NON-MARKOVIAN OPEN QUANTUM SYSTEMS

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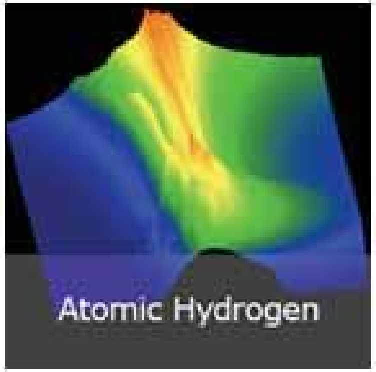

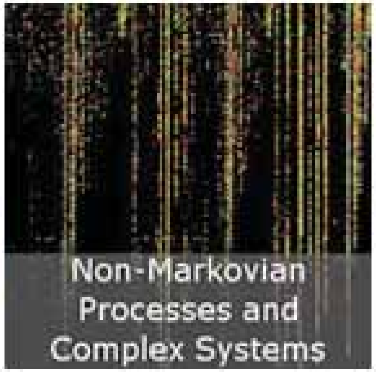

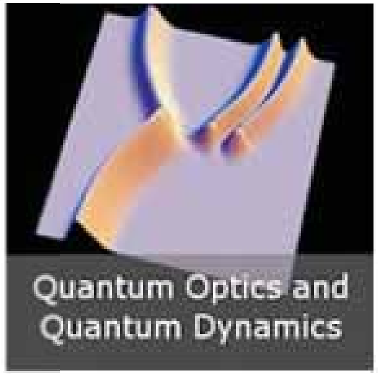

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Non-Markovian Processes and Complex Systems Group

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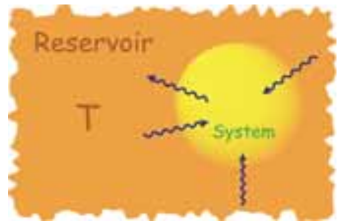

2. Local in time master equations

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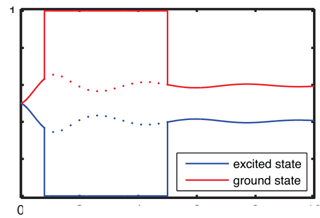

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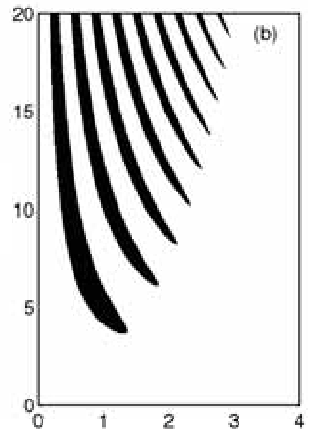

4. Measures of non-Markovianity

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1. General Framework: Open Quantum Systems

- Open vs closed systems
- Dynamical map
- Semigroup
- Lindblad equation
- Derivations
- Example

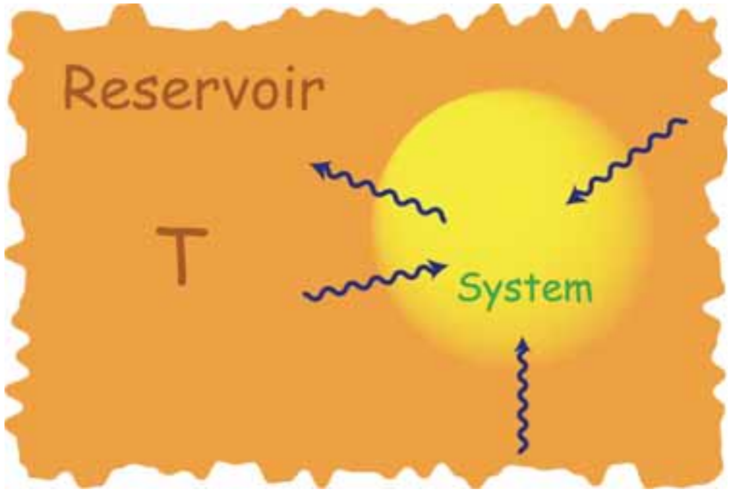
Literature

H.-P. Breuer and F. Petruccione:
The Theory of Open Quantum Systems
(Oxford University Press, Oxford, 2002)

C.W. Gardiner and P. Zoller:
Quantum Noise
(Springer, 2004)

U.Weiss
Quantum Dissipative Systems
(World Scientific, 1999)

Open quantum systems



Any realistic quantum system coupled to its environment

- The open system exchanges energy and information with its environment
- We are interested in: how does the interaction influence the open system, equation of motion?
- Not interested in the evolution of large environment
- E.g., radiation field - matter interaction, cavity-QED, quantum optics, dissipation of energy,...

Closed system dynamics

Pure state, state vector:

$$|\Psi\rangle$$

Schrödinger's Equation:

$$i\hbar\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle$$

Solution:

$$|\Psi(t)\rangle = e^{-iHt/\hbar}|\Psi(0)\rangle = U(t)|\Psi(0)\rangle$$

Mixed state, density matrix:

$$\rho = \sum_i P_i(t)|\Psi\rangle\langle\Psi|$$

Liouville - von Neumann equation:

$$i\hbar\frac{d\rho}{dt} = [H, \rho]$$

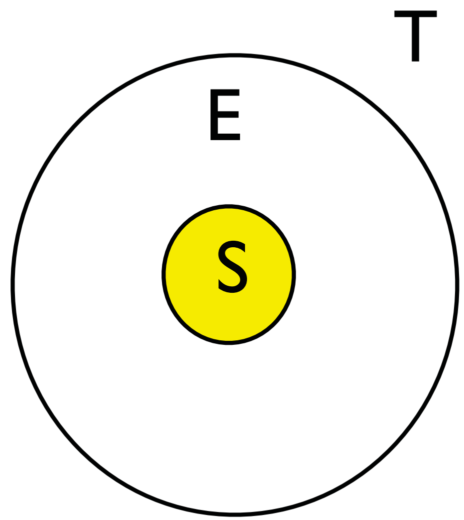
Solution:

$$\rho(t) = U(t)\rho(0)U^\dagger(t)$$

Deterministic, reversible time evolution

Open system dynamics

Total system closed: ρ_T



Open system: $\rho_S = \mathrm{Tr}_{\mathrm{E}} \rho_T$

Environment: $\rho_E = \mathrm{Tr}_{\mathrm{S}} \rho_T$

No initial correlations:

$$\rho_T(0) = \rho_S(0) \otimes \rho_E(0)$$

Total system Hamiltonian

$$H = H_S \otimes \mathbb{I}_E + \mathbb{I}_S \otimes H_E + H_I$$

Evolution of the open system and dynamical map

$$\rho_S(t) = \mathrm{Tr}_{\mathrm{E}} \left[U(t) \rho_S(0) \otimes \rho_E(0) U^\dagger(t) \right] = \Phi_t \rho_S(0)$$

Note: partial trace: $\rho_S = \mathrm{Tr}_E \rho_T = \sum_i {}_E\langle \varphi_i | \rho_T | \varphi_i \rangle_E$

Dynamical map

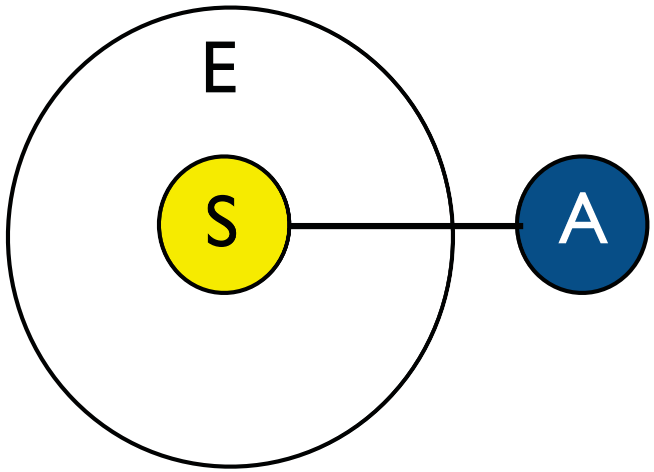
Linear dynamical map for open system

$$\Phi_t : \quad \rho_S(0) \rightarrow \rho_S(t) = \Phi_t \rho_S(0)$$

- Trace preserving
- Positive (P)
- Completely positive (CP)

  $\Phi \otimes I_n$ positive in extended space

  $(\Phi \otimes I_n)\rho_{SA} \geqslant 0$

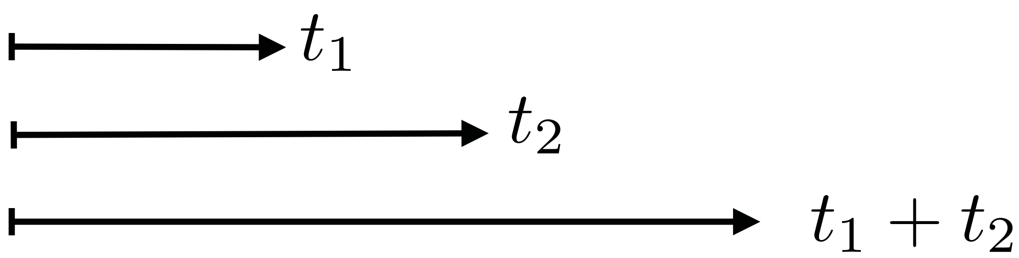


Specific properties and master equation...

Semigroup, Lindblad generator

$$\Phi_t : \quad \rho_S(0) \rightarrow \rho_S(t) = \Phi_t \rho_S(0)$$

- Semigroup property: $\Phi_{t_1+t_2} = \Phi_{t_1}\Phi_{t_2}$



It follows that:

- Dynamical map: $\Phi_t = e^{\mathcal{L}t}$
- Lindblad generator $\mathcal{L}$

$$\mathcal{L}\rho_S = -i[H, \rho_S] + \sum_k \gamma_k \left(A_k \rho_S A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

And the master equation...

Lindblad equation

Lindblad-Gorini-Kossakowski-Sudarshan master equation (1975)

unitary part — dissipator (non-unitary part)

$$\frac{d}{dt}\rho_S(t) = -i\,[H, \rho_S(t)] + \mathcal{D}(\rho_S(t))$$

$$\mathcal{D}(\rho_S) \equiv \sum_k \gamma_k \left(A_k \rho_S A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

decay rate — jump operators

- Lindblad or jump operators: A_k
- Decay rate constants: $\gamma_k \geqslant 0$
- Guarantees physical validity of the solution (CP)

(semigroup: master equation has to be of this form and validity guaranteed)

Microscopic derivation

Microscopic derivation

- Total system Hamiltonian (system S; environment or bath B, total system SB)

$$H = H_S \otimes \mathbb{I}_B + \mathbb{I}_S \otimes H_B + H_I$$

- Total system evolution in interaction picture (L-vN)

$$i\hbar \frac{d\rho_{SB}}{dt} = [H_I, \rho_{SB}]$$

- Integral form

$$\rho_{SB}(t) = \rho_{SB}(0) - \frac{i}{\hbar}\int_0^t ds[H_I(s), \rho_{SB}(s)]$$

- Plug this into L-vN (weak system-environment interaction)

$$\frac{d\rho_{SB}(t)}{dt} = -\frac{i}{\hbar}[H_I(t), \rho_{SB}(0)] - \frac{1}{\hbar^2}\int_0^t ds[H_I(t), [H_I(s), \rho_{SB}(s)]] + O\left(\frac{1}{\hbar^3}\right)$$

Microscopic derivation

$$\frac{d\rho_{SB}(t)}{dt} = -\frac{i}{\hbar}[H_I(t), \rho_{SB}(0)] - \frac{1}{\hbar^2}\int_0^t ds[H_I(t), [H_I(s), \rho_{SB}(s)]] + O\left(\frac{1}{\hbar^3}\right)$$

- Factorized initial condition
  $\varrho_{SB}(0) = \varrho_S(0) \otimes \varrho_B(0)$
- Trace over the bath gives

$$\frac{d\varrho_S}{dt}(t) = -\int_0^t ds \mathrm{Tr}_B\{[H_I(t), [H_I(s), \varrho_{SB}(s)]]\}$$

with $\mathrm{Tr}_B[H_I(t), \varrho_{SB}(0)] = 0$

- Stationary, macroscopic environment
  $\varrho_B(0) = \varrho_B$
- **Born approximation** (weak coupling)
  $\varrho_{SB}(t) \approx \varrho_S(t) \otimes \varrho_B$

Microscopic derivation

- Born approximation gives...

$$\frac{d\varrho_S(t)}{dt} = -\int_0^t ds \mathrm{Tr}_B\{[H_I(t),[H_I(s),\varrho_S(s)\otimes\varrho_B]]\}$$

- **Markov I**: no dependence on previous states (short reservoir correlation/memory time τ_B)

$$\varrho_S(s) \approx \varrho_S(t)$$

Redfield equation

$$\frac{d\varrho_S(t)}{dt} = -\int_0^t ds \mathrm{Tr}_B\{[H_I(t),[H_I(s),\varrho_S(t)\otimes\varrho_B]]\}$$

- **Markov II**: induced SB correlations decay fast, allows to extend the time integration to infinity...

Microscopic derivation

Born-Markov equation

$$\frac{d\varrho_S}{dt}(t) = -\int_0^\infty ds \mathrm{Tr}_B\{[H_I(t),[H_I(t-s),\varrho_S(t)\otimes\varrho_B]]\}$$

- Born approximation (weak coupling)
  $\varrho_{SB}(t) \approx \varrho_S(t) \otimes \varrho_B$
- Markov approximation: time scales
  $\tau_B \ll \tau_S$
- The contribution to the integral in the Redfield equation from short time interval during which the system state does not change very much
- Exact solution of Born-Markov not necessarily easy
- Not yet in Lindblad form...

Microscopic derivation

Born-Markov equation

$$\frac{d\varrho_S}{dt}(t) = -\int_0^\infty ds \mathrm{Tr}_B\{[H_I(t),[H_I(t-s),\varrho_S(t)\otimes\varrho_B]]\}$$

How to go from here to Lindblad form?

- Interaction Hamiltonian

$$H_I = \sum_\alpha A_\alpha \otimes B_\alpha$$

- Defining the eigenoperators of the system

$$A_\alpha(\omega) = \sum_{\epsilon'-\epsilon=\omega} \Pi(\epsilon) A_\alpha \Pi(\epsilon')$$

where $\Pi(\epsilon)$ projects to eigenspace of H_S with eig. value ϵ

allows to write the master equation as...

Microscopic derivation

$$\frac{d\varrho_s}{dt}(t) = \sum_{\omega,\omega'}\sum_{\alpha,\beta} e^{i(\omega'-\omega)t}\Gamma_{\alpha\beta}(\omega)[A_\beta(\omega)\varrho_S(t)A_\alpha^\dagger(\omega') - A_\alpha^\dagger(\omega')A_\beta(\omega)\varrho_S(t)] + \text{h.c.}$$

with

$$\Gamma_{\alpha\beta}(\omega) \equiv \int_0^\infty ds e^{i\omega s}\langle B_\alpha^\dagger(t)B_\beta(t-s)\rangle$$

and reservoir correlation function

$$\langle B_\alpha^\dagger(t)B_\beta(t-s)\rangle \equiv \mathrm{Tr}_B\{B_\alpha^\dagger(t)B_\beta(t-s)\varrho_B\}$$

- Real and imaginary parts

$$\Gamma_{\alpha\beta}(\omega) = \frac{1}{2}\gamma_{\alpha\beta}(\omega) + iS_{\alpha\beta}(\omega)$$

- For stationary reservoir, homogeneous in time

$$\langle B_\alpha^\dagger(t)B_\beta(t-s)\rangle = \langle B_\alpha^\dagger(s)B_\beta(0)\rangle$$

- Almost in the Lindblad form...
- One more approximation: fastly oscillating terms average out: secular approximation...

Microscopic derivation

$$\frac{d\varrho_S}{dt}(t) = -i[H_{LS}, \varrho_S(t)] + L\varrho_S(t)$$

- Lamb shift term (energy renormalization)

$$H_{LS} = \sum_{\omega} \sum_{\alpha,\beta} S_{\alpha\beta}(\omega) A_\alpha^\dagger(\omega) A_\beta(\omega)$$

- Dissipator

$$L\varrho_S = \sum_{\omega} \sum_{\alpha,\beta} \gamma_{\alpha\beta} \left[A_\beta(\omega)\varrho_S A_\alpha^\dagger(\omega) - \frac{1}{2}\{A_\alpha^\dagger(\omega)A_\beta(\omega), \varrho_S\} \right]$$

- By diagonalizing the dissipator, we finally obtain Lindblad form

$$L\varrho_S = \sum_{\omega} \sum_{\alpha} \gamma_\alpha(\omega) \left[A_\alpha(\omega)\varrho_S A_\alpha^\dagger(\omega) - \frac{1}{2}\{A_\alpha^\dagger(\omega)A_\alpha(\omega), \varrho_S\} \right]$$

So far:

- Time-evolution of the total system
- Tracing over the environment
- Born, Markov, and secular approximations

↓

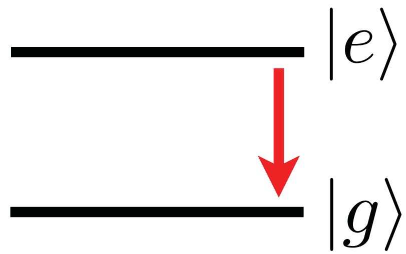
Lindblad master equation
(Markovian, semigroup)

Example: two-level atom in vacuum

$$\frac{d\varrho}{dt} = -i[H, \varrho] + \Gamma \left[\sigma_- \varrho \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \varrho\} \right]$$

- System energy

  $H = \omega_0 \sigma_z$



$\sigma_- = |g\rangle\langle e|$

- Lindblad operator

  $\sigma_- = |g\rangle\langle e|$

- Decay rate Γ

- Exponential decay from excited state

$$\rho_{ee(t)} = e^{-\Gamma t} \rho_{ee(0)}$$

$$\rho_{gg(t)} = \rho_{gg}(0) + (1 - e^{-\Gamma t})\rho_{ee(0)}$$

$$\rho_{eg(t)} = e^{-\Gamma t/2} \rho_{eg(0)}$$

II. Non-Markovian open systems: local in time master equations

- Projection operator techniques
- Nakazima-Zwanzig (memory kernel)
- TCL (time-convolutionless)
- Example

Open systems: Beyond semigroup

$$\frac{d\varrho_S(t)}{dt} = \mathcal{L}_S \varrho_S(t)$$

- Semigroup iff generator $\mathcal{L}_s$ in Lindblad form

$$\frac{d\varrho_S(t)}{dt} = \mathcal{L}_S(t)\varrho_S(t)$$

- Time-dependent generator $\mathcal{L}_S$ (t)

$$\frac{d\varrho_S(t)}{dt} = \int_o^t ds \mathcal{K}_S(t-s)\varrho_S(s)$$

- Memory kernel $\mathcal{K}_S(t-s)$

Nakazima-Zwanzig projection operator technique

- Total system Hamiltonian

$$H = H_0 + \alpha H_I$$

- Total system equation of motion (interaction picture)

$$\frac{d\varrho}{dt}(t) = -i\alpha[H_I(t), \varrho(t)] \equiv \alpha\mathcal{L}(t)\varrho(t)$$

here $\mathcal{L}$ is the total system Liouville superoperator

Basic idea: project to relevant and irrelevant parts of the total system

$\mathcal{P}\varrho \equiv \mathrm{Tr}_B[\varrho] \otimes \varrho_B$ Relevant part

$\mathcal{Q}\varrho = \varrho - \mathcal{P}\varrho$ Irrelevant part

Nakazima-Zwanzig projection operator technique

$$\mathcal{P}\varrho \equiv \mathrm{Tr}_B[\varrho] \otimes \varrho_B$$ Relevant part

$$\mathcal{Q}\varrho = \varrho - \mathcal{P}\varrho$$ Irrelevant part

Here, ϱ_B is a fixed environmental state (stationary env.)

The projection superoperators have the properties:

$$\mathcal{P} + \mathcal{Q} = \mathrm{I}$$

$$\mathcal{P}^2 = \mathcal{P}$$

$$\mathcal{Q}^2 = \mathcal{Q}$$

$$[\mathcal{P}, \mathcal{Q}] = 0$$

Nakazima-Zwanzig projection operator technique

The task: derive equation of motion for the relevant part, that is: for $\varrho_S(t) = Tr_B \varrho(t)$

$$\frac{d\varrho}{dt}(t) = -i\alpha[H_I(t), \varrho(t)] \equiv \alpha\mathcal{L}(t)\varrho(t)$$

$$\downarrow$$

$$\frac{\partial}{\partial t}\mathcal{P}\varrho = \alpha\mathcal{P}\mathcal{L}\varrho \qquad \frac{\partial}{\partial t}\mathcal{Q}\varrho = \alpha\mathcal{Q}\mathcal{L}\varrho$$

or by inserting $\mathcal{P} + \mathcal{Q} = \mathrm{I}$ between Liouvillean and density matrix

$$\frac{\partial}{\partial t}\mathcal{P}\varrho = \alpha\mathcal{P}\mathcal{L}\mathcal{P}\varrho + \alpha\mathcal{P}\mathcal{L}\mathcal{Q}\varrho$$

$$\frac{\partial}{\partial t}\mathcal{Q}\varrho = \alpha\mathcal{Q}\mathcal{L}\mathcal{P}\varrho + \alpha\mathcal{Q}\mathcal{L}\mathcal{Q}\varrho$$

Nakazima-Zwanzig projection operator technique

$$\frac{\partial}{\partial t}\mathcal{P}\varrho = \alpha\mathcal{P}\mathcal{L}\mathcal{P}\varrho + \alpha\mathcal{P}\mathcal{L}\mathcal{Q}\varrho \qquad \frac{\partial}{\partial t}\mathcal{Q}\varrho = \alpha\mathcal{Q}\mathcal{L}\mathcal{P}\varrho + \alpha\mathcal{Q}\mathcal{L}\mathcal{Q}\varrho$$

- Coupled differential equations for the two parts

- The formal solution for the irrelevant part

$$\mathcal{Q}\varrho(t) = G(t,t_0)\mathcal{Q}\varrho(t_0) + \alpha\int_{t_0}^{t} ds G(t,s)\mathcal{Q}\mathcal{L}(s)\mathcal{P}\varrho(s)$$

where the propagator G is

$$G(t,t_0) \equiv T_{\leftarrow}\exp\left[\alpha\int_{t_0}^{t} ds\mathcal{Q}\mathcal{L}(s)\right]$$

Inserting the solution to the equation of the relevant part...

Nakazima-Zwanzig projection operator technique

...and using also (makes the first r.h.s. term to vanish)

$$\mathrm{Tr}_B[H_I(t_1)\cdots H_I(t_{2n+1})\varrho_B] = 0 \iff \mathcal{P}\mathcal{L}(t_1)\cdots\mathcal{L}(t_{2n+1})\mathcal{P} = 0$$

(odd moments of H_I vanish, valid for thermal env. state)

...finally gives the Nakazima-Zwanzig equation

$$\frac{\partial \mathcal{P}\varrho}{\partial t}(t) = \alpha \mathcal{P}\mathcal{L}(t)G(t,t_0)\mathcal{Q}\varrho(t_0) + \alpha^2 \int_{t_0}^{t} ds \mathcal{P}\mathcal{L}(t)G(t,s)\mathcal{Q}\mathcal{L}(s)\mathcal{P}\varrho(s)$$

Note:

- The 1st term on the r.h.s. contains initial correlations with the environment (vanish for initial product state)
- The 2nd term on the r.h.s. contain memory kernel

  $$K(t,s) = \alpha^2 \mathcal{P}\mathcal{L}(t)G(t,s)\mathcal{Q}\mathcal{L}(s)\mathcal{P}$$

- Exact equation for the relevant part, challenging to solve...

Nakazima-Zwanzig projection operator technique

...however, to 2nd order in the coupling constant α gives

$$\frac{\partial \varrho_S}{\partial t}(t) = -\alpha^2 \mathrm{Tr}_B \left\{ \int_{t_0}^{t} ds [H_I(t), [H_I(s), \varrho_s(s) \otimes \varrho_B]] \right\}$$

- This is the same as the previous equation after Born approximation

- Question:
  Is is possible to eliminate the memory kernel in the Nakazima-Zwanzig equation
- Is it possible to have local in time non-Markovian equation ?

Open systems: Beyond semigroup

$$\frac{d\varrho_S(t)}{dt} = \mathcal{L}_S \varrho_S(t)$$

- Semigroup iff generator $\mathcal{L}_s$ in Lindblad form

$$\frac{d\varrho_S(t)}{dt} = \mathcal{L}_S(t)\varrho_S(t)$$

- Time-dependent generator $\mathcal{L}_S$ (t)

$$\frac{d\varrho_S(t)}{dt} = \int_o^t ds \mathcal{K}_S(t-s)\varrho_S(s)$$

- Memory kernel $\mathcal{K}_S(t-s)$

Time convolutionless master equation (local in time, time dependent generator)

Time-convolutionless master equations

How to construct local in time generator?
(basic idea: introduce backward propagator)

- Start again from the formal solution for the irrelevant part

$$\mathcal{Q}\varrho(t) = G(t,t_0)\mathcal{Q}\varrho(t_0) + \alpha \int_{t_0}^{t} ds G(t,s)\mathcal{Q}\mathcal{L}(s)\mathcal{P}\varrho(s)$$

- Introduce backward propagator for the total system (s<t)

$$\varrho(s) = \bar{G}(t,s)(\mathcal{P}+\mathcal{Q})\varrho(t)$$

with (antichronological ordering)

$$\bar{G}(t,s) \equiv T_{\rightarrow} \exp\left[-\alpha \int_{s}^{t} ds' \mathcal{L}(s')\right]$$

- This gives for the irrelevant part

$$\mathcal{Q}\varrho(t) = G(t,t_0)\mathcal{Q}\varrho(t_0) + \alpha \int_{t_0}^{t} ds \underbrace{G(t,s)\mathcal{Q}\mathcal{L}(s)\mathcal{P}\bar{G}(t,s)}_{\Sigma(t)}(\mathcal{P}+\mathcal{Q})\varrho(t)$$

Time-convolutionless master equations

- Defining superoperator (depends on t only)

$$\Sigma(t) \equiv \alpha \int_{t_0}^{t} ds G(t,s)\mathcal{Q}\mathcal{L}(s)\mathcal{P}\bar{G}(t,s)$$

- We can write for irrelevant part as

$$\mathcal{Q}\varrho(t) = G(t,t_0)\mathcal{Q}\varrho(t_0) + \Sigma(t)\mathcal{P}\varrho(t) + \Sigma(t)\mathcal{Q}\varrho(t)$$

$$[\mathrm{I} - \Sigma(t)]\mathcal{Q}\varrho(t) = G(t,t_0)\mathcal{Q}\varrho(t_0) + \Sigma(t)\mathcal{P}\varrho(t)$$

- If the inverse of $\mathrm{I} - \Sigma(t)$ exists (small enough coupling)

$$\mathcal{Q}\varrho(t) = [\mathrm{I} - \Sigma(t)]^{-1}G(t,t_0)\mathcal{Q}\varrho(t_0) + [\mathrm{I} - \Sigma(t)]^{-1}\Sigma(t)\mathcal{P}\varrho(t)$$

Time evolution of Q depends on initial state and relevant part, also no dependence on previous point s.

Plugging this in for the equation for the relevant part...

Time-convolutionless master equations

$$\frac{\partial \mathcal{P}\varrho(t)}{\partial t} = \mathcal{K}(t)\mathcal{P}\varrho(t) + \mathcal{I}(t)\mathcal{Q}\varrho(t_0)$$

with time local generator and inhomogeneity

$$\mathcal{K}(t) = \alpha\mathcal{P}\mathcal{L}(t)[\mathrm{I} - \Sigma(t)]^{-1}\Sigma(t)\mathcal{P}$$

$$\mathcal{I}(t) = \alpha\mathcal{P}\mathcal{L}(t)[\mathrm{I} - \Sigma(t)]^{-1}G(t,t_0)\mathcal{Q}$$

- Exact local in time equation
- Generally complicated
- Geometric series and series expansion in coupling constant

$$[\mathrm{I} - \Sigma(t)]^{-1} = \sum_{n=0}^{+\infty}[\Sigma(t)]^n$$

$$\mathcal{K}(t) = \alpha\sum_{n=1}^{+\infty}\mathcal{P}\mathcal{L}(t)[\Sigma(t)]^n\mathcal{P} = \sum_{n=1}^{+\infty}\alpha^n K_n(t)$$

Time-convolutionless master equations

$$\frac{\partial \mathcal{P}\varrho(t)}{\partial t} = \mathcal{K}(t)\mathcal{P}\varrho(t) + \mathcal{I}(t)\mathcal{Q}\varrho(t_0)$$

$$\mathcal{K}(t) = \alpha \sum_{n=1}^{+\infty} \mathcal{P}\mathcal{L}(t)[\Sigma(t)]^n \mathcal{P} = \sum_{n=1}^{+\infty} \alpha^n K_n(t)$$

- Expanding also

$$\Sigma(t) = \sum_{k=1}^{+\infty} \alpha^k \Sigma_k(t)$$

gives, e.g.,

$$K_1(t) = \mathcal{P}\mathcal{L}(t)\mathcal{P} = 0 \qquad \text{1st order}$$

$$K_2(t) = \int_0^t dt_1 \mathcal{P}\mathcal{L}(t)\mathcal{L}(t_1)\mathcal{P} \qquad \text{2nd order TCL}$$

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Time-convolutionless master equations

The second order TCL leads to the following equation

$$\frac{d\rho_S(t)}{dt} = -\alpha^2 \int_0^t ds \mathrm{Tr}_B \left\{ [H_I(t), [H_I(s), \rho_S(t) \otimes \rho_B]] \right\}$$

- Comparing to 2nd order Nakazima-Zwanzig (previously)

$$\frac{d\rho_S(t)}{dt} = -\alpha^2 \int_0^t ds \mathrm{Tr}_B \left\{ [H_I(t), [H_I(s), \rho_S(s) \otimes \rho_B]] \right\}$$

Also similarity to Redfield equation...

Example: two-level atom in vacuum TCL2

$$\frac{d\varrho}{dt} = -i[H, \varrho] + \Gamma \left[\sigma_- \varrho \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \varrho\} \right]$$

- Decay rates

$$\text{Markov} \rightarrow \Gamma = \frac{1}{\pi}\int_0^{+\infty} ds \int d\omega J(\omega) \cos[(\omega - \omega_0)s] = \text{constant}$$

$$\text{TCL} \rightarrow \Gamma(t) = \frac{1}{\pi}\int_0^{t} ds \int d\omega J(\omega) \cos[(\omega - \omega_0)s]$$

Here, J is the spectral density of the Bosonic environment

- And the open system dynamics is

$$\text{Markov} \rightarrow \varrho_{ee}(t) = e^{-\Gamma t} \varrho_{ee}(0)$$

$$\text{TCL} \rightarrow \varrho_{ee}(t) = e^{-\int_0^t ds \Gamma(s)} \varrho_{ee}(0)$$

Example: two-level atom in vacuum TCL2

$$\frac{d\varrho}{dt} = -i[H, \varrho] + \Gamma \left[\sigma_- \varrho \sigma_+ - \frac{1}{2}\{\sigma_+ \sigma_-, \varrho\} \right]$$

- Decay rates

$$\text{Markov} \rightarrow \Gamma = \frac{1}{\pi} \int_0^{+\infty} ds \int d\omega J(\omega) \cos[(\omega - \omega_0)s] = \text{constant}$$

$$\text{TCL} \rightarrow \Gamma(t) = \frac{1}{\pi} \int_0^{t} ds \int d\omega J(\omega) \cos[(\omega - \omega_0)s]$$

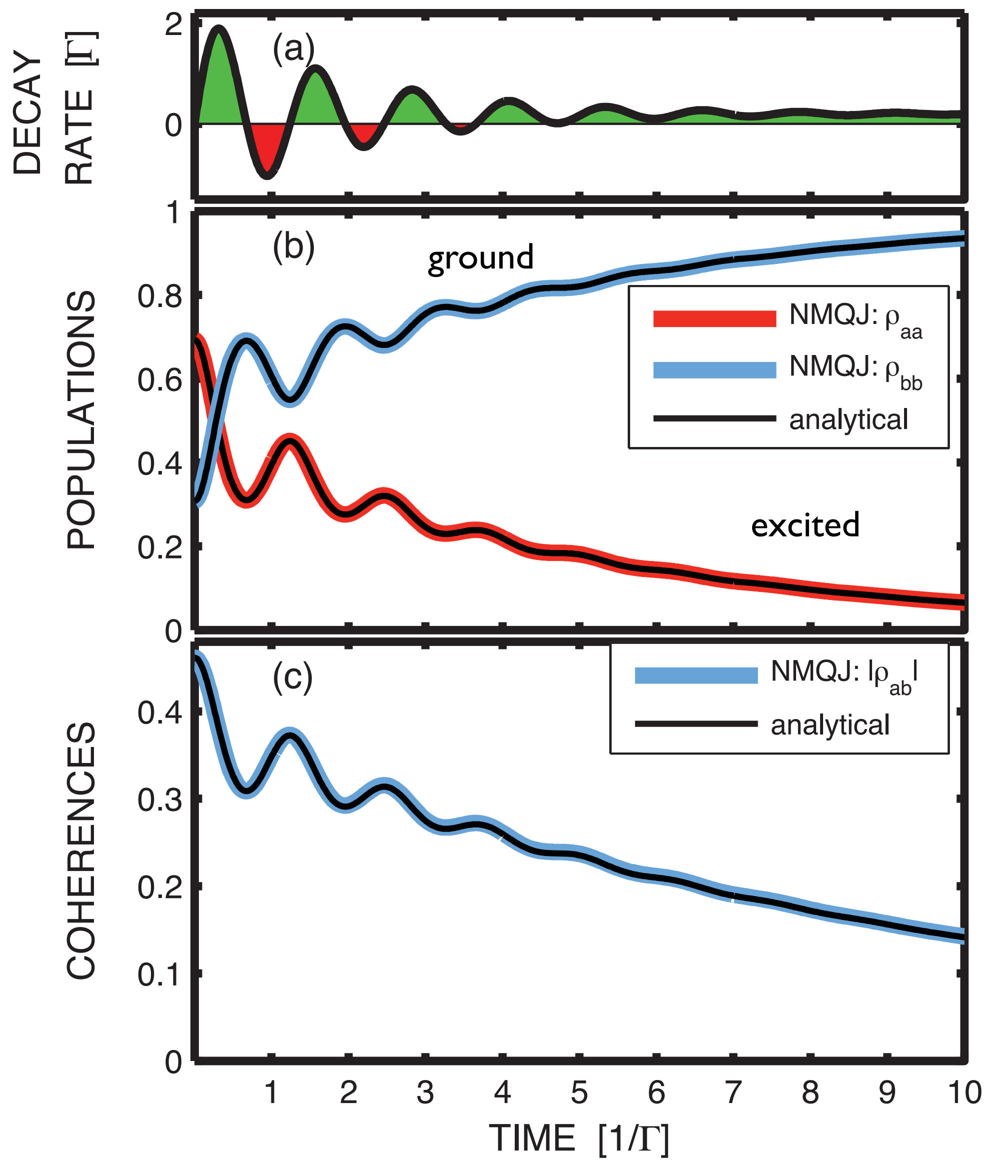
Here, J is the spectral density of the Bosonic environment

- And the open system dynamics is

$$\text{Markov} \rightarrow \varrho_{ee}(t) = e^{-\Gamma t} \varrho_{ee}(0)$$

$$\text{TCL} \rightarrow \varrho_{ee}(t) = e^{-\int_0^t ds \Gamma(s)} \varrho_{ee}(0)$$

Example: two-level atom in vacuum TCL2



- Decay rate oscillates having periods of negativity
- Excited state population oscillates
- Decoherence - re-coherence cycles

- Compare to Markovian, semigroup evolution: exponential decay

End of lecture 1

1. General framework: Open quantum systems
2. Local in time master equations

Next lecture:

Solving local in time equations by Markovian and non-Markovian quantum jumps