Linear and nonlinear response of three-level superconducting flux quantum circuits

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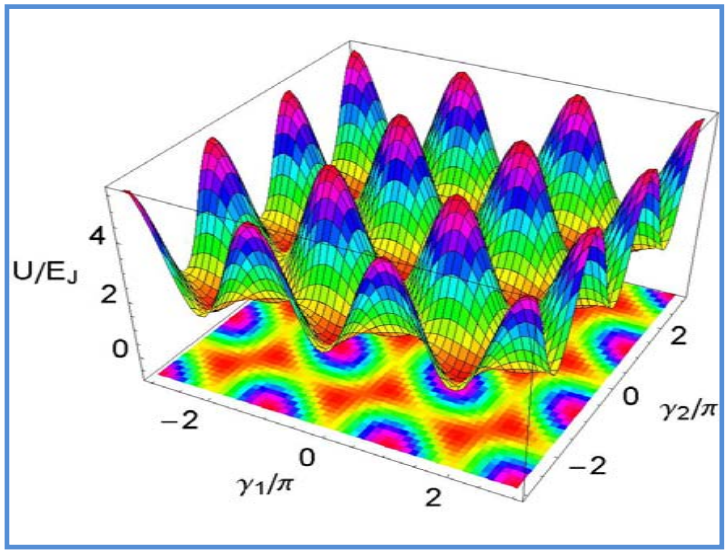
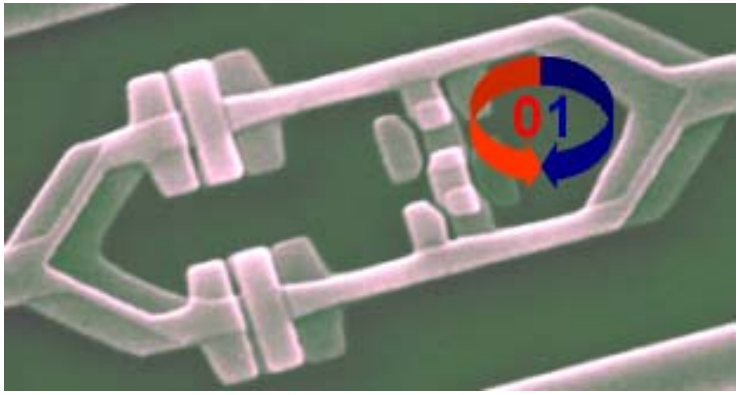
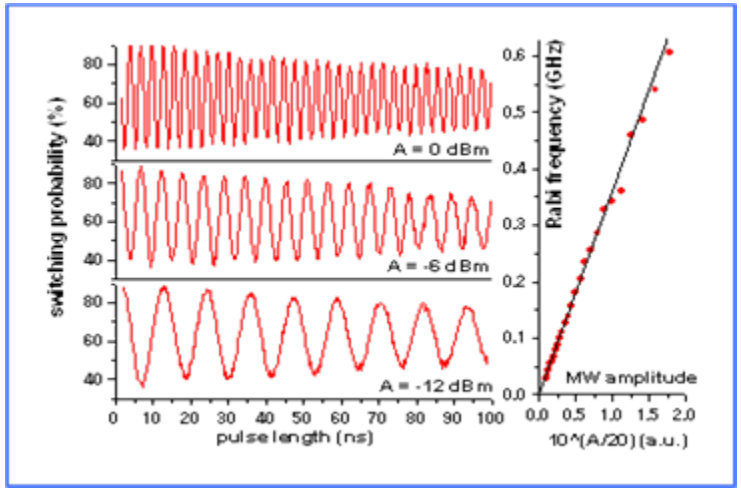
Superconducting flux quantum circuits



Potential energy

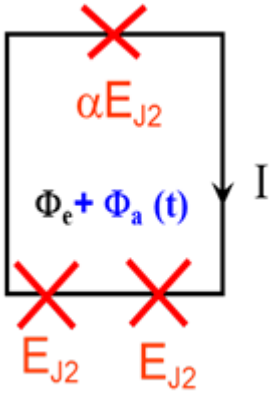


Delft, Jena, NTT, MIT, NEC-RIKEN



Delft and NEC, Science (2003)

Theoretical model



$$H_0 = \frac{P_p^2}{2M_p} + \frac{P_m^2}{2M_m} + U(\varphi_p, \varphi_m, f)$$

$$f = \frac{\Phi_e}{\Phi_0}$$

$$U = 2E_J\left(1 - \cos\varphi_p \cos\varphi_m\right) + \alpha E_J\left[1 - \cos\left(2\varphi_m + 2\pi f\right)\right]$$

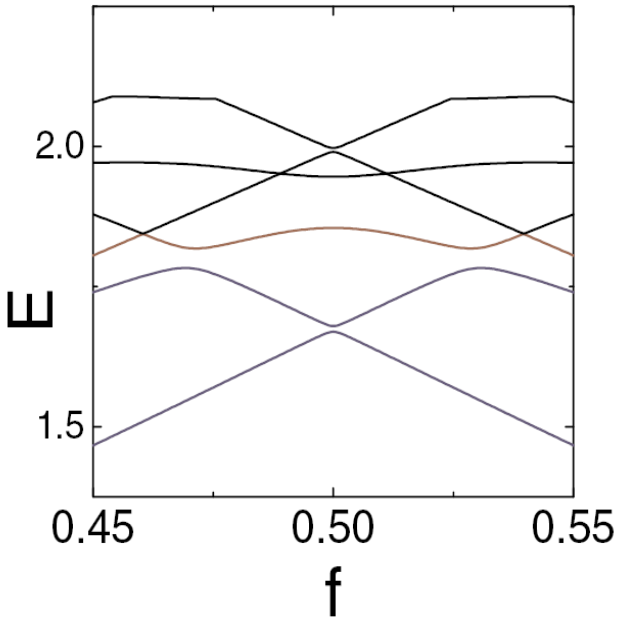
$$H_0 \mid n\rangle = \varepsilon_n \mid n\rangle$$

Parameters

$\alpha = 0.8, \quad E_J = 40\ E_C$

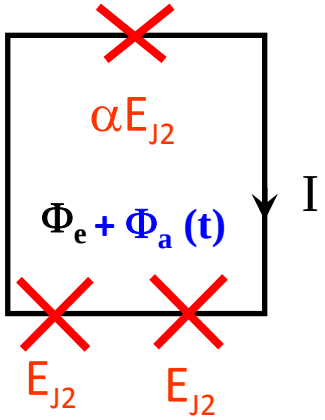
with

$E_C = e^2 / 2C$



Yu-xi Liu, J. Q. You, Wei, C. P. Sun, Nori, PRL **95**, 087001 (2005)

Microwave induced transitions



$$H_0 \,|\, n\rangle = \varepsilon_n \,|\, n\rangle$$

with

$$H_0 = \frac{P_p^2}{2M_p} + \frac{P_m^2}{2M_m} + U(\varphi_p, \varphi_m, f)$$

Time-dependent (ac) magnetic flux

$$\Phi_a(t) = \Phi_a^{(0)} \cos(\omega_{ij} t)$$

The Hamiltonian becomes into

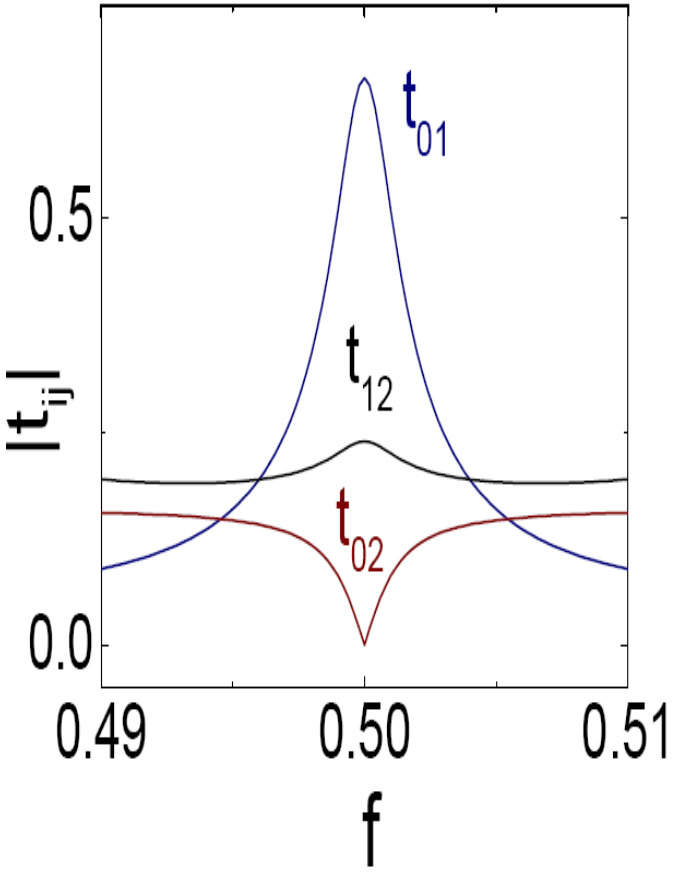
$$H = H_0 + V(t)$$

with

$$V(t) = -\frac{2\alpha\pi\Phi_a^{(0)} E_J}{\Phi_0} \sin\left(2\pi f + 2\varphi_m\right) \cos\left(\omega_{ij} t\right)$$

The transition elements are

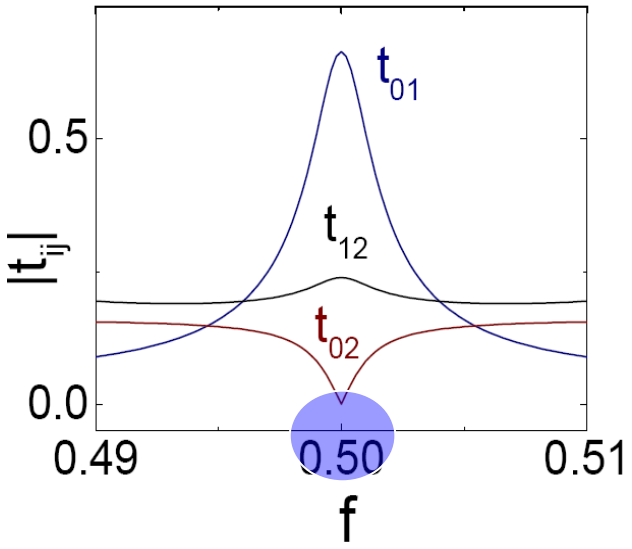
$$t_{ij} = -\frac{2\alpha\pi\Phi_a^{(0)} E_J}{\Phi_0} \left\langle i \,|\sin\left(2\pi f + 2\varphi_m\right)|\, j \right\rangle$$



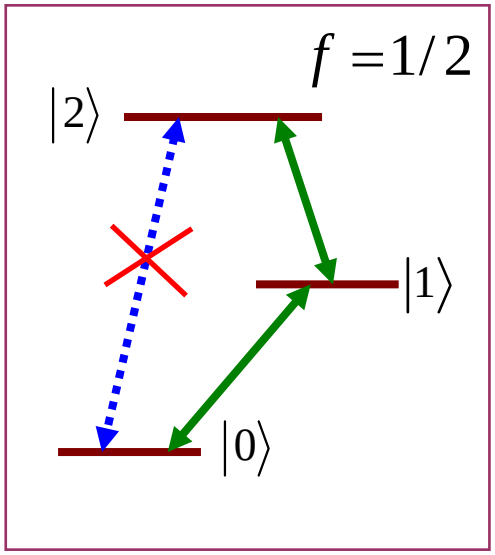
Transition elements versus reduced magnetic flux f

Liu, You, Wei, Sun, Nori, PRL **95**, 087001 (2005)

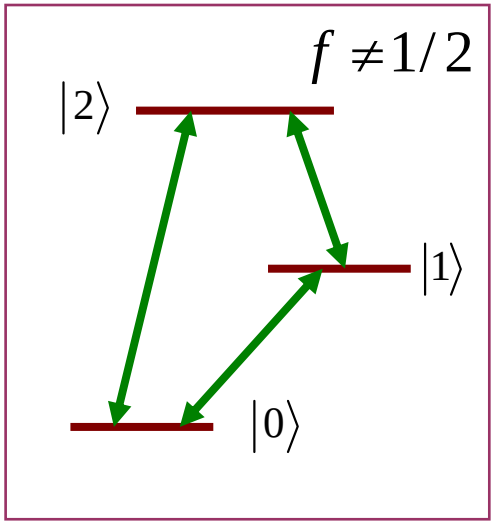
Diagram of three-level systems



Microwave induced transitions for the three lowest energy levels

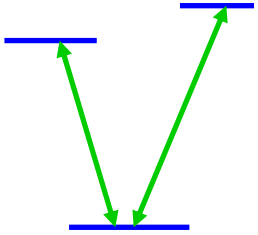


The transitions depend on the applied dc magnetic flux.

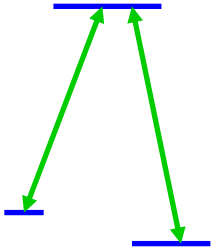


Liu, You, Wei, Sun, Nori, PRL **95**, 087001 (2005)

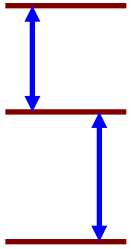
Transition diagrams of three-level systems



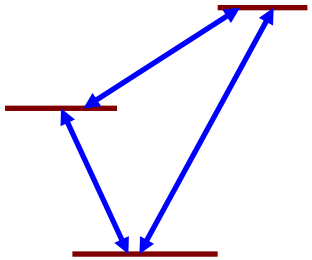
V - type



Λ- type



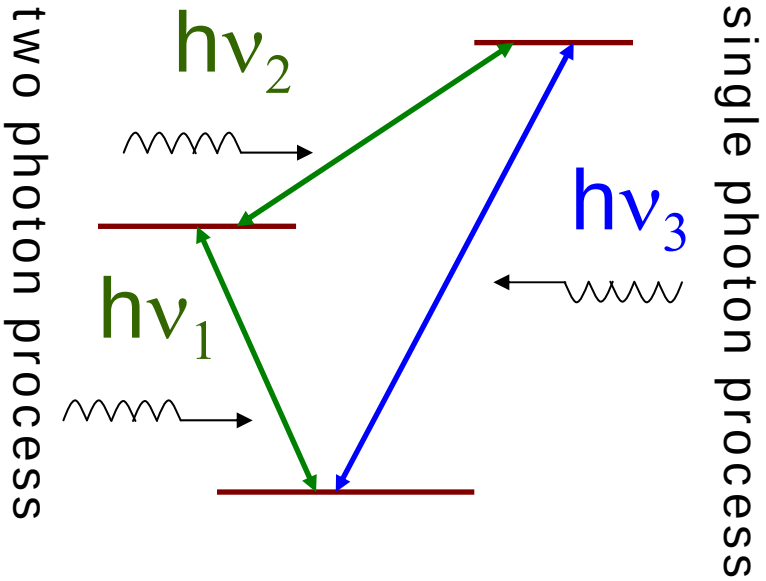
Ξ - type
or ladder
or cascade



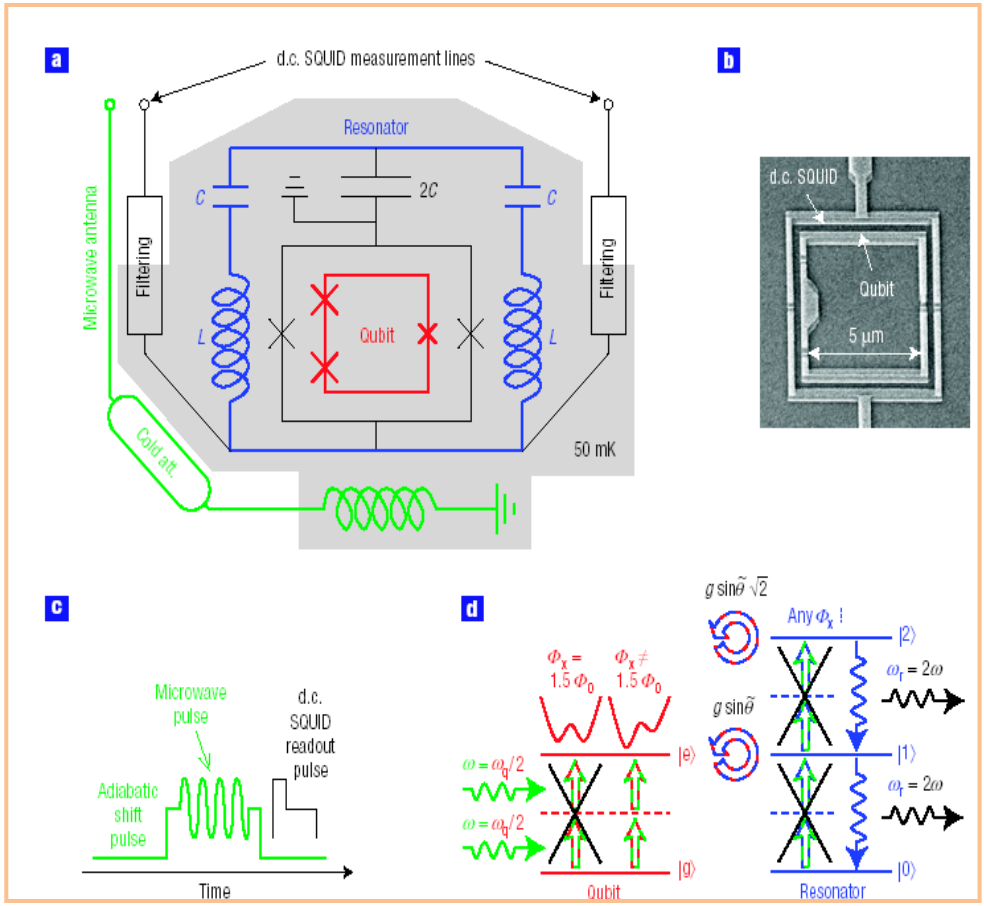
There is No Δ – type transitions in electric-dipole interaction between the natural atoms and the electric fields due to the selection rules.

In the superconducting quantum flux circuit, at the optimal point, the transitions for the lowest three levels form ladder type. However, when the reduced magnetic flux deviates from the optimal point $1/2$, we can get Δ-type transitions.

Coexistence of single- and two-photon processes

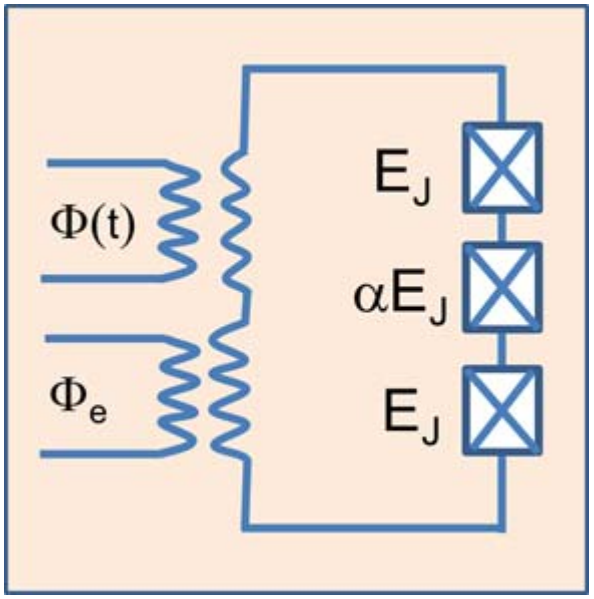


Liu, You, Wei, Sun, Nori, PRL **95**, 087001 (2005)

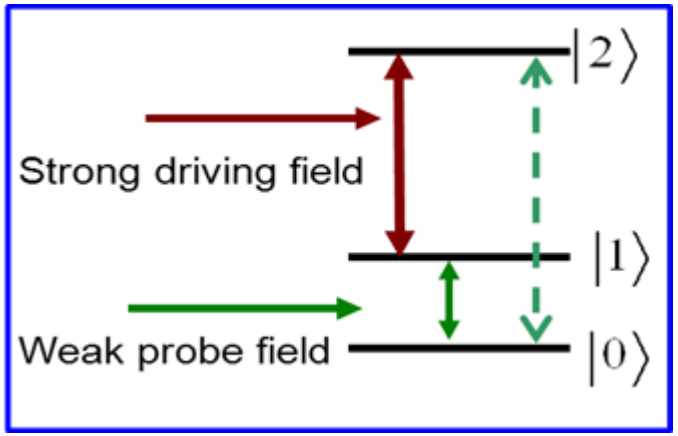


Nature Phys. 4, 686 (2008)

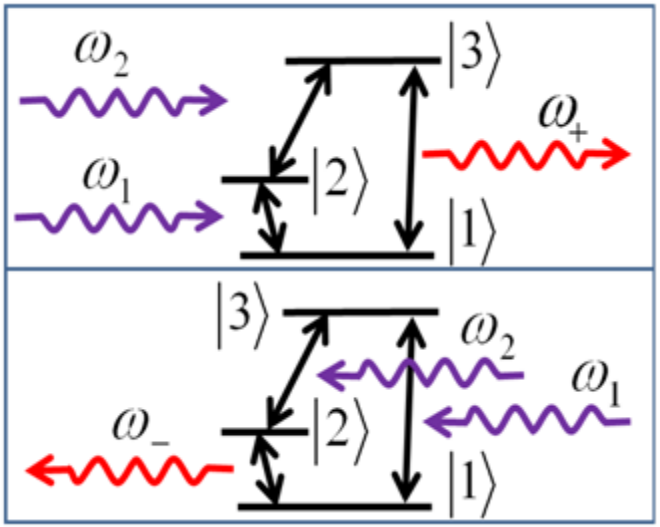
Linear and nonlinear phenomena in flux qubit circuits



Flux qubit circuits



Linear process



Nonlinear processes

Outline

1. Flux qubit circuits

2. Linear and nonlinear optical processes

   - Arharmonic oscillator and polarizations
   - Potential function and inversion symmetry
   - Quantum mechanical theory

3. EIT and ATS in flux qubit circuits

4. Three-wave mixing in flux qubit circuits

Anharmonic oscillator

Polarization of atomic ensemble is defined as

$$P = -Nex$$

N is the number density of atoms, e is the charge, x is the electron position

We assume that the equation of motion of the electron position is of the form

$$\frac{d^2x}{dt^2} - \underbrace{\Gamma}_{\text{damping}} \frac{dx}{dt} + \underbrace{\omega_0^2 x}_{\text{harmonic}} + \underbrace{ax^2}_{\text{anharmonic}} = \underbrace{\frac{F}{m}}_{\text{force}}$$

where a is a parameter that characterizes the strength of the nonlinearity

Polarizations

We assume that two electric fields

$$E = \varepsilon_1 \cos\omega_1 t + \varepsilon_2 \cos\omega_2 t$$

are applied to electron, then we have

$$F = qE$$

Weak driving field, we can use perturbation theory to obtain

$$x = x^{(1)} + x^{(2)} + x^{(3)} + \cdots \quad (x^{(1)} >> x^{(2)} >> x^{(3)} >> \cdots)$$

Polarization can be given as

$$P = \underbrace{Nex^{(1)}}_{P^{(1)}=\text{linear}} + \underbrace{Nex^{(2)}}_{P^{(2)}=\text{second-order}} + \underbrace{Nex^{(3)}}_{P^{(3)}=\text{third-order}} + \cdots$$

Linear and second-order polarizations

Frequency components	Polarization and position	Polarization processes
Single frequency	$P^{(1)} \propto x^{(1)}\left(\omega_i\right) \propto \varepsilon_i \exp\left(-i\omega_i t\right)$	Linear polarization
Sum-Difference-frequencies	$P^{(2)} \propto x^{(2)}\left(\omega_1 \pm \omega_2\right)$ $\propto \varepsilon_1 \varepsilon_2 \exp\left[-i\left(\omega_1 \pm \omega_2\right)t\right]$	Second-order nonlinear polarization
Second harmonic frequency	$\propto x^{(2)}\left(2\omega_i\right) \propto \varepsilon_i^{\,2} \exp\left[-i2\omega_i t\right]$	Second-order nonlinear polarization
Optical rectification	$\propto x^{(2)}\left(0\right) \propto F(\varepsilon_1^{\,2}, \varepsilon_2^{\,2})$	Second-order nonlinear polarization

Different potentials

$$\frac{d^2x}{dt^2} - \underbrace{\Gamma}_{\text{damping}}\frac{dx}{dt} + \underbrace{\omega_0^2 x}_{\text{harmonic}} + \underbrace{ax^2}_{\text{anharmonic}} = \underbrace{\frac{F}{m}}_{\text{force}}$$

Two electric fields

$$E = \varepsilon_1 \cos\omega_1 t + \varepsilon_2 \cos\omega_2 t$$

ax^3

Second-order polarization is nonzero

Zero

$$U_1(x) = \omega_0 x^2 + ax^3$$

$$\Downarrow$$

No symmetry

$$U_2(x) = \omega_0 x^2 + ax^4$$

$$\Downarrow$$

Even function

Quantum theory of nonlinear processes

The polarization can be calculated by

$$P = N\,\mathrm{Tr}\left(\rho p\right)$$

p is electric dipole moment of single atom, **ρ** is the density operator.

$$\rho = \rho^{(1)} + \rho^{(2)} + \rho^{(3)} + \cdots$$

Polarization can be given as

$$\begin{aligned} P &= P^{(1)} + P^{(2)} + P^{(3)} + \cdots \\ &= \mathrm{Tr}\left(\rho^{(1)} p\right) + \mathrm{Tr}\left(\rho^{(2)} p\right) + \mathrm{Tr}\left(\rho^{(3)} p\right) + \cdots \end{aligned}$$

Here we can find

$$\rho^{(i)} \propto \varepsilon^{i}$$

Outline

1. Flux qubit circuits

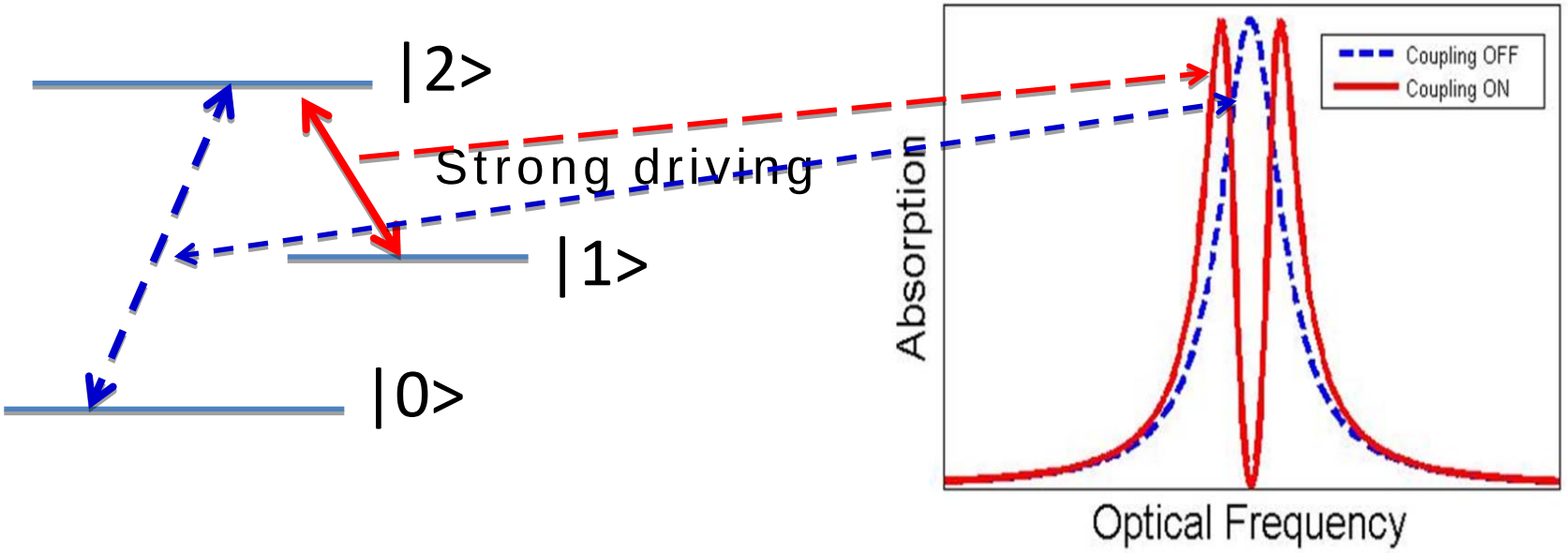
2. Linear and nonlinear optical processes

3. EIT and ATS in flux qubit circuits

   Linear response of driven three-level flux qubit circuits
   EIT and Aulter-Townes splitting (ATS)
   EIT and ATS in three-level superconducting flux circuits

4. Three-wave mixing in flux qubit circuits

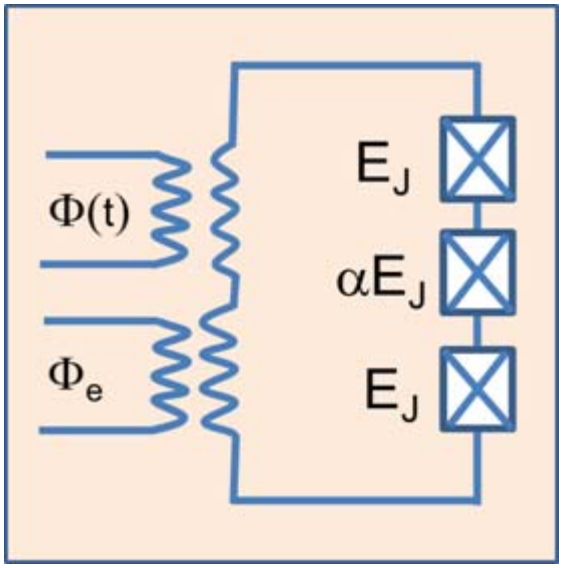
Electromagnetically induced transparency (EIT) and Autler-Townes splitting (ATS) in driven three-level systems

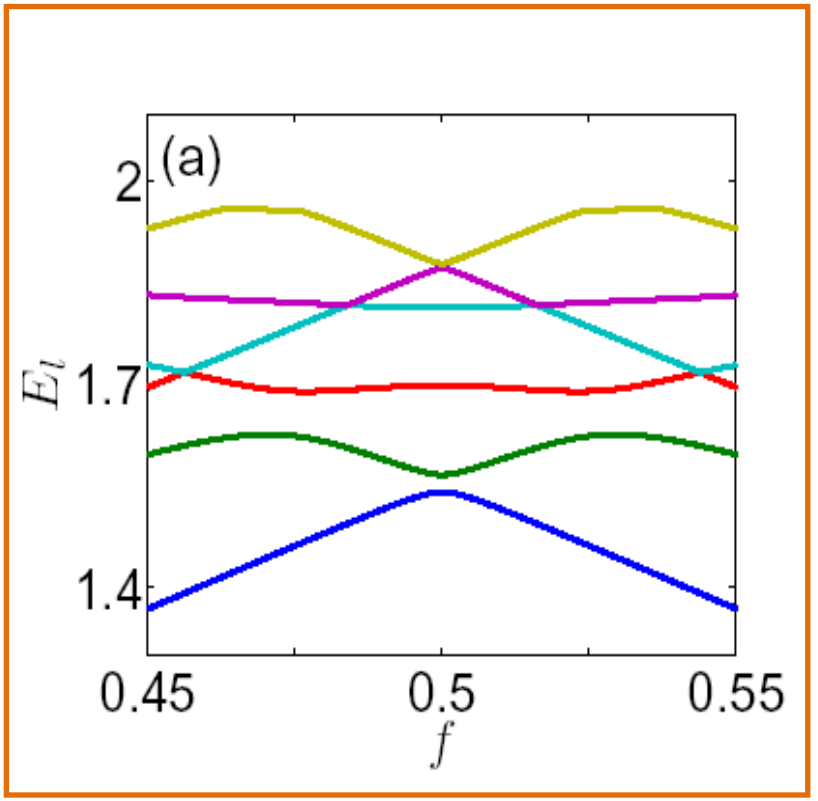


S. H. Autler and C. H. Townes, PR **100**, 703 (1955)
S. E. Harris , J. E. Field, and A. Imamoglu, PRL **64**, 1107 (1990)

Superconducting flux circuits

Three-junction system





$$H_0 = \frac{P_p^2}{2M_p} + \frac{P_m^2}{2M_m} + U(\varphi_p, \varphi_m, f)$$

$$U = 2E_J\left(1 - \cos\varphi_p \cos\varphi_m\right) + \alpha E_J\left[1 - \cos\left(2\varphi_m + 2\pi f\right)\right]$$

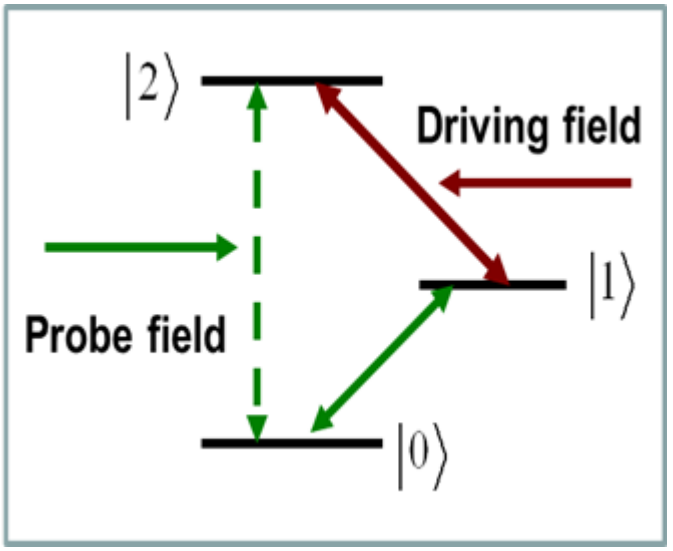
α = 0.7, E_J/E_c = 48

Yu-xi Liu *et al.*, New Journal of Physics **16**, 015031 (2014)

Theoretical model

Hamiltonian

$$H = \sum_{l=0}^{2} E_l \sigma_{ll} + \hbar \sum_i \omega_i' b_i^+ b_i + \hbar \left[\Omega_D \exp\left(i\omega_0 t\right) \sigma_{12} + \text{h.c.} \right] + \Phi_P(t) \left(\sum_{i=1}^{2} I_{0i} \sigma_{0i} + \text{h.c.} \right) + \hbar Q I$$



Schematic diagram

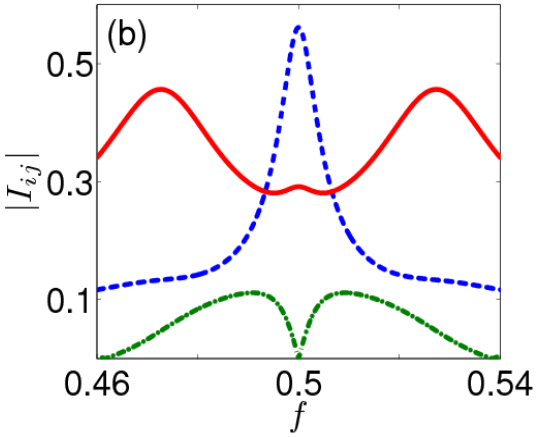
I is current operator which is defined as

$$I = \frac{\alpha I_0}{2\alpha + 1} \left[2 \sin\varphi_p \cos\varphi_m - \sin\left(2\pi f + 2\varphi_m\right) \right]$$

Linear response theory of driven three-level flux circuits

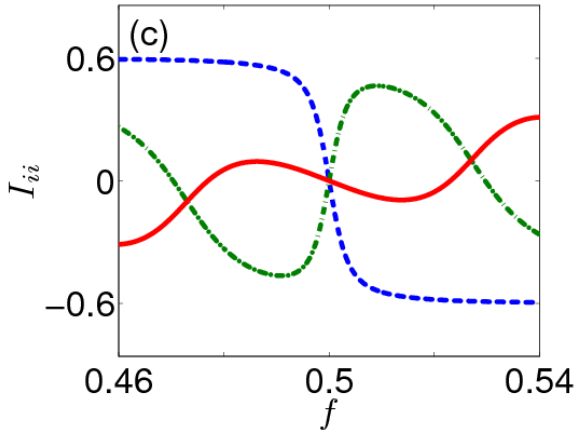
Susceptibility

$$\chi_q(\omega) = \int \mathrm{d}\tau \left\langle \frac{\delta I(t)}{\delta \Phi_P(t_1)} \right\rangle \exp(i\omega\tau)$$



Current operator

$$I = \sum_{i=0}^{2} I_{ij}\sigma_{ij} \quad \text{with} \quad \sigma_{ij} = |i\rangle\langle j|$$



which can be controlled by external magnetic field

Equation of motion

Goal:

Solutions of all operators

Calculation:

Straightforward, time consuming

Approximation:

first-order Markov approximation

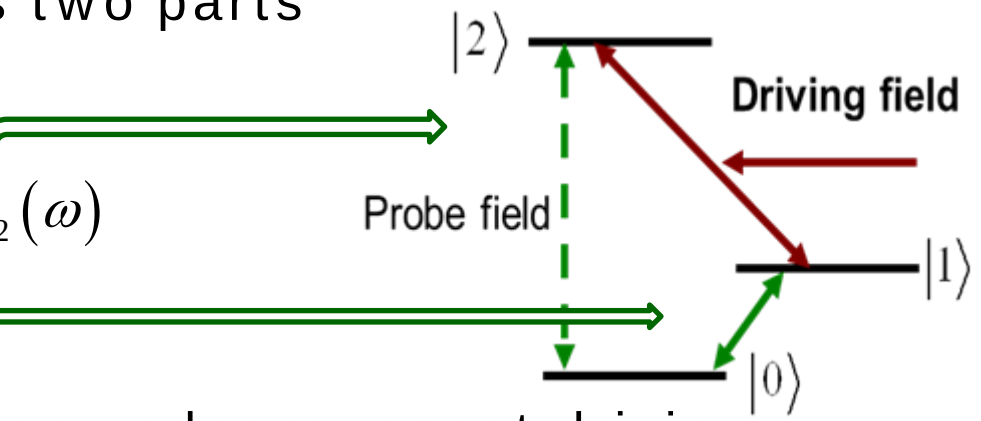
$$
\begin{aligned}
\frac{\partial \sigma_{lm}}{\partial t} &= -i\Delta(\sigma_{l2}\delta_{m2} - \sigma_{2m}\delta_{2l}) \\
&- i\Omega_{\mathrm{D}}(\sigma_{l2}\delta_{m1} - \sigma_{1m}\delta_{2l}) - i\Omega_{\mathrm{D}}^{*}(\sigma_{l1}\delta_{m2} - \sigma_{2m}\delta_{1l}) \\
&+ \frac{i}{\hbar}(\sigma_{l0}\delta_{m1} - \sigma_{1m}\delta_{0l})[\hbar Q + \Phi_{\mathrm{P}}(t)]I_{10}e^{i\omega_1 t} \\
&+ \frac{i}{\hbar}(\sigma_{l1}\delta_{m0} - \sigma_{0m}\delta_{1l})[\hbar Q + \Phi_{\mathrm{P}}(t)]I_{01}e^{-i\omega_1 t} \\
&+ \frac{i}{\hbar}(\sigma_{l0}\delta_{m2} - \sigma_{2m}\delta_{0l})[\hbar Q + \Phi_{\mathrm{P}}(t)]I_{20}e^{i\omega' t} \\
&+ \frac{i}{\hbar}(\sigma_{l2}\delta_{m0} - \sigma_{0m}\delta_{2l})[\hbar Q + \Phi_{\mathrm{P}}(t)]I_{02}e^{-i\omega' t} \\
&+ \frac{i}{\hbar}(\sigma_{l1}\delta_{m2} - \sigma_{2m}\delta_{1l})\hbar Q I_{21}e^{i\omega_0 t} \\
&+ \frac{i}{\hbar}(\sigma_{l2}\delta_{m1} - \sigma_{1m}\delta_{2l})\hbar Q I_{12}e^{-i\omega_0 t} \\
&+ \frac{i}{\hbar}(\sigma_{l0}\delta_{m0} - \sigma_{0m}\delta_{0l})\hbar Q I_{00} \\
&+ \frac{i}{\hbar}(\sigma_{l1}\delta_{m1} - \sigma_{1m}\delta_{1l})\hbar Q I_{11} \\
&+ \frac{i}{\hbar}(\sigma_{l2}\delta_{m2} - \sigma_{2m}\delta_{2l})\hbar Q I_{22},
\end{aligned}
$$

Hui-Chen Sun, Yu-xi Liu, Hou Ian, J. Q. You, E. Il'ichev, and Franco Nori, Phys. Rev. A 89, 063822 (2014)

Result of susceptibility

Susceptibility includes two parts

$$\chi_q(\omega) = \chi_{01}(\omega) + \chi_{02}(\omega)$$



Spectrum decomposition under resonant driving

$$\chi_{01}(\omega) = \frac{|I_{01}|^2}{\hbar}\frac{\delta_+ + i\gamma_{22}}{(\delta_- - \delta_+)(\delta_1 - \delta_+)} - \frac{|I_{01}|^2}{\hbar}\frac{\delta_- + i\gamma_{22}}{(\delta_- - \delta_+)(\delta_1 - \delta_-)}$$

$$\chi_{02}(\omega) = \frac{|I_{02}|^2}{\hbar}\frac{\delta_+ + i\gamma_{11}}{(\delta_- - \delta_+)(\delta_2 - \delta_+)} - \frac{|I_{02}|^2}{\hbar}\frac{\delta_- + i\gamma_{11}}{(\delta_- - \delta_+)(\delta_1 - \delta_-)}$$

$$\delta_1 = \omega - \omega_1$$

$$\delta_2 = \omega - \omega_2$$

here

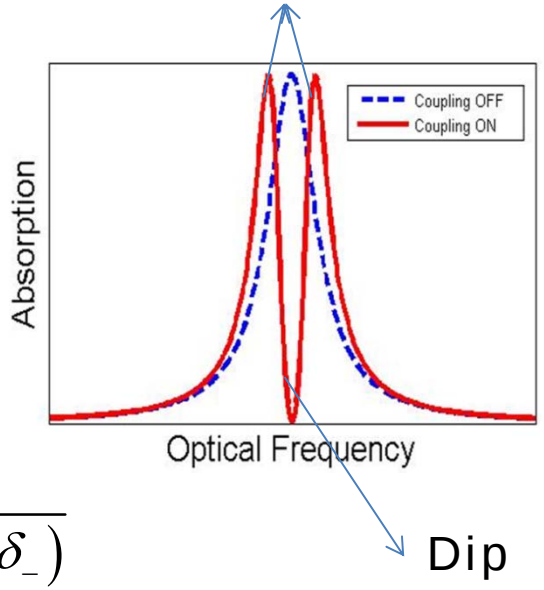
$$\delta_\pm = \frac{1}{2}\left(-i\gamma_{11} - i\gamma_{22} \pm \sqrt{4|\Omega_D|^2 - (\gamma_{11} - \gamma_{22})^2}\right)$$

Two maximum and a minimum values

Two maximum values

Two poles

$$\delta_{\pm} = \frac{1}{2}\left(-i\gamma_{11} - i\gamma_{22} \pm \sqrt{4\left|\Omega_D\right|^2 - \left(\gamma_{11} - \gamma_{22}\right)^2}\right)$$



When EIT or ATS is realized,

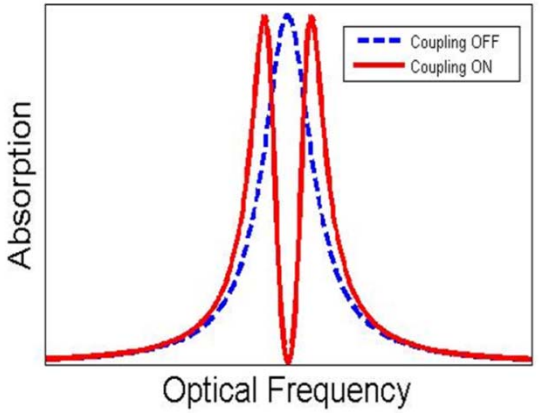
$$\chi_{01}(\omega) = \frac{\left|I_{01}\right|^2}{\hbar}\frac{\delta_+ + i\gamma_{22}}{(\delta_- - \delta_+)(\delta_1 - \delta_+)} - \frac{\left|I_{01}\right|^2}{\hbar}\frac{\delta_- + i\gamma_{22}}{(\delta_- - \delta_+)(\delta_1 - \delta_-)}$$

has minimum value for $\omega = \boldsymbol{\omega_1}$, thus we obtain a condition

$$\left|\Omega_D\right| > \gamma_{22}\sqrt{\frac{\gamma_{22}}{\gamma_{11} + 2\gamma_{22}}}$$

Comparisons of EIT and ATS

Both have similar absorption spectrum



Difference:

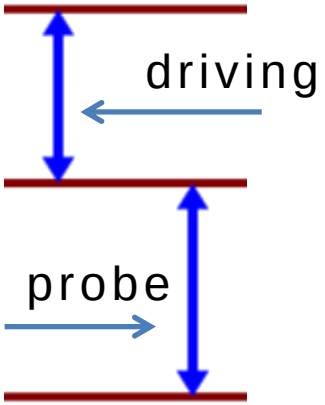
1、$|\Omega_D| > \gamma_{11}, \gamma_{22}$ for ATS, but $|\Omega_D| < \gamma_{11}, \gamma_{22}$ for EIT .

2、EIT is due to the interference, ATS is just superposition of two positive Lorentzian spectra.

C. C.ohen-Tannoudji *et al*., JPB **10**, 345 (1977)
T. Y. Abi-Salloum, PRA **81**, 053836 (2010).

Conditions for EIT and ATS

Ladder type transition



EIT in weak driving regime $|\Omega_D| < \frac{1}{2}|\gamma_{11} - \gamma_{22}|$

ATS in strong driving regime $|\Omega_D| > \frac{1}{2}|\gamma_{11} - \gamma_{22}|$

Combine with the dip condition

Condition of EIT $\gamma_{11} > 2\gamma_{22}$

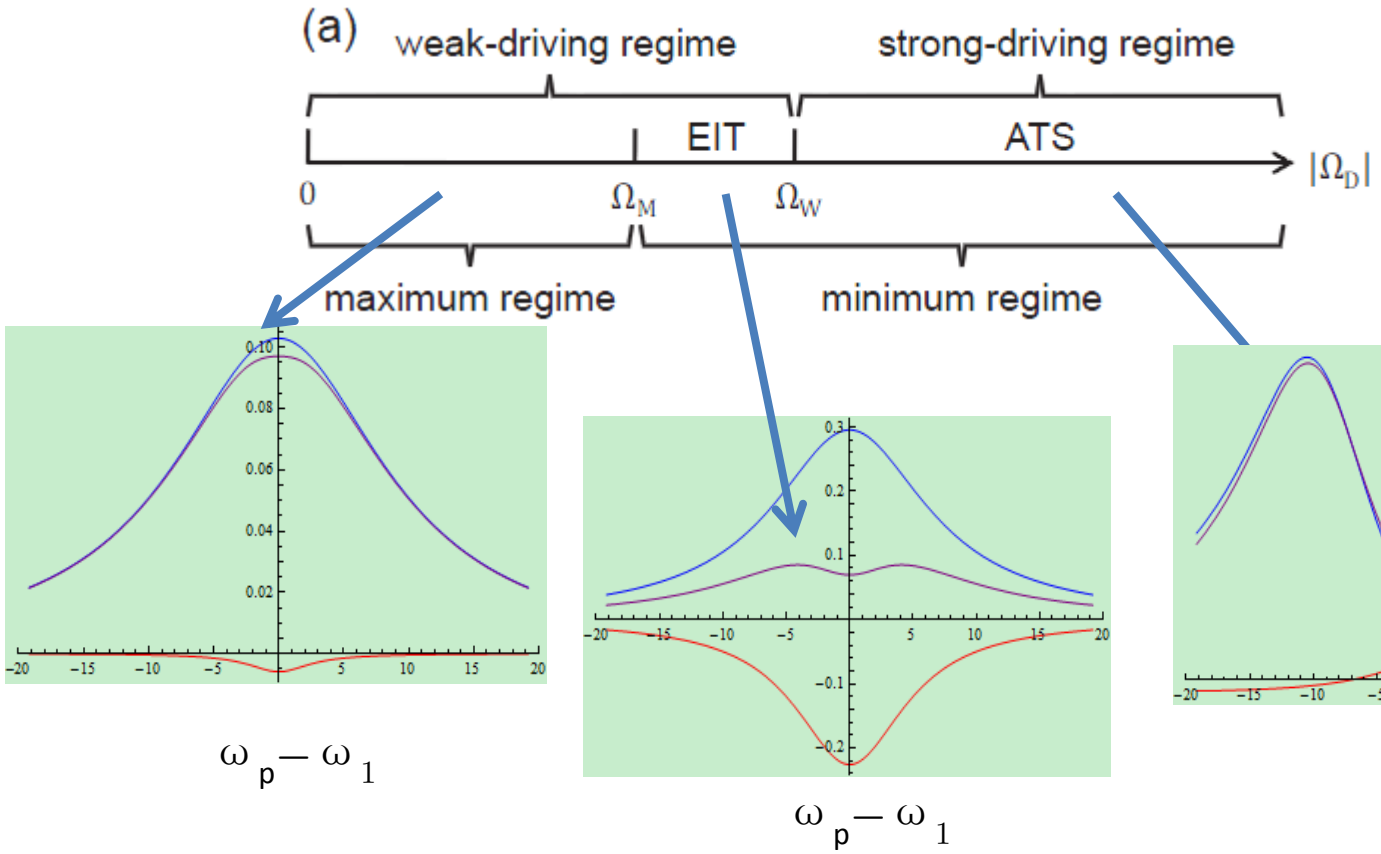
$$\begin{cases} |\Omega_D| < \frac{1}{2}|\gamma_{11} - \gamma_{22}| \\ |\Omega_D| > \gamma_{22}\sqrt{\frac{\gamma_{22}}{\gamma_{11} + 2\gamma_{22}}} \end{cases}$$

Condition of ATS

$$\begin{cases} |\Omega_D| > \frac{1}{2}|\gamma_{11} - \gamma_{22}| & \gamma_{11} > 2\gamma_{22} \\ |\Omega_D| > \gamma_{22}\sqrt{\frac{\gamma_{22}}{\gamma_{11} + 2\gamma_{22}}} & \gamma_{11} \le 2\gamma_{22} \end{cases}$$

EIT and ATS (I)

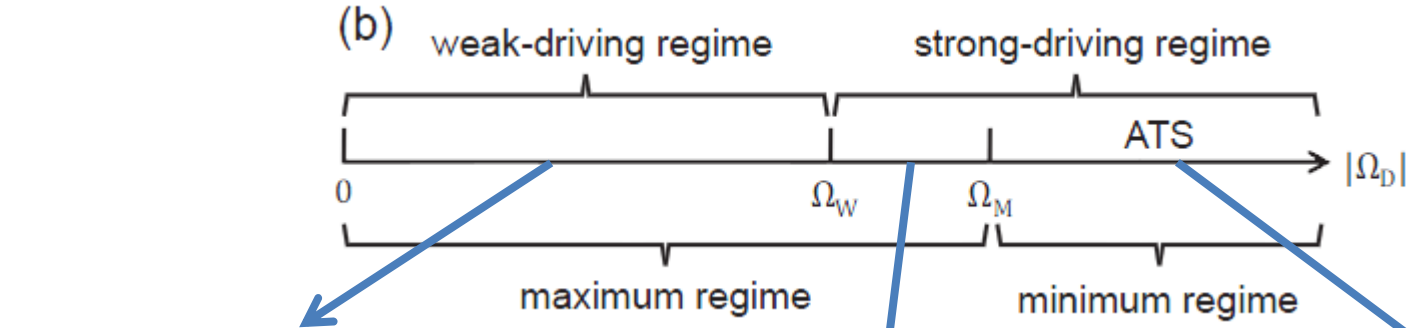
$\gamma_{11} > 2\gamma_{22}$, Ω_M (dip) $< \Omega_W$ (critical value for driving field)

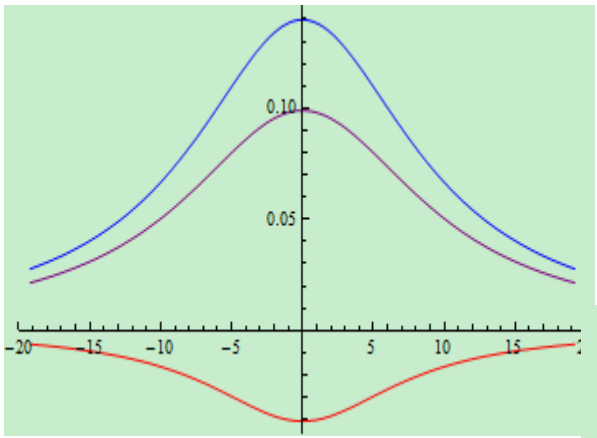


$$\chi_{01}(\omega) = \frac{|I_{01}|^2}{\hbar} \frac{\delta_+ + i\gamma_{22}}{(\delta_- - \delta_+)(\delta_1 - \delta_+)} - \frac{|I_{01}|^2}{\hbar} \frac{\delta_- + i\gamma_{22}}{(\delta_- - \delta_+)(\delta_1 - \delta_-)}$$

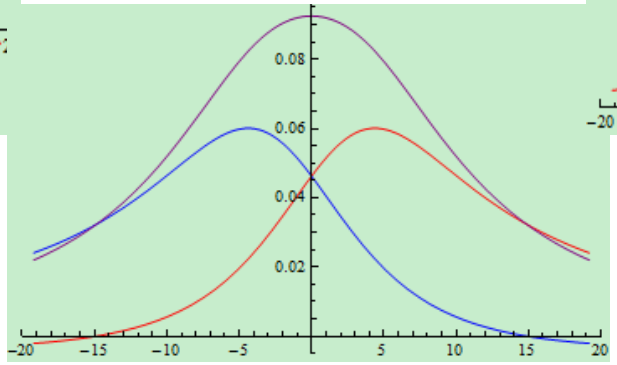
EIT and ATS (II)

$\gamma_{11} \leqslant 2\gamma_{22}$, Ω_M (dip) $\geqslant \Omega_W$ (critical value for driving field)

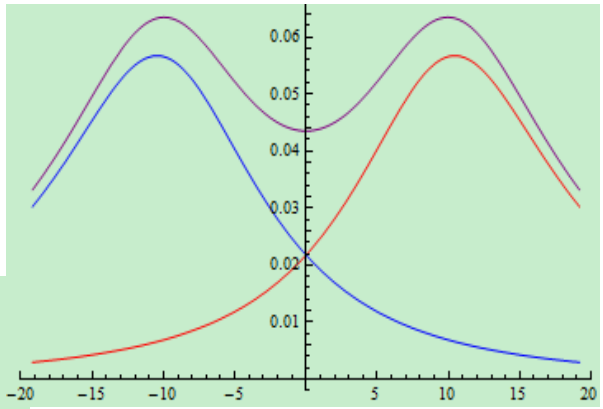




$\omega_p - \omega_1$



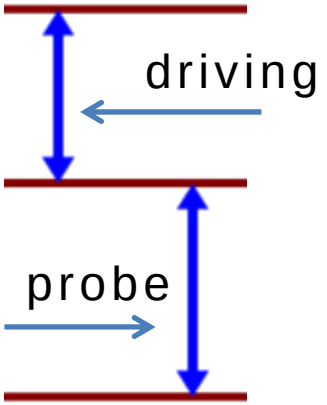
$\omega_p - \omega_1$



$\omega_p - \omega_1$

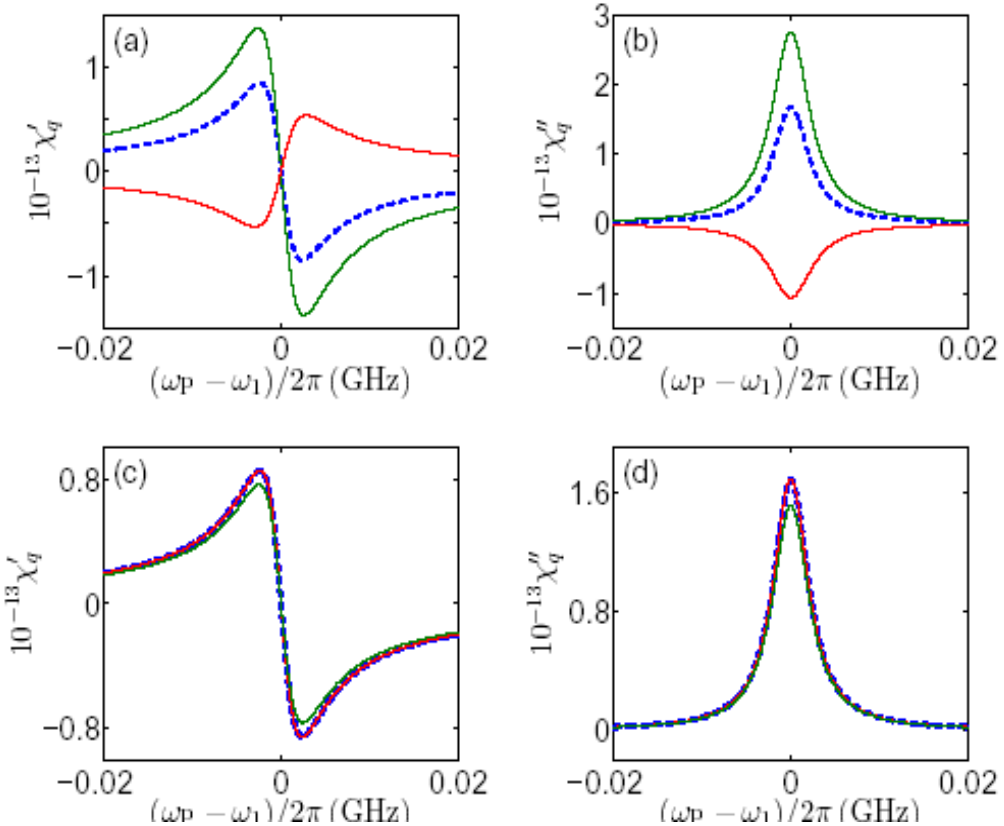
Three-level superconducting flux circuits at optimal point

Ladder type



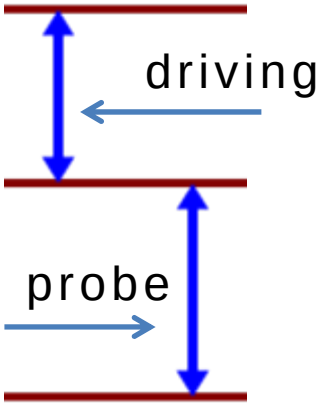
Weak driving regime

There is no dip

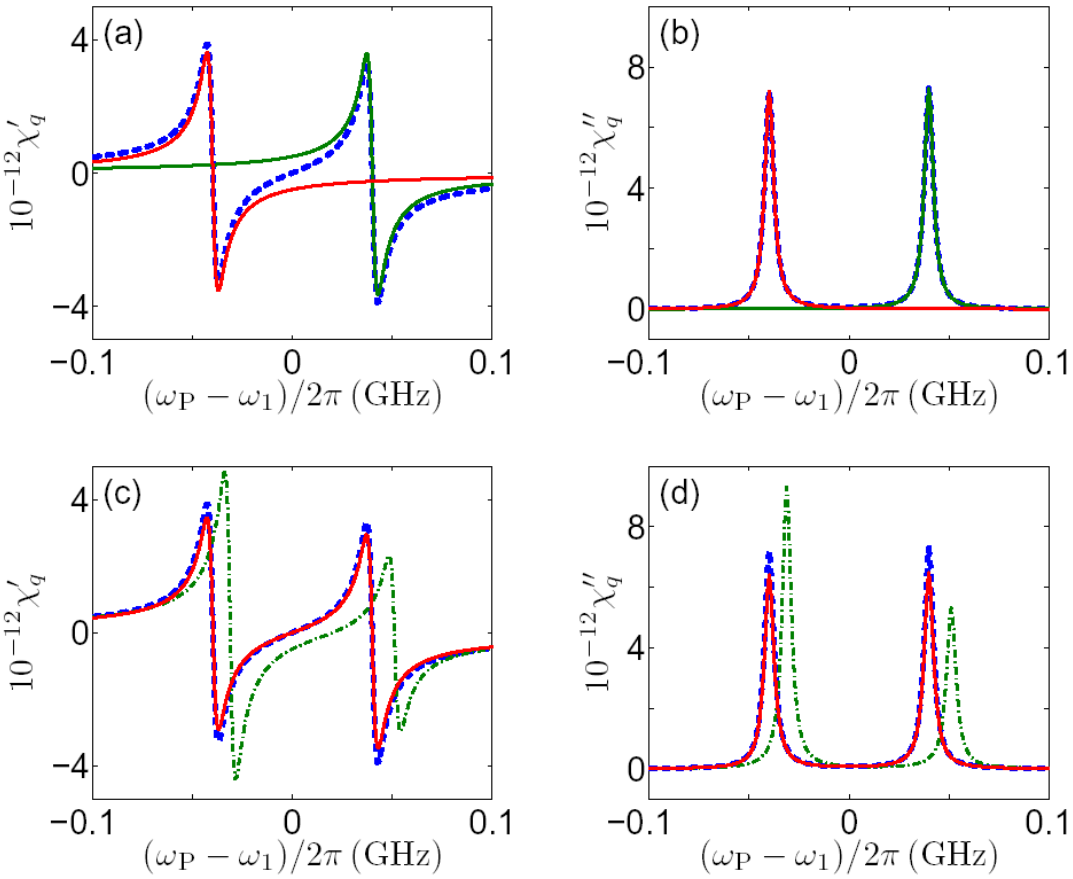


Three-level superconducting flux circuits at optimal point

Ladder type

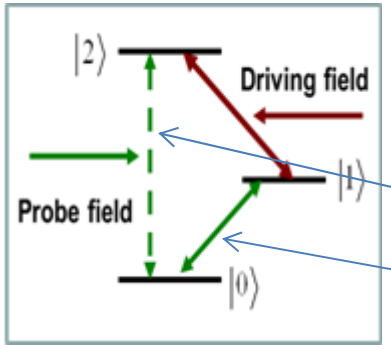


Strong driving regime: ATS

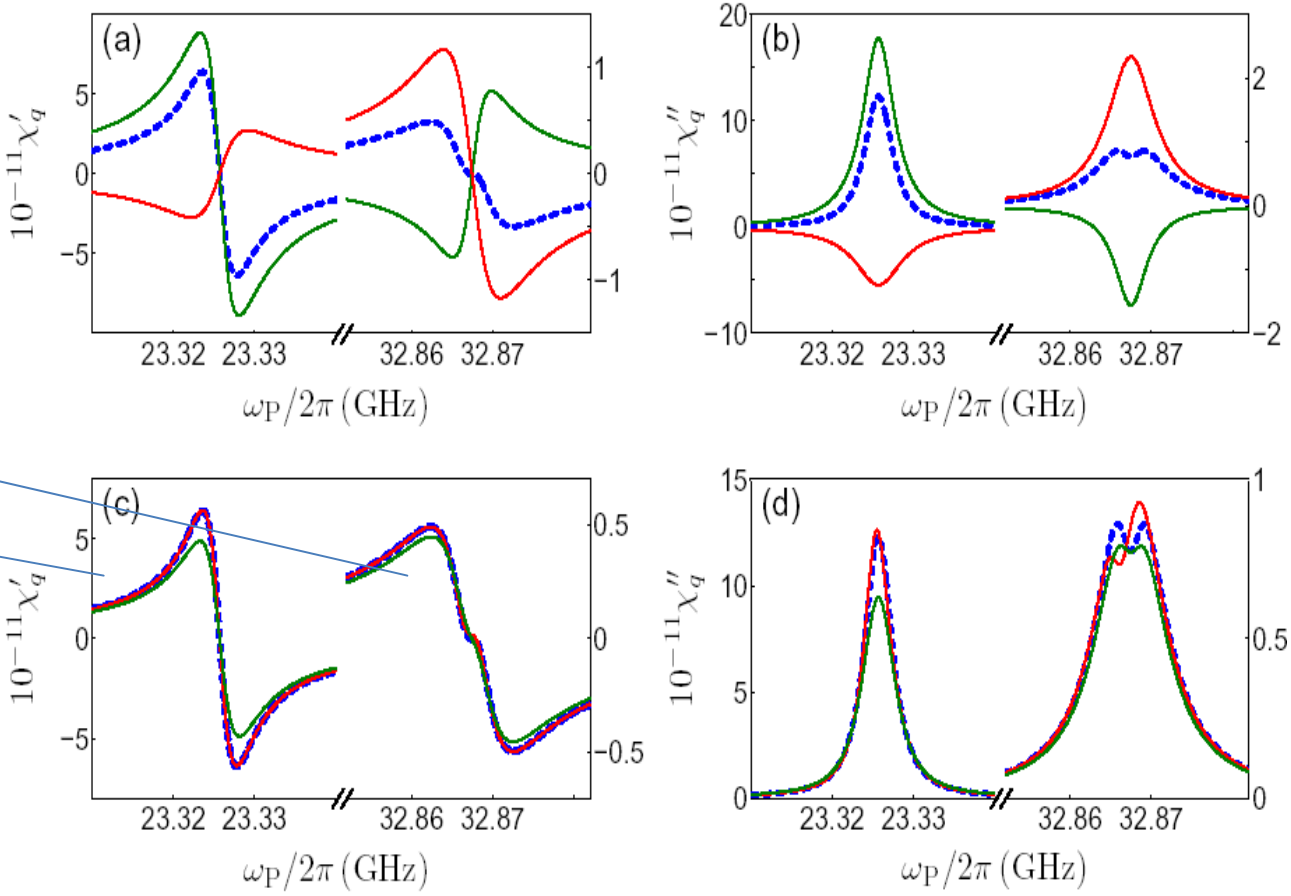


Three-level superconducting flux circuits not at optimal point

Cyclic type



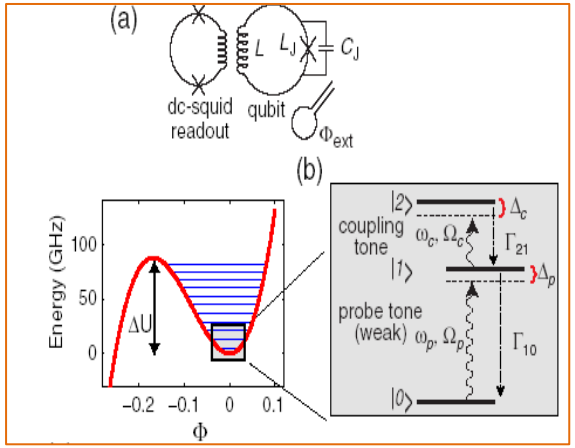
Weak driving regime: EIT



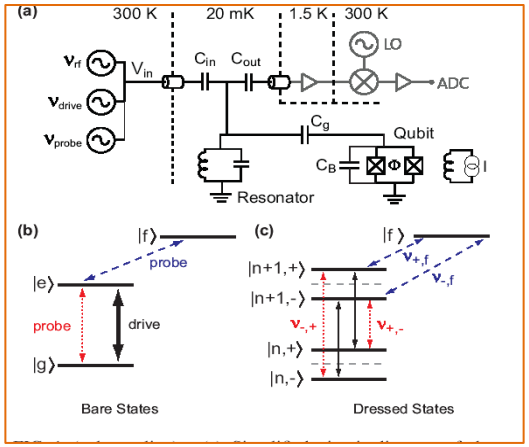
Hui-Chen Sun, Yu-xi Liu, Hou Ian, J. Q. You, E. Il'ichev, and Franco Nori, Phys. Rev. A 89, 063822 (2014)

ATS in superconducting quantum circuits

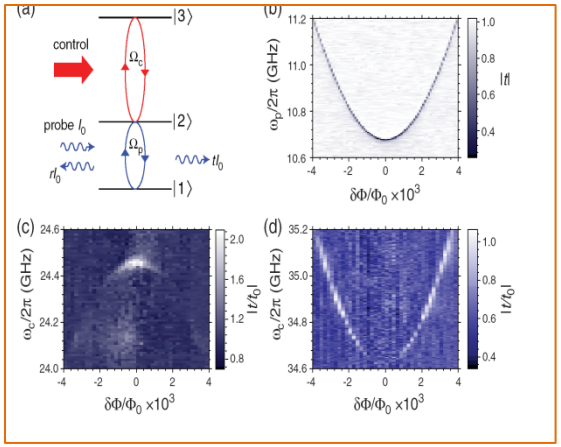
Up to now, there is only ATS in superconducting quantum circuits.



Phase



Transmon



flux

M. Baur *et al.*, PRL **102**, 243602 (2009), Transmon (Wallraff)
M. A. Sillanpaa *et al.*, PRL **103,** 193601 (2009), Phase (Simmonds and Hakonen)
A.A. Abdumalikov, Jr. *et al.*, PRL **104**, 193601 (2010), Flux (Tsai)
A.S. Novikov *et al.*, PRB **88**, 060503 (2013), Transmon (Wellstood)
I. C. Hoi *et al.*, New J Phys. **15,** 025011 (2013), Transmon (Delsing)

Outline

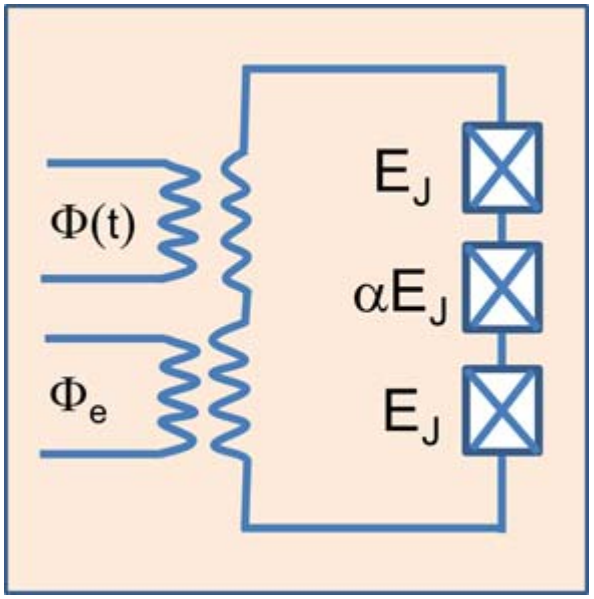
1. Flux qubit circuits

2. Linear and nonlinear optical processes

3. EIT and ATS in flux qubit circuits

4. Three-wave mixing in flux qubit circuits

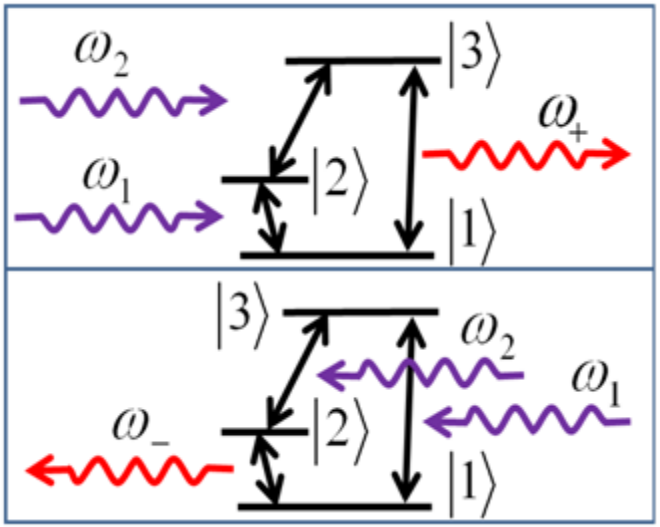
Nonlinear phenomena in flux qubit circuits



Flux qubit circuits

The reduce magnetic flux is not at the optimal point.

The symmetry of potential well is broken.

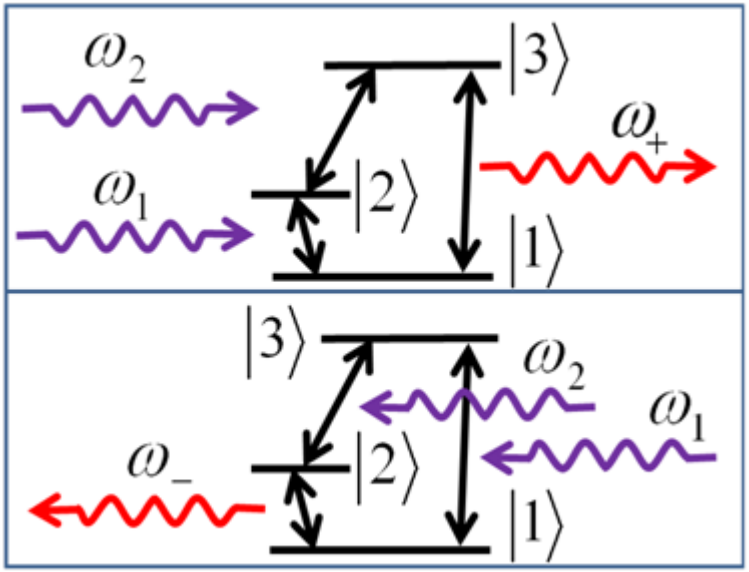


Nonlinear processes

Three-wave mixing in three-level flux qubit circuits

Two microwave fields are applied

$$\Phi(t) = \varepsilon_1 \cos\omega_1 t + \varepsilon_2 \cos\omega_2 t$$



Calculations for second-order polarization

$$P^{(2)} \propto \left\langle 3 \middle| x^{(2)}\left(\omega_1 \pm \omega_2\right) \middle| 1 \right\rangle \propto \varepsilon_1 \varepsilon_2 \exp\left[-i\left(\omega_1 \pm \omega_2\right)t\right]$$

Yu-xi Liu, H. C. Sun, Z. H. Peng, A. Miranowicz, J. S. Tsai, and F. Nori, Scientific Reports **4**, 7289 (2014)

Method: master equations

$$
\begin{aligned}
\dot{\rho}_{ij}(t) &= \frac{1}{i\hbar}[H_T, \rho(t)]_{ij} - \frac{1}{2}\Gamma_{ij}\widetilde{\rho}_{ij}(t), \; i \neq j, \\
\dot{\rho}_{11}(t) &= \frac{1}{i\hbar}[H_T, \rho(t)]_{11} + \gamma_{12}\widetilde{\rho}_{22}(t) + \gamma_{13}\widetilde{\rho}_{33}(t), \\
\dot{\rho}_{22}(t) &= \frac{1}{i\hbar}[H_T, \rho(t)]_{22} - \gamma_{12}\widetilde{\rho}_{22}(t) + \gamma_{23}\widetilde{\rho}_{33}(t), \\
\dot{\rho}_{33}(t) &= \frac{1}{i\hbar}[H_T, \rho(t)]_{33} - (\gamma_{13} + \gamma_{23})\widetilde{\rho}_{33}(t)
\end{aligned}
$$

Up to second-order of parameters of applied fields

$$\rho = \rho^{(0)} + \rho^{(1)} + \rho^{(2)} + \cdots$$

$\rho^{(0)}$ is the equilibrium state = ground state

Second-order Susceptibility

Second-order polarization

$$P^{(2)} = \mathrm{Tr}\left(\rho^{(2)} I\right)$$

Here / current operator of superconducting loop, it is equivalent to an electric-dipole operator

$$I = \frac{\alpha I_0}{2\alpha + 1}\left[2\sin\varphi_p \cos\varphi_m - \sin\left(2\pi f + 2\varphi_m\right)\right]$$

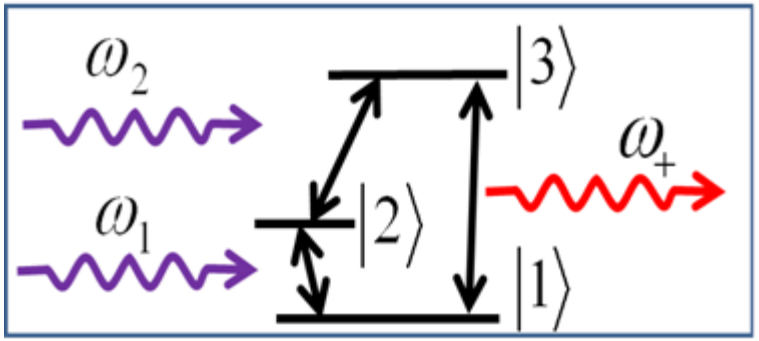
Second-order susceptibility:

$$\chi^{(2)} = \frac{P^{(2)}}{\Phi\left(\omega_1\right)\Phi\left(\omega_2\right)}$$

Generating sum-frequency

Two microwave with ω_1 and ω_2 are applied to three-level system.

Goal : $\omega_1+\omega_2$



Hamiltonian

$$H_T = \sum_{i=1}^{3} E_i \sigma_{ii} + \left[I_{12}(f)\Phi(\omega_1)\sigma_{12}\exp(i\omega_1 t) + I_{23}(f)\Phi(\omega_2)\sigma_{23}\exp(i\omega_2 t) + \text{h.c.} \right]$$

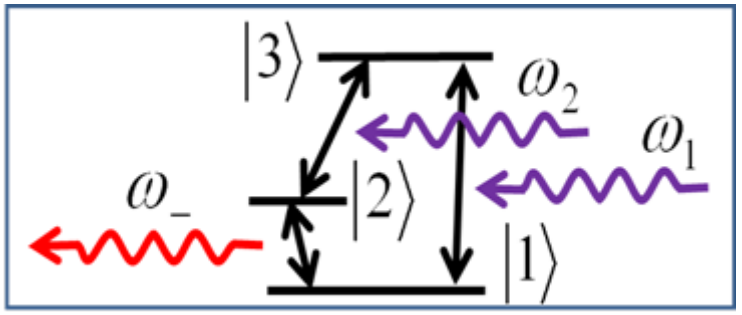
Second-order susceptibility

$$\chi_2(\omega_1+\omega_2) = \frac{I_{12}(f)I_{23}(f)I_{31}(f)}{(i\omega_1 - i\omega_{21} + \Gamma_{21})\left[i(\omega_1+\omega_2) - i\omega_{31} + \Gamma_{31} \right]}$$

Generating difference-frequency

Two microwave with ω_1 and ω_2 are applied to three-level system

Goal : $\omega_1 - \omega_2$



$$H_T = \sum_{i=1}^{3} E_i \sigma_{ii} + \left[I_{13}(f)\Phi(\omega_1)\sigma_{13}\exp(i\omega_1 t) + I_{23}(f)\Phi(\omega_2)\sigma_{23}\exp(i\omega_2 t) + \text{h.c.} \right]$$

Second-order susceptibility

$$\chi_2(\omega_1 - \omega_2) = \frac{I_{13}(f) I_{21}(f) I_{32}(f)}{(i\omega_1 - i\omega_{31} + \Gamma_{31})\left[i(\omega_1 - \omega_2) - i\omega_{21} + \Gamma_{21} \right]}$$

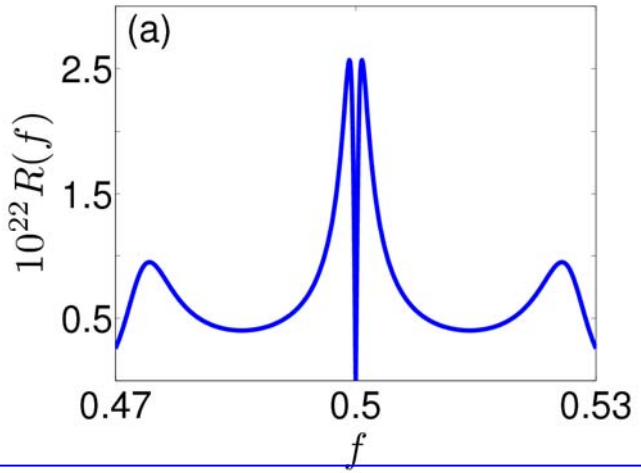
here

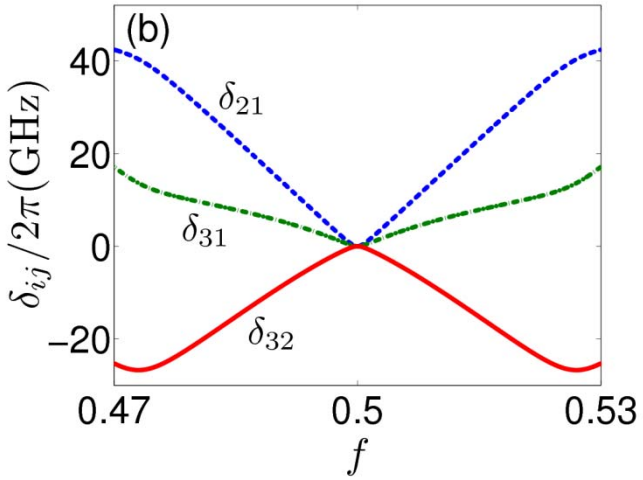
$$\Gamma_{21} = \gamma_{21} \ , \ \Gamma_{31} = \gamma_{31} + \gamma_{32} + \gamma_{33}$$

Susceptibility and tunability

$$R(f) = I_{13}(f) I_{21}(f) I_{32}(f)$$

$$\delta_{ji}(f) = \omega_{ji}(f) - \omega_{ji}^{(\mathrm{opt})},\ j > i$$



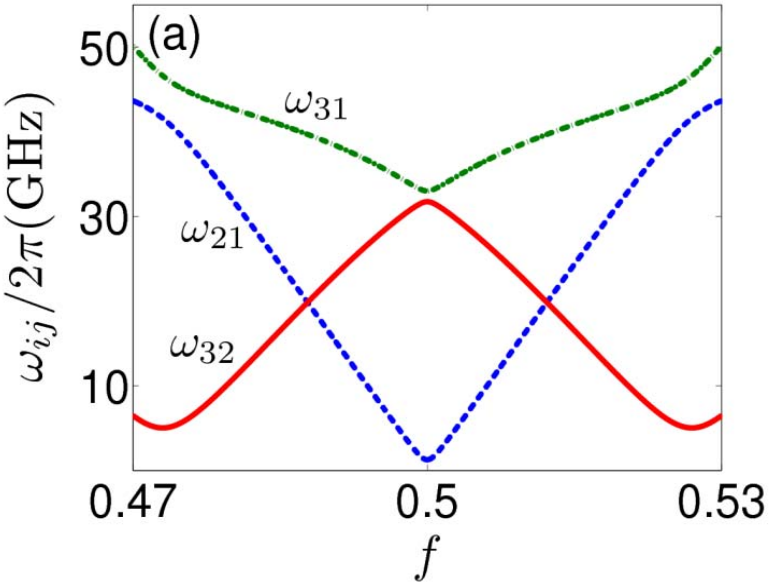


Parameters: E_J = 192 GHz, E_J/E_c = 48, α=0.8

Tunability of sum-frequency: 17 GHz; difference-frequency: either 42 GHz or 26 GHz.

At optimal point, the second-order susceptibility is zero, three-wave mixing does not occur.

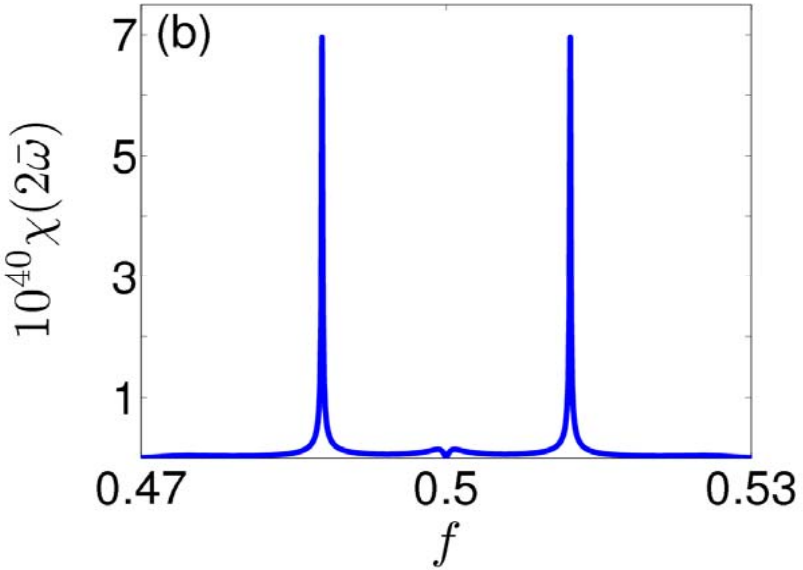
Second-harmonic generation



Frequency tunability

External fields: $\omega_1 = \omega_2 = \frac{\omega_{31}}{2} = \bar{\omega}$

$$\chi\left(2\bar{\omega}\right) = \frac{I_{12}(f)\,I_{23}(f)\,I_{31}(f)}{\left[i\left(\bar{\omega}-\omega_{21}\right)+\Gamma_{21}\right]\Gamma_{31}}$$



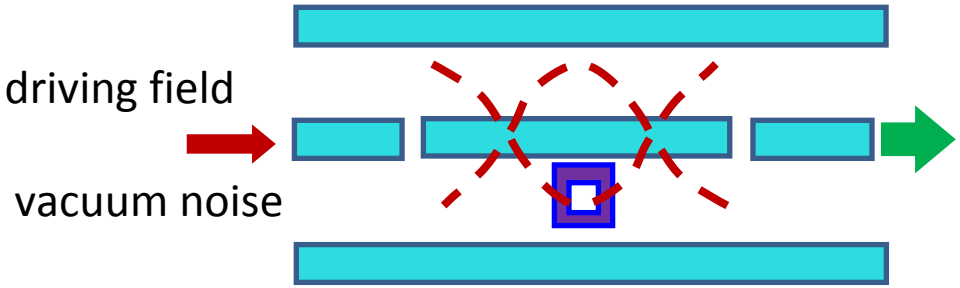
Conclusions:

Second-order susceptibility reaches maximum for $\omega_{12} = \omega_{23}$

Measurements

Input-output theory

$$\langle c_{\text{out}} \rangle = \langle c_{\text{in}} \rangle + \sqrt{\gamma_{13}} \langle \sigma_{13}(t) \rangle$$



Hamiltonian

$$H_{\text{int}} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[\sqrt{\gamma_{12}} a^{\dagger}(\omega) \sigma_{12} + \sqrt{\gamma_{23}} b^{\dagger}(\omega) \sigma_{23} + \sqrt{\gamma_{13}} c^{\dagger}(\omega) \sigma_{13} \right] + \text{h.c.}$$

Output of sum-frequency generation

$$\langle c_{\text{out}} \rangle = \frac{\sqrt{\gamma_{13}} I_{21} I_{32} \Phi(\omega_1) \Phi(\omega_2)}{\hbar^2 (i\omega_{21} - i\omega_1 + \Gamma_{21})(i\omega_{31} - i\omega_+ + \Gamma_{31})} \exp(-i\omega_+ t)$$

Yu-xi Liu, H. C. Sun, Z. H. Peng, A. Miranowicz, J. S. Tsai, and F. Nori, Scientific Reports **4**, 7289 (2014)

Microwave amplification and frequency conversion using a single three-level superconducting flux quantum circuit

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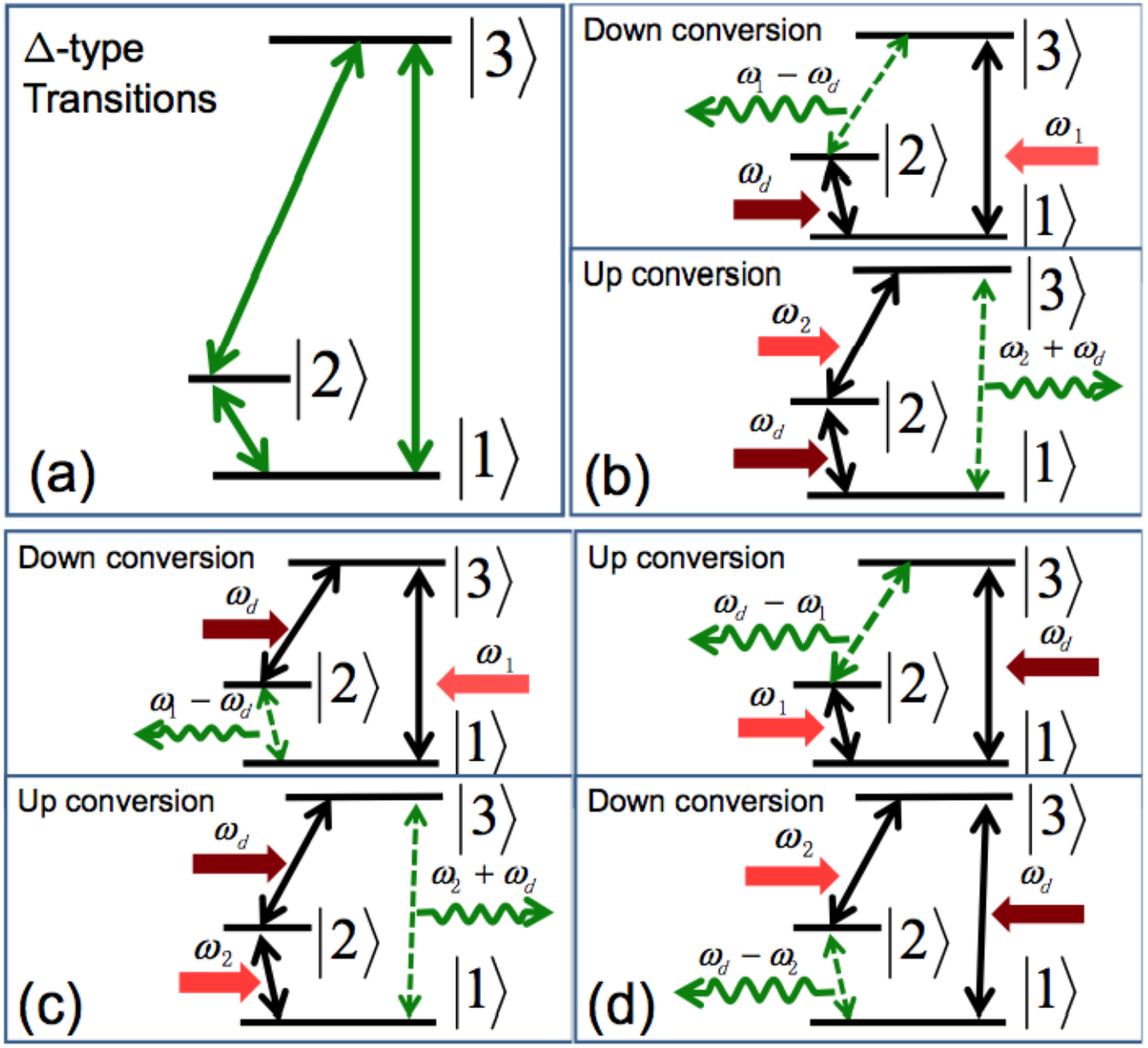
[1] *Institute of Microelectronics, Tsinghua University, Beijing 100084, China*
[2] *CEMS, RIKEN, Saitama 351-0198, Japan*
[3] *Tsinghua National Laboratory for Information Science and Technology (TNList), Beijing 100084, China*

(Dated: November 27, 2014)

A semiclassical treatment for both the microwave frequency conversion and the amplification is presented. In this study, a strong driving field and a weak probe field interact with a single three-level superconducting flux quantum circuit (SFQC), which has Δ-type transitions. Different from the parametric frequency conversion in nonlinear optical media, in our proposal, the real energy levels of the SFQC involve in the energy exchange when the frequency conversion is realized. Thus the efficiency of the conversion could be improved. We find that both the frequency down and up conversions can be realized by a single SFQC with different pump fields. The conversion efficiencies are calculated. This study provides a various methods for frequency conversion and microwave amplification. This study also lays a foundation for the generation of the entangled microwave pair using single artificial atom.

PACS numbers: 42.65.Es,42.65.Ky,75.30.Cr


Δ-type
Transitions
|3⟩
|2⟩
|1⟩
(a)
Down conversion
ω1 − ωd
ω1
ωd
(b)
Up conversion
ω2
ω2 + ωd
ωd
Down conversion
ωd
ω1
ω1 − ωd
(c)
Up conversion
ωd − ω1
ωd
ω1
(d)
Down conversion
ω2
ωd
ωd − ω2

Summary

1. Cyclic three-level flux qubit circuits provide new opportunity to control microwave photons at single-atom level.

2. EIT and ATS are two different phenomena. EIT is not easy to be observed with current design of superconducting quantum circuits.

3. Single superconducting quantum circuit can be used for amplification of weak microwave field, wave mixing, frequency conversion.

Acknowledgements

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J. Q. You and C. P. Sun, Beijing computational Science research Center, China

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A. Miranowicz, Adam Mickiewicz University, Poland

E. Ilichev, Institute of photonic technology, Germany

Thank you