Hybrid entanglement between quantum and classical states of light: generation and applications

The Brussels Journal (29 October 2007)

12 December, APCWQIS 2014, Tainan, Taiwan

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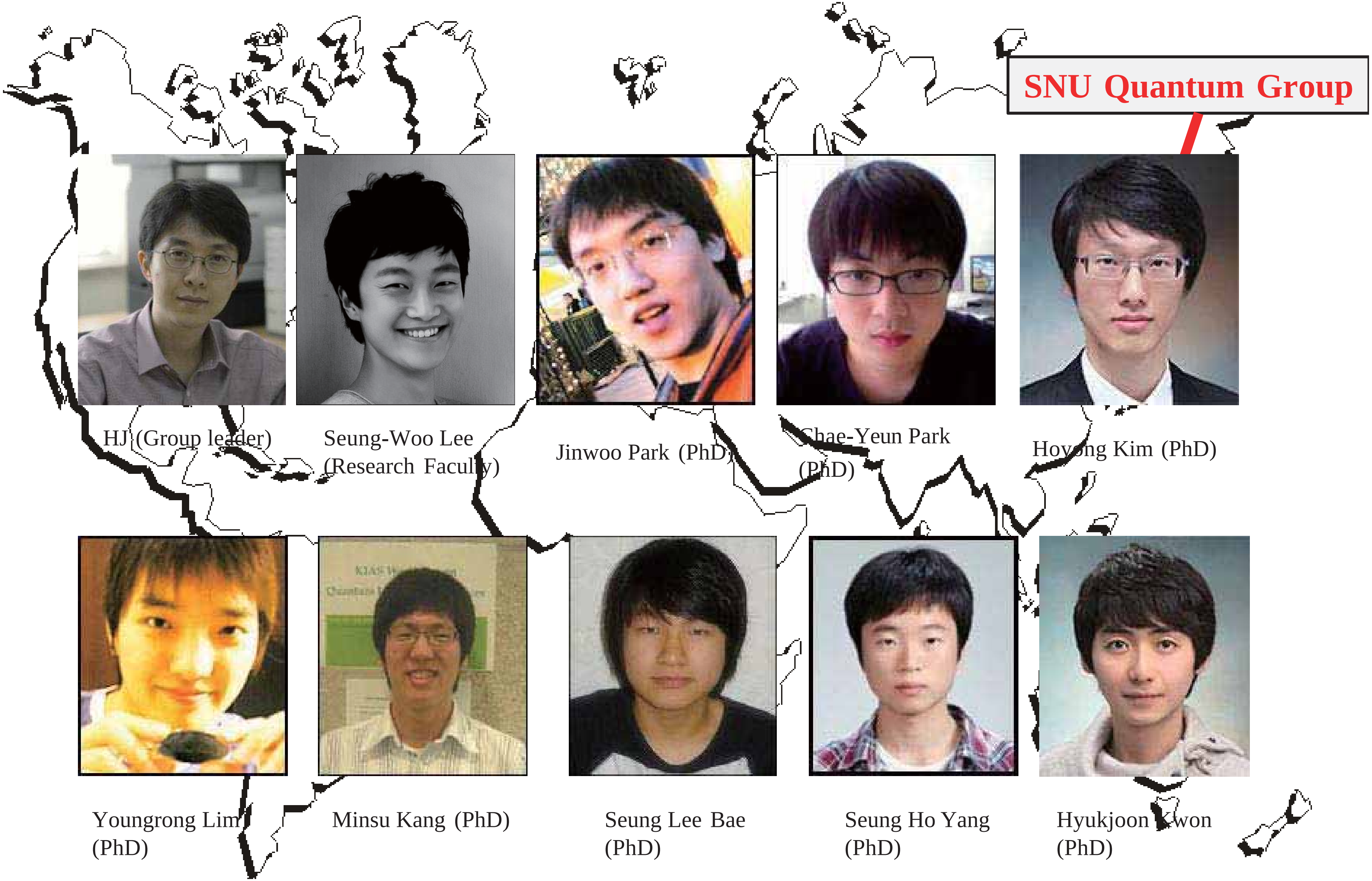
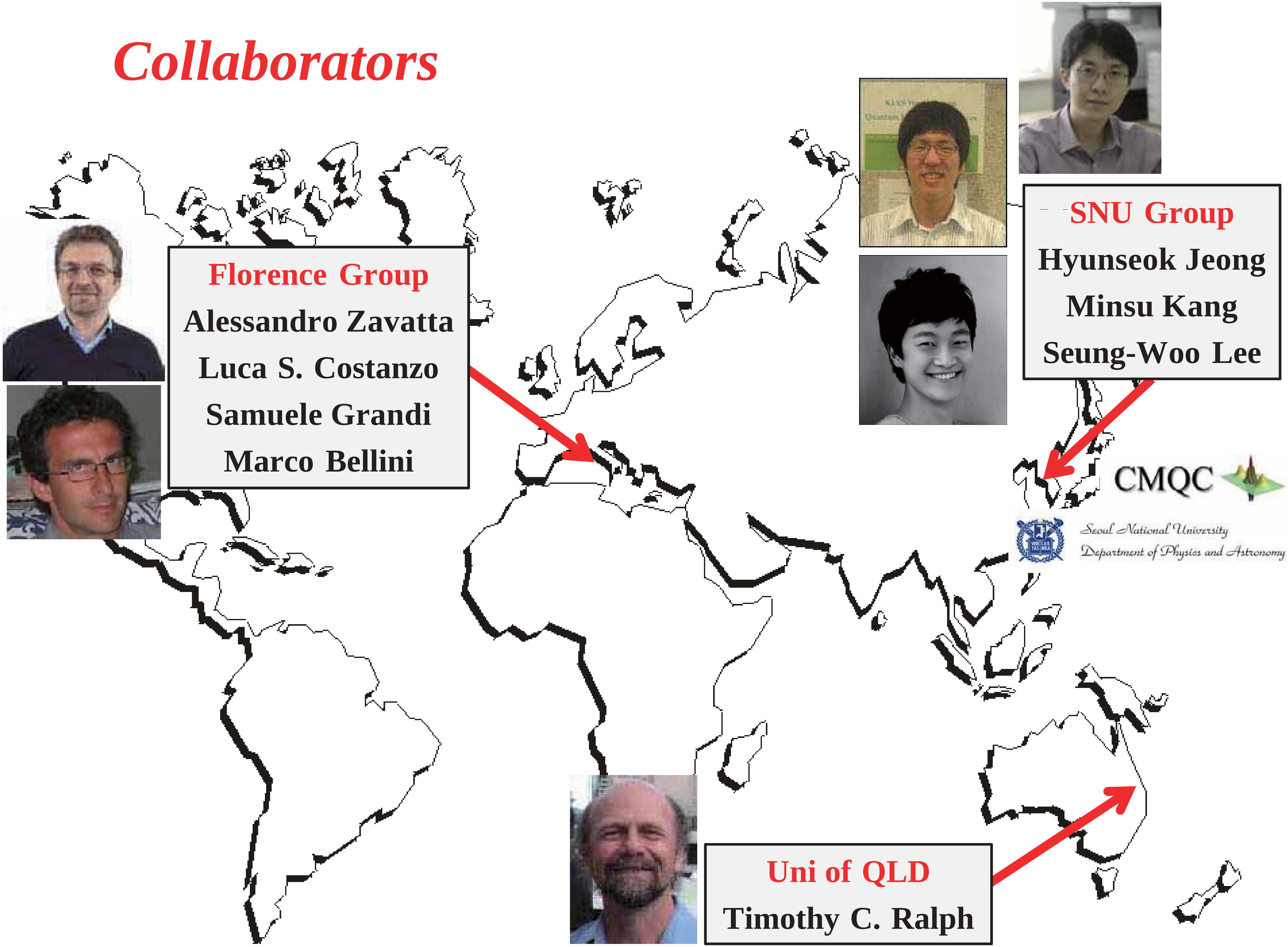


Collaborators



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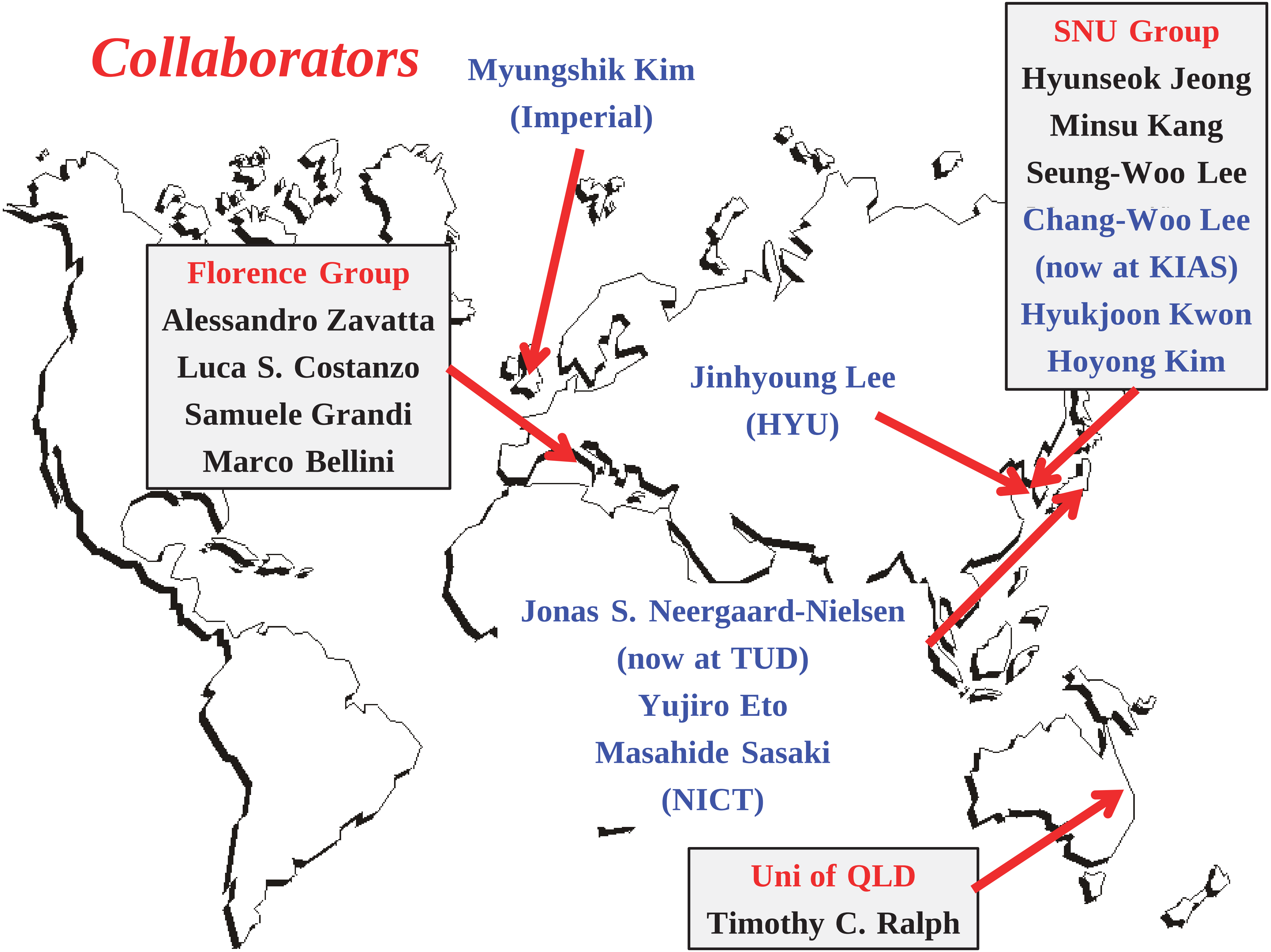
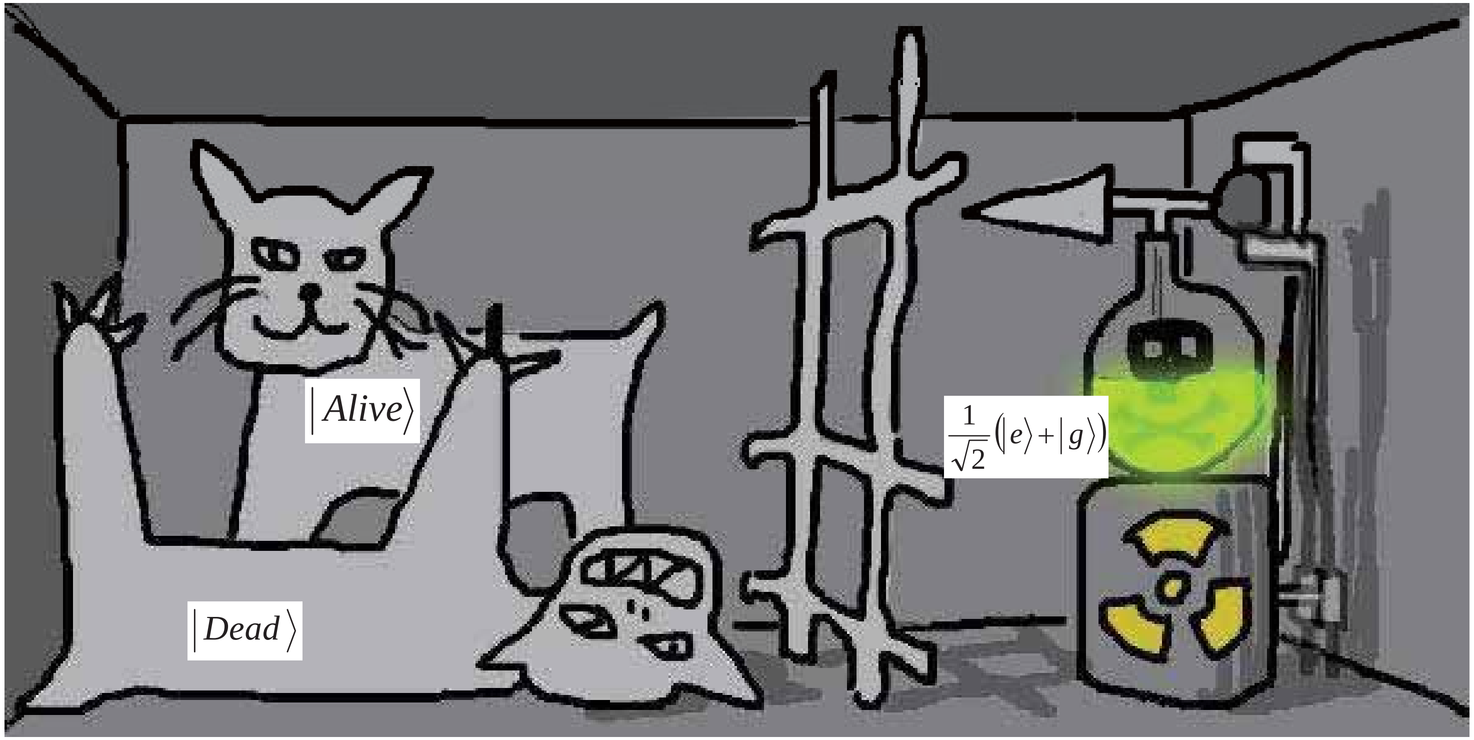


Contents

- Hybrid entanglement as a “Schrödinger cat” state
- All-optical quantum information processing using hybrid entanglement
- Generation scheme and experimental results – entangling quantum and classical states

Schrödinger's cat paradox

E. Schrödinger, *Naturwissenschaftern*. 23 (1935)



The Brussels Journal (29 October 2007)

$$\frac{1}{\sqrt{2}}(|e\rangle+|g\rangle)|Alive\rangle \longrightarrow \boxed{\frac{1}{\sqrt{2}}\left(|e\rangle|Alive\rangle+|g\rangle|Dead\rangle\right)}$$

Entanglement between "quantum" and "classical" systems

Coherent states and single photons

- **Coherent state:** $$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$ ***"Classical"***

✓ Coherent states are most classical among all pure states

- semi-classical descriptions available

- most robust against decoherence ("pointer states")

✓ The two coherent states | α > and | -α > are "classically" (or macroscopically) distinguishable for α >>1, *i.e.*, they can be well discriminated by a homodyne measurement (HD) with limited efficiency.

(For 70% of HD efficiency: D≈99.7% for α=1.6 and D>99.9% for α=2.0.)

- **Single photon:** $|1\rangle$ ***"Non-classical"***

✓ Discrete light quantum containing the minimum quantized amount of energy available at a given frequency.

✓ Negative values in well-known quasi-probability distributions such as the Wigner function.

Hybrid entanglement

- ***Hybrid entanglement between a single photon and a coherent state:***

$$\frac{1}{\sqrt{2}}\left(|H\rangle|\alpha\rangle+|V\rangle|-\alpha\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|e\rangle|Alive\rangle+|g\rangle|Dead\rangle\right)$$

- **The closest optical analogy to Schrödinger's *Gedankenexperiment* when α is reasonably large.**

Can we quantify "quantum macroscopicity"?

"*What is the correct measure of 'Schrödinger's-cattiness'? Ideally, one would like* ***a quantitative measure which corresponds to our intuitive sense****; I shall attempt one below, but would emphasize that the choice between this and a number of similar and perhaps equally plausible definitions is, with one important exception (see below),* ***very much a matter of personal taste****, and that I very much doubt that 50 years from now anything of importance will be seen to have hung on it.*"

(A. J. Leggett, J. Phys.: Condens. Matter 14 (2002) R415–R451)

Proposed measures and criteria

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[13] C.-W. Lee, H. Jeong, Phys. Rev. Lett. 106 (2011) 220401.
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[16] P. Sekatski, N. Sangouard, N. Gisin, Phys. Rev. A 89 (2014) 012116.

References taken from

H. Jeong, M. Kang and H. Kwon, "*Characterizations and quantifications of macroscopic quantumness and its implementations using optical fields*" (Review article) Special Issue on Macroscopic Quantumness, Optics Communications 337, 12–21 (2015)

A general measure for bosonic systems

C.-W. Lee and H. Jeong, ***Phys. Rev. Lett.*** **106, 220401 (2011)**

$$\mathcal{I}(\rho) = \frac{1}{2\pi^M}\int d^2\boldsymbol{\xi} \sum_{m=1}^{M}\left[|\xi_m|^2 - 1\right]|\chi(\boldsymbol{\xi})|^2$$
$$= \frac{\pi^M}{2}\int d^2\boldsymbol{\alpha}\, W(\boldsymbol{\alpha}) \sum_{m=1}^{M}\left[-\frac{\partial^2}{\partial\alpha_m\partial\alpha_m^*} - 1\right] W(\boldsymbol{\alpha})$$

- It can be applied to *any* harmonic oscillator systems such as light fields.
- Independent of the decomposition of the component states.
- For an ***arbitrary*** state, it simultaneously quantifies (1) *how far-separate the component states of the superposition are* ***and*** (2) *the degree of genuine quantum coherence between the component states against their classical mixture.*

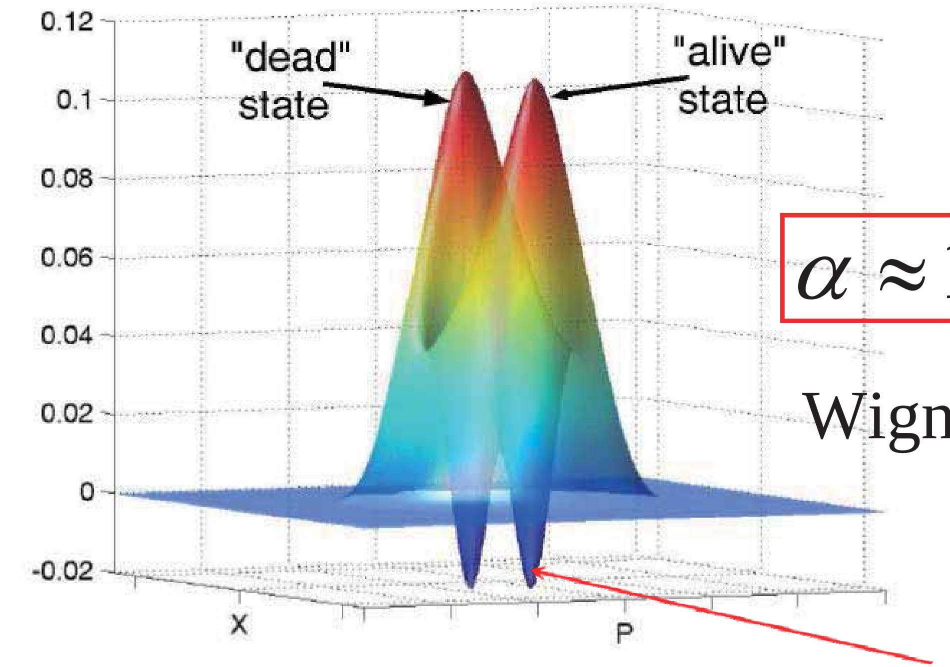
Superposition of coherent states

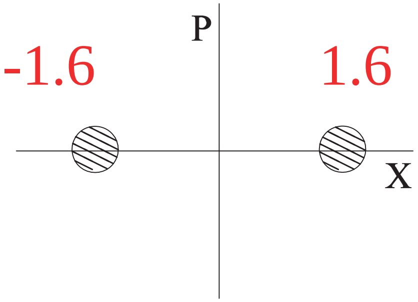
- **"Schrödinger cat" states of light:**

$$|cat\rangle = N\left(|\alpha\rangle + e^{i\varphi}|-\alpha\rangle\right)$$

$\alpha >> 1$

$$\hat{X} = \hat{a} + \hat{a}^{+}$$
$$\hat{P} = -i(\hat{a} - \hat{a}^{+})$$





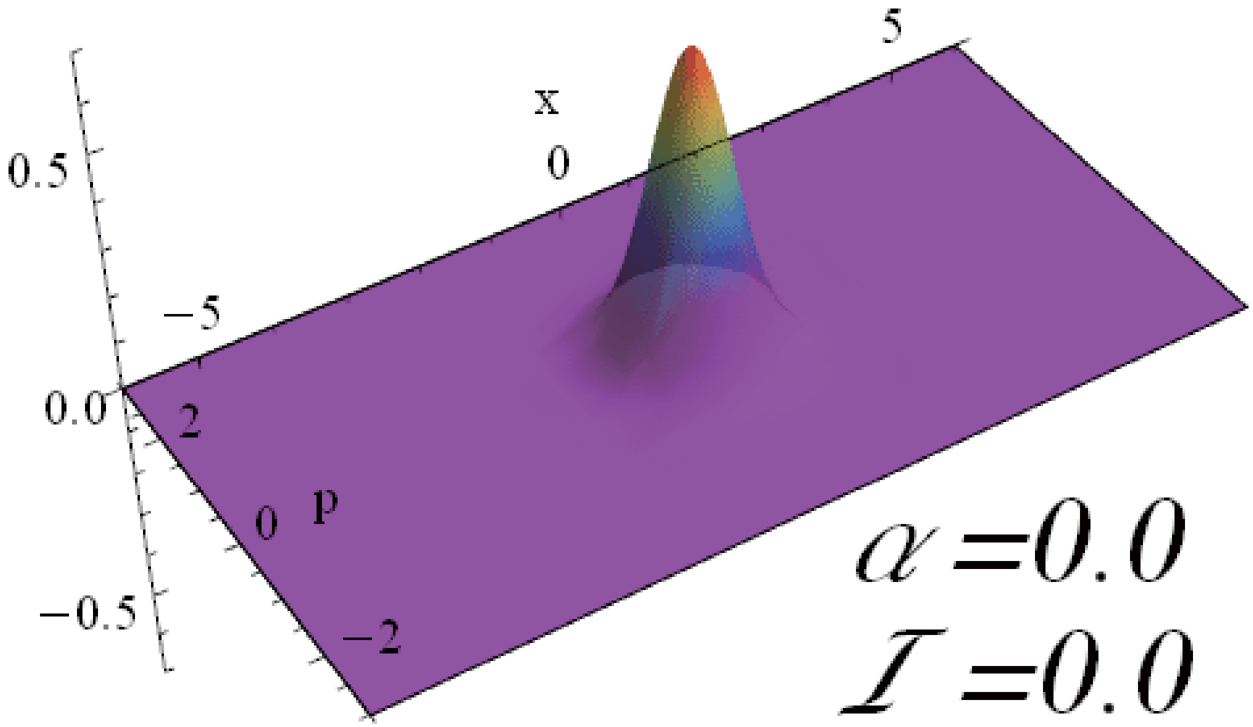
$\alpha \approx 1.6$

Wigner function

Evidence of quantum interference

A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri and Ph. Grangier, *Nature* 448, 784 (2007)

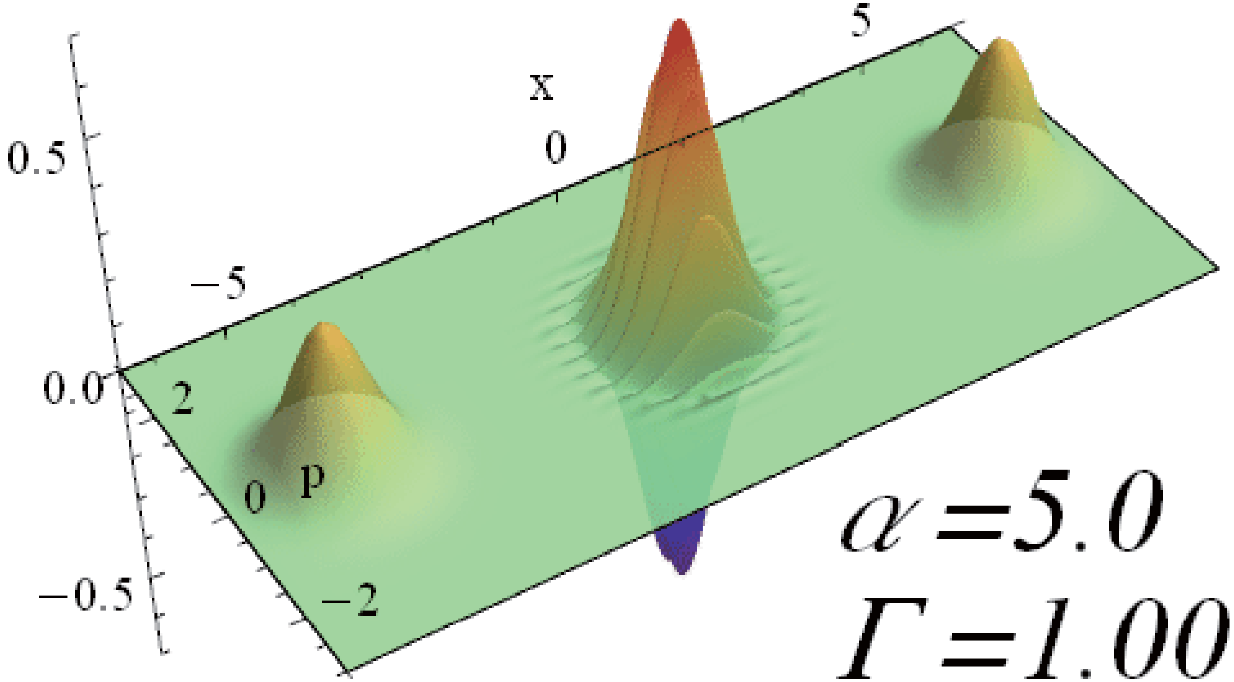
Macroscopic quantumness I of $|\alpha\rangle + |-\alpha\rangle$ with increasing α



Wigner function

Macroscopic quantumness I of

$$\rho_{\rm scs} = N_\Gamma[|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha| + \Gamma(|\alpha\rangle\langle-\alpha| + |-\alpha\rangle\langle\alpha|)]$$



Wigner function

$\alpha = 5.0$

$\Gamma = 1.00$

$I = 25.0$

Macroscopically quantum?

- Coherent state: $I(|\alpha\rangle) = 0$ regardless of the value of α.
- Hybrid entanglement

$$\frac{1}{\sqrt{2}}\left(|H\rangle|\alpha\rangle + |V\rangle|-\alpha\rangle\right)$$

  - **maximum value** $I = |\alpha|^2 + 1$

- Well known states in the "Schrödinger-cat family" with the maximum values of "macroscopic quantumness" I:
  - ✓ Superposition of coherent states: $|\alpha\rangle + |-\alpha\rangle$
  - ✓ NOON states: $|n\rangle|0\rangle + |0\rangle|n\rangle$
  - ✓ GHZ state: $|H\rangle^N + |V\rangle^N$

- **“Micro-macro” entanglement or “macro-macro” entanglement as analogies of Schrödinger’s thought experiment**

[24] P. Sekatski, N. Sangouard, M. Stobińska, F. Bussieres, M. Afzelius, N. Gisin, Phys. Rev. A 86 (2012) 060301(R).

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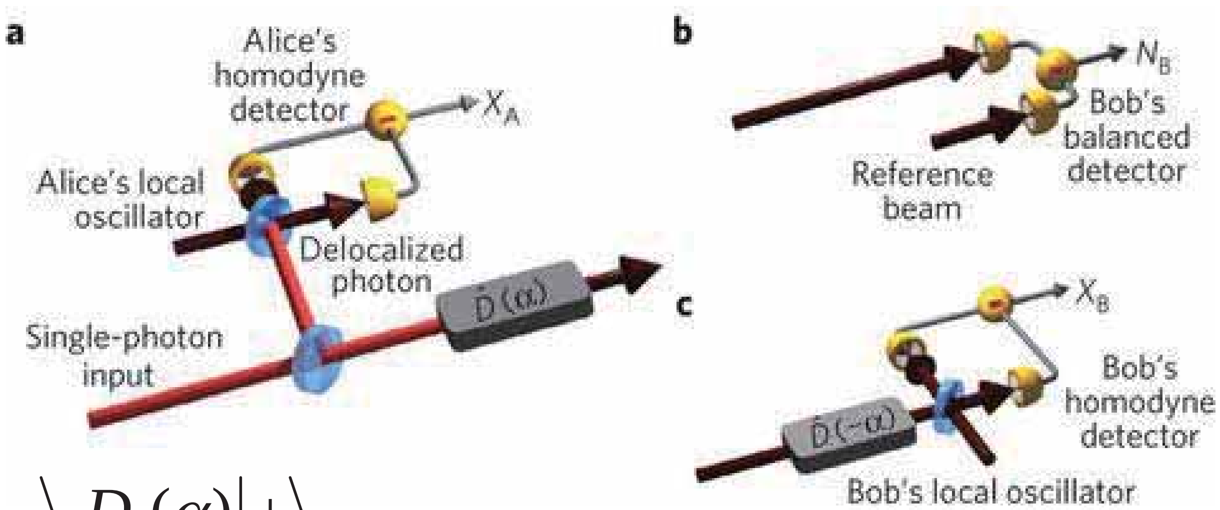
[160] A.I. Lvovsky, R. Ghobadi, A. Chandra, A.S. Prasad, C. Simon, Nat. Phys. 9 (2013) 541.

Micro-macro entanglement

N. Bruno *et al.*, Nature Physics 9, 545 (2013); A. I. Lvovsky *et al*. Nature Physics 9, 541 (2013)



$$D_1(\alpha)\left(|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2\right)$$
$$=D_1(\alpha)\left(|+\rangle_1|-\rangle_2-|-\rangle_1|+\rangle_2\right)$$
$$=D_1(\alpha)|+\rangle_1 D_2(\alpha)|-\rangle_2-D_1(\alpha)|-\rangle_1 D_2(\alpha)|+\rangle_2$$

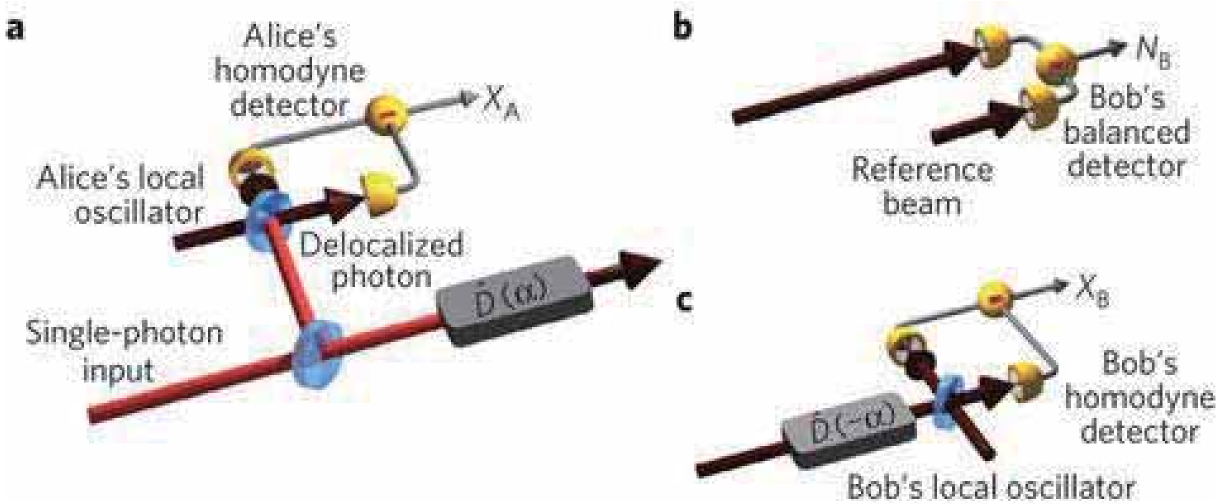
$$|\pm\rangle=|0\rangle\pm|1\rangle$$

- **$D(\alpha)(|0\rangle+|1\rangle)$ and $D(\alpha)(|0\rangle-|1\rangle)$ are distinguishable by a single-shot photon number measurement with a high probability [Sekatski *et al*., PRA 012116 (2014)].**

Micro-macro entanglement

N. Bruno *et al.*, Nature Physics 9, 545 (2013); A. I. Lvovsky *et al.* Nature Physics 9, 541 (2013)

$$D_1(\alpha)\left(|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2\right)$$
$$=D_1(\alpha)\left(|+\rangle_1|-\rangle_2-|-\rangle_1|+\rangle_2\right)$$



- **However, if so, even a coherent state**

$$|\alpha\rangle = D(\alpha)\left(|0\rangle+|1\rangle\right)+D(\alpha)\left(|0\rangle-|1\rangle\right)$$

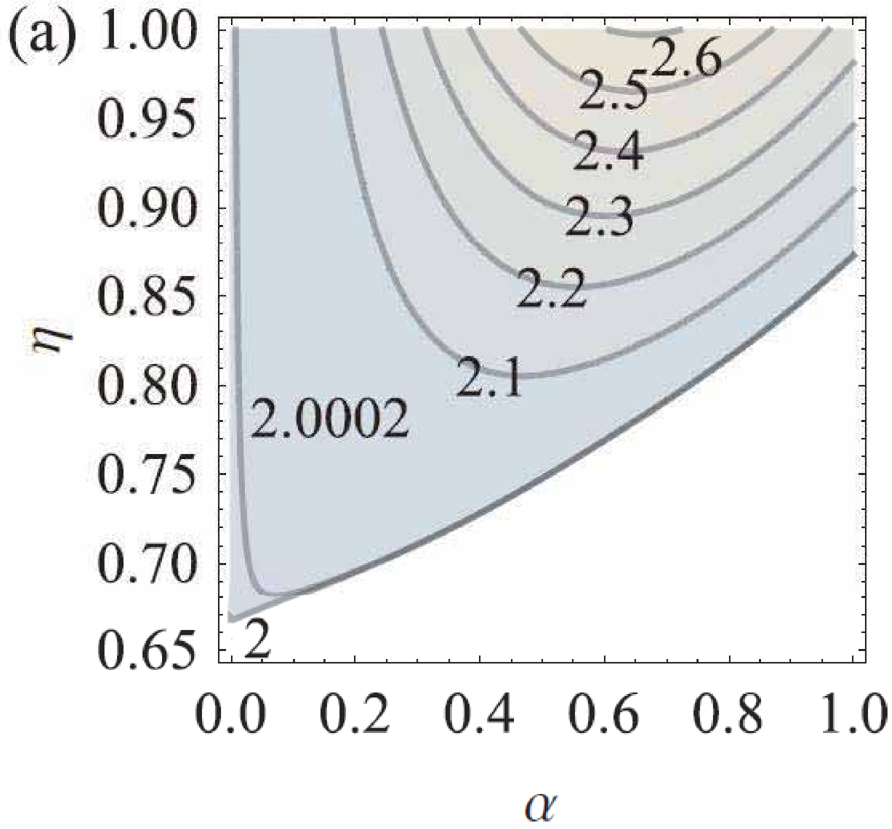
should be interpreted as a macroscopic superposition state. Is it acceptable?

- **The value of macroscopic quantumness is only I=1 regardless of the value of α.**

H. Jeong, M. Kang and H. Kwon, Optics Communications 337, 12–21 (2015)
http://dx.doi.org/10.1016/j.optcom.2014.07.012 (2014)

Usefulness for a loophole-free Bell test

- **Required detection efficiency: > 0.67**



$$\frac{1}{\sqrt{2}}\left(|H\rangle|\alpha\rangle+|V\rangle|-\alpha\rangle\right)$$

H. Kwon and H. Jeong, "Violation of the Bell-Clauser-Horne-Shimony-Holt inequality using imperfect photodetectors with optical hybrid states," **Phys. Rev. A 88, 052127 (2013)**.

Contents

- Hybrid entanglement as a “Schrödinger cat” state
- **All-optical quantum information processing using hybrid entanglement**
- Generation scheme and experimental results

Schemes for all-optical quantum computation

Single photon qubits

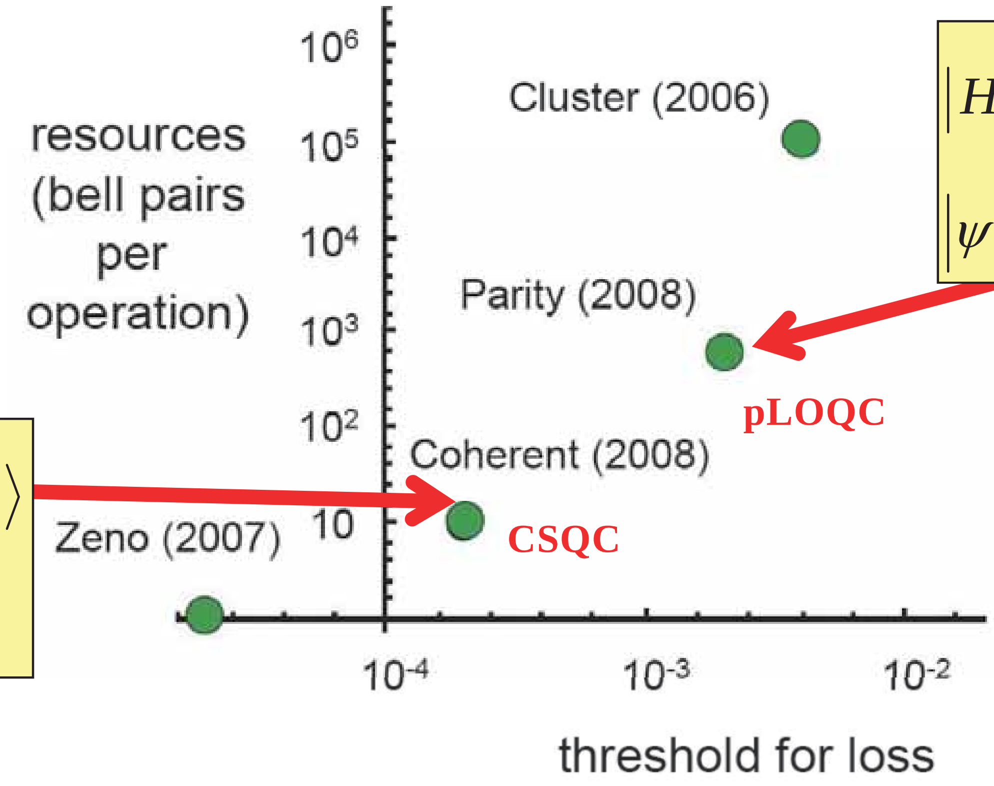
$$|H\rangle \rightarrow |0_L\rangle, \quad |V\rangle \rightarrow |1_L\rangle$$

$$|\psi_{qubit}\rangle = a|H\rangle + b|V\rangle$$

Coherent state qubits

$$|\alpha\rangle \rightarrow |0_L\rangle, \quad |-\alpha\rangle \rightarrow |1_L\rangle$$

$$|\psi_{qubit}\rangle = a|\alpha\rangle + b|-\alpha\rangle$$



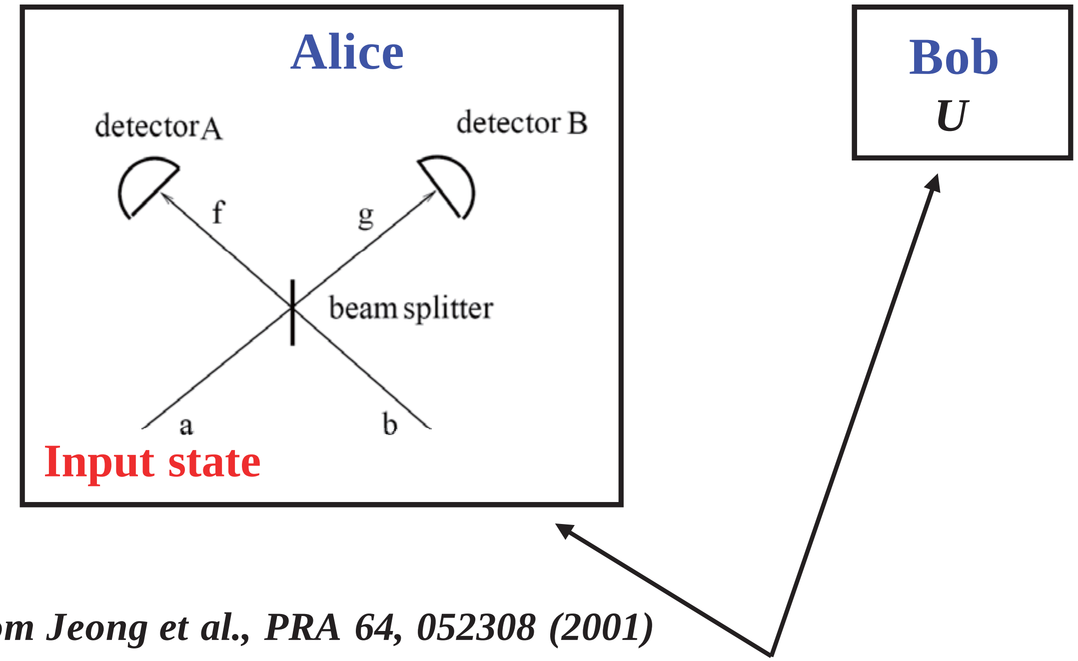
"The best compromise for medium scale quantum computing (100s of logical operations) appear to be the CSQC scheme and the Parity State scheme."

T. C. Ralph and G. J. Pryde, Progress in Optics, Ed. E.Wolf, 54, 209 (2009)

Teleportation and quantum gates using coherent state qubits

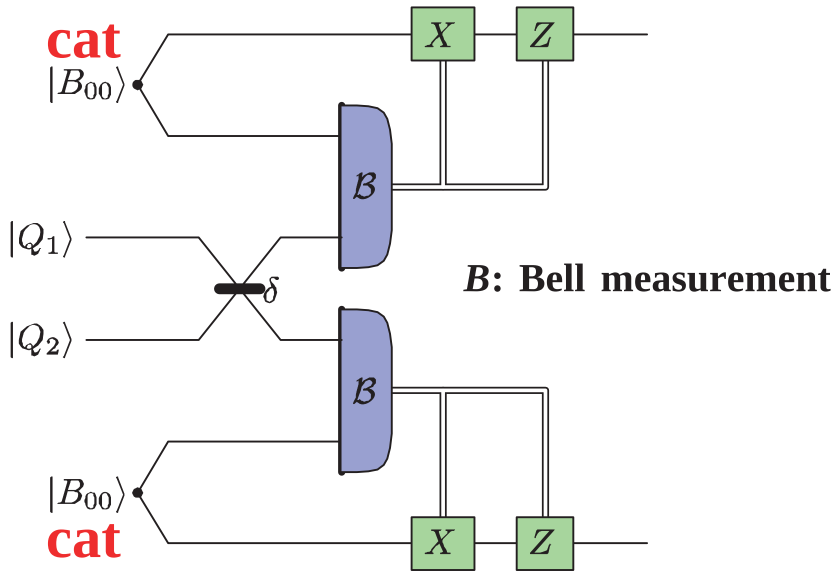
(Jeong *et al.*, PRA 2001, Jeong and Kim, PRA 2002, Ralph *et al.*, Proc. SPIE 2002, Ralph *et al.*, PRA 2003)

Nearly deterministic Bell measurement



From Jeong et al., PRA 64, 052308 (2001)

$$N_{-}(|\alpha\rangle|-\alpha\rangle - |-\alpha\rangle|\alpha\rangle)$$



$$|B_{00}\rangle = N_{+}(|\alpha\rangle|\alpha\rangle + |-\alpha\rangle|-\alpha\rangle)$$

From Ralph et al., PRA 68, 042319 (2003)

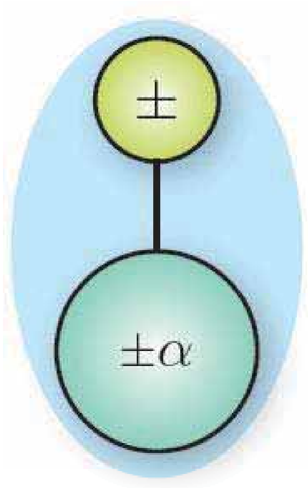
- **Single qubit rotation (*Z*-rotation) cannot be realized deterministically.**
- **Sensitive to photon losses**
- **Optimized value α~1.6 [Lund *et al.* PRL 2008]**

New approach:

S.-W. Lee and H. Jeong, "Near-deterministic quantum teleportation and resource-efficient quantum computation using linear optics and hybrid qubits," **Phys. Rev. A 87, 022326 (2013).**

$$a|H\rangle|\alpha\rangle + b|V\rangle|-\alpha\rangle$$

“Optical hybrid qubit”

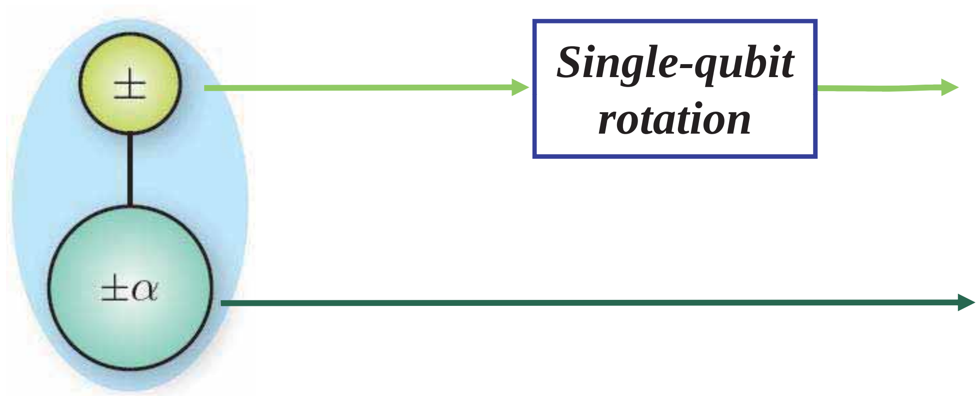


“Single-photon” mode

“Coherent-state” mode

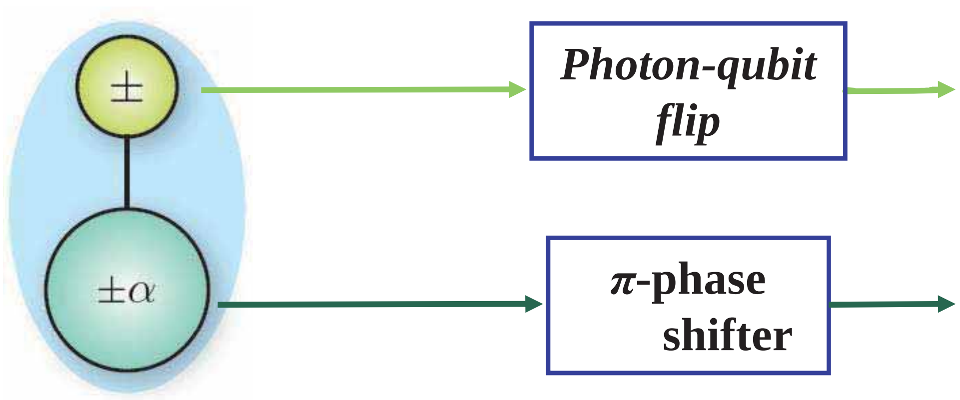
Single-qubit operations

Arbitrary Z rotations



$$a|H\rangle|\alpha\rangle + b|V\rangle|-\alpha\rangle \longrightarrow a|H\rangle|\alpha\rangle + be^{i\phi}|V\rangle|-\alpha\rangle$$

Pauli X (bit flip)



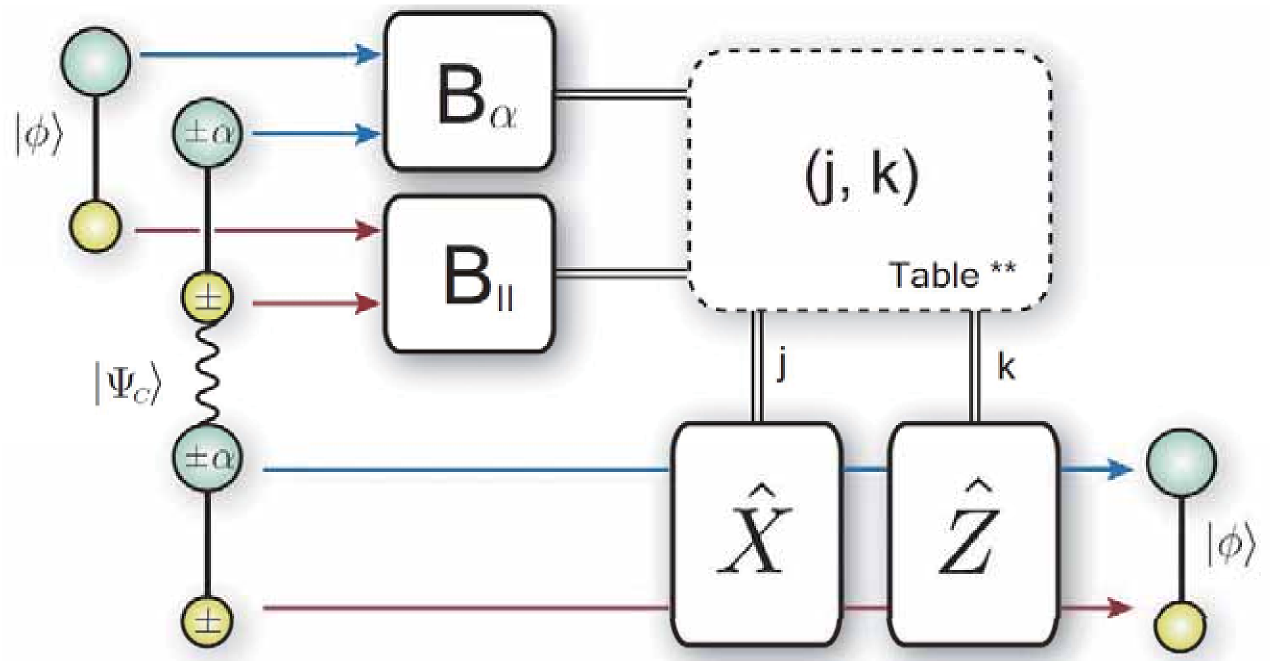
$$|H\rangle \Leftrightarrow |V\rangle$$

$$|\alpha\rangle \Leftrightarrow |-\alpha\rangle$$

$$a|H\rangle|\alpha\rangle + b|V\rangle|-\alpha\rangle \longrightarrow a|V\rangle|-\alpha\rangle + b|H\rangle|+\alpha\rangle$$

Near-deterministic teleportation

S.-W. Lee and H. Jeong, Phys. Rev. A 87, 022326 (2013)



Failure probability

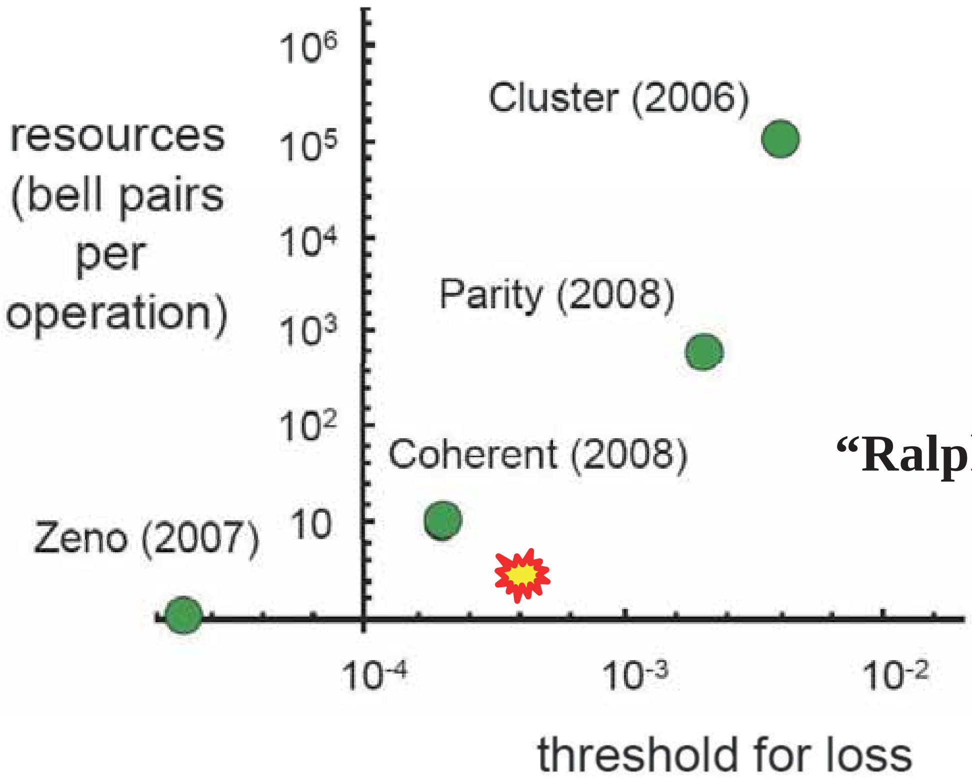
$$P_f = e^{-2\alpha^2}/2$$

$$P_f \sim 0.6\% \text{ for } \alpha = 1.5$$

$$P_f \sim 0.02\% \text{ for } \alpha = 2$$

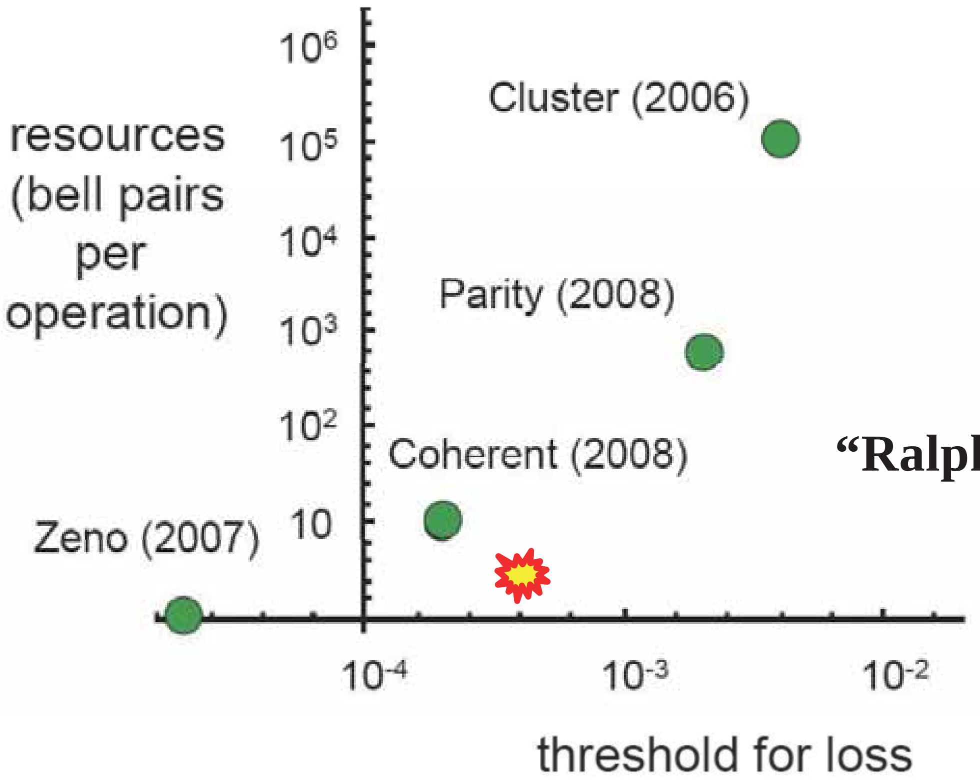
→ Universal gate operations can be performed using "gate teleportation" in a nearly deterministic manner.

Comparison with previous schemes



“Ralph-Pryde diagram”

Comparison with previous schemes



"Ralph-Pryde diagram"

The new approach seems to outperform the previous ones with hybrid entangled states of $\alpha \approx 1$ as resources.

S.-W. Lee and H. Jeong, Phys. Rev. A 87, 022326 (2013)

Necessary resource states

- "Hybrid" quantum information processing:

Hybrid pairs

$$\frac{1}{\sqrt{2}}\left(|H\rangle|\alpha\rangle+|V\rangle|-\alpha\rangle\right)$$

- Optimized amplitude: $\alpha \approx 1.1$

[S.-W. Lee and H. Jeong, Phys. Rev. A 87, 022326 (2013)]

Necessary resource states

- "Hybrid" quantum information processing:

Hybrid pairs

$$\frac{1}{\sqrt{2}}\left(|0\rangle|\alpha\rangle+|1\rangle|-\alpha\rangle\right)$$

- Optimized amplitude: $\alpha \approx 1.1$

[H. Kim, S.-W. Lee and H. Jeong, in preparation]

Generation of hybrid entanglement

- Cross-Kerr nonlinearity

[62] C.C. Gerry, Phys. Rev. A 59 (1999) 4095.
[72] H. Jeong, Phys. Rev. A 72 (2005) 034305.

Generation of hybrid entanglement

- Cross-Kerr nonlinearity

[62] C.C. Gerry, Phys. Rev. A 59 (1999) 4095.
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- Highly nontrivial and demanding

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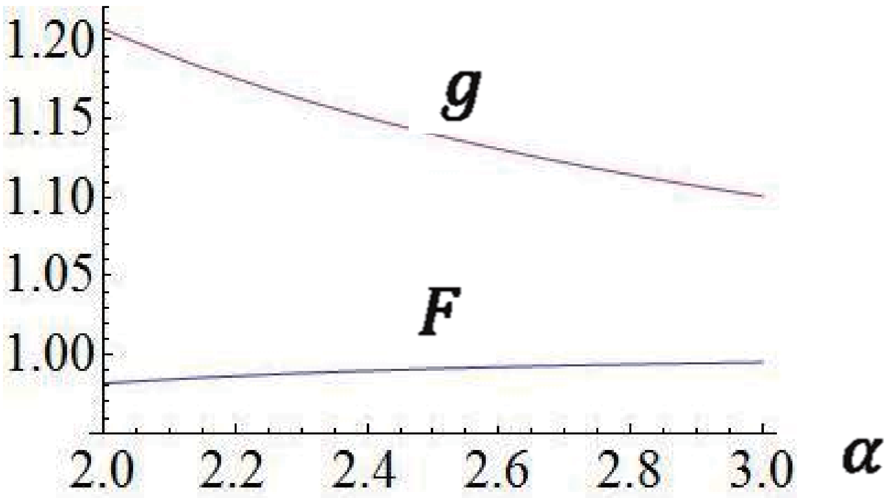
- **Generation scheme and experimental results**

Single photon added coherent state

$$\frac{\hat{a}^\dagger|\alpha\rangle}{\sqrt{1+\alpha^2}} \approx |g\alpha\rangle$$

$$g = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{\alpha^2}}$$

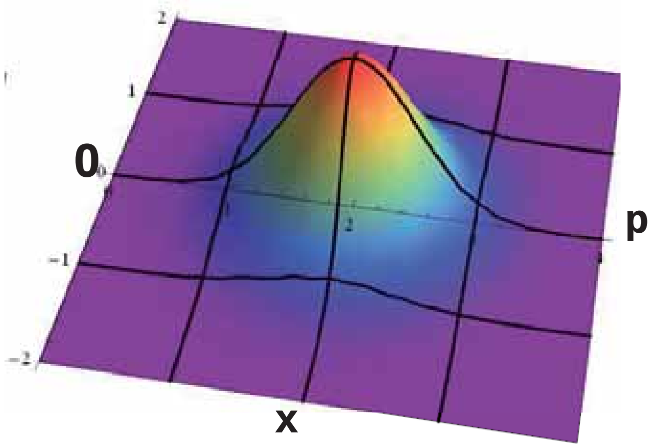


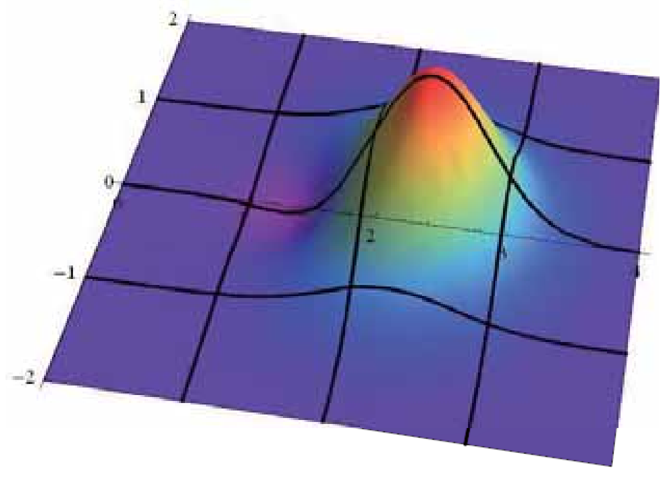
$\alpha = 2 \longrightarrow g\alpha = 2.414 \quad F = 0.98$

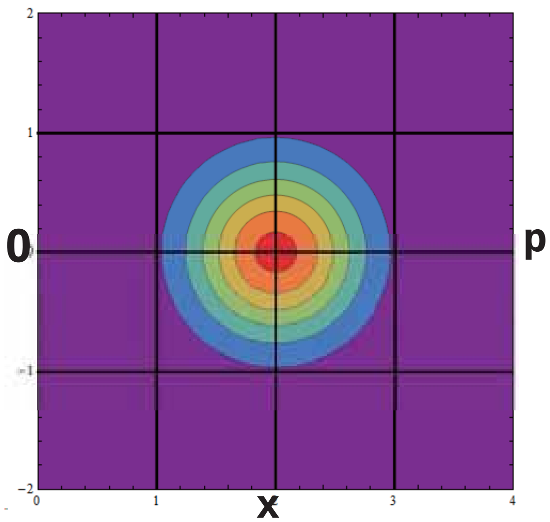
$\alpha = 4 \longrightarrow g\alpha = 4.236 \quad F = 0.998$

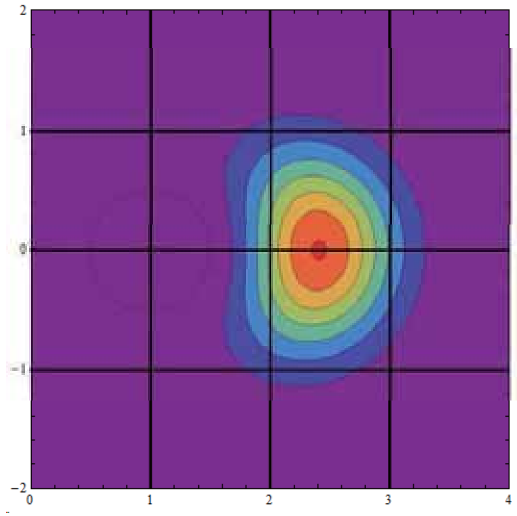
Amplifying Coherent State by Photon Addition

$|\alpha\rangle$ $\qquad\qquad$ $\hat{a}^{\dagger}|\alpha\rangle$

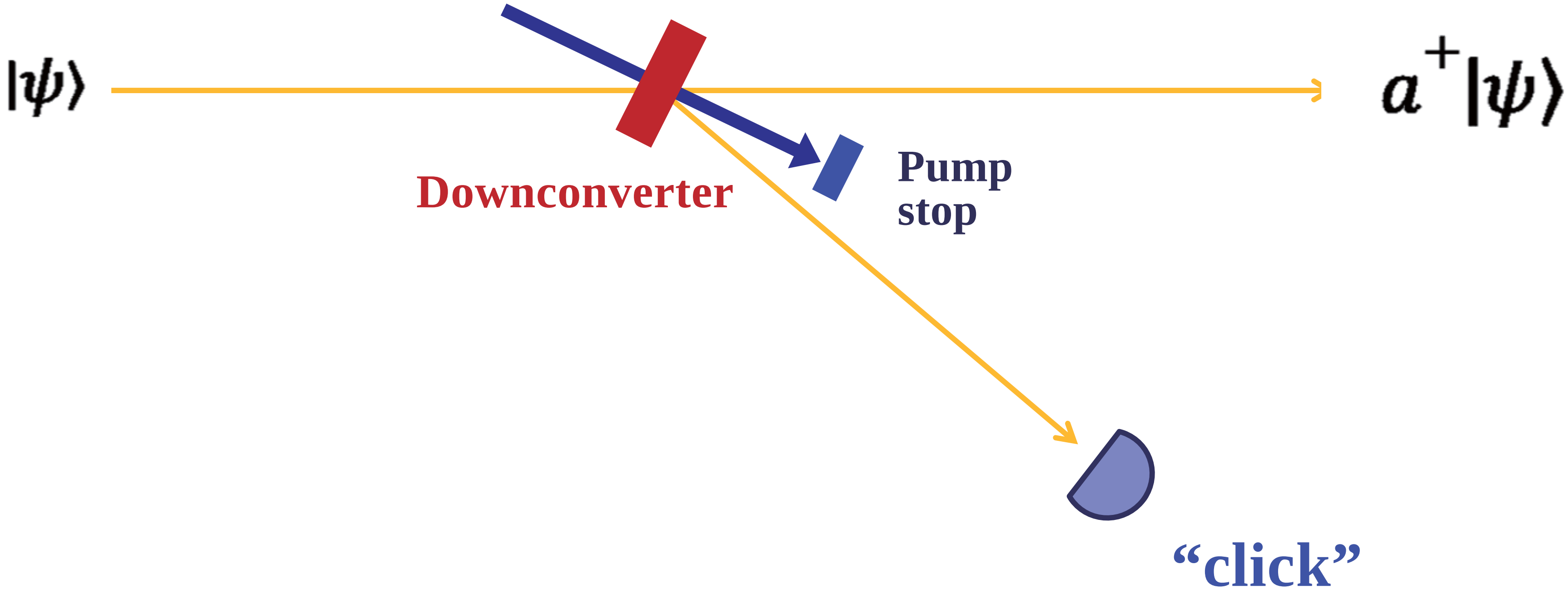






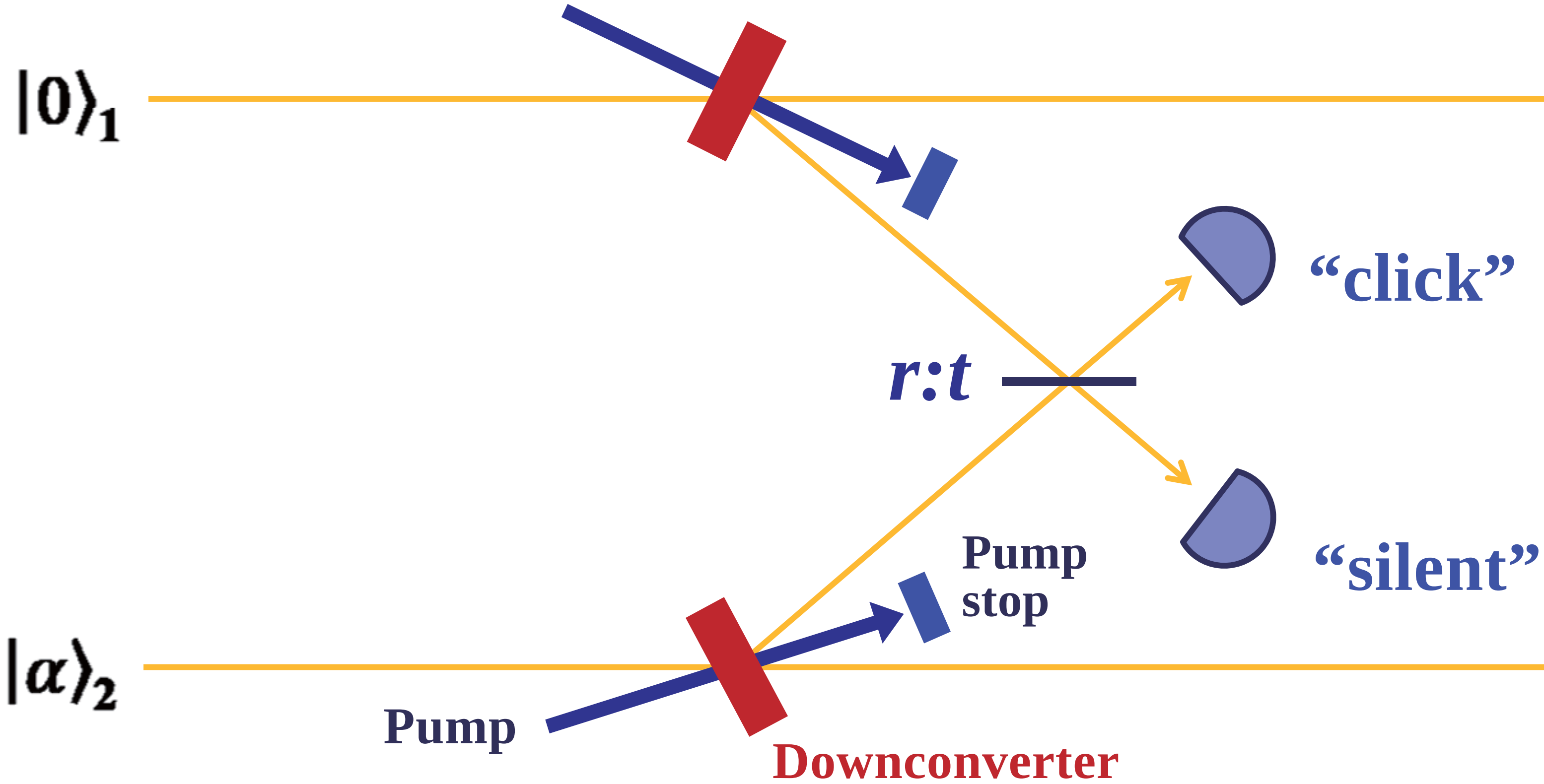


Single photon addition

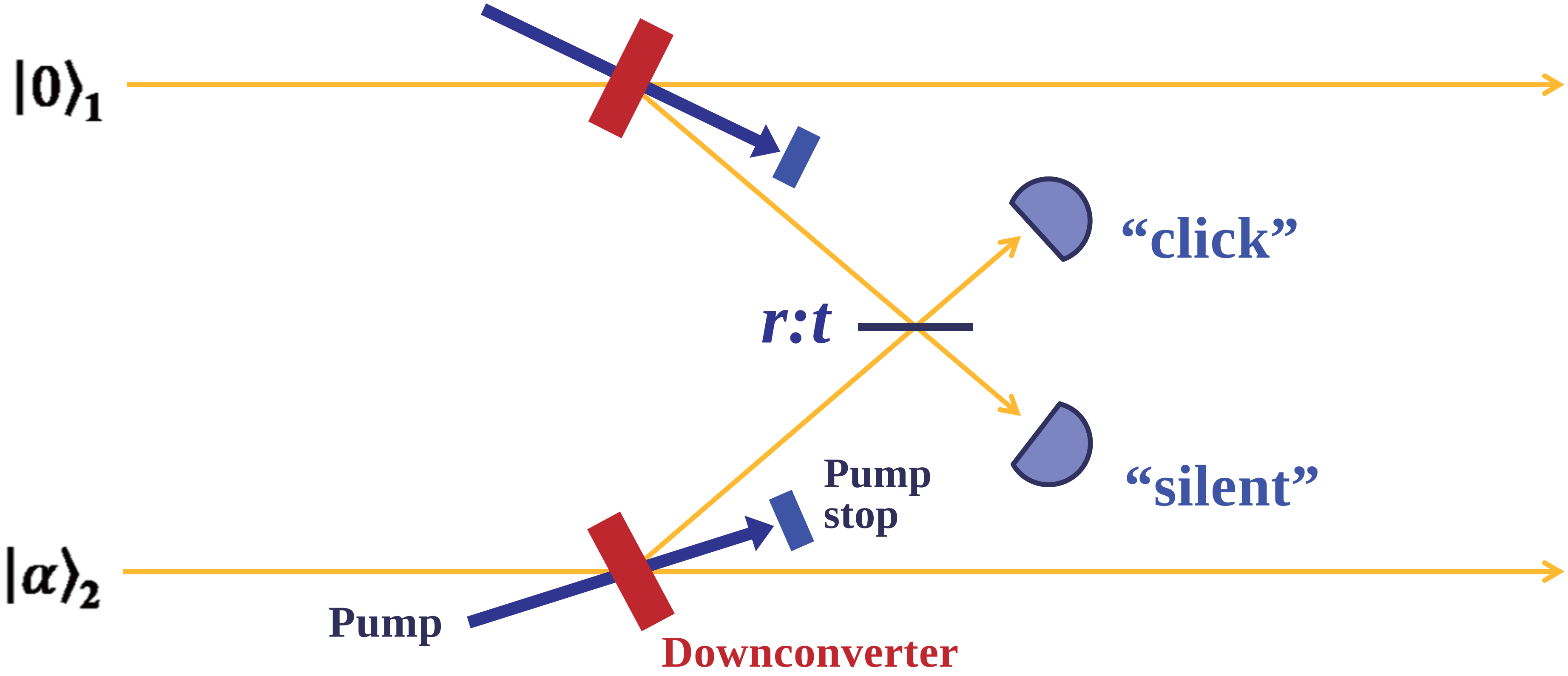


[Zavatta *et al.*, Science, 2004]

Generation of hybrid entanglement

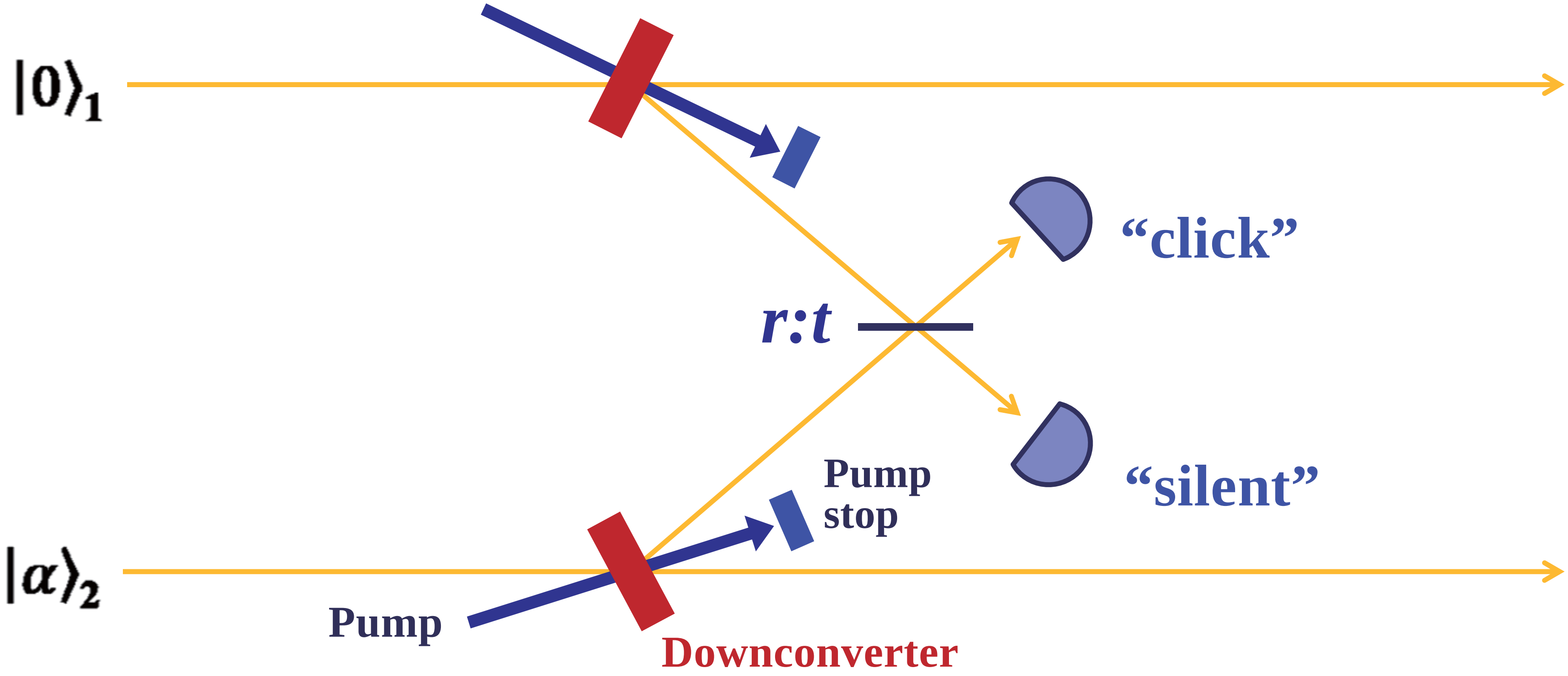


Generation of hybrid entanglement



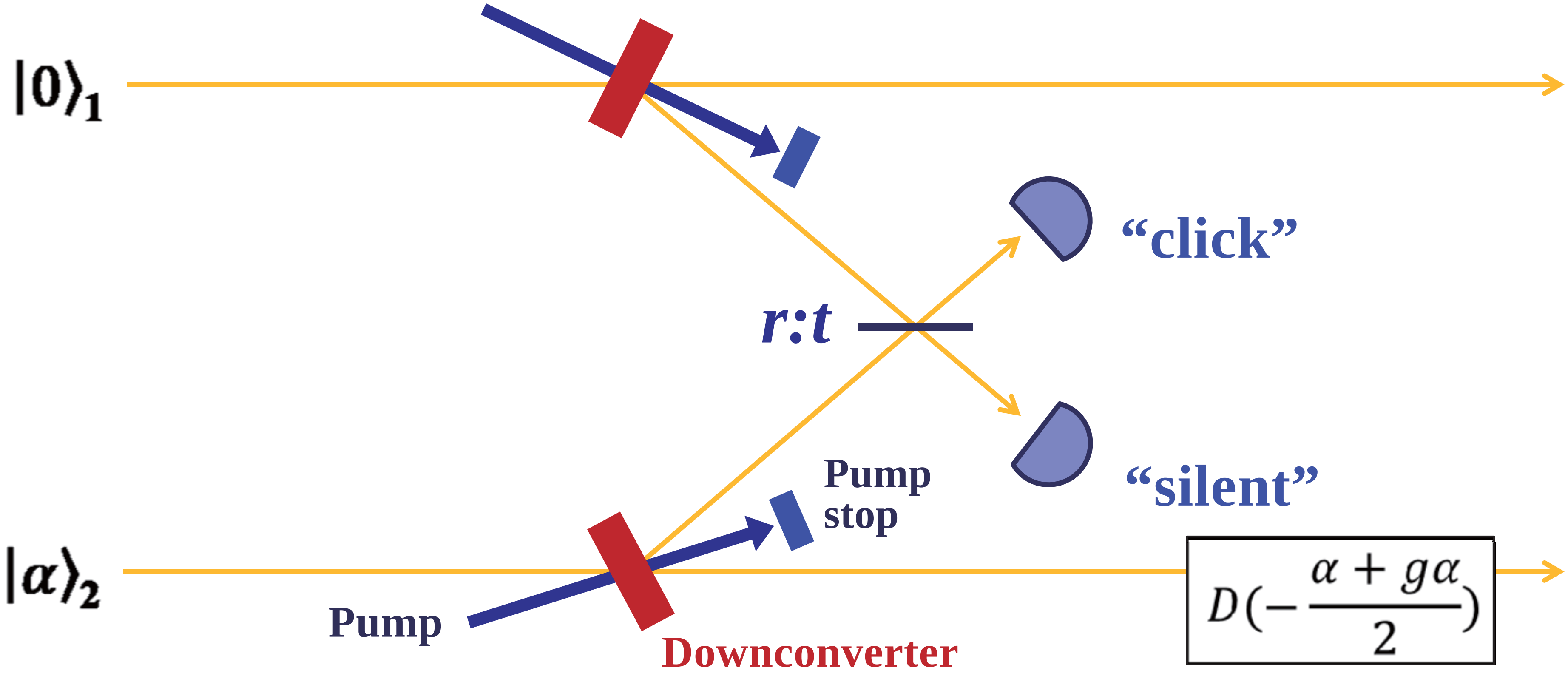
$$ra^{+}|0\rangle|\alpha\rangle + t|0\rangle a^{+}|\alpha\rangle$$

Generation of hybrid entanglement



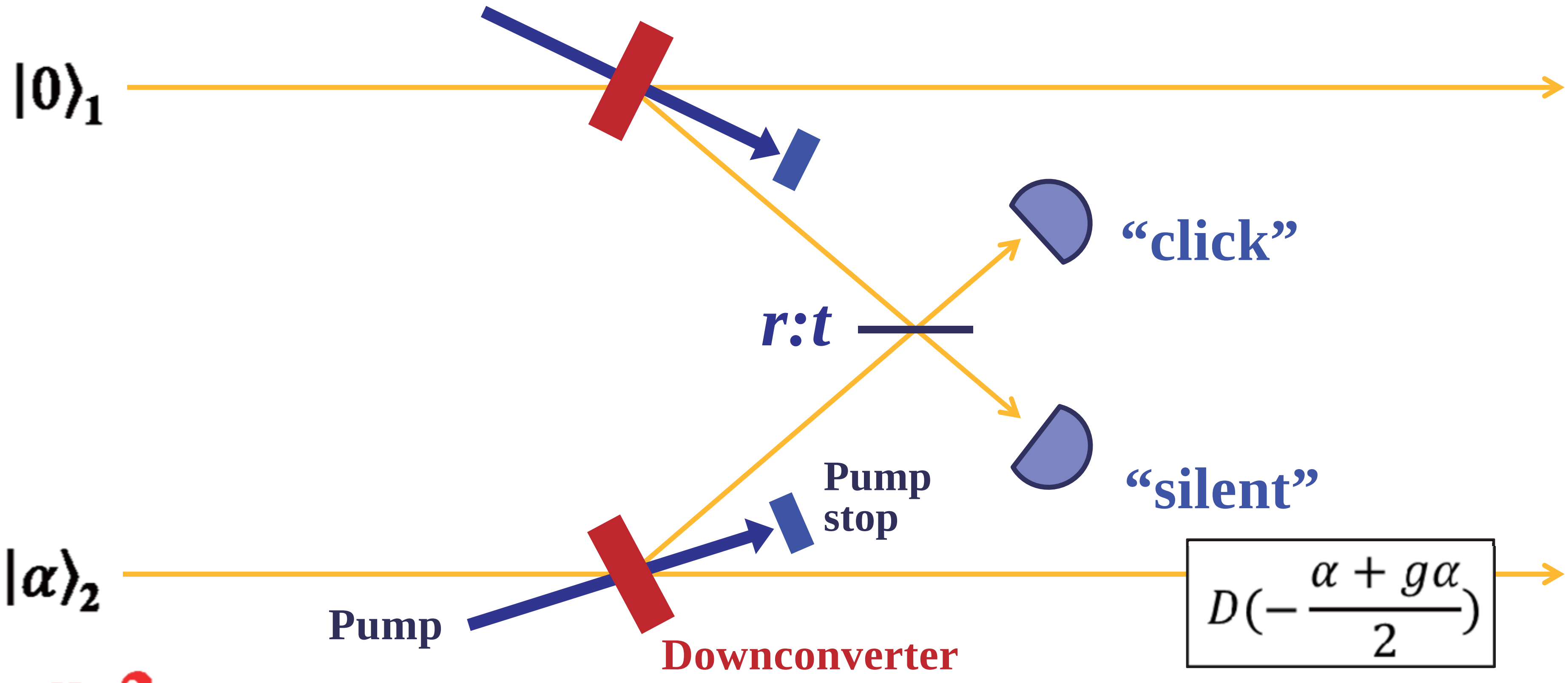
$$r a^{+} |0\rangle|\alpha\rangle + t|0\rangle a^{+} |\alpha\rangle \approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle \quad \text{with } g>1$$

Generation of hybrid entanglement



$$\approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle$$

Generation of hybrid entanglement

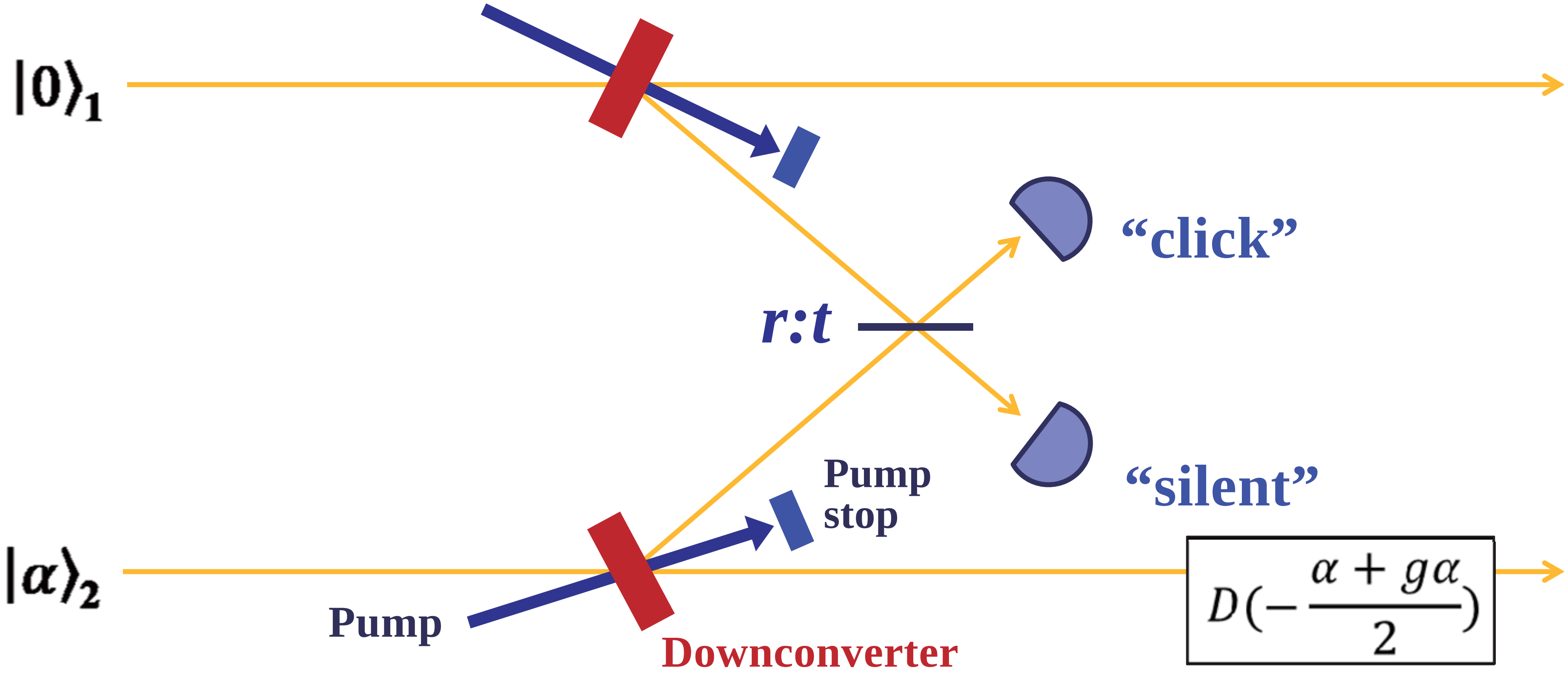


$\alpha=2$

$$\approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle \quad \rightarrow \; |0\rangle|\alpha_f\rangle + |1\rangle|-\alpha_f\rangle$$

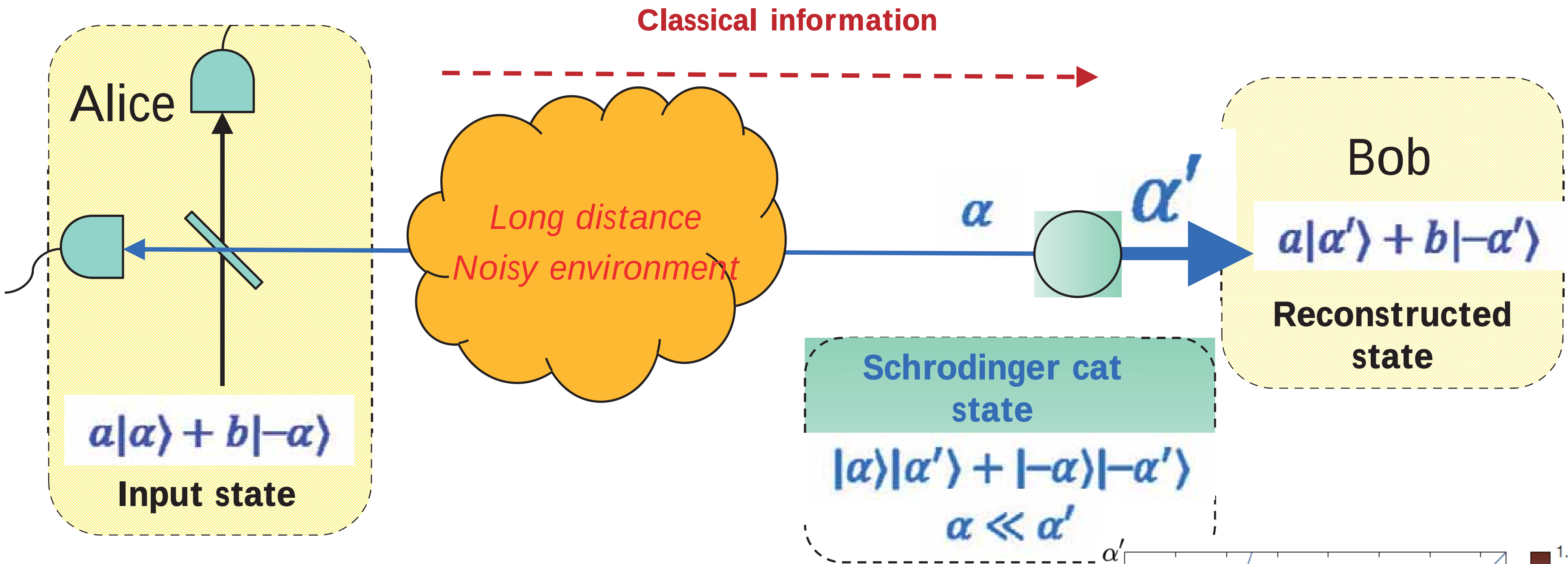
$F > 0.99$ for $\alpha_f = 0.21$

Generation of hybrid entanglement

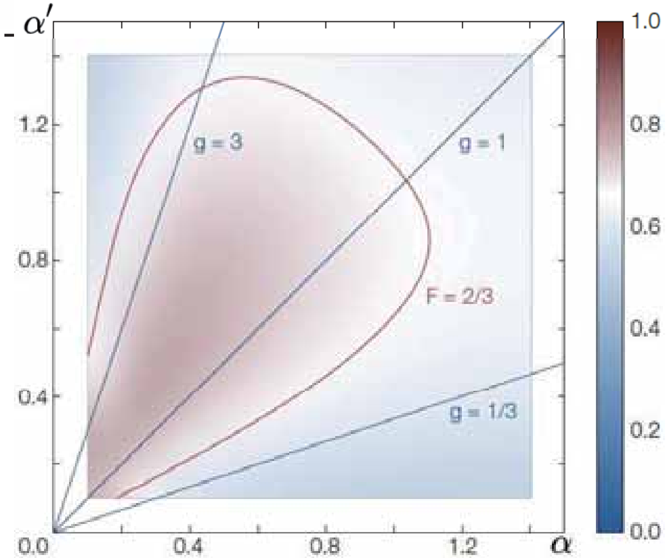


$$\approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle \quad \rightarrow \quad |0\rangle|\alpha_f\rangle + |1\rangle|-\alpha_f\rangle$$

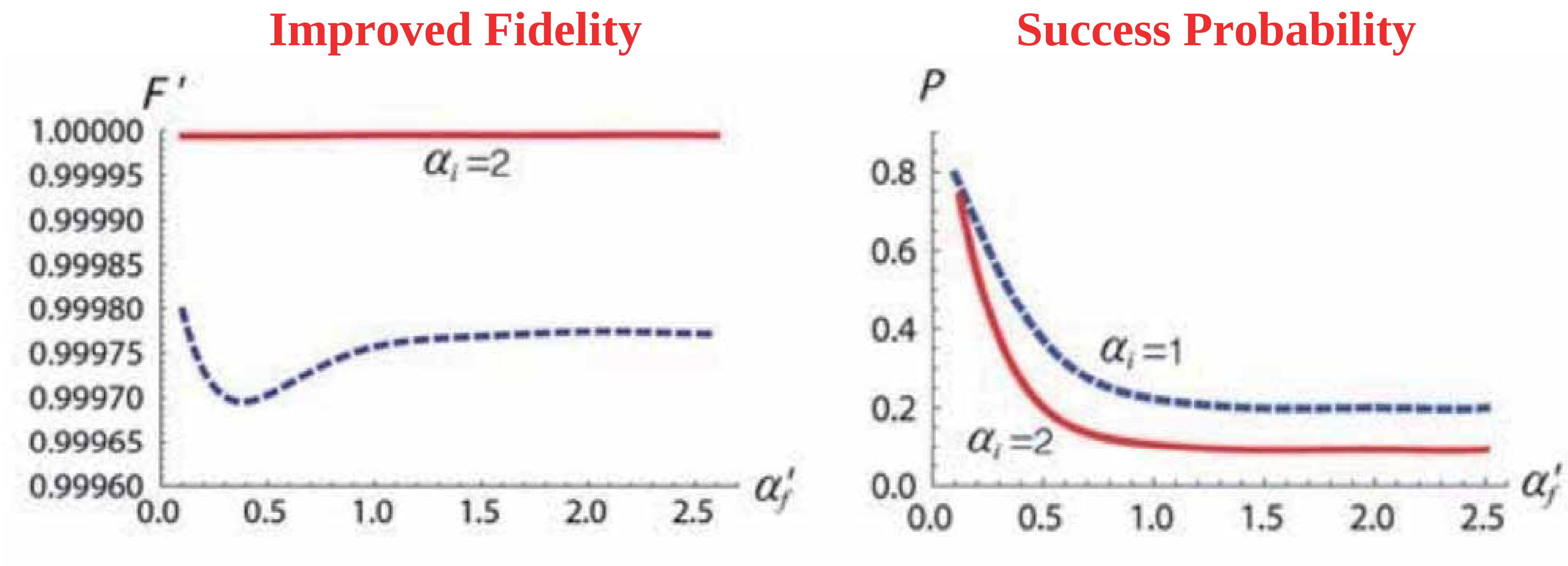
Tele-amplification of coherent-state qubits



J. S. Neergaard-Nielsen, Y. Eto, C.-W. Lee, H. Jeong and M. Sasaki, *Nature Photonics* (2013)



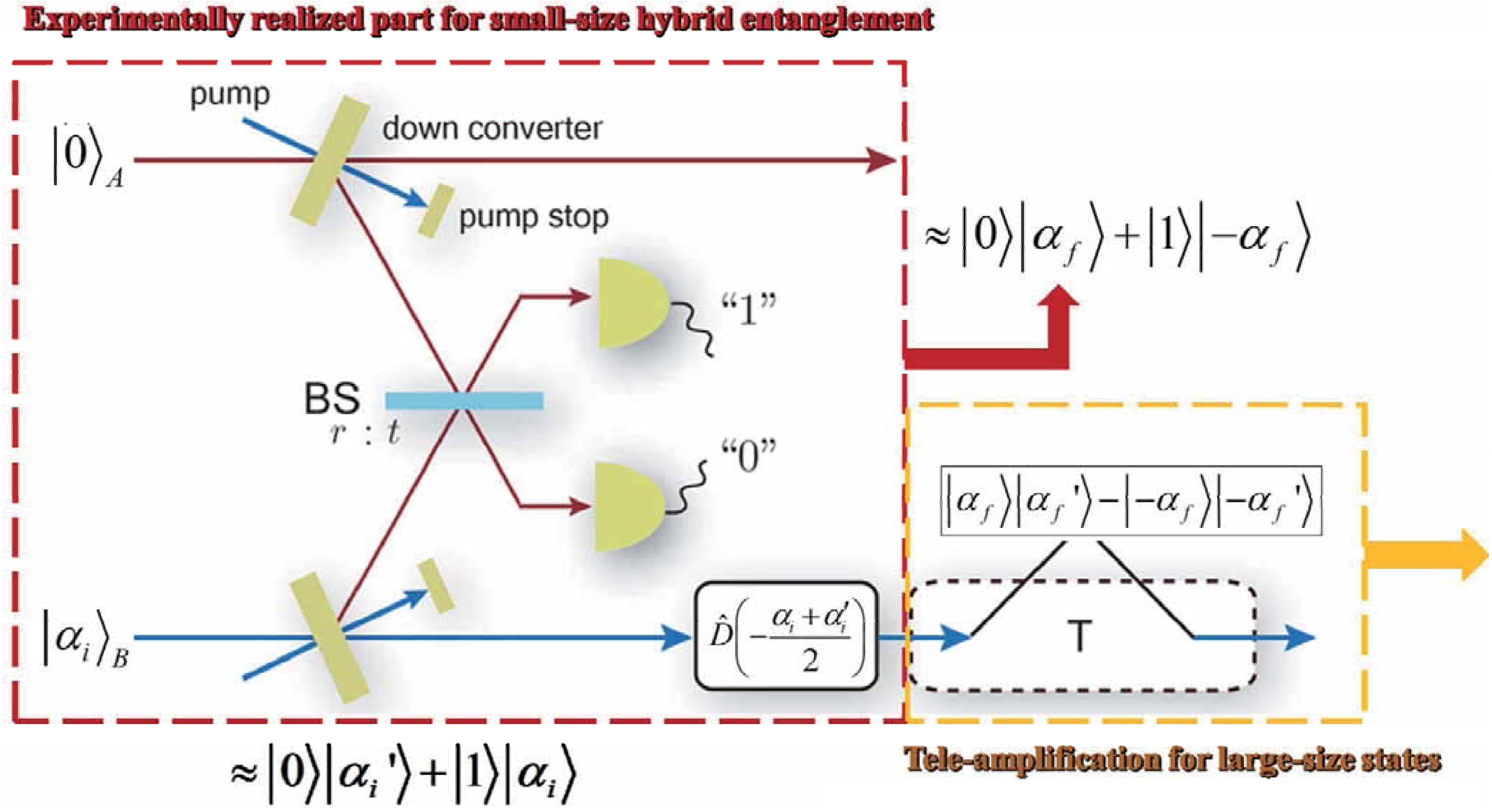
Tele-amplification of hybrid entanglement



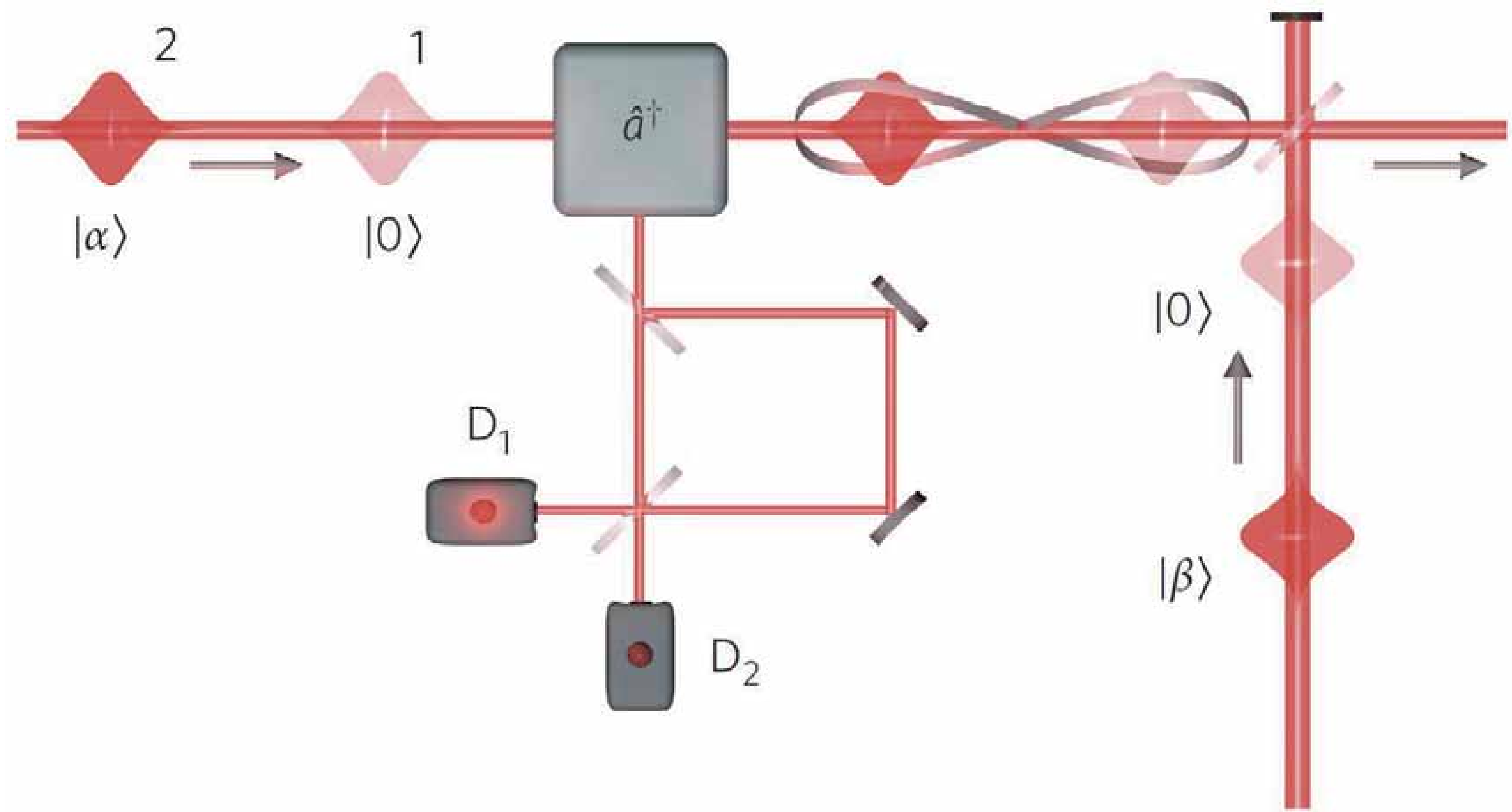
H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L.S. Costanzo, S. Grandi, T.C. Ralph & M. Bellini, Nature Photonics 8, 564 (2014).

Entangling quantum and classical states of light

Hyunseok Jeong, Alessandro Zavatta, Minsu Kang, Seung-Woo Lee, Luca S. Costanzo, Samuele Grandi, Timothy C. Ralph & Marco Bellini, Nature Photonics 8, 564 (2014).



Experimental setup



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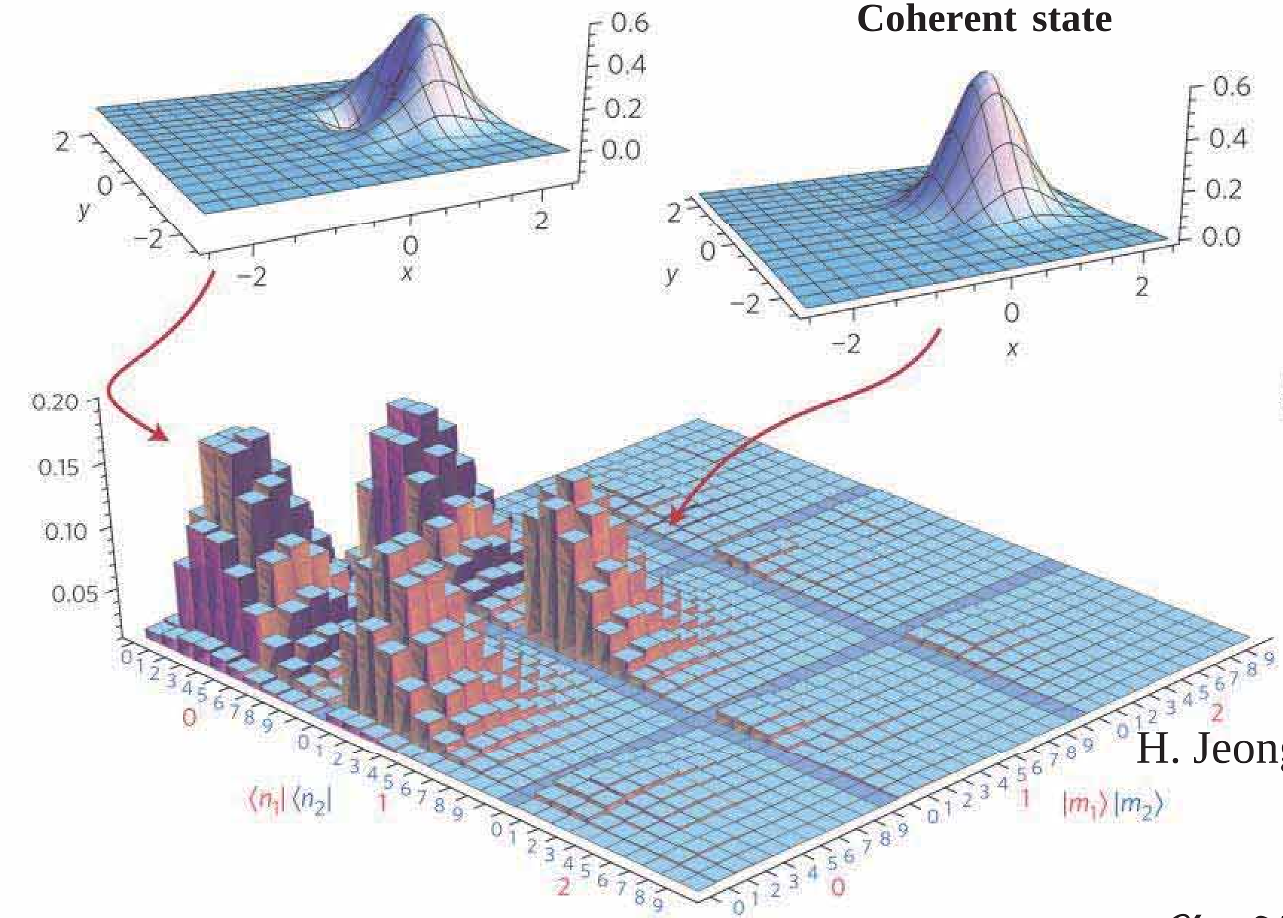
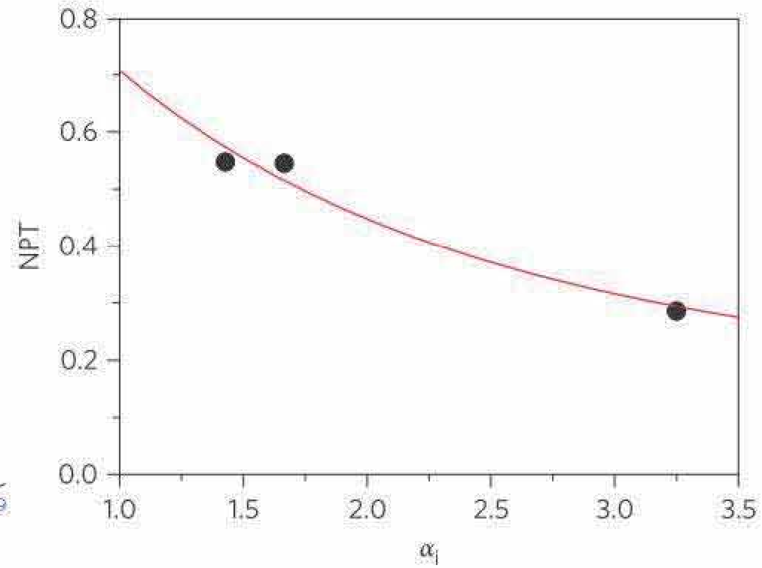
State tomography and entanglement

Photon added coherent state

Coherent state





H. Jeong *et al*. Nature Photonics 8, 564 (2014)

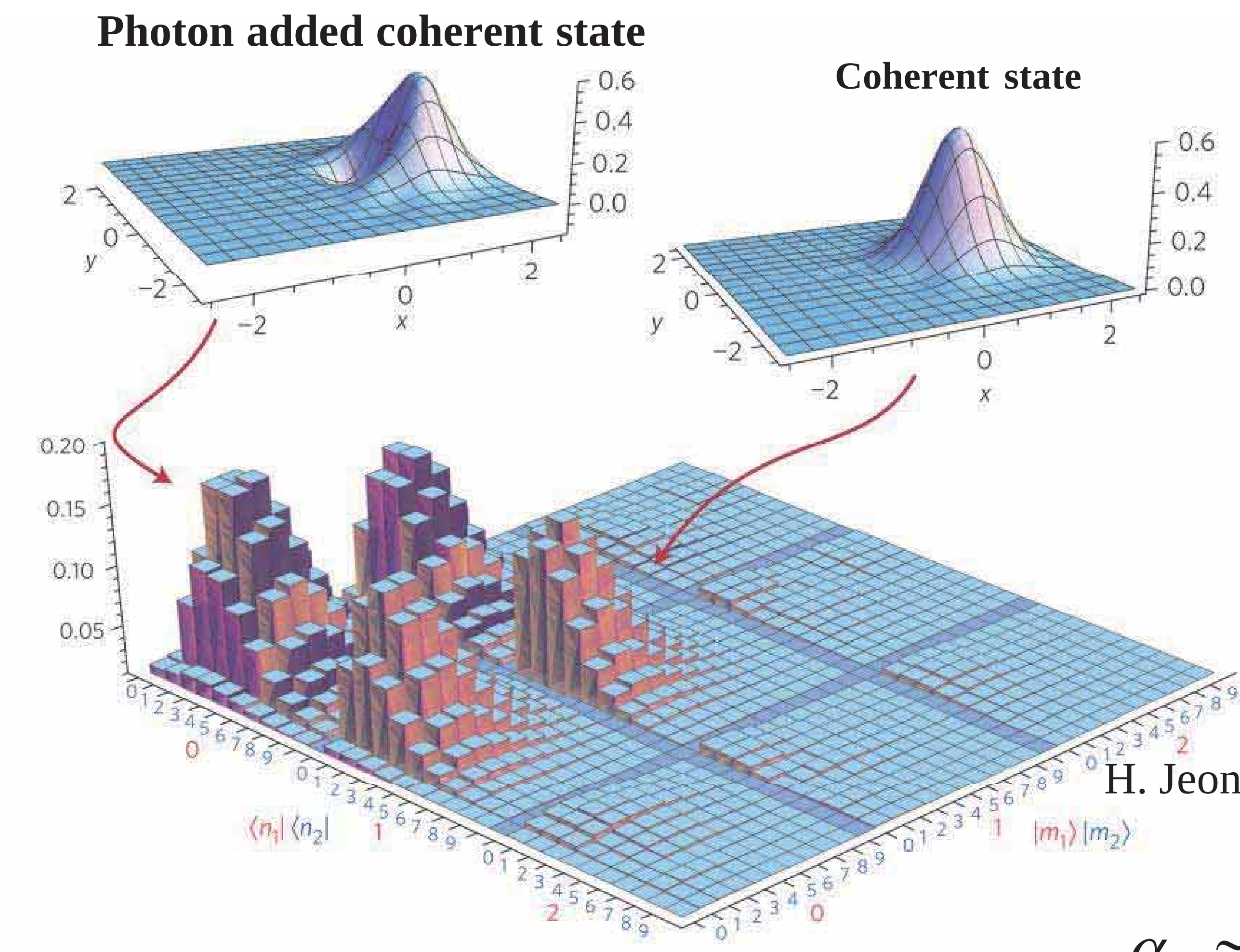
$\alpha_i \approx 1$ ⇒

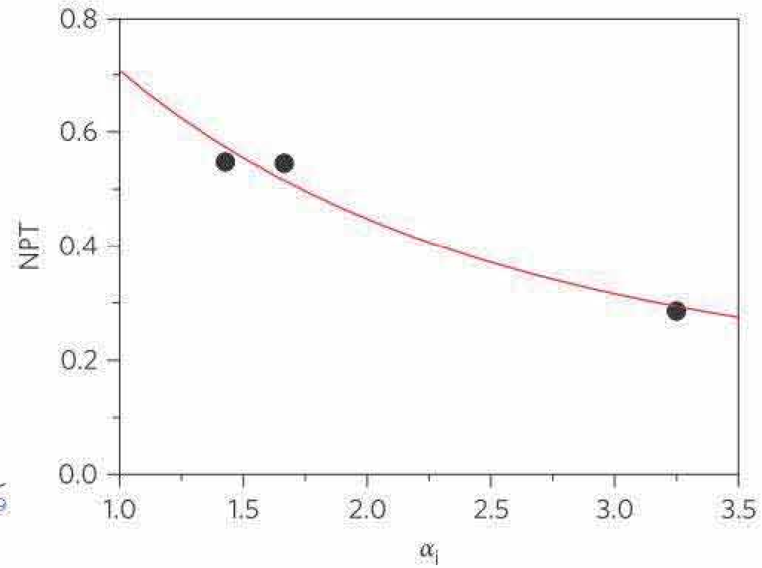
$\alpha_f \approx 0.31$

$F \approx 0.76$

$\mathrm{NPT} \approx 0.45$

State tomography and entanglement





H. Jeong *et al*. Nature Photonics 8, 564 (2014)

$\alpha_i \approx 1$

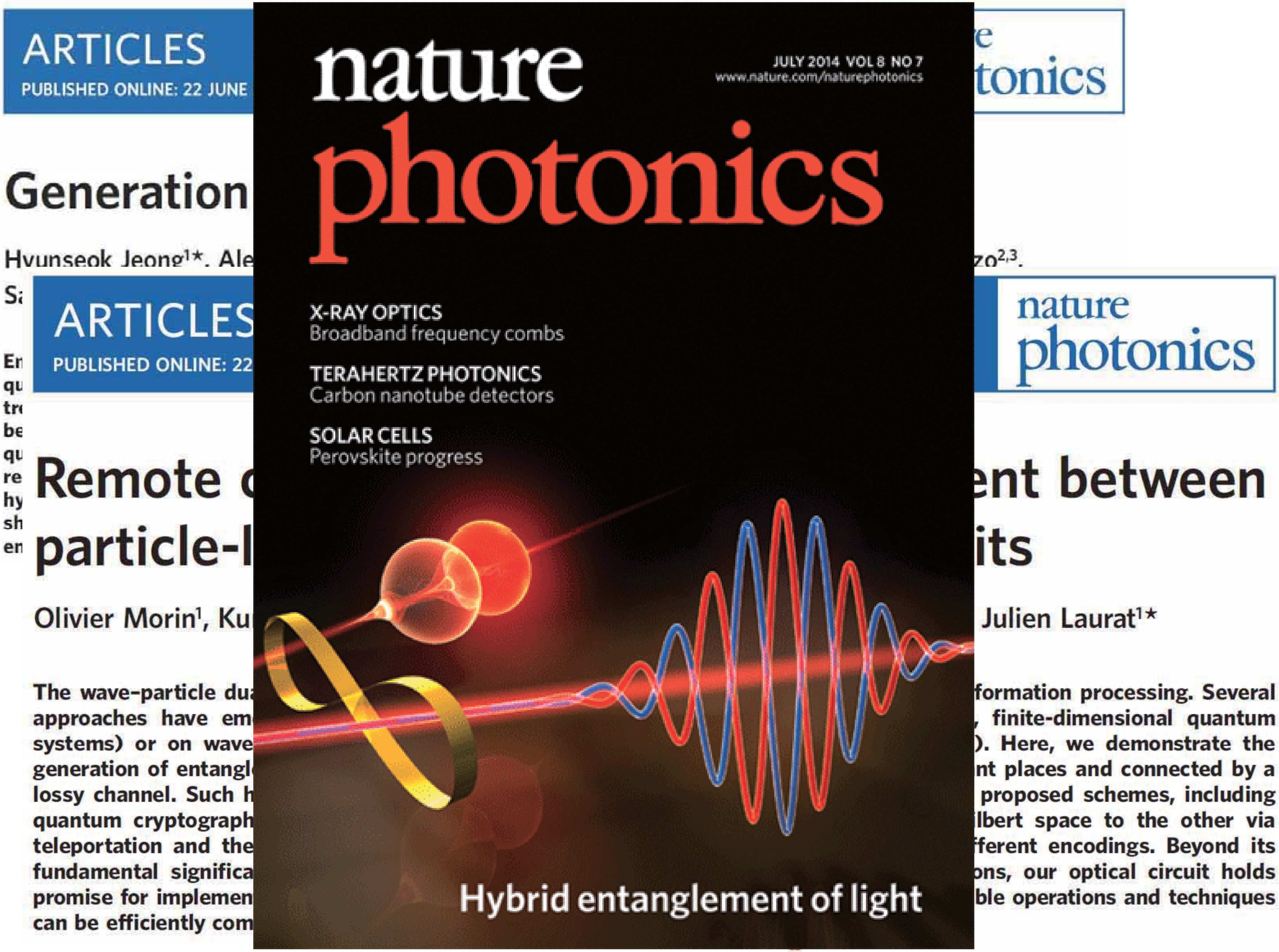
$\alpha_f \approx 0.31$

$F \approx 0.76$

$\mathrm{NPT} \approx 0.45$

$\alpha_f \approx 1$

Next challenge



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ent between
particle-l
its
Olivier Morin[1], Ku
Julien Laurat[1]*
The wave-particle du
formation processing. Several
approaches have em
, finite-dimensional quantum
systems) or on wave
). Here, we demonstrate the
generation of entangl
nt places and connected by a
lossy channel. Such h
proposed schemes, including
quantum cryptograph
lbert space to the other via
teleportation and the
fferent encodings. Beyond its
fundamental significa
ons, our optical circuit holds
promise for implemen
ble operations and techniques
can be efficiently com

Remarks

- We have studied hybrid entanglement of light:
  - ➢ Interesting – an analogy of Schrödinger's cat
  - ➢ Useful for optical quantum information processing
  - ➢ We've made it (in a small scale).

THANK YOU