General Aspects on Classical and Quantum Correlation- II

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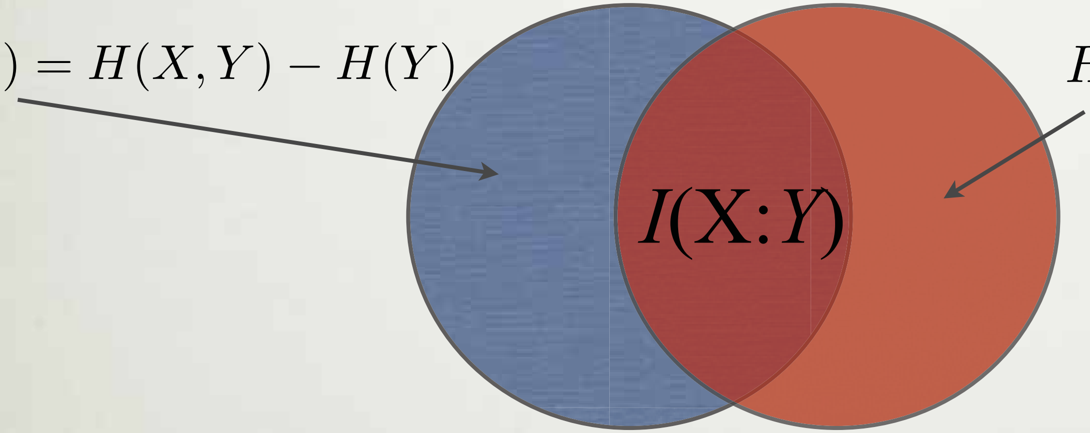
- II-Applications and thermodynamical aspects

- Review
    - Classical Mutual Information
    - Measurement process
    - Local accessible and Local Inaccessible Mutual Information
- Thermodynamical aspects
    - Redefinition of Discord and measurement interpretation
    - Correlation as a mean to produce Work
    - Quantum Deficit
    - Irreversibility of entanglement
    - Distribution of Correlation

Mutual Information (Classical)



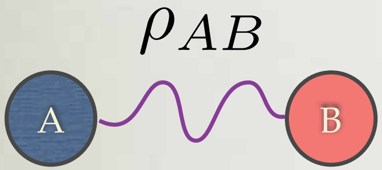
$$H(X) = -\sum_j p(x_j)\log_2 p(x_j) \qquad H(Y) = -\sum_k p(y_k)\log_2 p(y_k)$$

$$H(X,Y) = -\sum_{j,k} p(x_j, y_k)\log_2 p(x_j, y_k)$$

$$I(X:Y) \equiv H(X:Y) = H(X) + H(Y) - H(X,Y)$$

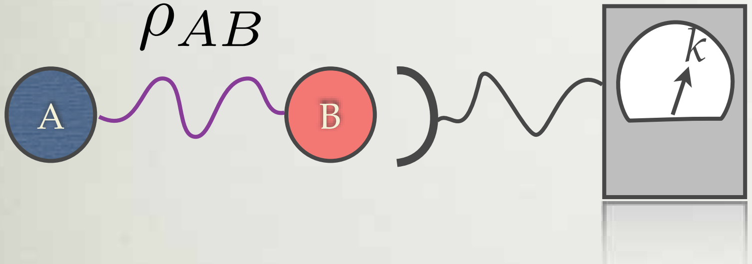
Post and pre-selected states

Post and pre-selected states



Post and pre-selected states

Measurement on B with outcome k



$$\rho^k_{AB} = \frac{\Pi_k \rho_{AB} \Pi_k}{p_k} \qquad \Pi_k = |\phi_k\rangle\langle\phi_k|_B$$

$$p_k = Tr\{\Pi_k \rho_{AB}\}$$

Post and pre-selected states

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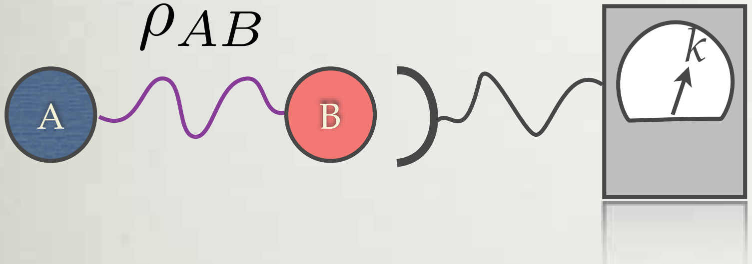

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$$\boxed{\rho_A^k = \frac{Tr_B\{\Pi_k \rho_{AB} \Pi_k\}}{p_k}}$$

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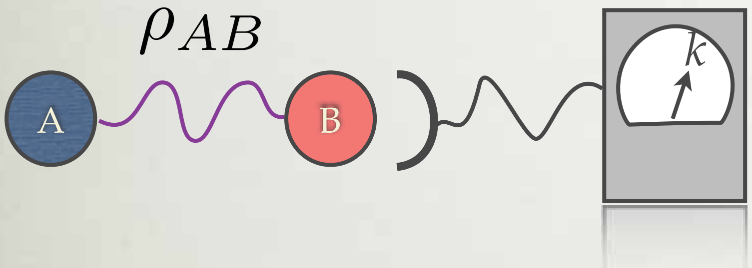

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$$p_k = Tr\{\Pi_k \rho_{AB}\}$$

$$\rho_A^k = \frac{Tr_B\{\Pi_k \rho_{AB} \Pi_k\}}{p_k}$$

Post-selected state

$$\rho_A = \sum_k p_k \rho_A^k = Tr_B\{\rho_{AB}\}$$

Pre-selected state

(Quantum) Mutual Information

$$S(A:B) = S_A - S(A|B)$$

$$J_{A|B} = S(\rho_A) - \sum_j p_j S(\rho_A^j)$$

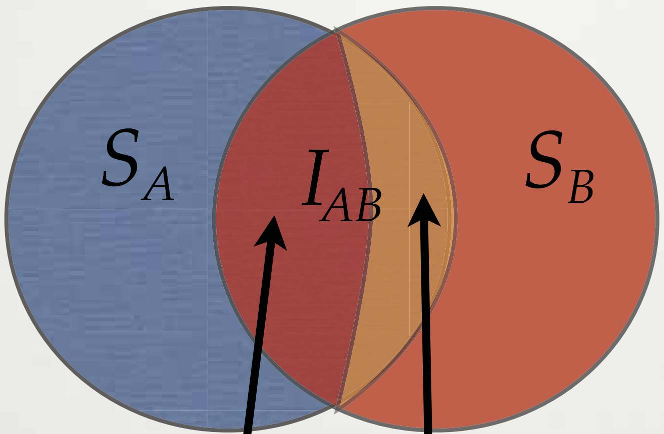
$$p_j = \mathrm{Tr}_{AB}\left\{\Pi_j^B \rho_{AB} \Pi_j^B\right\}, \quad \rho_A^j = \frac{\mathrm{Tr}_B\{\Pi_j^B \rho_{AB} \Pi_j^B\}}{p_j}$$

$$\boxed{J_{AB}^{\leftarrow} = \max_{\{\Pi_j^B\}} \left[S(\rho_A) - \sum_j p_j S(\rho_A^j) \right]}$$

Classical Correlation

L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

Local accessible and Inaccessible information



$$I_{AB} = S_A + S_B - S_{AB}$$

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right],$$

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$ (Quantum Discord)

(Entanglement of Formation)

$$E_{\mathcal{F}}(\rho_{AB}) = E_{AB} = \min_{\mathcal{E}} \left\{ \sum_i p_i S(\rho_i^A) \right\} \qquad \mathcal{E} = \{p_i, |\psi_i^{AB}\rangle\}$$

Redefinition

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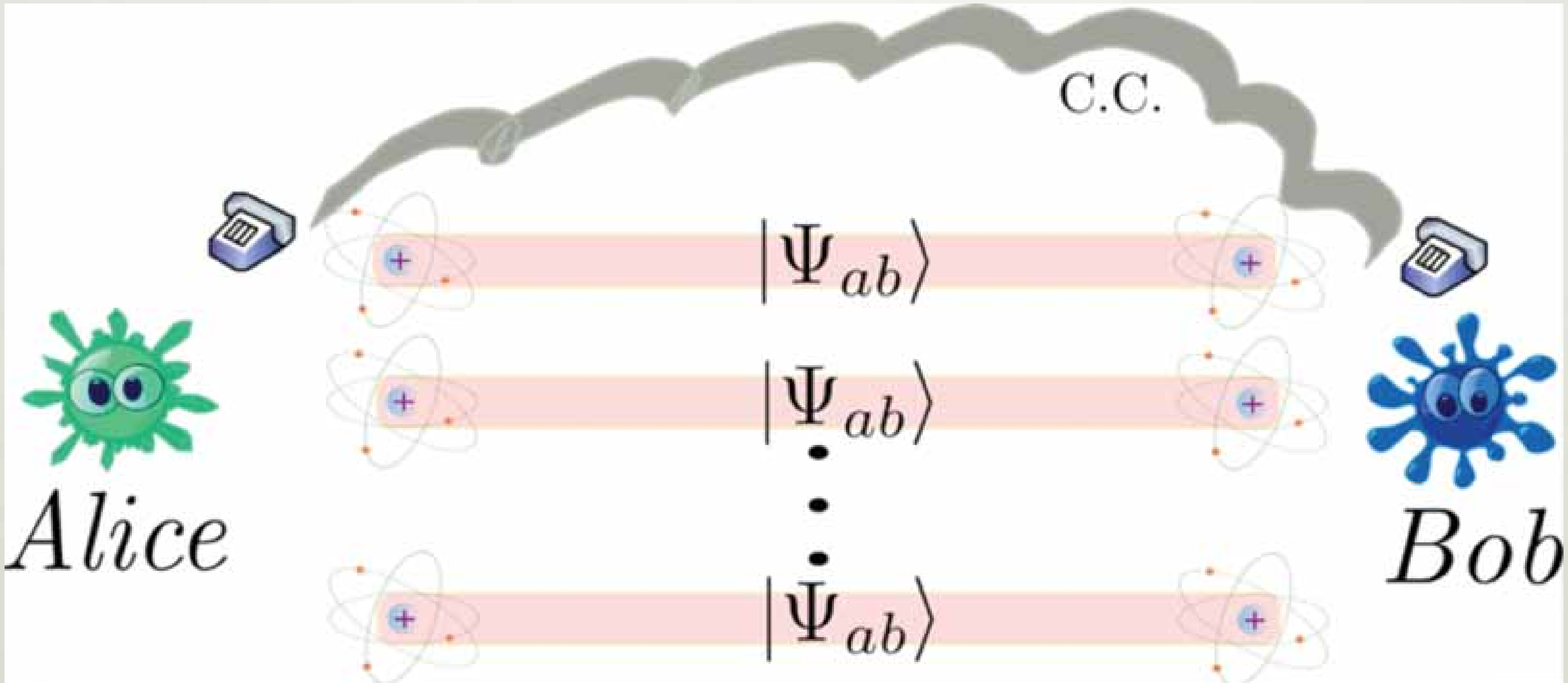
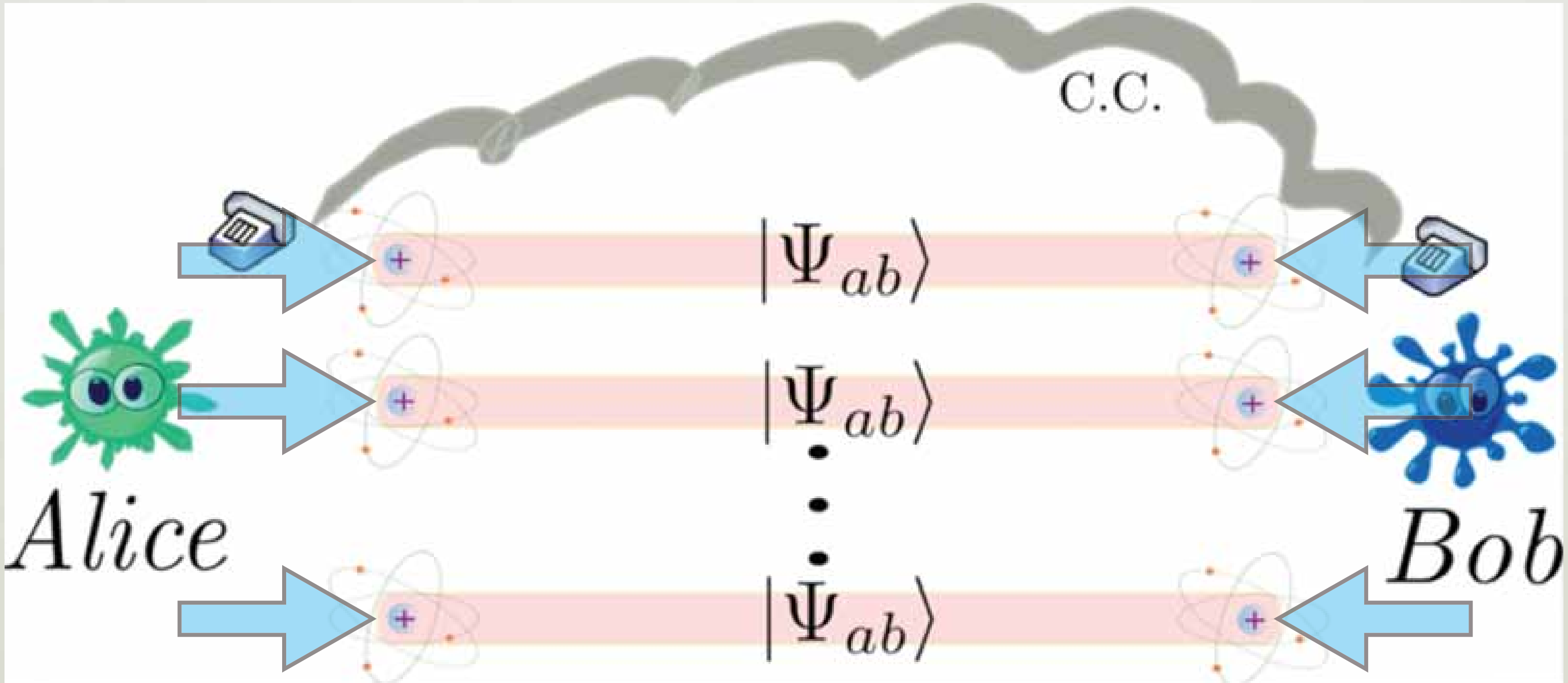
$$I_{AB} = S_A - S_{A|B} \qquad \boxed{\delta^{\leftarrow}_{AB} = S(A|B)_q - S_{A|B}}$$

Entanglement manipulation



Local Operation and Classical Communication (LOCC)

Entanglement manipulation



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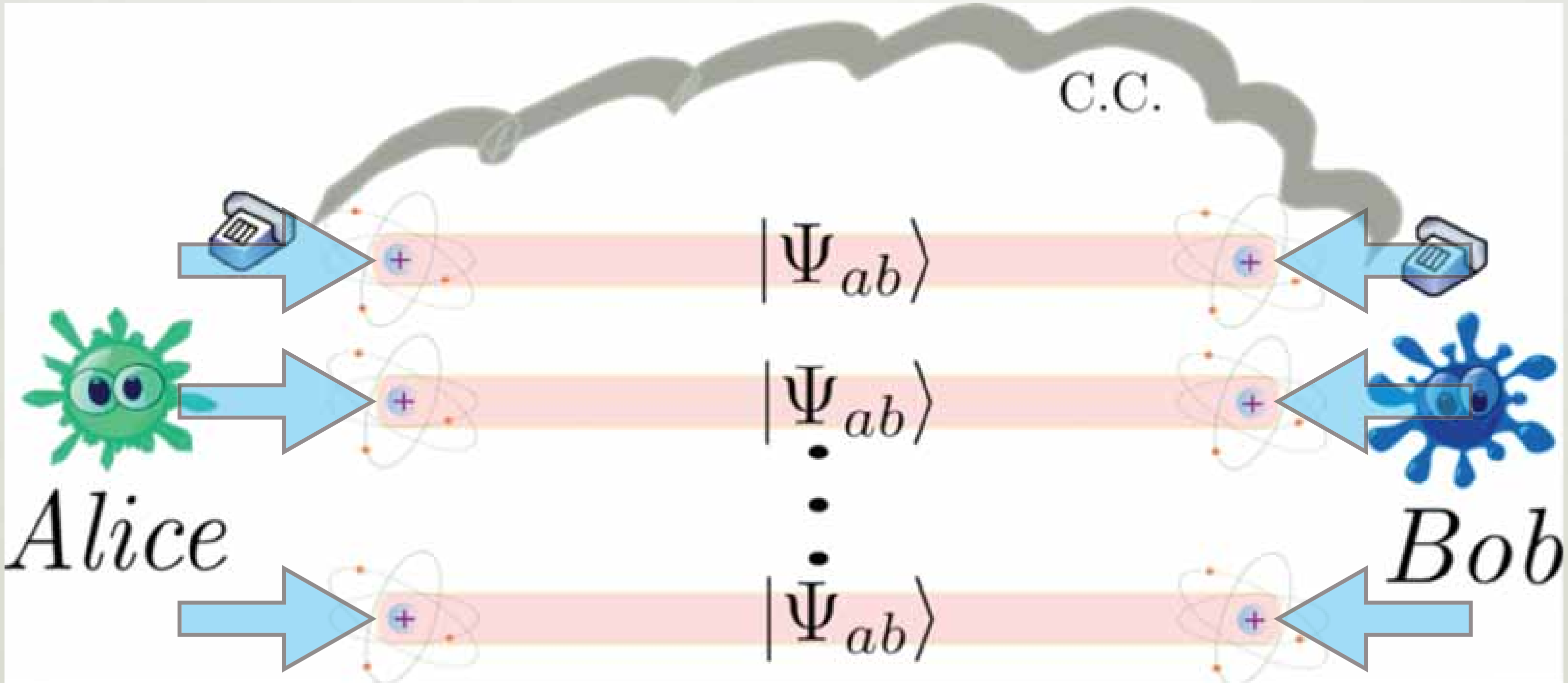

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Simmilarities

Thermodynamics

Entanglement

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<u>Thermodynamics</u>

- *A system's entropy cannot decrease if the system is closed.*
- *Energy is a resource for doing work*

<u>Entanglement</u>

Simmilarities

Thermodynamics

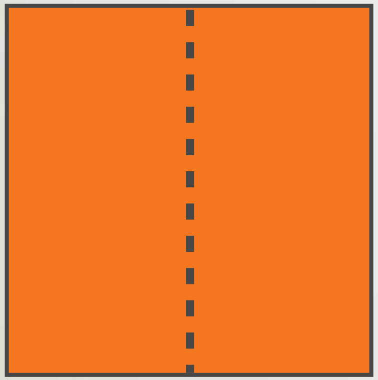
- *A system's entropy cannot decrease if the system is closed.*
- *Energy is a resource for doing work*

Entanglement

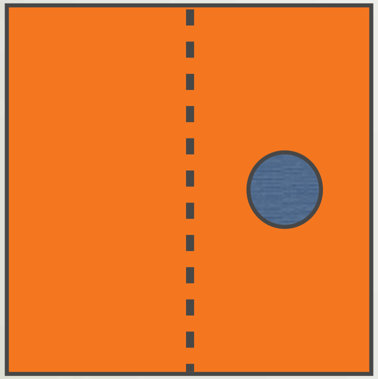
- *The entanglement of a bipartite system cannot be increased by LOCC.*
- *Entanglement <u>can</u> be a resource for doing work*

Szilard Thermal Machine

Szilard Thermal Machine



Szilard Thermal Machine



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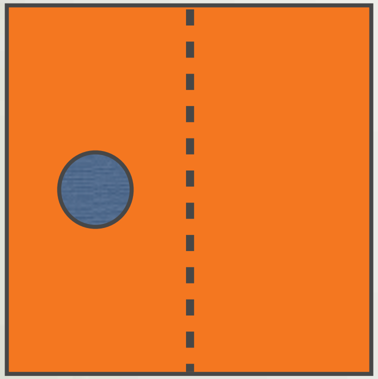
Szilard Thermal Machine



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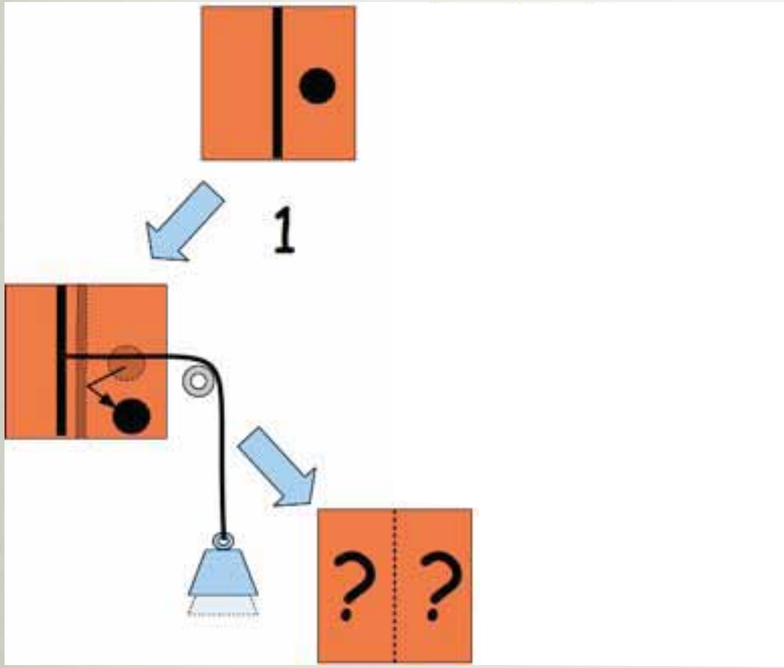
Szilard Thermal Machine



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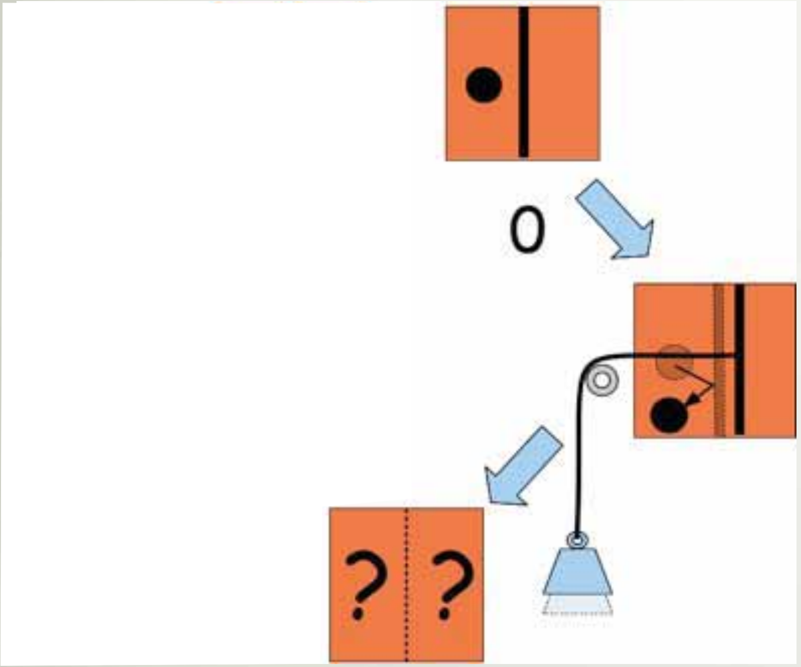

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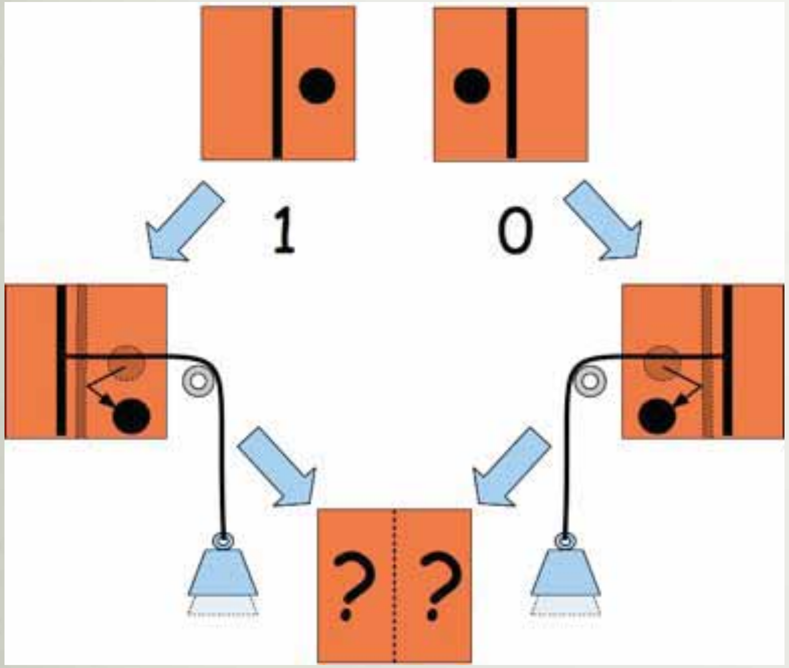

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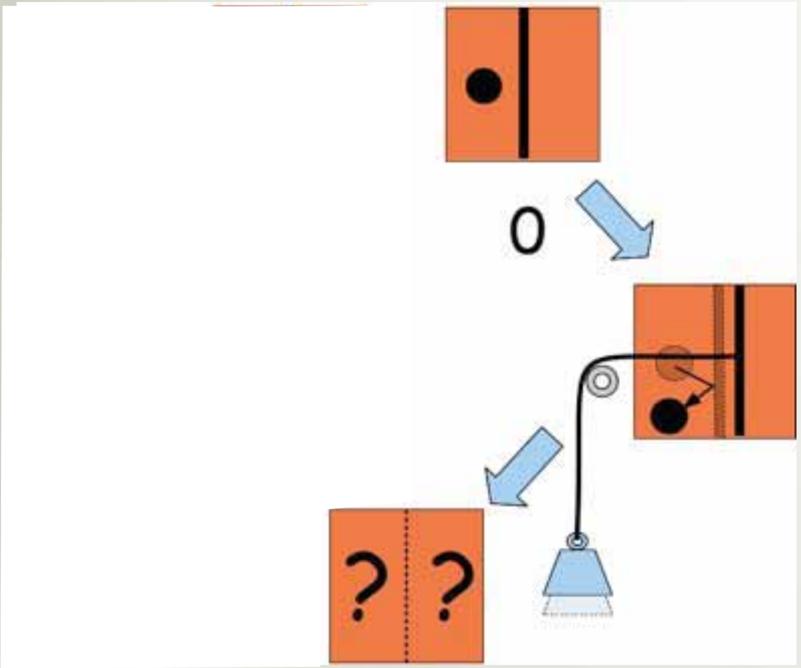
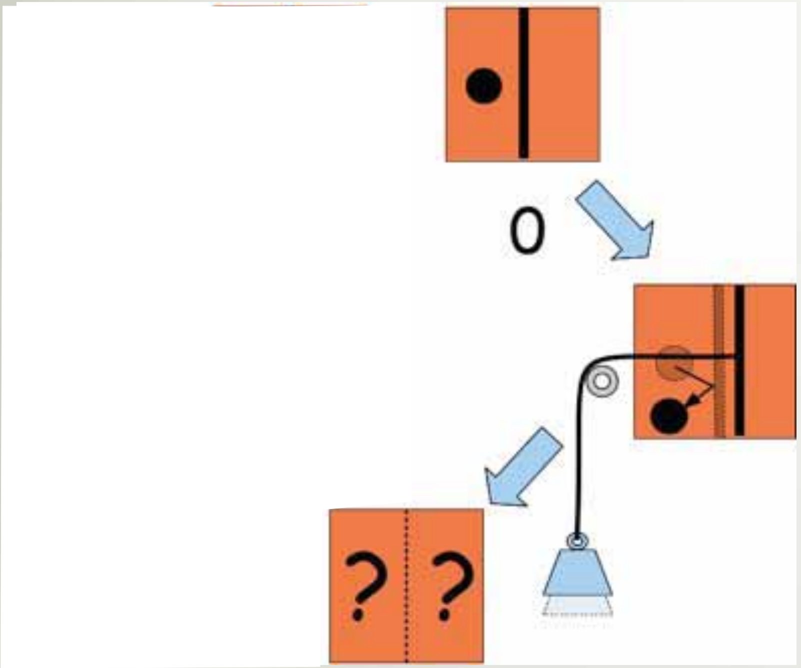


Infinitesimal work

$$\delta W = F\delta x$$

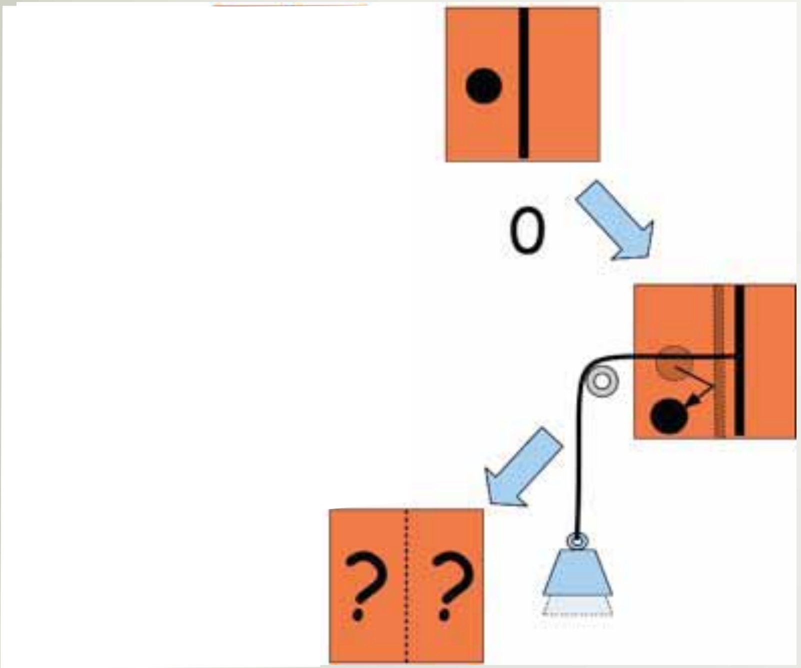
Szilard Thermal Machine



Infinitesimal work

$$\delta W = PA\delta x$$

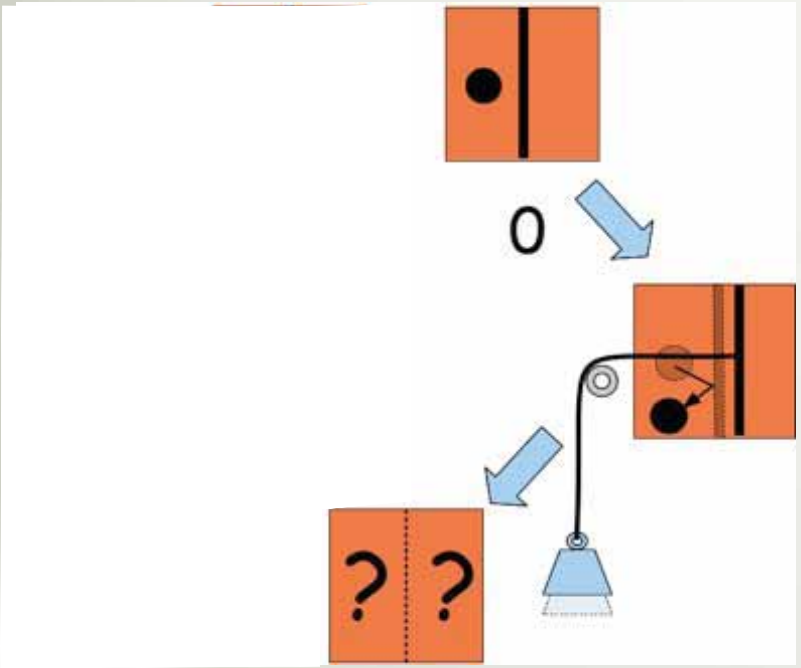
Szilard Thermal Machine



Infinitesimal work

$$\delta W = P\delta V$$

Szilard Thermal Machine



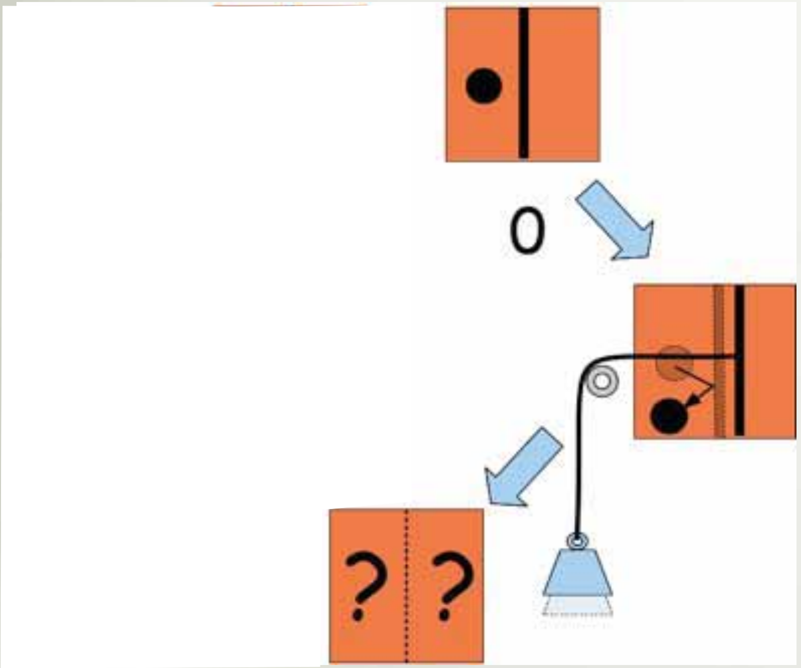
Infinitesimal work

Consider 1-molecule gas as ideal

$$PV = Nk_BT, \quad N = 1$$

$$\delta W = P\delta V$$

Szilard Thermal Machine



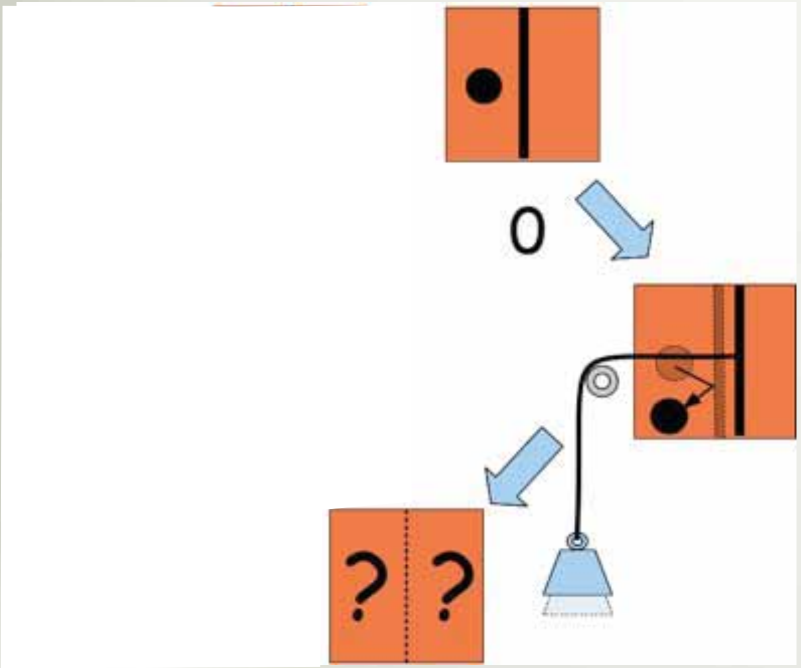
Infinitesimal work

Consider 1-molecule gas as ideal

$$PV = Nk_BT, \quad N = 1$$

$$\delta W = \frac{k_BT}{V}\delta V$$

Szilard Thermal Machine



Infinitesimal work

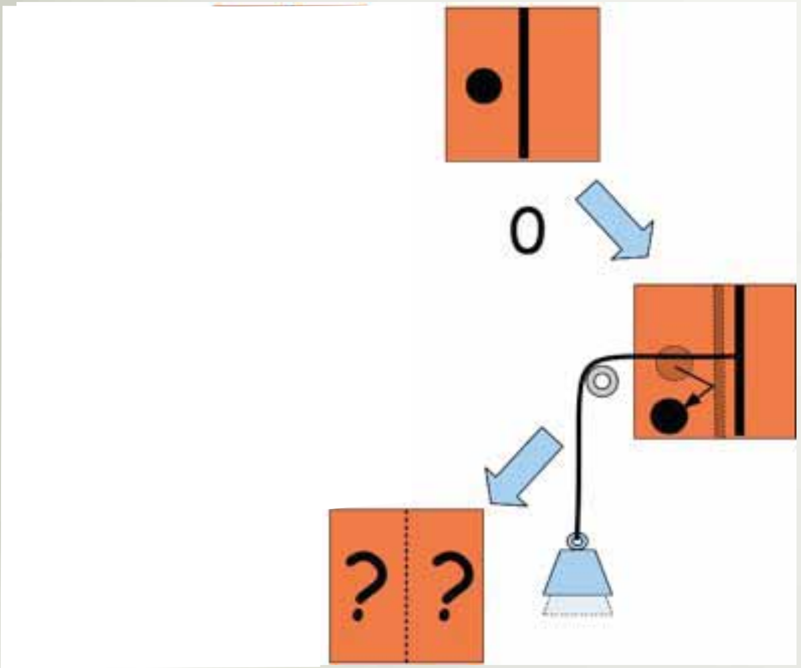
Consider 1-molecule gas as ideal

$$PV = Nk_BT, \quad N = 1$$

$$\delta W = \frac{k_BT}{V}\delta V$$

$$W = \int_{V_1}^{V_2} \frac{k_BT}{V}\delta V$$

Szilard Thermal Machine



Infinitesimal work

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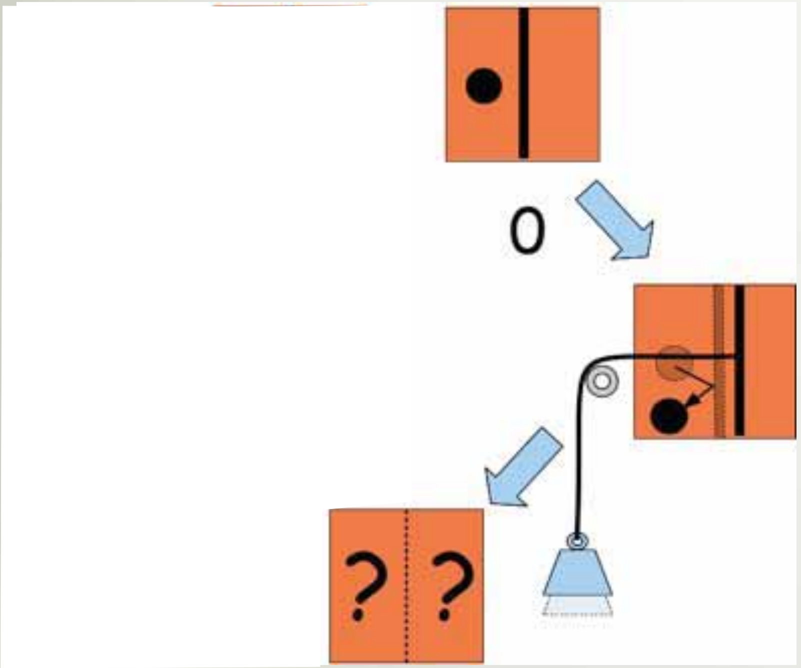
$$W = k_BT\ln\frac{V_2}{V_1}$$

$$V_2 = 2V_1$$

Szilard Thermal Machine



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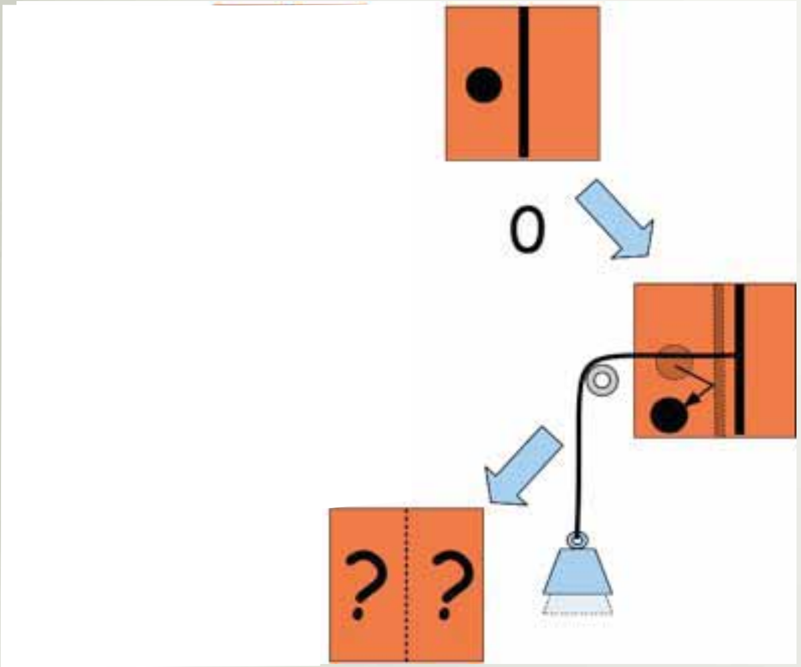
$$W = k_BT \ln 2$$

Szilard Thermal Machine



Infinitesimal work

Consider 1-molecule gas as ideal

$$PV = Nk_BT, \quad N = 1$$

$$\delta W = \frac{k_B T}{V}\delta V$$

$$W = k_B T \ln \frac{V_2}{V_1} \qquad V_2 = 2V_1$$

$$W = \frac{k_B T}{\log_2 e}\log_2 2 = 1 \qquad \frac{k_B T}{\log_2 e} = 1$$

1 bit of work was extracted from the system

We cannot extract work if the state is not known

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We can if there exists classical correlation

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Two bits: (00) or (11) with equal probability

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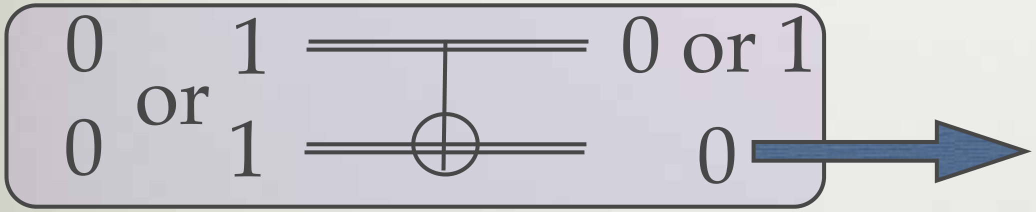

can be used for work extraction

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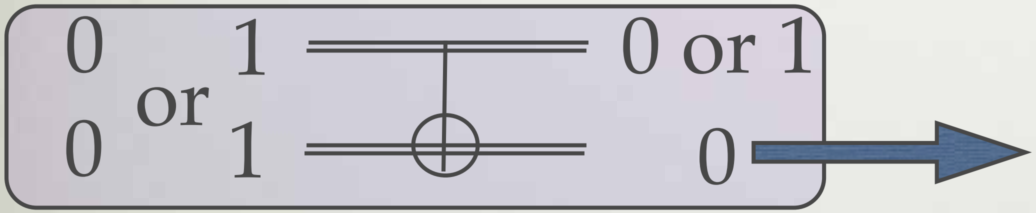


can be used for work extraction

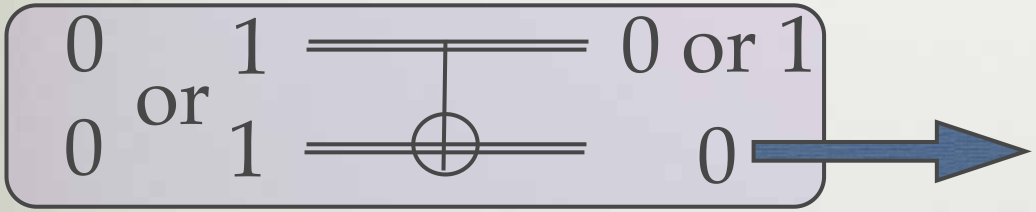
For random variable X with *n-bits*

$$W_c = n - H(X)$$

We cannot extract work if the state is not known

We can if there exists classical correlation

Two bits: (00) or (11) with equal probability



can be used for work extraction

For random variable X with *n-bits*

$$W_c = n - H(X)$$

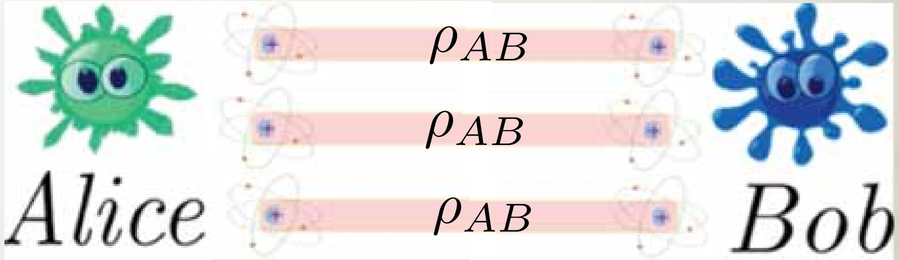
For state ρ encoded in *n-qubits*

$$W_T = n - S(\rho)$$

Work extraction

Work extraction

- Alice and Bob share many copies of a state ρ_{AB}



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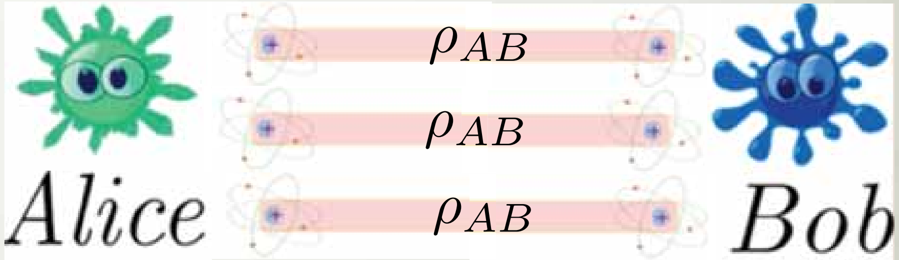

- They use the information about ρ_{AB} to do work through a Szilard thermal machine.

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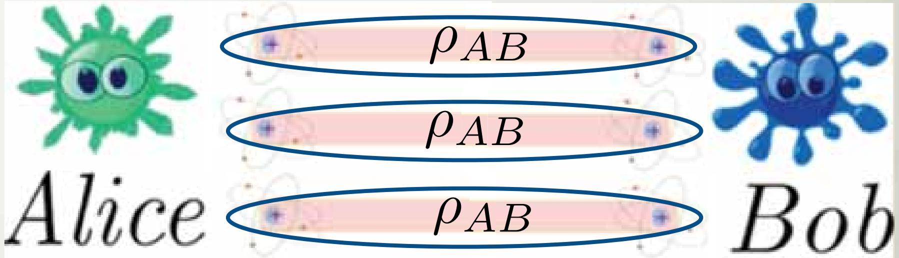

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- If they operate globally $\longrightarrow$ W_T

Work extraction

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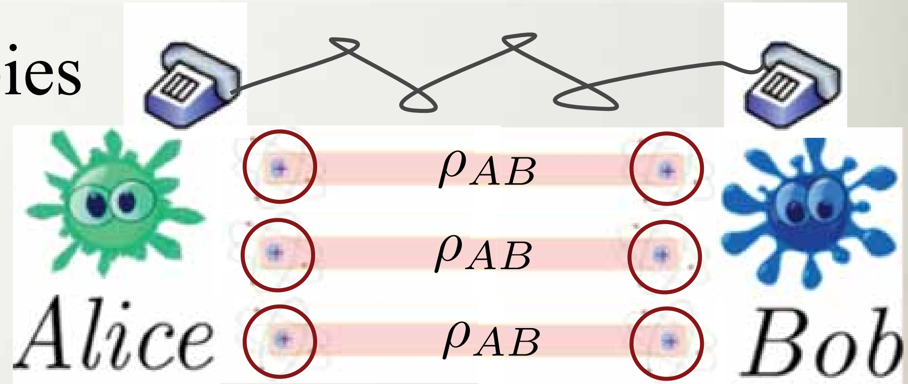

- They use the information about ρ_{AB} to do work through a Szilard thermal machine.

- If they operate globally $\longrightarrow$ W_T

- If they operate locally through LOCC $\longrightarrow$ W_l

Quantum deficit

$$W_T = [n - S(\rho_{AB})]$$

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$$W_l = [n_A - S(\rho_A)] + [n_B - S(\rho_B)],\ n_A + n_B = n$$

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$$\Delta = W_T - W_l$$

Quantity of non-localizable information

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Quantity of non-localizable information

$$\Delta^{\leftarrow}_{BA} = \min_{\{P_j\}} S(\rho'_A) - S(\rho_{AB})$$

one-way quantum deficit

Quantum deficit

$$W_T = [n - S(\rho_{AB})]$$

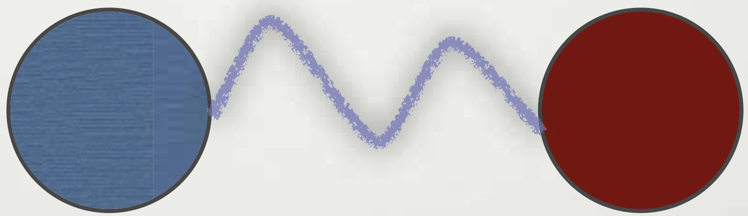
$$W_l = [n_A - S(\rho_A)] + [n_B - S(\rho_B)], \; n_A + n_B = n$$

$\Delta = W_T - W_l$ Quantity of non-localizable information

$\Delta^{\leftarrow}_{BA} = \min\limits_{\{P_j\}} S(\rho'_A) - S(\rho_{AB})$ one-way quantum deficit

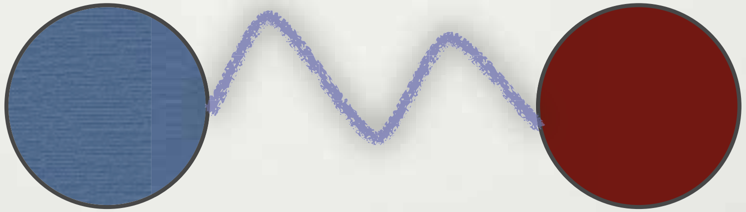
$\widetilde{\Delta}^{\leftarrow}_{BA}(\rho_{AB}) = \frac{1}{n} \Delta^{\leftarrow}_{BA}(\rho^{\otimes n}_{AB})$ regularized 1-way q. deficit

Maxwell Demons



Maxwell Demons

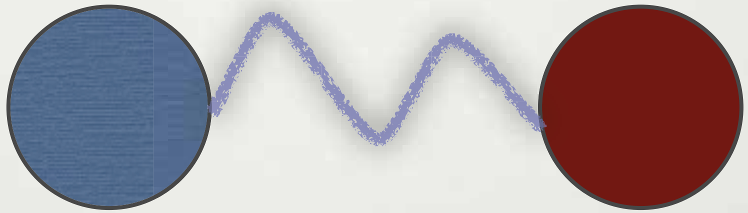
Classical



Maxwell Demons

Classical

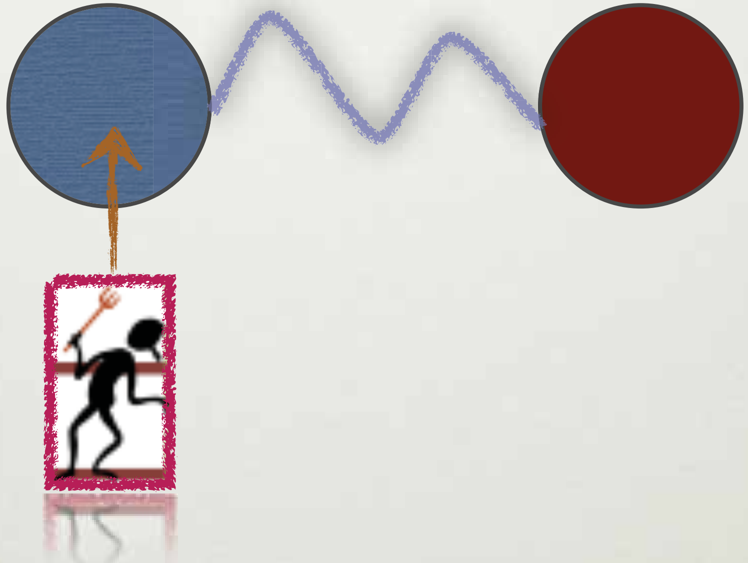
Quantum



Maxwell Demons

Classical

Quantum



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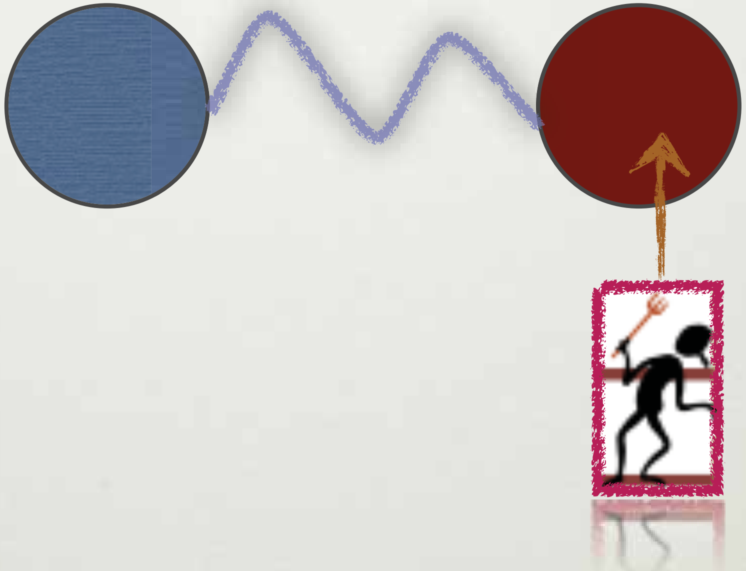
Quantum



Maxwell Demons

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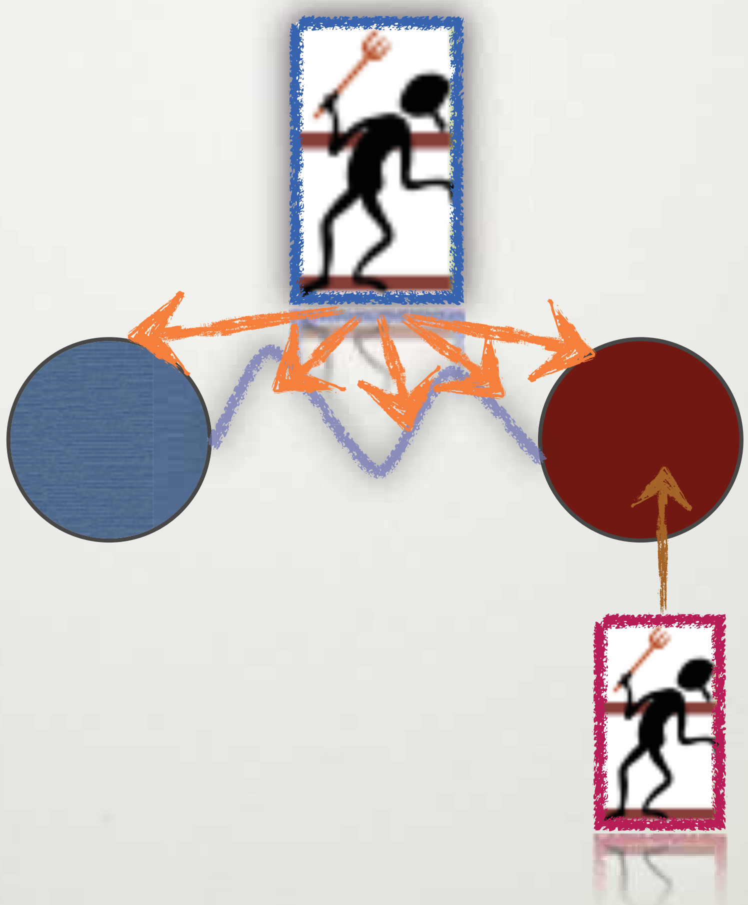

Maxwell Demons

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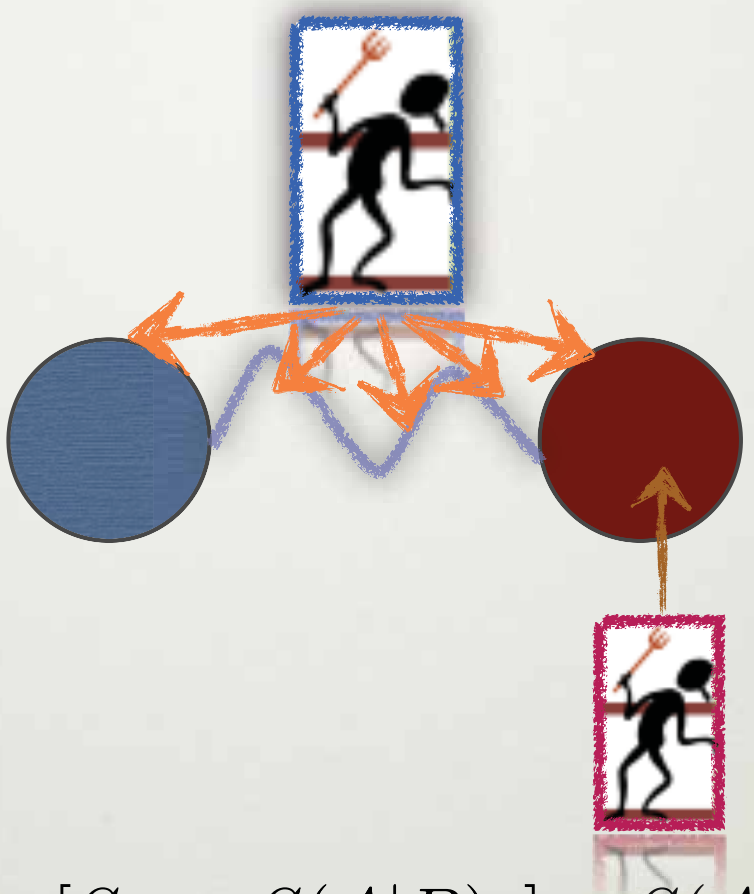
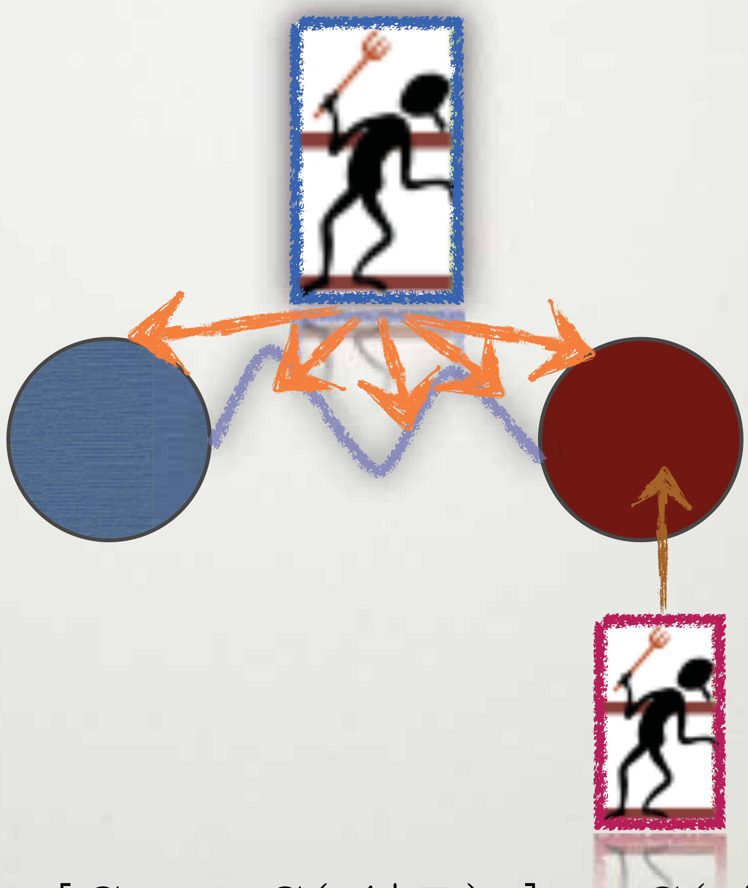


$$\Delta W/k_BT = [S_B + S(A|B)_q] - S(A,B)$$

Maxwell Demons

Classical

Quantum

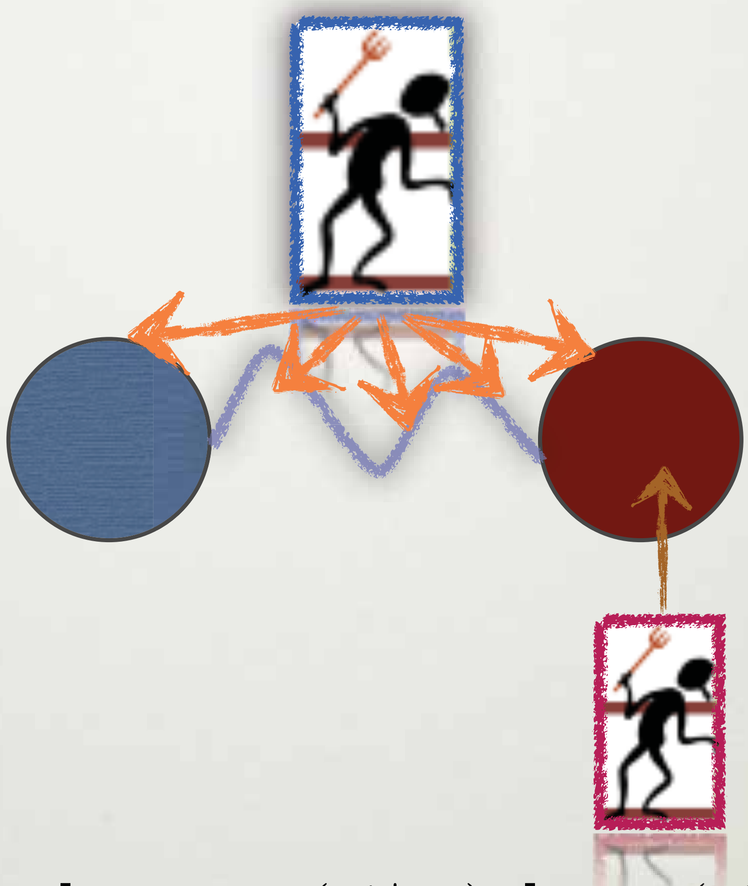


$$\Delta W/k_B T = [S_B + S(A|B)_q] - S(A,B)$$
$$\Delta W/k_B T = S(A|B)_q - S_{A|B}$$

Maxwell Demons

Classical

Quantum



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$$\Delta W = k_B T \delta^{\leftarrow}_{AB}$$

Regularized versions

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Entanglement manipulation

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Entanglement as resource for QIT

Unit of entanglement $$|\text{e-bit}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

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- teleport 1 qubit
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<u>With 1 e-bit:</u>

- teleport 1 qubit
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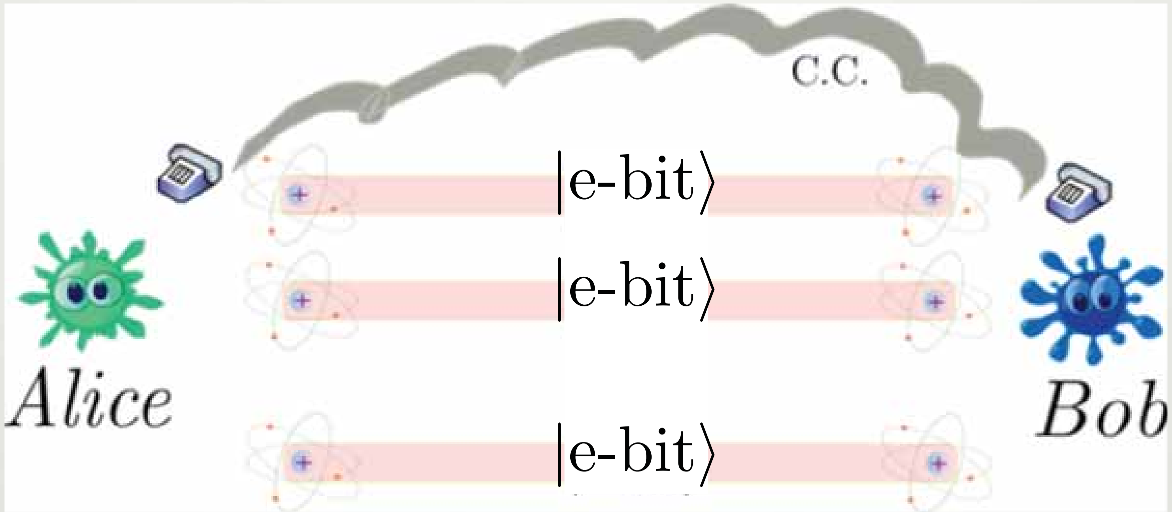
<u>Manipulating states by LOCC:</u>

- e-bits can be converted in other entangled states
- distillable states can be converted in e-bits

Entanglement Dilution



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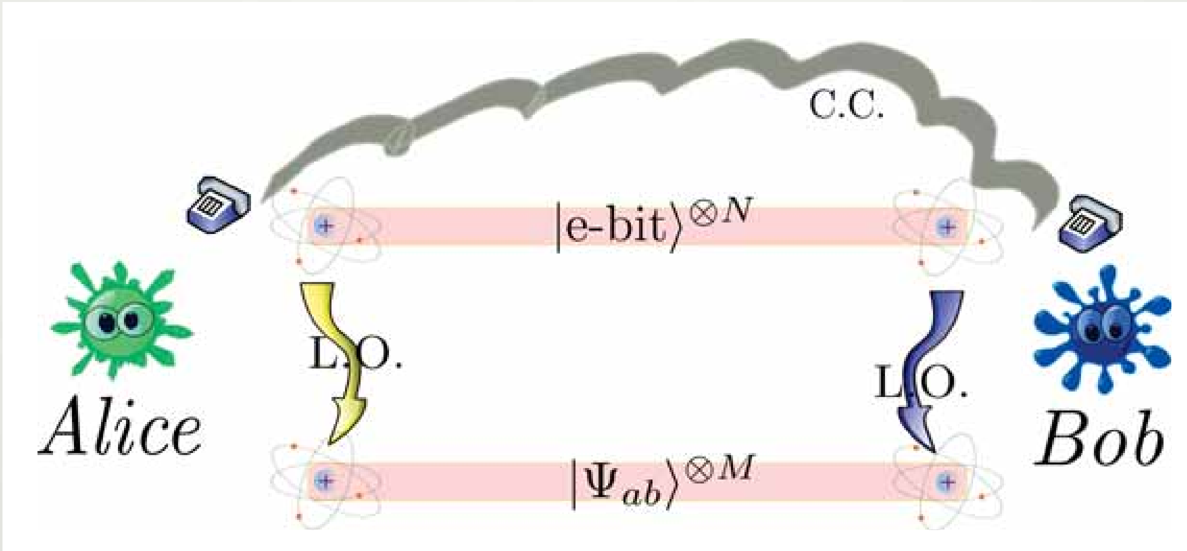

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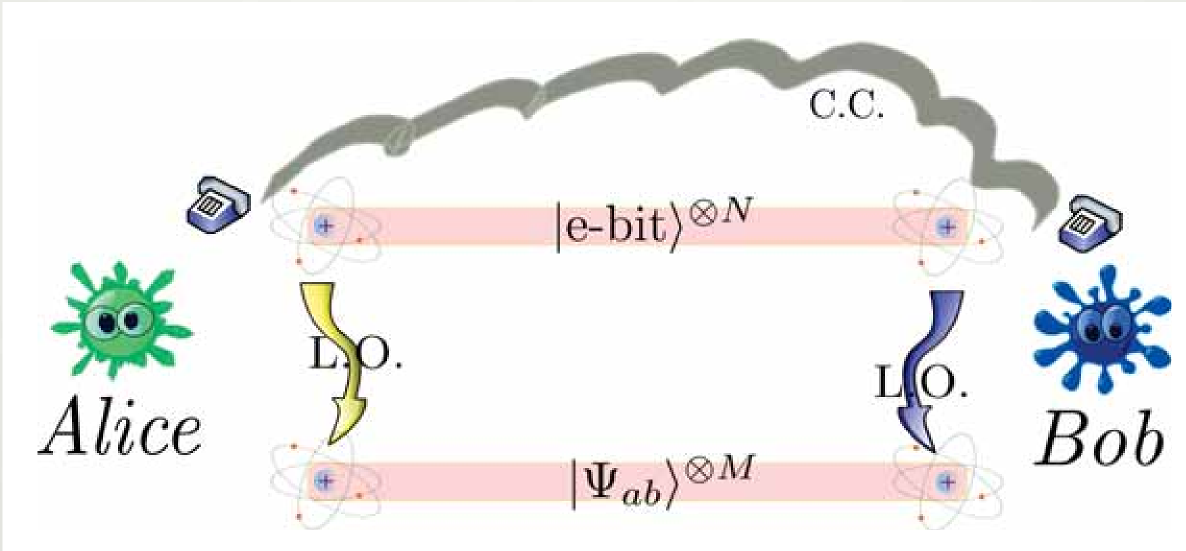

$$M = N/E^{\mathcal{C}} \qquad \text{(Ent. Cost)}$$

[Bennett et al PRA 96]

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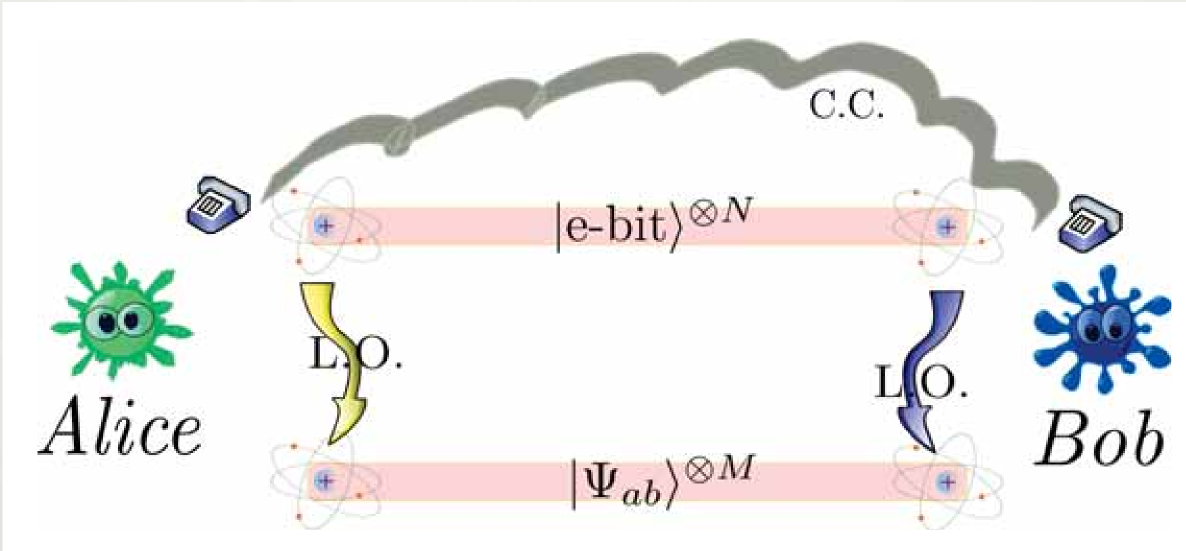
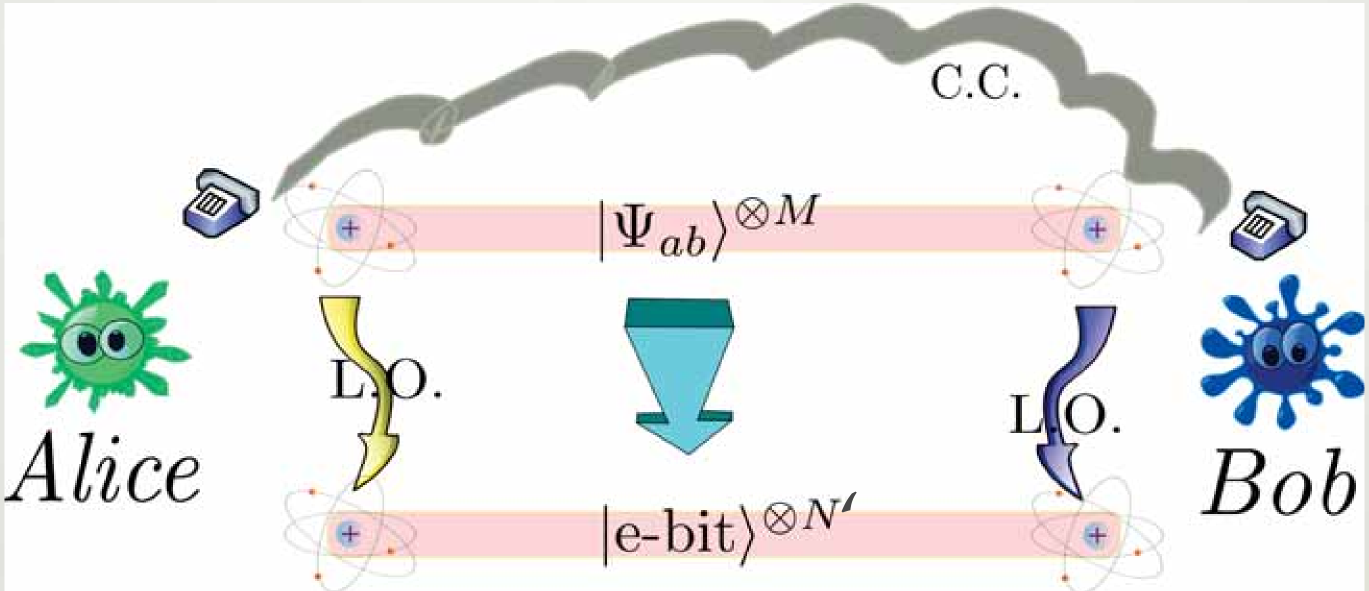


$$M = N/E^{\mathcal{C}} \qquad \text{(Ent. Cost)}$$

$$E^{\mathcal{C}} := \text{ best rate } \frac{M}{N} = S(\rho_a^r)$$

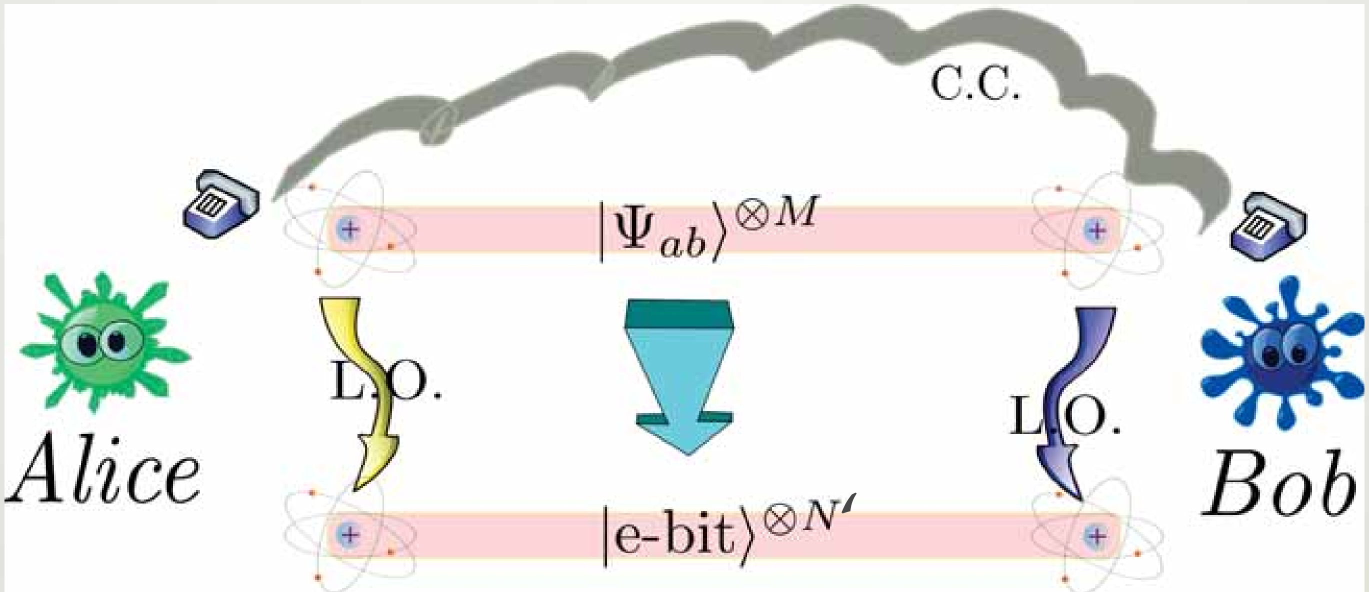
[Bennett et al PRA 96]

Entanglement Distillation



(Distillable Ent.)

Entanglement Distillation



$$N' = ME^{\mathcal{D}} = NE^{\mathcal{D}}/E^{\mathcal{C}}$$

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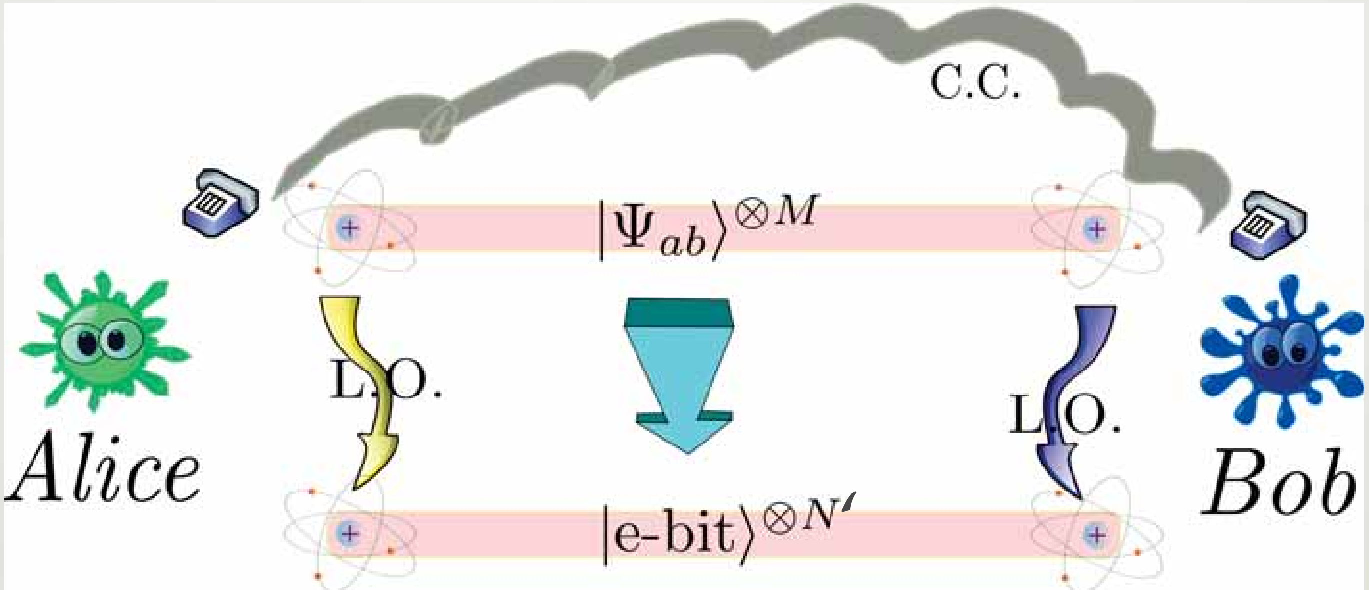

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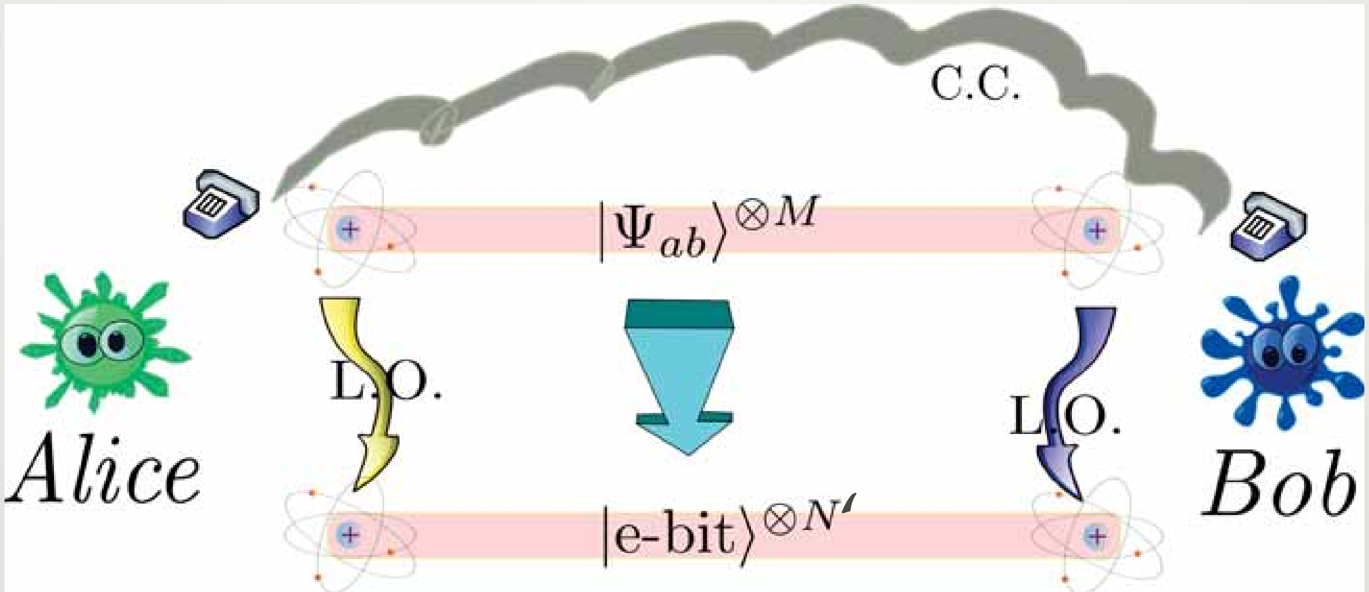

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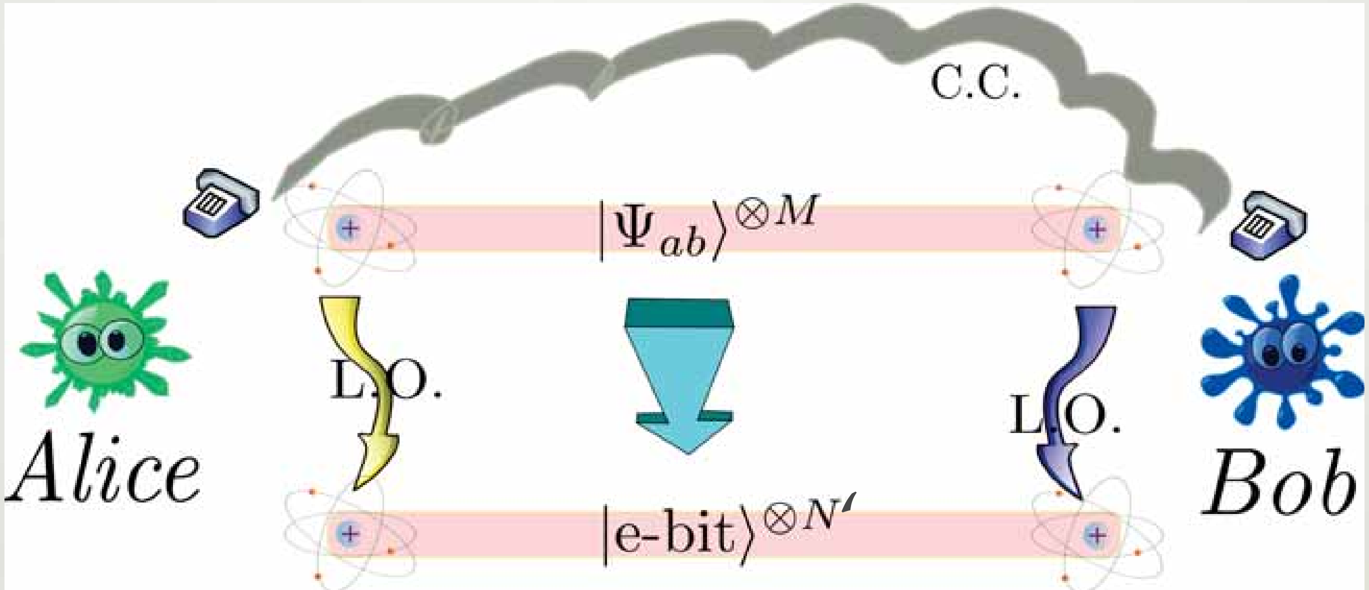

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The amount of entanglement was conserved

LOCC Manipulations are *reversible* over pure states

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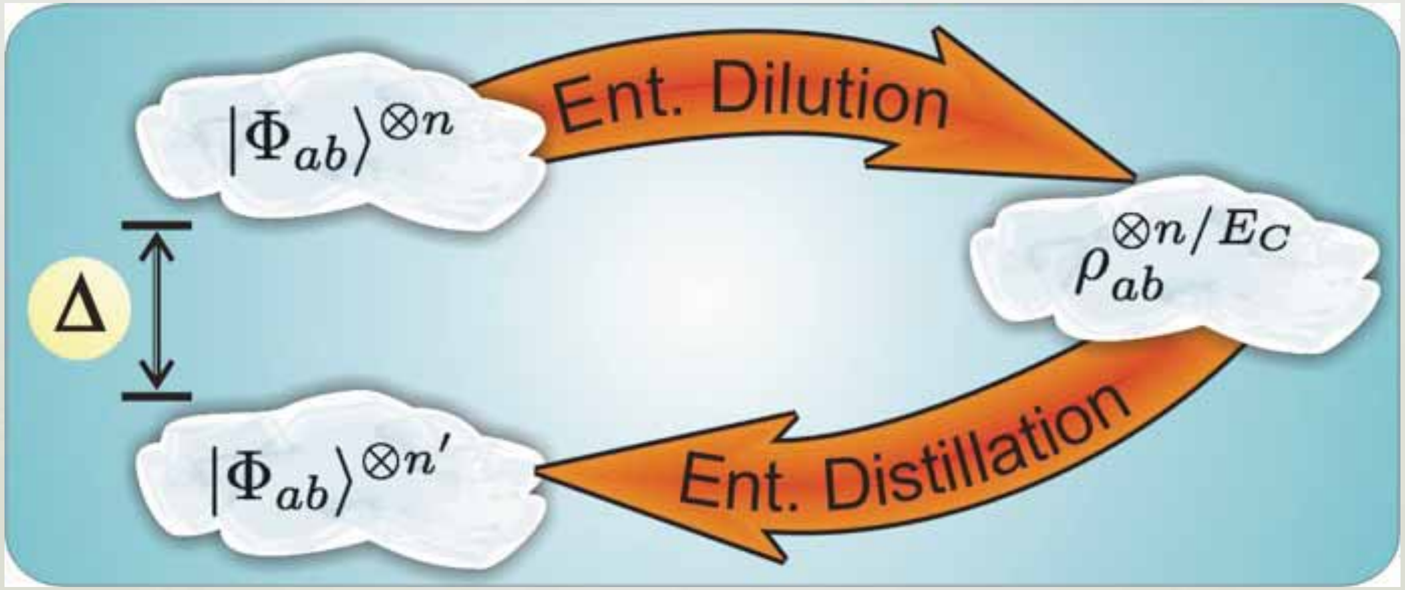

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Entanglement Irreversibility

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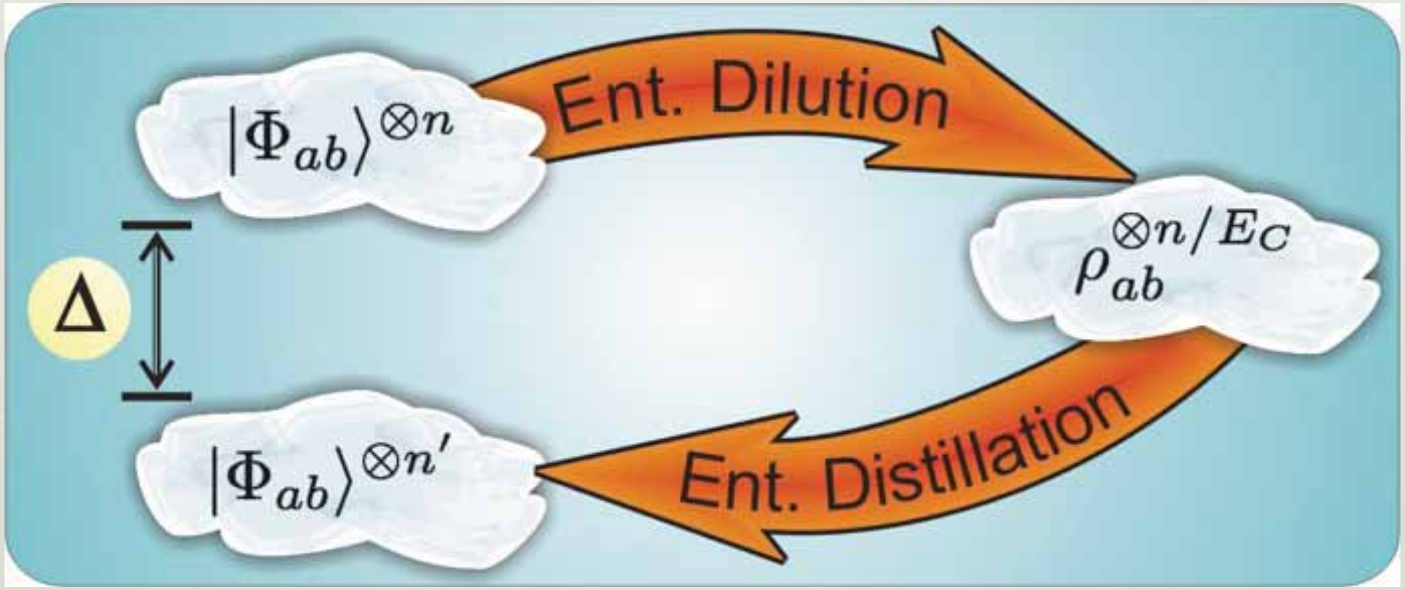

$$n' \neq n$$

Widely believed that only pure states are reversible

Vidal PRL 01,02 Vollbrecht PRA04, Yang PRL 05

If

$$E^{\mathcal{C}}(\rho_{ab}) = \frac{1}{n} E^{\mathcal{F}}(\rho_{ab}^{\otimes n}) \qquad \text{Type A}$$

$$E^{\mathcal{D}}(\rho_{ab}) = \max_{V} \frac{1}{k} I^{\mathcal{C}}\left(V\rho^{\otimes k}\right) \qquad \text{Type B}$$

for finite *n* and *k* then entanglement is irreversible

Marcio F. Cornelio, MCO and Felipe F. Fanchini, PRL 107, 020502 (2011).

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V is a LOCC over *k* copies of ρ_{ab}

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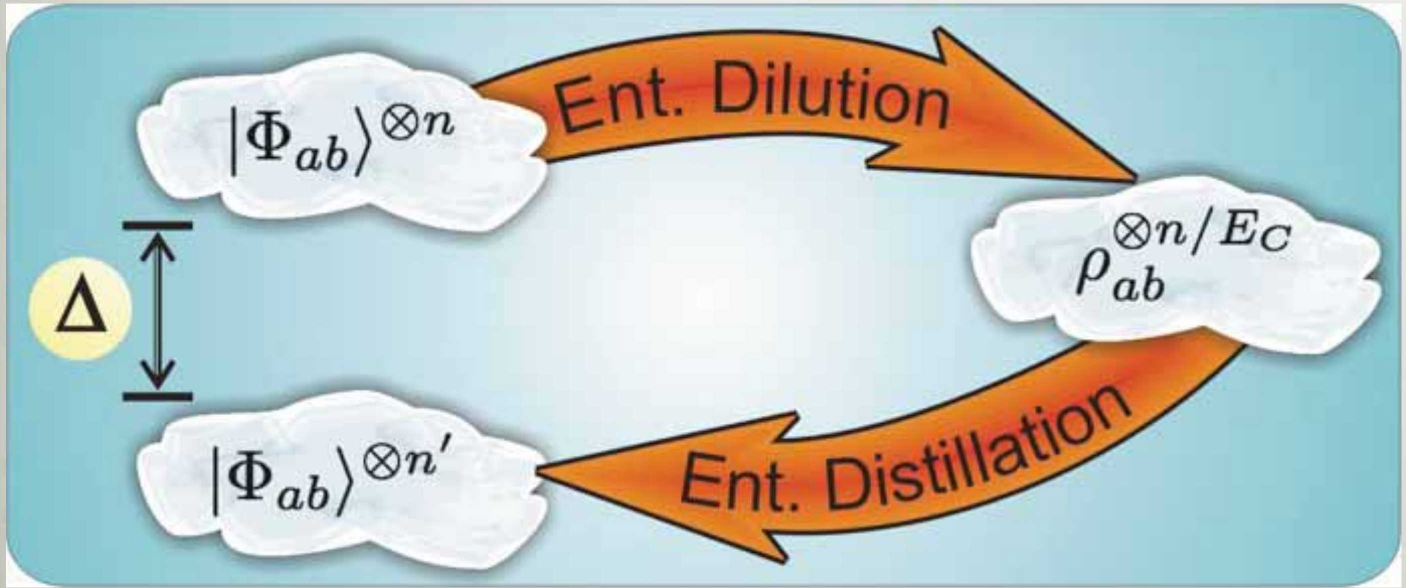
Irreversibility

$$E^{\mathcal{C}}(\rho_{ab}^{(\sigma)}) - E^{\mathcal{D}}(\rho_{ab}^{(\sigma)}) \geq \Delta_{a|c}(\sigma_{ac})$$ in general

$$E^{\mathcal{C}}(\rho_{ab}^{(\sigma)}) - E^{\mathcal{D}}(\rho_{ab}^{(\sigma)}) = \Delta_{a|c}(\sigma_{ac})$$ for certain classes of states



c: environment

Entanglement

Trade-off between the bipartite entanglement of *A* with *B* and the entanglement of *A* with *C*.

If $$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0_A, 0_B\rangle + |1_A, 1_B\rangle\right)$$

There is no way to *A* get entangled to *C* without decreasing entanglement with *B*.

Concurrence

Entanglement

Trade-off between the bipartite entanglement of A with B and the entanglement of A with C.

$$\mathcal{E}_{A|BC} = \mathcal{E}_{A|B} + \mathcal{E}_{A|C} + \tau_{ABC}$$

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V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A 61, 052306 (2000)

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Correlation between two stochastic variables: (X, Y)

$$H(X:Y) \equiv H(X) + H(Y) - H(X,Y)$$

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So $$H(X:Z) = H(X:Y) - H(X|Z)$$

increases linearly with $H(X{:}Y)$, being only constrained by

$$H(X:Z) \leq H(X:Y)$$

Quantum Systems

- Extension of classical form

$$S(A : B) \equiv I_{AB} = S_A + S_B - S_{AB}$$

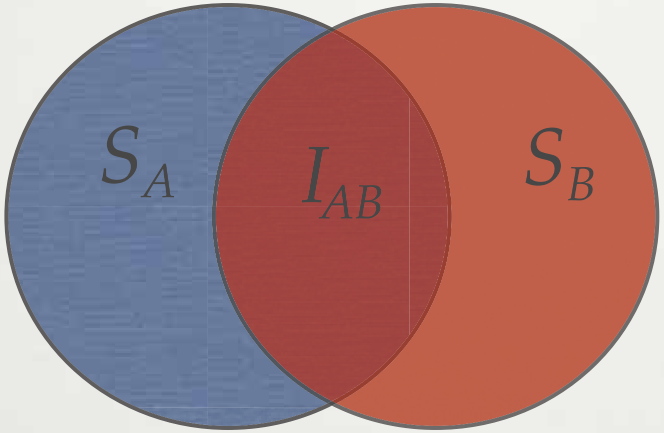
Not always subadditive

$$S(A : B, C) \not\leq S(A : B) + S(A : C)$$

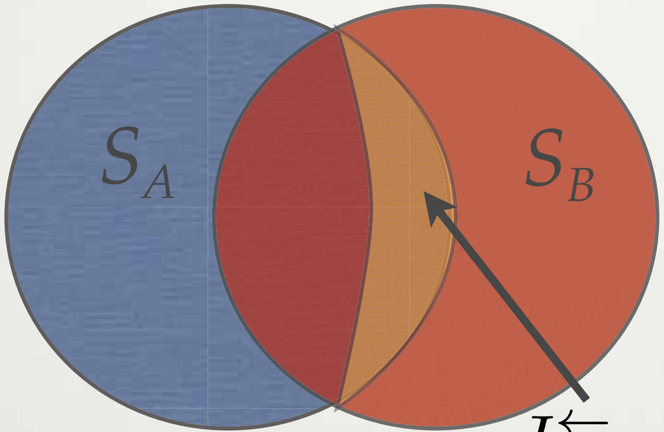
Proper form

$$S(A : B) = S_A - S(A|B)$$

Local accessible and Inaccessible information



Local accessible and Inaccessible information



(CC)

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right]$$

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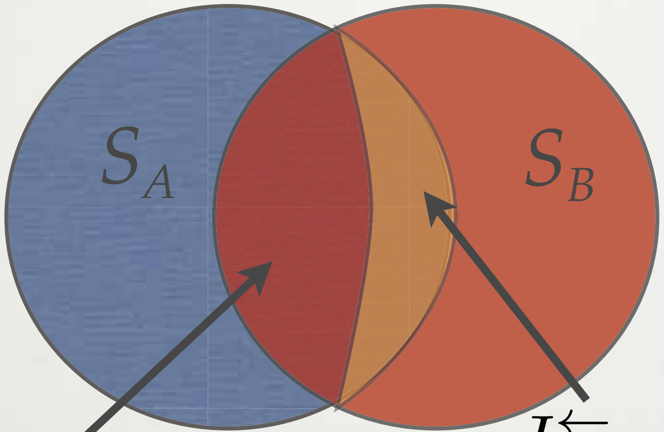

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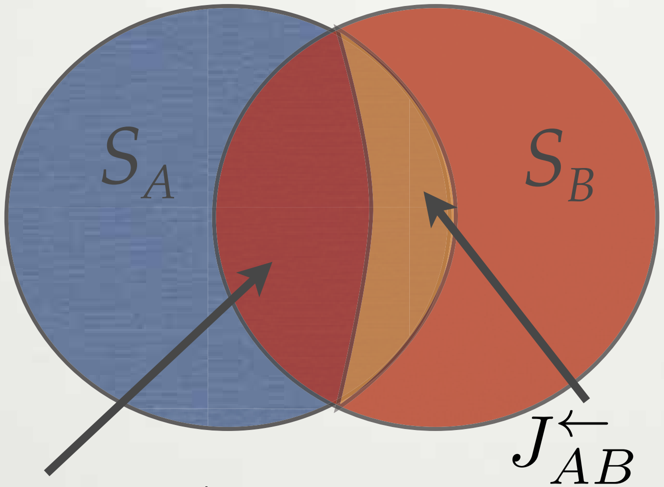
(QD)

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$

Local accessible and Inaccessible information



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(QD)

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Discrepancy: $\boxed{\Delta^{\leftarrow}_{AB} \equiv J^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AB}}$ $\quad -I_{AB} \leq \Delta^{\leftarrow}_{AB} \leq I_{AB}$

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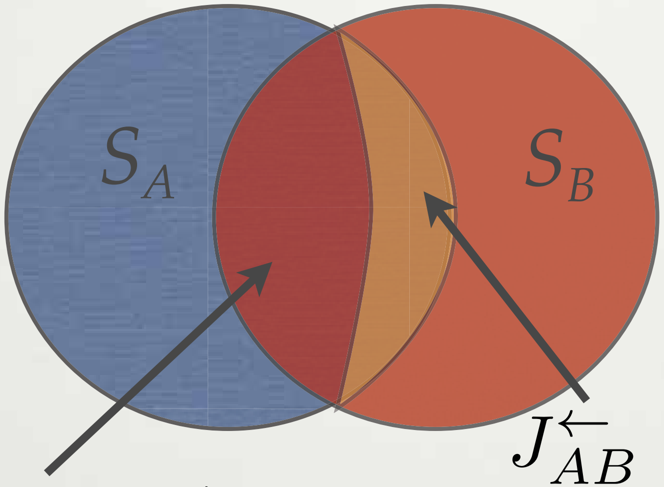

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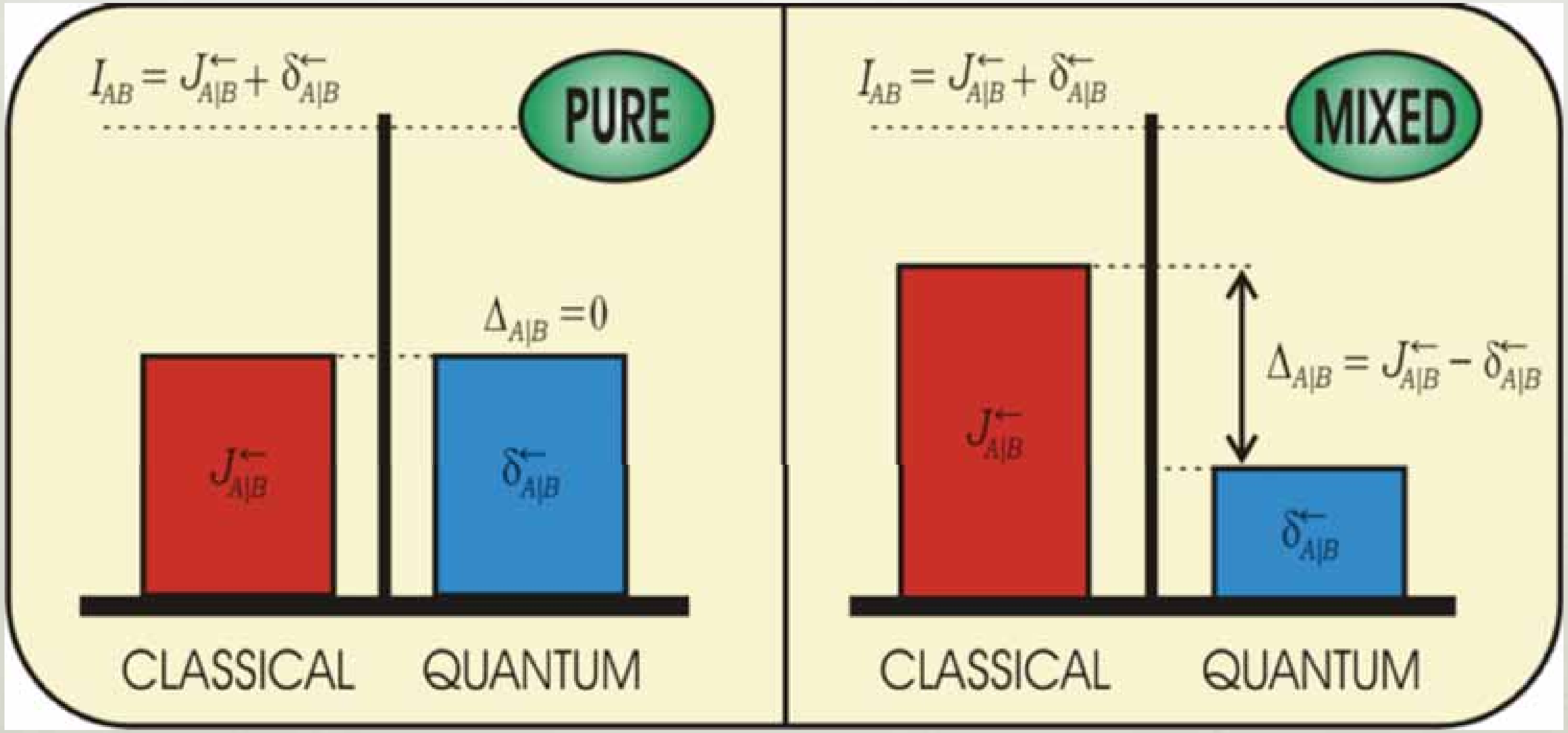
Discrepancy: $\boxed{\Delta^{\leftarrow}_{AB} \equiv J^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AB}}$

$$-I_{AB} \leq \Delta^{\leftarrow}_{AB} \leq I_{AB}$$

Balance between the gain in work extraction by the use of global operations over local ones, and the work extracted locally only.

Correlation Discrepancy



EOF Monogamy

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$$E_{AB} = \delta^{\leftarrow}_{AC} + S_{A|C}$$

$$E_{AB} = E_F\left(\rho_{ab}\right) = \min_{\mathcal{E}}\left\{\sum_i p_i E_F\left(|\varphi_i\rangle\right)\right\}$$

M. Koashi and A. Winter, PRA 69, 022309 (2004)

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$$E_{AB} + E_{AC} + S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC} = S_A$$

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} + E_{AC} + (S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC}) = E_{A(BC)}$$

EOF Monogamy

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$$E_{AB} + E_{AC} + \tau_A = E_{A(BC)}$$

$$\tau_A = (S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC}) = \frac{1}{2}[\Delta^{\leftarrow}_{AB} + \Delta^{\leftarrow}_{AC}]$$

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EOF <u>not</u> monogamous if

$$S_A < S_q(A|B) + S_q(A|C) \leq 2S_A$$

$$S_q(A|i) = \min_{\{\Pi_k\}} \sum_k p_k S(\rho_{A|k}), \qquad \rho_{A|k} = \frac{\mathrm{Tr}_i(\Pi^i_k \rho_{Ai} \Pi^i_k)}{\mathrm{Tr}_{Ai}(\Pi^i_k \rho_{Ai} \Pi^i_k)}, \quad i = B, C$$

Example

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$$\tau_{ABC} \equiv \tau_A + \tau_B + \tau_C = \Delta_{\circlearrowright} \equiv \Delta^{\leftarrow}_{BA} + \Delta^{\leftarrow}_{CB} + \Delta^{\leftarrow}_{AC}$$

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$$|GHZ\rangle = \theta|\uparrow\uparrow\uparrow\rangle + \phi|\downarrow\downarrow\downarrow\rangle \quad \Rightarrow \quad \tau_{ABC} > 0$$

Example

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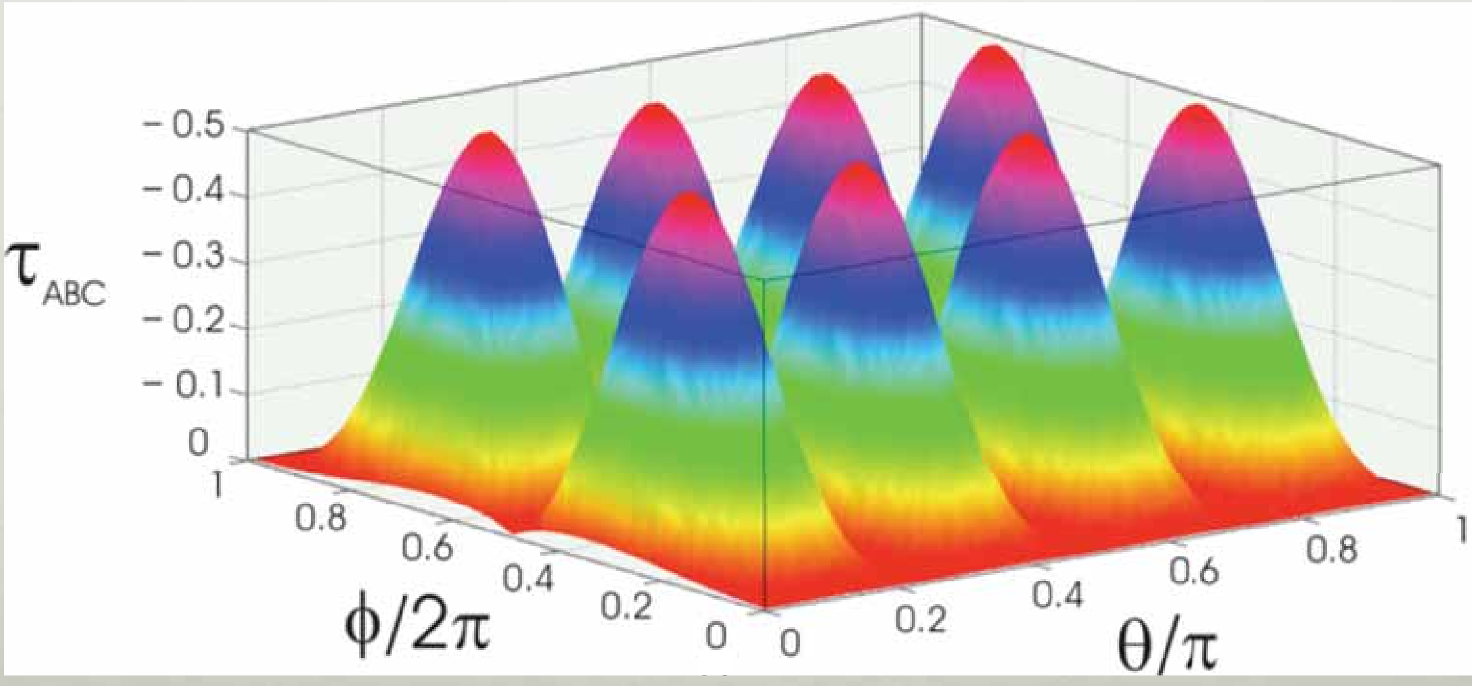
$$|GHZ\rangle = \theta|\uparrow\uparrow\uparrow\rangle + \phi|\downarrow\downarrow\downarrow\rangle \Rightarrow \tau_{ABC} > 0$$

$$|W\rangle = \alpha|\uparrow\uparrow\downarrow\rangle + \beta|\uparrow\downarrow\uparrow\rangle + \gamma|\downarrow\uparrow\uparrow\rangle \Rightarrow \tau_{ABC} < 0$$



Conclusions

- Classical Mutual Information
- Measurement process
- Local accessible and Local Inaccessible Mutual Information
- Thermodynamical aspects
- Correlation as a mean to produce Work
- Quantum Deficit and Quantum Discord
- Irreversibility of entanglement
- Distribution of Correlation