Photon Production from the Quantum Vacuum: Taking a break from the bits

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6th Winter School on Quantum Information Science & Quantum Phenomena

Essence of the Quantum Vacuum:

$$[x, p] = i\hbar$$

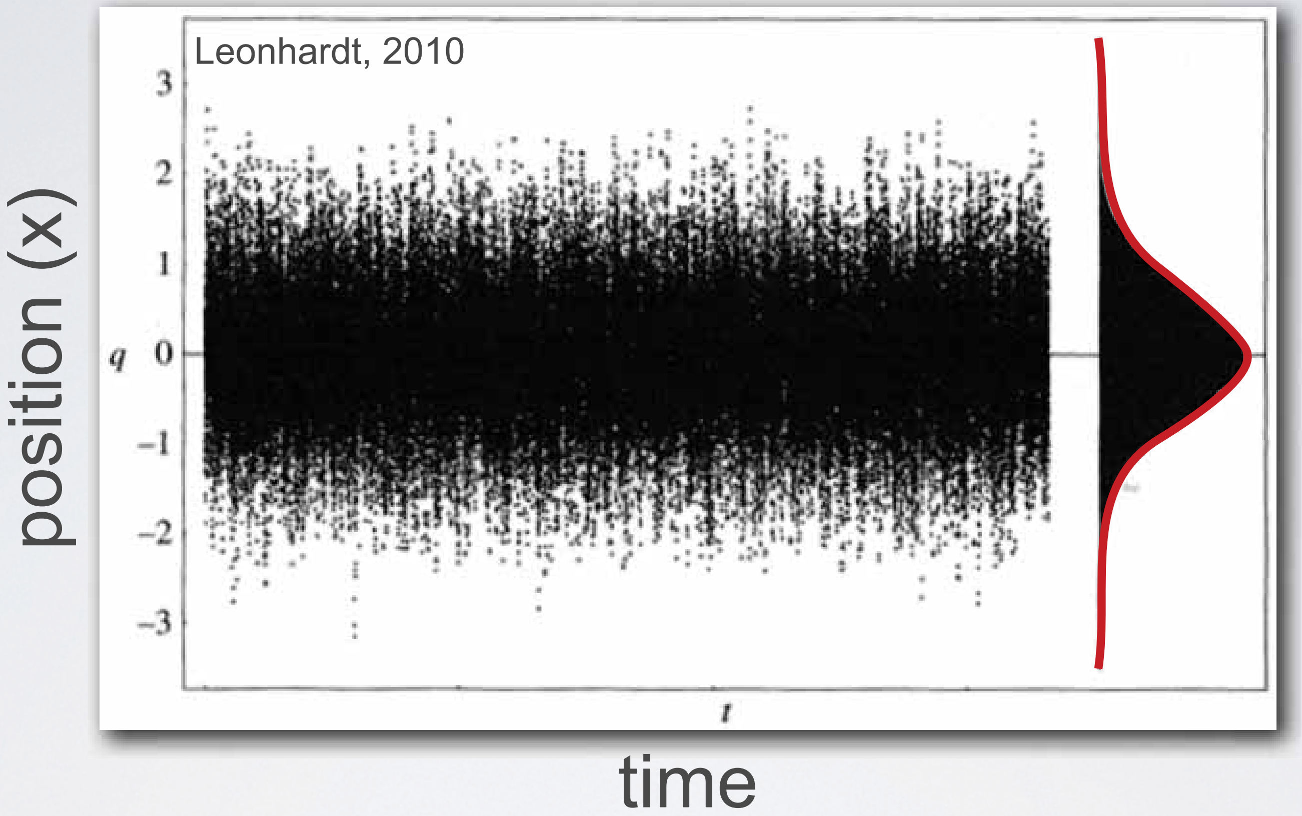
$$+$$

$$\left\langle (\Delta A)^2 \right\rangle \left\langle (\Delta B)^2 \right\rangle \geq \frac{1}{4} \left| \langle [A, B] \rangle \right|^2$$

$$\Downarrow$$

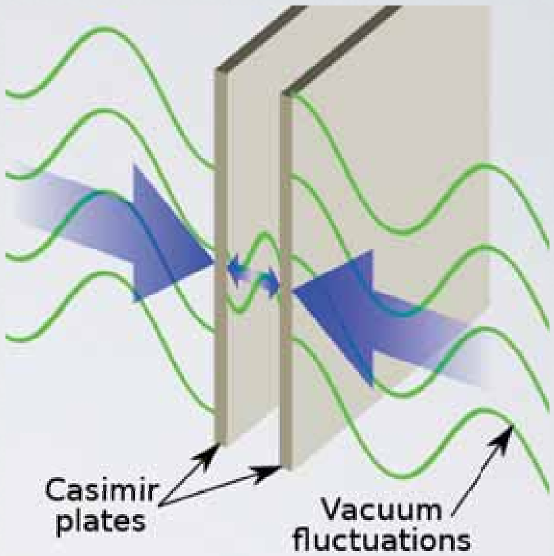
$$\boxed{\left\langle (\Delta x)^2 \right\rangle \left\langle (\Delta p)^2 \right\rangle \geq \frac{\hbar^2}{4}}$$

The quantum vacuum state is ALWAYS fluctuating:



Origin of some of the most important physical processes in the universe.

spontaneous emission



Casimir effect

large-scale structure

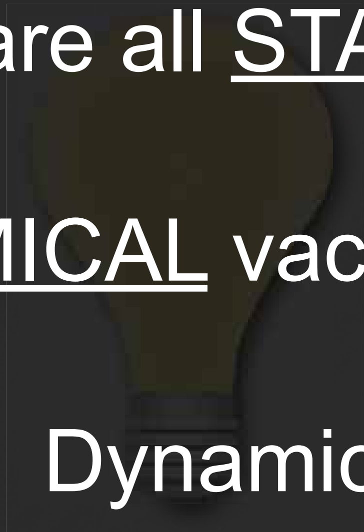
Λ

cosmological constant

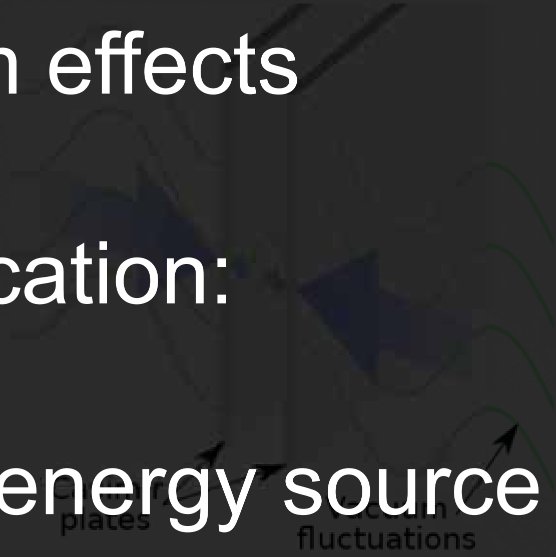
These are all <u>STATIC</u> vacuum effects

<u>DYNAMICAL</u> vacuum amplification:

Dynamics driven by energy source

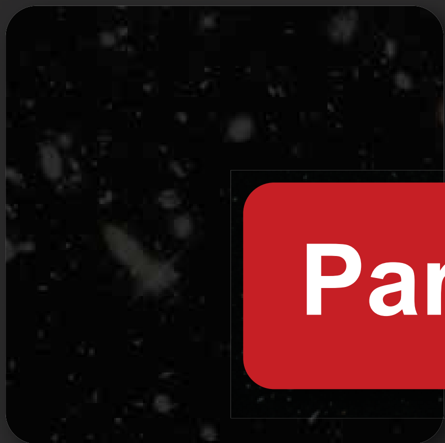


spontaneous emission



Casimir effect

Particle production

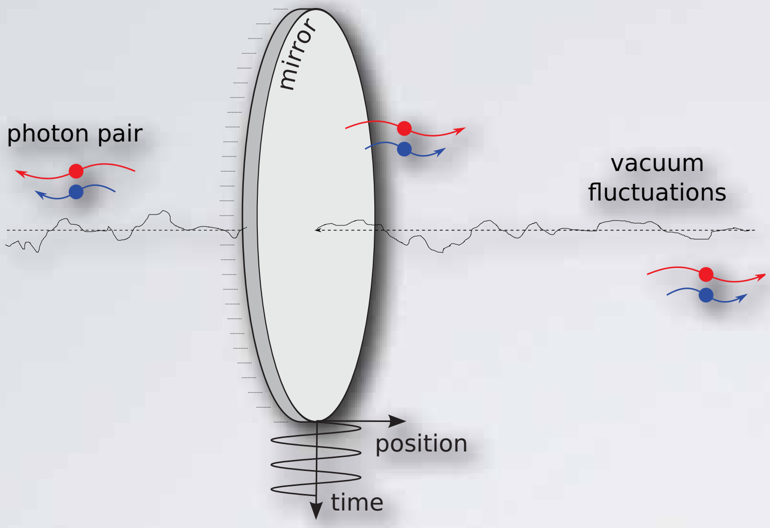


large-scale structure

Λ

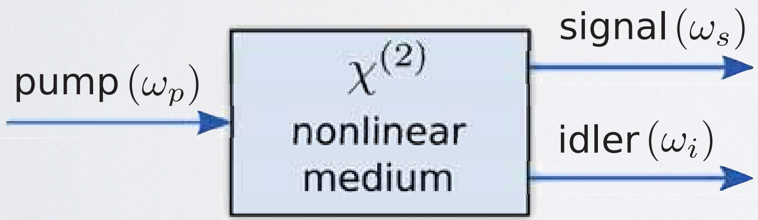
cosmological constant

Dynamical Quantum Vacuum Effects:

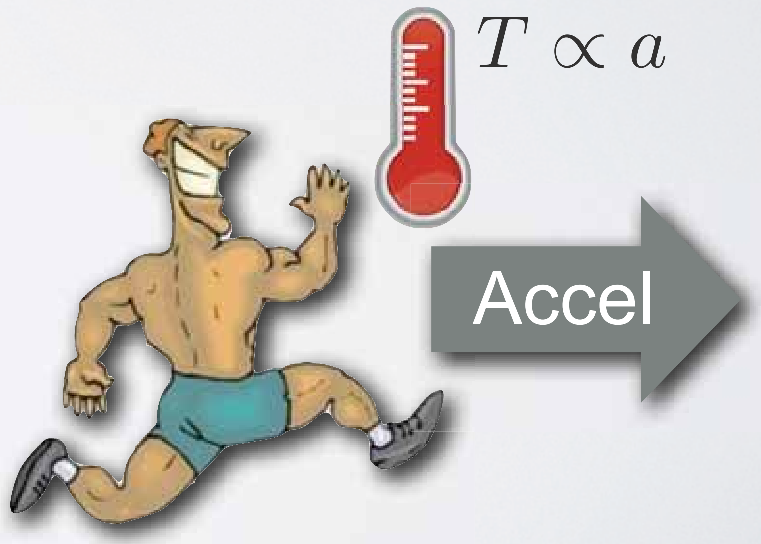


Dynamical Casimir

Hawking Radiation



Parametric Amplifier



Unruh Effect

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Why so hard to detect?

Dynamical Casimir: $\langle N \rangle \propto v/c$

- Need small mass / massless mirror.

Unruh effect: $k_\mathrm{B} T_\mathrm{U} \propto a/c$

- Requires very large accelerations.
- Detector must be accelerating.

Hawking radiation: $k_\mathrm{B} T_\mathrm{H} \propto \kappa/c$

- For small black hole $T_\mathrm{H} \sim 10^{-9}$ K.
- No way to verify photons come from black hole.

Goal #1:

#1: Introduce vacuum photon amplification effects, highlighting relationship to quantum parametric amplifier.

I. Standard quantum optics effect.

II. Workhorse of quantum optics, circuit-QED,...

nonclassical states, wave-particle duality, quantum erasers, quantum teleportation, heralded photons...

III. Contains the basic ingredients of all vacuum amplifiers.

Goal #2:

#2: Demonstrate the use of superconducting circuits to realize these effects.

- Low noise and dissipation.
  - I. Photons can travel 10km before dissipation.
  - II. Can maintain entanglement
- Controllability on the single-photon level.
  - I. Single photons on demand.
  - II. Controlled generation of quantum cavity states.
  - III. Nonlinearities on the single-photon level.
- Single-shot photon detection (PRL **107** 217401)
  - I. Can detect and measure single photon pairs

Parametric amplifiers:

Castellanos-Beltran, Nat. Phys. **4**, 929 (2008)

Bergeal, Nature **465**, 64 (2010)

Roch, PRL (In Press) (2012), arXiv:1202.1315

Hawking radiation:

Nation, PRL **103**, 087004 (2009)

Dynamical Casimir Effect:

Johansson, PRL **103**, 143003 (2009)

Wilson, Nature **479**, 376 (2011)

Lähteenmäki, arXiv:1111.5608 (2011)

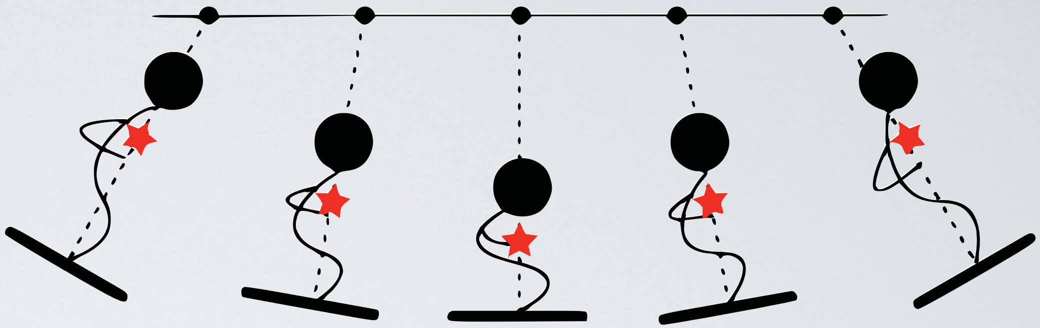
Rev. Mod. Phys. 84, 1 (2012)

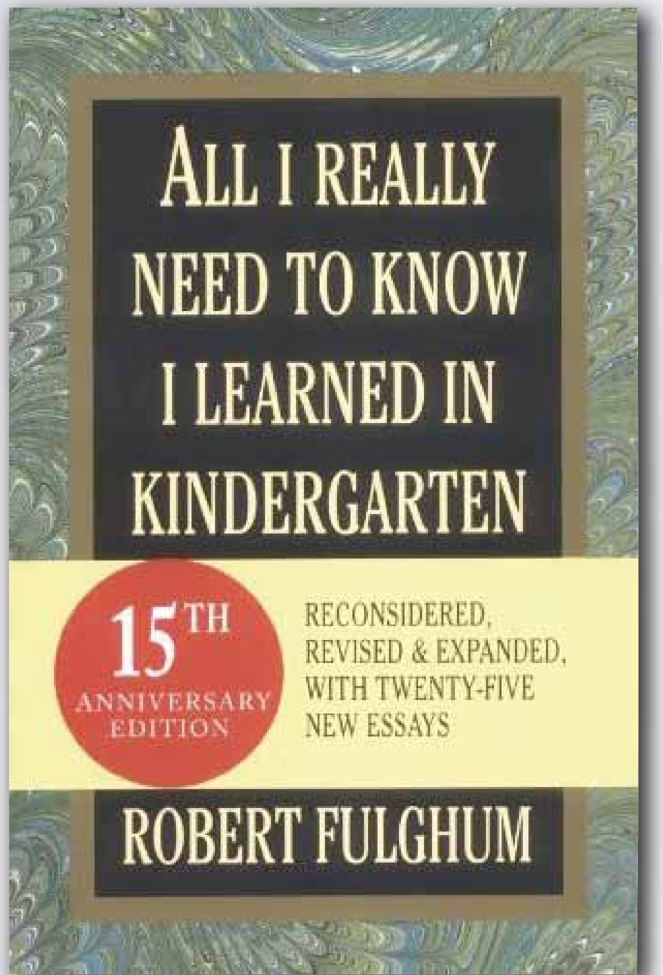
Classical Parametric Amplifier

Def: A parametric amplifier is a system that amplifies an input by varying a parameter (frequency) of the system.

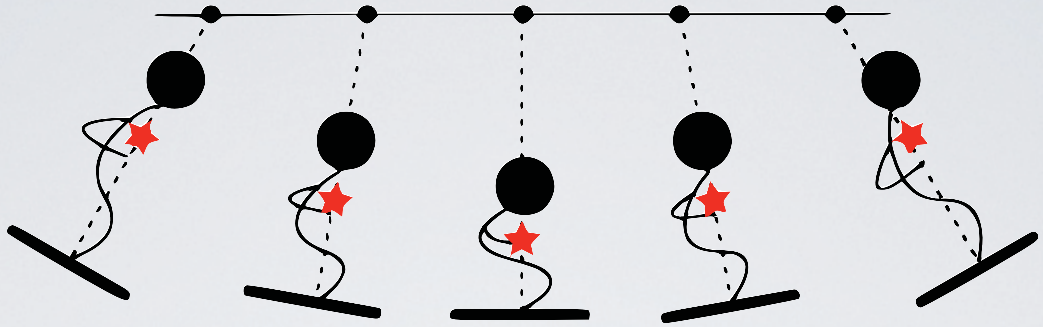
So easy, a child can understand!

Classical Parametric Amplifier





Classical Parametric Amplifier



Harmonic oscillator: $\theta(t) = \theta(0)\cos(\omega_s t) + \frac{L(0)}{m\omega_s l}\sin(\omega_s t)$

Modulate center of mass (★): $\omega_s(t) = \omega_s(0) + \epsilon\sin\left(\omega_{\rm cm} t\right)$

if $\omega_{\rm cm} = 2\omega_s$:

$$\theta(t) = \theta(0)\underline{e^{\epsilon t/2}}\cos(\omega_s t) + \frac{L(0)}{m\omega_s l}\underline{e^{-\epsilon t/2}}\sin(\omega_s t)$$

Occurs only if system is initially displaced.

In classical physics: $\theta(0) = L(0) = 0$

Quantum physics: $[\theta, L] \neq 0$ $\rightarrow$ $\theta(0) = \cancel{L}(0) = 0$

Can parametrically amplify any state, even the vacuum!

Closely related to particle production in quantum field theory.

Prelude to quantum amplification

Harmonic oscillator: $H = p^2/(2m) + m\omega^2 x^2/2$

$$[x, p] = m\,[x, \dot{x}] = i\hbar$$

Work in Heisenberg picture: $\ddot{x} + \omega^2 x = 0$

Decompose into creation and destruction operators:

complex conjugate

$$x(t) = f(t)a + \bar{f}(t)a^\dagger$$

“mode function” (not operator)

Mode functions satisfy: $\ddot{f}(t) + \omega^2 f(t) = 0$

Plug $x(t)$ into commutator:

demand = 1 for all time

$$\frac{m}{i\hbar}[x,\dot{x}] = \frac{m}{i\hbar}\left(f(t)\dot{\bar{f}}(t) - \bar{f}(t)\dot{f}(t)\right)[a,a^\dagger] = 1$$

Define inner-product: (Klein-Gordon inner-product)

$$\langle f,g\rangle \equiv \frac{im}{\hbar}\left[\bar{f}(t)\dot{g}(t) - g(t)\dot{\bar{f}}(t)\right]$$

Gives:

orthogonal

$$\langle f,f\rangle = 1 \qquad \langle f,\bar{f}\rangle = 0$$

$$\boxed{a^\dagger = -\langle \bar{f},x\rangle \qquad a = \langle f,x\rangle}$$

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ground state $\underset{\longrightarrow}{\overset{\longleftarrow}{a = \langle f, x \rangle}}$ mode function

Ground state: $a|0\rangle = 0$

Want ground state as eigenstate of H:

$$
\begin{aligned}
H|0\rangle &= \left(\frac{m\dot{x}^2}{2} + \frac{m\omega^2 x^2}{2}\right)|0\rangle \\
&= \frac{m}{\sqrt{2}}\overline{\left[\dot{f}(t)^2 + \omega^2 f(t)^2\right]}|2\rangle + \frac{m}{2}\left[\left|\dot{f}(t)\right|^2 + \omega^2\left|f(t)\right|^2\right]|0\rangle.
\end{aligned}
$$

Solutions: ($x_{\text{zp}} = \sqrt{\hbar/2m\omega}$)

$$f(t) = x_{\text{zp}} \exp(-i\omega t) \qquad \bar{f}(t) = x_{\text{zp}} \exp(+i\omega t)$$

"positive frequency" solution

"negative frequency" solution

thus:

$$x(t) = x_{\rm zp}\left(\underline{e^{-i\omega t}}a + \underline{e^{+i\omega t}}a^\dagger\right)$$

let us now modulate frequency (like the swing):

$$\ddot{x} + \omega(t)^2 x = 0$$

in: $\omega(t\to-\infty)=\omega_{\rm in}$	out: $\omega(t\to\infty)=\omega_{\rm out}$
$a_{\rm in}$	$a_{\rm out}$
$\vert 0\rangle_{\rm in}$	$\vert 0\rangle_{\rm out}$

$$f_{\rm in}(t)\big|_{t\to-\infty} \sim \exp\left(-i\omega_{\rm in}t\right) \qquad f_{\rm out}(t)\big|_{t\to+\infty} \sim \exp\left(-i\omega_{\rm out}t\right)$$

In general,

$$x(t) = f_{\rm in}(t)a_{\rm in} + \bar{f}_{\rm in}(t)a^\dagger_{\rm in} = f_{\rm out}(t)a_{\rm out} + \bar{f}_{\rm out}(t)a^\dagger_{\rm out}$$

Interested in “out” state given “in” is vacuum

- I know f_{in}

need 2 linearly independent solutions:

$$\langle f, \bar{f} \rangle = 0 \quad \Rightarrow \quad f_{\text{out}} = \alpha f_{\text{in}} + \beta \bar{f}_{\text{in}}$$

using $a_{\text{out}} = \langle f_{\text{out}}, x \rangle$:

$$a_{\text{out}} = \alpha a_{\text{in}} - \bar{\beta} a_{\text{in}}^{\dagger} \quad ; \quad |\alpha|^2 - |\beta|^2 = 1$$

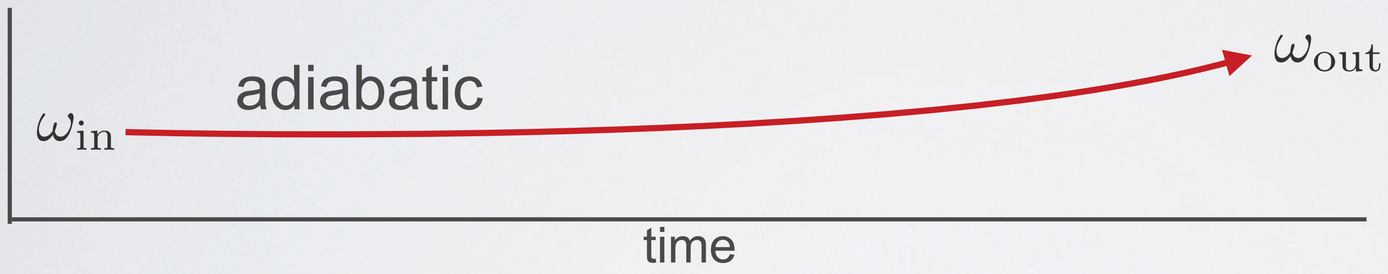
Bogoliubov transformation

All quantum amplifiers can be cast as Bogoliubov transformations.

if “in” state $|0\rangle_{\text{in}}$, then particle # at “out” state:

$$N_{\text{out}} = \langle 0|a^{\dagger}_{\text{out}} a_{\text{out}}|0\rangle_{\text{in}} = |\beta|^2$$

Particle # at “out” determined by negative frequency ($a^{\dagger}_{\text{in}}$) coefficient



$$|\beta|^2 \approx 0$$

$$|\beta|^2 = \frac{1}{4}\frac{\omega_{\text{in}}}{\omega_{\text{out}}}\left(1 - \frac{\omega_{\text{out}}}{\omega_{\text{in}}}\right)^2$$

Take home message:

All amplifiers described by Bogoliubov transform:

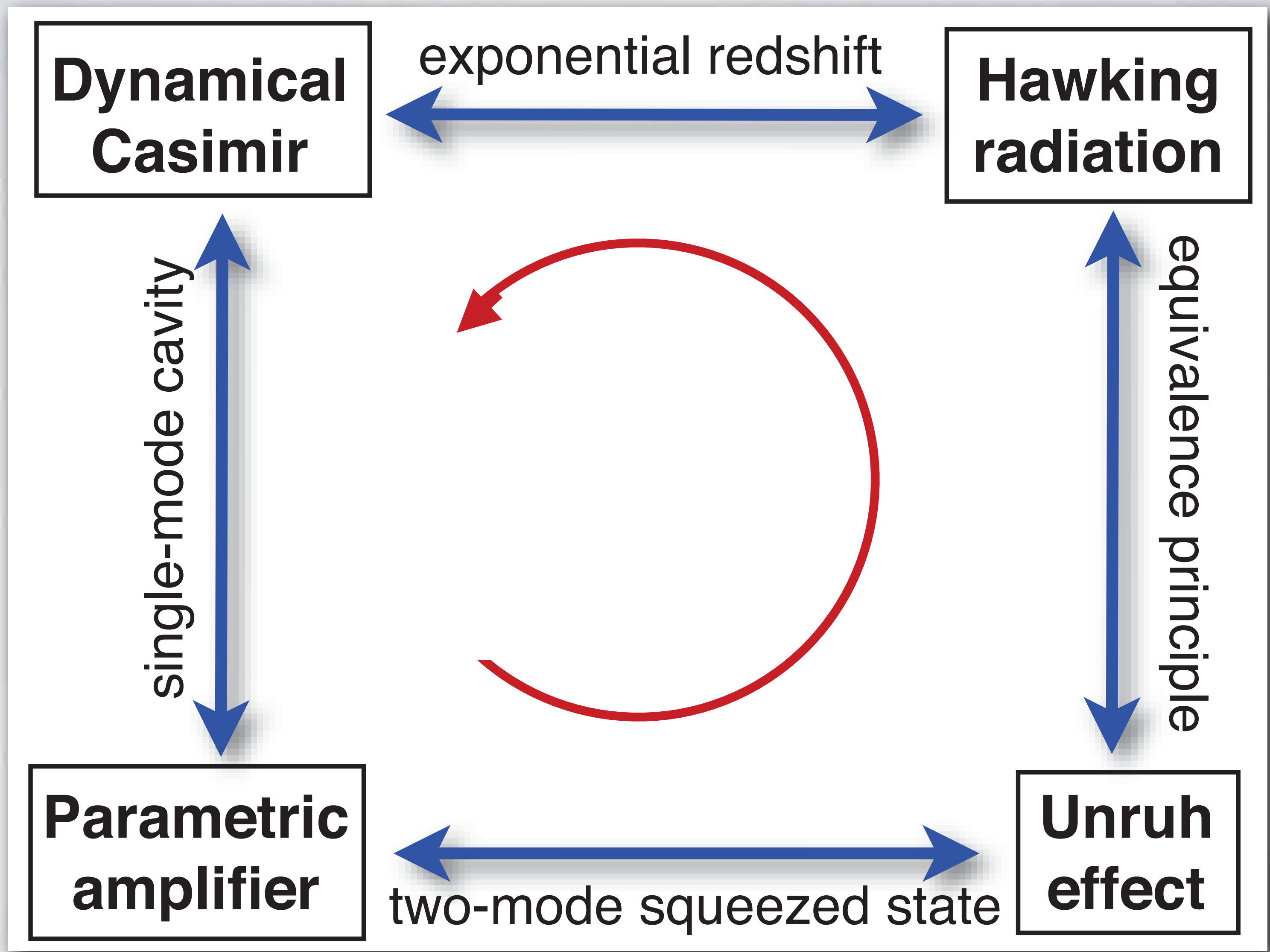
$$a_{\text{out}} = \alpha a_{\text{in}} - \bar{\beta} a_{\text{in}}^{\dagger} \qquad |\alpha|^2 - |\beta|^2 = 1$$

Vacuum defined via positive-frequency ($ae^{-i\omega t}$) components by choice of mode function.

Particles at "output" given by coefficient of negative-frequency "input state": $\bar{\beta} a_{in}^{\dagger}$

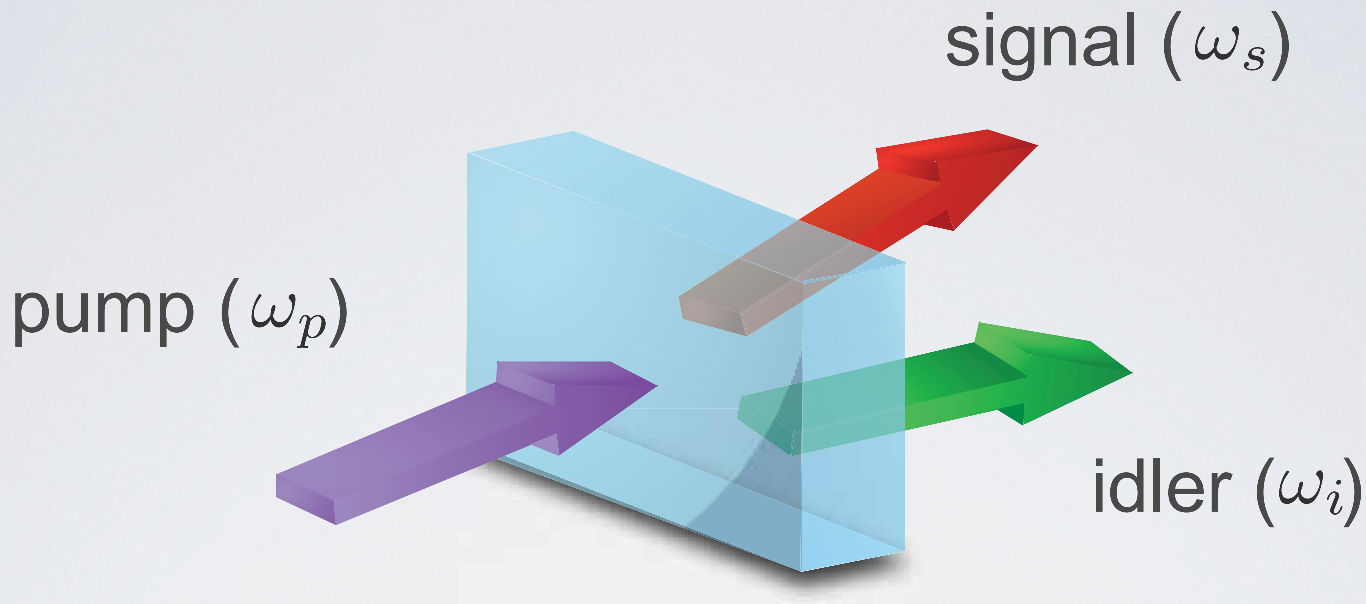
Nonzero β generated by modulating mode frequency non-adiabatically.

Vacuum Photon Production Methods



Quantum Parametric Amplifier

Simplest nonlinear photon interaction



Energy conservation: $\omega_p = \omega_s + \omega_i$

Optical par. amp. needs nonlinear medium (BBO,KTP)

Works for any system with effective $\chi^{(2)}$ nonlinearity.

Will assume pump mode is classical drive,unaffected by loss of photons.

i.e. unlimited, fixed amplitude supply of energy

Hamiltonian (rotating frame): $H = i\hbar\eta(b_{\mathrm{s}}^{\dagger}b_{\mathrm{i}}^{\dagger} - b_{\mathrm{s}}b_{\mathrm{i}})$

Pump amplitude in coupling constant: η (frequency)

Two modes of operation:

Degenerate (DPA): $\omega_s = \omega_i$

Non-degenerate (NDPA): $\omega_s \neq \omega_i$

DPA

Solve Heisenberg Eq. of Motion for $b_s = b_i = b$

$$b(t) = b(0)\cosh(2\eta t) + b(0)^\dagger \sinh(2\eta t)\,,$$

Bogoliubov transformation:

$$\alpha = \cosh(2\eta t) \qquad\qquad \underline{\beta = \sinh(2\eta t)}$$

If mode initially in ground state:

$$N = \left\langle b^\dagger(t) b(t) \right\rangle = |\beta|^2 = \sinh^2(2\eta t)$$

# of particles from vacuum

DPA

If $b_s = b_i = b$ $\Rightarrow$ $\omega_p = 2\omega_b$ (like the swing)

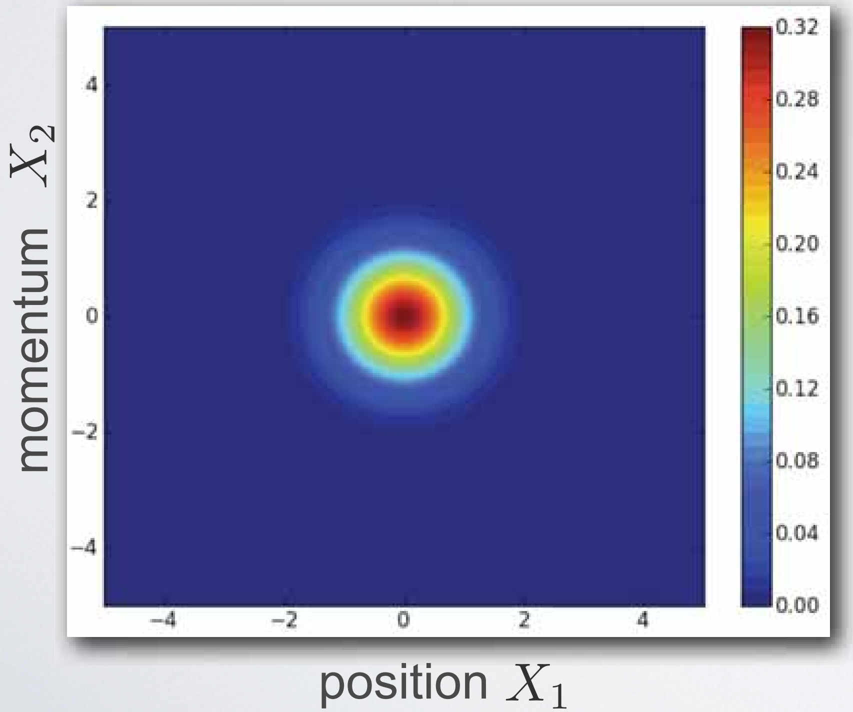
let $X_1 = b + b^\dagger$, $X_2 = -i(b - b^\dagger)$

squeezing parameter

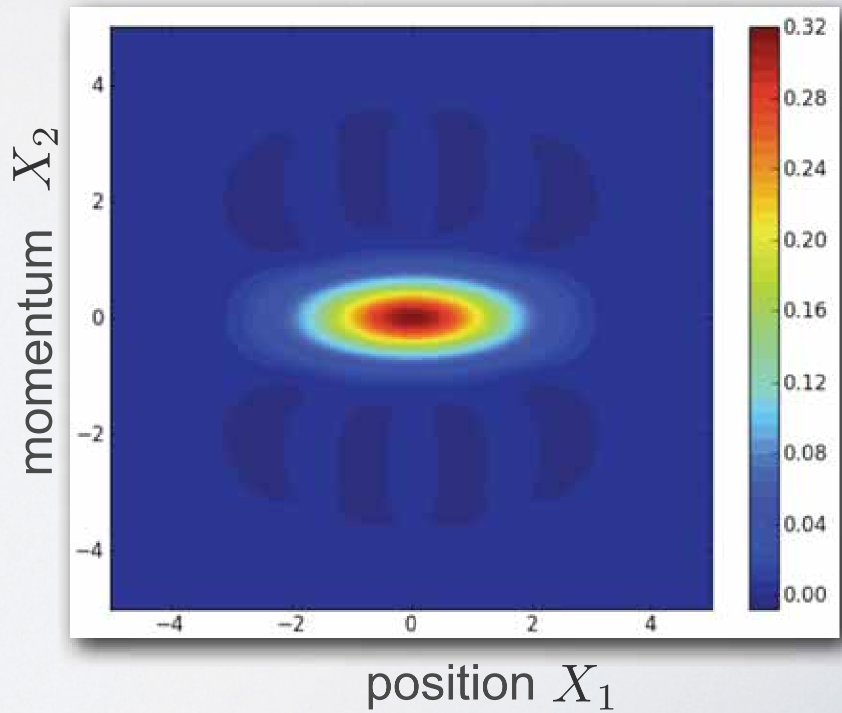
$$X_1(t) = e^{2\eta t} X_1(0) \qquad X_2(t) = e^{-2\eta t} X_2(0)$$

Squeezed State

Ground State



Squeezed state: $2\eta t = 0.5$



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NDPA

More general case: $\omega_s \neq \omega_i$

Heisenberg eqs. of motion:

$$b_s(t) = b_s(0)\cosh(\eta t) + b_i^\dagger(0)\sinh(\eta t)$$

$$b_i(t) = b_i(0)\cosh(\eta t) + b_s^\dagger(0)\sinh(\eta t)$$

$$N_s = N_i = \sinh^2(\eta t)$$

In Schrödinger picture: two-mode squeezed state

$$|\Psi(t)\rangle = \frac{1}{\cosh \eta t}\sum_{n=0}^{\infty}(\tanh \eta t)^n \, |n\rangle_s \otimes |n\rangle_i$$

squeezing parameter

Example of EPR state; mode-mode correlations stronger than classically allowed.

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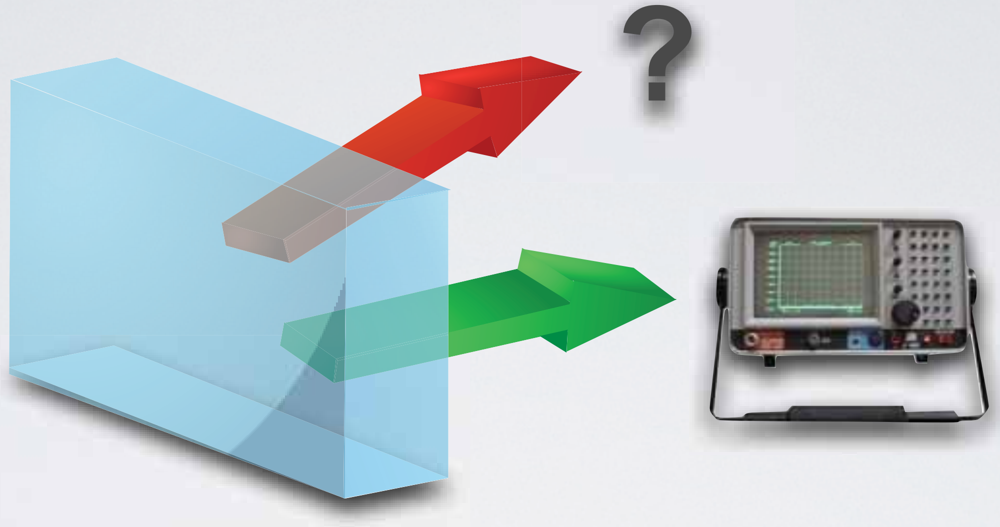
NDPA

What if only single-mode is observable?



Can do a partial trace, or we can think about it ...

Energy conservation → Implicitly know energy of other mode

Missing information ↔ Entropy

Find max. entropy constrained by energy

NDPA

Gives entropy for single-mode thermal state:

$$S = -\ln\left[1 - e^{-\hbar\omega_s/k_{\mathrm{B}}T(t)}\right] - \frac{\hbar\omega_s}{k_{\mathrm{B}}T(t)}\left[1 - e^{\hbar\omega_s/k_{\mathrm{B}}T(t)}\right]^{-1}$$

Temperature given via:

$$\tanh^2 \eta t = \exp\left[-\frac{\hbar\omega_s}{k_{\mathrm{B}}T(t)}\right]$$

Temperature is determined by squeezing parameter.

Bipartite system so both modes in thermal state.

Unruh Effect

Simplest particle production method: just accelerate!

Accelerating system called “Observer”

Need two coordinate systems (constant a):

$a = 0$: Minkowski Coordinates (ct, x)

$a > 0$: Rindler Coordinates $(c\tau, \xi)$

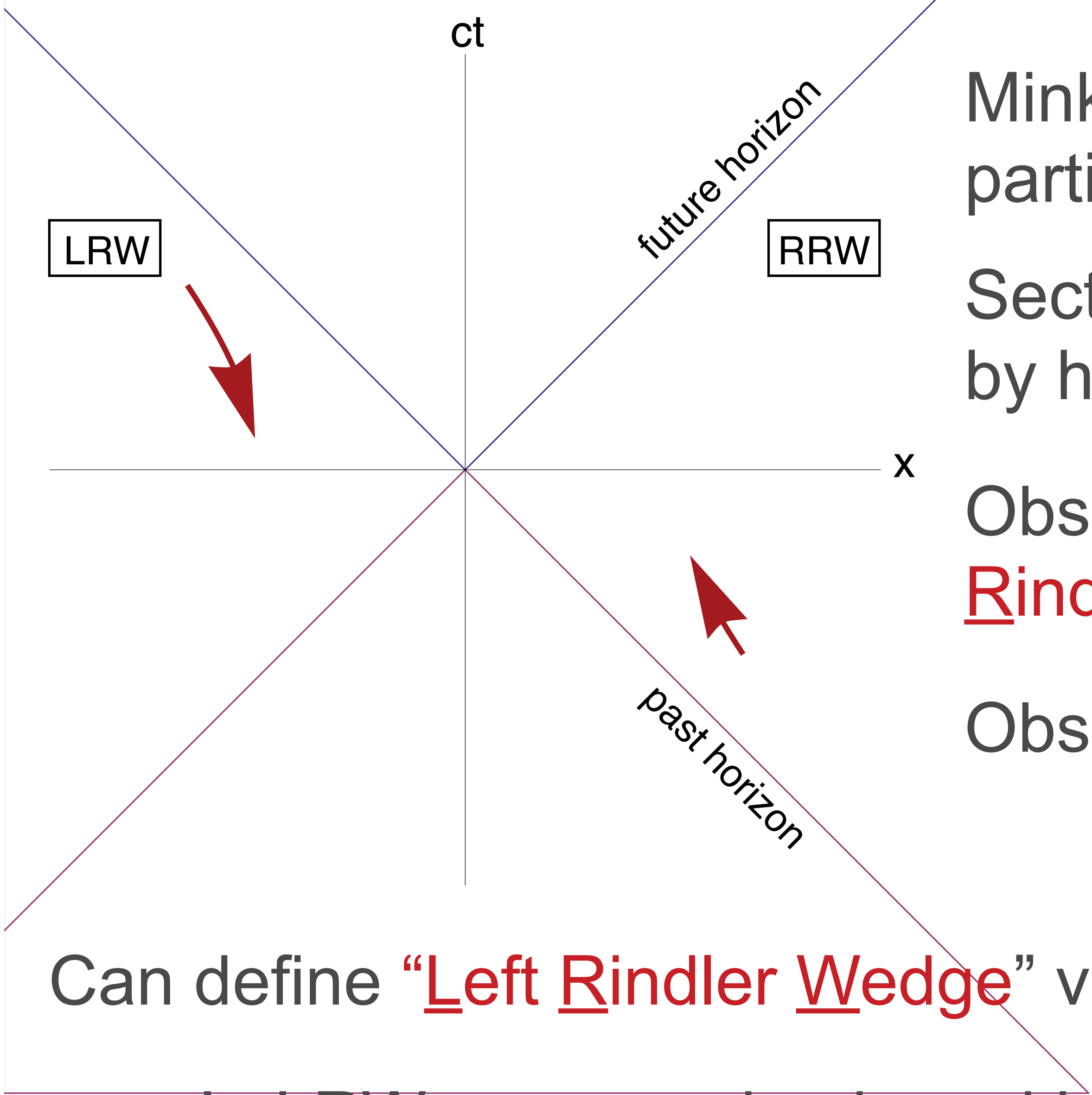
Rindler coordinates defined by relations:

$$ct = \xi \sinh\left(\frac{a\tau}{c}\right) \quad x = \xi \cosh\left(\frac{a\tau}{c}\right)$$

$$\xi = c^2/a$$

$\alpha = a/c$ acceleration frequency

Rindler coordinates are hyperbolas in Mi
coordinates:



Minkowski space-
partitioned into tw

Sections causally
by horizon at

Observer confine
Rindler Wedge"

Observer reaches

Can define "Left Rindler Wedge" via:

In LRW, moves backward in Minkowski time

Goal: Determine what Minkowski vacuum looks like in observers (Rindler) frame.

Follow harmonic oscillator example:

I. Find mode functions & vacuum states in both space-times.

II. Calculate Bogoliubov transformation linking operators in both space-times.

II. Evaluate number operator and determine quantum state.

Minkowski Space-time: (inertial observer)

Consider scalar-field: $\phi = \sum_j u^{\mathrm{M}}_{\omega_j} a^{\mathrm{M}}_{\omega_j} + \bar{u}^{\mathrm{M}}_{\omega_j} a^{\mathrm{M},\dagger}_{\omega_j}$

Solution to wave equation: $\omega_j = c|k_j| \quad -\infty \leq j \leq \infty$

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right]\phi = 0 \qquad u^{\mathrm{M}}_{\omega_j} = \frac{1}{\sqrt{4\pi\omega_j}} e^{ik_j x - i\omega_j t}$$

Minkowski time

Vacuum state:

$$|0\rangle^{\mathrm{M}} = \prod_j |0_{\omega_j}\rangle^{\mathrm{M}} \qquad a^{\mathrm{M}}_{\omega_j}|0\rangle^{\mathrm{M}} = 0 \quad \forall j$$

Rindler Space-time: (accel. observer)

Defines vacuum in Rindler coordinates $(c\tau, \xi)$

RRW:

proper time

$$|0\rangle^{\mathrm{R}} = \prod_j |0_{\omega_j}\rangle^{\mathrm{R}} \qquad u^{\mathrm{R}}_{\omega_j} \propto \exp\left(-i\omega_j \tau\right)$$

LRW has its own independent vacuum state:

$$|0\rangle^{\mathrm{L}} = \prod_j |0_{\omega_j}\rangle^{\mathrm{L}} \qquad u^{\mathrm{L}}_{\omega_j} \propto \exp\left(i\omega_j \tau\right)$$

Positive and negative frequency switch roles in LRW!

Need both LRW & RRW to get complete Minkowski

Guess the Bogoliubov transformation:

Remember vacuum defined by positive freq. terms

$$a^{\mathrm{M}}_{\omega_j} = C_1 a^{\mathrm{R}}_{\omega_j} + C_2 a^{\mathrm{L},\dagger}_{\omega_j} \qquad |C_1|^2 - |C_2|^2 = 1$$

positive frequency
in LRW

Minkowski vacuum should have particles when viewed by Rindler observer!

Single mode transformations:

$$b^{(1),\mathrm{M}}_{\omega_j} = a^{\mathrm{R}}_{\omega_j} \cosh(r) + a^{\dagger,\mathrm{L}}_{\omega_j} \sinh(r)$$
$$b^{(2),\mathrm{M}}_{\omega_j} = a^{\mathrm{L}}_{\omega_j} \cosh(r) + a^{\dagger,\mathrm{R}}_{\omega_j} \sinh(r)$$

Operators transform exactly like NDPA!

$\tanh r = \exp(-\pi\omega_j/\alpha)$ squeezing parameter

Same two-mode squeezed state:

$$|0_{\omega_j}\rangle^{\mathrm{M}} = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (\tanh r)^n \underbrace{|n_{\omega_j}\rangle^{\mathrm{L}}}_{\text{particles in LRW}} \otimes \underbrace{|n_{\omega_j}\rangle^{\mathrm{R}}}_{\text{particles in RRW}}$$

Thermal state determined by squeezing parameter:

$$\tanh^2(r) = e^{-2\pi\omega/\alpha} = \exp\left(-\frac{\hbar\omega}{k_\mathrm{B} T_\mathrm{U}}\right)$$

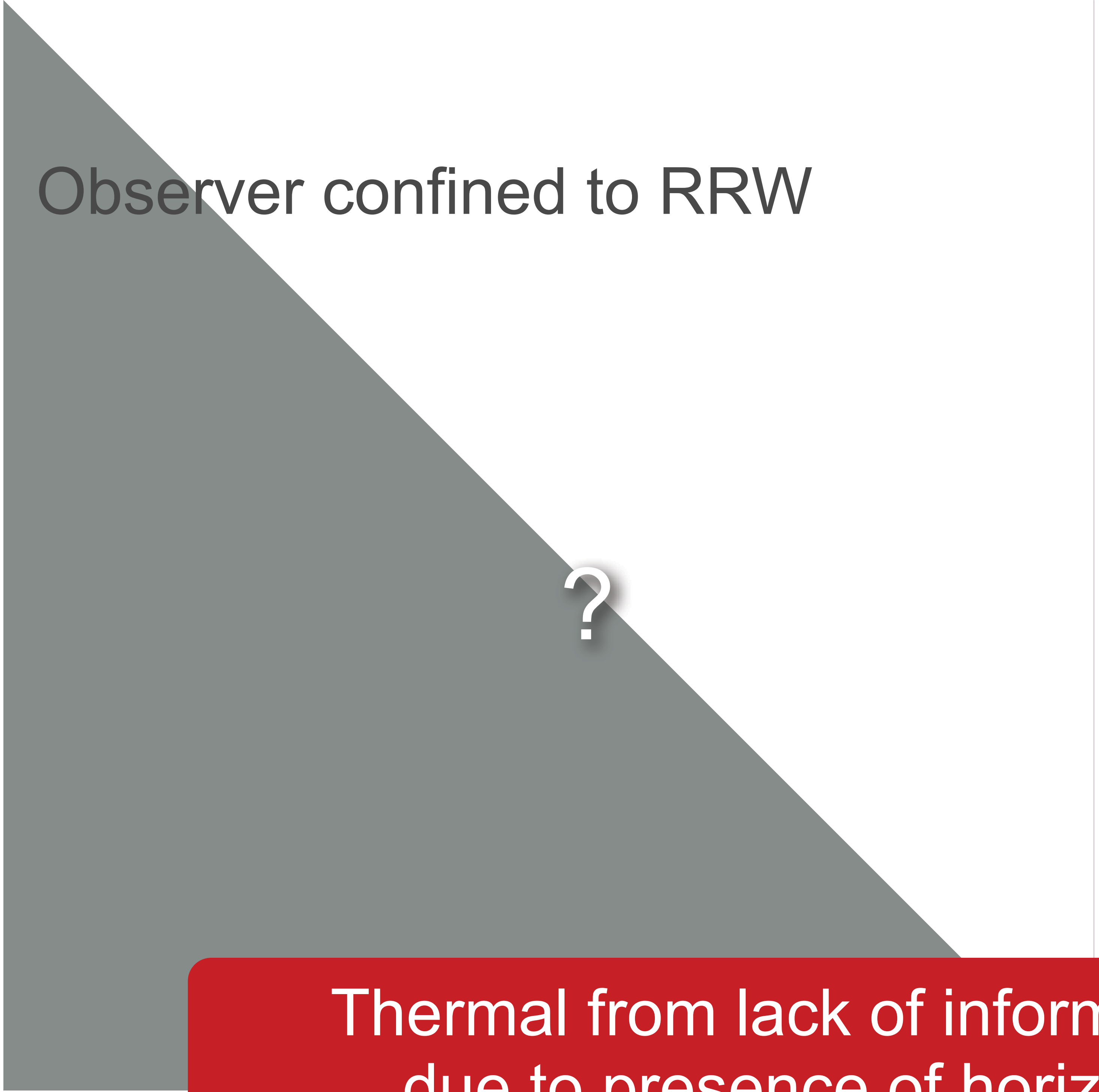
Defines Unruh temperature:

$$T_\mathrm{U} = \frac{\hbar\alpha}{2\pi k_\mathrm{B}}$$ depends only on accel. freq: $\alpha = a/c$

Same temperature for all frequencies; real thermal state (not true for NDPA)

Why thermal?

Observer confined to RRW



Thermal from lack of information due to presence of horizon.

How do mode frequencies transform?

Plane wave: $\phi(x,t) = \exp\left[-i\Omega\left(t - x/c\right)\right]$

Rewrite in Rindler coordinates:

$$\phi(\tau) = \exp\left[i\frac{\Omega}{\alpha}\left(e^{-\alpha\tau}\right)\right]$$

mode frequency
exponentially
red-shifted

Red-shifting gives rise to thermal spectrum.

- General property of horizons

Where does energy come from?

Have assumed constant acceleration.

Acceleration driven by fixed amplitude, classical unlimited source of energy.

→ Exact same assumption as parametric amplifier.

<u>Note:</u>

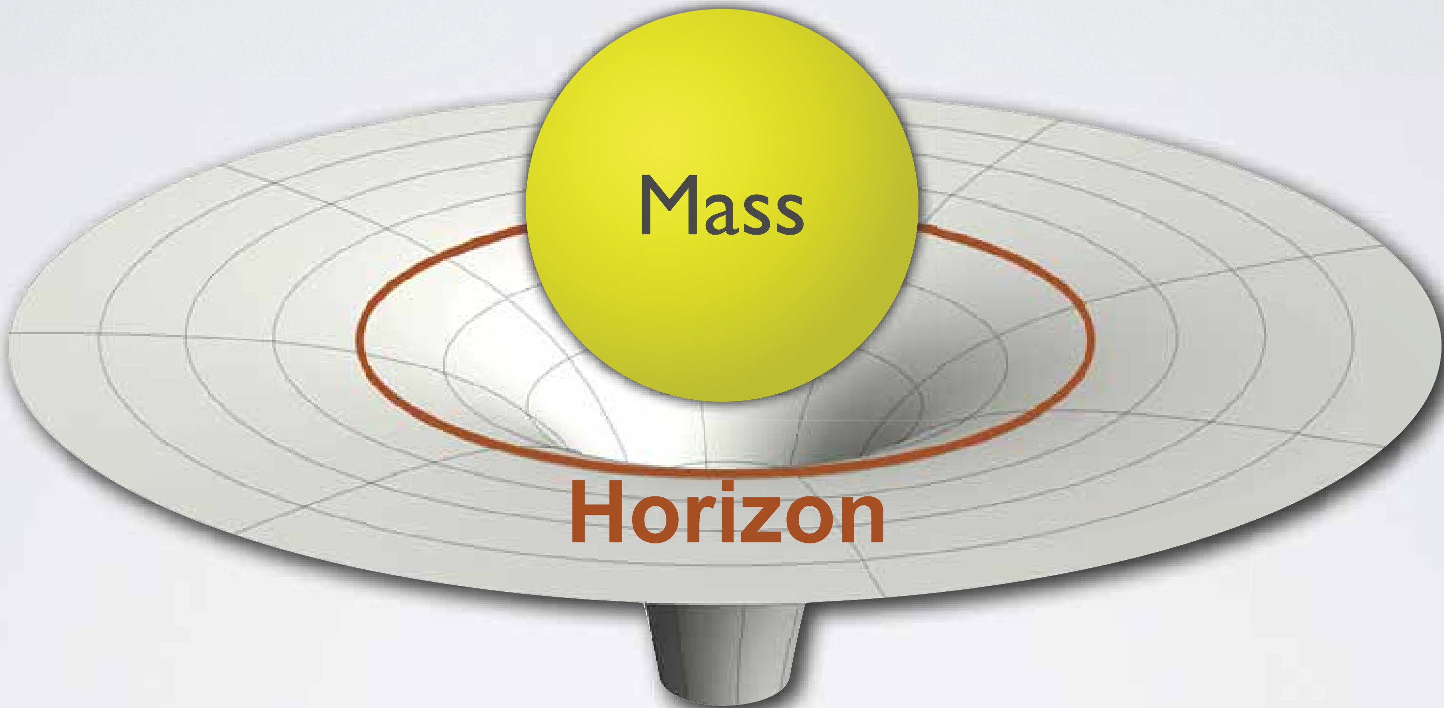
Acceleration → Temperature → Decoherence

Testbed for relativistic quantum information.

Hawking Radiation

Black Hole: Region of space-time separated from the rest of the universe by a horizon inside which gravity is so strong that not even light can escape.

Schwarzschild black hole; depends only on mass.



Horizon acts like unidirectional surface

can go inside; can not come out

→ causally disconnected from outside


slow
water
fast
water
Horizon

Horizons in the kitchen

Horizon located at Schwarzschild radius:

$$r_s = 2GM/c^2 \quad \Rightarrow \quad A_{\rm BH} = 4\pi r_s^2$$

Use Einstein $E = Mc^2$ for energy conservation relation

$$dE = c^2 dM = \frac{\kappa c^2}{8\pi G} dA_{\rm BH}$$

surface gravity: $\kappa = \dfrac{c^4}{4GM}$

The force/mass needed to keep a small test mass stationary at the horizon as viewed at infinity.

Horizon unidirectional; mass can only increase:

$$dA_{\mathrm{BH}} \geq 0$$

Looks like 1st-law of thermodynamics: $dS \geq 0$

$$dE = c^2 dM = \frac{\kappa c^2}{8\pi G} dA_{\mathrm{BH}}$$

$$\Downarrow$$

$$dE = T_{\mathrm{H}} dS_{\mathrm{BH}} = \frac{\hbar \kappa}{8\pi k_{\mathrm{B}} c} \lambda dS_{\mathrm{BH}}$$

Implies that a black hole has a temperature

Hawking (1974) showed that it does: (Hawking temp.)

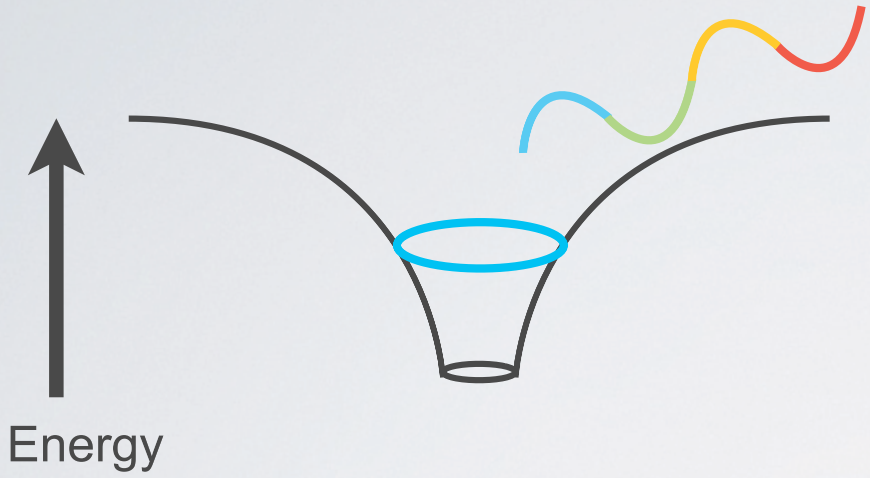
$$T_{\mathrm{H}} = \frac{\hbar\gamma}{2\pi k_{\mathrm{B}}} \qquad \gamma = \kappa/c$$ characteristic frequency

Hawking radiation as particle production:

It is the Unruh effect! Viewed by a different observer.

Hawking radiation viewed at infinity

$\gamma = \kappa/c$ includes redshift factor:



$$\kappa = Va\big|_{r=r_s}$$

$$V(r) = \sqrt{1 - r_s/r}$$

$$a(r) = \frac{GM}{r^2\sqrt{1-\frac{r_s}{r}}}$$

What does observer stationary at horizon see?

$$r \to r_s \qquad T \to \frac{\hbar(a/c)}{2\pi k_{\mathrm{B}}} = \frac{\hbar\alpha}{2\pi k_{\mathrm{B}}}$$

Recover Unruh effect

Einstein's Equivalence Principle in Action!

Where does energy come from?

Energy in Hawking radiation must be balanced by loss of mass:

$$\underbrace{(Mc^2 - E_H)}_{\text{Black hole energy}} + \underbrace{E_H}_{\text{radiation energy}}$$

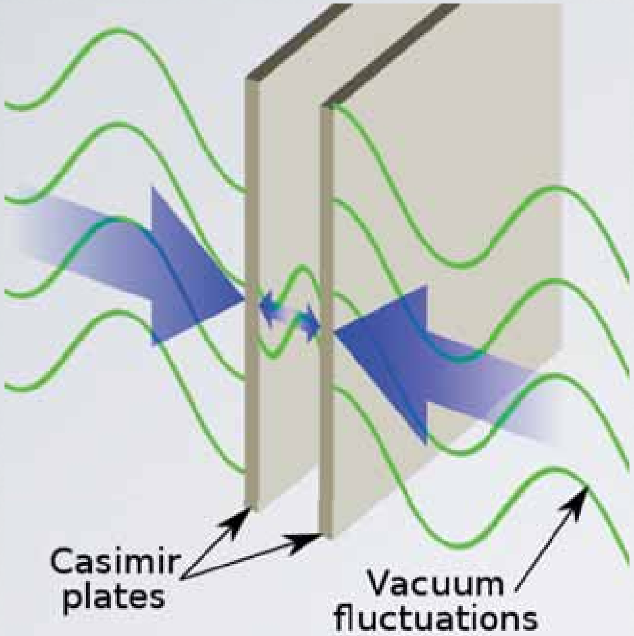
Black hole mass assumed fixed in calculation

- Energy source classical & fixed amplitude

Valid only for large black holes: $\delta M/M \ll 1$

Dynamical Casimir Effect
(DCE)

Dynamical Casimir (Moore) effect:



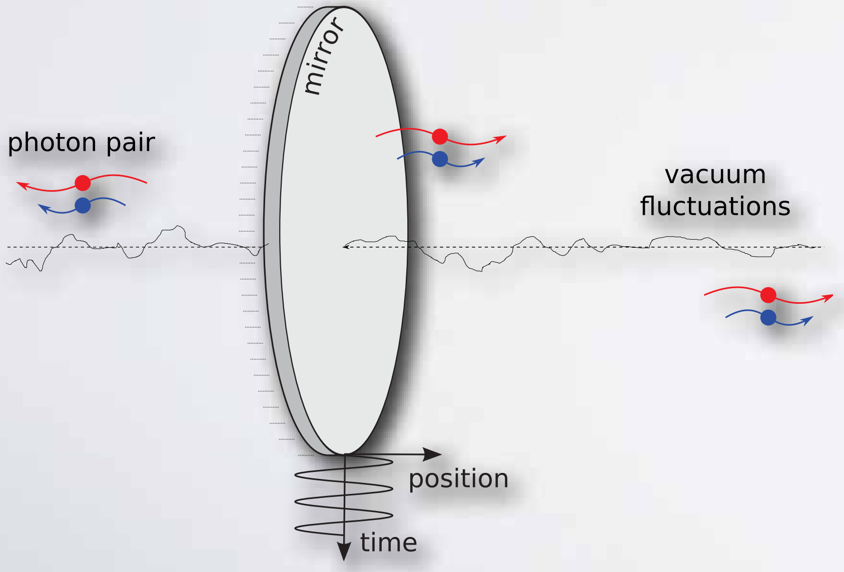
- Boundary conditions lead to discrete mode structure.

- “Shake” one of the mirrors:

$$\omega_n = \frac{2\pi c_0}{\lambda_n}$$ Modulates frequency

changes wavelength

1D Mirror - Fulling & Davies:



- Continuum of modes

- Path modifies exponent of modes negative frequency term

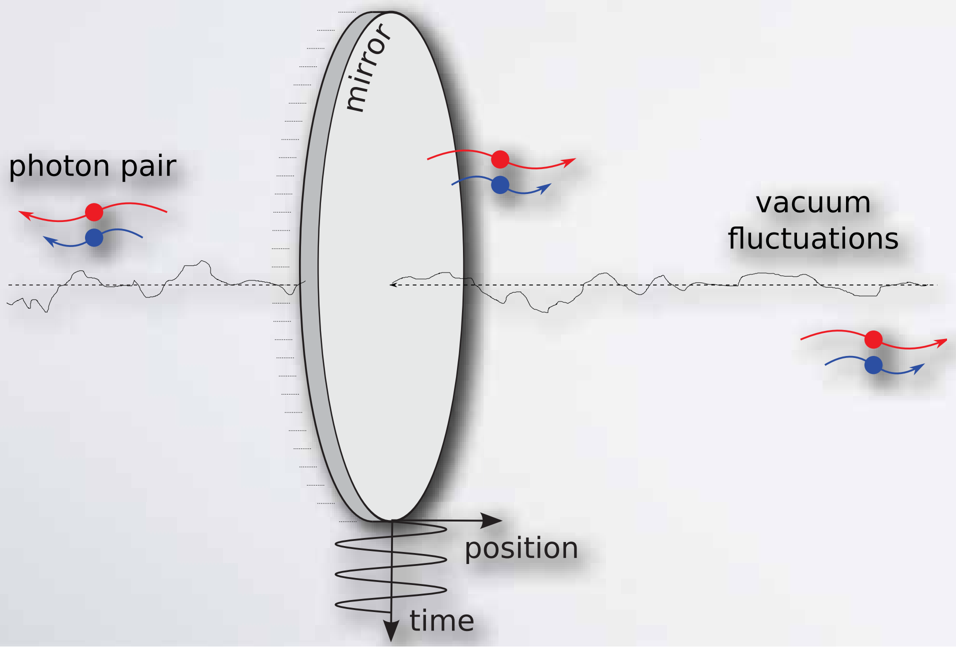
$$e^{i\omega u} \rightarrow e^{i\omega(u-\tau_u)}$$ path dependent

Can choose any mirror trajectory you like.

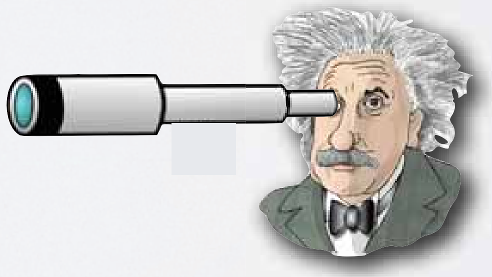
Experimentalist: Sinusoidal

Theorist: $x(t) = -t - A\exp\left(-\kappa t\right) + B$

→ Exponential red-shift of frequency & thermal state

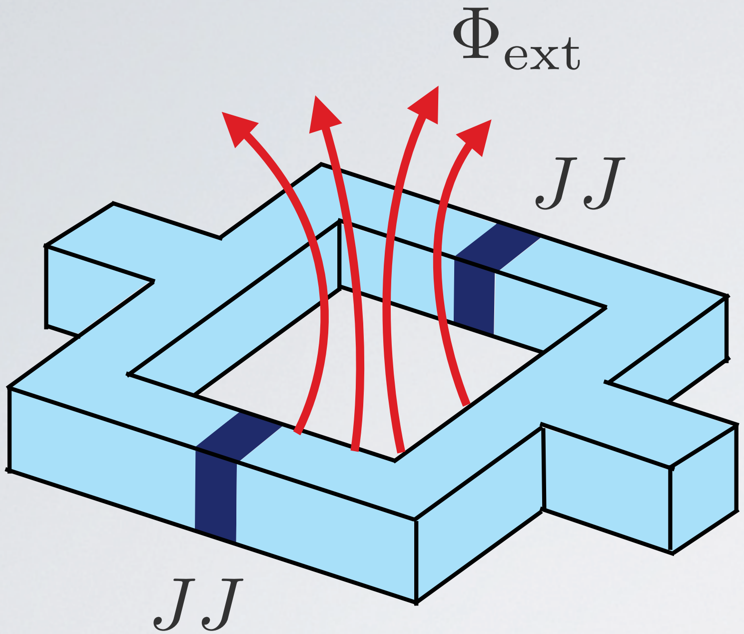


$$T = \frac{\hbar\gamma}{2\pi k_B}$$



Dynamical Casimir Effect
in Circuit-QED

dc-SQUID



Flux through loop changes characteristic energy scale

$$E_{\mathrm{J}}\left(\Phi_{\mathrm{ext}}\right) = 2E_{\mathrm{J}}\left|\cos\left(\pi\frac{\Phi_{\mathrm{ext}}}{\Phi_0}\right)\right|$$

Also can be thought of as nonlinear inductor:

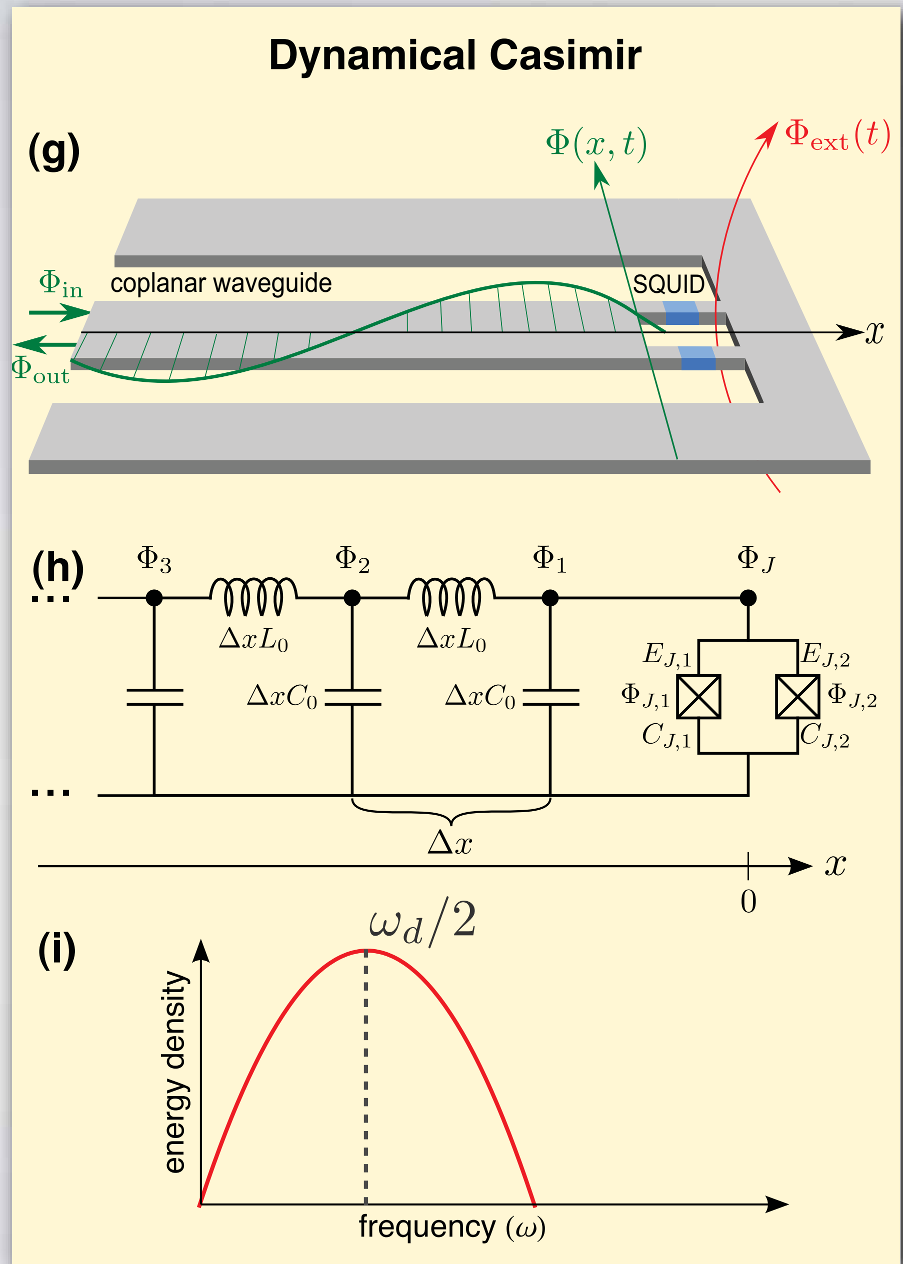
$$L\left(\Phi_{\mathrm{ext}}\right) = \frac{\Phi_0}{2\pi I_{\mathrm{c}}}\sec\left(\pi\frac{\Phi_{\mathrm{ext}}}{\Phi_0}\right)$$

Recall LC-oscillator:

$$\omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad \omega\left(\Phi_{\mathrm{ext}}\right) = \frac{1}{\sqrt{L\left(\Phi_{\mathrm{ext}}\right)C}}$$

Dynamical Casimir (1D type):

Wilson et al., Nature 479, 376 (2011)



SQUID gives perfect mirror boundary condition:

$$\Phi(x = L^0_{\text{eff}}, t) = 0$$

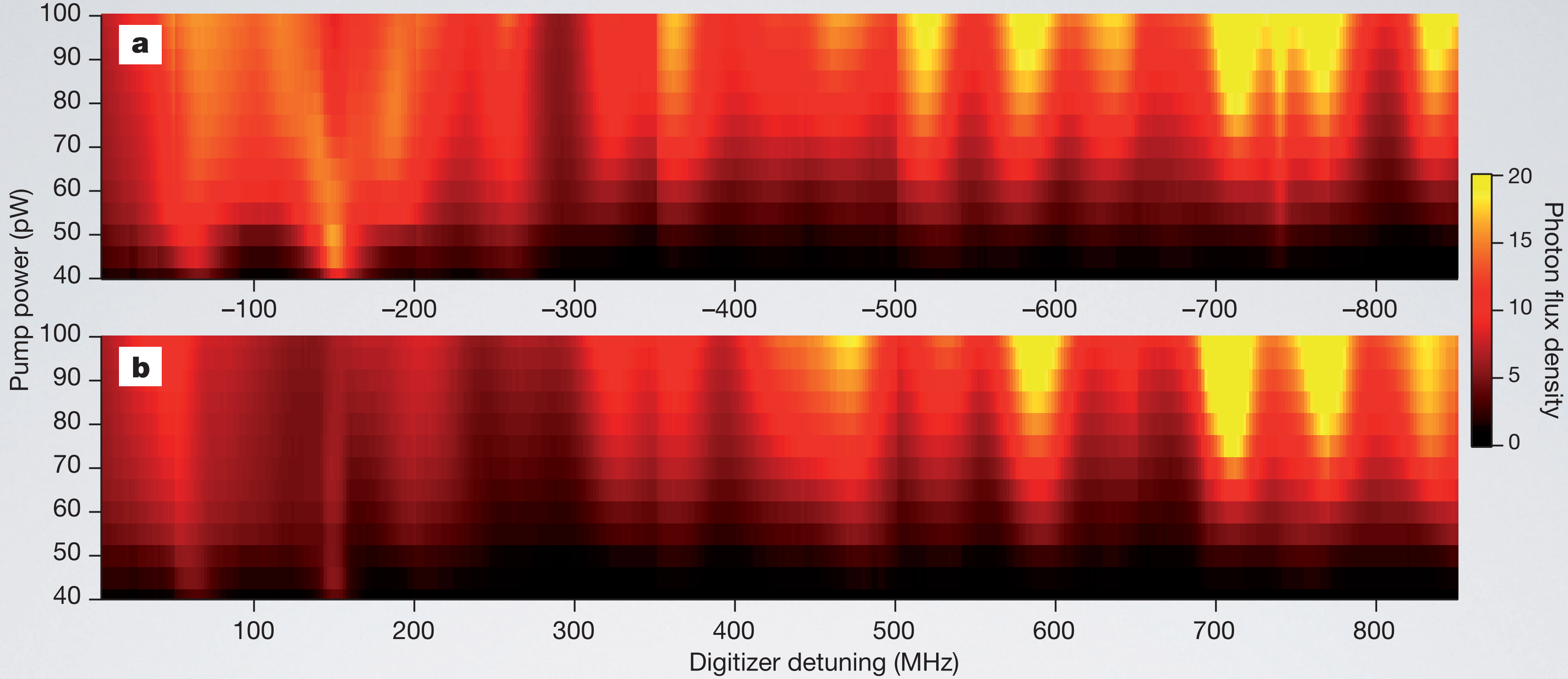
Sinusoidal modulation:

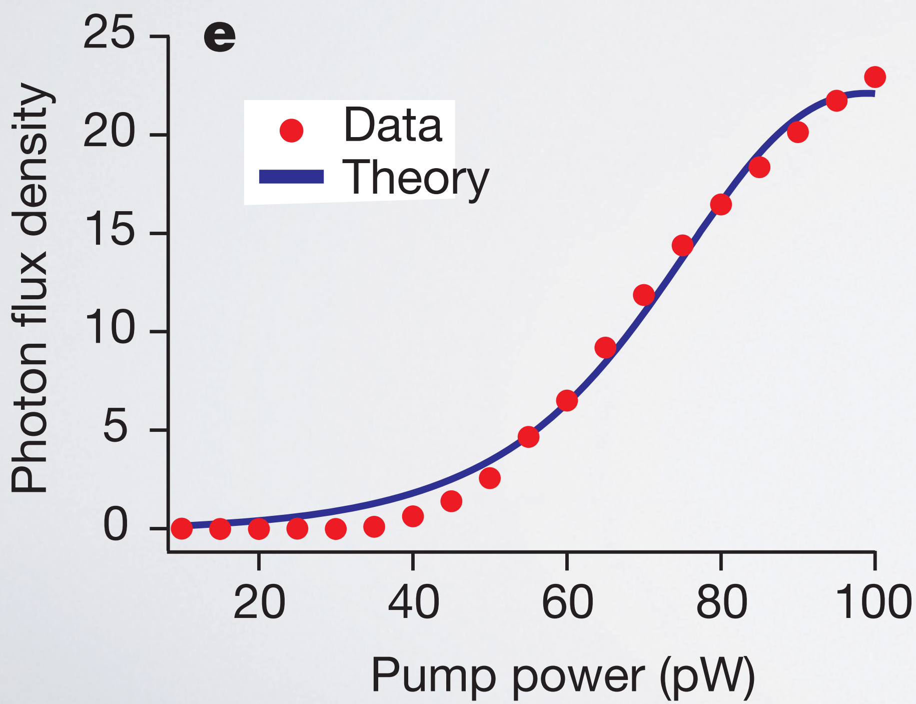
$$E_{\text{J}} = E^0_{\text{J}} + \delta E_{\text{J}} \cos(\omega_{\text{d}} t)$$

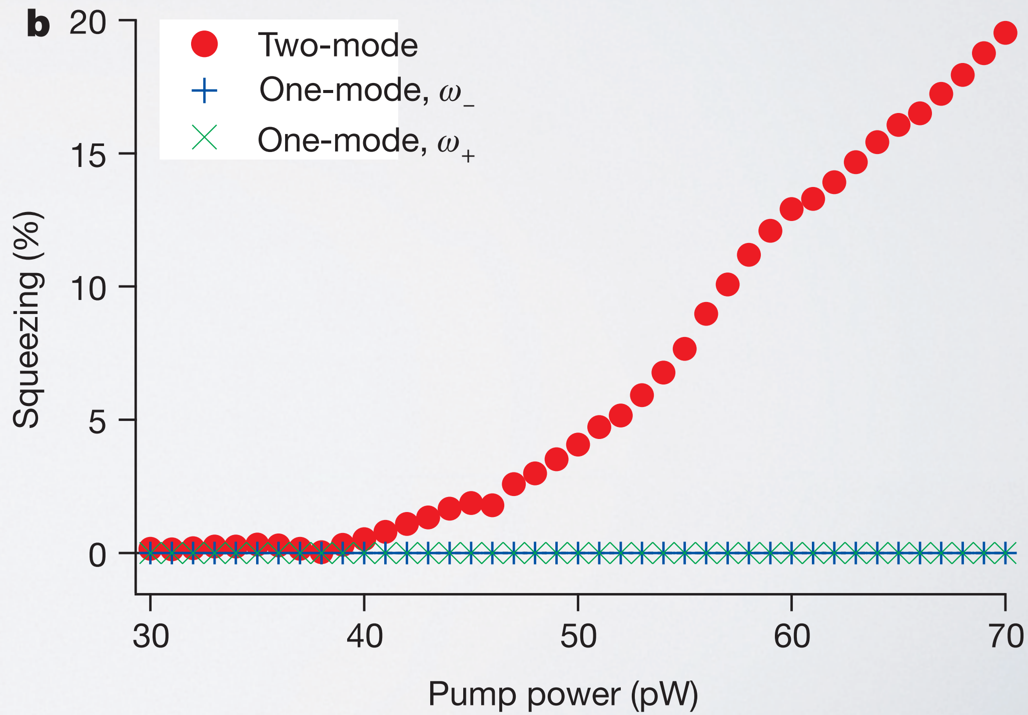
Effective length change:

$$\delta L_{\text{eff}} = L^0_{\text{eff}} \delta E_{\text{J}} / E^0_{\text{J}}$$

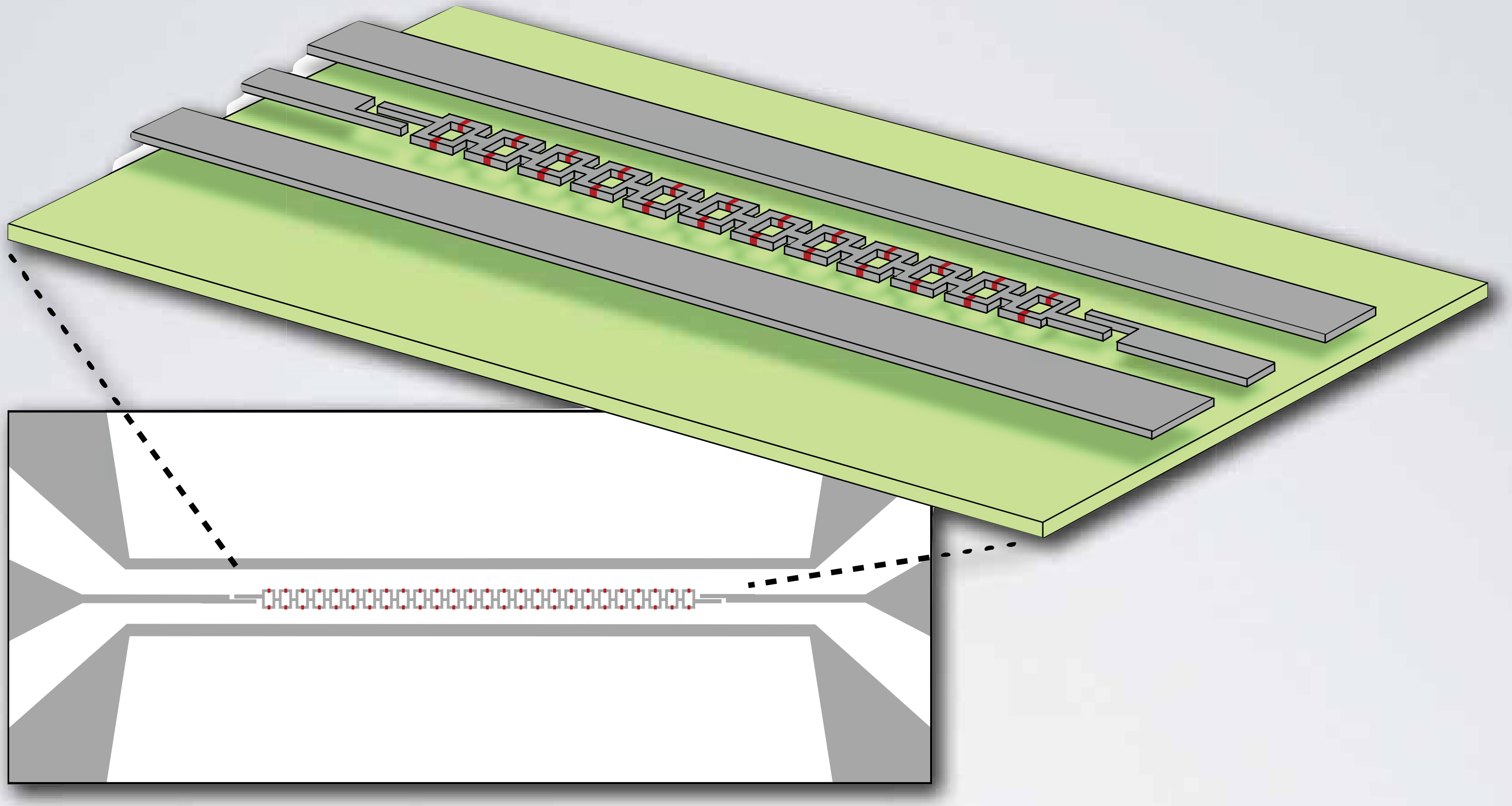
Spectrum symmetric about: $\omega_d/2$


a
b
Pump power (pW)
100
90
80
70
60
50
40
−100
−200
−300
−400
−500
−600
−700
−800
100
200
300
400
500
600
700
800
Digitizer detuning (MHz)
Photon flux density
20
15
10
5
0


e
Data
Theory
Photon flux density
25
20
15
10
5
0
20
40
60
80
100
Pump power (pW)


b
Two-mode
One-mode, ω_
One-mode, ω+
Squeezing (%)
20
15
10
5
0
30
40
50
60
70
Pump power (pW)

Dynamical Casimir (cavity type):



Microwave photons travel in medium (not vacuum)

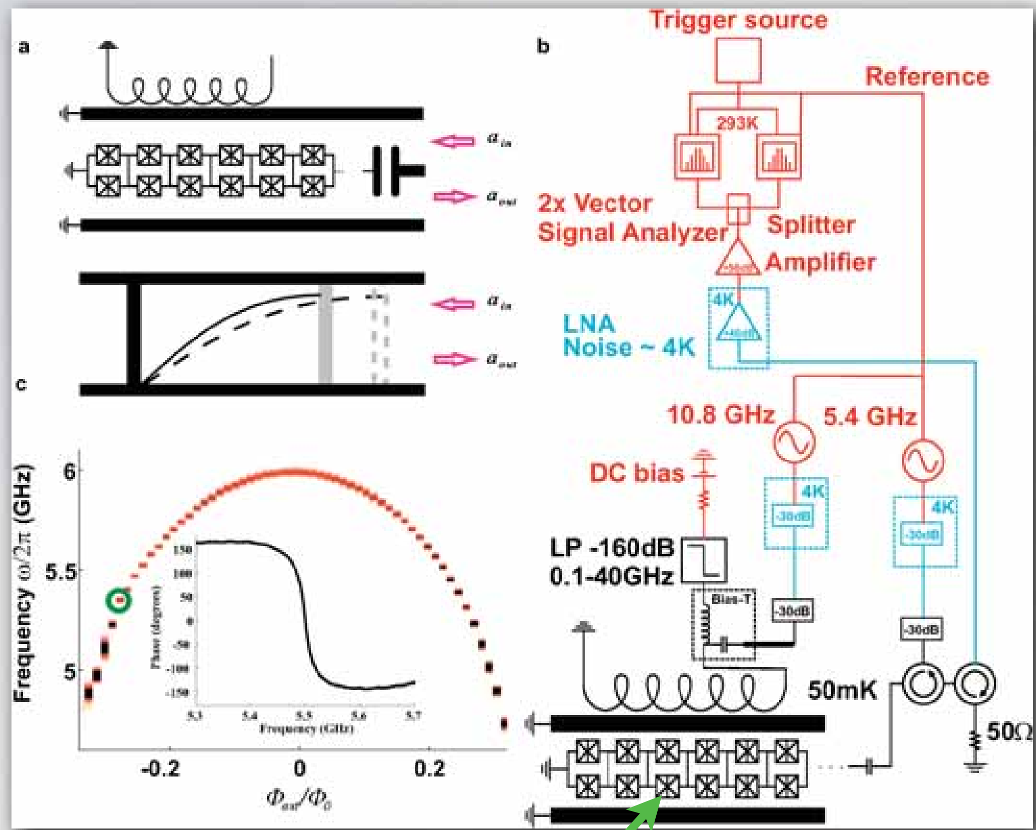
$$\omega_n = \frac{2\pi c_{\mathrm{sq}}}{\lambda_n} \qquad c_{\mathrm{sq}} = 1/\sqrt{l\,(\Phi_{\mathrm{ext}})\,c}$$

fixed

per unit length

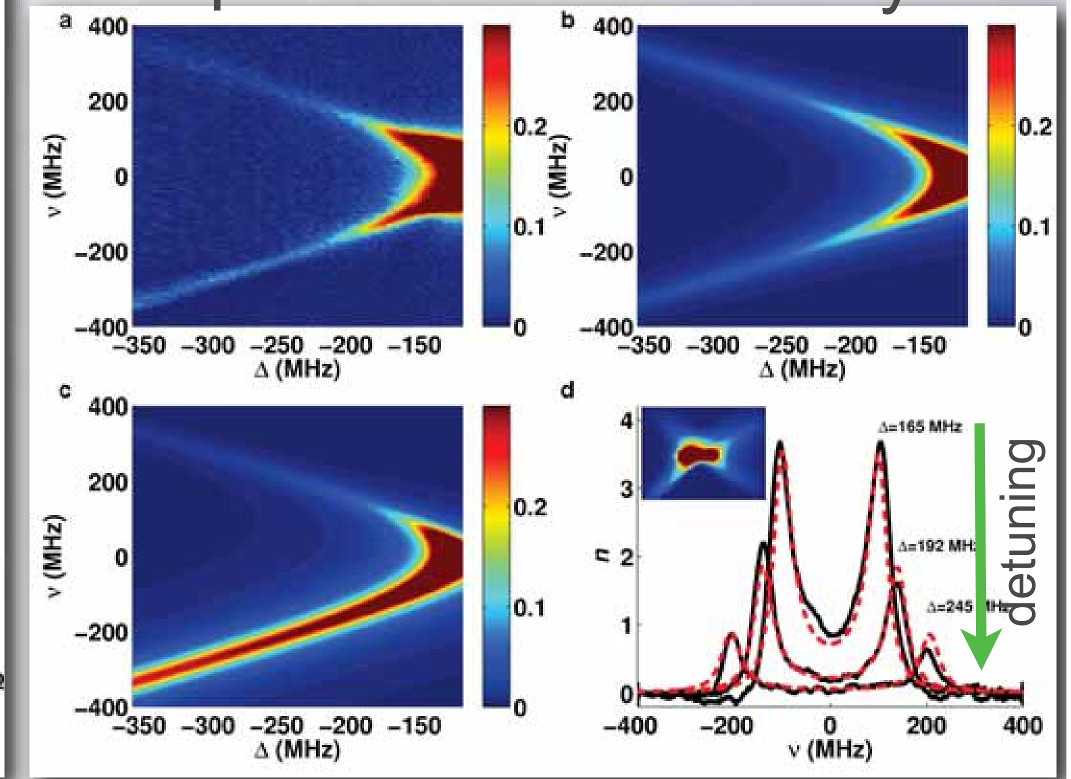
Dynamical Casimir:

Lähteenmäki et al., arXiv:1111.5608 (2011)



Experiment

Theory



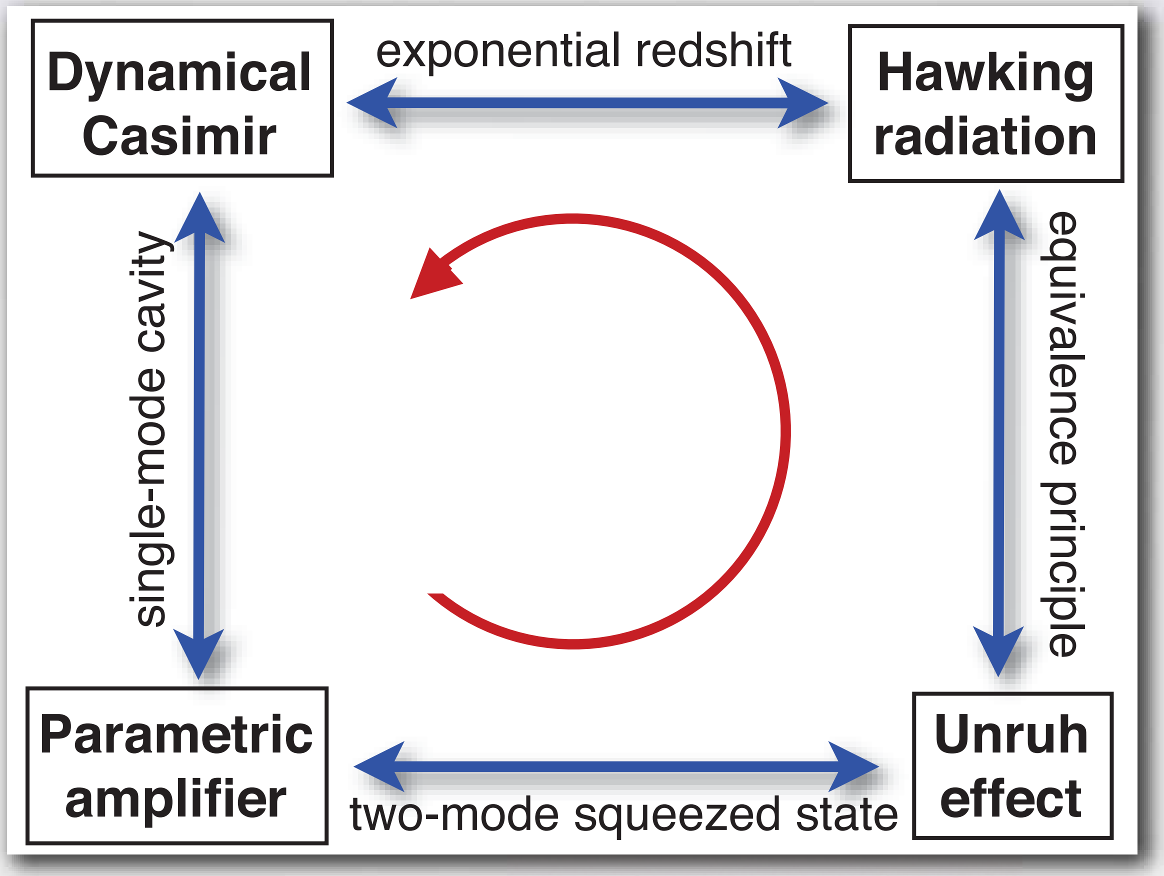
Classical

Cavity: 250 SQUIDs
tunable by bias line

Similar to Castellanos-Beltran 2007 & 2008

For single cavity mode:

$$H_{\mathrm{DCE}} = \frac{\delta L_{\mathrm{eff}}}{4 d_{\mathrm{eff}}} \frac{\hbar \omega_d}{2} \left[\left(a^{\dagger}\right)^2 - a^2 \right]$$ DPA Hamiltonian

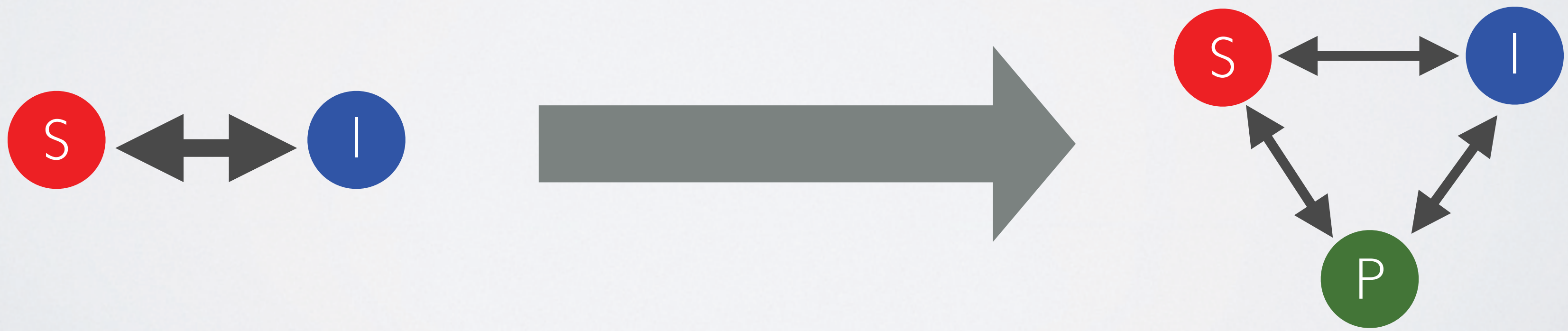


Summary

Can learn a lot by playing outside!

Surprising generality to quantum vacuum amplifcation.

- Described by Bogoliubov transforms
- Modulate system frequencies
- Same quantum states
- * - Same classical fixed amplitude energy source approximation.



Thermal character of states vanishes!