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Entanglement Web Expansion

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collaborators:

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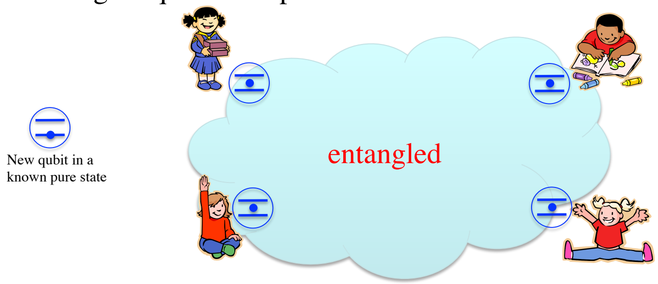
What is the problem?

Generating a large-scale entanglement at a time is difficult.

→ (1) Put a new qubit (new qubits) to the existing entangled state.
and/or
(2) Merge two existing entangled states into one.

A trivial (but not easy to do) method:
bring all qubits at a place and recreate the desired state.



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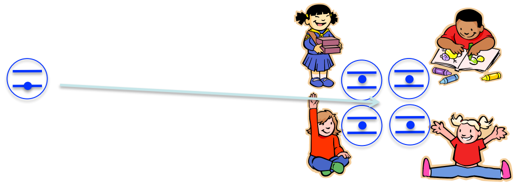

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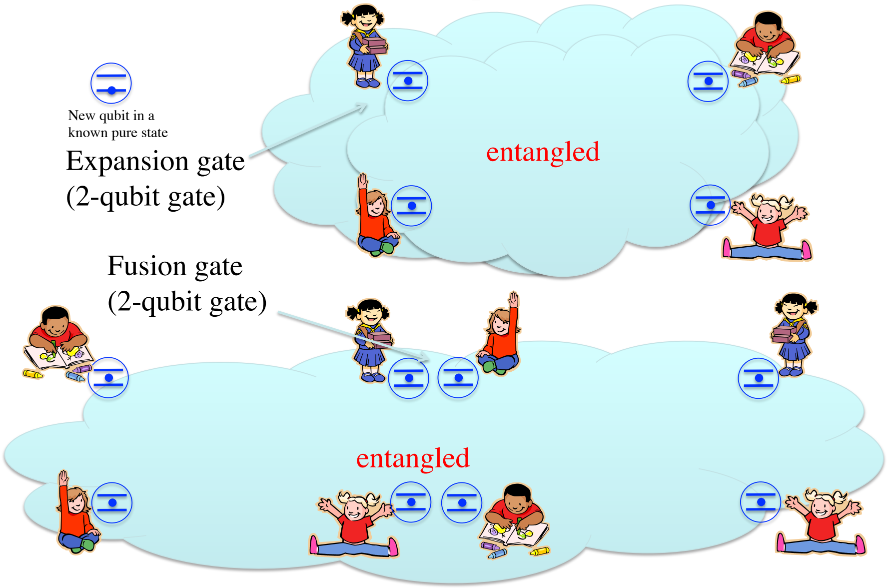
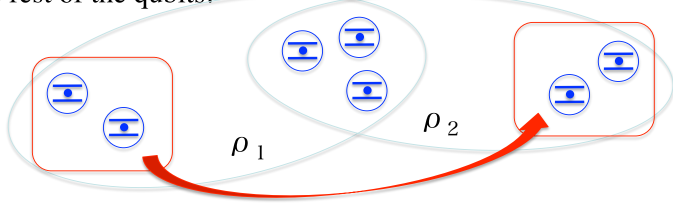


Previous works

When two entangled states share a subset of qubits in the same marginal state, then the two states can be converted by only touching the rest of the qubits.



GHZ_n and GHZ_{n+m} satisfies this. → deterministic exp/merge possible.

C_n and C_{n+m} satisfies this. → deterministic exp/merge possible.

Open questions ↓

W_n and W_{n+m} do not. → Is probabilistic expansion possible?

$Dicke_n$ and $Dicke_{n+m}$ do not. → Is probabilistic expansion possible?

If so, how large is the maximum probability?
Possible within linear optics? If so, how is the probability?

Marginal (partial) density operator of an entangled system

Marginal density operator: $\hat{\rho}^{(A)}_{\rm marginal} = {\rm Tr}_B\left[\hat{\rho}^{(AB)}\right]$ (9)

For $\hat{\rho}^{(AB)} = \hat{\rho}^{(A)} \otimes \hat{\rho}^{(B)} \Rightarrow \hat{\rho}^{(A)}_{\rm marginal} = \hat{\rho}^{(A)}$ (10)

For a Bell state $\Rightarrow \hat{\rho}^{(A)}_{\rm marginal} = \hat{\rho}^{(B)}_{\rm marginal} = \dfrac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2}$ (11)

By the way, a Bell state can be transferred to any Bell state by only rotating qubit A (or B).

For a maximally entangled state $\dfrac{|11\rangle_{AB} + |22\rangle_{AB} + \cdots |NN\rangle_{AB}}{\sqrt{N}}$

$\Rightarrow \hat{\rho}^{(A)}_{\rm marginal} = \hat{\rho}^{(B)}_{\rm marginal} = \dfrac{|1\rangle\langle 1| + |2\rangle\langle 2| + \cdots + |N\rangle\langle N|}{N}$ (12)

The marginal density operator of a maximally entangled state is a maximally mixed state.

An MES is transferred to any other MES by only rotating system A (or B).

For GHZ_N states $\Rightarrow \hat{\rho}^{(A)}_{\rm marginal} = \hat{\rho}^{(B)}_{\rm marginal} = \dfrac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2}$ (13)

This means that GHZ_N can be enlarged to GHZ_{N+1} by interacting a component qubit with an attached ancilla qubit.

Our work

[1] T. Tashima, et.al.: PRA77 (2008) 030302.
Proposal: An expansion gate (using ordinary BSs) for $W_n \rightarrow W_{n+2}$

[2] T. Tashima, et.al.: NJP11 (2009) 023024.
Proposal of an optimal expansion gate for expanding $W_n \rightarrow W_{n+1}$

[3] T. Tashima, et.al.: PRL102 (2009) 130502.
Proposal and experimental: A fusion gate that fuses $W_2 + W_2 \rightarrow W_3$

[4] T. Tashima, et.al.: PRL105 (2010) 210503.
Experimental demonstration of [1] ($W_1 \rightarrow W_3$ and $W_2 \rightarrow W_4$)

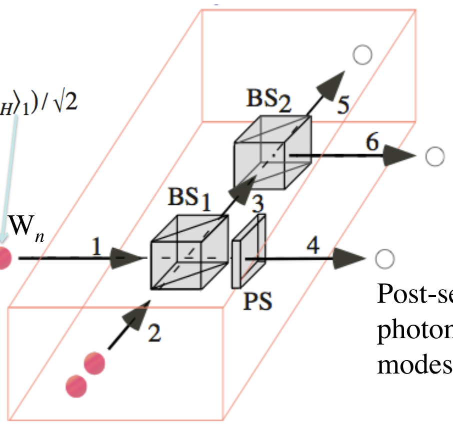
[5] R. Ikuta, et.al.: PRA83 (2011) 012314.
Proposal of an optimal gate for $W_n \rightarrow W_{n+m}$ with a Fock-state resource

[6] S. K. Ozdemir, et.al.: NJP (2011) 103003.
Proposal of a fusioning W states with/without recycling

[7] T. Kobayashi, et.al.: NJP (2014) 023005.
Proposal of a universal gates that transforms Dicke(nm)→Dicke(kj)

$W_2 =$
$(|1_H\rangle_0|1_V\rangle_1 + |1_V\rangle_0|1_H\rangle_1)/\sqrt{2}$

One photon from W_n



Post-select single-photon counting in modes 4, 5 and 6.

Prepare two horizontally polarized photons

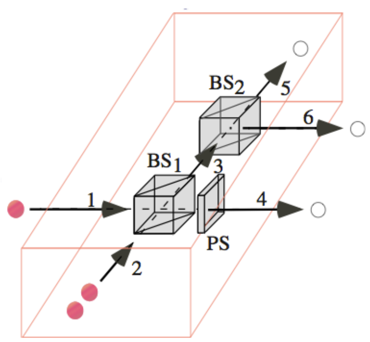
Then, photons 4, 5, 6 and the remaining n-1 photons form W_{n+2}.

[1] Phys. Rev. A77, 030302(R) (2008): "Elementary optical gate for expanding an entanglement web"
T. Tashima, S. K. Ozdemir, T. Yamamoto, M. Koashi & NI

polarization. We assume that the BS1 is polarization independent, namely, the transformation for V polarization has the same form. Using these relations, we see that the initial states $|1_{H(V)}\rangle_1 \otimes |2_H\rangle_2 = 2^{-1/2}\hat{a}^{\dagger}_{1H(V)}(\hat{a}^{\dagger}_{2H})^2|0\rangle$ evolve as

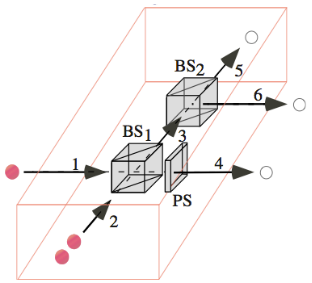
$$|1_H\rangle_1|2_H\rangle_2 \rightarrow \frac{\sqrt{3}}{2\sqrt{2}}|3_H\rangle_3|0\rangle_4 + \underline{\frac{1}{2\sqrt{2}}|2_H\rangle_3|1_H\rangle_4} - \frac{1}{2\sqrt{2}}|1_H\rangle_3|2_H\rangle_4 - \frac{\sqrt{3}}{2\sqrt{2}}|0\rangle_3|3_H\rangle_4,$$

$$|1_V\rangle_1|2_H\rangle_2 \rightarrow \frac{1}{2\sqrt{2}}|1_V 2_H\rangle_3|0\rangle_4 + \underline{\frac{1}{2}|1_H 1_V\rangle_3|1_H\rangle_4} + \frac{1}{2\sqrt{2}}|1_V\rangle_3|2_H\rangle_4 - \underline{\frac{1}{2\sqrt{2}}|2_H\rangle_3|1_V\rangle_4} - \frac{1}{2}|1_H\rangle_3|1_H 1_V\rangle_4 - \frac{1}{2\sqrt{2}}|0\rangle_3|1_V 2_H\rangle_4. \quad (1)$$



For the gate operation to be successful, there must be two photons in mode 3 and one photon in mode 4. Hence we are interested only in the underlined terms. The states $|2_H\rangle_3$ and $|1_H 1_V\rangle_3$ appearing in the underlined terms are transformed at BS_2 as

: for expanding an entanglement web"



$$|2_H\rangle_3 \to \frac{1}{2}|2_H\rangle_5|0\rangle_6 + \underline{\frac{1}{\sqrt{2}}|1_H\rangle_5|1_H\rangle_6} + \frac{1}{2}|0\rangle_5|2_H\rangle_6,$$

$$|1_H1_V\rangle_3 \to \frac{1}{2}|1_H1_V\rangle_5|0\rangle_6 + \underline{\frac{1}{2}|1_H\rangle_5|1_V\rangle_6} + \underline{\frac{1}{2}|1_V\rangle_5|1_H\rangle_6}$$

$$+ \frac{1}{2}|0\rangle_5|1_H1_V\rangle_6. \qquad (2)$$

Clearly, only the underlined terms in Eq. (2) contributes to the successful operation. Therefore, if we postselect the successful events, the action of the gate is given by the following state transformations:

$W_2 =$
$(|1_H\rangle_0|1_V\rangle_1+|1_V\rangle_0|1_H\rangle_1)/\sqrt{2}$

$$|1_H\rangle_1|2_H\rangle_2 \to \frac{1}{4}|1_H\rangle_4|1_H\rangle_5|1_H\rangle_6, \qquad (3)$$

$$|1_V\rangle_1|2_H\rangle_2 \to \frac{1}{4}|1_H\rangle_4|1_H\rangle_5|1_V\rangle_6 + \frac{1}{4}|1_H\rangle_4|1_V\rangle_5|1_H\rangle_6$$

$$+ \frac{1}{4}|1_V\rangle_4|1_H\rangle_5|1_H\rangle_6, \qquad (4)$$

W_4

[1] Phys. Rev. A77, 030302(R
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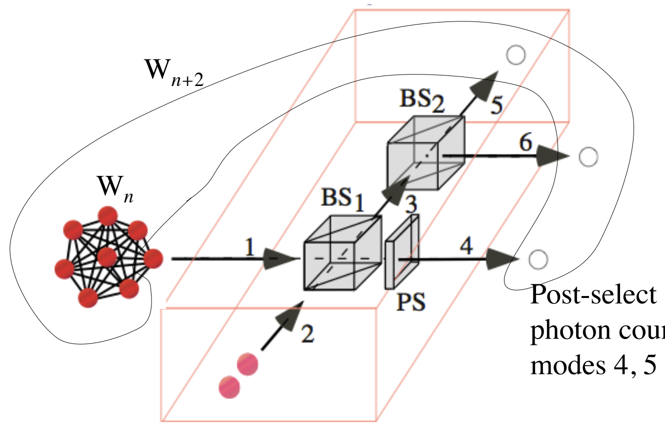

Post-select single-photon counting in modes 4, 5 and 6.

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[1] Phys. Rev. A77, 030302(R) (2008): "Elementary optical gate for expanding an entanglement web"
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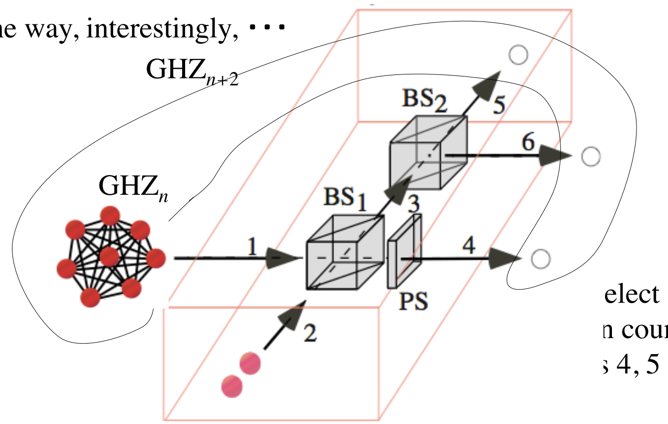
By the way, interestingly, ···



elect single-
n counting in
; 4, 5 and 6.

Prepare one horizontally and one vertically polarized photons

Then, photons 4, 5, 6 and the remaining n-1 photons form GHZ_{n+2} .

[1] Phys. Rev. A77, 030302(R) (2008): "Elementary optical gate for expanding an entanglement web"
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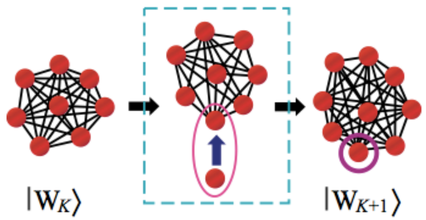
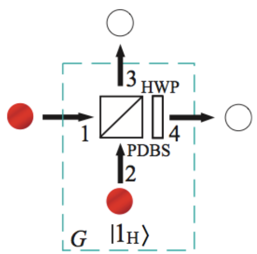




$\mu = (5-\sqrt{5})/10$
$\nu = (5+\sqrt{5})/10$

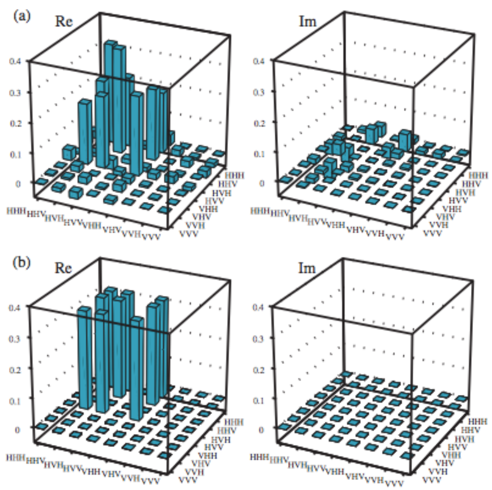
[2] NJP 11, 023024 (2009): "Local expansion of photonic W state using a polarization dependent beamsplitter,"
T. Tashima, S. K. Ozdemir, T. Yamamoto, M. Koashi & NI

EPR + EPR → W3

In other words,

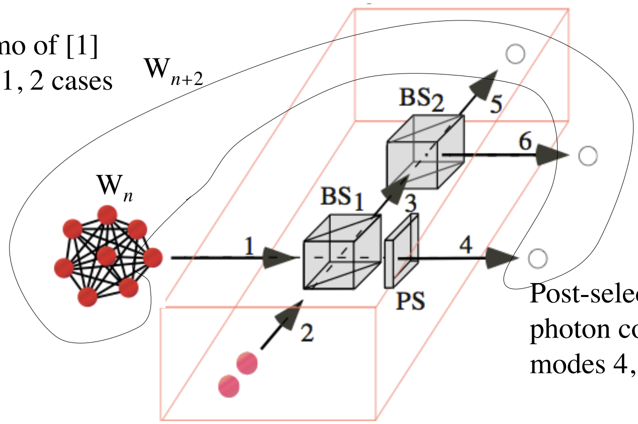
W2 + W2
→ W3 (+ detecting 1 photon)

$$\mu = (7 + \sqrt{17})/16$$
$$\nu = 1/2$$



[3] PRL 102, 130502 (2009): "Local Transformation of Two Einstein-Podolsky-Rosen Photon Pairs into a Three-Photon W State"
T. Tashima, T. Wakatsuki, S. K. Ozdemir, T. Yamamoto, M. Koashi & NI

Demo of [1]
$n = 1, 2$ cases



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[4] PRL 105, 210503 (2010): "Demonstration of Local Expansion Toward Large-Scale Entangled Webs"
T. Tashima, T. Kitano, S. K. Ozdemir, T. Yamamoto, M. Koashi & NI

Check whether method [1] gives the largest probability.

・Thoery of the largest probability within linear optics.

・Actually, the prob given by [1] is not the largest, but very close to it.

[5] PRA83, 012314 (2011): "Optimal local expansion of W states using linear optics and Fock states"
R. Ikuta, T. Tashima, S. K. Ozdemir, T. Yamamoto, M. Koashi & NI

Some *failure* cases provide recycling of the photons.

→ Let's recycle them!

→ Theory of clever way of recycling
（slightly complicated）

[6] NJP 13 103003 (2011): "An optical fusion gate for W-states"
S. K. Ozdemir, E. Matsunaga, T. Tashima, T. Yamamoto, M. Koashi & NI

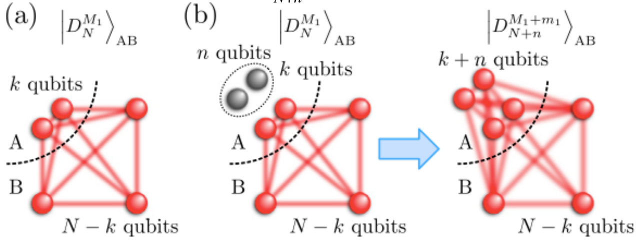
An N-qubit Dicke state with M_1 excitations is the equally weighted superposition of all permutations of N-qubit product states with M_1 spin-up ($|1\rangle$) and $M_0 = N - M_1$ spin-down ($|0\rangle$), and is written as

$$|D_N^{M_1}\rangle = (C_N^{M_1})^{1/2} \hat{P} \, |M_0, M_1\rangle , \qquad (1)$$

Ex) $D_4^2 = (|0011\rangle + |1100\rangle + |0101\rangle + |1010\rangle + |0110\rangle + |1001\rangle) / \sqrt{6}$

Scenario: Alice can use k qubits from D_N^{M1} and combine n new qubits.

Question: Can Alice and Bob hold D_{N+n}^{M1+m1}?



Answer: (assuming that N, M_1, k, m_1 are reasonable, then) It is possible.

(But, we only showed the existence of such gate.)

[7] NJP 16, 023005 (2014): "Universal gates for transforming multipartite entangled Dicke states"

T. Kobayashi, R. Ikuta, S. K. Ozdemir, M. Tame, T. Yamamoto, M. Koashi & NI

Summary : Progress in Expansion of Entanglement Web

[1] (Theory) Elementary optical gate for expanding an entanglement web, PRA77 (2008) 030302
First proposal of an expansion gate (using ordinary BSs) for $W_n \rightarrow W_{n+2}$

[2] (Theory) Local expansion of photonic W state using a polarization-dependent beamsplitter, NJP11 (2009) 023024
Proposal of an optimal gate for expanding $W_n \rightarrow W_{n+1}$

[3] (Experiment) Local Transformation of Two Einstein-Podolsky-Rosen Photon Pairs into a Three-Photon W State, PRL102 (2009) 130502
Proposal and demonstration of a fusion gate that fuses $W_2 + W_2 \rightarrow W_3$

[4] (Experiment) Demonstration of Local Expansion Toward Large-Scale Entangled Webs, PRL102 (2009) 130502
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[5] (Theory) Optimal local expansion of W states using linear optics and Fock states, PRA83 (2011) 012314
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[6] (Theory) An optical fusion gate for W-states, NJP (2011) 103003
Proposal of a fusion gate for W states with/without recycling

[7] (Theory) Universal gates for transforming multipartite entangled Dicke states, NJP (2014) 023005
Proposal a universal gates that transforms $\mathrm{Dicke}^n_m \rightarrow \mathrm{Dicke}^k_j$