# APCWQIS 2014 at TAINAN

# Photon Number Combs

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cf. JK et al., Optics Comm. (2014), arXiv 2010
Imoto, Yamamoto, Munro, ...
Shapiro, PK Lam,

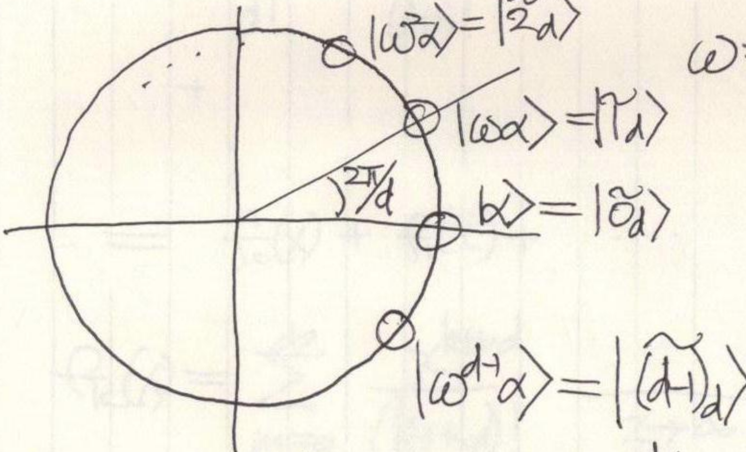
$\pi$ - day

$e$-day $e$-time

Feb. 7, 18:28 at KIAS

$e \cong 2.71828\,18284590\cdots$

$$\{|0\rangle, |1\rangle\} \underset{}{\overset{H}{\rightleftarrows}} \{|+\rangle, |-\rangle\}$$




$$\omega = e^{\frac{2\pi i}{d}}$$

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \Big\{ \underbrace{\sqrt{d}\, e^{-\frac{|\alpha|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha^{k+md}}{\sqrt{(k+md)!}} |k+md\rangle}_{\equiv |k_d\rangle} \Big\}$$

$$e^x = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{d-1}}{(d-1)!}$$

$$+ \frac{x^d}{d!} + \frac{x^{d+1}}{(d+1)!} + \cdots$$

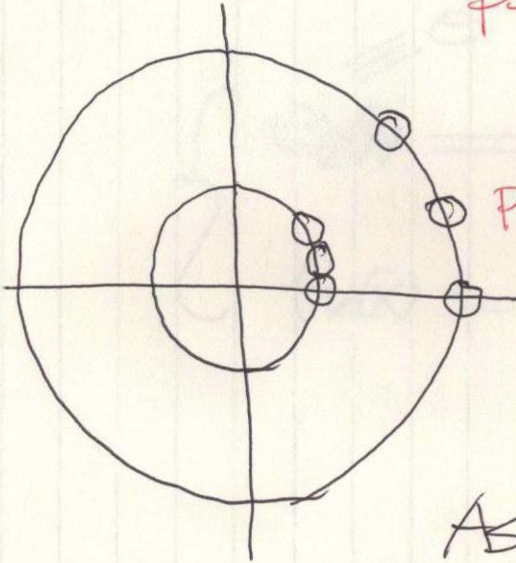
$$+ \cdots$$

$$\vdots$$

$$= f_0(x) + f_1(x) + \cdots + f_{d-1}(x)$$

$$f_k(x) = \sum_{m=0}^{\infty} \frac{x^{k+md}}{(k+md)!} \xrightarrow[x\to\infty]{} \frac{e^x}{d}$$

$$\langle k_d | k_d \rangle = \frac{d}{e^{|\alpha|^2}} \sum_{m=0}^{\infty} \frac{|\alpha|^{2(k+md)}}{(k+md)!} \xrightarrow[|\alpha|^2\to\infty]{} 1$$



pseudo-phase: NORMAL

$$\left\{ \begin{array}{l} |\tilde{l}_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{k=0}^{d-1} \omega^{lk} |k_d\rangle = |\omega^l \alpha\rangle \\ \text{pseudo-number: ORTHOGONAL} \\ |k_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{l=0}^{d-1} \omega^{-kl} |\tilde{l}_d\rangle \end{array} \right.$$

"EXACT !!!"

As $|\alpha| \to$ bigger,

$$\left\{ \begin{array}{l} \text{ORTHOGONALITY of } |\tilde{l}_d\rangle \longrightarrow \text{better} \\ \text{NORMALITY of } |k_d\rangle \longrightarrow \text{better} \end{array} \right.$$

$$\boxed{O\left(e^{-|\alpha|^2 \frac{4\pi^2}{d^2}}\right)}$$

$$e_\ell(x) \equiv e^{\omega^\ell x}$$

$$\left\{\begin{aligned} e_\ell(x) &= \sum_{k=0}^{d-1} \omega^{\ell k} f_k(x) \\ f_k(x) &= \sum_{\ell=0}^{d-1} \omega^{-k\ell} e_\ell(x) \end{aligned}\right.$$

"exact!!!"

$$\{|0\rangle, |1\rangle\}$$

$$\{|+\rangle, |-\rangle\}$$

$$Z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = (-1)^0 |0\rangle\langle 0| + (-1)^1 |1\rangle\langle 1|$$

$$= \omega^{\hat{n}}$$

$$CZ = (-1)^{\hat{n}_c \cdot \hat{n}_t} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} Z^0 & \\ & Z^1 \end{pmatrix}$$

$$\{|0_d\rangle, |1_d\rangle, \cdots, |(d-1)_d\rangle\}$$

$$\{|\tilde{0}_d\rangle, |\tilde{1}_d\rangle, \cdots, |\widetilde{(d-1)}_d\rangle\}$$

$$\quad \| \qquad\qquad \| \qquad\qquad\qquad \|$$

$$|\alpha\rangle \qquad |\omega\alpha\rangle \qquad |\omega^{d-1}\alpha\rangle$$

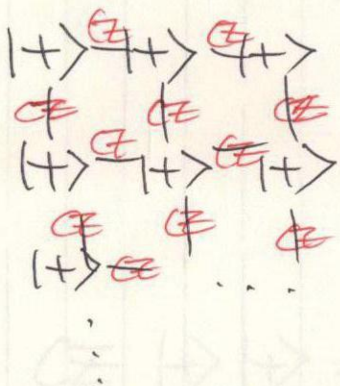
$$Z_d = \begin{pmatrix} \omega^0 & & & & \\ & \omega^1 & & & \\ & & \omega^2 & & \\ & & & \ddots & \\ & & & & \omega^{d-1} \end{pmatrix} = \sum_{l=0}^{d-1} \omega^l |l_d\rangle\langle l_d| = \omega^{\hat{n}}$$

$$CZ_d = \omega^{\hat{n}_c \cdot \hat{n}_t}$$

$$= e^{\frac{2\pi i}{d} \hat{n}_c \hat{n}_t}$$

$$CZ_{12}\,|+\rangle_1|+\rangle_2 = \frac{|0\rangle|+\rangle + |1\rangle|-\rangle}{\sqrt{2}}$$

$$CZ_{12}\,CZ_{23}\,|+\rangle_1|+\rangle_2|+\rangle_3 = \frac{|+\rangle|0\rangle|+\rangle + |-\rangle|1\rangle|-\rangle}{\sqrt{2}}$$




: cluster states

$$\omega^{\hat{n}_1 \hat{n}_2} |\alpha\rangle_1 |\alpha\rangle_2 = \omega^{\hat{n}_1 \hat{n}_2} \sum_{k=0}^{d-1} \frac{|k_d\rangle_1}{\sqrt{d}} |\alpha\rangle_2$$

$CZ_d$: generalized

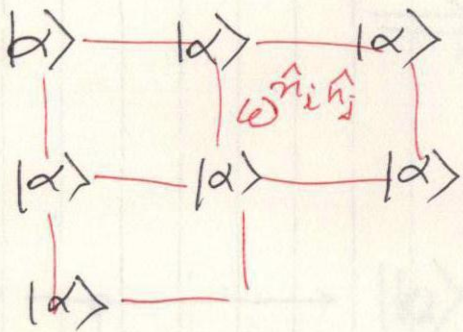
$$= \sum_{k=0}^{d-1} \frac{|k_d\rangle_1}{\sqrt{d}} \omega^{k \hat{n}_2} |\alpha\rangle_2 = |\omega^k \alpha\rangle_2 = |\tilde{k}_d\rangle_2$$

$$= \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k_d\rangle_1 |\tilde{k}_d\rangle_2 = \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} |\tilde{l}_d\rangle_1 |l_d\rangle_2$$

maximal entanglement.

cf. $CZ_{12} |+\rangle_1 |+\rangle_2 = \frac{1}{\sqrt{2}} \left( |0\rangle_1 |+\rangle_2 + |1\rangle_1 |-\rangle_2 \right) = \frac{1}{\sqrt{2}} \left( |+\rangle_1 |0\rangle_2 + |-\rangle_1 |1\rangle_2 \right)$

# quDit cluster state



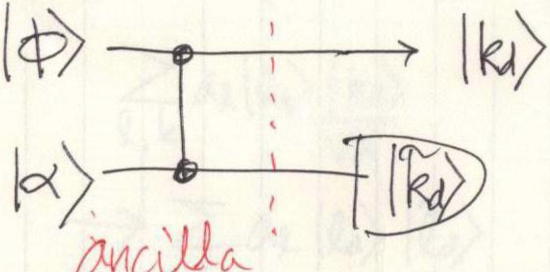

$$\mathcal{O}\left(e^{-|\alpha|^2 \cdot \frac{4\pi^2}{d^2}}\right) \quad \& \quad e^{\frac{2\pi i}{d}\hat{n}_1 \hat{n}_2} = e^{i\chi L \hat{n}_1 \hat{n}_2}$$

$$\frac{2\pi}{d} = \chi L$$

$|\alpha|$

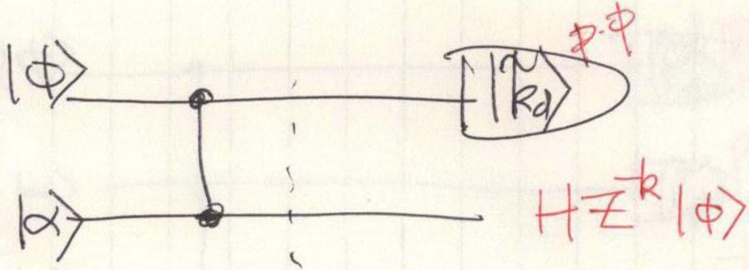
$|\tilde{k}_\alpha\rangle = |\omega^k \alpha\rangle$ : pseudo-phase measurement.

$\Longrightarrow$ HOMODYNE




: pseudo-number measurement.

# ONE-STEP TELEPORTATION

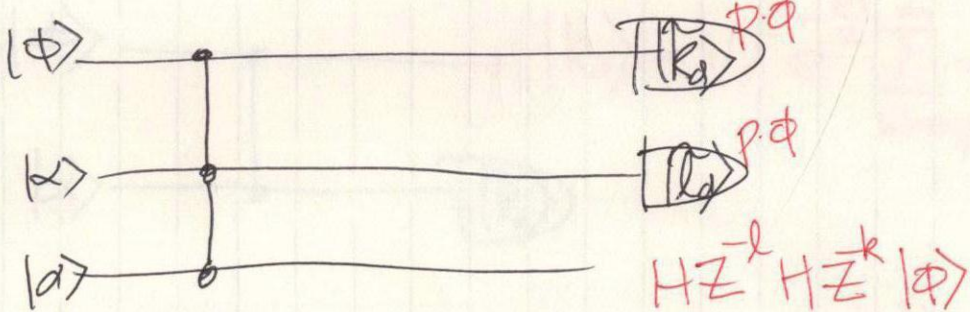



$$\sum_{l,k} a_l |l_d\rangle \frac{|k_d\rangle}{\sqrt{d}}$$

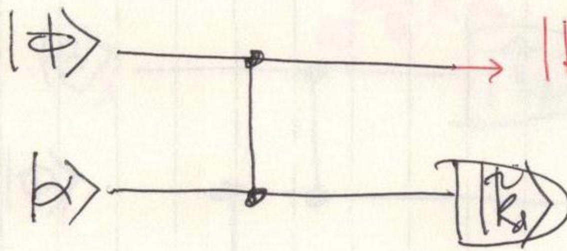
$$\to \sum_{l} a_l |l_d\rangle |\tilde{l}_d\rangle$$

$$\to \sum a_l \omega^{-kl} |\tilde{l}_d\rangle = H Z^{-k} \underline{\sum a_l |l_d\rangle}$$

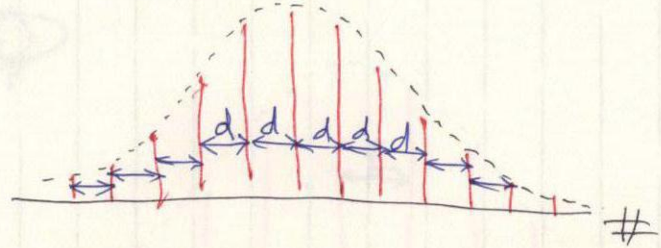
# (TWO-STEP) QUDIT TELEPORTATION



# PSEUDO-NUMBER MEASUREMENT




$$|k_d\rangle = \sqrt{d}\, e^{-\frac{|\alpha|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha^{k+md}}{\sqrt{(k+md)!}} |k+md\rangle$$




"photon number comb"

cf. frequency comb

Thank you so much

謝謝大家