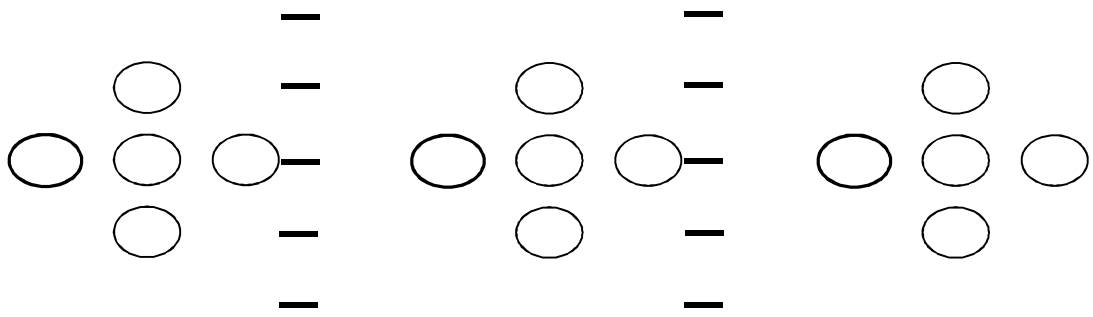
2D cluster state

- Created by applying CZ gates to each pair with edge

$$|C\rangle \equiv \bigotimes_{\langle i,j \rangle} CZ_{ij} \ (|+\rangle|+\rangle \cdots |+\rangle)$$

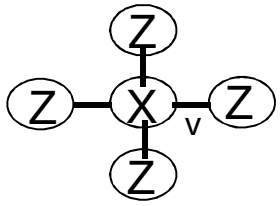
[Raussendorf&Briegel '01]



$$|+\rangle \equiv (|0\rangle + |1\rangle)/\sqrt{2}$$

- Ground state of 5-body Hamiltonian

$$H = -\sum_v K_v = -\sum_v$$



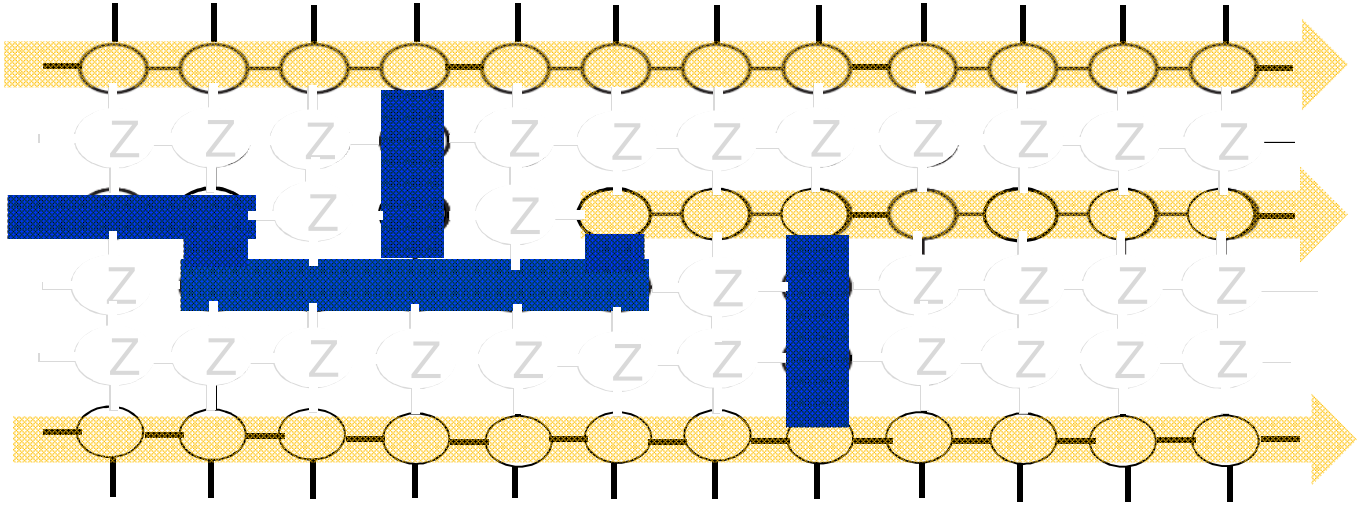
(except boundary spins)

X, Y, Z Pauli matrices

Resource for universal quantum computation

[Raussendorf&Briegel '01]

- Carve out entanglement structure by local Z measurement



(1) Measurement along each wire simulates one-qubit evolution (gates)

(2) Measurement along each bridge simulates two-qubit gate (CNOT)

⇒ Universal measurement-based quantum computation (MBQC)

Other states for universal QC?

- The first known resource state is the 2D cluster state
  - Other 2D *graph/cluster* states* on regular lattices: triangular, honeycomb, kagome, etc. [*Van den Nest et al.* '06]
  - A few other states from tensor network construction and TriCluster state [*Gross & Eisert* '07, '10] [*Chen et al.* '09]
  - Family of 2D Affleck-Kennedy-Lieb-Tasaki states [***Wei***, *Affleck & Raussendorf '11&12; Miyake '11;* ***Wei*** *'13,* ***Wei***, *Haghnagadar &Raussendorf '14*]

☹ *So far still no complete characterization for resource states*

Resource states from ground states?

- Unique ground states of certain gapped Hamiltonians?

$$H|\Psi_{\text{resource}}\rangle = E_0|\Psi_{\text{resource}}\rangle$$

  - ➔ If so, create resources by cooling!
  - ➔ Desire simple and short-ranged (nearest nbr) 2-body Hamiltonians
    - ❖ Cluster states require few-body (e.g. 5-body) interactions! [*Nielsen* '06]
    - ❖ AKLT states are ground state of two-body interacting Hamiltonians (possibly gapped) [*AKLT* '87,88] [*Garcia-Saez,Murg,**Wei*** '12]

- What about thermal states for quantum computation?

$$\rho_{\text{resource}} = e^{-\beta H}/\text{Tr}(e^{-\beta H})$$ useful?

[*Li,Browne,Kwek,Raussendorf &**Wei*** '11]

Main motivations here

- Universal resource states from certain phases of matter?

  ➔ it's a difficult question in general, but some examples:

  - Browne et al.--- percolation
  - Bartlett et al. --- (1) Cluster in B field; (2) deformed AKLT
  - Murao et al. --- interacting cluster at finite T
  - Li et al. --- thermal states for QC

- Can we can characterize regions in the phase diagram by the quantum computational power of the equilibrium states?

- System parameter vs. temperature "phase diagram" in terms of universal quantum computation?

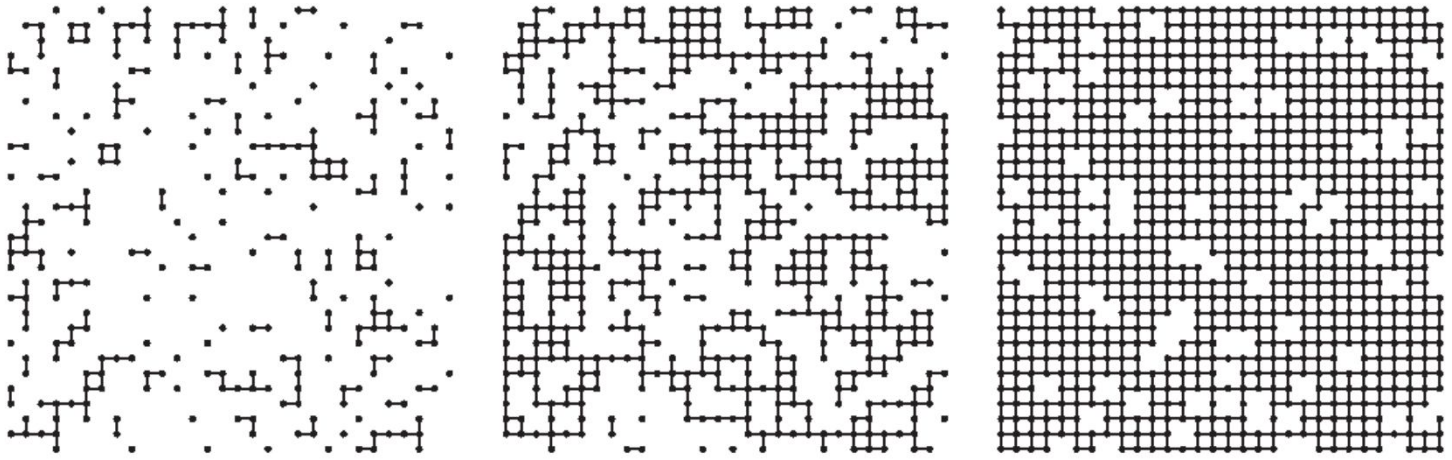
Some examples why transitions in quantum computational power make sense…

Cluster state and percolation

- Cluster-state at faulty square lattice:

[*Browne, Elliot, Flammia, Merkel, Miyake,Short* '08]

cf [*Gross, Eisert, Schuch,Perez-Garcia rowne, '*07]



p_{occupy} : system parameter➔ QC possible if $p_{occupy} > p_{perco.threshold}$

➔ Transition in quantum computational power

Cluster phase?

- Doherty and Bartlett: Cluster Hamiltonian in B field

$$H = -\sum_v K_v - B\sum_v X_v$$

[*Doherty, Bartlett* '09]

➔ can be mapped to $H = -\sum_{\square} \bar{Z}\bar{Z}\bar{Z}\bar{Z} - B\sum_{\mu} \bar{X}_{\mu}$

with known phase transition at |B|=1

- They argue that the phase |B|<1 is characterized by fidelities of universal gates

[*Doherty, Bartlett* '09]

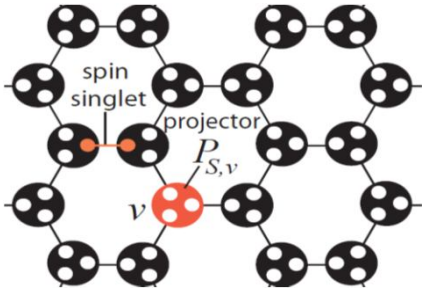
➔ transition in quantum computational power

- Question: what about finite temperature T?

Deforming AKLT state

- Spin-3/2 AKLT state on honeycomb lattice is universal

[***Wei***, *Affleck & Raussendorf '11; Miyake '11*]



- Deformed AKLT also universal (in a range of deformation)

$$|\psi(a)\rangle \propto \left(D(a)^{-1}\right)^{\otimes N} |\psi_{\mathrm{AKLT}}\rangle$$

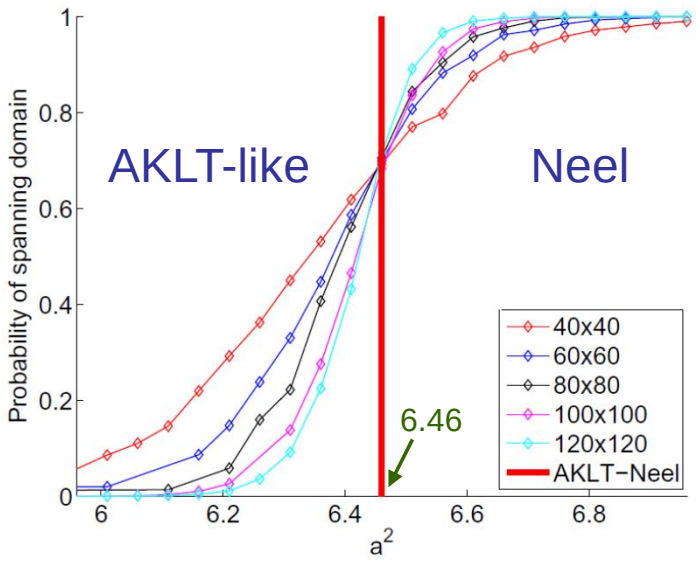
[*Darmawan,Brennen,Bartlett '12*]

➔ Transition to non-universal coincides with transition to Neel order

➔ Q: finite T?

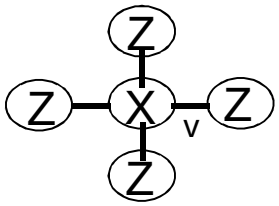
$$D(a) = \frac{\sqrt{3}}{a}(|+3/2\rangle\langle+3/2| + |-3/2\rangle\langle-3/2|) + |1/2\rangle\langle 1/2| + |-1/2\rangle\langle -1/2|$$

$$H = \sum_{\langle i,j\rangle} D(a)_i \otimes D(a)_j \; h_{ij}^{[\mathrm{AKLT}]} \; D(a)_i \otimes D(a)_j$$



Interacting cluster Hamiltonian at finite T

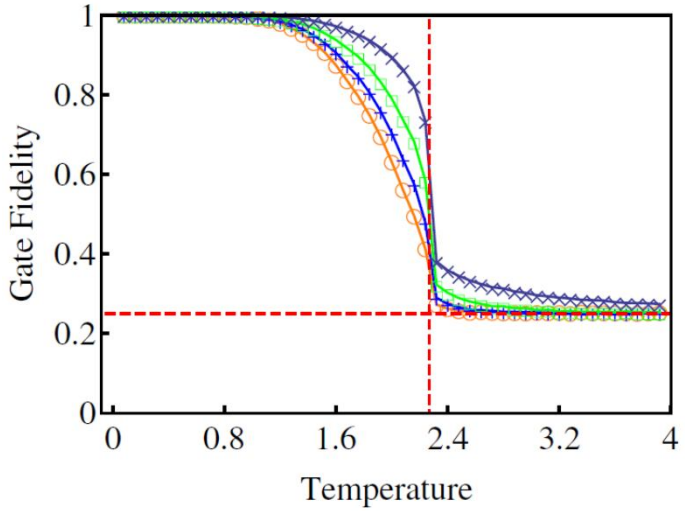
$$H = -\sum_v K_v = -\sum_v$$



➔ $H' = -\sum_{\langle u,v \rangle} K_u K_v \rightarrow \sum_{\langle u,v \rangle} X_u X_v$ (mapped to classical Ising by CZ)

[*Fujii, Nakata, Ohzeki, Murao '13*]

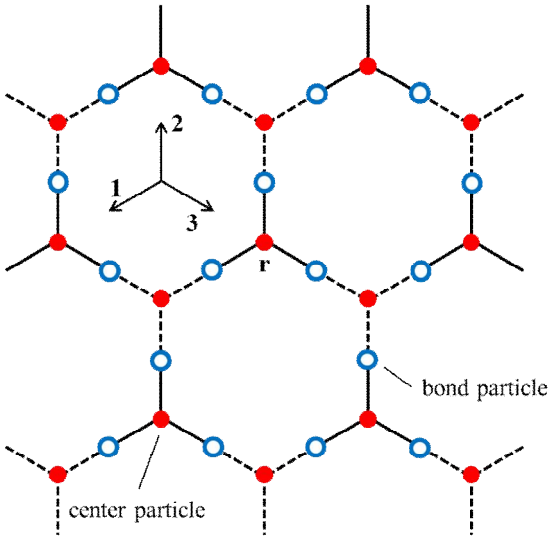
- Gate fidelity shows a transition at classical Ising transition T_c



➔ Q: can we add local field to vary Hamiltonian?

$$H'' = -\sum_{\langle u,v \rangle} K_u K_v - B \sum_v X_v$$

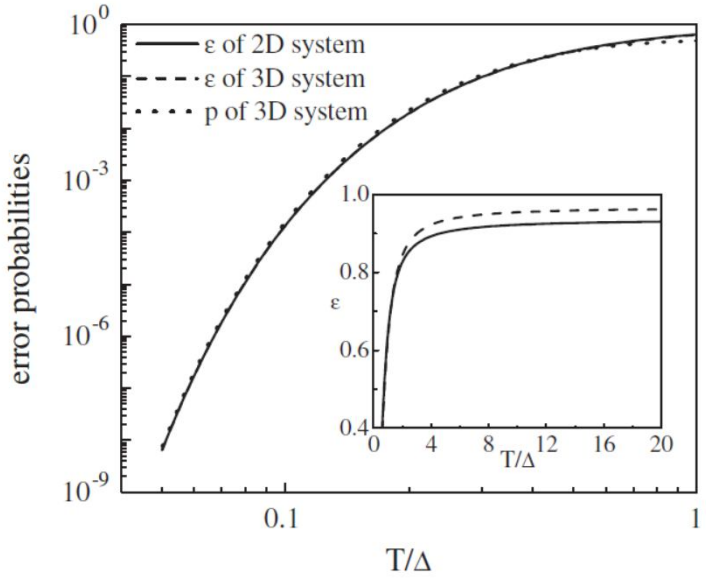
Thermal states as resource



- 2D & 3D spin Hamiltonians

[*Li, Browne, Kwek, Raussendorf,* ***Wei*** '11]

$$V_{\text{line}} = \Delta(S^x_{\mathbf{c}} A^x_{\mathbf{b}} + S^y_{\mathbf{c}} A^y_{\mathbf{b}} + S^z_{\mathbf{c}} A^z_{\mathbf{b}})$$
$$V_{\text{dash}} = \Delta(S^x_{\mathbf{c}} B^x_{\mathbf{b}} + S^y_{\mathbf{c}} B^y_{\mathbf{b}} + S^z_{\mathbf{c}} B^z_{\mathbf{b}})$$

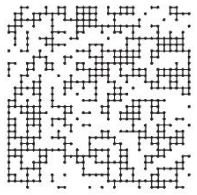


➔ Thermal states can be used for universal QC, if T/Δ is such that error below threshold

Q: can we vary some system parameter or local field?

Goal: Phase diagram with temperature and other system parameter

- Models illustrated:
  - Percolation: vary probability only



  - Cluster with B field and deformed AKLT: only at zero T

$$H = -\sum_{v} K_v - B\sum_{v} X_v$$

$$H = \sum_{\langle i,j \rangle} D(a)_i \otimes D(a)_j \; h_{ij}^{[\mathrm{AKLT}]} \; D(a)_i \otimes D(a)_j$$

  - Interacting cluster at finite T: mapped to classical Ising

$$H' = -\sum_{\langle u,v \rangle} K_u K_v \rightarrow -\sum_{\langle u,v \rangle} X_u X_v$$

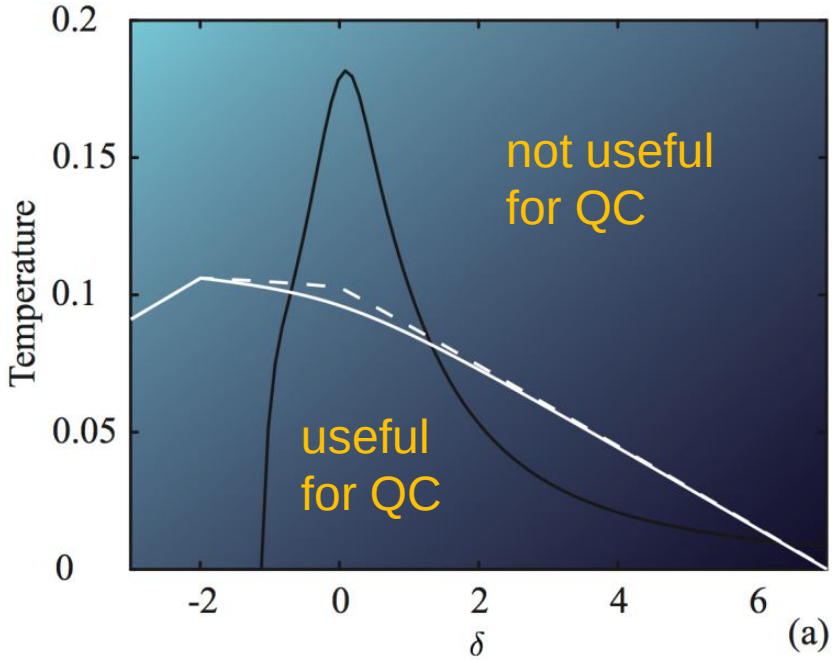
  - Thermal states for universal QC: Hamiltonian Heisenberg-like

- Varying both T & some system parameter?
  ➔ difficult problem: Hamiltonian not solvable in general

Goal: Phase diagram of quantum computational power

- Another question: Does transition in quantum computational power necessarily coincide with transition in the phase of matter?

- Will construct two models to investigate these



[***Wei***,*Li*,*Kwek* ‘14]

Outline

I. Introduction

- quantum computation by local measurement
- notion of transitions in quantum computational power

II. Example Hamiltonians: Building blocks

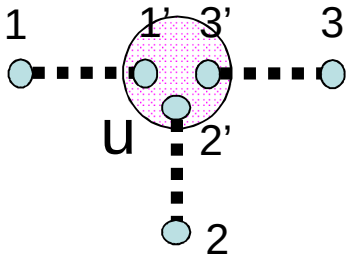
III. 2D/3D structure

- Phase diagrams for quantum computational power

IV. Summary

First: Toy model

- 3 spin-1/2 at sides and 1 spin-3/2 at center *u*



$$|\text{Singlets}\rangle \equiv (|01\rangle - |10\rangle)_{11'}(|01\rangle - |10\rangle)_{22'}(|01\rangle - |10\rangle)_{33'}$$

Mathematically project joint state of (1'2'3') to their symmetric subspace (i.e. virtual to physical)

$$\Pi_S^{[1'2'3'\to u]} = \left|+\frac{3}{2}\right\rangle\left\langle 000\right| + \left|-\frac{3}{2}\right\rangle\left\langle 111\right| + \left|+\frac{1}{2}\right\rangle\left\langle W\right| + \left|-\frac{1}{2}\right\rangle\left\langle \overline{W}\right|$$

$$|\overline{W}\rangle \equiv \frac{1}{\sqrt{3}}(|110\rangle + |101\rangle + |011\rangle) \leftrightarrow \left|-\frac{1}{2}\right\rangle$$

$$|W\rangle \equiv \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle) \leftrightarrow \left|\frac{1}{2}\right\rangle$$

Spin 3/2 and three virtual qubits

- Addition of angular momenta of 3 spin-1/2's

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1 \qquad \frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}$$

Symmetric subspace

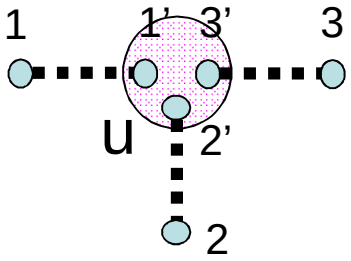
- The four basis states in the symmetric subspace

$$|111\rangle \equiv |\downarrow\downarrow\downarrow\rangle \leftrightarrow \left|-\frac{3}{2}\right\rangle \qquad |\overline{W}\rangle \equiv \frac{1}{\sqrt{3}}(|110\rangle + |101\rangle + |011\rangle) \leftrightarrow \left|-\frac{1}{2}\right\rangle$$

$$|000\rangle \equiv |\uparrow\uparrow\uparrow\rangle \leftrightarrow \left|\frac{3}{2}\right\rangle \qquad |W\rangle \equiv \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle) \leftrightarrow \left|\frac{1}{2}\right\rangle$$

Toy model: building block

- 3 spin-1/2 at sides and 1 spin-3/2 at center *u*



$$|\text{Singlets}\rangle \equiv (|01\rangle - |10\rangle)_{11'}(|01\rangle - |10\rangle)_{22'}(|01\rangle - |10\rangle)_{33'}$$

$$\Pi_S^{[1'2'3'\to u]} = \left|+\tfrac{3}{2}\right\rangle\langle 000| + \left|-\tfrac{3}{2}\right\rangle\langle 111| + \left|+\tfrac{1}{2}\right\rangle\langle W| + \left|-\tfrac{1}{2}\right\rangle\langle \overline{W}|$$

➔ Form an AKLT-like state:

$$|\psi_{\text{AKLT}}\rangle \equiv \Pi_S^{[1'2'3'\to u]}|\text{Singlets}\rangle$$

GHZ state: $|\uparrow\uparrow\uparrow\Downarrow\rangle - |\downarrow\downarrow\downarrow\Uparrow\rangle$

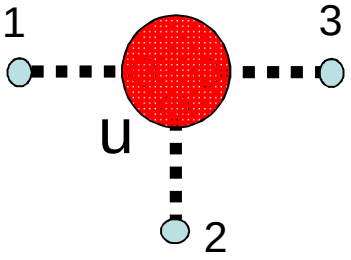
$$= |000\rangle_{123}\otimes\left|-\tfrac{3}{2}\right\rangle_u - |111\rangle_{123}\otimes\left|\tfrac{3}{2}\right\rangle_u - |W\rangle_{123}\otimes\left|-\tfrac{1}{2}\right\rangle_u + |\overline{W}\rangle_{123}\otimes\left|\tfrac{1}{2}\right\rangle_u$$

- Unique ground state of

Projector to total S=2 joint subspace of spins *i* and *u*

$$H = \sum_{i=1}^{3} P_{i,u}^{[S=2]} = \sum_i \left[\frac{1}{2}\vec{S}_u\cdot\vec{s}_i + \frac{5}{8}\right]$$

Toy model Hamiltonians



$$H = \sum_{i=1,2,3} \vec{S}_u \cdot \vec{s}_i$$

spin 3/2

spin 1/2

- Exactly diagonalizable

  ➔ Finite energy gap = 1

- Next: Allow the Hamiltonians to vary

$$H_1 = \sum_{i=1,2,3} [S_u^x \, s_i^x + S_u^y \, s_i^y + (1+\delta) S_u^z \, s_i^z]$$

$$H_2 = \sum_{i=1,2,3} \left(\vec{S}_u \cdot \vec{s}_i\right) - d_z \left(S_u^z\right)^2$$

Toy model is for 4 spins.

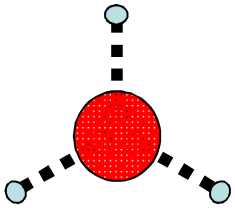
Q: how do we scale to 2D or 3D structure for universal quantum computation?

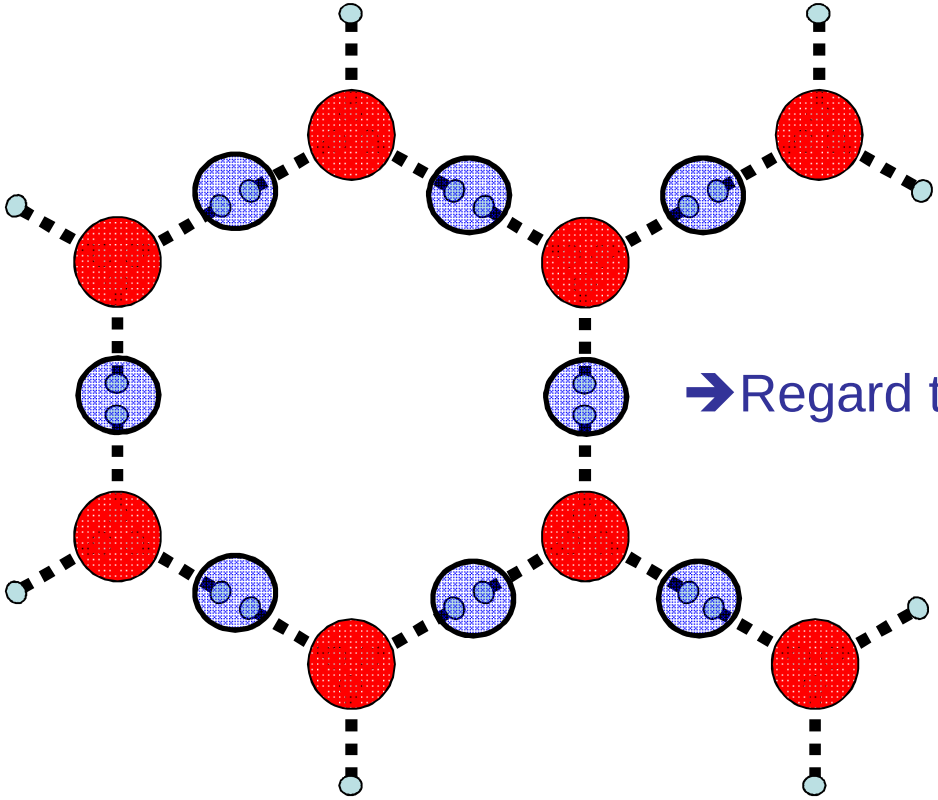
Ans. Use toy model as a building block

➔ Patch up 2D or 3D structure

2D or 3D spin models

- Use this as a building block to construct spin systems with a spectral gap





➔Regard two bond qubits as a spin-3/2

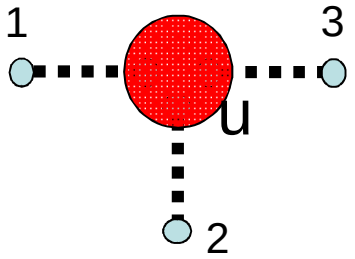
: bond particle

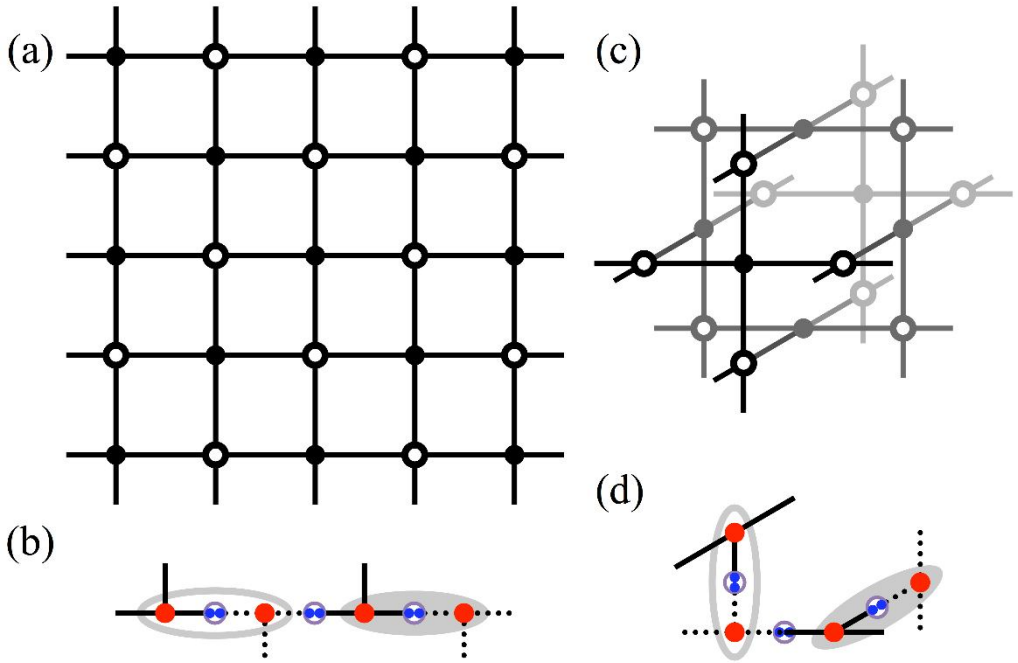
: center particle

- The spectral property is inherited from toy model

2D or 3D structure

- Use this as a building block to construct spin systems with a spectral gap

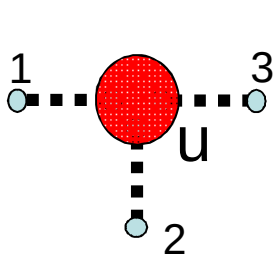


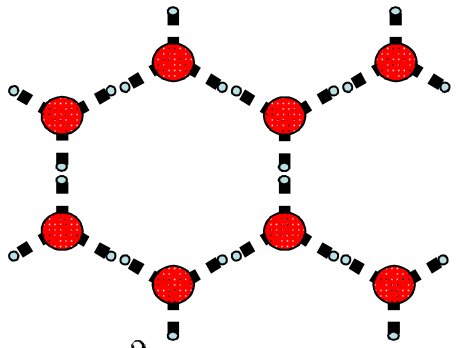


$$H_1 = \sum_{i=1,2,3} [S_u^x \, s_i^x + S_u^y \, s_i^y + (1+\delta) S_u^z \, s_i^z]$$

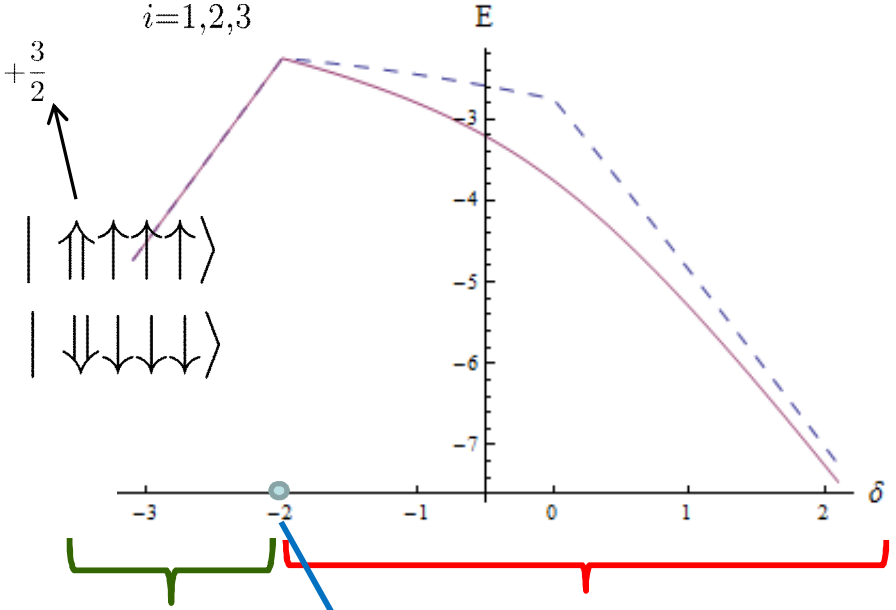
$$H_2 = \sum_{i=1,2,3} \left(\vec{S}_u \cdot \vec{s}_i\right) - d_z \left(S_u^z\right)^2$$

Two spin models





$$H_1 = \sum_{i=1,2,3} \left[S_u^x \, s_i^x + S_u^y \, s_i^y + (1+\delta) S_u^z \, s_i^z \right]$$

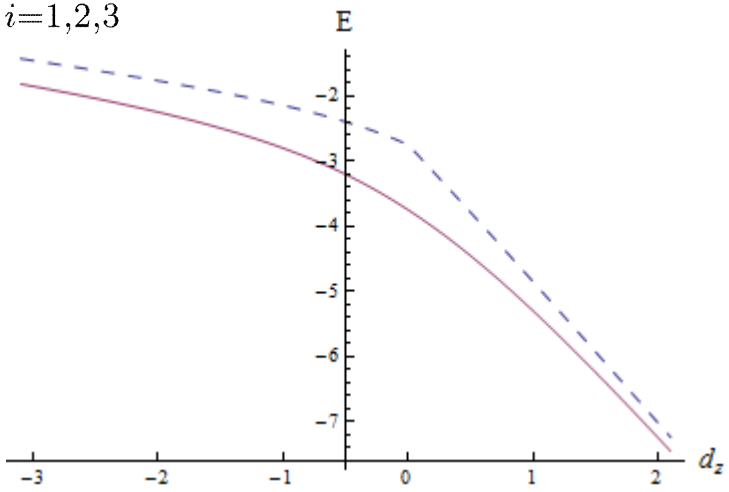


Ferro-like
(degenerate)

AKLT-like(gapped)

1st-order phase transition

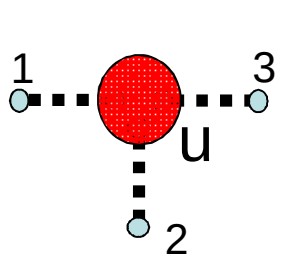
$$H_2 = \sum_{i=1,2,3} \left(\vec{S}_u \cdot \vec{s}_i \right) - d_z \left(S_u^z \right)^2$$

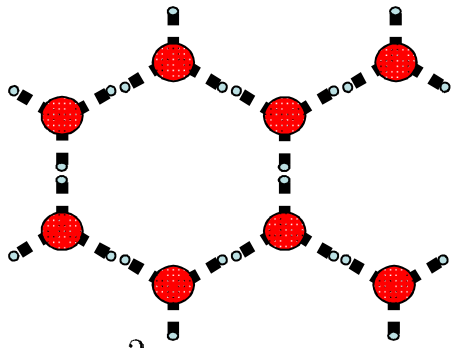


AKLT-like(gapped)
➔ no phase transitiion

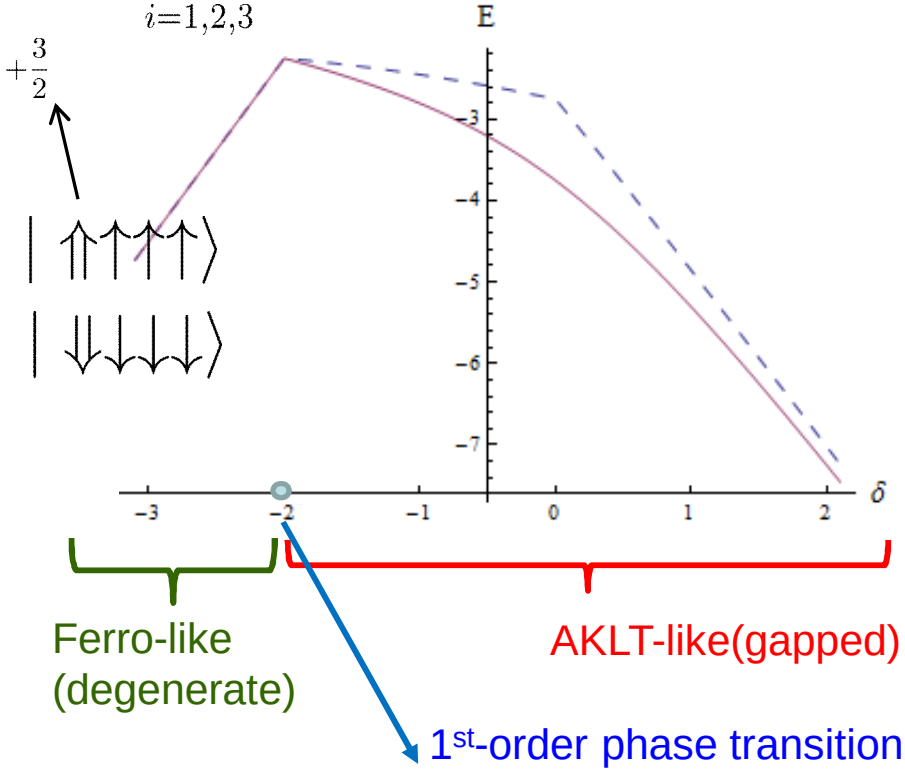
- No transitions at finite T (free energy analytic) $f = -T \log \left[\mathrm{Tr}(e^{-\beta H}) \right]$

Two spin models





$$H_1 = \sum_{i=1,2,3} [S_u^x s_i^x + S_u^y s_i^y + (1+\delta) S_u^z s_i^z]$$



Ferro-like
(degenerate)

AKLT-like(gapped)

1st-order phase transition

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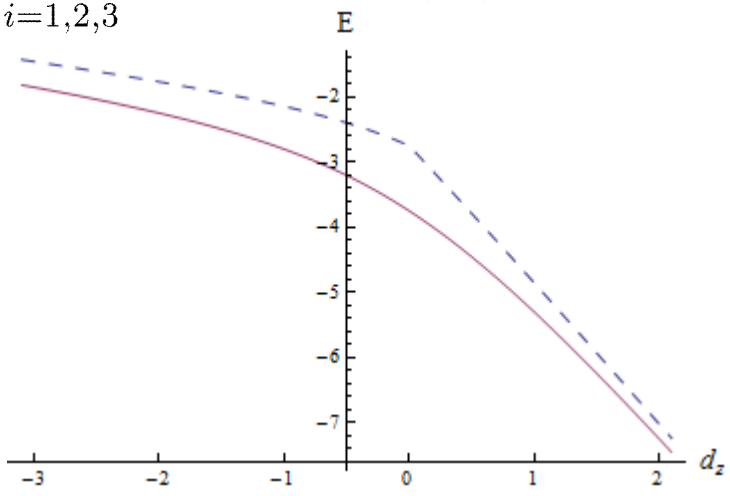

AKLT-like(gapped)
➔ no phase transitiion

- What about transitions in quantum computational power?

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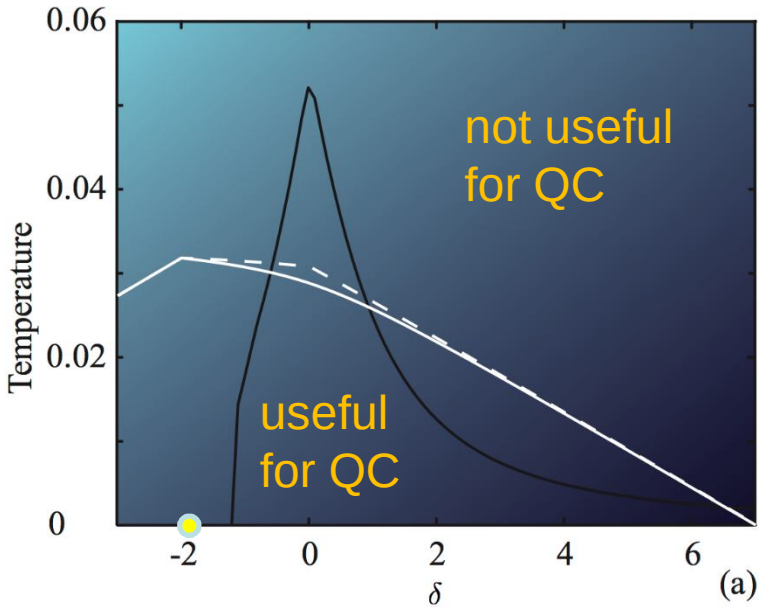
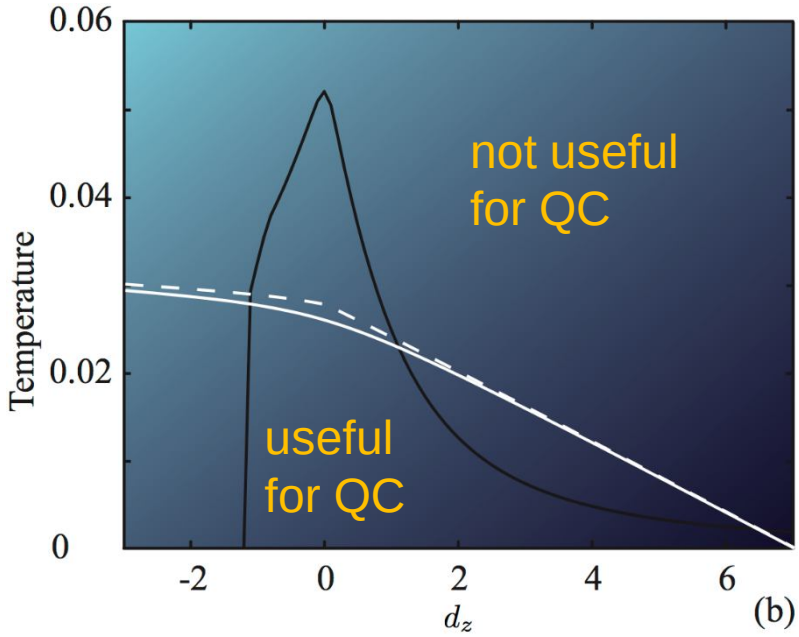
Computational phase (in 2D)

Model 1

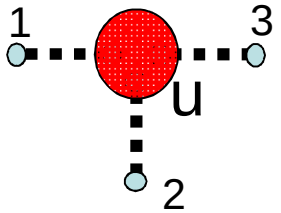


Model 2

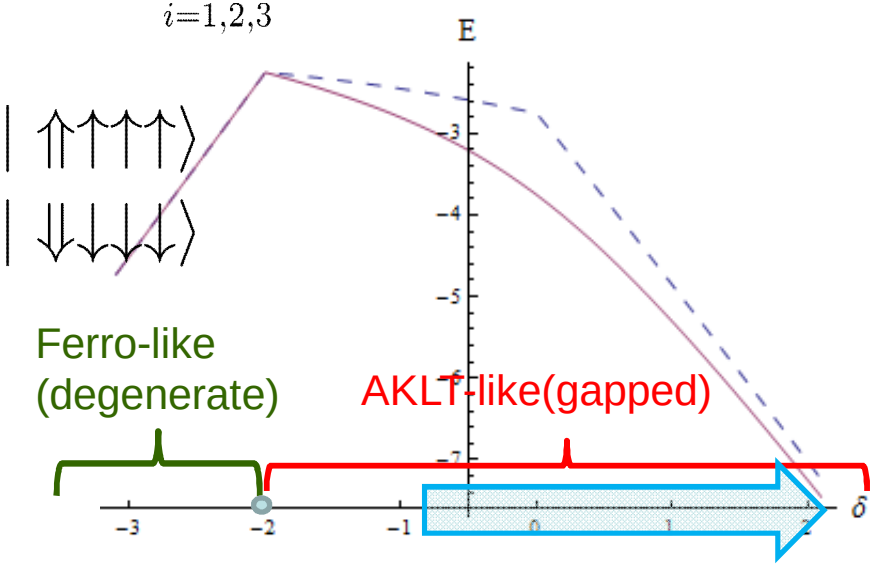


- How to understand the computational phase in δ (d_z) –vs.-T plane?

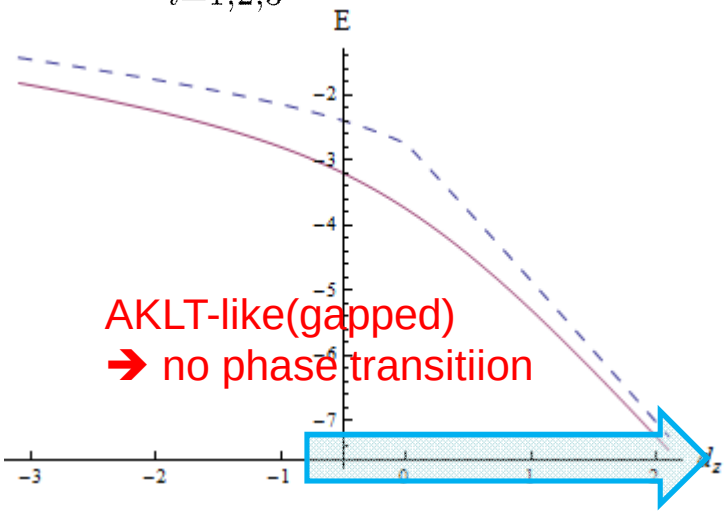
Zero temperature



$$H_1 = \sum_{i=1,2,3} [S_u^x s_i^x + S_u^y s_i^y + (1+\delta) S_u^z s_i^z]$$



$$H_2 = \sum_{i=1,2,3} \left(\vec{S}_u \cdot \vec{s}_i\right) - d_z \left(S_u^z\right)^2$$

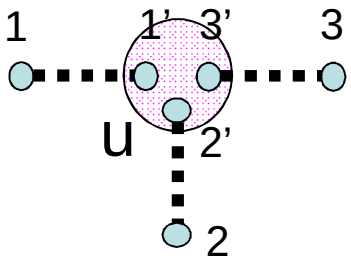


❖ Wavefunction:

$$|\Psi(\delta)\rangle = -(|3/2,111\rangle - |-3/2,000\rangle) + \frac{-2\delta + \sqrt{9+4\delta^2}}{3}(|1/2,\overline{W}\rangle - |-1/2,W\rangle)$$

➔ For $\delta, d_z \geq -\sqrt{3}/2$, can filter out a GHZ $|\Uparrow\downarrow\downarrow\downarrow\rangle - |\Downarrow\uparrow\uparrow\uparrow\rangle$ with probability=1 via generalize measurement (next slides)

Distill a four-spin GHZ state (@ $\delta=d_z=0$)



$$H_1 = \sum_{i=1,2,3} [S_u^x s_i^x + S_u^y s_i^y + (1+\delta) S_u^z s_i^z]$$

$$H_2 = \sum_{i=1,2,3} \left(\vec{S}_u \cdot \vec{s}_i\right) - d_z \left(S_u^z\right)^2$$

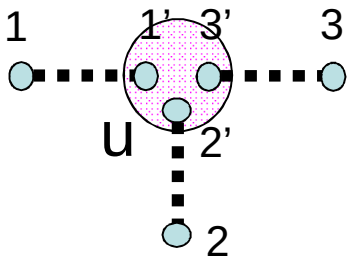
- Central spin has 4 levels: how to reduce to 2 levels?
  - ➔ Use projection? $P_z = \left|\frac{3}{2}\right\rangle_u \left\langle\frac{3}{2}\right| + \left|-\frac{3}{2}\right\rangle_u \left\langle-\frac{3}{2}\right|$

$$|000\rangle_{123} \otimes \left|-\frac{3}{2}\right\rangle_u - |111\rangle_{123} \otimes \left|\frac{3}{2}\right\rangle_u - |W\rangle_{123} \otimes \left|-\frac{1}{2}\right\rangle_u + |\overline{W}\rangle_{123} \otimes \left|\frac{1}{2}\right\rangle_u$$

$$\xrightarrow{P_z} \quad |000\rangle_{123} \otimes \left|-\frac{3}{2}\right\rangle_u - |111\rangle_{123} \otimes \left|\frac{3}{2}\right\rangle_u$$

- What if the projection does not succeed?

Distill a four-spin GHZ state (@ $\delta=d_z=0$)



$$H_1 = \sum_{i=1,2,3} [S_u^x s_i^x + S_u^y s_i^y + (1+\delta) S_u^z s_i^z]$$

$$H_2 = \sum_{i=1,2,3} \left(\vec{S}_u \cdot \vec{s}_i\right) - d_z \left(S_u^z\right)^2$$

- Central spin has 4 levels: how to reduce to 2 levels?

$$|\psi\rangle_{123u} = |000\rangle_{123} \otimes \left|-\frac{3}{2}\right\rangle_u - |111\rangle_{123} \otimes \left|\frac{3}{2}\right\rangle_u - |W\rangle_{123} \otimes \left|-\frac{1}{2}\right\rangle_u + |\overline{W}\rangle_{123} \otimes \left|\frac{1}{2}\right\rangle_u$$

➔ Use projection? $P_z = \left|\frac{3}{2}\right\rangle_u \left\langle\frac{3}{2}\right| + \left|-\frac{3}{2}\right\rangle_u \left\langle-\frac{3}{2}\right|$

$$|\psi\rangle_{123u} \xrightarrow{P_z} |000\rangle_{123} \otimes \left|-\frac{3}{2}\right\rangle_u - |111\rangle_{123} \otimes \left|\frac{3}{2}\right\rangle_u$$

- What if the projection does not succeed?

Generalized measurement (POVM)

➔ *x* and *y* axes are also good :

$$F_{u,z} = \sqrt{\frac{2}{3}}\left(\left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_z + \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_z\right)$$

$$F_{u,x} = \sqrt{\frac{2}{3}}\left(\left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_x + \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_x\right)$$

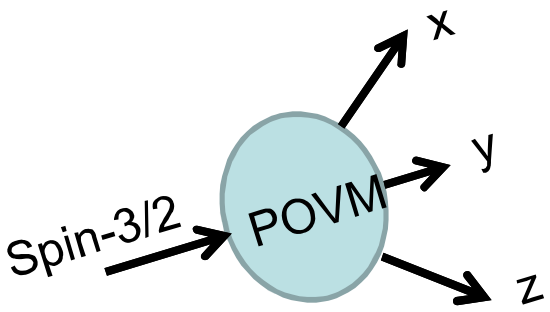
$$F_{u,y} = \sqrt{\frac{2}{3}}\left(\left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_y + \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_y\right)$$

[***Wei***,*Affleck & Raussendorf* '11; *Miyake* '11]

u: site index for center particles

➔ Three elements satisfy: $F_{u,x}^\dagger F_{u,x} + F_{u,y}^\dagger F_{u,y} + F_{u,z}^\dagger F_{u,z} = I_u$

- POVM outcome a_u={*x*,*y*, or *z*} is random



➔ effective 2-level system

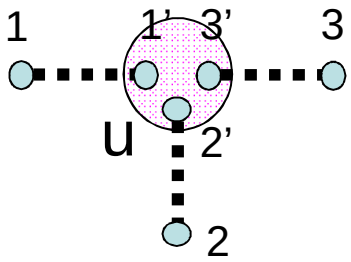
$$\left|\frac{3}{2}\right\rangle_{a_u} \leftrightarrow \left|000\right\rangle_{a_u}, \quad \left|-\frac{3}{2}\right\rangle_{a_u} \leftrightarrow \left|111\right\rangle_{a_u}$$

➔ a_u : new quantization axis

$$\overline{Z} \equiv \left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_{a_u} - \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_{a_u} \qquad \overline{X} \equiv \left|\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_{a_u} + \left|-\frac{3}{2}\right\rangle\left\langle+\frac{3}{2}\right|_{a_u}$$

➔ state becomes $|\Phi\rangle \longrightarrow F_{u,a_u}|\Phi\rangle$

Distill a four-spin GHZ state (@ $\delta=d_z=0$)



$$H_1 = \sum_{i=1,2,3} [S_u^x s_i^x + S_u^y s_i^y + (1+\delta) S_u^z s_i^z]$$

$$H_2 = \sum_{i=1,2,3} \left(\vec{S}_u \cdot \vec{s}_i\right) - d_z \left(S_u^z\right)^2$$

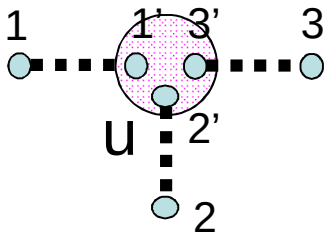
- POVM outcome a_u={x,y.z} indicates a randomly chosen quantization axis and the state becomes

$$F_{u,a_u}|\psi_{\mathrm{AKLT}}\rangle = |(+3/2)_u (111)_{123}\rangle_{a_u} - |(-3/2)_u (000)_{123}\rangle_{a_u}$$

  - Re-label states @ site u: $|(+3/2)_u\rangle \to |1_u\rangle,\ -|(-3/2)_u\rangle \to |0_u\rangle$

➔Post-POVM state is a GHZ state: $|(0)_u (000)_{123}\rangle + |(1)_u (111)_{123}\rangle$

Distill a four-spin GHZ state (@ $\delta=d_z\neq 0$)



$$|\Psi(\delta)\rangle = -(|3/2,111\rangle - |-3/2,000\rangle) + \underbrace{\frac{-2\delta+\sqrt{9+4\delta^2}}{3}}_{1/a}(|1/2,\overline{W}\rangle - |-1/2,W\rangle)$$

$$D(a) = |+3/2\rangle\langle+3/2| + |-3/2\rangle\langle-3/2| + a(|1/2\rangle\langle 1/2| + |-1/2\rangle\langle -1/2|)$$

- Use combination of filtering D(a) and POVM F's

$$F_x'(a) = \frac{1}{a}F_x D(a)$$

$$F_y'(a) = \frac{1}{a}F_y D(a)$$

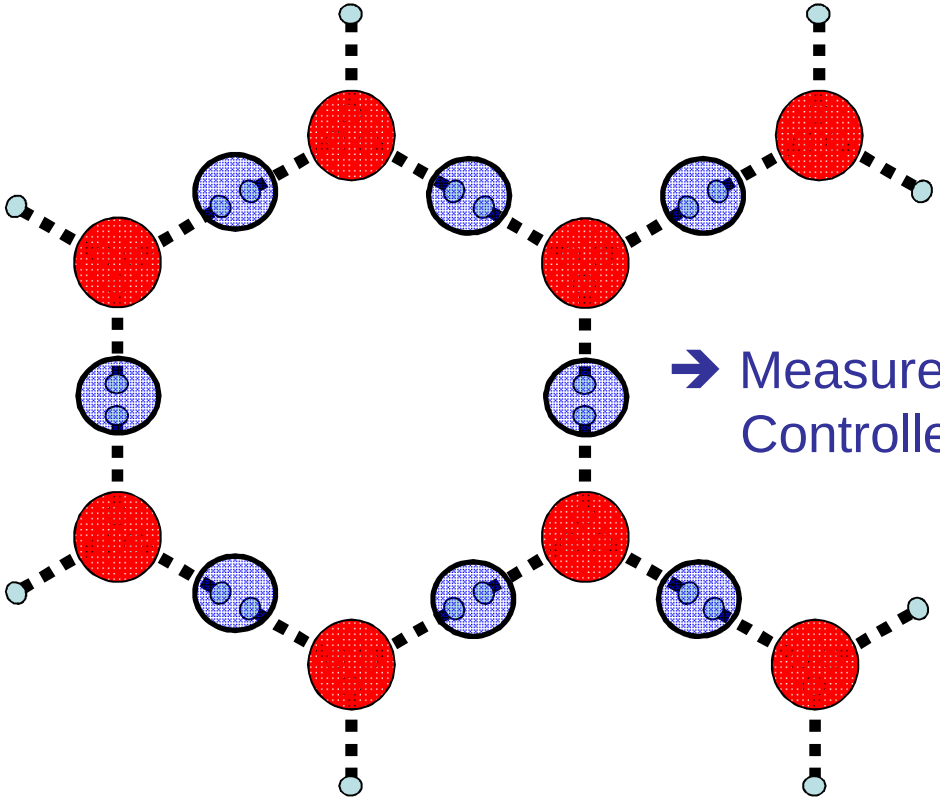
$$F_z'(a) = \sqrt{\frac{3a^2-1}{2a^2}}F_z D(a)$$

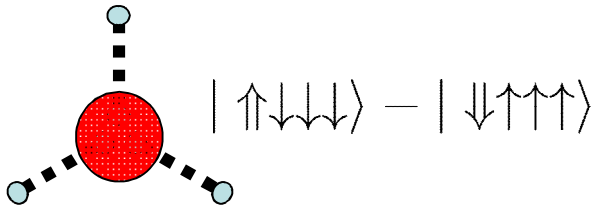
➔ Distill GHZ state with certainty for $a \geq 1/\sqrt{3}$ ($\delta, d_z \geq -\sqrt{3}/2$)

- For $a < 1/\sqrt{3}$, need a different POVM but succeeds with

$p = (1-3a^2)/(1+a^2)$ ➔ Percolation consideration for cluster state

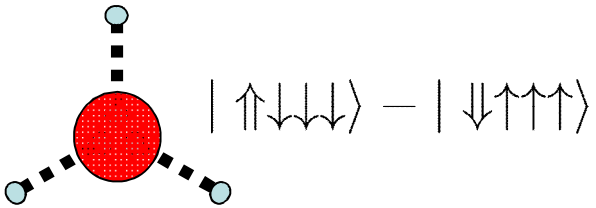
GHZ network + CZ

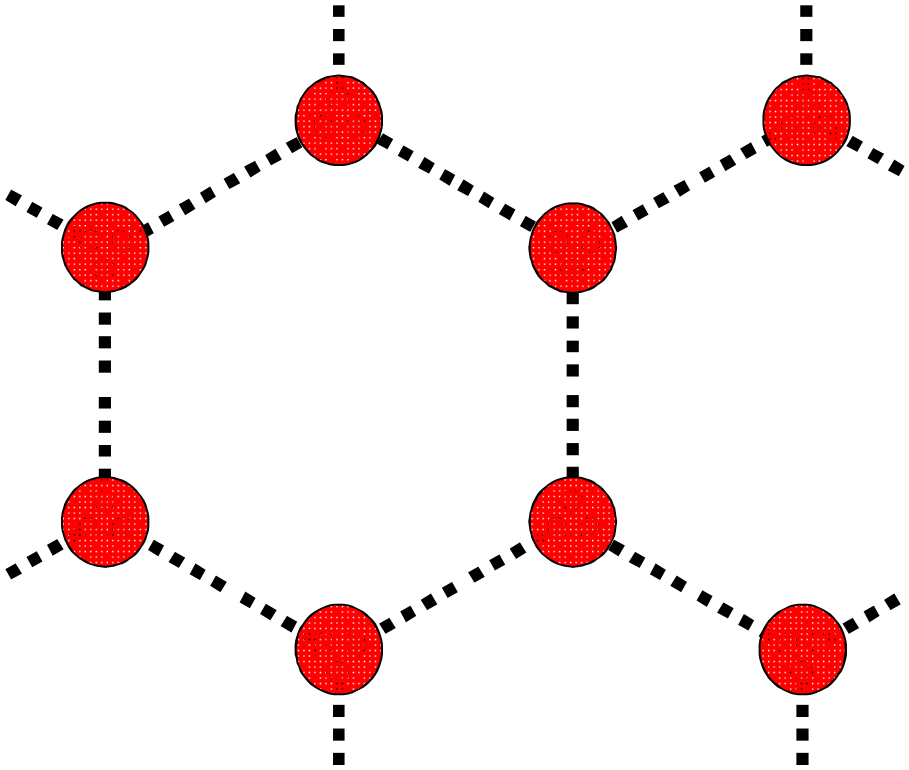




➔ Measurement of two virtual qubits enacts Controlled-Z gate btwn nbr center particles

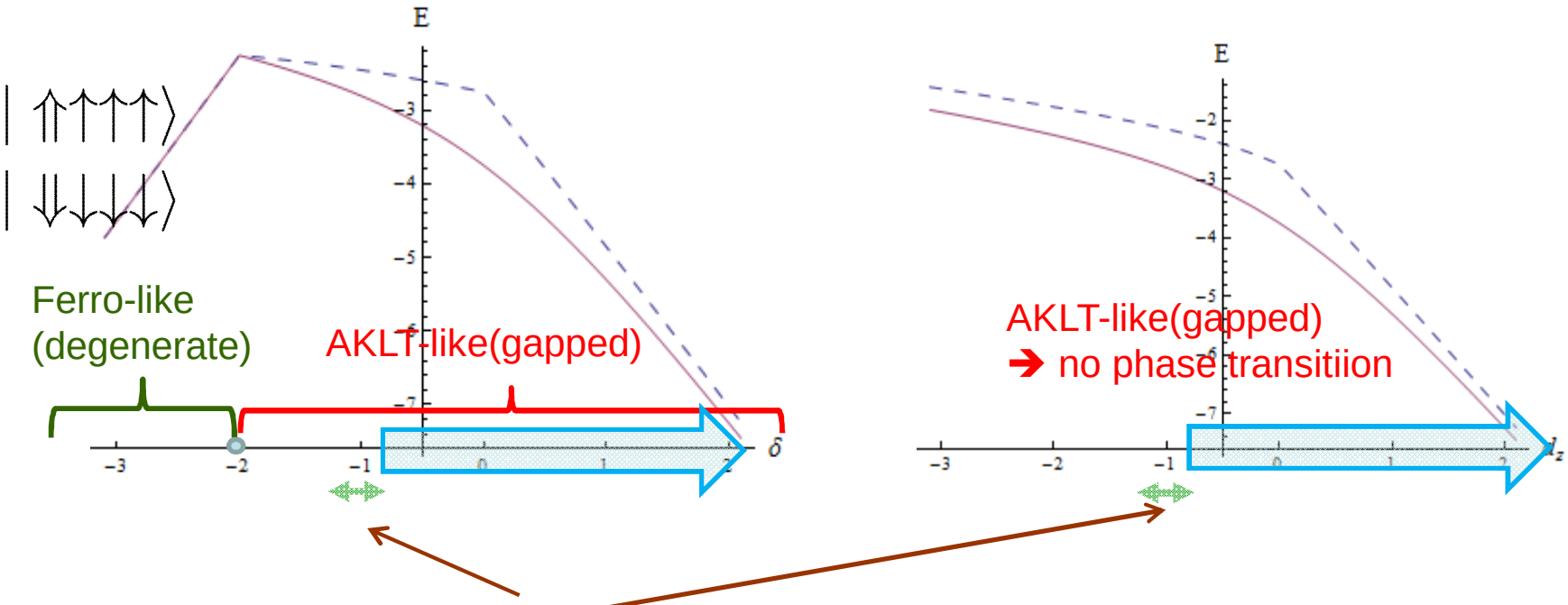
GHZ network + CZ ➔ cluster state





- Cluster state (universal for QC) is created (for $\delta, d_z \geq -\sqrt{3}/2$) on honeycomb lattice with unit probability

Zero temperature (δ,d_z near -1)

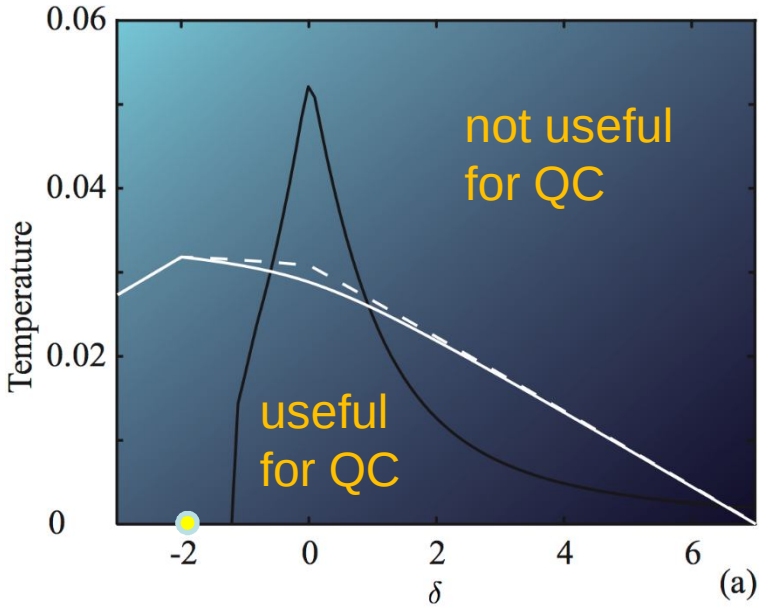


➔ For $-1.288 \lesssim \delta, d_z < -\sqrt{3}/2$, can filter out a GHZ with probability > percolation threshold

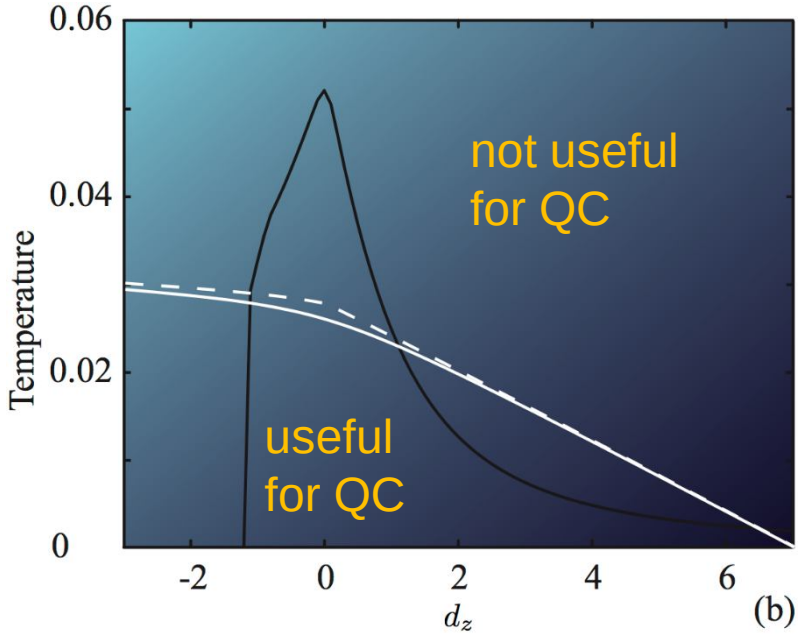
➔ Connected 2D cluster state on faulty honeycomb lattice

Finite T diagram (T=0 understood)

Model 1 (2D)



Model 2 (2D)



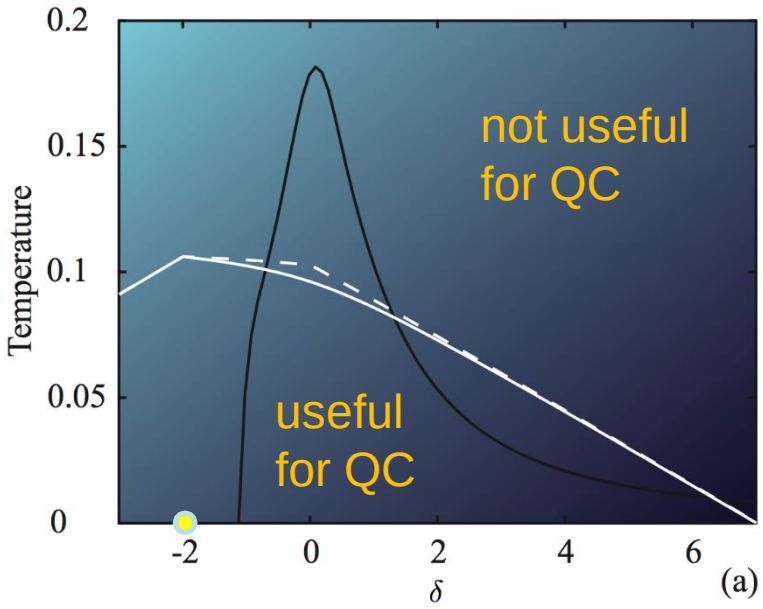
- Model exactly solvable ➔ GHZ fraction at finite T is known
- Use techniques from fault-tolerant quantum computation

  ➔ to locate temperature where error rate = FT threshold

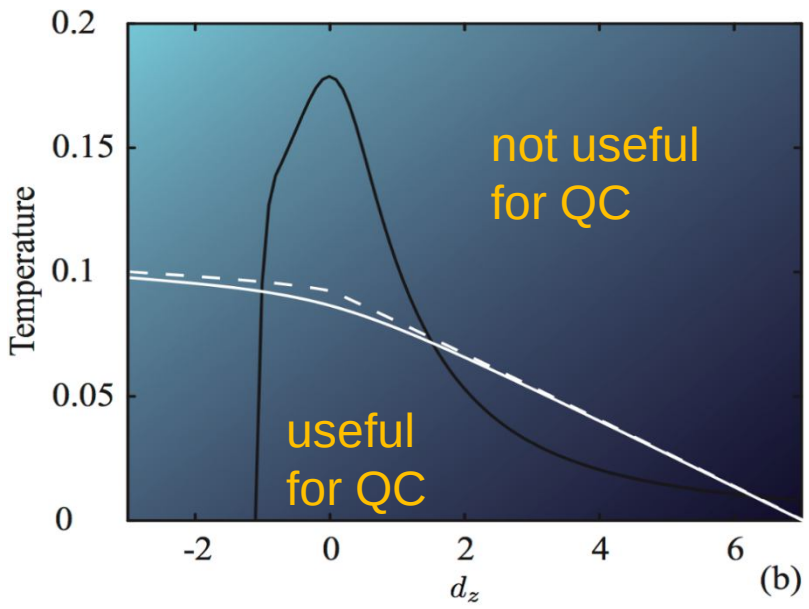
  ➔ transition temperature

3D is more robust

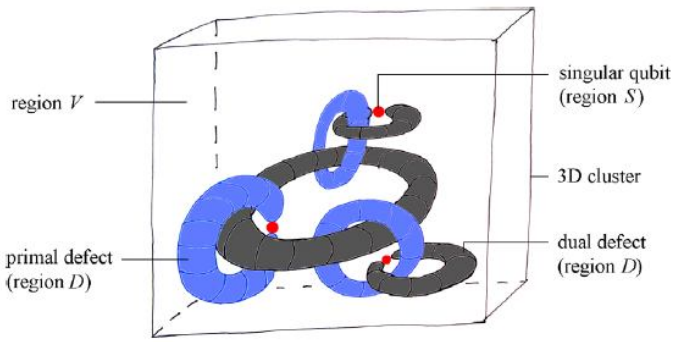
Model 1

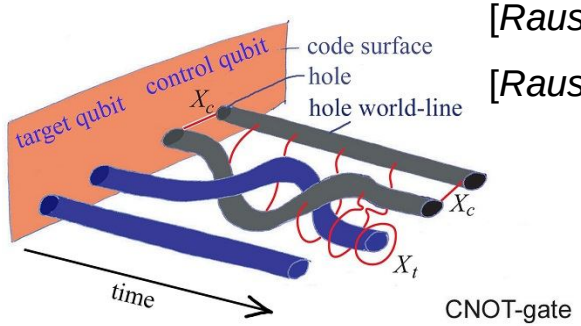


Model 2



- Can create 3D cluster state ➔ topological protection for QC
  (Measurement-based version of so-called surface code QC)





[*Raussendorf,Harrington,Goyal* '06]

[*Raussendorf, Harrington*, PRL (2007)]

Summary

- Introduce notion of “computational phase” via resource states in measurement-based quantum computation
- Explicitly construct two model spin-3/2 Hamiltonians
  ➔ QC phase diagram of T-vs. δ (or d_z)
- Transitions in quantum computational power need NOT coincide with transitions in phases of matter

Main Refs.: Li, Browne, Kwek, Raussendorf & **Wei**, PRL 107,060501 (2011)

Wei, Li & Kwek, PRA 89,0502315 (2014)

Related Refs.: **Wei**, Affleck,Raussendorf, PRL 106,070501 (2011)

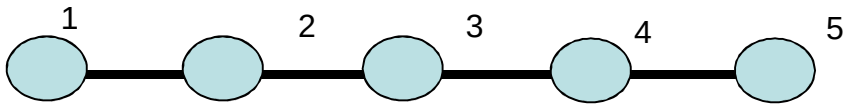
Wei, PRA 88,062307 (2013); **Wei** et al, PRA 90, 042333 (2014)

Garcia-Saez,Murg,**Wei**, PRB 88, 245118 (2013)

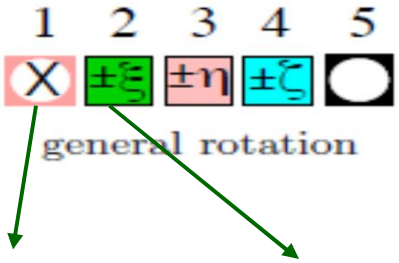
Supplementary slides

One-qubit gate $U = e^{-i\frac{\zeta}{2}\sigma_x} e^{-i\frac{\eta}{2}\sigma_z} e^{-i\frac{\xi}{2}\sigma_x}$

input $|\psi_{\text{in}}\rangle$ output $|\psi_{\text{out}}\rangle \sim U|\psi_{\text{in}}\rangle$



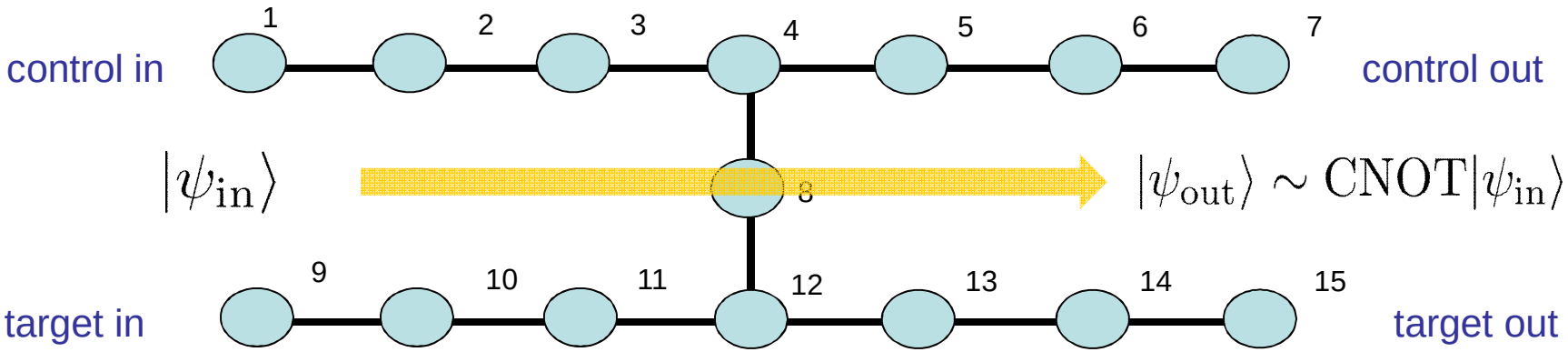
Measurement pattern:



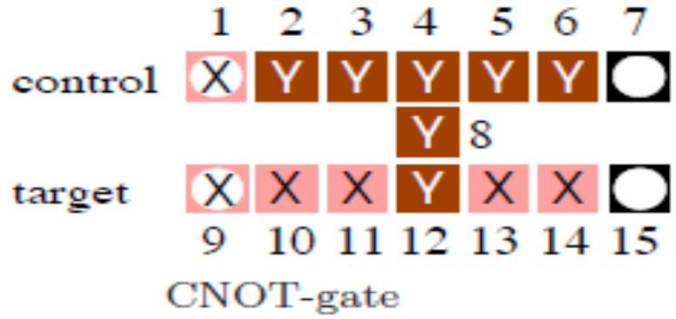
Observables: σ_x $\cos(\xi)\,\sigma_x \pm \sin(\xi)\,\sigma_y$

CNOT gate

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



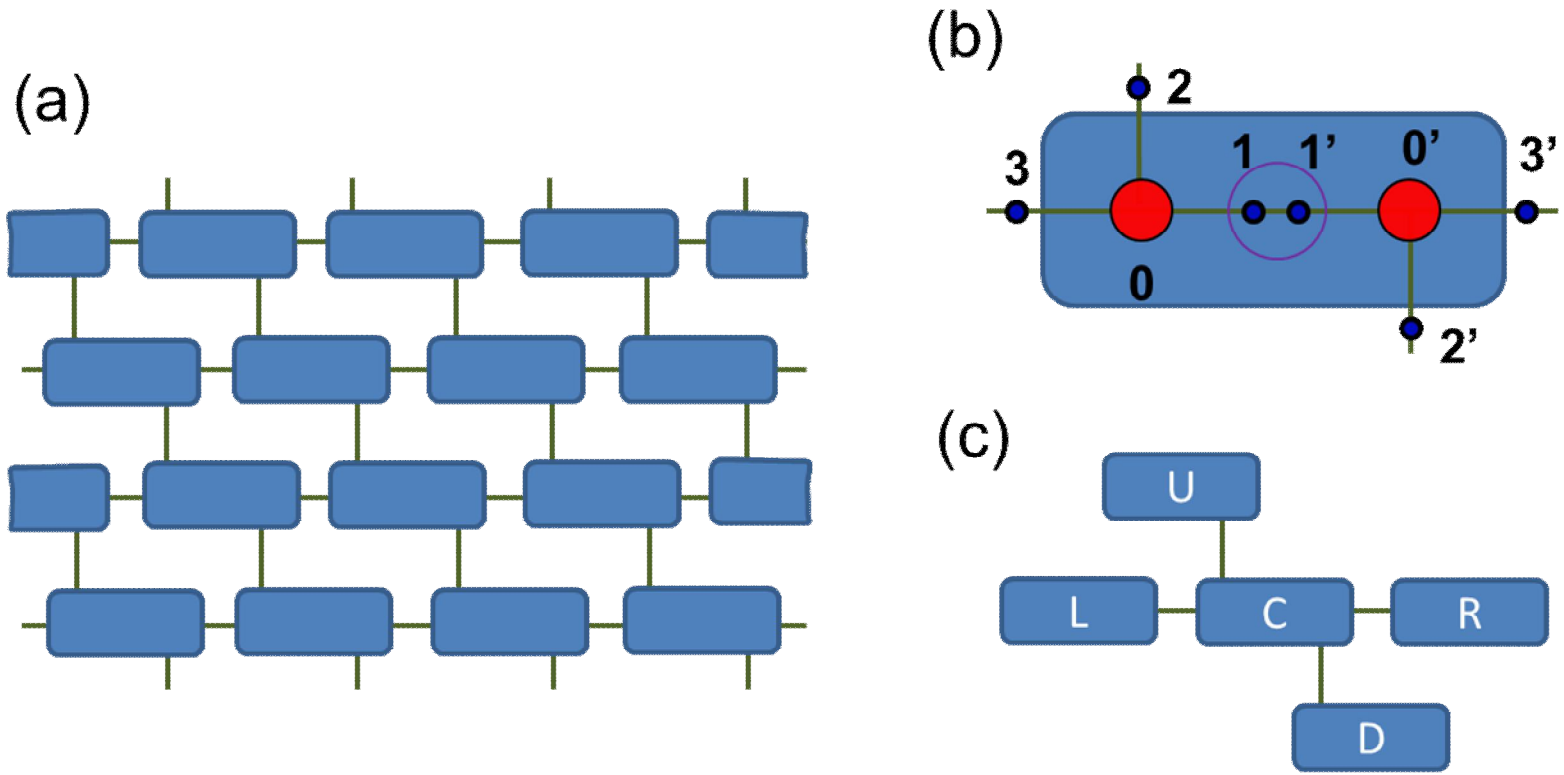
Measurement pattern



➔ simulates CNOT (via entanglement between wires)

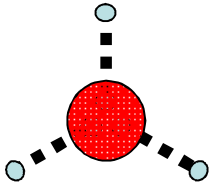
Generating a cluster state

- Example: 2D cluster state on square lattice



Creating a cluster state

- After POVM on center particles,
  each block is an effective 4-qubit GHZ state



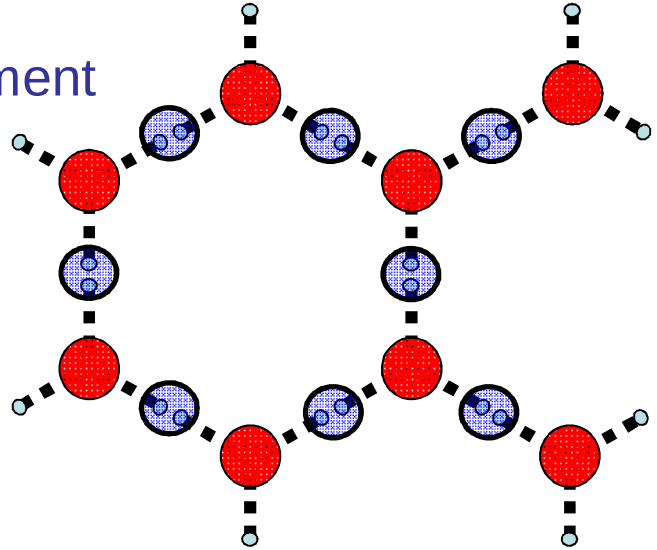
$$\longrightarrow |0000\rangle + |1111\rangle$$

- Perform measurement on the bond particles

➔Effective joint measurement on the
two virtual qubits (e.g. Bell-state measurement
or in 2-qubit cluster state basis)

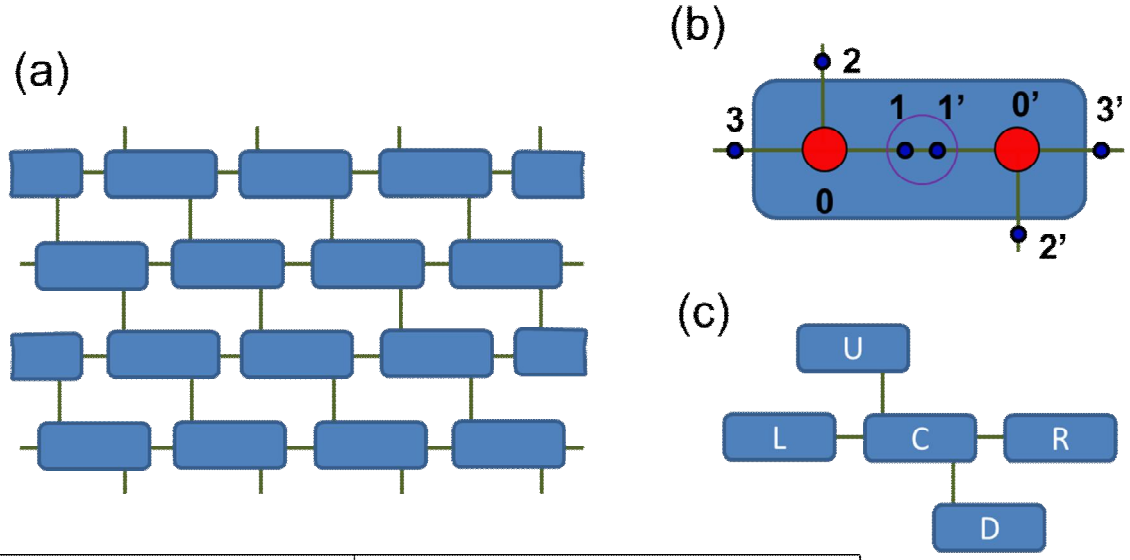
➔Induce control-phase gate between
two center qubits (up to Z gates)

➔Give rise to a cluster state on
a hexagonal (honeycomb) lattice



Error analysis

[***Wei***, *Li*, *Kwek*, PRA '14]



errors on 0-3 or 0'-3'	errors on C, U, L, D, and R
X_0	I (no error)
$X_{0'}$	$X_C \equiv Z_U Z_L Z_D Z_R$
X_1 or $X_{1'}$	$Z_U Z_L$
X_2	Z_U
$X_{2'}$	Z_D
X_3	Z_L
$X_{3'}$	Z_R
Z_i (for all i)	Z_C

Table 1: Consequence of qubit errors (from 0-3 and 0'-3' of block C in Fig. b on logical cluster-state qubits. The logical error X_C is equivalent to $Z_U Z_L Z_D Z_R$ due to the cluster-state stabilizer operator $X_C Z_U Z_L Z_D Z_R$.

Error analysis

[***Wei***, *Li*, *Kwek*, PRA '14]

- As goal is to investigate intrinsic property of quantum computational power, assume error caused by finite T (i.e. assume perfect measurement)

$$\rho_T = \frac{e^{-H/T}}{\mathrm{Tr}\, e^{-H/T}}$$

$$\rho_{GHZ} = U_y F_x \rho_T F_x^\dagger U_y^\dagger + U_x F_y \rho_T F_y^\dagger U_x^\dagger + F_z \rho_T F_z^\dagger$$

$$\rho'_{GHZ} = \prod_{K\in\{K\}} \frac{1}{2}([I]+[K])\rho_{GHZ} = \sum_{\sigma\in\{\sigma\}} p_\sigma [\sigma] |GHZ\rangle\langle GHZ|$$

$$p_z \simeq 2(p_{Z_0} + 2p_{X_1} + p_{X_2} + p_{X_3} + 3p_{Z_0X_1} + 2p_{Z_0X_2} + 2p_{Z_0X_3})$$

- 2D: transition at

$$p'_z \approx 10^{-7} \Rightarrow p_z \approx \frac{1}{3}10^{-7}k(p_l)$$

- 3D:

$$\frac{p_l}{24.9\%} + \frac{p_z}{2.93\%} \approx 1$$

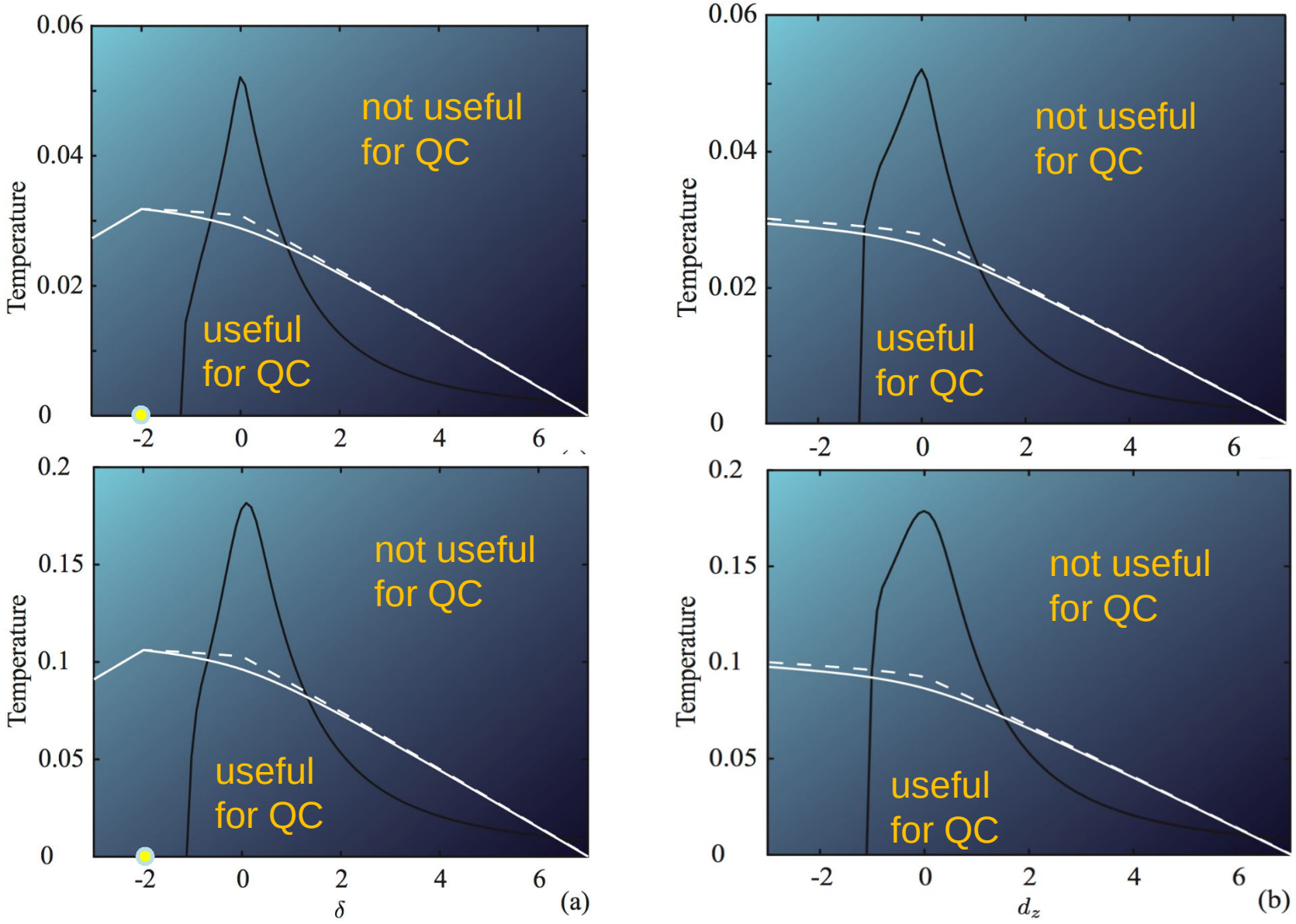
Computational phases

Model 1

Model 2

2D:

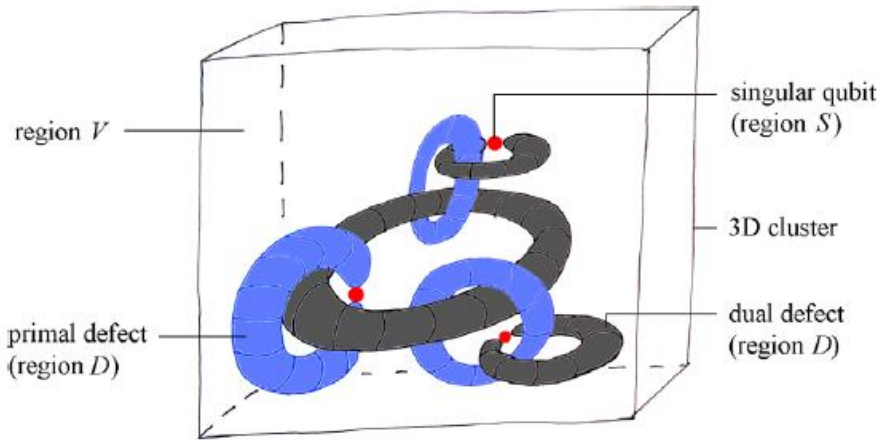
3D:

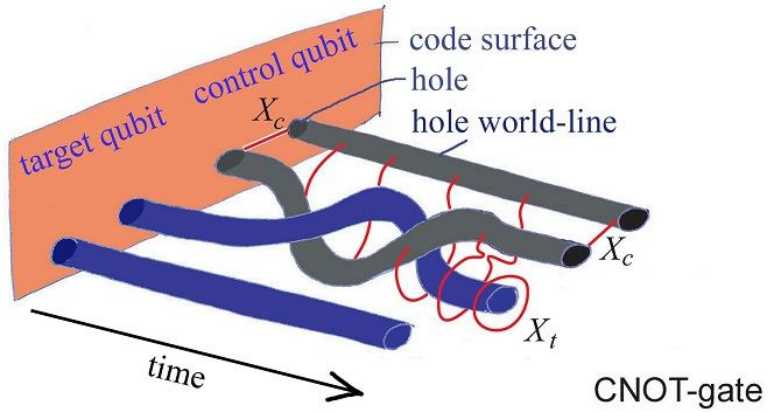


Fault tolerance at 3D

- Builds upon Raussendorf-Harrington-Goyal scheme on 3D cluster state

[Ann of Phys 321, 2242 (2006)]





➔ Error threshold: 1.4% for depolarizing error and 0.11 % (later improved to 0.75%) on preparation-, gate-, storage-, and measurement errors

[Raussendorf & Harrington, PRL (2007)]