LECTURE III
QUANTIFYING AND APPLYING NON-MARKOVIANITY

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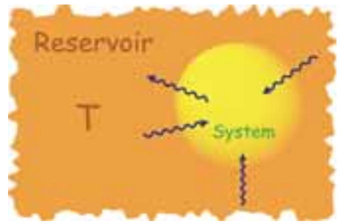

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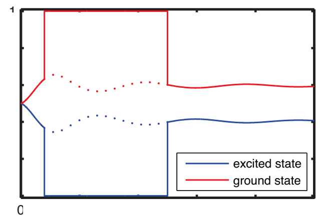
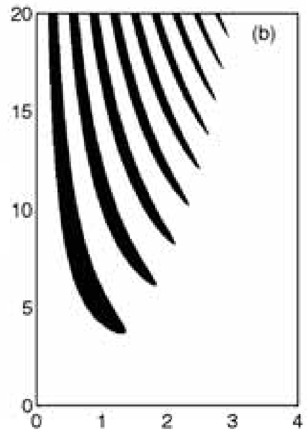


Lecture III



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IV Measures of non-Markovianity

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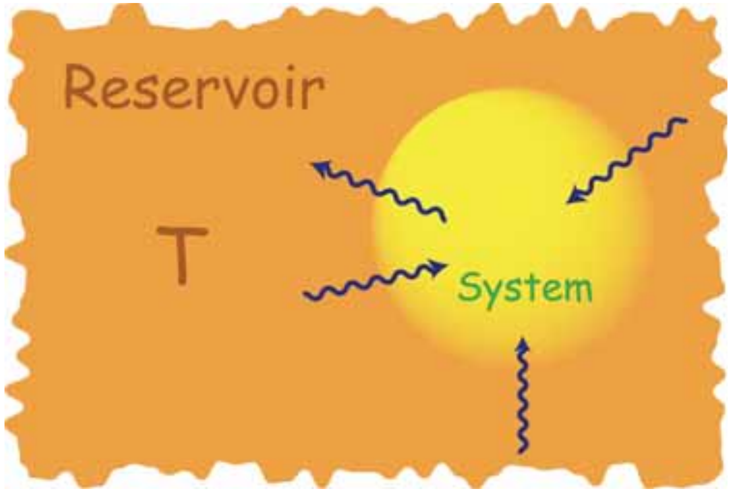
Background

Any realistic quantum system coupled to its environment



- The open system exchanges energy and information with its environment
- Thermalization
- Simple example: 2-level atom on excited state coupled to vacuum electromagnetic field (T=0) - spontaneous emission of a photon

Consequence: decoherence

- Quantum systems lose their quantum properties (e.g. quantum superpositions lost)
- Transition from quantum to classical world
- From Schrödinger equation and wave functions to master equations and density matrices

- Lindblad master equation (1975, Markovian, semigroup)

unitary part → $-i[H,\rho_S(t)]$; dissipator (non-unitary part) → $\mathcal{D}(\rho_S(t))$

$$\frac{d}{dt}\rho_S(t) = -i\,[H,\rho_S(t)] + \mathcal{D}(\rho_S(t))$$

$$\mathcal{D}(\rho_S) \equiv \sum_k \gamma_k \left(A_k \rho_S A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S - \frac{1}{2}\rho_S A_k^\dagger A_k \right)$$

decay rate → γ_k; jump operators → A_k

Background

Non-Markovian dynamics: quantum memory effects

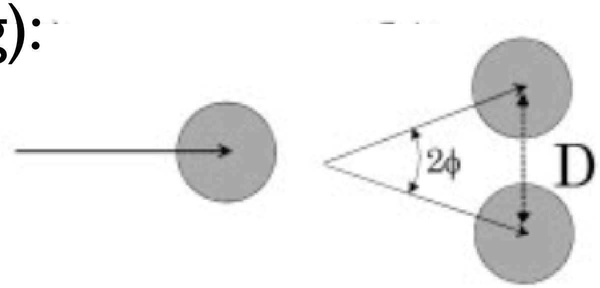
- Memoryless Markovian dynamics (e.g. exponential decay) is an approximation of more general theory of non-Markovian open system dynamics
- Classical vs quantum stochastic processes
- Eg. atom spontaneously emitting a photon: free space vs photonic band gap materials
- Is the loss of quantum properties a one-way process, role of memory effects?
- Can we control and exploit memory effects?

Background

Early experimental work on decoherence (Markovian):

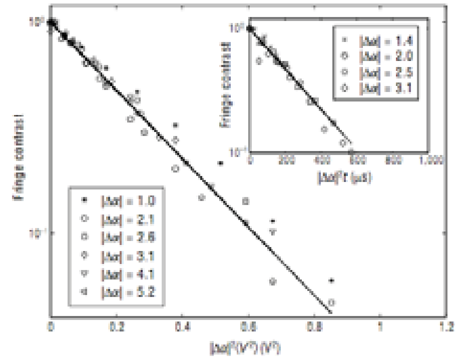
Haroche and co-workers, PRL 1996 (observing):

- Light field in a cavity, preparation of a Schrödinger's cat-like state
- Measuring decoherence by atoms



Wineland and co-workers, Nature 2000 (engineered):

- Trapped ions, superpositions of coherent states and Fock-states
- Engineering the decoherence by applying noise to trap electrodes



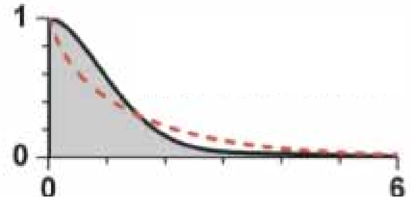
Often decoherence considered as an obstacle, for example to quantum information... however, can be also exploited...

Background

No universally agreed definition of non-Markovianity for quantum processes

Earlier non-Markovianity associated, e.g., to:

- Anything not following Lindblad-equation
- Non-exponential decay
- Memory-kernel equations
- Negative decay rates...



$$\frac{d\rho(t)}{dt} = \int_0^t k(t')\mathcal{L}\rho(t-t')dt'$$

Is it possible to define and quantify non-Markovianity

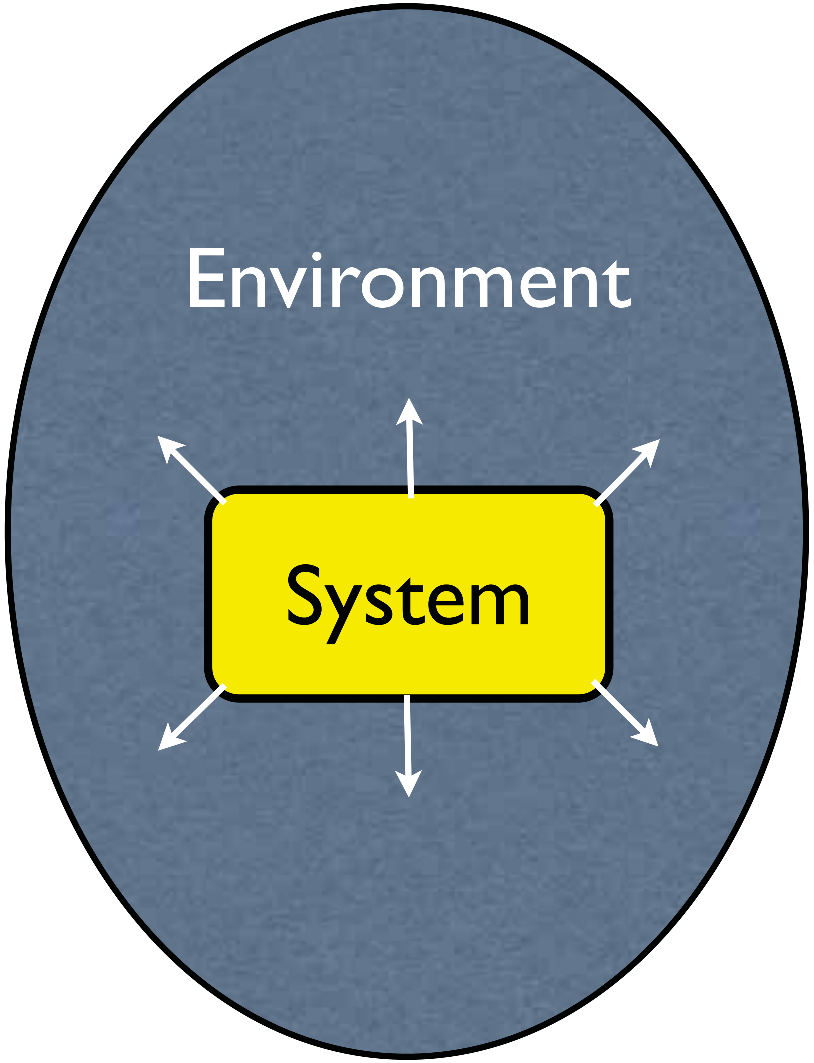
- Independently of the used mathematical formalism
- Intuitively clear interpretation
- Physically motivated approach (instead of formal mathematical approach)
- What does memory mean in quantum dynamics?

Our starting point: information flow and its direction between the system and environment...

Information flow: Markovian case

Cartoon of the information flow: Markovian case (no memory).

"Small" system.
"Large" environment.



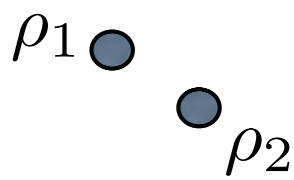
System-environment interaction
➜
System reaches steady state

Any initial state of the system leads to same steady state
➜
The system loses information on its initial state.

Markovian system: Information flow from the system to the environment.

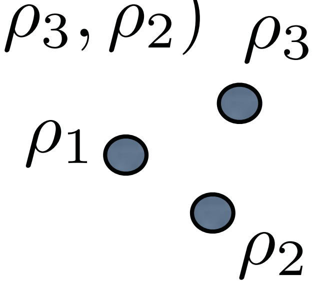
Distance measures for quantum states

Q: What is a suitable distance measure to quantify non-Markovianity?



Criteria for arbitrary distance measure: $L(\rho_1, \rho_2)$

- Symmetric? $L(\rho_1, \rho_2) = L(\rho_2, \rho_1)$
- Contractivity? $L(\Phi_t(\rho_1), \Phi_t(\rho_2)) \leqslant L(\rho_1, \rho_2)$
- Triangle equality? $L(\rho_1, \rho_2) \leqslant L(\rho_1, \rho_3) + L(\rho_3, \rho_2)$



Distance measures for quantum states

Choices (for example):

$$L(\rho_1, \rho_2) = L(\rho_2, \rho_1)$$

$$L(\Phi(\rho_1), \Phi(\rho_2)) \leqslant L(\rho_1, \rho_2)$$

$$L(\rho_1, \rho_2) \leqslant L(\rho_1, \rho_3) + L(\rho_3, \rho_2)$$

Relative entropy:

$$S(\rho_1 || \rho_2) = \mathrm{Tr}[\rho_1(\log \rho_1 - \log \rho_2)]$$

- Not symmetric
- Does not satisfy triangular inequality
- Singularities

Fidelity:

$$F(\rho_1, \rho_2) = \mathrm{Tr}\sqrt{\rho_1^{1/2} \rho_2 \rho_1^{1/2}}$$

- Does not satisfy triangular inequality

Hilbert-Schmidt distance:

$$\mathrm{HS}(\rho_1, \rho_2) = \sqrt{\mathrm{Tr}[(\rho_1 - \rho_2)^2]}$$

- Generally not contractive

Trace distance:

$$D(\rho_1, \rho_2) = \frac{1}{2}\mathrm{Tr}|\rho_1 - \rho_2|$$

- OK

Trace distance

Distance measure for two states ρ_1 and ρ_2:
Trace distance D:

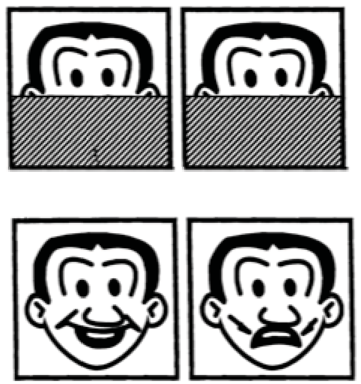
$$D(\rho_1, \rho_2) = \frac{1}{2}\mathrm{Tr}\,|\rho_1 - \rho_2| \qquad 0 \leq D \leq 1$$

- For identical states $D = 0$, for orthogonal states $D = 1$
- Physical interpretation: measure of distinguishability

The max probability to distinguish the two states is equal to $\frac{1}{2}(1 + D)$

- In terms of information:

  The larger D, the higher the probability to distinguish, more information which state we have



(Nielsen, Chuang)

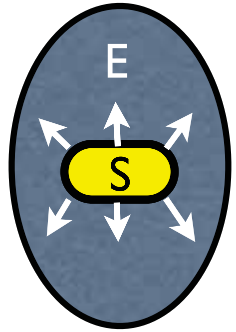
- Invariant under unitary transformations
- Contractive for all CPT-maps Φ

$$D(\Phi\rho_1, \Phi\rho_2) \leq D(\rho_1, \rho_2)$$

Information flow and distinguishability of states (2)

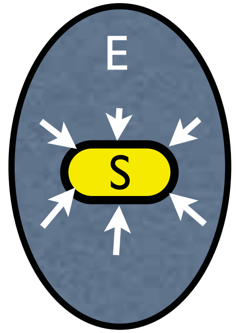
Notation:
$\sigma(t) = dD(t)/dt$ the change of the trace distance

In terms of the information flow:



$\sigma(t) < 0$
Decreasing trace distance:
More and more difficult to distinguish the different states.
➜
The information flows from the system to the environment.

$\sigma(t) > 0$
Increasing trace distance:
More and more easy to distinguish the different states.



➜
The information flows from the environment to the system.

The aim: quantify memory by calculating the reverse flow of information from the environment to the system.

Measure for non-Markovianity

Non-Markovianity: whenever period of increase of trace distance: backflow of information from the environment to the open system

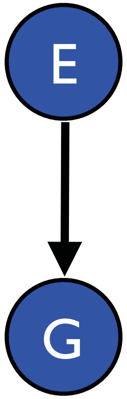
Allows to define a measure for non-Markovianity:

$$\mathcal{N}(\Phi) = \max_{\rho_{1,2}(0)} \int_{\sigma>0} dt\, \sigma(t, \rho_{1,2}(0)).$$

- Gives the total increase of the trace distance during the time evolution
- The total amount of information that has flown from the environment to the system during the time evolution.

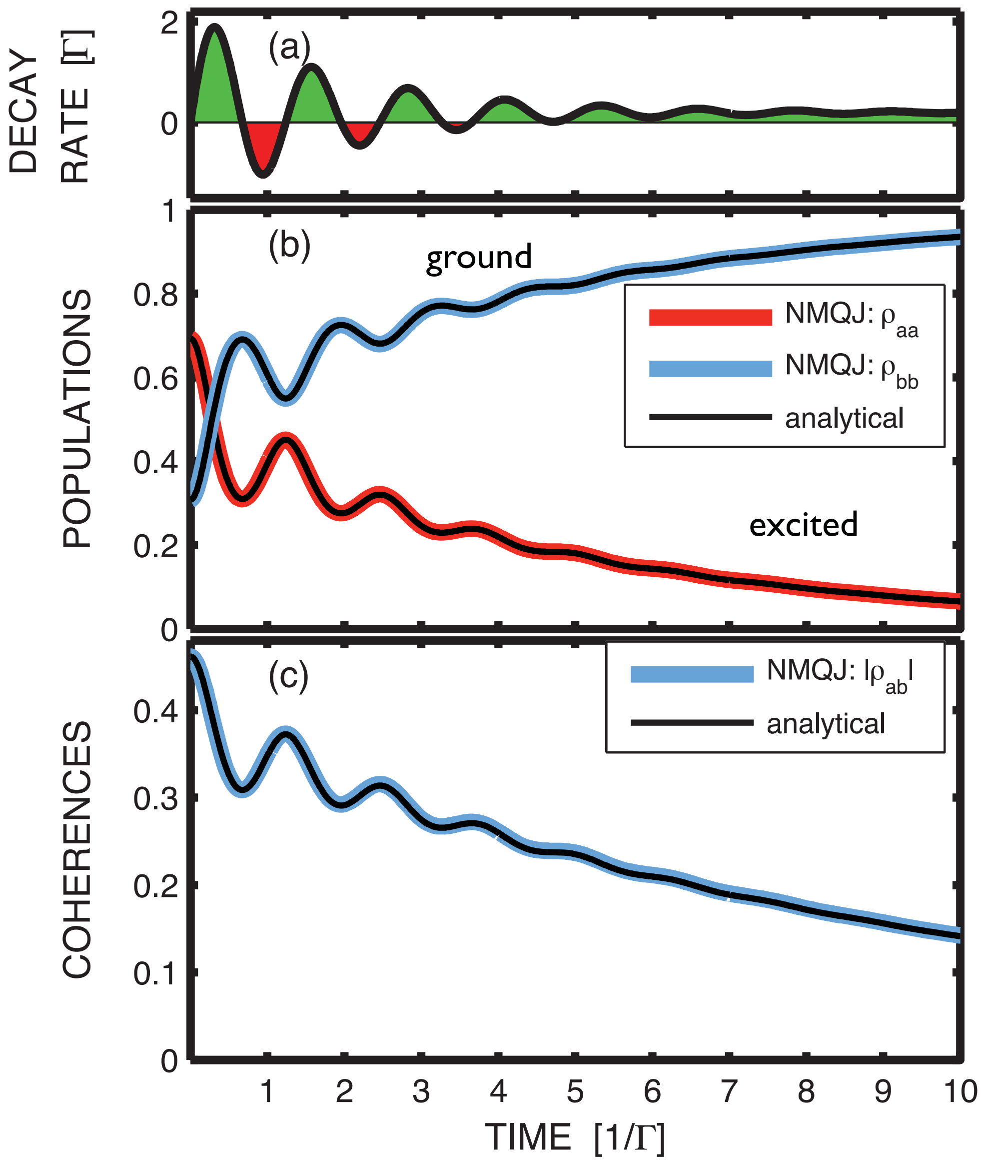
General definition, independent of the used formalism to solve the open system dynamics.

Example: 2-level atom



Two-level atom with Lorentzian spectral density

$$\dot{\rho}(t) = \frac{1}{i}\lambda(t)[\sigma_+\sigma_-,\rho(t)] + \Delta(t)\sigma_-\rho(t)\sigma_+ - \frac{1}{2}\Delta(t)\{\rho(t),\sigma_+\sigma_-\}.$$



- Decay rate oscillates having periods of negativity
- Excited state population oscillates

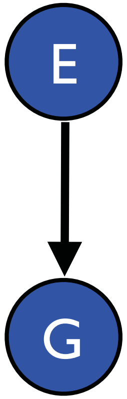
$$\rho_{aa}(t) = e^{-D_1(t)}\rho_{aa}(0)$$

- Decoherence - re-coherence cycles

$$\rho_{ab}(t) = e^{-D_1(t)/2}\rho_{ab}(0)$$

$$D_i(t) = \int_0^t ds\Delta_i(s)$$

Example: 2-level atom



+ electromagnetic environment with Lorentzian spectral density

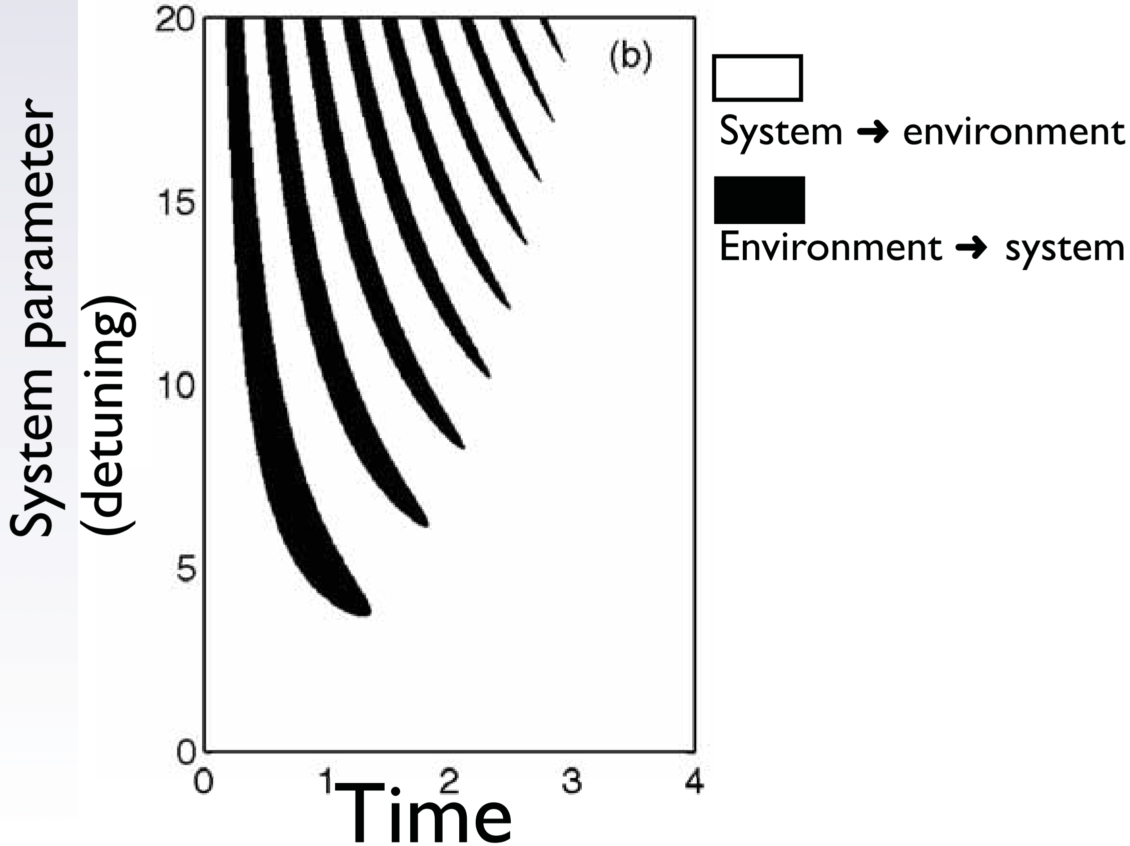
Change of the trace distance

$$\sigma(t, \rho_{1,2}(0)) = -\gamma(t)\exp[-\Gamma(t)],$$

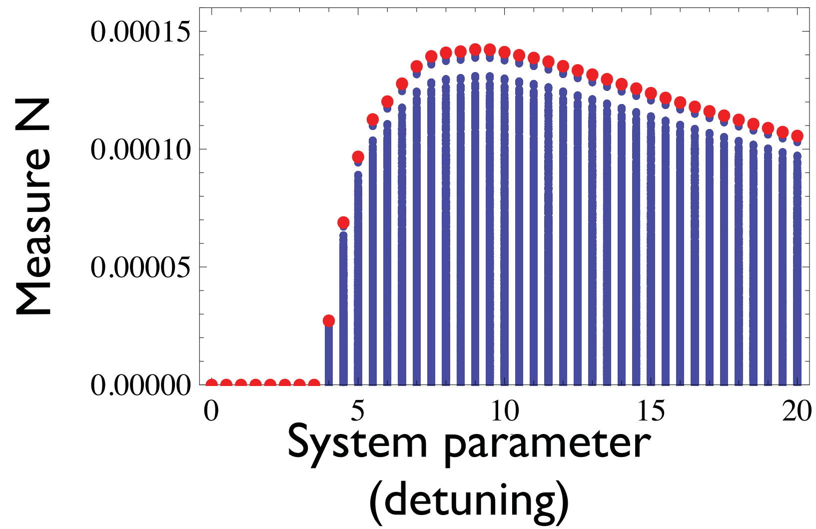
decay rate

The sign of the decay rate gives directly the direction of the information flow.

Information flow



For which system parameter strongest memory?

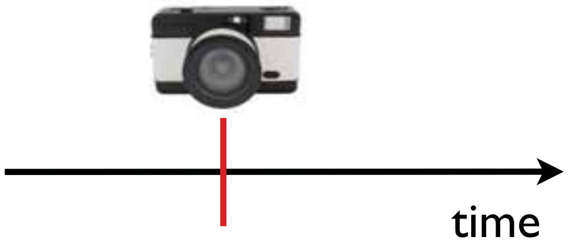


Breuer, Laine, Piilo
Phys. Rev. Lett. 103, 210401 (2009)

Other measures

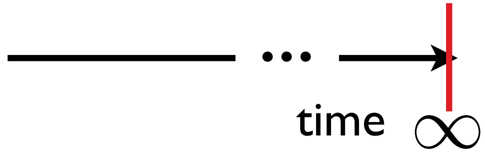
Wolf, Eisert, Cubitt, Cirac: PRL 2008

- Markovianity vs non-Markovianity from a snapshot of the evolution.



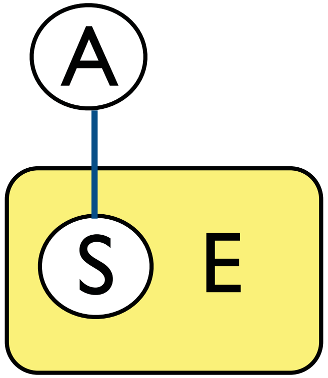
Chruscinski, Kossakowski, Pascazio: PRA 2010

- Asymptotic state dependence from initial conditions



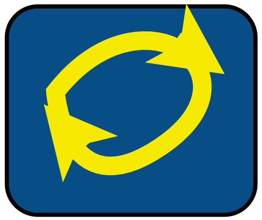
Rivas, Huelga, Plenio: PRL 2010

- Entanglement based measure (ancilla, system)
- Non-divisibility based measure
  $\Phi(t,0) = \Phi(t,s)\Phi(s,0)$



Our view: PRL 2009:

- Recycling of info between S and E, quantify backflow



Xiao-Ming Lu, Xiaoguang Wang, C. P. Sun: PRA 2010

- Quantum Fisher Information based measure

Bylicka, Chruscinski, Maniscalco: arXiv1301:2585

- Definition based on channel capacity

...also others exist

Other measures

Comparing to Wolf, Eisert, Cubitt, Cirac: PRL 2008

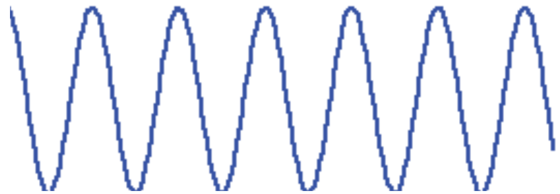
- Markovianity vs non-Markovianity from a snapshot of the evolution.
- Non-zero value when the state can have reached by non-Markovian evolution, zero when both Markovian and non-Markovian possible

Pure dephasing and rephasing evolution:

$$\rho_{++}(t) = \rho_{++}(0) \qquad \rho_{--}(t) = \rho_{--}(0)$$

$$\rho_{-+}(t) = g(t)\rho_{-+}(0) \qquad \rho_{+-}(t) = g(t)\rho_{+-}(0)$$

$$g(t) = \frac{1}{2}\left(1 + \cos^2(\omega t)\right)$$



Snapshot measure = 0:

at all points of time, the map can be written as an element of Markovian semigroup ⟶ snapshot measure = 0

$$\Phi(t_0, 0) = \exp(\mathcal{L}) \qquad \mathcal{L}\rho = \frac{\Gamma}{2}(\sigma_3\rho\sigma_3 - \rho) \quad \Gamma = -\ln g(t_0)$$

Our trace distance measure:

Laine, Piilo, Breuer: Physical Review A 81, 062115 2010

$\mathcal{N} = \infty$ (decoherence-recoherence-decoherence cycles till time ∞)

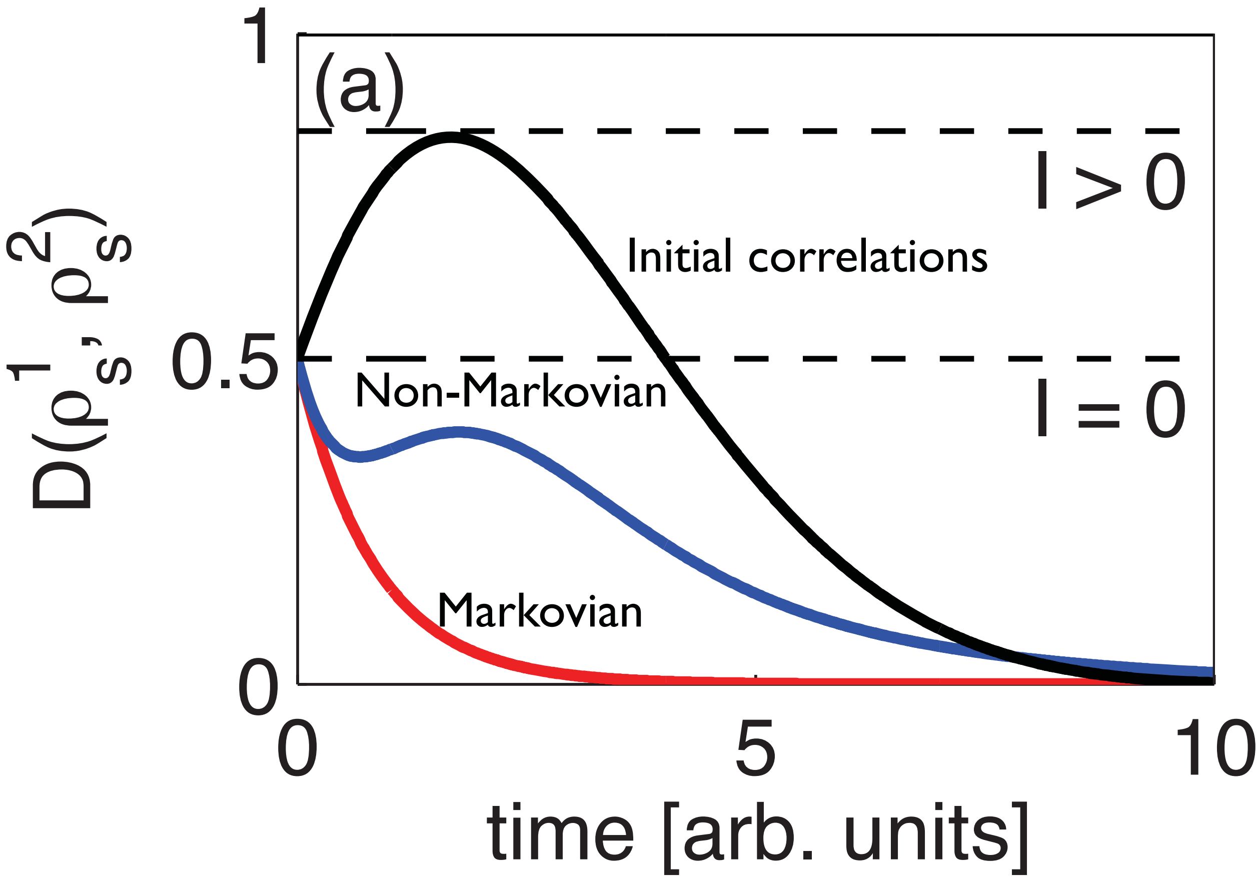
Other measures

Semigroup	Divisibility	Information backflow (trace distance meas.)
$\Phi(t_1+t_2,0)=\Phi(t_1,0)\Phi(t_2,0)$	$\Phi(t,0)=\Phi(t,s)\Phi(s,0)$	$\mathcal{N}(\Phi)$

- Semigroup: constant decay rates
- Semigroup broken when decay rates become time dependent
- Divisibility broken when at least one of the decay rates becomes negative
- Trace distance measure: backflow of information
- Can break divisibility but no backflow of information

Markovian - non-Markovian - initial correlations

General classification based on the information flow



Laine, Piilo, Breuer
Europhys. Lett. 92, 60010 (2010)

V Applications of non-Markovianity

- Controlling the memory effects
- Nonlocal memory effects
- Non-Markovian probes

Non-Markovian dephasing

$$\frac{d\rho(t)}{dt} = -i\frac{\epsilon(t)}{2}\left[\sigma_z, \rho\right] + \frac{\gamma(t)}{2}\left(\sigma_z \rho \sigma_z - \rho\right)$$

- Time evolution and the map: two state system $|H\rangle, |V\rangle$

$$\begin{aligned} \rho_{\mathrm{H,H}}(t) &= \rho_{\mathrm{H,H}}(0), \quad \rho_{\mathrm{V,V}}(t) = \rho_{\mathrm{V,V}}(0), \\ \rho_{\mathrm{H,V}}(t) &= \kappa^*(t)\rho_{\mathrm{H,V}}(0), \quad \rho_{\mathrm{V,H}}(t) = \kappa(t)\rho_{\mathrm{V,H}}(0) \end{aligned}$$

- Connection between the decoherence function $\kappa(t)$ and the rates in the master equation

$$\kappa(t) = \exp\left(-\int_0^{t'} \gamma(t') + i\epsilon(t')dt'\right)$$

$$\epsilon(t) = -\Im\left[\frac{\dot{\kappa}(t)}{\kappa(t)}\right], \qquad \gamma(t) = -\Re\left[\frac{\dot{\kappa}(t)}{\kappa(t)}\right]$$

Non-Markovian dephasing

$$\frac{d\rho(t)}{dt} = -i\frac{\epsilon(t)}{2}[\sigma_z, \rho] + \frac{\gamma(t)}{2}(\sigma_z \rho \sigma_z - \rho)$$

Single photons in dephasing reservoir

- The system: polarization states of the photon: $|H\rangle$ $|V\rangle$
- Environment: frequency degrees of freedom $\{|\omega\rangle\}_{\omega\in\mathbb{R}}$ $|\omega_i\rangle$
- Dephasing by quartz plates (birefringent material)

Markovian - non-Markovian transition

- Initial total system states

$$|\psi_{1,2}(0)\rangle = |\varphi_{1,2}\rangle \otimes |\chi\rangle$$

- Initial open system states (polarization)

$$|\varphi_{1,2}\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$$

- Initial environmental states (frequency distribution)

$$|\chi\rangle = \int d\omega \underbrace{f(\omega)}_{\text{Modified by the FP cavity}}|\omega\rangle$$

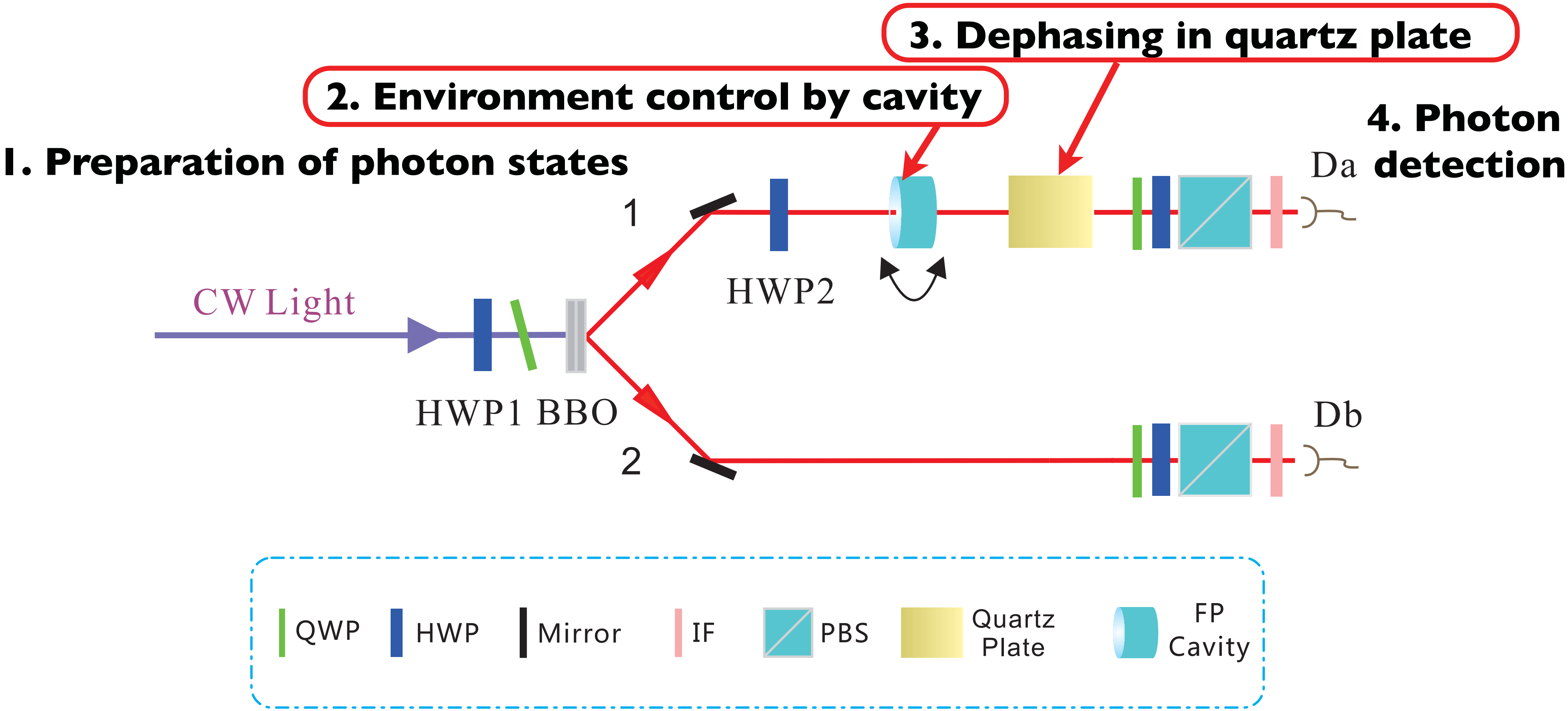
- Total system evolution in the quartz plate

$$U(t)|\lambda\rangle \otimes |\omega\rangle = e^{in_\lambda \omega t}|\lambda\rangle \otimes |\omega\rangle$$

- H and V acquire different phase due to the different refraction indices (birefringent quartz plate)

Liu, L. Li, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
Nature Physics 7, 931-934 (2011).

Experimental setup

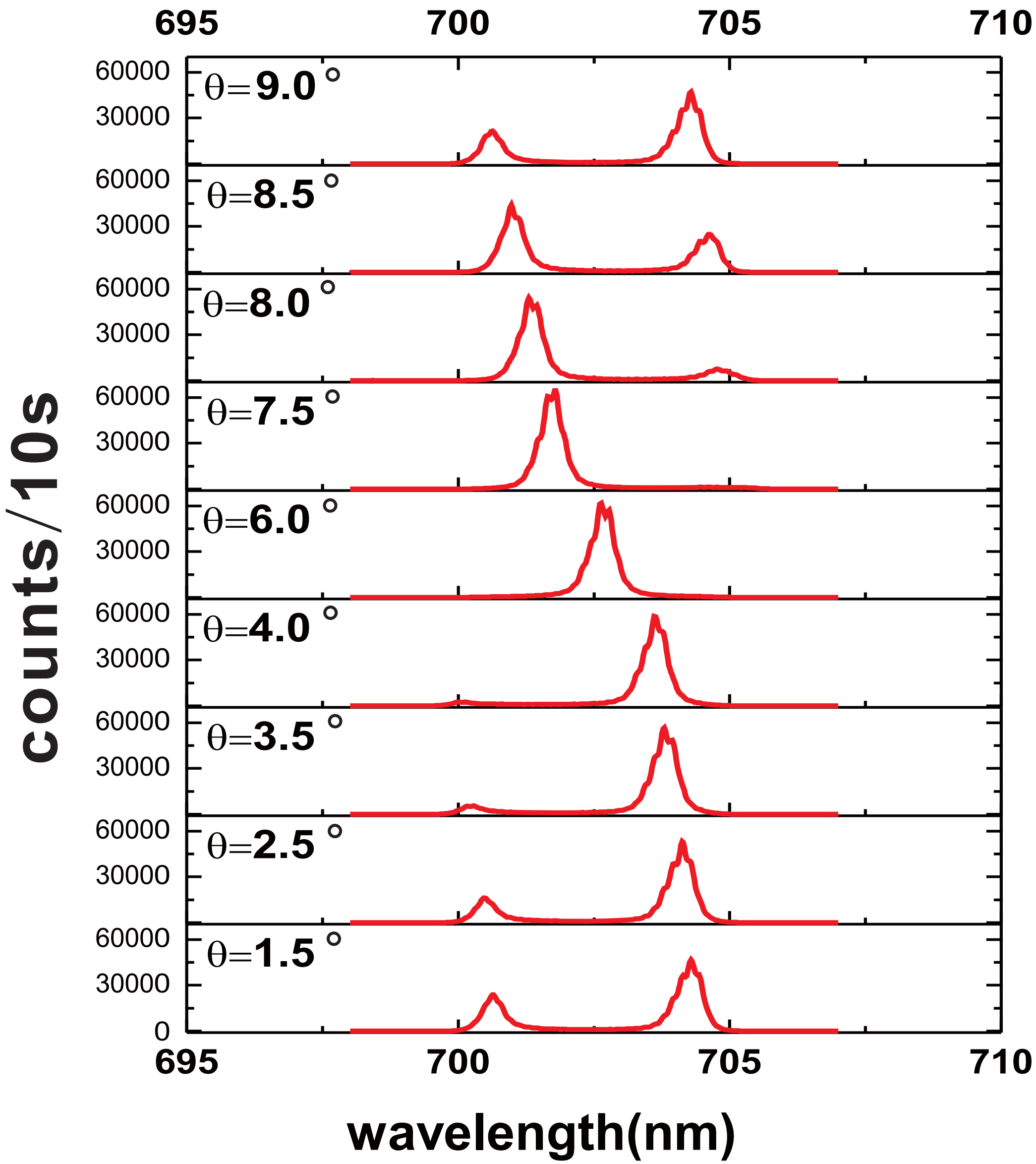


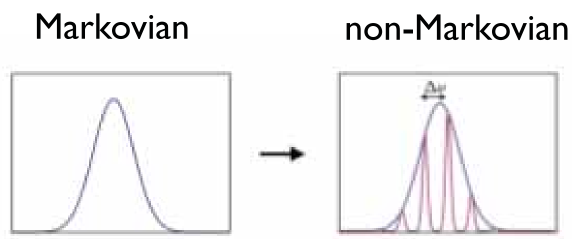
- The tilting of the FP cavity modifies the frequency spectrum.

Liu, L. Li, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
Nature Physics 7, 931-934 (2011).

Markovian - non-Markovian transition experiment

Environment control - frequency spectrum





Tilting of the cavity modifies the initial environmental state

Non-Markovianity

- The optimal trace distance $D(\rho_1(t), \rho_2(t)) = |\kappa(t)|$.

$$D(\rho_1(t), \rho_2(t)) = \sqrt{a^2 + |\kappa(t)b|^2}$$
$$b = \rho_1^{12}(0) - \rho_2^{12}(0)$$
$$a = \rho_1^{11}(0) - \rho_2^{11}(0)$$

- Two Gaussian peaks, relative weights

$$A_1 = \frac{1}{1+A}, \; A_2 = \frac{A}{1+A}$$

- Decoherence function

$$|\kappa(t)| = \frac{e^{-\frac{1}{2}\sigma^2(\Delta n t)^2}}{1+A}\sqrt{1 + A^2 + 2A\cos(\Delta\omega \cdot \Delta n t)}$$

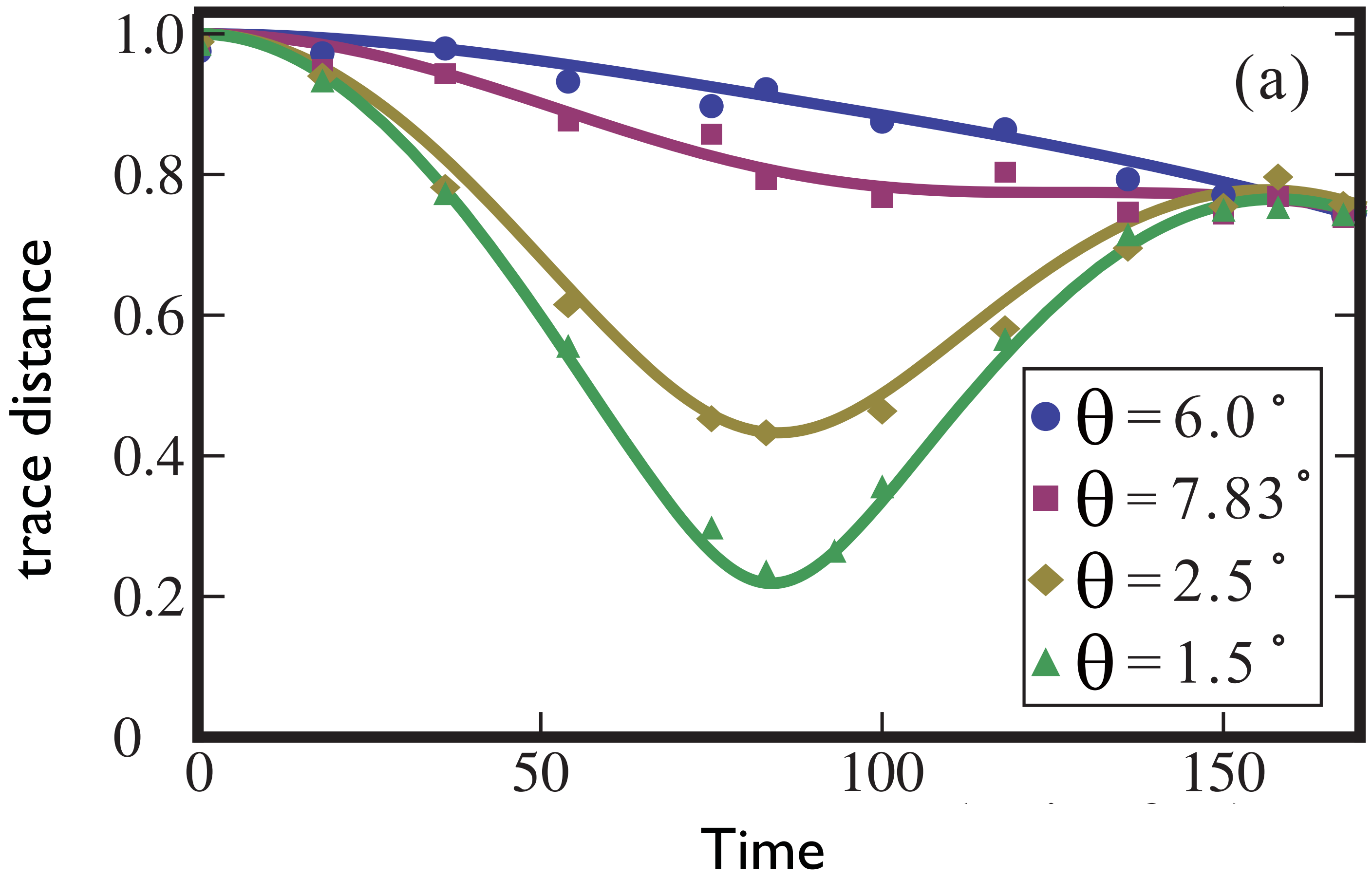
- One peak $A = 0$

$|\kappa(t)|$ monotonically decreasing

Liu, L. Li, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
Nature Physics 7, 931-934 (2011).

Markovian - non-Markovian transition experiment

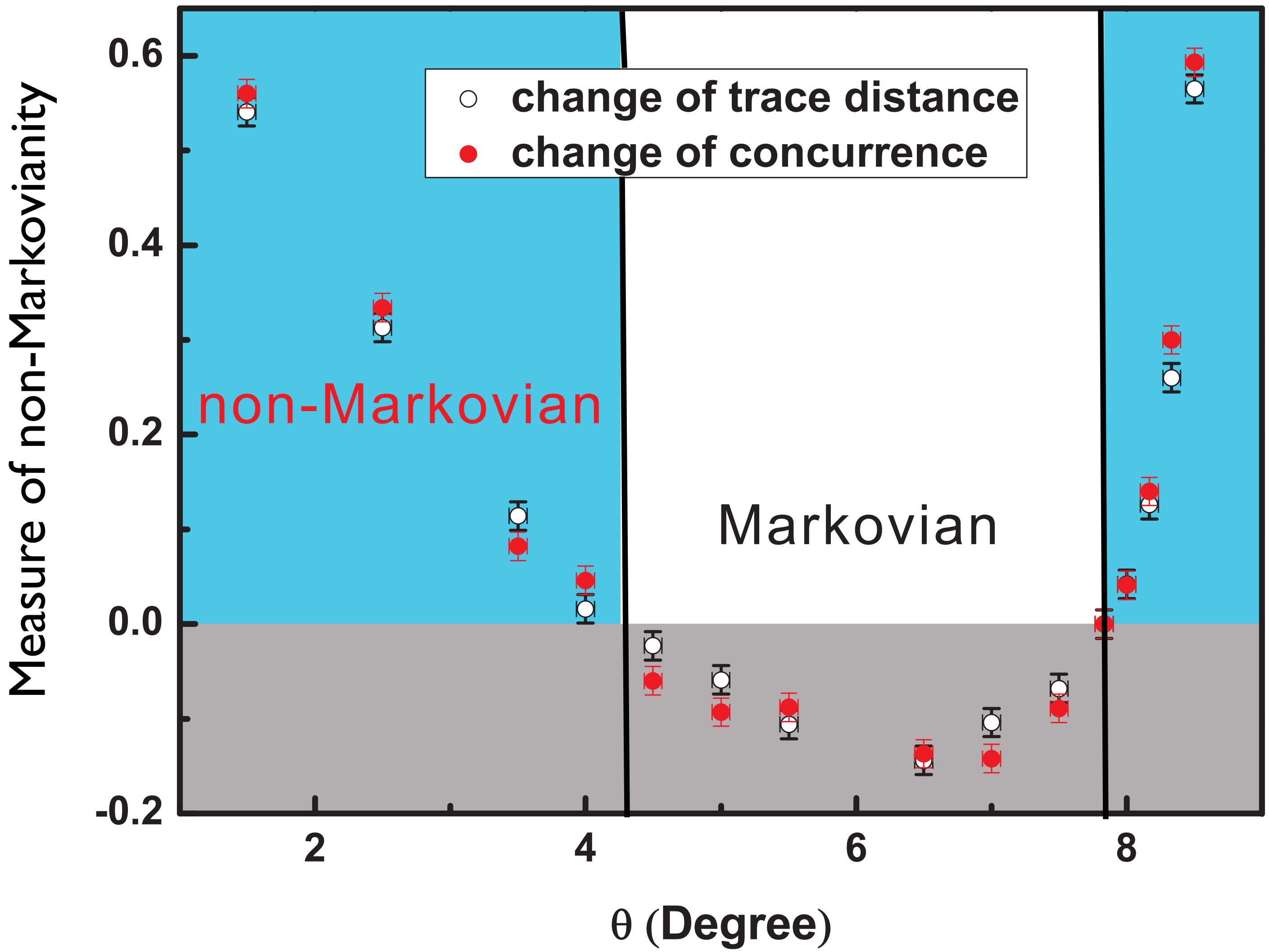
Trace distance dynamics - transition from monotonic to non-monotonic behavior



points - experiments
solid line - theory

Liu, L. Li, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
Nature Physics 7, 931-934 (2011).

Markovian - non-Markovian transition



Experimental control on the amount and direction of the information flow between the system and the environment.

Liu, L. Li, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
Nature Physics 7, 931-934 (2011).

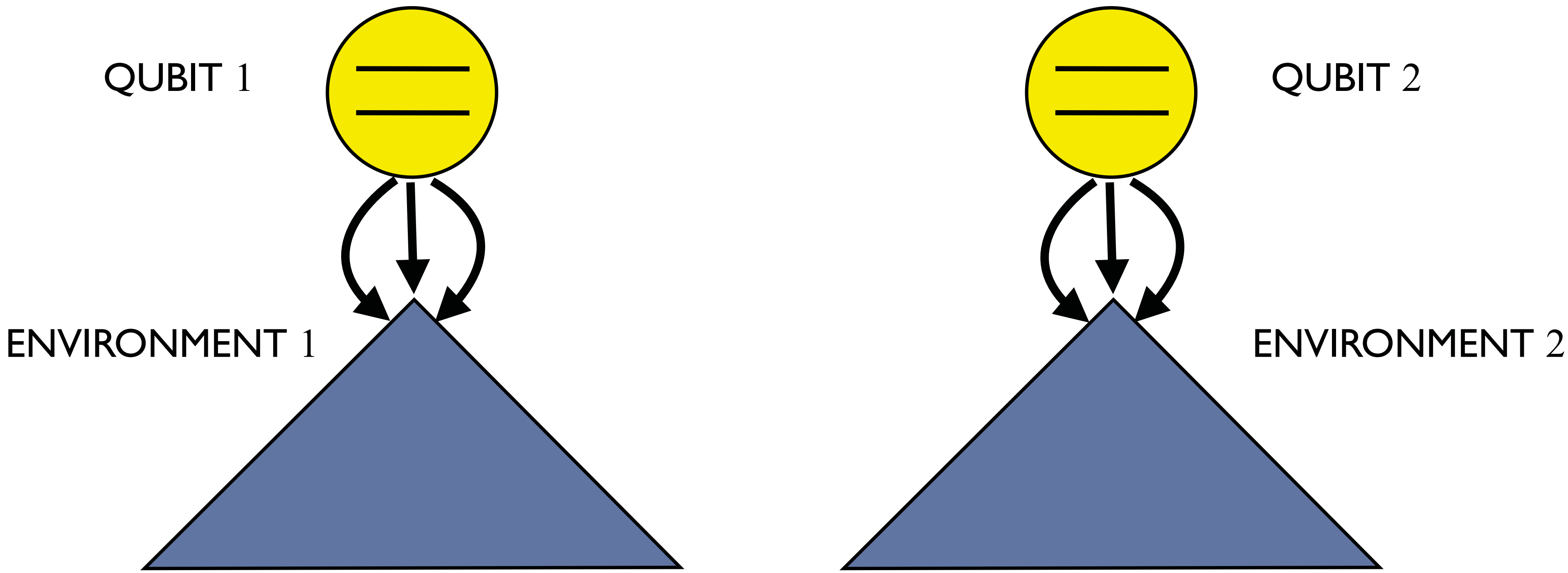
Nonlocal memory effects and a non-Markovian quantum probe

Laine, Breuer, Piilo, C.-F. Li, Guo,
Phys. Rev. Lett. 108, 210402 (2012)

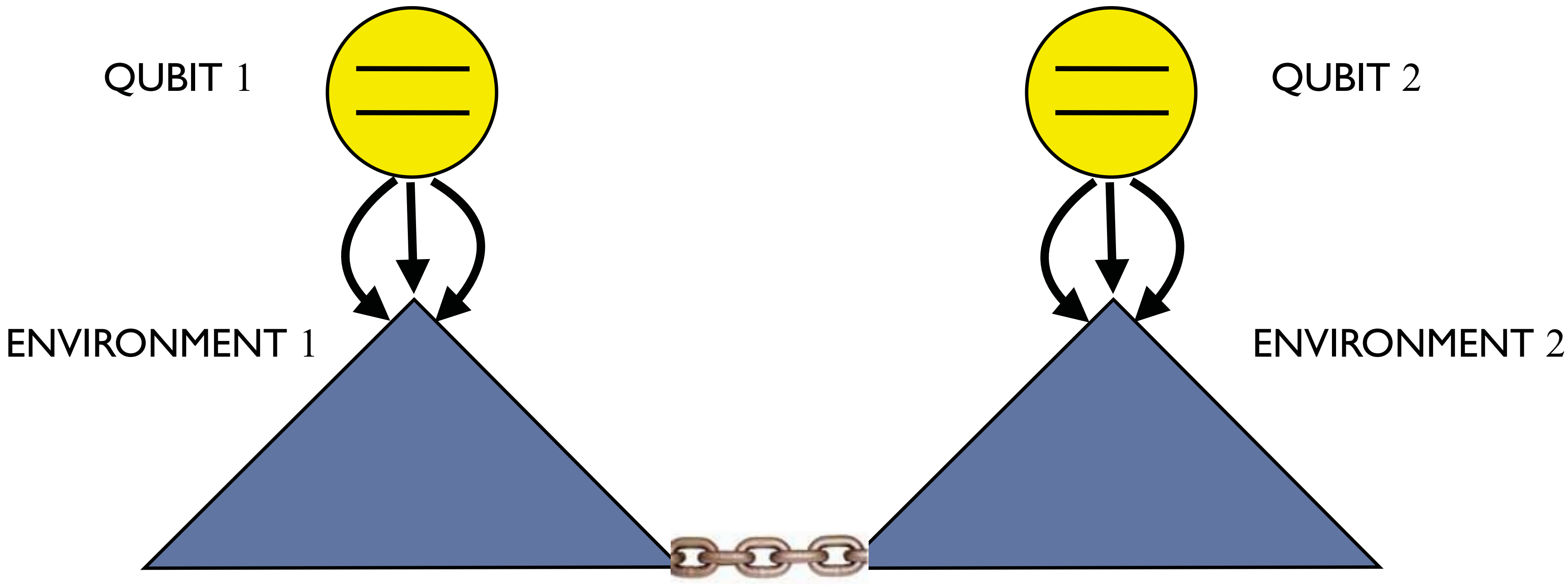
Liu, Cao, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
”Photonic realization of nonlocal memory effects and non-Markovian quantum probes”,
arXiv:1208.1358

Cartoon of the system

2-qubits interacting with their local environments



2-qubits interacting with their local environments



- What happens when the environments are initially correlated?

$$\rho_S^{12}(t) = \Phi_{12}(t)(\rho_S^{12}(0))$$
$$= \mathrm{tr}_E\left[\left(U_1(t)\otimes U_2(t)\right)\rho_S^{12}(0)\otimes\rho_E^{12}(0)\left(U_1^\dagger(t)\otimes U_2^\dagger(t)\right)\right]$$

Nonlocal memory effects

- Considered open system: two photons
  - polarizations H,V
  - initial pure state

$$|\psi_{12}\rangle = a\,|HH\rangle + b\,|HV\rangle + c\,|VH\rangle + d\,|VV\rangle$$

- Environment: frequency degrees of freedom

$$|\chi\rangle = \int d\omega_1 d\omega_2\, g(\omega_1,\omega_2)\, |\omega_1,\omega_2\rangle$$

  - joint probability distribution

$$P(\omega_1,\omega_2) = |g(\omega_1,\omega_2)|^2$$

- Initial system-environment product state

$$|\Psi(0)\rangle = |\psi_{12}\rangle \otimes \int d\omega_1 d\omega_2\, g(\omega_1,\omega_2)\, |\omega_1,\omega_2\rangle$$

Nonlocal memory effects

- General dephasing for 2 qubits (photons, polarization)

$$\rho_S^{12}(t) = \begin{pmatrix} |a|^2 & ab^*\kappa_2(t) & ac^*\kappa_1(t) & ad^*\kappa_{12}(t) \\ ba^*\kappa_2^*(t) & |b|^2 & bc^*\Lambda_{12}(t) & bd^*\kappa_1(t) \\ ca^*\kappa_1^*(t) & cb^*\Lambda_{12}^*(t) & |c|^2 & cd^*\kappa_2(t) \\ da^*\kappa_{12}^*(t) & db^*\kappa_1^*(t) & dc^*\kappa_2^*(t) & |d|^2 \end{pmatrix}$$

- Local states for qubit 1 and 2 [trace out qubit 2 (1)]

$$\rho_1(t) = \begin{pmatrix} |a|^2 + |b|^2 & (ac^* + bd^*)\kappa_1(t) \\ (ca^* + db^*)\kappa_1^*(t) & |c|^2 + |d|^2 \end{pmatrix}$$

$$\rho_2(t) = \begin{pmatrix} |a|^2 + |c|^2 & (ab^* + cd^*)\kappa_2(t) \\ (ba^* + dc^*)\kappa_2^*(t) & |b|^2 + |d|^2 \end{pmatrix}$$

- **Global** 2-qubit decoherence functions: $\kappa_{12}(t), \Lambda_{12}(t)$
- **Local** decoherence functions: $\kappa_1(t), \kappa_2(t)$

Nonlocal memory effects

- ...however, the open system map is a product of local maps

$$\Phi_{12}(t) = \Phi_1(t) \otimes \Phi_2(t)$$

if and only if $\kappa_{12}(t) = \kappa_1(t)\kappa_2(t)$ $\Lambda_{12}(t) = \kappa_1(t)\kappa_2^*(t)$

- Local interaction Hamiltonian for photon i

$$H_i = -\int d\omega_i\, \omega_i \Big[n_V |V\rangle\langle V| + n_H |H\rangle\langle H| \Big] \otimes |\omega_i\rangle\langle\omega_i|$$

$$H_{\text{int}}(t) = \chi_1(t) H_1 + \chi_2(t) H_2$$

- Decoherence functions

$$\kappa_1(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n \omega_1 t_1}$$

$$\kappa_2(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n \omega_2 t_2}$$

$$\kappa_{12}(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n (\omega_1 t_1 + \omega_2 t_2)}$$

$$\Lambda_{12}(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n (\omega_1 t_1 - \omega_2 t_2)}$$

Frequency correlations give nonlocal map and non-Markovian dynamics

Nonlocal memory effects

- Consider
  - Gaussian two-photon frequency distribution
  - Variance C
  - Correlation coefficient K
  - initial open system states

$$|\psi_{12}^{\pm}\rangle = (|HH\rangle \pm |VV\rangle)/\sqrt{2}$$

- ...trace distance dynamics of the open system

$$D(t) = \exp\left[-\frac{1}{2}\Delta n^2 C\left(t_1^2 + t_2^2 - 2|K|t_1 t_2\right)\right]$$

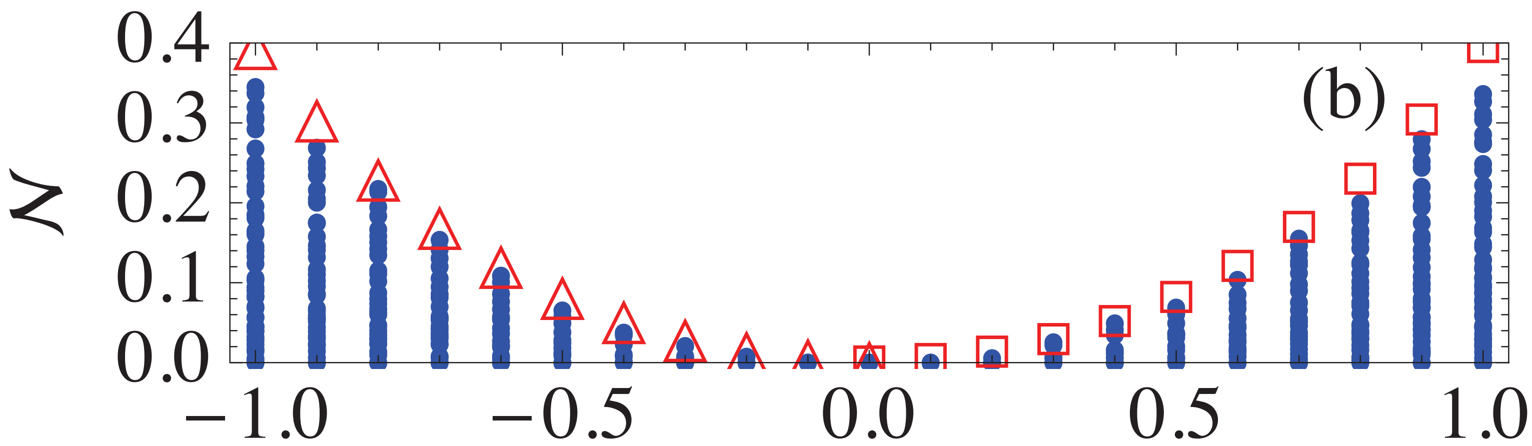
- ...and the non-Markovianity measure is

$$\mathcal{N} = e^{-\frac{1}{2}C_{11}(\Delta n T)^2}\left[e^{\frac{1}{2}C_{11}(\Delta n T)^2 K^2} - 1\right]$$

Direct connection between the amount of non-Markovianity of the open system and the correlations between the local environments

Laine, Breuer, Piilo, C.-F. Li, Guo: Phys. Rev. Lett. 108, 210402 (2012)

Correlations, anticorrelations, and non-Markovianity



Correlation coefficient K

(frequency distributions)

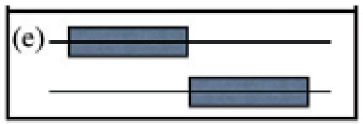
△ $(1/\sqrt{2})(|HH\rangle \pm |VV\rangle)$

□ $(1/\sqrt{2})(|HV\rangle \pm |VH\rangle)$

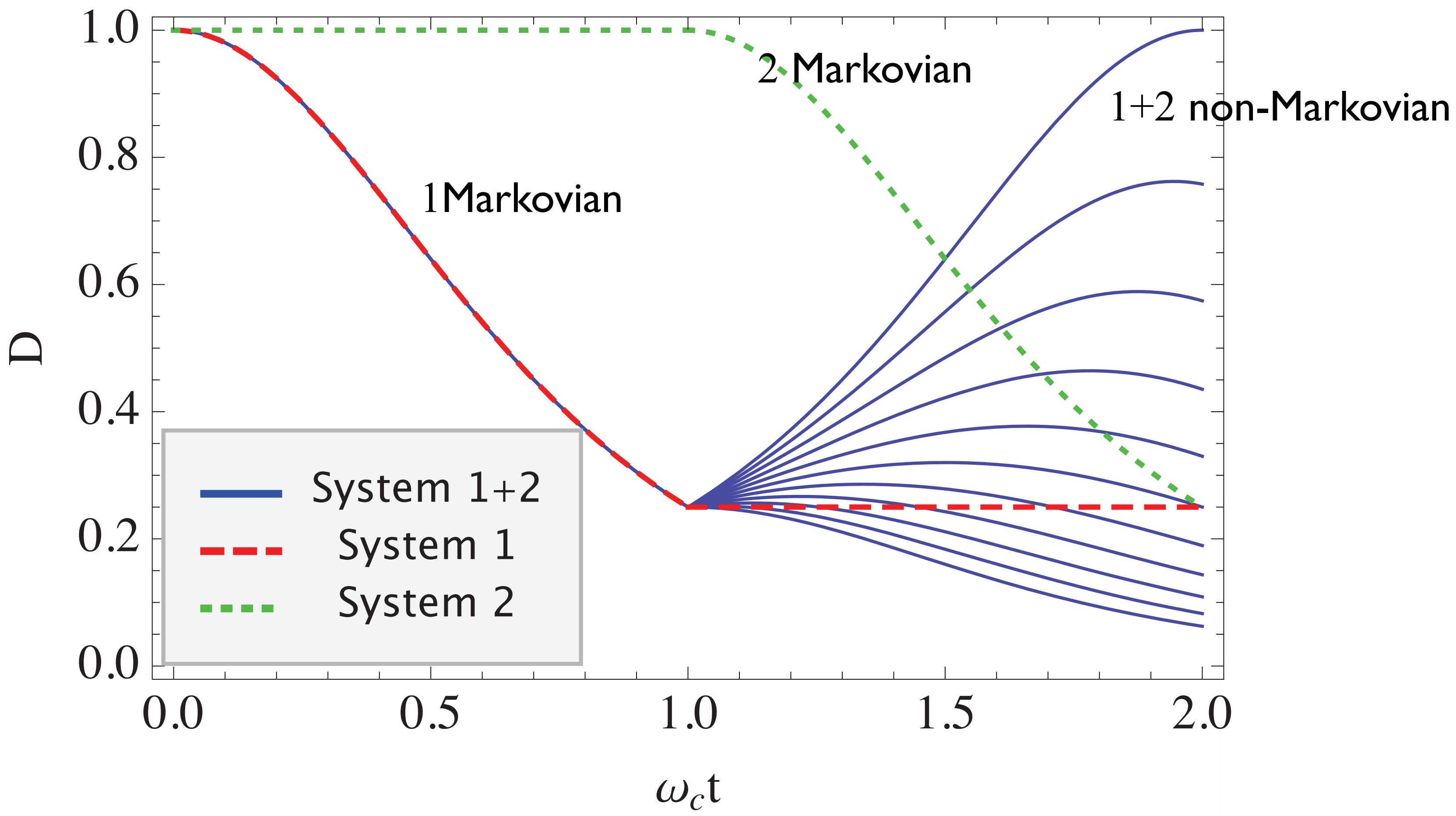
Max non-Markovianity for

- Correlated frequency, anticorrelated polarization
- Anticorrelated frequency, correlated polarization

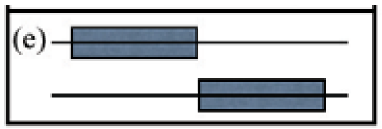
Laine, Breuer, Piilo, C.-F. Li, Guo,
Phys. Rev. Lett. 108, 210402 (2012)



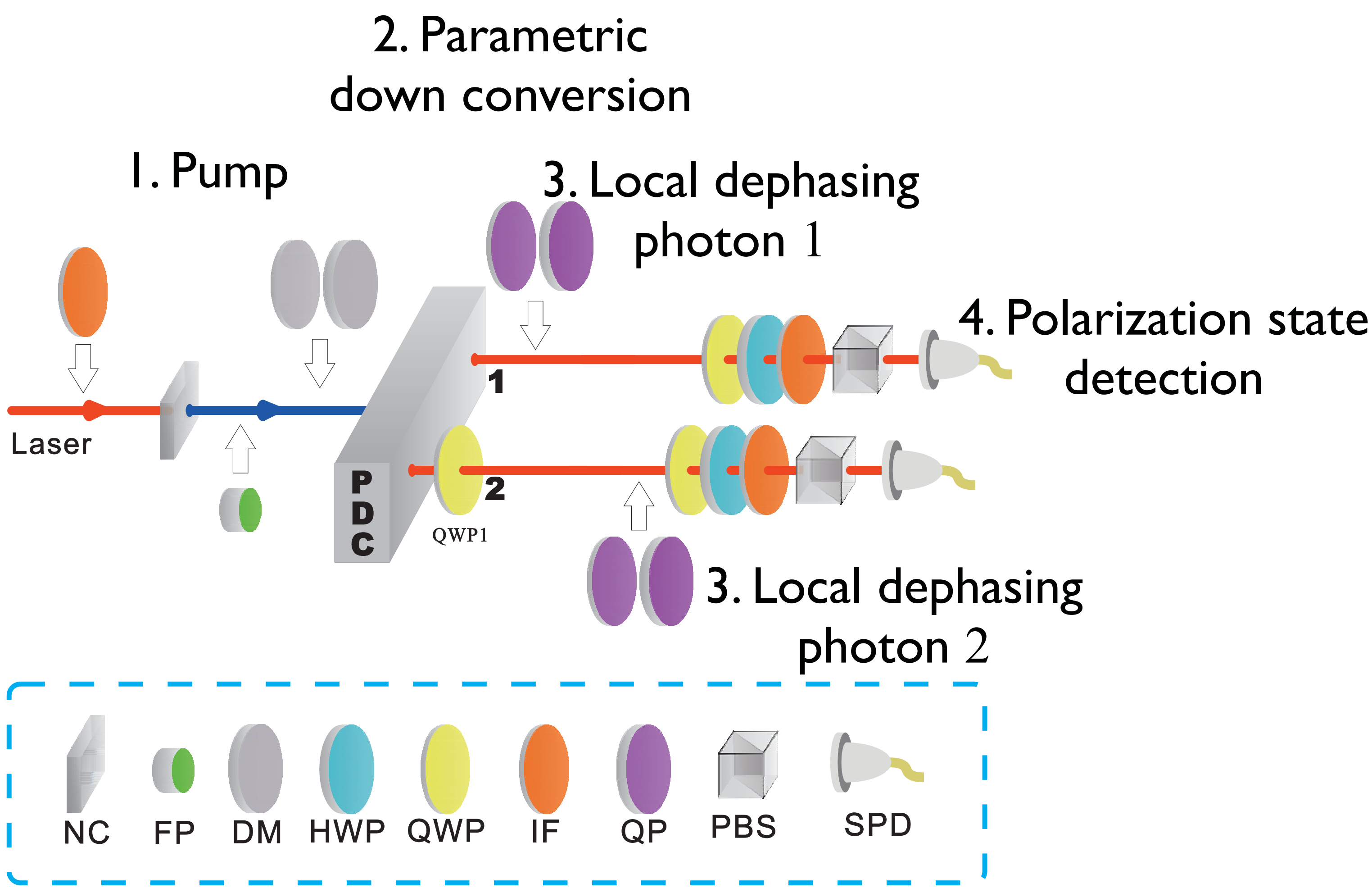
Local Markovian dynamics - global non-Markovian dynamics



Laine, Breuer, Piilo, C.-F. Li, Guo,
Phys. Rev. Lett. 108, 210402 (2012)



Experimental setup

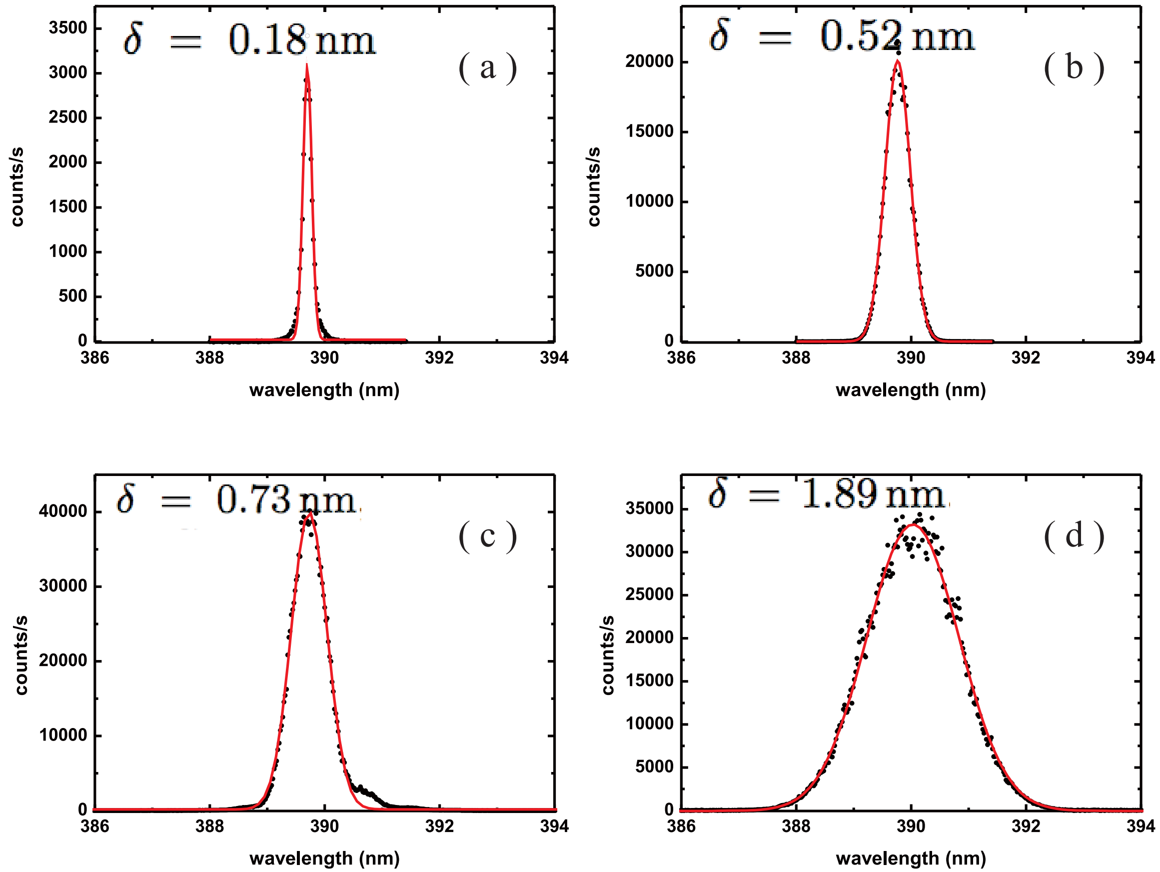


Liu, Cao, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
"Photonic realization of nonlocal memory effects and non-Markovian quantum probes",
arXiv:1208.1358

Nonlocal memory effects experiment

Control of the pump width...

Intensity



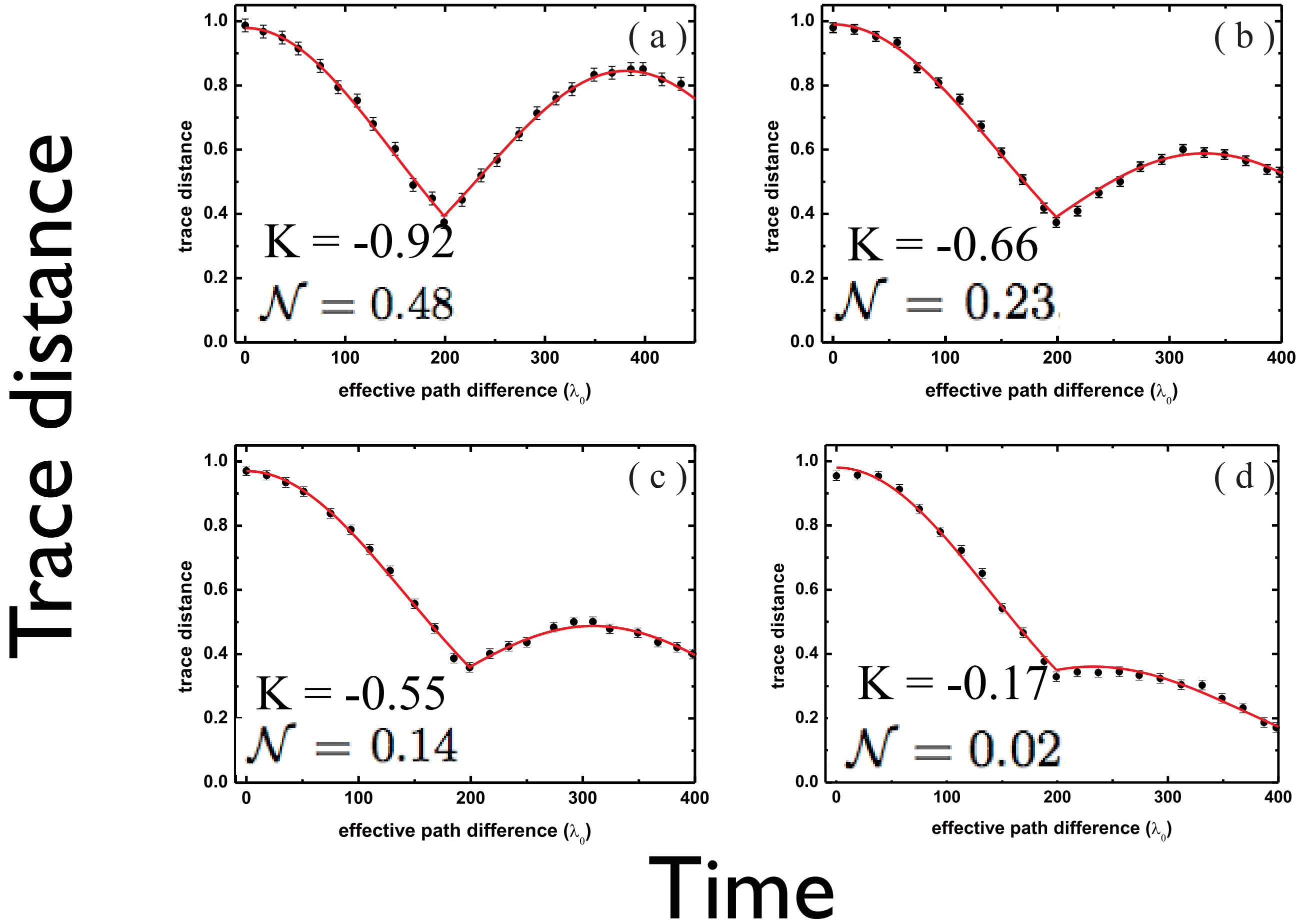
Wavelength

...this controls the frequency anticorrelations of the down converted photon pair (energy conserved, $w_0 = w_1 + w_2$)

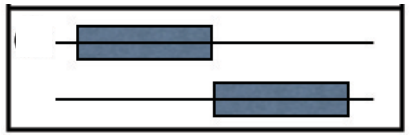
Liu, Cao, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
”Photonic realization of nonlocal memory effects and non-Markovian quantum probes”,
arXiv:1208.1358

Nonlocal memory effects experiment

Amount of frequency correlations K and the degree of non Markovianity N



solid line - theory
points - experiments



- Correlations in frequency induce non-Markovian polarization dynamics
- By measuring polarization we probe the frequency correlations

Liu, Cao, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
”Photonic realization of nonlocal memory effects and non-Markovian quantum probes”, arXiv:1208.1358

Applications of the information flow based measure

- **Energy transfer in light-harvesting complexes**
  P. Rebentrost and A. Aspuru-Guzik
  J. Chem. Phys. 134, 101103 (2011).
- **Spin environments**
  T. J. G. Apollaro, C. Di Franco, F. Plastina, and M. Paternostro
  Phys. Rev. A 83, 032103 (2011)
- **Cold atomic gases**
  P. Haikka, S. McEndoo, G. de Chiara, M. Palma, and S. Maniscalco
  PRA 2011
- **Chaotic quantum systems**
  M. Znidaric, C. Pineda, I. Garcia-Mata
  PRL 2011
- **Ising model in transverse field**
  Non-Markovianty pinpoints the quantum phase transition from paramagnetic to ferromagnetic ground state
  Haikka, Maniscalco
  PRA 2012

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•
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Simple to take home point:

MARKOVIAN:
Constant info flow rate from the system to the environment.

TIME-DEPENDENT MARKOVIAN:
Time-dependent info flow rate from the system to the environment.

NON-MARKOVIAN:
Recycling of info from the environment to the system.

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