Spin Filtering: how to write and read quantum information on mobile qubits

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Asia-Pacific conference on quantum information, Tainan, December 2014

Quantum computers

Conventional computers: information in **bits**,

0 or 1, +1 or -1, ⬆ or ⬇

Quantum computers: information in **Qubits**,

Electron described by **spinor**:

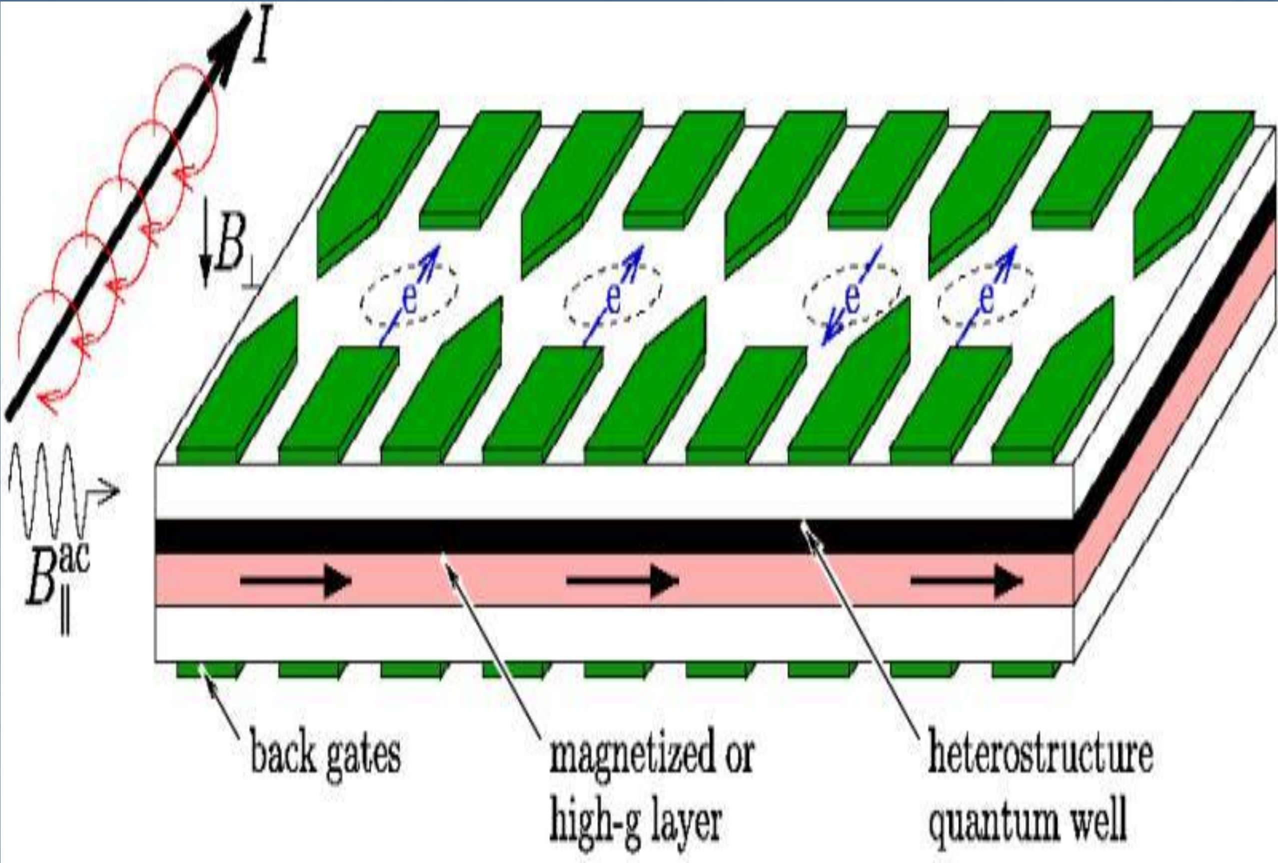
$$\psi = \cos\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{i\gamma} \sin\alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Complex numbers

Spinor is an eigenvector of $\mathbf{n} \cdot \boldsymbol{\sigma}$, the spin component along $\mathbf{n}$

$\alpha, \gamma \Longleftrightarrow \mathbf{n}$

Static qubits:



Here we discuss **mobile (**or **flying)** qubits, in **mesoscopic semiconductor** devices

The Aharonov-Bohm (AB) Effect

- Classical Physics, e.g. Lorentz force

$$m\left(\frac{d^2\vec{r}}{dt^2}\right) = -e\left[\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right]$$

- Quantum Physics, Schrödinger equation

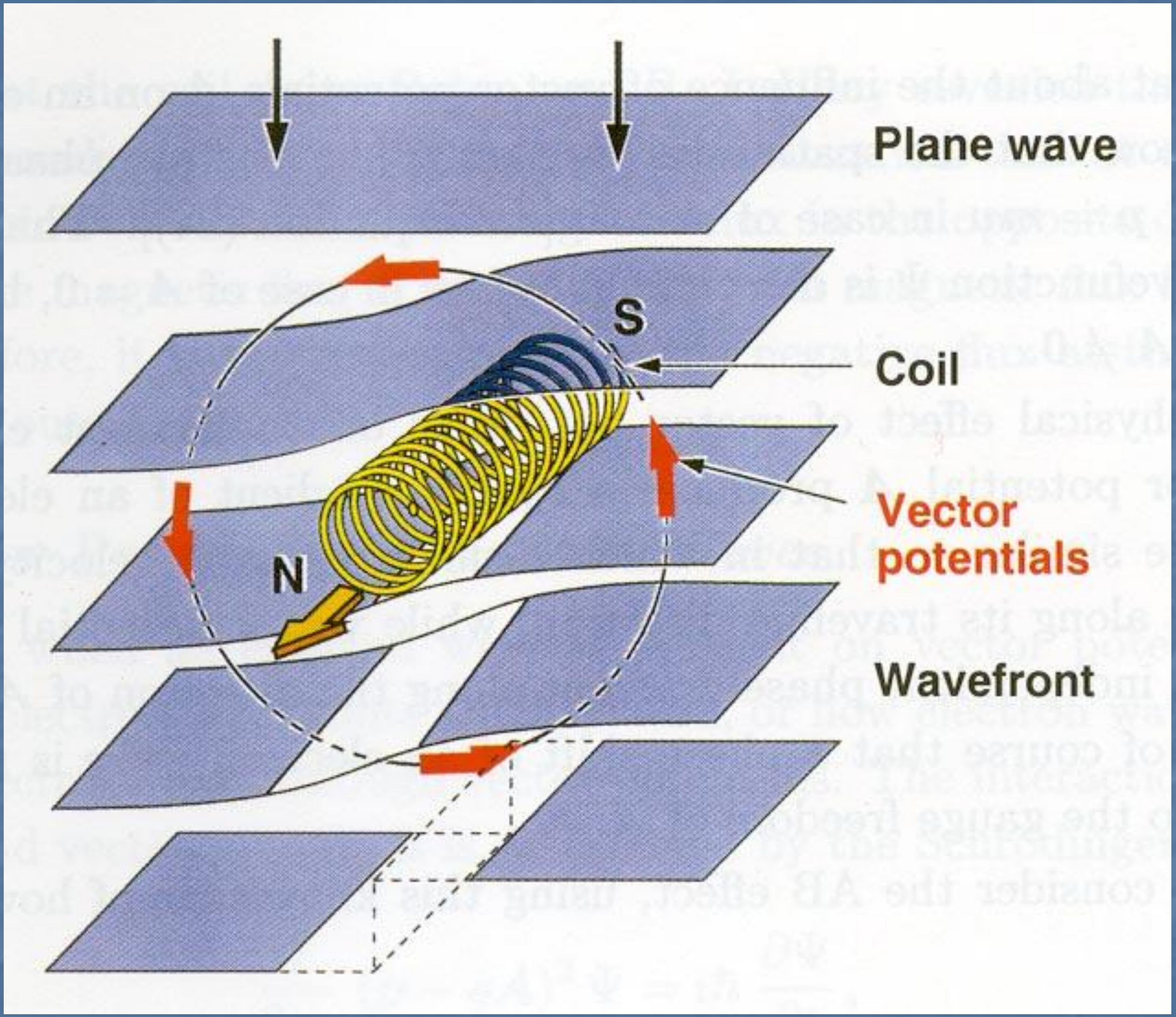
$$(H_o + V)\Psi = \frac{i\hbar\partial\Psi}{\partial t}$$

  - $V = -eED$ and $H_o = \frac{p^2}{2m}$, where $\boxed{\vec{p} = m\vec{v} + \frac{e\vec{A}}{c}}$
  - with E electric field, D electrode's separation

- Aharonov and Bohm (AB), *Phys.Rev.* **115**, 485 (1959)

- Phase Shift $\Delta\phi = \frac{1}{\hbar}\int L dt = \frac{1}{\hbar}\int\left(m\vec{v} + \frac{e\vec{A}}{c}\right).d\vec{s} - \frac{1}{\hbar}\int eEDdt$

A. Tonomura



Quantum mechanics:

Particle-wave duality

Schrödinger's wave equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$

Dirac's equation: spin and spinor

$$(i\hbar\gamma^\mu\partial_\mu - mc)\,\psi = 0$$

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t) + \boxed{\frac{\hbar}{(2M_0c)^2}\nabla V(\mathbf{r})(\hat{\boldsymbol{\sigma}}\times\hat{\mathbf{p}})}\,\Psi(\mathbf{r},t)$$

Spin-orbit interactions

Dirac:

$$\hat{H}_{SO} = \frac{\hbar}{(2M_0c)^2} \nabla V(\mathbf{r})(\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}).$$

Entin-Wohlman, Gefen, Meir, Oreg (1989, 1992)

$$\frac{1}{2m}(\mathbf{p} + e\,\mathbf{A}_{\text{s.o.}}/c)^2, \quad \mathbf{A}_{\text{s.o.}} = (\hbar/4mc)\boldsymbol{\sigma} \times \mathbf{E}$$

A spinor ψ entering from the left and travelling a distance L along the x-axis will be multiplied by the 2x2 unitary matrix

$$e^{-i\alpha\sigma_y}$$

$$\alpha = \alpha_R = k_R L$$

Aharonov-Casher

Rotation of spin direction around y-axis

Rashba Spin-orbit interactions

Dirac:

$$\hat{H}_{SO} = \frac{\hbar}{(2M_0c)^2} \nabla V(\mathbf{r})(\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}).$$

Rashba: 2DEG, confined to a plane by an asymmetric potential along z:

$$\mathcal{H}_{\mathrm{R}} = \frac{\hbar k_R}{m}(p_y \sigma_x - p_x \sigma_y)$$

Strength of Rashba term can be tuned by gate voltage!

A spinor ψ entering from the left and travelling a distance *L* along the *x*-axis will be multiplied by the 2x2 unitary matrix

$$e^{-i\alpha\sigma_y}$$

$$\alpha = \alpha_R = k_R L$$

Rotation of spin direction around *y*-axis

Dresselhaus Spin-orbit interactions

Dresselhaus: originates from bulk inversion asymmetry of the crystal structure:

$$\frac{\gamma}{2}\Big[p_z^2(p_x\sigma_x - p_y\sigma_y) + p_xp_y(p_x\sigma_y - p_y\sigma_x)\Big].$$

Linear Dresselhaus:

$$\mathcal{H}_{\mathrm{SO}}^{\mathrm{D}} = \alpha_D(p_x\sigma_x - p_y\sigma_y).$$

$$\mathcal{H}_{\mathrm{SO}} = U_p \cdot \sigma,$$

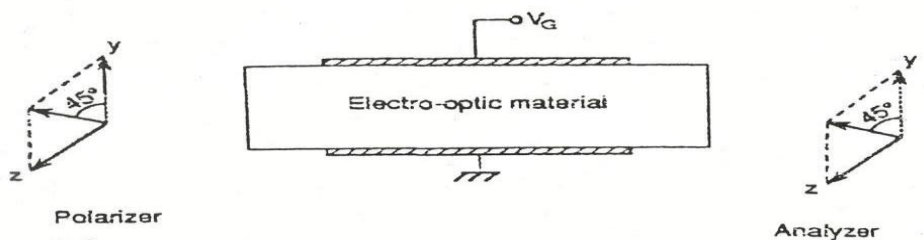
$$U_p = \Big[\alpha_D p_x + \alpha_R p_y, -(\alpha_R p_x + \alpha_D p_y)\Big].$$

Spin field effect transistor

Electronic analog of the electro-optic modulator

Supriyo Datta and Biswajit Das
School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

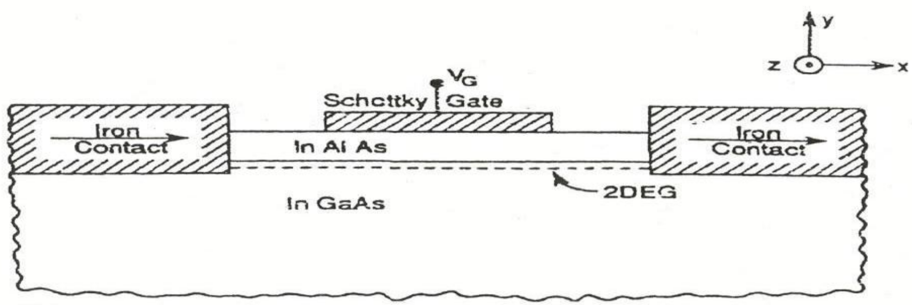
Appl. Phys. Lett. 56 (7), 665 12 February 1990



(a)

$$\underset{(45^\circ\ \text{pol.})}{\begin{pmatrix}1\\1\end{pmatrix}} = \underset{(z\ \text{pol.})}{\begin{pmatrix}1\\0\end{pmatrix}} + \underset{(y\ \text{pol.})}{\begin{pmatrix}0\\1\end{pmatrix}} .$$

$$P_0 \propto \left| (1\ \ 1)\begin{pmatrix}e^{ik_1L}\\ e^{ik_2L}\end{pmatrix}\right|^2 = 4\cos^2\frac{(k_1-k_2)L}{2} .$$



(b)

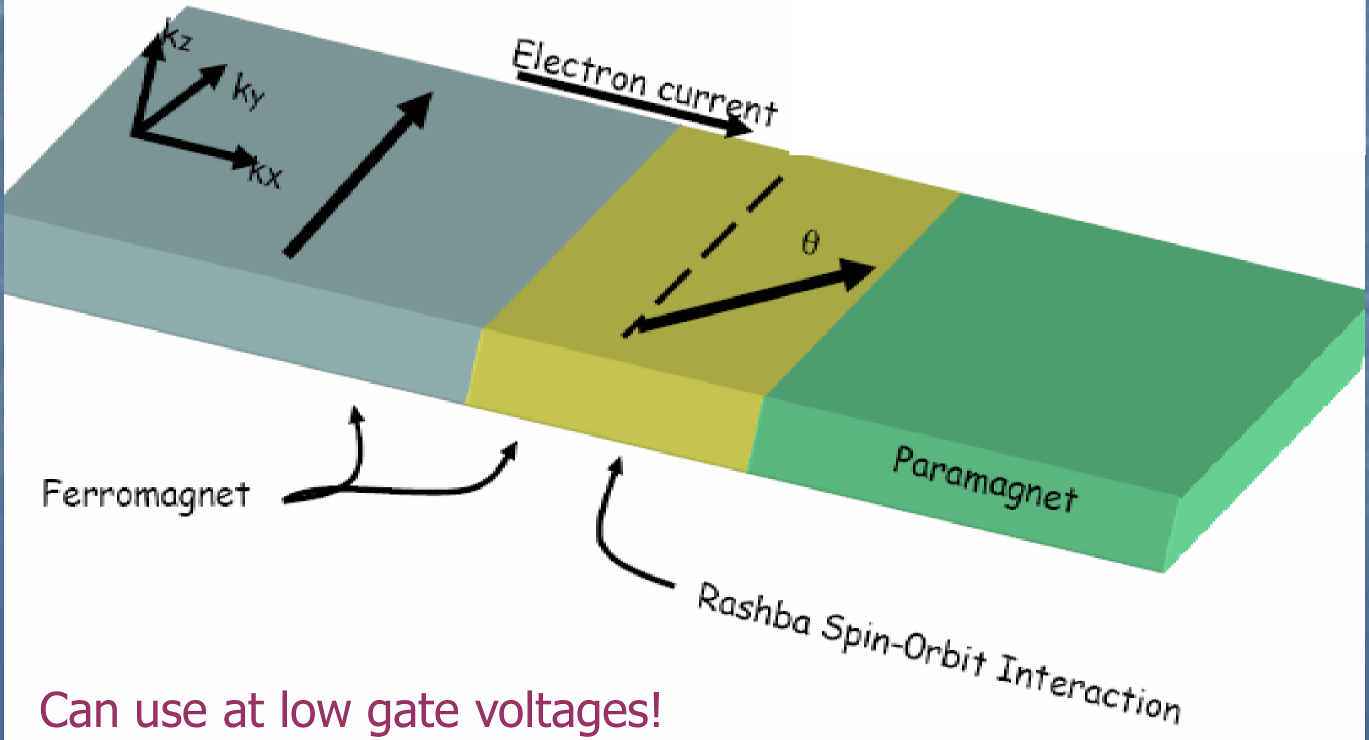
FIG. 1. (a) Electro-optic modulator; (b) proposed electron wave analog of the electro-optic modulator.

$$\underset{(+x\ \text{pol.})}{\begin{pmatrix}1\\1\end{pmatrix}} = \underset{(+z\ \text{pol.})}{\begin{pmatrix}1\\0\end{pmatrix}} + \underset{(-z\ \text{pol.})}{\begin{pmatrix}0\\1\end{pmatrix}} .$$

Das and Datta (1990): The **Spin field effect transistor**

Tunable with gate voltage

$$H_{SO} = \lambda(\vec{p} \times \hat{z}) \cdot \vec{s}$$



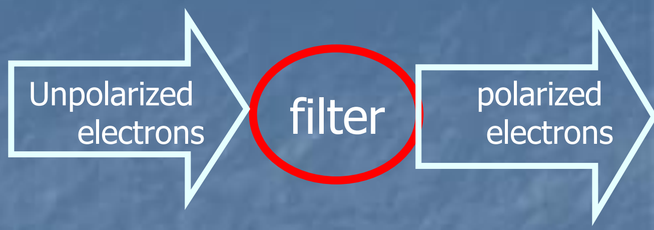
Can use at low gate voltages!

‘Writing’ on spinor: Spin filtering

Work with **mobile** electrons,
Generate **spin-polarized** current out of an unpolarized source

Spin filtering:

Generate **spin-polarized** current out of an **unpolarized** source



Earlier work: usually calculate **spin-dependent conductance**, and generate **partial** polarization, which varies with parameters.

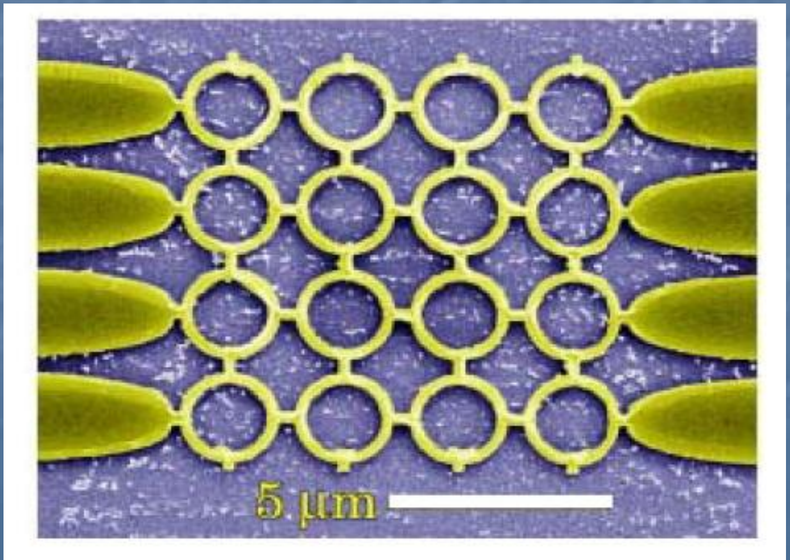
Our aim: obtain **full** polarization, in a **tunable** direction → **quantum networks**

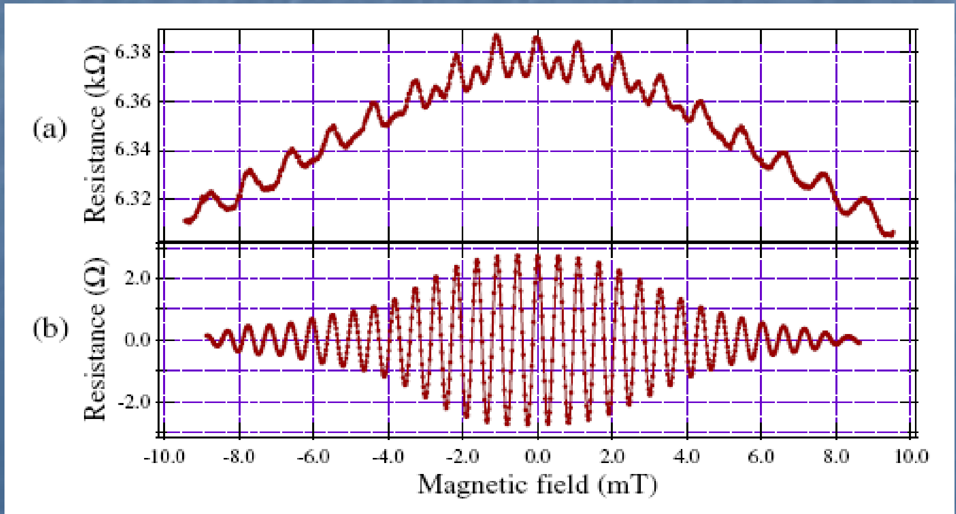
Quantum networks

PRL **97,** 196803 (2006) PHYSICAL REVIEW LETTERS week ending 10 NOVEMBER 2006

Experimental Demonstration of the Time Reversal Aharonov-Casher Effect

Tobias Bergsten,[1,3] Toshiyuki Kobayashi,[1] Yoshiaki Sekine,[1] and Junsaku Nitta[1,2,3]
[1]*NTT Basic Research Labs, 3-1 Morinosato-Wakamiya, Atsugi-shi, Kanagawa 243-0198, Japan*
[2]*Graduate School of Engineering, Tohoku University, 6-6-02 Aramaki-Aza Aoba, Aoba-ku, Sendai 980-8579, Japan*
[3]*CREST-Japan Science and Technology Agency, Kawaguchi Center Building, 4-1-8, Hon-cho, Kawaguchi-shi, Saitama 332-0012, Japan*





PHYSICAL REVIEW B **74**, 041302(R) (2006)

Experimental realization of a ballistic spin interferometer based on the Rashba effect using a nanolithographically defined square loop array

Takaaki Koga,[1,2,3,*] Yoshiaki Sekine,[2] and Junsaku Nitta[2,3,†]
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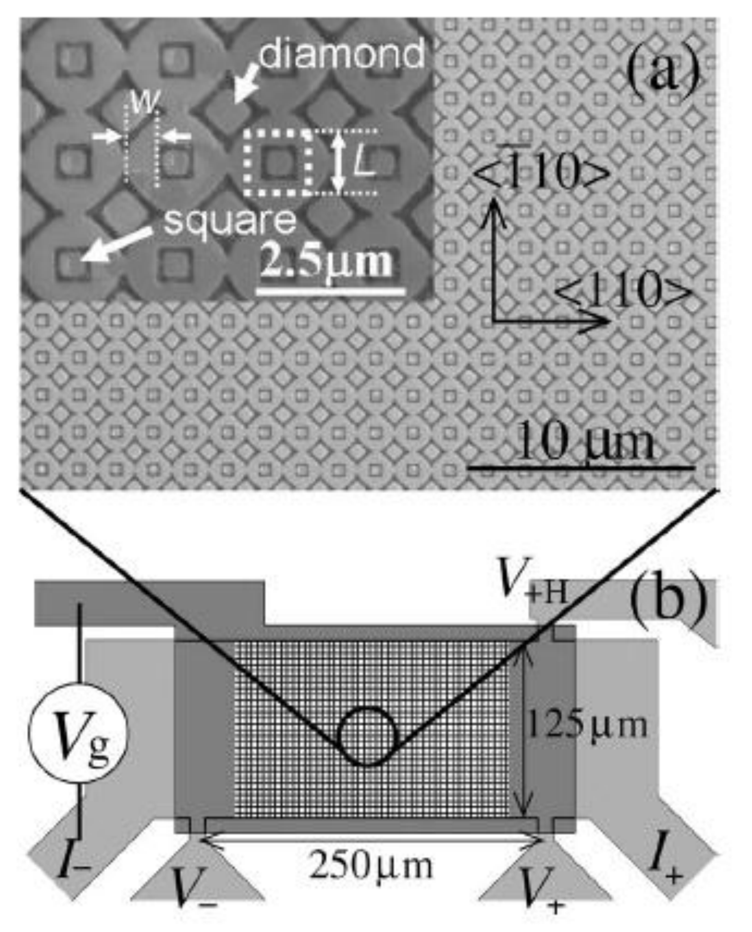
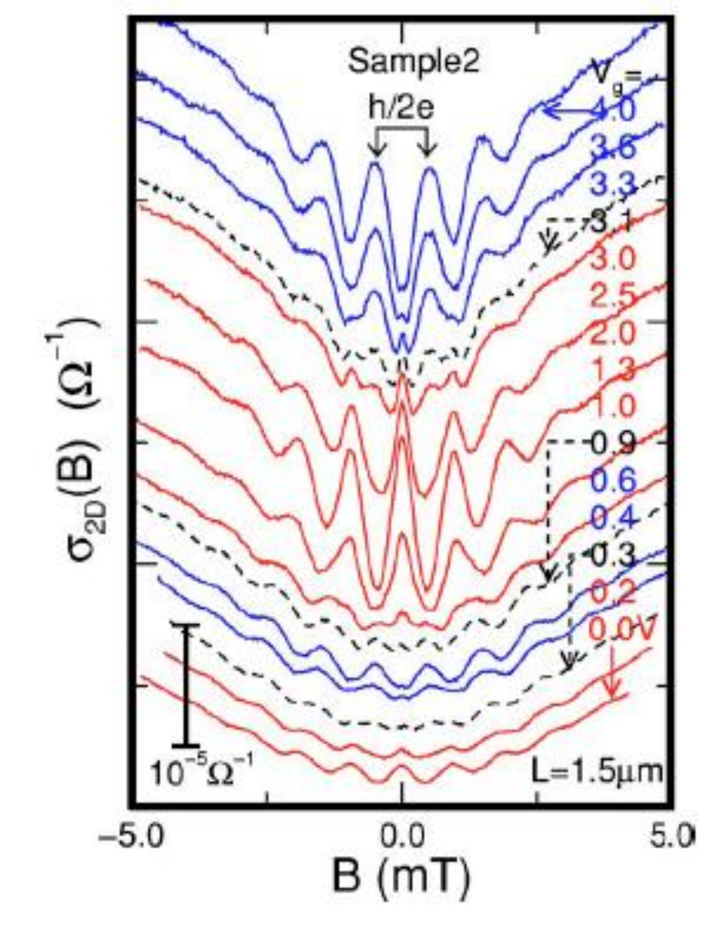
[3]*CREST, Japan Science and Technology Agency, 4-1-8, Honchou, Kawaguchi, Saitama 332-0012, Japan*

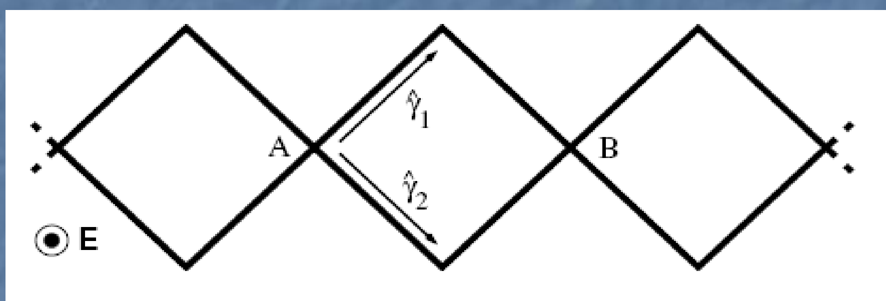




VOLUME 93, NUMBER 5 PHYSICAL REVIEW LETTERS week ending 30 JULY 2004

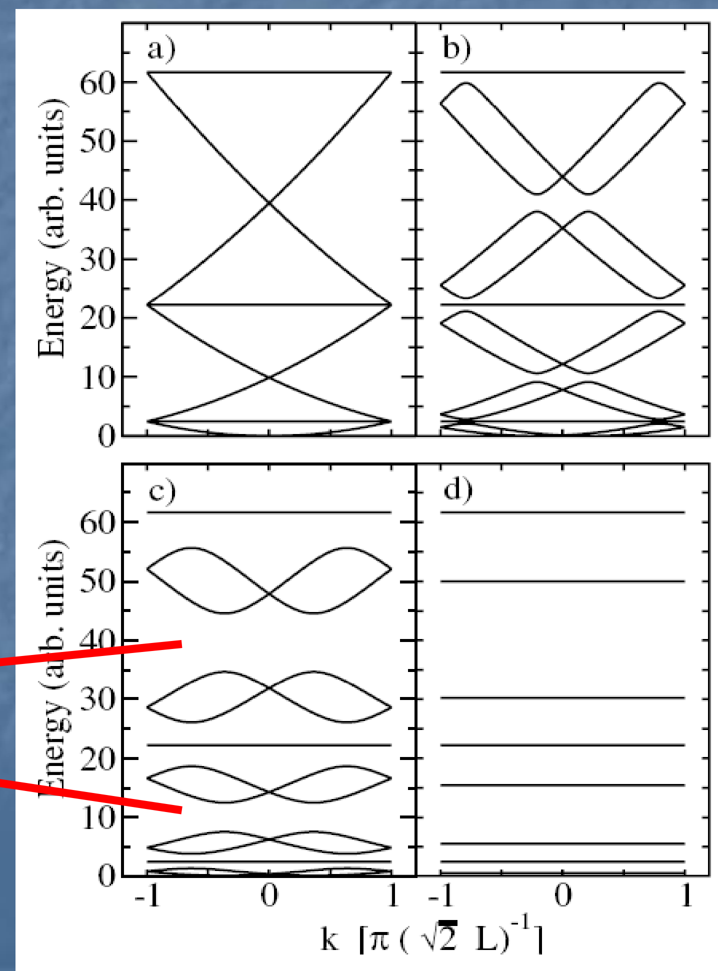
Rashba-Effect-Induced Localization in Quantum Networks

Dario Bercioux,[1] Michele Governale,[2] Vittorio Cataudella,[1] and Vincenzo Marigliano Ramaglia[1]
[1]Coherentia-INFM and Dipartimento di Scienze Fisiche, Università degli studi "Federico II," I-80126 Napoli, Italy
[2]NEST-INFM and Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy



k →

k= momentum of electrons along the chain



Gaps: SO cage

Evanescent modes

PHYSICAL REVIEW B **78**, 125328 (2008)

Spin filtering by a periodic spintronic device

Amnon Aharony,[1,*] Ora Entin-Wohlman,[1,*] Yasuhiro Tokura,[2] and Shingo Katsumoto[3]
[1]*Department of Physics and the Ilse Katz Center for Meso- and Nano-Scale Science and Technology, Ben Gurion University, Beer Sheva 84105, Israel*
[2]*NTT Basic Research Laboratories, NTT Corporation, Atsugi-shi, Kanagawa 243-0198, Japan*
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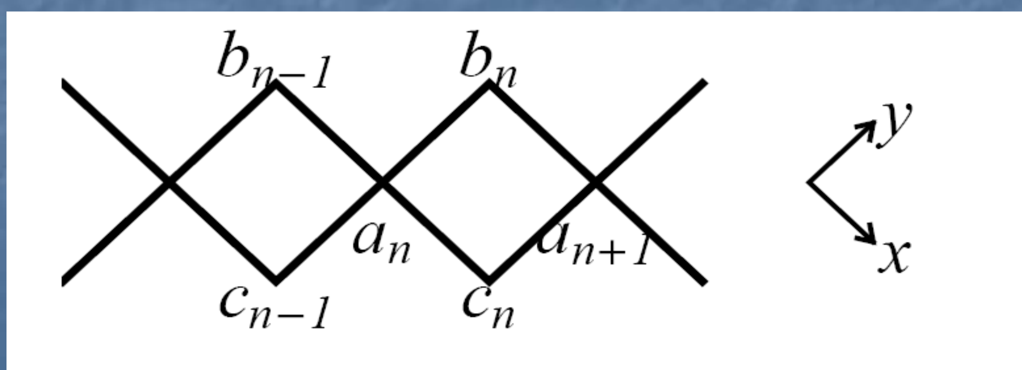
(Received 27 May 2008; revised manuscript received 3 September 2008; published 30 September 2008)

For a linear chain of diamondlike elements, we show that the Rashba spin-orbit interaction (which can be tuned by a perpendicular gate voltage) and the Aharonov-Bohm flux (due to a perpendicular magnetic field) can combine to select only one propagating ballistic mode, for which the electronic spins are fully polarized along a direction that can be controlled by the electric and magnetic fields and by the electron energy. All the other modes are evanescent. For a wide range of parameters, this chain can serve as a spin filter.



Earlier work concentrated on spin-dependent conductance, averaged over electron energies, did not concentrate on spin filtering

Our aim: use simplest quasi-1D model to generate spin filtering

Our main conclusion: can achieve **full filtering** provided we use **both** spin-orbit and Aharonov-Bohm

We use tight-binding quantum networks,

$$(\epsilon - \epsilon_u)|\psi(u)\rangle = -\sum_v \widetilde{U}_{uv}|\psi(v)\rangle$$

2-component spinor at node *u*

$$\widetilde{U}_{uv} \equiv J_{uv}U_{uv}$$

2x2 unitary matrix, representing hopping from *v* to *u*

Continuum versus tight-binding networks:
AA + Ora Entin-Wohlman, J. Phys. Chem. **113**, 3676 (2009); ArXiv: 0807.4088

General solution:

$$\psi_a(n) = \sum_{i=1}^{4} A_i e^{i q_i \bar{L} n} \chi_a(q, \mu)$$

4 solutions, which appear in pairs, $\pm q_i$

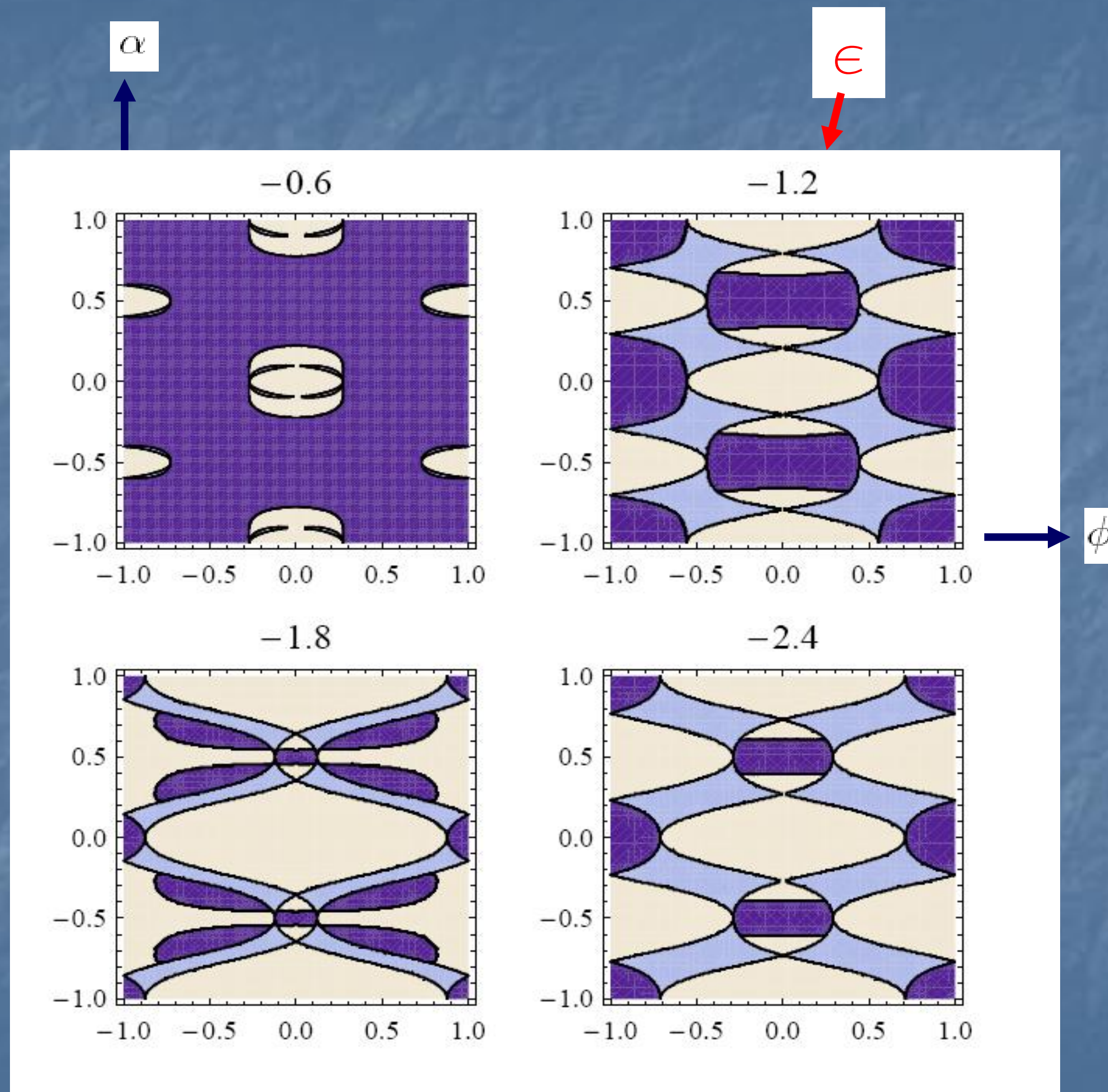
Real *q*: ¶running ¶solution.
Complex *q*: evanescent solution.

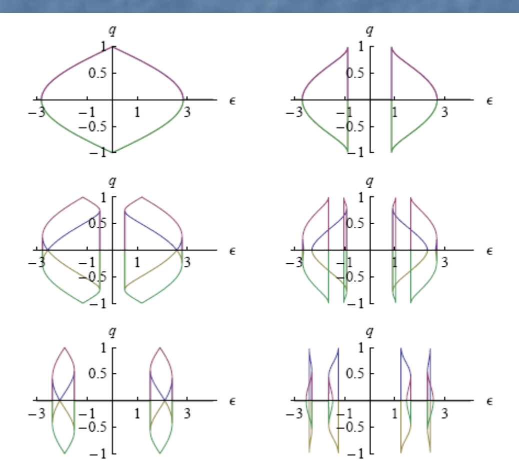
Ballistic conductance = $(e^2/h)g(E_F)$
g= number of solutions which
run from left to right: *g*= 0, 1 or 2

For a broad range of parameters,
there is only **one** running solution, and
then the electrons are fully polarized!

Ballistic conductance g

$(e^2/h)g(E_F)$





To obtain full filtering –
Must break both
Time reversal symmetry
(magnetic field)
And **reflection symmetry**
(electric field)

Problems:

*How to **realize** long chain?*

*How to **read information from** spinor?*

PHYSICAL REVIEW B 84, 035323 (2011)

Filtering and analyzing mobile qubit information via Rashba–Dresselhaus–Aharonov–Bohm interferometers

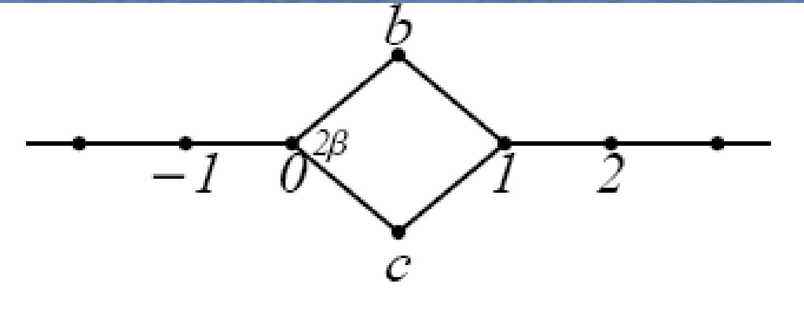
Amnon Aharony,[1,*] Yasuhiro Tokura,[2] Guy Z. Cohen,[3,†] Ora Entin-Wohlman,[1,‡] and Shingo Katsumoto[4]

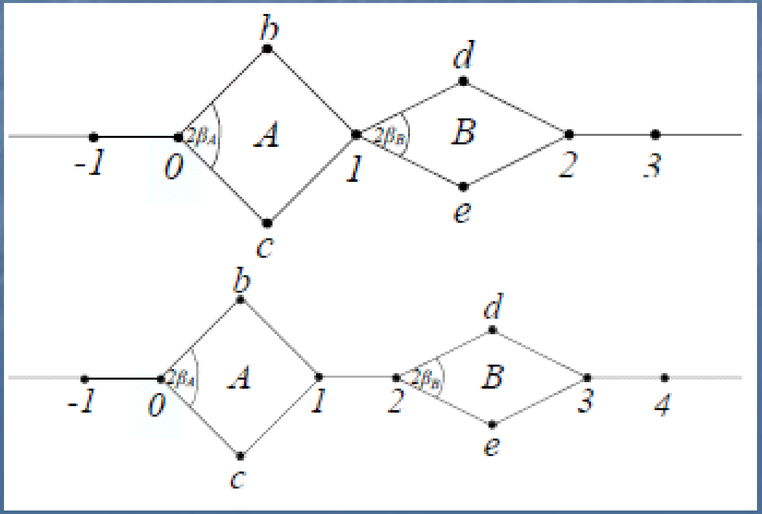
Problems:

*How to **realize** long chain?*

*How to **read information from** spinor?*

Single or double diamond





Single loop interferometer

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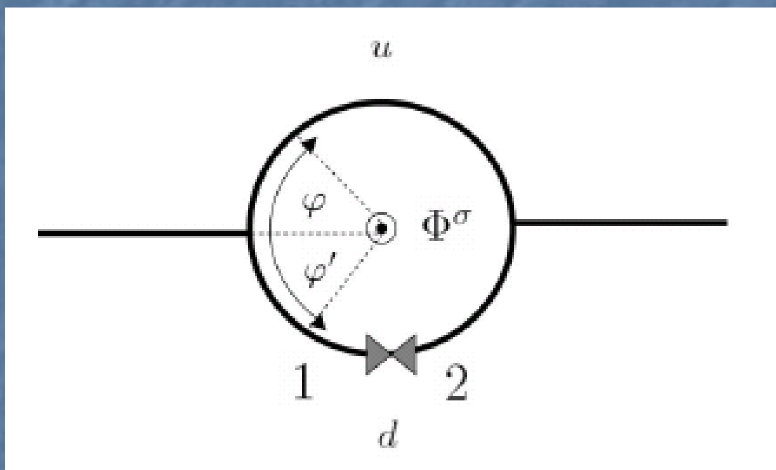
PHYSICAL REVIEW B 74, 115329 (2006)

Zero-conductance resonances and spin filtering effects in ring conductors subject to Rashba coupling

R. Citro, F. Romeo, and M. Marinaro



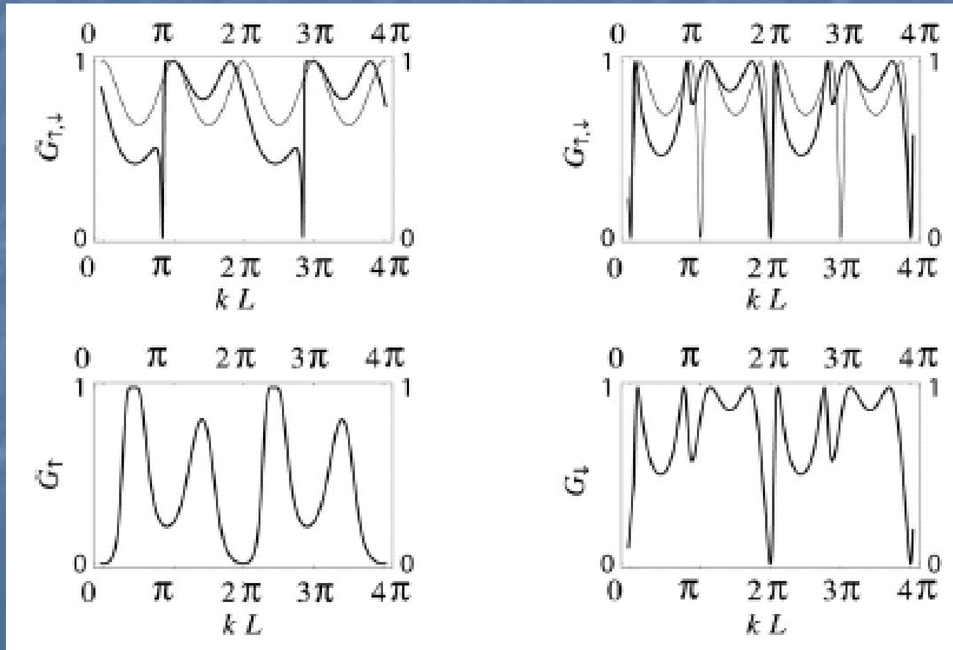


FIG. 2. The spin-resolved conductance G_σ as a function of kL.

PHYSICAL REVIEW A 75, 032107 (2007)

Non-Abelian gauge field theory of the spin-orbit interaction and a perfect spin filter

Naomichi Hatano
Institute of Industrial Science, University of Tokyo, Komaba, Meguro, Tokyo 153-8505, Japan

Ryōen Shirasaki
Department of Physics, Yokohama National University, Tokiwadai, Hodogaya-ku, Yokohama, Kanagawa 240-8501, Japan

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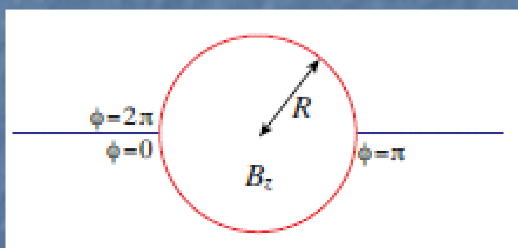
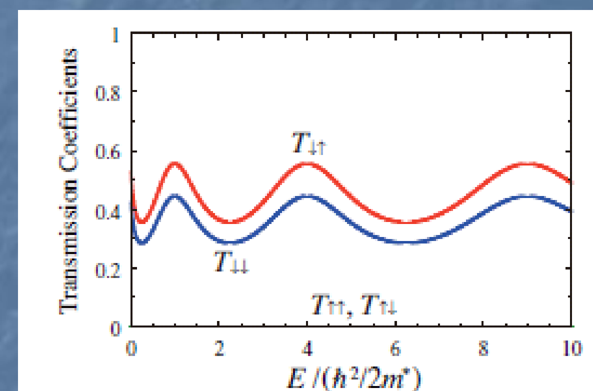
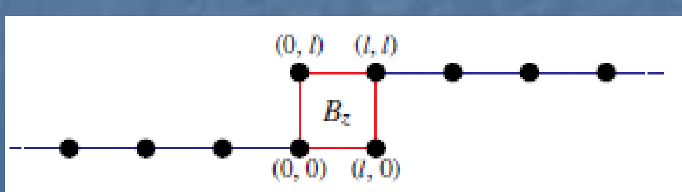
Theory and Computer Simulation Center, National Institute for Fusion Science, Oroshi-cho, Toki, Gifu 509-5292, Japan



$$2\pi\varphi_B = \frac{\pi}{2} + 2m\pi \quad \text{and} \quad 2\pi\varphi_R = \frac{\pi}{2} + 2n\pi$$

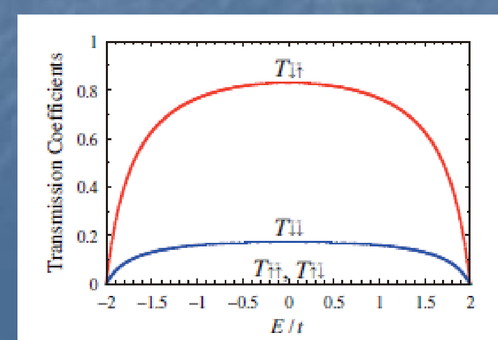




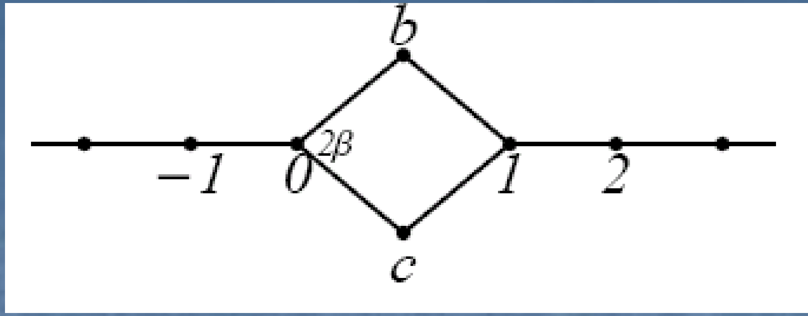
$$2\pi\varphi_B = 2\pi\varphi_R = \frac{\pi}{2}$$

$$l^2 B_z = \frac{\Phi_0}{4},$$

$$\theta l = 2 \arcsin \frac{1}{2^{1/4}} = 1.997\ldots.$$



Single diamond



$$(\epsilon - \epsilon_u)\psi(u) = -\sum_v J_{uv} U_{uv} \psi(v)$$

Tight-binding

$$(\epsilon - \epsilon_0)|\psi(0)\rangle = -\big(\widetilde{U}_{0b}|\psi(b)\rangle + \widetilde{U}_{0c}|\psi(c)\rangle\big) - j|\psi(-1)\rangle$$
$$(\epsilon - \epsilon_1)|\psi(1)\rangle = -\big(\widetilde{U}^{\dagger}_{b1}|\psi(b)\rangle + \widetilde{U}^{\dagger}_{c1}|\psi(c)\rangle\big) - j|\psi(2)\rangle,$$
$$(\epsilon - \epsilon_b)|\psi(b)\rangle = -\big(\widetilde{U}^{\dagger}_{0b}|\psi(0)\rangle + \widetilde{U}_{b1}|\psi(1)\rangle\big),$$
$$(\epsilon - \epsilon_c)|\psi(c)\rangle = -\big(\widetilde{U}^{\dagger}_{0c}|\psi(0)\rangle + \widetilde{U}_{c1}|\psi(1)\rangle\big).$$

Eliminate B and c →

$$(\epsilon - \epsilon_0 - \gamma_b - \gamma_c)|\psi(0)\rangle = \boxed{\mathbf{W}}|\psi(1)\rangle - j|\psi(-1)\rangle,$$
$$(\epsilon - \epsilon_1 - \gamma_b - \gamma_c)|\psi(1)\rangle = \boxed{\mathbf{W}^{\dagger}}|\psi(0)\rangle - j|\psi(2)\rangle,$$

$$\mathbf{W} \equiv \gamma_{0b1} U_{0b} U_{b1} + \gamma_{0c1} U_{0c} U_{c1}$$

$$\mathbf{W} = \gamma_b U_b + \gamma_c U_c$$

Non unitary

Scattering theory

Electron from left:

$$|\psi(n)\rangle = e^{ikna}|\chi_{in}\rangle + re^{-ikna}|\chi_r\rangle, \quad n \leq 0,$$
$$|\psi(n)\rangle = te^{ik(n-1)a}|\chi_t\rangle, \quad n \geq 1,$$

$$t|\chi_t\rangle = \mathcal{T}|\chi_{in}\rangle, \quad r|\chi_r\rangle = \mathcal{R}|\chi_{in}\rangle$$

Transmission:

$$\mathcal{T} = 2ij\sin(ka)\mathbf{W}^\dagger\left(Y\mathbf{1} - \mathbf{W}\mathbf{W}^\dagger\right)^{-1},$$

Reflection:

$$\mathcal{R} = -\mathbf{1} - 2ij\sin(ka)X_1(Y\mathbf{1} - \mathbf{W}\mathbf{W}^\dagger)^{-1}.$$

$$\mathcal{T} = 2ij\sin(ka)\mathbf{W}^\dagger\left(Y\mathbf{1} - \underline{\mathbf{W}\mathbf{W}^\dagger}\right)^{-1},$$

$$\mathcal{R} = -\mathbf{1} - 2ij\sin(ka)X_1(Y\mathbf{1} - \underline{\mathbf{W}\mathbf{W}^\dagger})^{-1}.$$

$$\mathbf{W} = d + \mathbf{b}\cdot\boldsymbol{\sigma}, \Rightarrow \mathbf{W}\mathbf{W}^\dagger = A + \mathbf{B}\cdot\boldsymbol{\sigma}$$

$$\mathbf{W} = \gamma_b U_b + \gamma_c U_c, \Rightarrow \mathbf{W}\mathbf{W}^\dagger = \gamma_b^2 + \gamma_c^2 + \gamma_b\gamma_c(u + u^\dagger),$$

$$u \equiv U_b U_c^\dagger = e^{-i\phi + i\boldsymbol{\omega}\cdot\boldsymbol{\sigma}}$$

Unitary matrix transforming spinor after a full walk around the loop

$$\mathbf{W}\mathbf{W}^\dagger = \gamma_b^2 + \gamma_c^2 + \gamma_b\gamma_c(u + u^\dagger),$$

$$u \equiv U_b U_c^\dagger = e^{-i\phi + i\boldsymbol{\omega}\cdot\boldsymbol{\sigma}}$$

$$\mathbf{W}\mathbf{W}^\dagger = A + \mathbf{B}\cdot\boldsymbol{\sigma}$$

$$A = \gamma_b^2 + \gamma_c^2 + 2\gamma_b\gamma_c \cos\omega\cos\phi,$$
$$\mathbf{B} = 2\gamma_b\gamma_c \sin\omega\sin\phi\,\hat{\mathbf{n}}.$$

$$\Rightarrow \quad \mathbf{W}\mathbf{W}^\dagger|\pm\hat{\mathbf{n}}\rangle = \lambda_\pm|\pm\hat{\mathbf{n}}\rangle \qquad \hat{\mathbf{n}}\cdot\boldsymbol{\sigma}|\hat{\mathbf{n}}\rangle = |\hat{\mathbf{n}}\rangle$$

$$\lambda_\pm = A \pm |\mathbf{B}| = \gamma_b^2 + \gamma_c^2 + 2\gamma_b\gamma_c\cos(\phi\pm\omega).$$

$$\mathbf{W}\mathbf{W}^\dagger|\pm\hat{\mathbf{n}}\rangle=\lambda_\pm|\pm\hat{\mathbf{n}}\rangle$$

$$\mathbf{W}^\dagger\equiv\sqrt{\lambda_-}|-\mathbf{n}'\rangle\langle-\mathbf{n}|+\sqrt{\lambda_+}|\hat{\mathbf{n}}'\rangle\langle\hat{\mathbf{n}}|$$

$$\mathbf{B}'=2\mathrm{Re}[d^*\mathbf{b}]-2[\mathrm{Re}(\mathbf{b})\times\mathrm{Im}(\mathbf{b})]\equiv|\mathbf{B}|\hat{\mathbf{n}}'$$

$$|\chi_{in}\rangle=c_+|\hat{\mathbf{n}}\rangle+c_-|-\hat{\mathbf{n}}\rangle \Rightarrow t|\chi_t\rangle=c_+t_+|\hat{\mathbf{n}}'\rangle+c_-t_-|-\hat{\mathbf{n}}'\rangle$$

$$t_\pm=\frac{2ij\sin(ka)}{Y-\lambda_\pm}\sqrt{\lambda_\pm}.$$

Full filtering if one eigenvalue vanishes!

$$\lambda_-=0 \Rightarrow t|\chi_t\rangle=c_+t_+|\hat{\mathbf{n}}'\rangle \Rightarrow T=T_+|c_+|^2$$

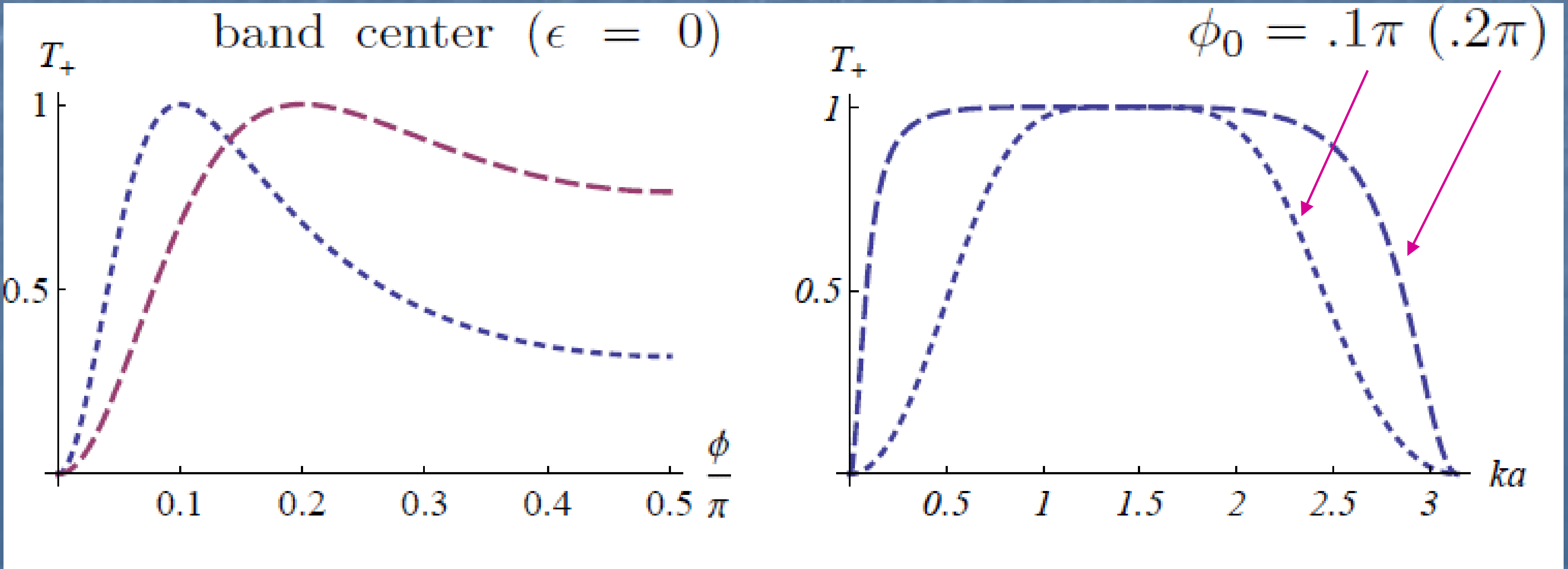
Full filtering if one eigenvalue vanishes!

$$\lambda_\pm = A \pm |\mathbf{B}| = \gamma_b^2 + \gamma_c^2 + 2\gamma_b\gamma_c \cos(\phi \pm \omega).$$

$\lambda_- = 0$ **only when** $\gamma_b = \gamma_c \equiv \gamma \quad \text{and} \quad \cos(\phi - \omega) = -1$

$\Rightarrow$ $\omega = \phi + \pi$ $\Rightarrow$ $\lambda_+ = 4\gamma^2 \sin^2 \phi$

T depends only on ϕ



“Reading” spin information

Incoming electrons polarized, $|\chi_{in}\rangle \equiv |\hat{\mathbf{n}}_0\rangle$

$$|c_+|^2 = |\langle\hat{\mathbf{n}}|\hat{\mathbf{n}}_0\rangle|^2 = \frac{1}{2}[1 + \hat{\mathbf{n}}_0 \cdot \hat{\mathbf{n}}]$$

$$T = T_+ |c_+|^2$$

⇒ Can measure the projection of the incoming polarization on that of the filter

Rashba spin orbit

$$\mathcal{H}_{\mathrm{R}} = \frac{\hbar k_R}{m}(p_y\sigma_x - p_x\sigma_y)$$

$$U_{uv} = \exp[i\frac{\pi BL}{\Phi_0}\hat{\gamma}_{uv} \times \hat{z} \cdot \mathbf{r}_u + ik_{SO}L\hat{\gamma}_{uv} \times \hat{z} \cdot \boldsymbol{\sigma}]$$

$$U_{0b} = \exp(i\alpha\sigma_1), \quad U_{b1} = \exp(-i\phi/2 - i\alpha\sigma_2)$$
$$U_{0c} = \exp(-i\alpha\sigma_2), \quad U_{c1} = \exp(i\phi/2 + i\alpha\sigma_1)$$

$$\sigma_1 = \sin\beta\sigma_x - \cos\beta\sigma_y$$

$$\sigma_2 = \sin\beta\sigma_x + \cos\beta\sigma_y$$

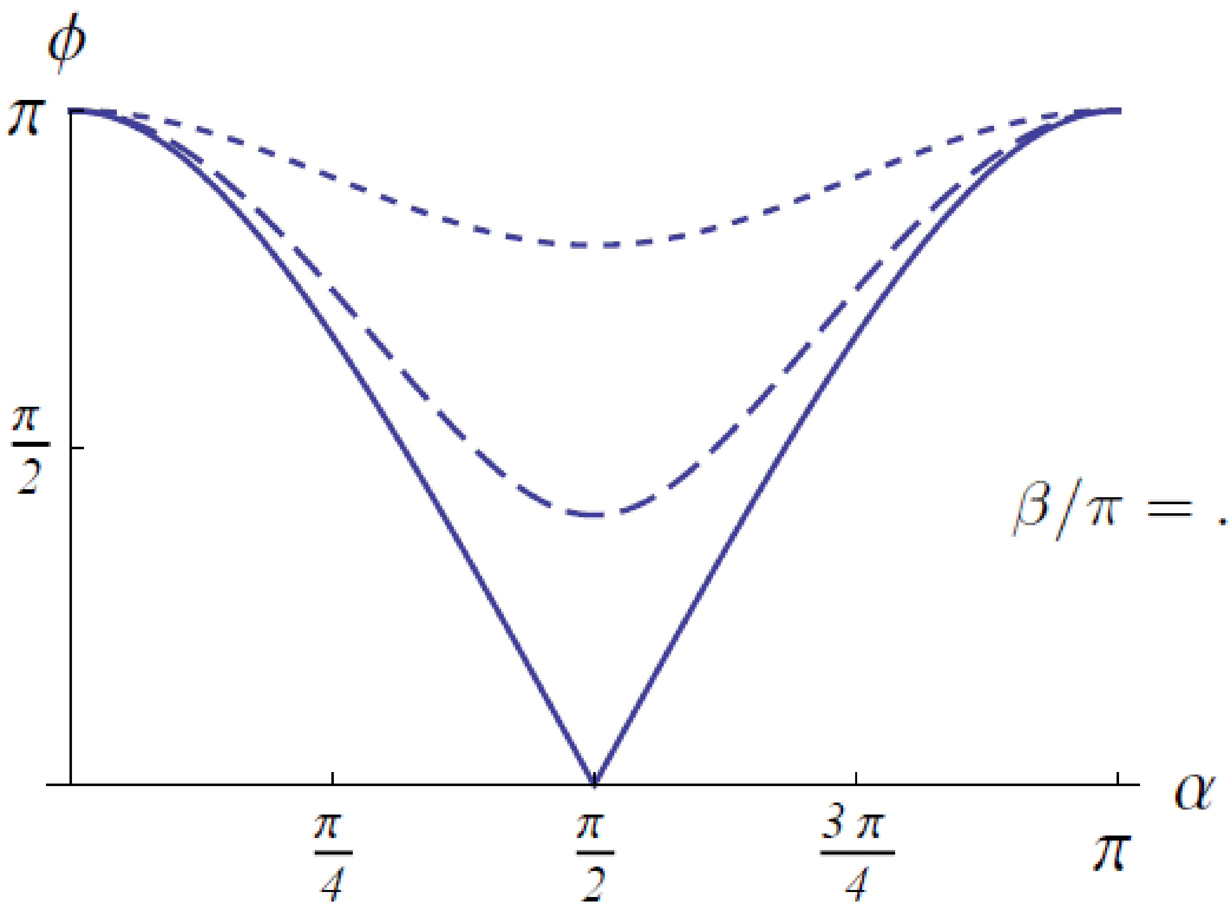
$$\alpha = \alpha_R = k_R L \equiv \alpha_1/\cos\beta$$

$$\lambda\_ = 0 \Rightarrow \cos(\phi/2) = s^2\sin(2\beta)$$

$$c = \cos\alpha, \; s = \sin\alpha$$

$\lambda_{-} = 0 \Rightarrow \cos(\phi/2) = s^2 \sin(2\beta)$

Independent of energy!



$\beta/\pi = .25,\ .15,\ .05.$

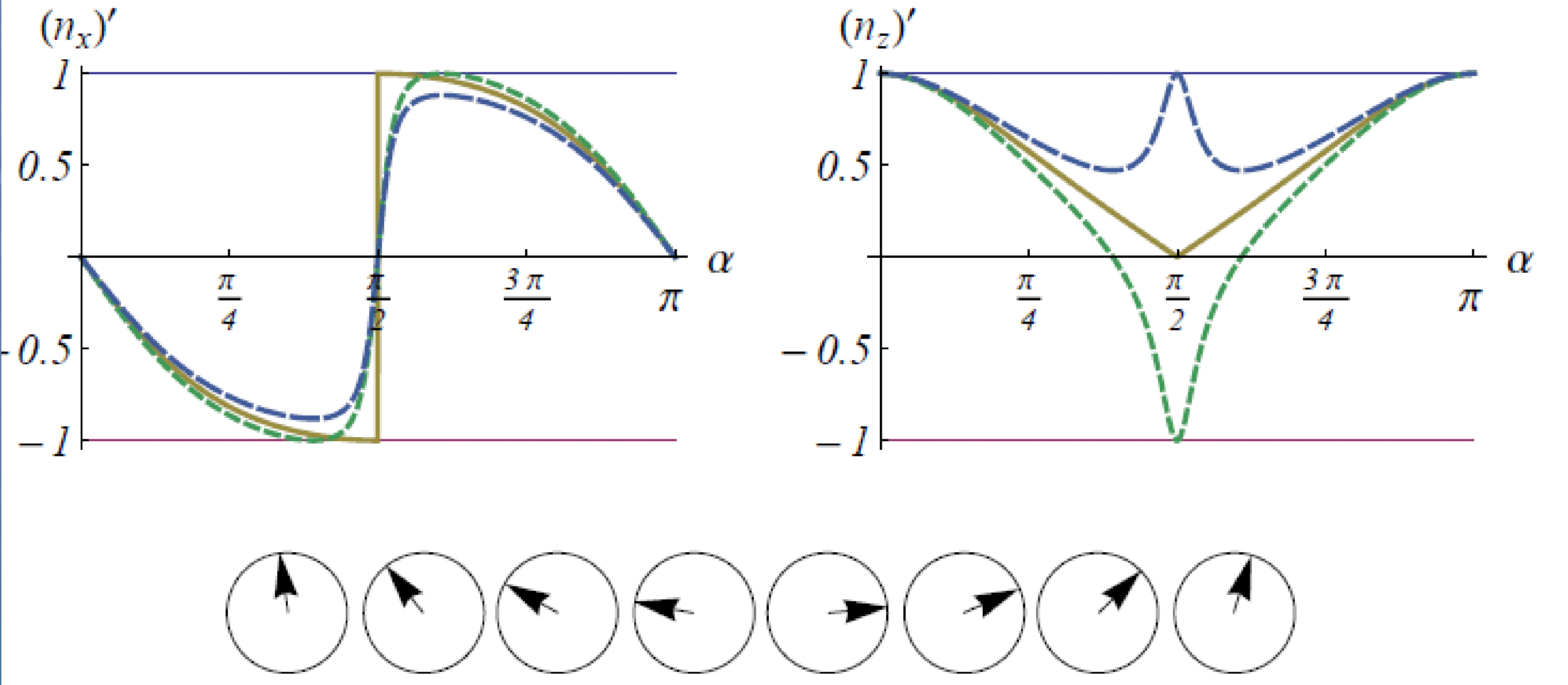
$c = \cos\alpha,\ s = \sin\alpha$

To obtain full filtering –
Must break both
Time reversal symmetry
(magnetic field)
And **reflection symmetry**
(electric field)

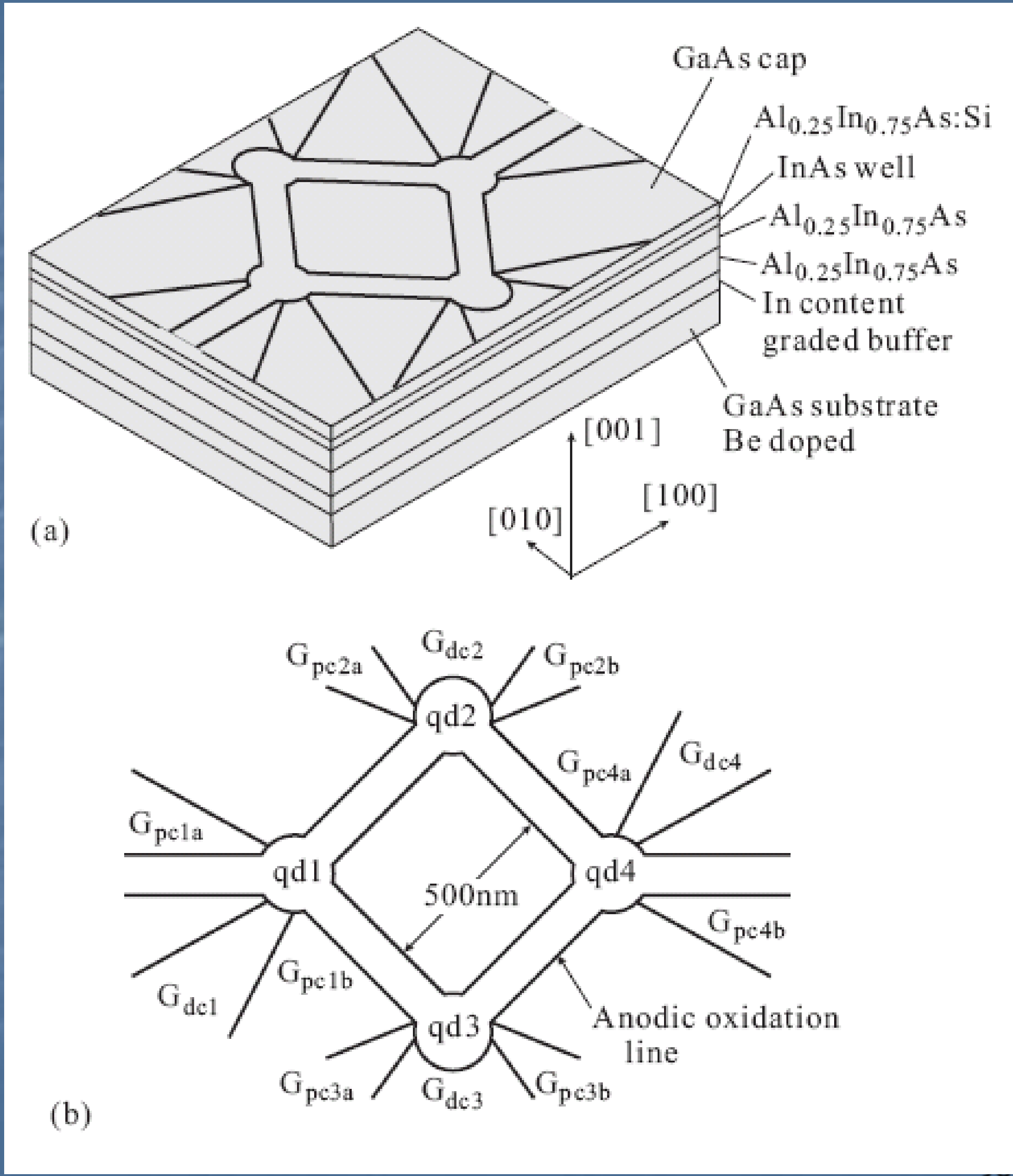
$$\hat{\mathbf{n}} = \left(2cs\cos\beta, 0, c^2 - s^2\cos(2\beta)\right)/\sqrt{1 - s^4\sin^2(2\beta)}$$

Independent of energy!

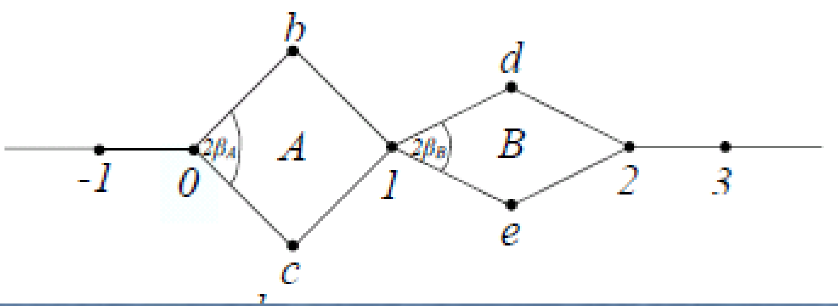
$$\hat{\mathbf{n}}' = (-\hat{n}_x, 0, \hat{n}_z).$$



Experimental realization



Two diamonds

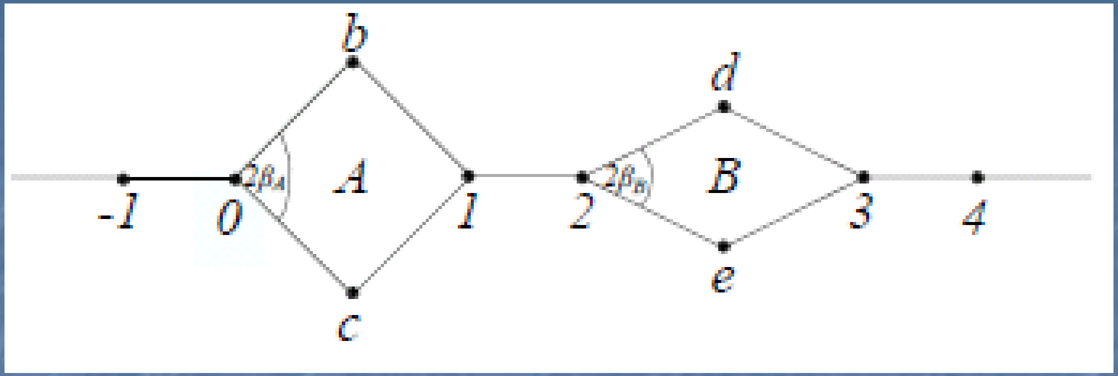


$$\mathcal{T} = 2ij\sin(ka)\mathbf{W}_B^\dagger\left[Z_1X_0X_2\mathbf{1} + X_0\mathbf{W}_B\mathbf{W}_B^\dagger + X_2\mathbf{W}_A^\dagger\mathbf{W}_A\right]^{-1}\mathbf{W}_A^\dagger$$
$$= 2ij\sin(ka)\mathbf{W}_B^\dagger\frac{Z_1X_0X_2 + X_0A_B + X_2A_A - (X_0\mathbf{B}_B + X_2\mathbf{B}'_A)\cdot\boldsymbol{\sigma}}{(Z_1X_0X_2 + X_0A_B + X_2A_A)^2 - (X_0\mathbf{B}_B + X_2\mathbf{B}'_A)^2}\mathbf{W}_A^\dagger$$

$A_A = A_B$ and $\mathbf{B}_A = \mathbf{B}'_B$

$$\mathcal{T} = \frac{2ij\sin(ka)\lambda_+}{z_1X_0X_2 + (X_0 + X_2)\lambda_+}|\hat{\mathbf{n}}_A\rangle\langle\hat{\mathbf{n}}_A|$$

Same incoming and outgoing spin, large transmission



$$\mathcal{T} = -2ij\sin(ka)J_0\mathbf{W}_B^\dagger U^\dagger\left[\Delta X_0 X_3 \mathbf{1} + z_1 X_0 U \mathbf{W}_B \mathbf{W}_B^\dagger U^\dagger + z_2 X_3 \mathbf{W}_A^\dagger \mathbf{W}_A + \mathbf{W}_A^\dagger \mathbf{W}_A U \mathbf{W}_B \mathbf{W}_B^\dagger U^\dagger\right]^{-1}\mathbf{W}_A^\dagger$$

Can rotate the spins between the two diamonds

$$\mathbf{W}_A^\dagger = \sqrt{\lambda_{A+}}|\hat{\mathbf{n}}'_A\rangle\langle\hat{\mathbf{n}}_A|$$

$$\widetilde{\mathbf{W}}_B^\dagger = \sqrt{\lambda_{B+}}|\hat{\mathbf{n}}'_{BU+}\rangle\langle\hat{\mathbf{n}}_{BU+}|$$

$$\mathcal{T} = t|\hat{\mathbf{n}}'_{BU+}\rangle\langle\hat{\mathbf{n}}_A|$$

$$t = \frac{-2ij\sin(ka)J_0\langle\hat{\mathbf{n}}_{BU+}|\hat{\mathbf{n}}'_{A+}\rangle\sqrt{\lambda_{A+}\lambda_{B+}}}{\Delta X_0 X_3 + z_1 X_0 \lambda_{B+} + z_2 X_3 \lambda_{A+} + \lambda_{A+}\lambda_{B+}}$$

Datta-Das spin FET without ferromagnets!

Are there materials for this device?

$\mathrm{Al_{0.25}In_{0.75}As}$ barrier layer

$$k_R = m^*\alpha/\hbar^2 = 9\times 10^6\,\mathrm{m}^{-1}$$

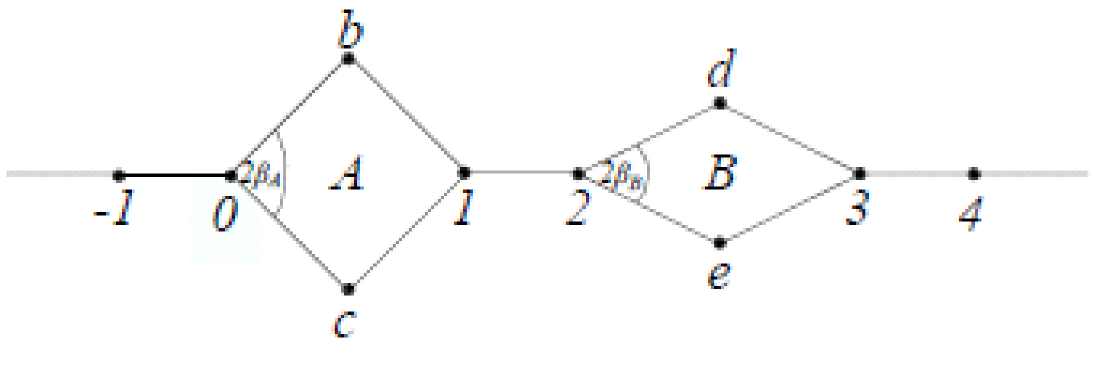
quaternary InGaAsP/InGaAs heterointerface

$$k_R = 5.55\times 10^6\,\mathrm{m}^{-1}$$

$L = 300\mathrm{nm}$ → $\alpha_R \sim 1.6 - 2.7$

How to confirm filtering?

•Use double interferometer as a Datta-Das device.



Datta-Das spin FET without ferromagnets!

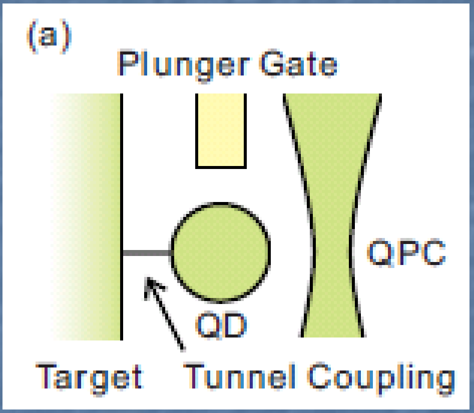
How to confirm filtering?

- Use side quantum dot:

PHYSICAL REVIEW B 79, 195313 (2009)

Detection of spin polarization with a side-coupled quantum dot

Tomohiro Otsuka,* Eisuke Abe, Yasuhiro Iye, and Shingo Katsumoto



How to confirm filtering?

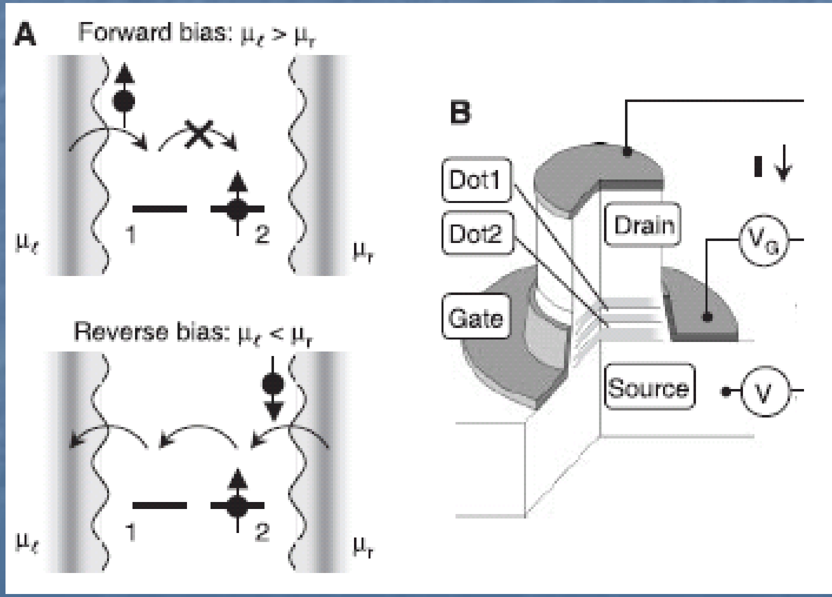
•Use rectification by Pauli exclusion:

Current Rectification by Pauli Exclusion in a Weakly Coupled Double Quantum Dot System

K. Ono,[1] D. G. Austing,[2,3] Y. Tokura,[2] S. Tarucha[1,2,4]*

SCIENCE VOL 297 23 AUGUST 2002

1313



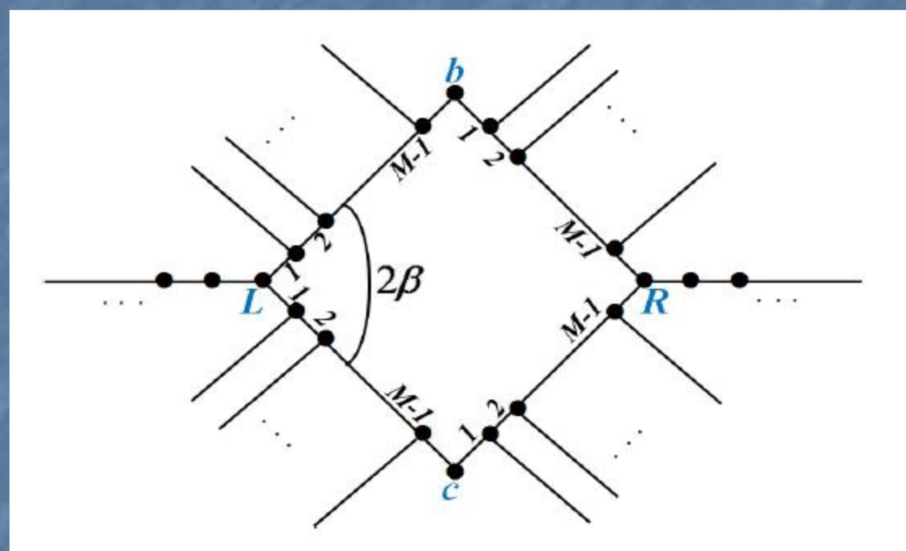
More recent results

* Stability against leaking?

PHYSICAL REVIEW B 87, 205438 (2013)

Robustness of spin filtering against current leakage in a Rashba-Dresselhaus-Aharonov-Bohm interferometer

Shlomi Matityahu,[1] Amnon Aharony,[1,2,3,*] Ora Entin-Wohlman,[1,2,3] and Shingo Katsumoto[4]



$$(\epsilon - \widetilde{\epsilon}_{uv})\,|\psi_n^{uv}\rangle = -J_{uv}\left(U_{uv}^{\dagger}|\psi_{n-1}^{uv}\rangle + U_{uv}|\psi_{n+1}^{uv}\rangle\right)\,,$$

$$\widetilde{\epsilon}_{uv} = -\frac{|J_{x,uv}|^2 e^{ika}}{j}$$

$$\lambda_{\pm} = A \pm B = \gamma_b^2 + \gamma_c^2 + 2\gamma_b\gamma_c \cos(\phi \mp \omega).$$

$$\gamma_b = |\gamma_b| e^{i\delta_b} \text{ and } \gamma_c = |\gamma_c| e^{i\delta_c}$$

$$\begin{aligned} \lambda_{LR,\pm} &= A_{LR} \pm B_{LR} \\ &= |\gamma_b|^2 + |\gamma_c|^2 + 2|\gamma_b||\gamma_c| \cos(\widetilde{\phi} \mp \omega). \end{aligned}$$

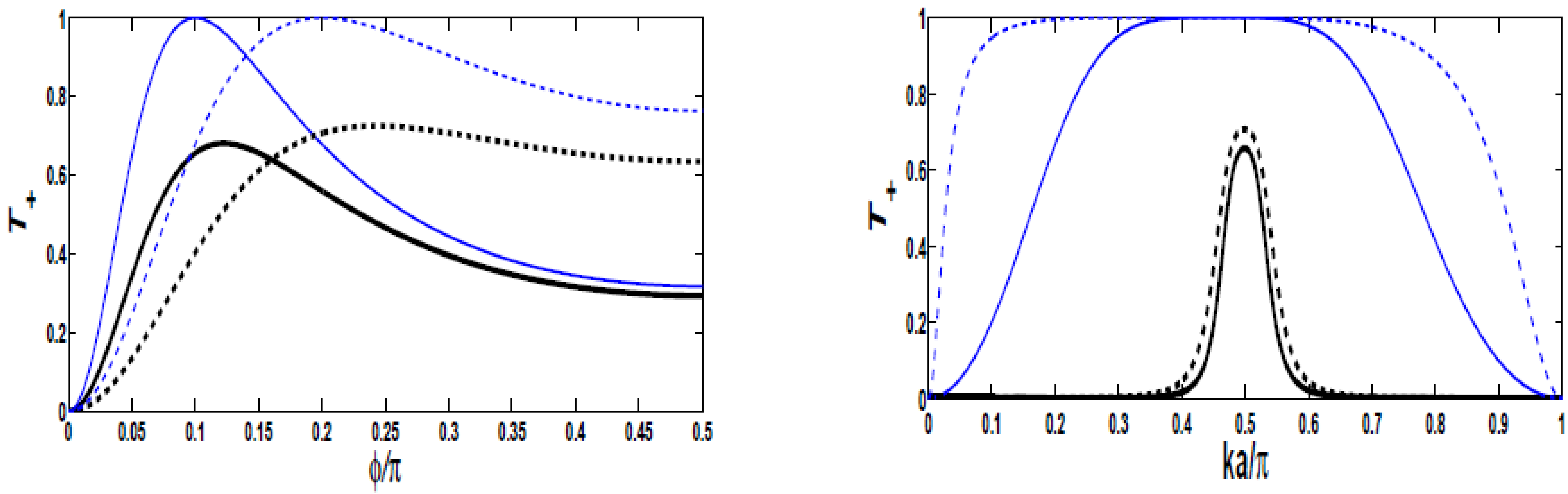
$$\widetilde{\phi} = \phi + \delta_c - \delta_b$$

Filtering:

$$|\gamma_b| = |\gamma_c| \equiv \gamma, \quad \cos(\widetilde{\phi} + \omega) = -1.$$

Leaking breaks time reversal symmetry!

No need for magnetic field



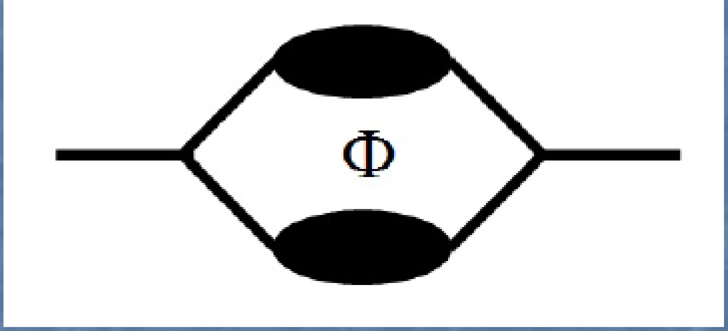
T+
0
0.2
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
φ/π
T+
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ka/π

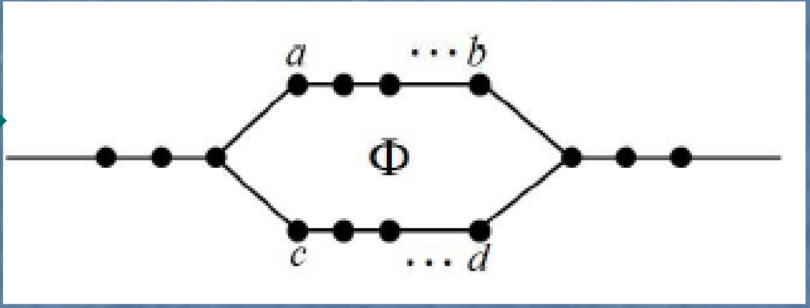
**

New Journal of Physics
The open access journal for physics

Spin filtering in a Rashba–Dresselhaus–Aharonov–Bohm double-dot interferometer

Shlomi Matityahu[1], Amnon Aharony[1,2,3,6],
Ora Entin-Wohlman[1,2,3] and Seigo Tarucha[4,5]





$$\tilde{\varepsilon}_{uv} = \varepsilon_{uv} - J_{uv}\frac{\sin[k_{uv}(M-1)]}{\sin(k_{uv}M)} \qquad (uv = ab, cd),$$

$$\tilde{J}_{uv} = J_{uv}\frac{\sin k_{uv}}{\sin(k_{uv}M)} \qquad (uv = ab, cd),$$

$$\tilde{\varepsilon}^2_{ab} - \tilde{J}^2_{ab} = \tilde{\varepsilon}^2_{cd} - \tilde{J}^2_{cd},$$

$$\omega = \alpha_{cd} - \alpha_{ab} = \phi + \pi.$$

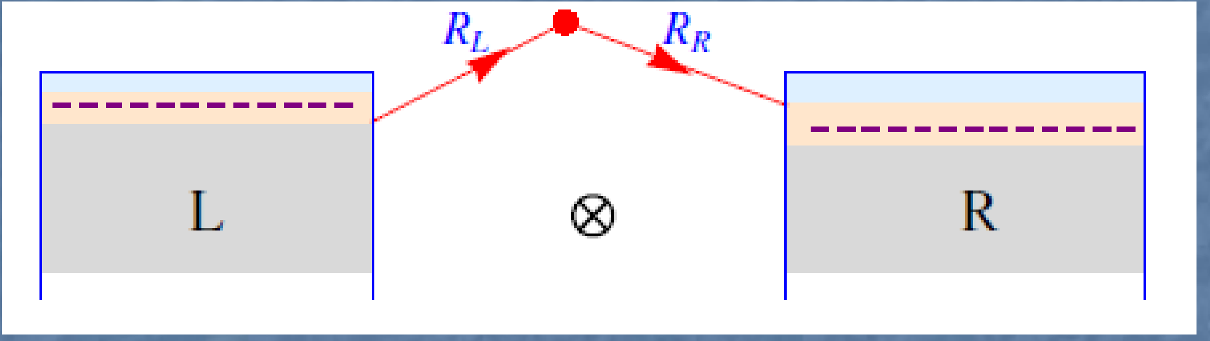
Need to tune only 2 voltages!

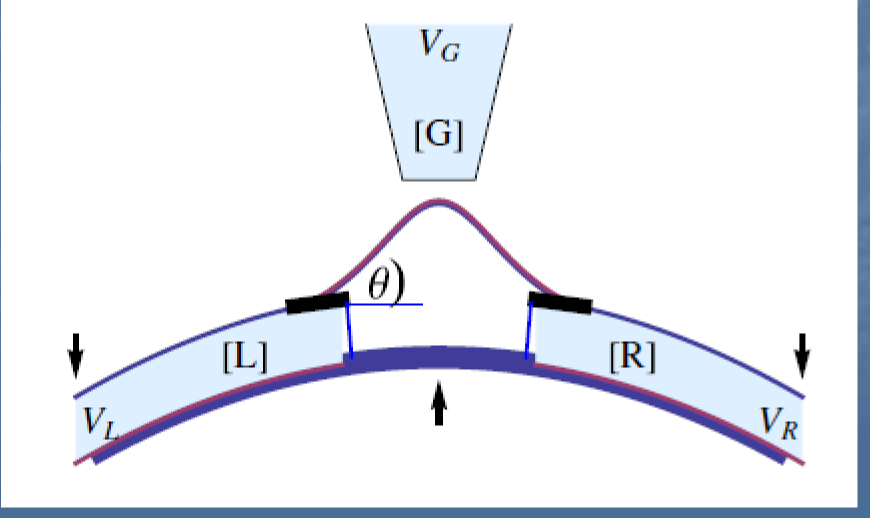
***

PRL **111,** 176602 (2013) PHYSICAL REVIEW LETTERS week ending 25 OCTOBER 2013

Suspended Nanowires as Mechanically Controlled Rashba Spin Splitters

R. I. Shekhter,[1] O. Entin-Wohlman,[2,3,*] and A. Aharony[2,3]





$$V_{k(p)} = -J_{L(R)} \exp[-i\psi_{L(R)}],$$

$$\psi_L = \phi_L - \alpha(x_L\sigma_y - y_L\sigma_x),$$
$$\psi_R = \phi_R - \alpha(x_R\sigma_y + y_R\sigma_x).$$

Misbalanced spin population in the leads yields spin-split currents from the wire

vibrations ⇨

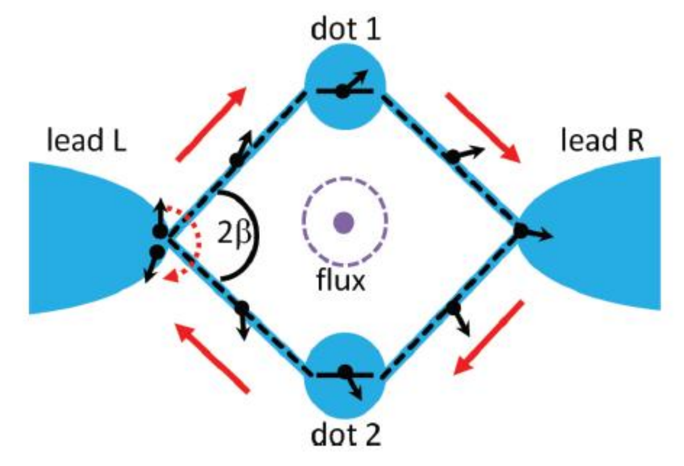
$$\frac{G_{\rm spin}/G_0}{\sin^2(\alpha d)\cos^2(\theta_0)} = \begin{cases} 1 - \frac{\beta\omega}{6}\frac{H^2}{H_0^2} & \beta\omega \ll 1 \\ \exp[-H^2/H_0^2], & \beta\omega \gg 1. \end{cases}$$

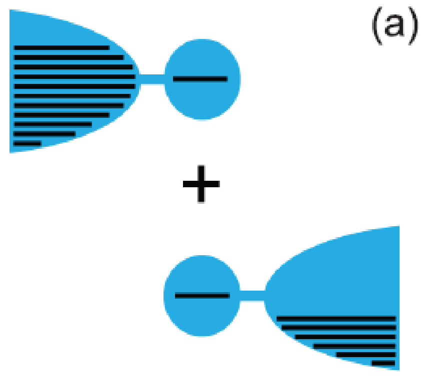
****

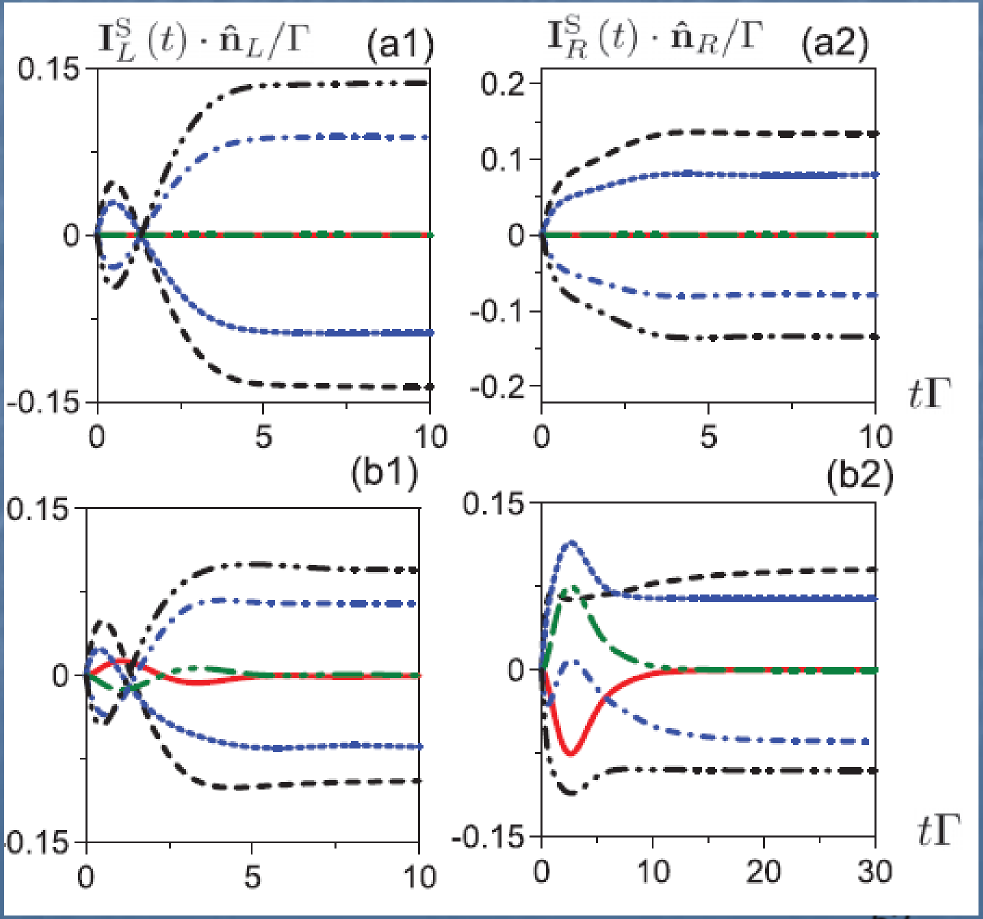
PHYSICAL REVIEW B 90, 165422 (2014)

Real-time dynamics of spin-dependent transport through a double-quantum-dot Aharonov-Bohm interferometer with spin-orbit interaction

Matisse Wei-Yuan Tu,[1] Amnon Aharony,[2,3,*] Wei-Min Zhang,[1,†] and Ora Entin-Wohlman[2,3]







Conclusions:

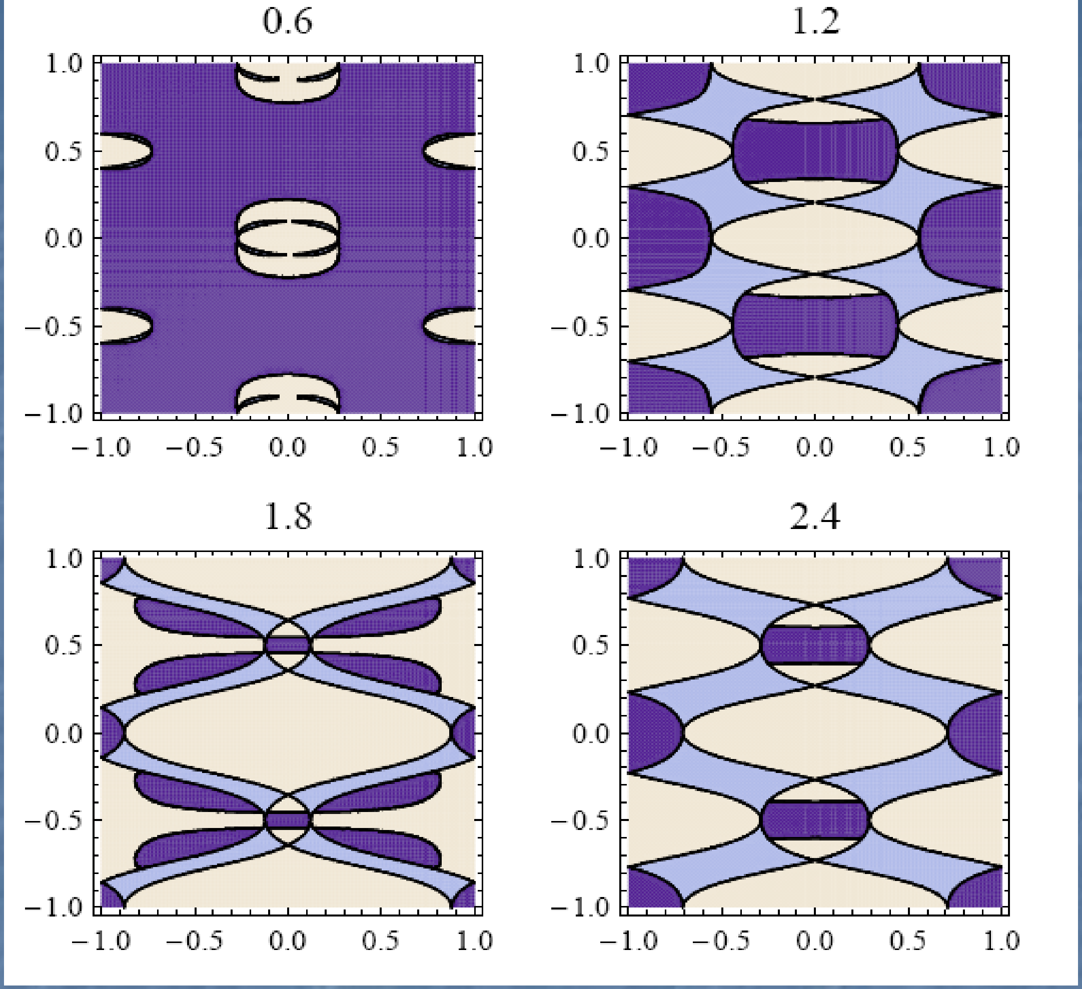
Need **both** Aharonov-Bohm and spin-orbit to maintain **full** filtering.

Spin is **sensitive to parameters**: small changes in parameters switch the direction of the filtered spin.

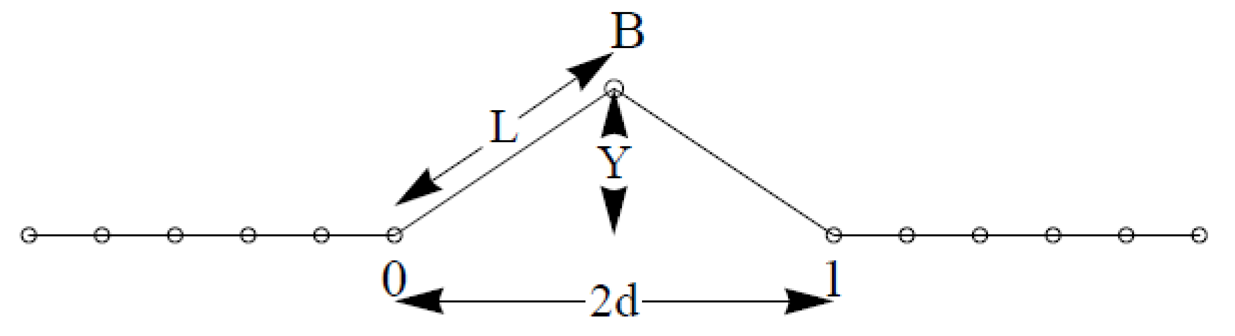
Can work at **fixed small magnetic field**, with small changes in electric field or in electron energy.

Double diamond = **Datta-Das** spin FET.

*** Results robust against leaks, ** can use double dot, *** can use vibrating molecule, **** time evolution generates spin currents in the leads.**



Thank you



$$(\epsilon - \tilde{\epsilon}_0)|0\rangle = -W|1\rangle - J|-1\rangle\ , \qquad (\epsilon - \tilde{\epsilon}_1)|1\rangle = -W^\dagger|0\rangle - J|2\rangle$$

$$W = J_{0B}J_{B1}/(\epsilon - \epsilon_B) = j^2 e^{i\phi}[\cos(\alpha L) + i\sin(\alpha L)\sigma_1][\cos(\alpha L) + i\sin(\alpha L)\sigma_2]/(\epsilon - \epsilon_B)$$
$$= j^2 e^{i\phi}\{\cos^2(\alpha L) + i\sin(2\alpha L)\cos\beta\sigma_y - \sin^2(\alpha L)[\cos(2\beta) + i\sin(2\beta)\sigma_z]\}/(\epsilon - \epsilon_B)$$

$$\tilde{\epsilon}_m = \epsilon_m - j^2/(\epsilon - \epsilon_B)$$

$$\overline{W} = e^{-(H/H_0)^2}\Big(1 - \frac{(2\alpha d)^2}{2!} + 2i\alpha d\sigma_y + \sigma_z \frac{\lambda}{d}\frac{H}{H_0}\frac{(2\alpha d)^2}{2!}\Big)$$

$$\mathcal{S}(\epsilon, Y) = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2iJ\sin(ka)\begin{bmatrix} [\epsilon - \Sigma_1^r(\epsilon)]\mathbf{1} & J_{\text{eff}}\overline{W} \\ J_{\text{eff}}\overline{W}^\dagger & [\epsilon - \Sigma_0^r(\epsilon)]\mathbf{1} \end{bmatrix}\mathcal{D}(\epsilon)^{-1}$$

$$\mathcal{D} = (\epsilon - \Sigma_1^r)(\epsilon - \Sigma_0^r)\mathbf{1} - [J_{\text{eff}}]^2\overline{WW^\dagger}$$

$$\overline{WW^\dagger} \approx 1 + 4[\lambda H/(dH_0)](\alpha d)^2\sigma_z$$

More to do:

How to **measure**?

Add **Zeeman** field ±Aharonov-Casher? Berry phase?

Dissipation: stochastic noise? phonons? Dephasing?

Add **e-e interactions?**

How can we **combine** beams to perform **computing?**

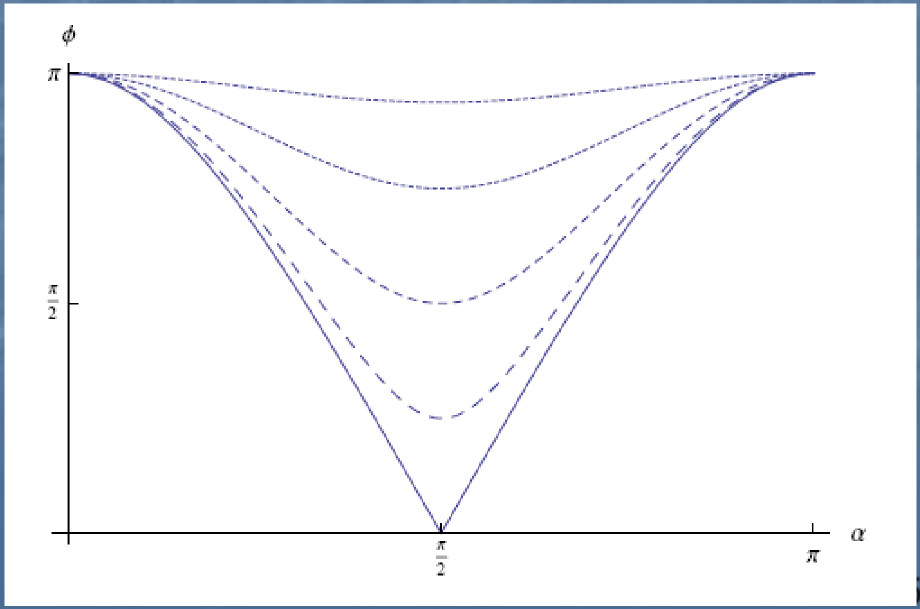
Choose parameters so that

$$\mathbf{W}^\dagger|\chi_-^{\mathbf{n}}\rangle = 0$$

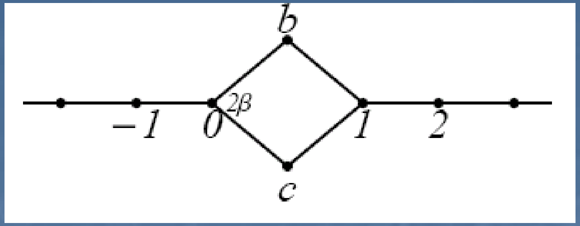
$$t|\chi_t\rangle = c_+ t_+ |\chi_+^{\mathbf{n}'}\rangle$$

Full filtering!

$$\cos(\phi/2) = s^2 \sin(2\beta)$$



Eliminate *b, c:*



$$(\epsilon - \epsilon_0 - \gamma_b - \gamma_c)|\psi(0)\rangle = \mathbf{W}|\psi(1)\rangle - j|\psi(-1)\rangle,$$
$$(\epsilon - \epsilon_1 - \gamma_b - \gamma_c)|\psi(1)\rangle = \mathbf{W}^\dagger|\psi(0)\rangle - j|\psi(2)\rangle,$$

$$\mathbf{W} = \gamma_b U_{0b} U_{b1} + \gamma_c U_{0c} U_{c1} = d + b_y \sigma_y + b_z \sigma_z,$$
$$d = a_+[c^2 - s^2 \cos(2\beta)],$$
$$b_y = -2i a_+ cs \cos\beta, \quad b_z = i a_- s^2 \sin(2\beta),$$

Non-unitary!

$$c = \cos\alpha,\ s = \sin\alpha$$

$$a_\pm = \gamma_b e^{-i\phi/2} \pm \gamma_c e^{i\phi/2}.$$

$$\gamma_j = J^2/(\epsilon - \epsilon_j),\ j = b, c$$

Electron from left:

$$\psi(n) = e^{ikna}\chi_{in} + r e^{-ikna}\chi_r, \quad n \le 0,$$
$$\psi(n) = t e^{ik(n-1)a}\chi_t, \quad n \ge 1,$$

$$\Rightarrow \quad t|\chi_t\rangle = \mathcal{T}|\chi_{in}\rangle$$

Spin-polarized electric currents in quantum transport

O. Entin-Wohlman,[1,2,*] A. Aharony,[1,*] Y. Tokura,[3] and Y. Avishai[1]

arXiv:0911.1347v2

Generalized Landauer formula

$$I_{x}^{j} = \int \frac{dE}{2\pi} \sum_{a} f_{a}(E) \mathrm{Tr}\Big\{ \delta_{a,L}\sigma_{j} - \sum_{nn'} \mathcal{M}_{Ln',an}(E)\sigma_{j} \Big\}$$

$$= \int \frac{dE}{2\pi} \Big(f_{L}(E) - f_{R}(E) \Big) \mathrm{Tr} \sum_{nn'} \mathcal{M}_{Ln',Rn}(E)\sigma_{j} \ , \quad (6)$$

$$\mathcal{M}_{Ln',an}(E) \equiv \boldsymbol{\mathcal{S}}_{Ln',an}(E)\boldsymbol{\mathcal{S}}^{\dagger}_{Ln',an}(E)$$

$$\mathcal{M}_{Ln',Rn}(E) = \frac{1}{2}\Big(\mathcal{T}_{n'n}(E) + \mathbf{V}_{n'n}(E) \cdot \boldsymbol{\sigma} \Big)$$

$$t|\chi_t\rangle = \mathcal{T}|\chi_{in}\rangle$$

$$\mathcal{T} = 2ij\sin(ka)\mathbf{W}^\dagger\left(Y - \mathbf{W}\mathbf{W}^\dagger\right)^{-1}.$$

$$Y = (X+\epsilon_0)(X+\epsilon_1), \quad X = \gamma_b + \gamma_c + je^{-ika}.$$

$$\mathbf{W}\mathbf{W}^\dagger = A + \mathbf{B}\cdot\boldsymbol{\sigma}.$$

$$\mathbf{W}\mathbf{W}^\dagger|\chi^{\mathbf{n}}_\pm\rangle = \lambda_\pm|\chi^{\mathbf{n}}_\pm\rangle$$

$$\mathbf{n}\cdot\boldsymbol{\sigma}|\chi^{\mathbf{n}}_\pm\rangle = \pm|\chi^{\mathbf{n}}_\pm\rangle$$

$$\mathbf{n} = \mathbf{B}/|\mathbf{B}| = \left(2cs\cos\beta, \ 0, \ c^2 - s^2\cos(2\beta)\right)/\sqrt{1 - s^4\sin^2(2\beta)}$$

Depends only on Rashba and on AB flux!

$$\lambda_\pm = A \pm |\mathbf{B}|.$$

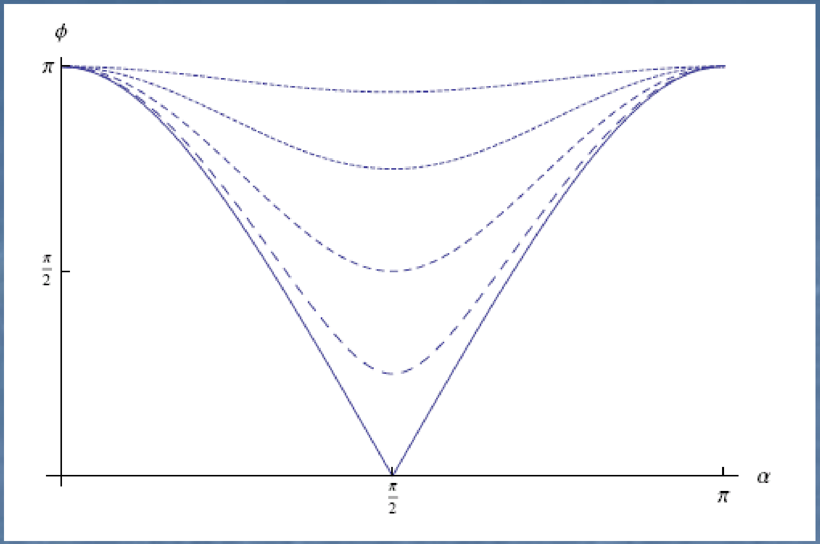
$$|\chi_{in}\rangle = c_+|\chi_+^{\mathbf{n}}\rangle + c_-|\chi_-^{\mathbf{n}}\rangle$$

$$\Rightarrow\quad t|\chi_t\rangle = c_+t_+|\chi_+^{out}\rangle + c_-t_-|\chi_-^{out}\rangle$$

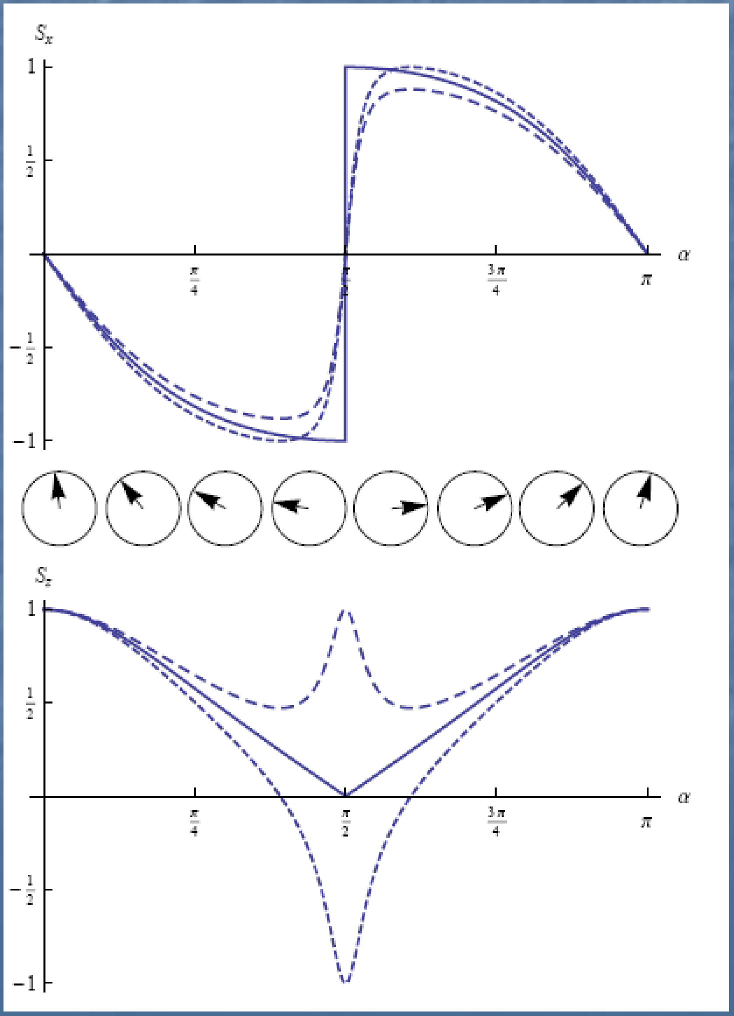
$$|\chi_\pm^{out}\rangle = \mathbf{W}^\dagger|\chi_\pm^{\mathbf{n}}\rangle/\sqrt{|\lambda_\pm|}$$

$$|\chi_\pm^{out}\rangle \equiv |\chi_\pm^{\mathbf{n}'}\rangle$$

$$\mathbf{n}' = (-n_x, 0, n_z)$$



Polarization of outgoing spins

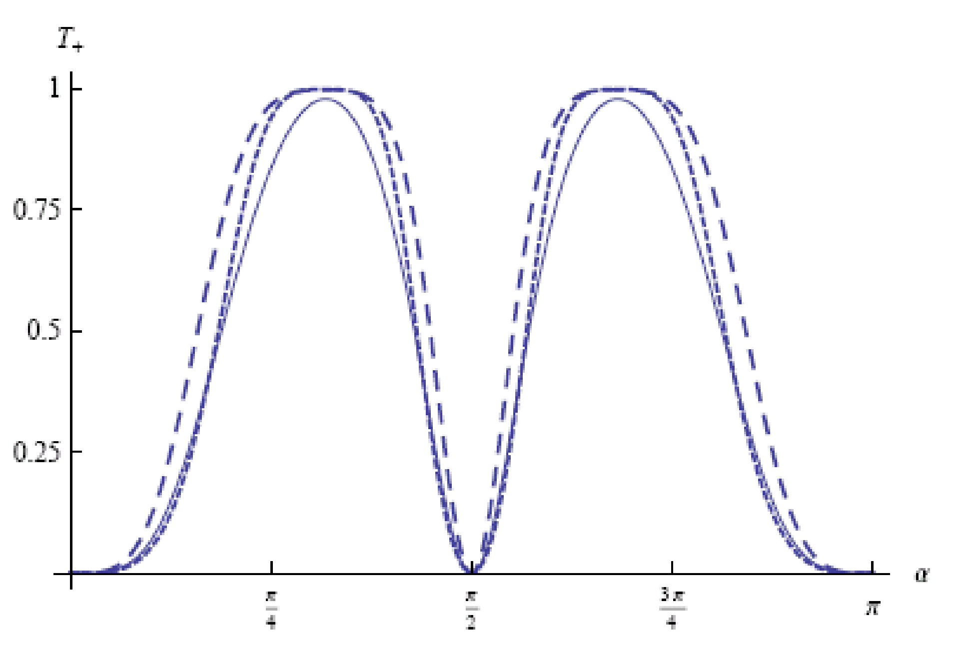


$$|\chi_{in}\rangle = c_+|\chi_+^{\mathbf{n}}\rangle + c_-|\chi_-^{\mathbf{n}}\rangle$$

$$t|\chi_t\rangle = c_+ t_+ |\chi_+^{\mathbf{n}'}\rangle$$

$$|c_+|^2 = |\langle|\chi_+^{\mathbf{n}}|\chi_{in}\rangle|^2$$

$$|c_+|^2 = \frac{1}{2}[1 + \mathbf{s}\cdot\mathbf{n}]$$



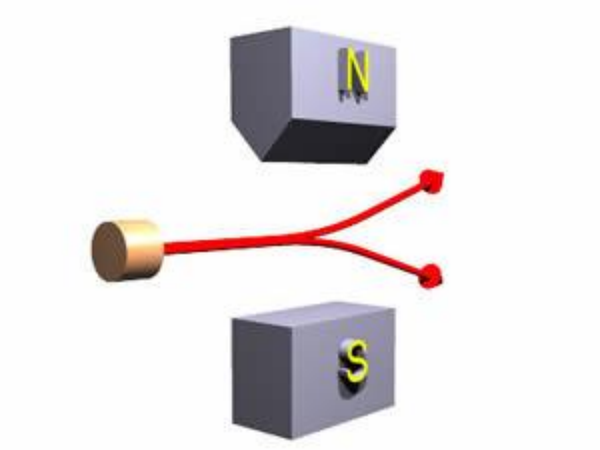
Transmitted currrent
Proportional to $|c_+|^2$:
Can measure incoming
spin polarization
Via measurements
of the transmission!

READING ¶

‘Writing’ on spinor: Spin filtering:

Work with mobile electrons,
Generate spin-polarized current out of an unpolarized source

Textbook method: Stern-Gerlach splitting



Based on **Zeeman** splitting,
Requires large fields, separation of beams not easy due to uncertainty

Writing and reading spin information on mobile electronic qubits

Amnon Aharony

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