phase diagram in D class charactarized by ν (left) and BDI class characterized by winding number W (right)

Two ingredients in generating more MFs [3]

1. Time reversal symmetry
2. Long-range interactions

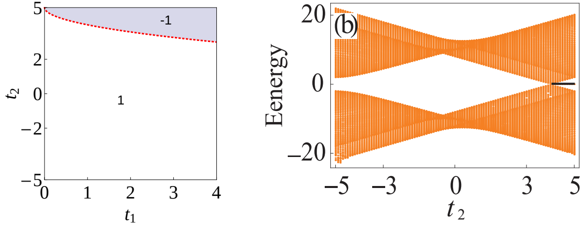
Both introduce additional difficulties to the practical experiments

[3] Niu, Chung, Hsu, Mandal, Raghu, and Chakravarty, PRB **85**, 035110 (2012)

Driving protocol

$$\hat{H}(t) = \begin{cases} \hat{H}_1 = \hat{H}(\phi_1, \phi_2), & \text{for} \quad t \in [nT, (n+1/2)T) \\ \hat{H}_2 = \hat{H}(\phi_2, \phi_1), & \text{for} \quad t \in [(n+1/2)T, (n+1)T) \end{cases}$$

- For each of the half periods, the system belongs to D class, where at most one pair of MFs can be formed



Phase diagram in D class characterized by ν (left) and the corresponding energy spectrum

Role of periodic driving in $\hat{H}_{\text{eff}}$

$$[\hat{H}(t) - i\hbar\partial_t]|\phi_n(t)\rangle = \varepsilon_n|\phi_n(t)\rangle \Leftrightarrow \hat{U}_T|\phi_n(0)\rangle = e^{\frac{-i}{\hbar}\hat{H}_{\text{eff}}T}|\phi_n(0)\rangle$$

1. Restore time-reversal symmetry

$$\hat{\bar{\mathcal{K}}}\hat{U}_T\hat{\bar{\mathcal{K}}}^{-1} = e^{\frac{i\hat{H}_1 T}{2\hbar}}e^{\frac{i\hat{H}_2 T}{2\hbar}} = \hat{U}_T^\dagger,$$

with $\hat{\bar{\mathcal{K}}} \equiv \hat{\mathcal{K}}\hat{G}$ and $\hat{G} = e^{-i\frac{\phi_1+\phi_2}{2}\sum_l c_l^\dagger c_l}$.

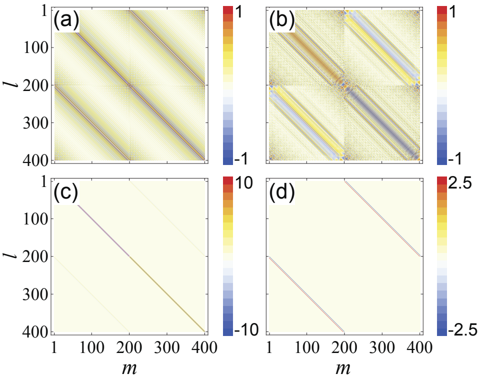
2. Synthesize longer-range interaction (Baker-Campbell-Hausdorff formula)

$$\hat{U}_T = e^{-\frac{iT}{2\hbar}\hat{H}_2}e^{-\frac{iT}{2\hbar}\hat{H}_1} \equiv e^{\frac{-i}{\hbar}\hat{H}_{\text{eff}}T}$$

$$\hat{H}_{\text{eff}} = \frac{\hat{H}_1+\hat{H}_2}{2} - \frac{iT}{8\hbar}[\hat{H}_2, \hat{H}_1] - \frac{T^2}{96\hbar^2}\left[\hat{H}_2 - \hat{H}_1, [\hat{H}_2, \hat{H}_1]\right] + \cdots$$

Even the interactions in $\hat{H}_1$ and $\hat{H}_2$ are short ranged, the ones in $\hat{H}_{\text{eff}}$ may be long ranged due to the commutators.

Numerical confirmation

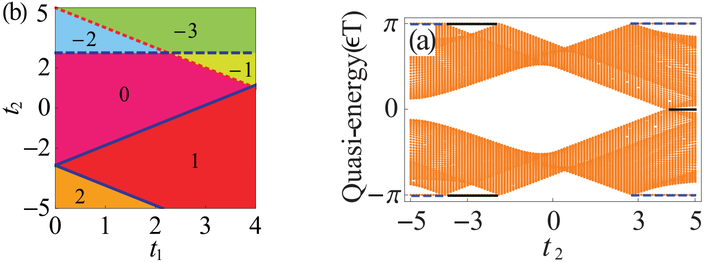


coefficients of Hamiltonian expanded in the operator basis $(c_1, \cdots, c_N, c_1^\dagger, \cdots, c_N^\dagger)^T$ when $T = 0.2$ (a) and 2.0 (b) and the real (c) and the imaginary (d) of static case.

1. Interaction range is enhanced for the driven case (a,b) comparing with the static case (c,d)
2. The expansion coefficients of $\hat{H}_{\text{eff}}$ are real, which confirms the restoring of time-reversal symmetry

Multiple MFs

1. Multiple MFs can be generated

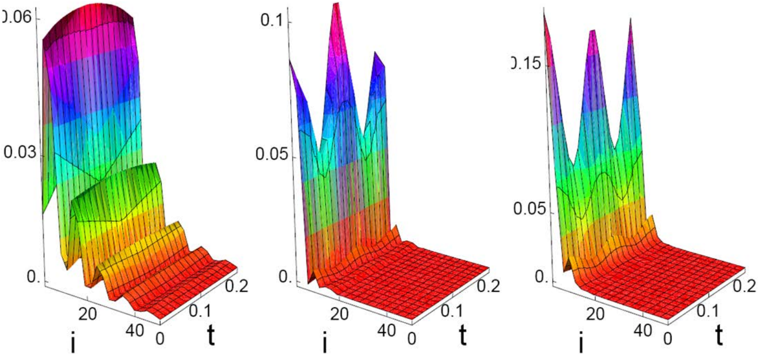


Phase diagram and quasi-energy spectrum of the periodically driven system. $T = 0.2$

2. We may go further by increasing T, which induces longer-range interactions in $\hat{H}_{\text{eff}}$

t_2	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
$T = 0.5$	2	4	4	3	3	2	0	0	0	1	1	2	2	4	4	4	3
$T = 1.0$	6	6	7	7	6	3	3	2	1	1	2	5	5	6	7	7	6
$T = 2.0$	13	13	12	11	9	8	8	1	1	3	4	7	11	10	13	13	12

Spatial distribution of MFs



The evolution of the distribution of the generated three MFs over the lattices

- All the generated MFs are confined in the edge of the lattice during the time evolutioin

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Conclusions

- The number of the MFs may be greatly enhanced and widely tuned by periodic driving
  1. The enhanced signal is more robust against experimental disorder, and contaminations from thermal excitations
  2. It supplies a novel way to identify if the signal originates from MFs by observing the change of the signal in response of the tuning of the driving coefficients
- The generation of a tunable number of MFs is expected to offer another dimension for experimental studies

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Summary

- We may manipulate (quasi-)energy spectrum by periodic driving such that novel quantum phase transition is triggered
- Periodically driven system may exhibit novel properties with no analogue to its static correspondence

Thank you for your attention