General Aspects on Classical and Quantum Correlation

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- I-Fundamental aspects and general relations

- II-Applications and thermodynamical aspects

- Classical systems
  - Information and Shannon Entropy
  - Classical Correlation
- Quantum systems
  - von Neumann Entropy and correlation
  - Entanglement of pure states
  - Entanglement of mixed states
  - Quantum correlation with zero entanglement

Information

Ex:Statistical problem with P_0 possibilities:

- without any previous information about the problem;

  (probability: $p_0 = 1/P_0$)

- If we obtain further information we can achieve a situation where only one of the P_0 possibilities is actually realized.
- The larger the uncertainty, the larger will be P_0 and the larger will be the required information to realize a single selection.

Required Information

1. $I_0 = 0$, with P_0 possibilities equally probable.
2. $I_0 \neq 0$, with $P_0 = 1$: a single possible result

$$I_1 \equiv kf(P_0)$$

Two independent problems:

P_{01} *possibilities and* P_{02} *possibilities*

 # total possibilities: $P_0 = P_{01}P_{02}$

- *Additivity of required information:*

$$I = I_{01} + I_{02} = k[f(P_{01}) + f(P_{02})] = kf(P_{01}P_{02}) = kf(P_0)$$

Shannon

$$f(P_0) = ln(P_0)$$

- For $k = \log_2 e \rightarrow I = \log_2 P_0$: Bits:{0,1}
- <u>*Example*</u>:

Consider a binary word with *n* elements:

0111001010...0

total possibilities:

$$P = 2^n \rightarrow I = k\, ln(P) = kn\, ln(2) = \log_2 P$$

I = # of bits necessary to represent *P* possibilities

Generalization

Message: *N* cels with *l 0s* and *m 1s*/

$$N = l + m \rightarrow p_0 = \frac{l}{N}, p_1 = \frac{m}{N}$$

# of possibilities in the message:

$$P = \frac{N!}{l!m!} \rightarrow I = \log_2 N! - \log_2 l! - \log_2 m!$$

long messages $\qquad (l, m, N \gg 1)$

$$\log_2 N! \approx N(\log_2 N - 1)$$

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$$I \approx -N\left(p_0 \log_2 p_0 + p_1 \log_2 p_1\right)$$

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long messages $(l, m, N \gg 1)$

$$i = \frac{I}{N} = -\left(p_0 \log_2 p_0 + p_1 \log_2 p_1\right)$$

Shannon Entropy

For M symbols $x_1, x_2, ..., x_M$

occurring with probabilities: $p(x_1), p(x_2), ..., p(x_M)$

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$$H(X) \equiv i = -\sum_{j=1}^{M} p(x_j) \log_2 p(x_j)$$

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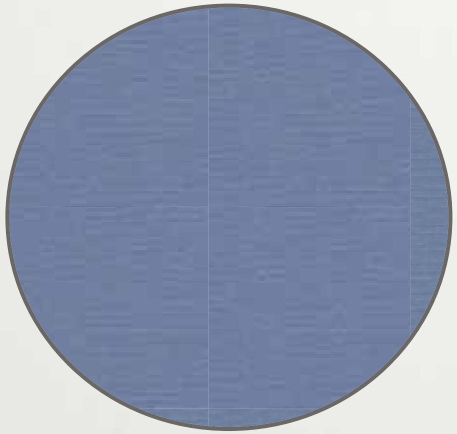

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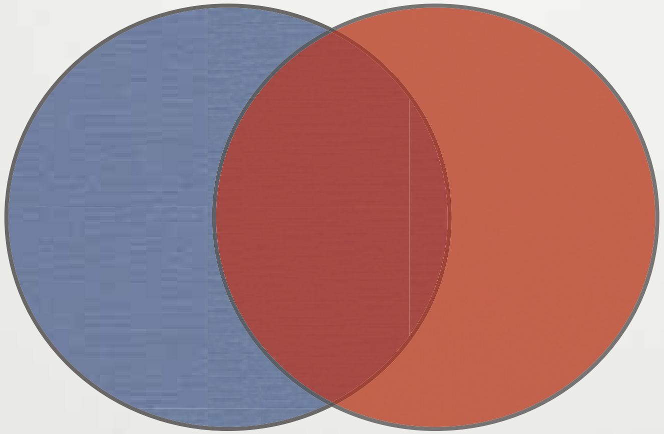

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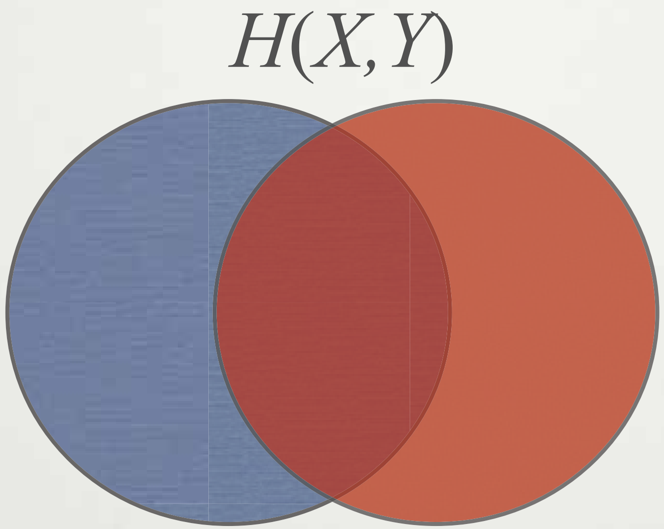

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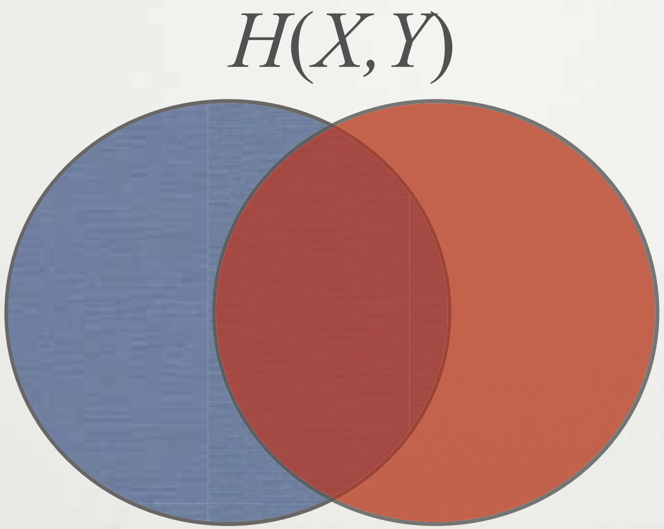

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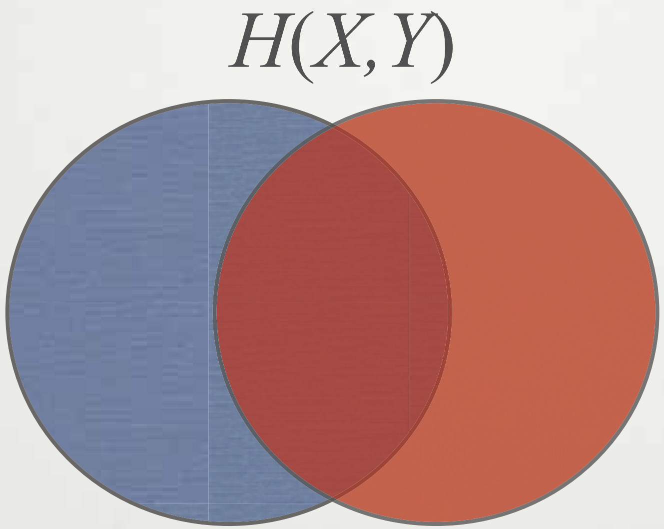

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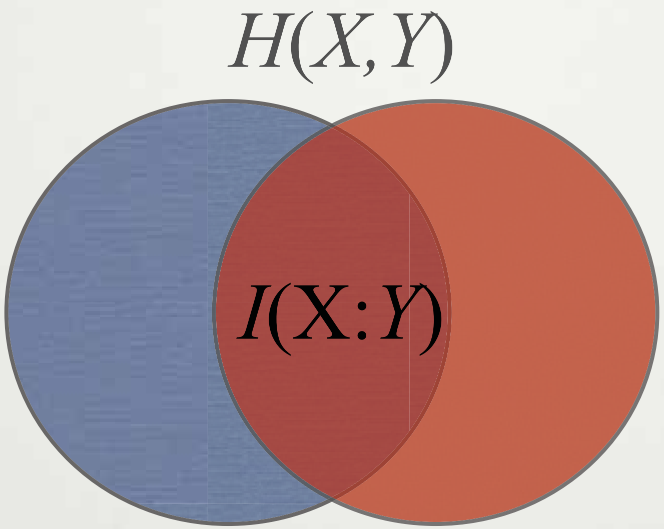

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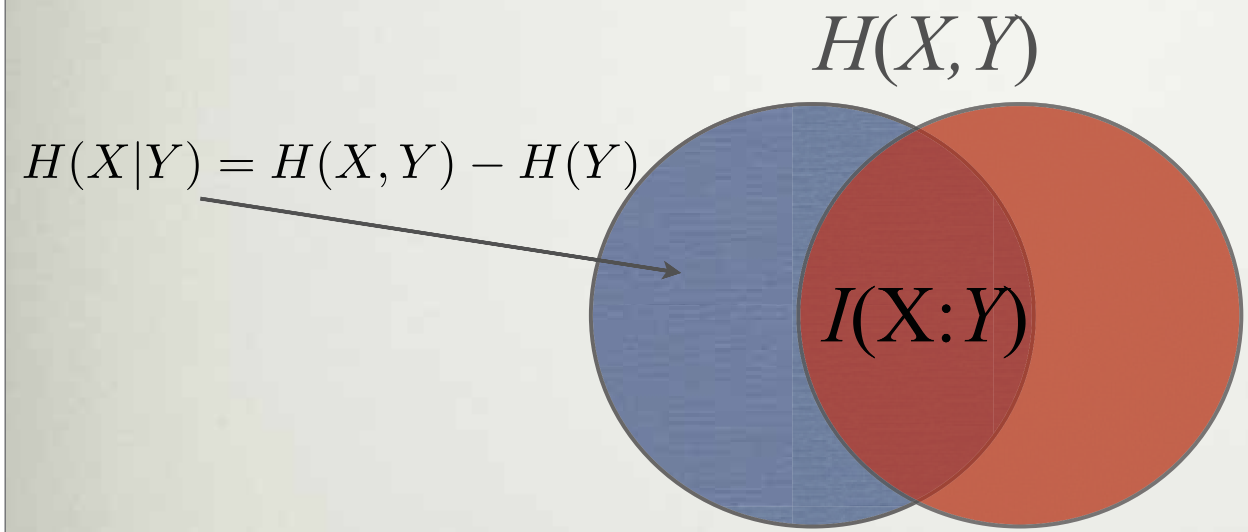

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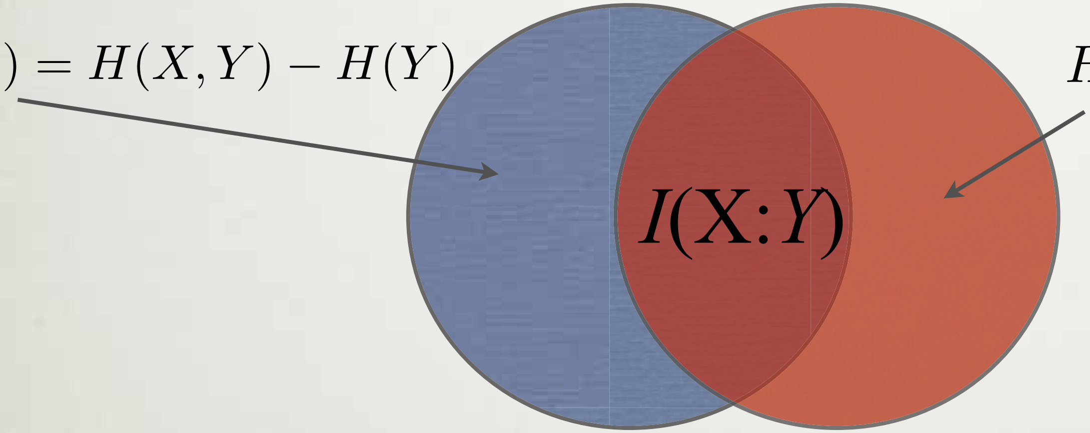

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- Correlation can be distributed at will.

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So $$H(X:Z) = H(X:Y) - H(X|Z)$$

increases linearly with $H(X{:}Y)$, being only constrained by

$$H(X:Z) \leq H(X:Y)$$

Quantum Systems

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$$\begin{array}{ccc} p(x_j) & \longrightarrow & \rho \\ \sum_j & \longrightarrow & Tr\{\} \end{array}$$

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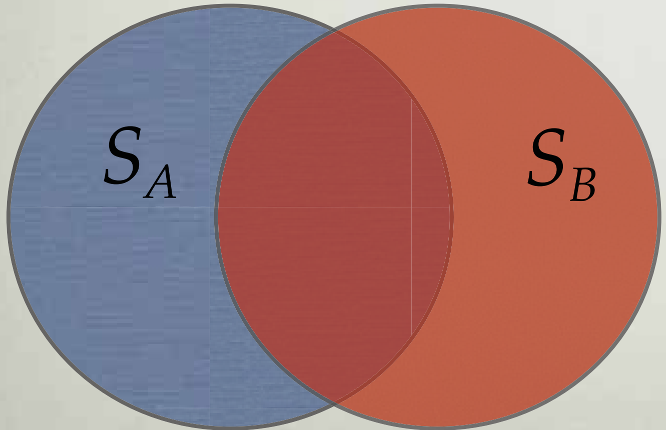

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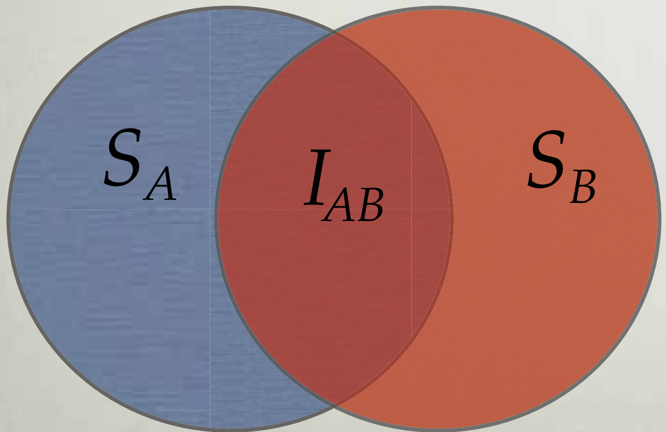

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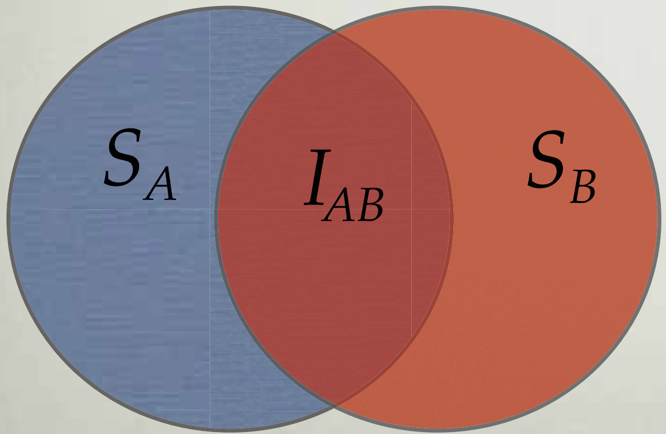

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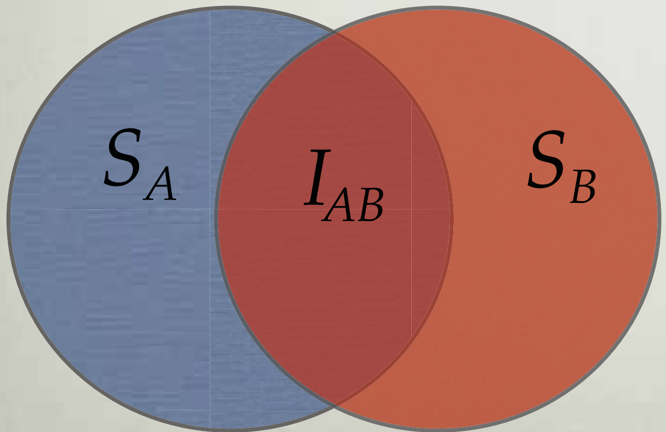

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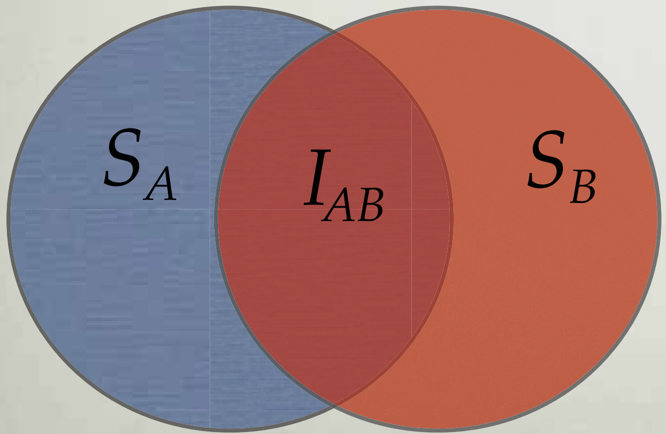

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$I_{AB} \neq 0 \longleftrightarrow$ Entanglement

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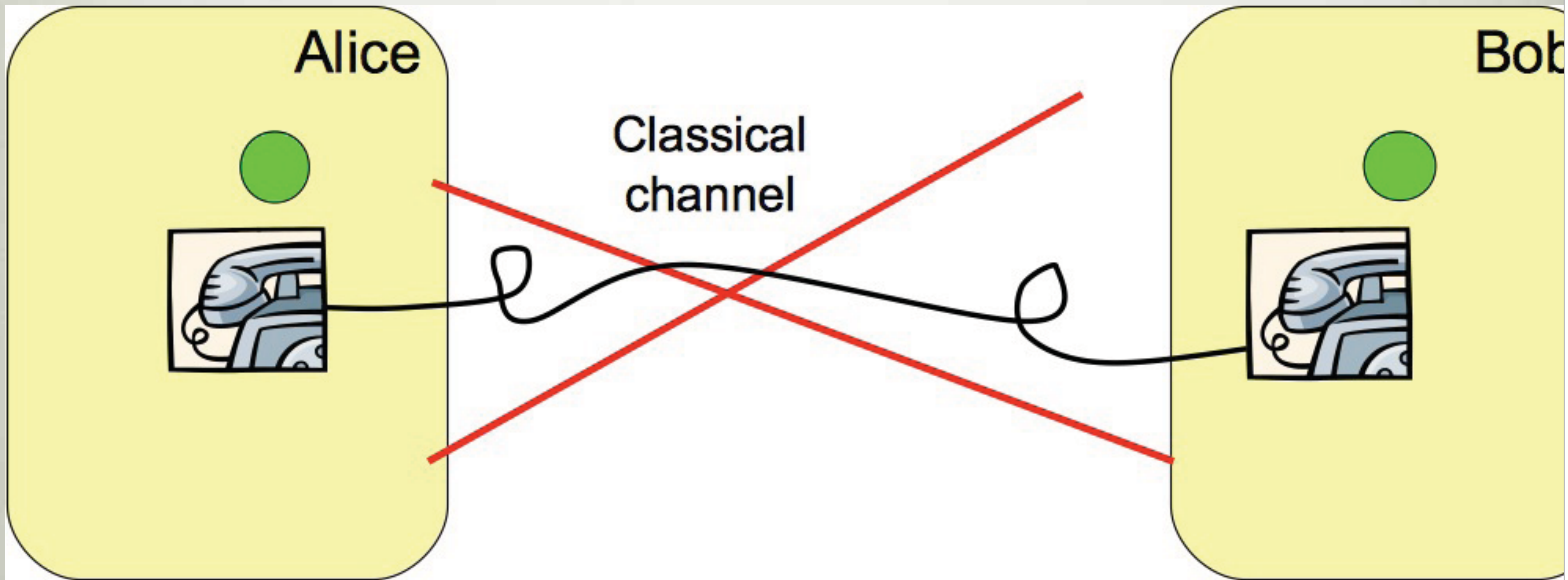
State teleportation

$$|\psi\rangle = a|0\rangle + b|1\rangle, a, b?$$



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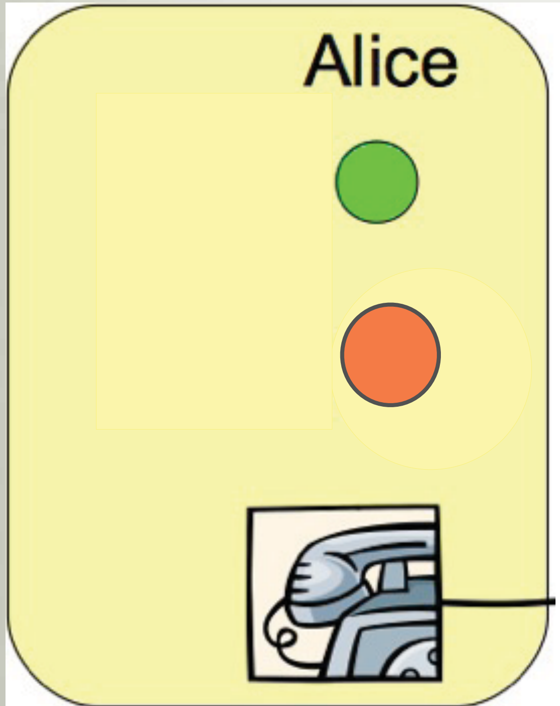
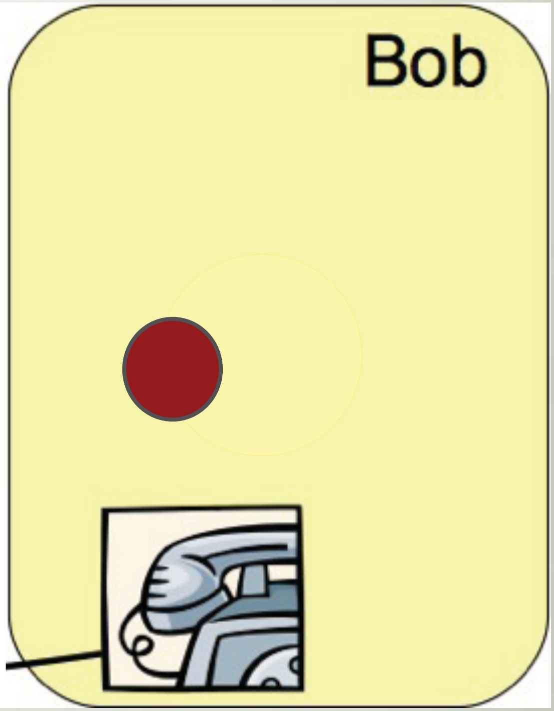


$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$



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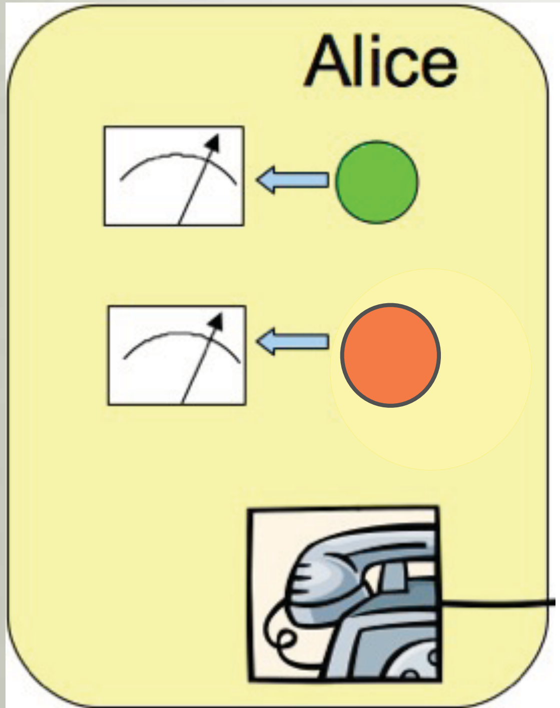

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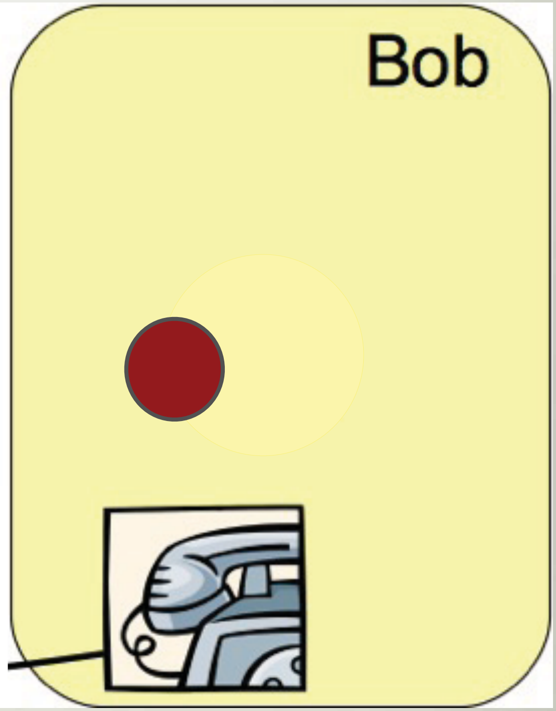

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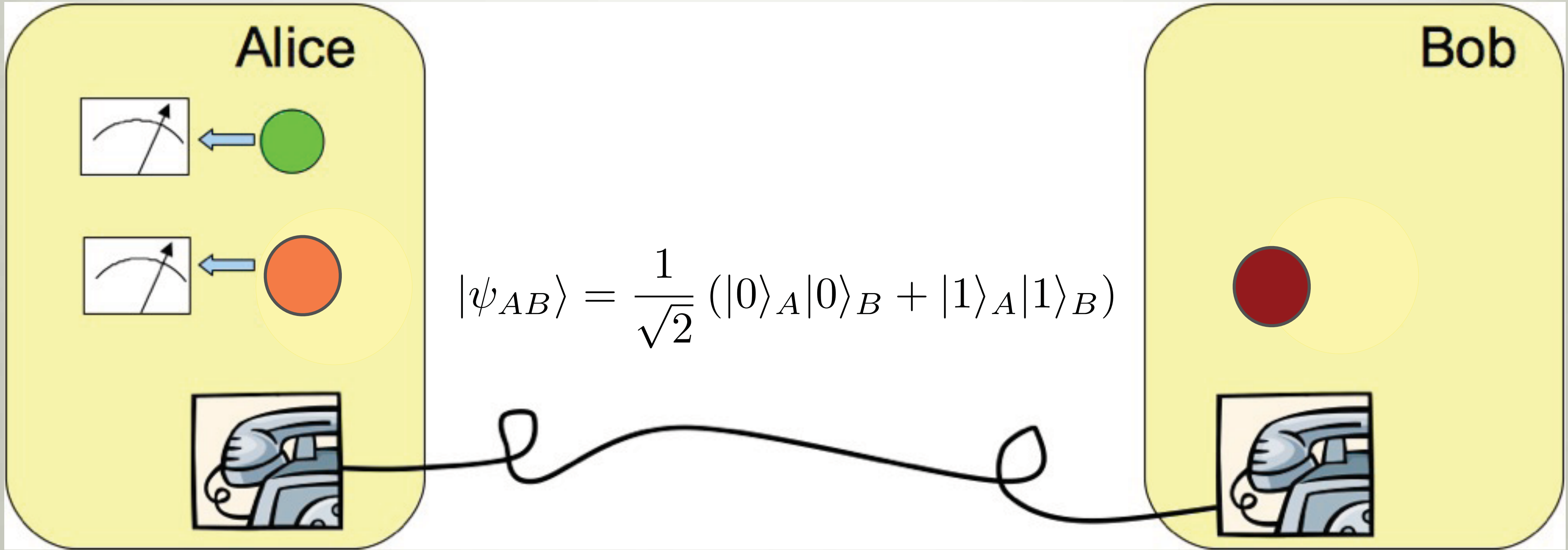

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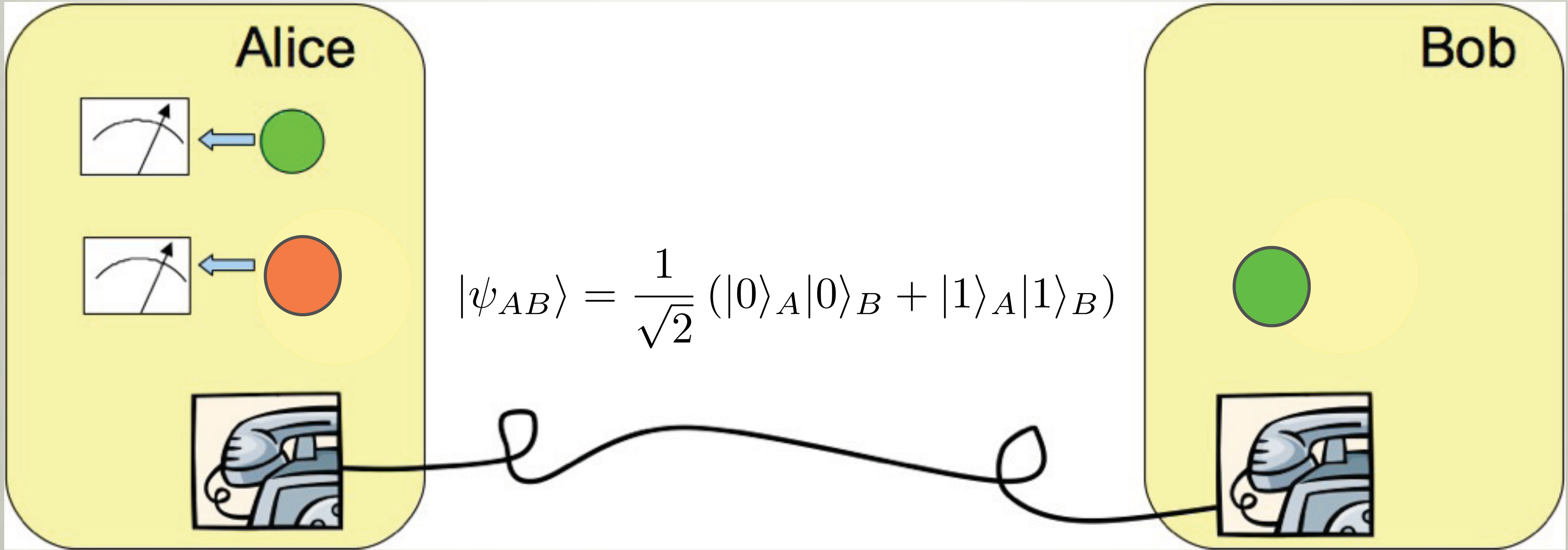

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XY-model with transverse magnetic

$$H = -\frac{J}{2}(1+\gamma)\sum_{i=1}^{N} \sigma_i^x \sigma_{i+1}^x - \frac{J}{2}(1-\gamma)\sum_{i=1}^{N} \sigma_i^y \sigma_{i+1}^y - h\sum_{i=1}^{N} \sigma_i^z$$

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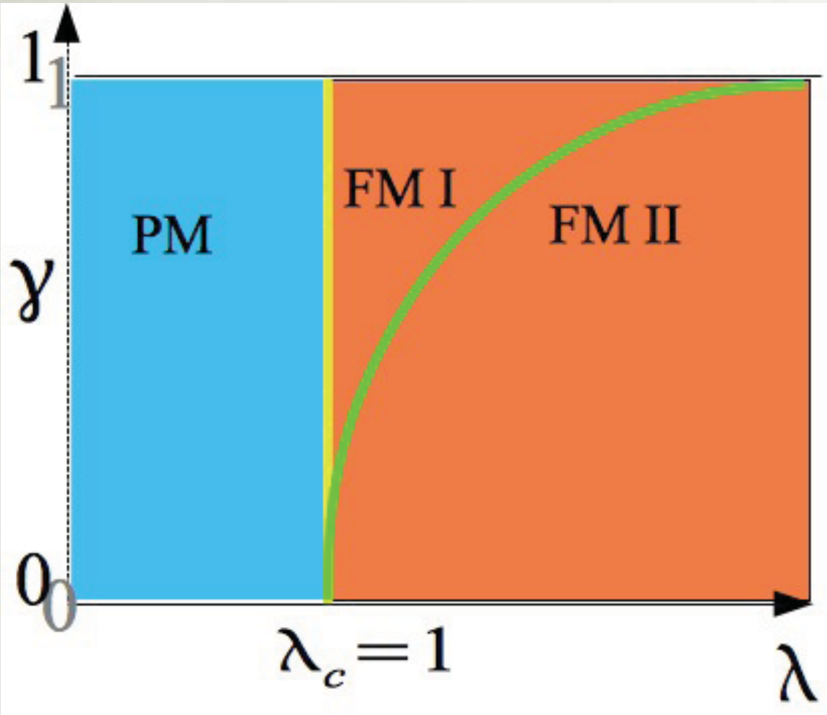

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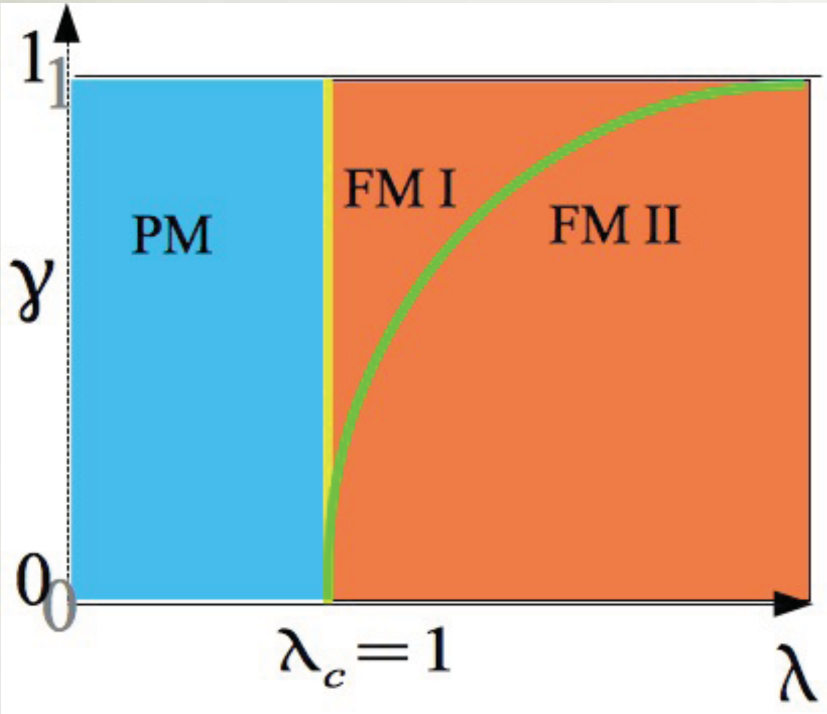

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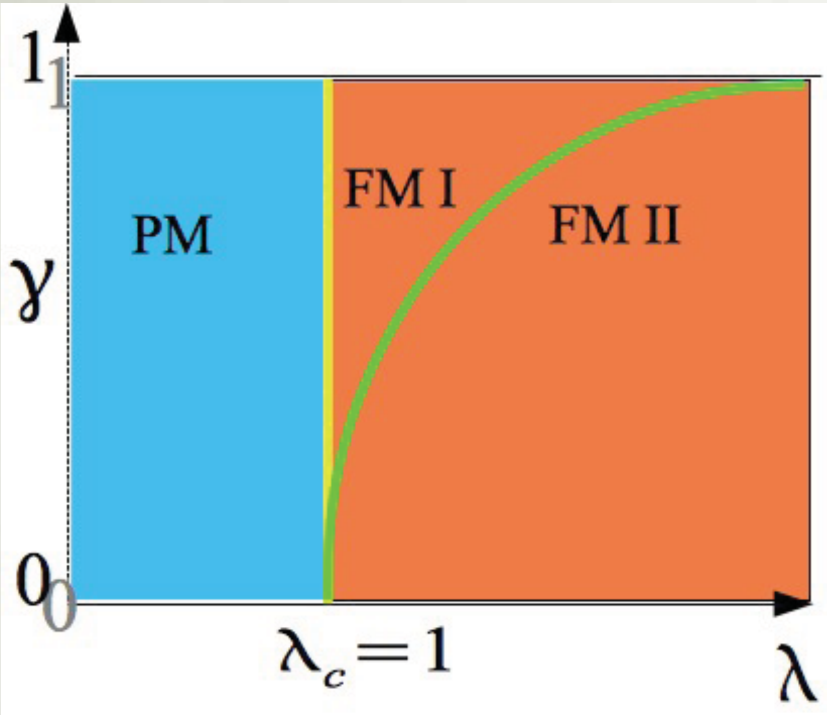
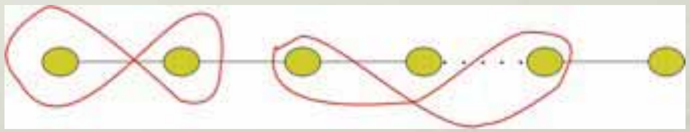
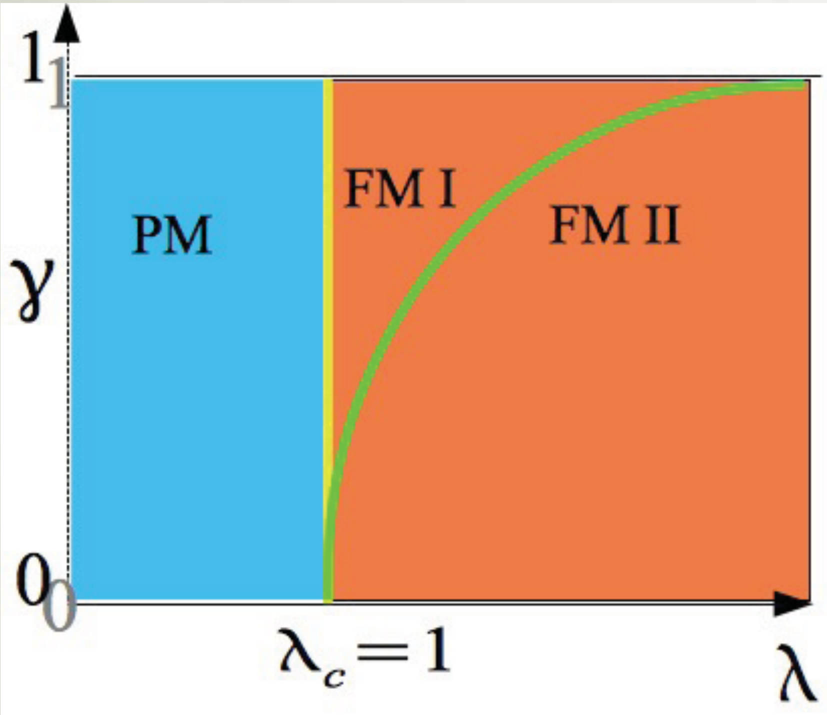


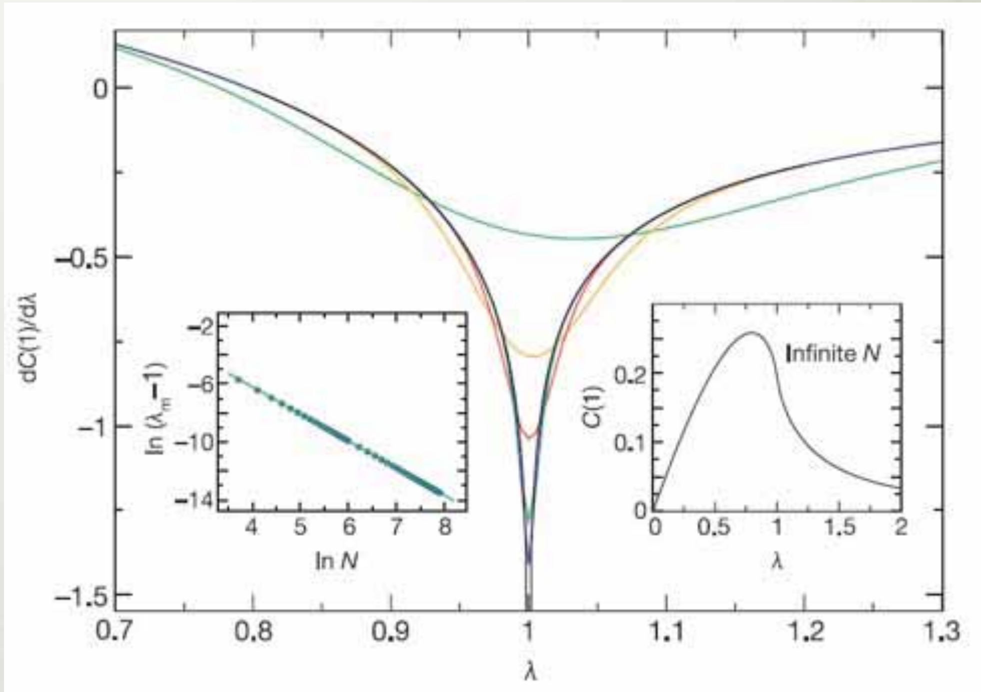
$\lambda = J/h$ (Ising) $\gamma = 1$

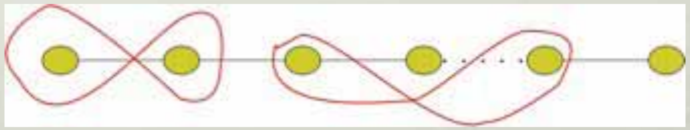


XY-model with transverse magnetic

$$H = -\frac{J}{2}(1+\gamma)\sum_{i=1}^{N}\sigma_i^x\sigma_{i+1}^x - \frac{J}{2}(1-\gamma)\sum_{i=1}^{N}\sigma_i^y\sigma_{i+1}^y - h\sum_{i=1}^{N}\sigma_i^z$$







$\lambda = J/h$ (Ising) $\gamma = 1$

A. Osterloh, Luigi Amico, G. Falci & Rosario Fazio, Nature (London) **416**, 608 (2002).
T. J. Osborne and M. A. Nielsen, Phys. Rev. A **66**, 032110 (2002).

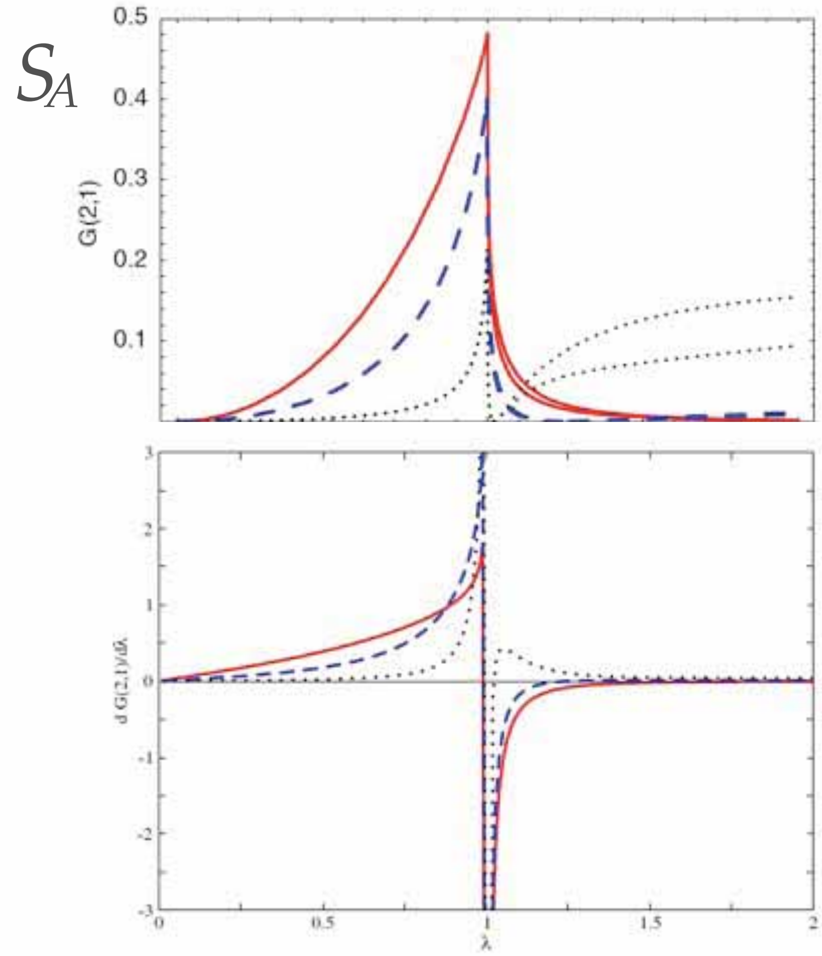
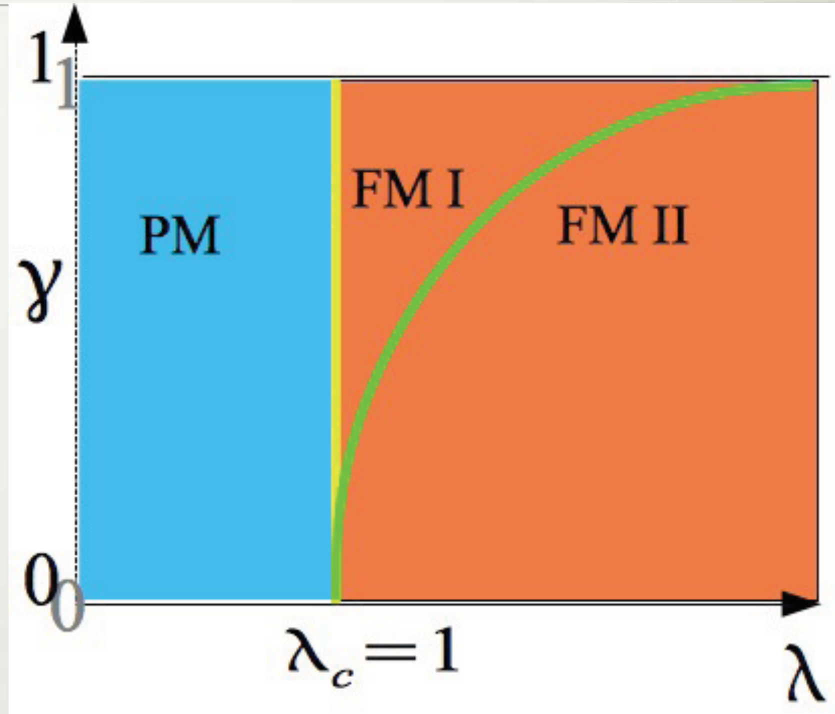
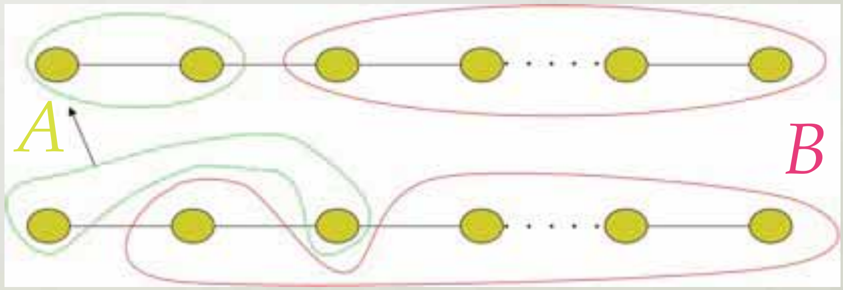


FIG. 2 (color online). Derivative of the lower bound of $\mathcal{G}(2,1)$ for three values of anisotropy: $\gamma = 1$ (red solid line), 0.6 (blue dashed line), and 0.2 (black dotted line). The second phase transition is also imprinted for the $\gamma = 0.2$ as the curve crosses the abscissa at $\lambda = 1/\sqrt{1-\gamma^2}$.





T.R. de Oliveira, G. Rigolin, MCO, and E. Miranda, PRL **97,** 170401 (2006)

$$|S_A - S_B| \leq S_{AB} \leq S_A + S_B$$

$$\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}| \text{ (pure): } Tr\{\rho_{AB}\} \longrightarrow S_{AB} = 0$$

$$I_{AB} = 2S_A$$

$S_A \neq 0$?

$$S_A = H(C) = -\sum_k |c_k|^2 \log(|c_k|^2)$$

$I_{AB} \neq 0 \longleftrightarrow$ Entanglement

$$|\psi_{AB}\rangle = \sum_k c_k |\psi_A^k\rangle \otimes |\phi_B^k\rangle$$

$$\rho_A = Tr_B\{|\psi_{AB}\rangle\langle\psi_{AB}|\} = \sum_k |c_k|^2 |\psi_A^k\rangle\langle\phi_A^k|$$

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ρ_{AB} mixed: $I_{AB} \neq 0 \not\longleftrightarrow$ Entanglement

Entanglement of Formation

Entanglement of Formation

$$\rho_{AB} = \sum_i p_i |\psi_i^{AB}\rangle\langle\psi_i^{AB}|$$

Entanglement of Formation

$$\rho_{AB} = \sum_i p_i |\psi_i^{AB}\rangle\langle\psi_i^{AB}| \longrightarrow S_A$$

Entanglement of Formation

$$\rho_{AB} = \sum_i p_i |\psi_i^{AB}\rangle\langle\psi_i^{AB}| \longrightarrow S_A$$

but

$$\rho_{AB} = \sum_i q_i |\phi_i^{AB}\rangle\langle\phi_i^{AB}|$$

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$$\rho_{AB} = \sum_i p_i |\psi_i^{AB}\rangle\langle\psi_i^{AB}| \rightarrow S_A$$

but

$$\rho_{AB} = \sum_i q_i |\phi_i^{AB}\rangle\langle\phi_i^{AB}| \rightarrow S'_A$$

which one?

Entanglement of Formation

$$\rho_{AB} = \sum_i p_i |\psi_i^{AB}\rangle\langle\psi_i^{AB}| \rightarrow S_A$$

but

$$\rho_{AB} = \sum_i q_i |\phi_i^{AB}\rangle\langle\phi_i^{AB}| \rightarrow S_A'$$

which one?

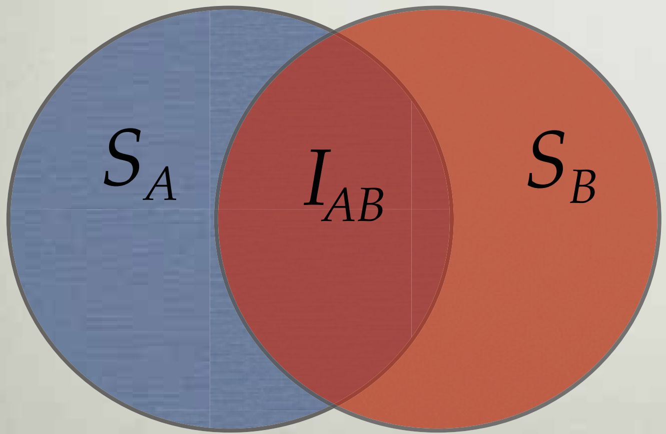
$$E_{\mathcal{F}}(\rho_{AB}) = E_{AB} = \min_{\mathcal{E}} \left\{ \sum_i p_i S(\rho_i^A) \right\} \qquad \mathcal{E} = \{p_i, |\psi_i^{AB}\rangle\}$$

Quantum Systems

$$p(x_j) \longrightarrow \rho$$
$$\sum_j \longrightarrow Tr\{\}$$

$$S_A = S(\rho_A) = -Tr\rho_A \log \rho_A \quad S_B = S(\rho_B) = -Tr\rho_B \log \rho_B$$

$$S_{AB} = S(\rho_{AB}) = -Tr\rho_{AB} \log \rho_{AB}$$



<u>Mutual Information</u>

$$I_{AB} \equiv S(A:B) = S_A + S_B - S_{AB}$$

$$S(A:B) = S_A - S(A|B)$$

Quantum Systems

$$p(x_j) \longrightarrow \rho$$
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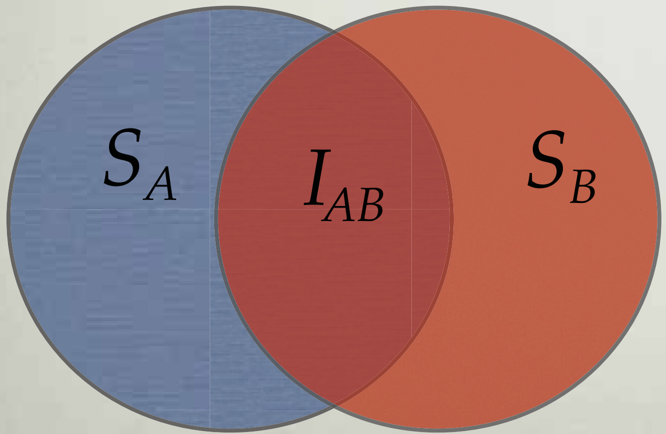
$$S_A = S(\rho_A) = -Tr\rho_A \log \rho_A \quad S_B = S(\rho_B) = -Tr\rho_B \log \rho_B$$

$$S_{AB} = S(\rho_{AB}) = -Tr\rho_{AB} \log \rho_{AB}$$



Mutual Information

$$I_{AB} \equiv S(A:B) = S_A + S_B - S_{AB}$$

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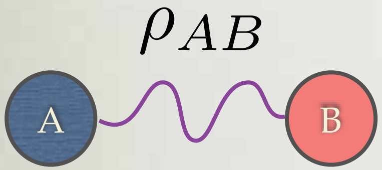
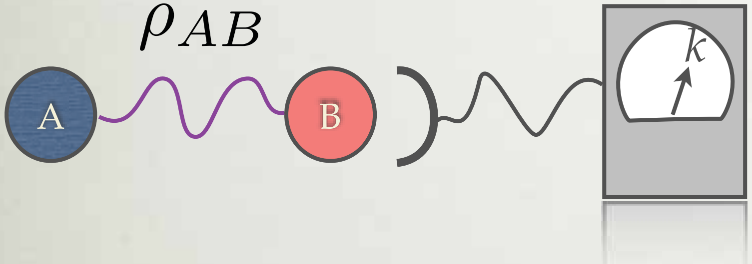
Post and pre-selected states

Post and pre-selected states



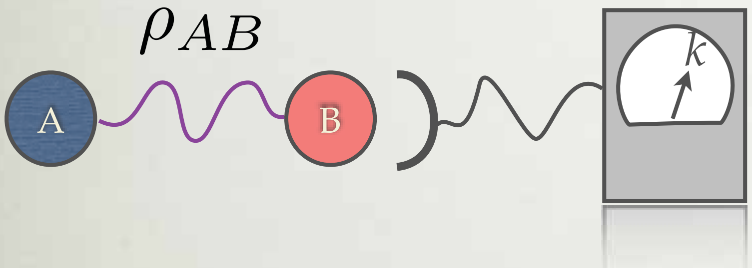
Post and pre-selected states

Measurement on B with outcome k



Post and pre-selected states

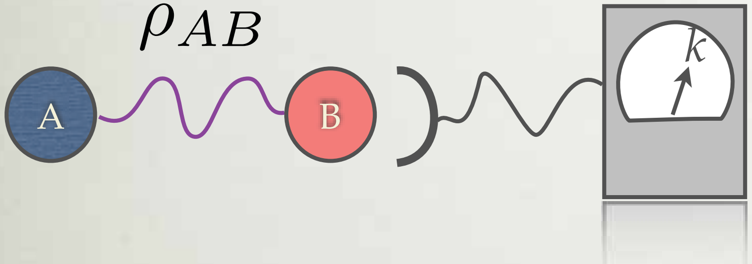
Measurement on B with outcome k



$$\rho^k_{AB} = \frac{E_k \rho_{AB} E_k^\dagger}{Tr\{E_k^\dagger E_k \rho_{AB}\}}$$

Post and pre-selected states

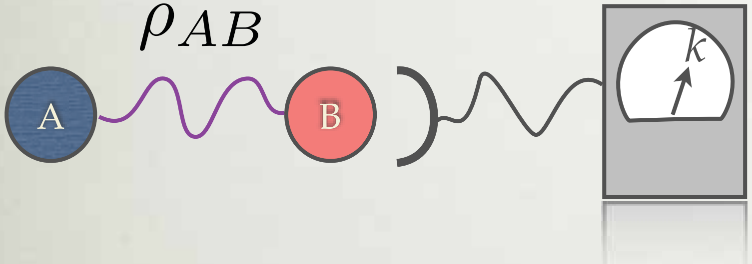
Measurement on B with outcome k



$$\rho^k_{AB} = \frac{\Pi_k \rho_{AB} \Pi_k}{Tr\{\Pi_k \rho_{AB}\}} \qquad \Pi_k = |\phi_k\rangle\langle\phi_k|_B$$

Post and pre-selected states

Measurement on B with outcome k

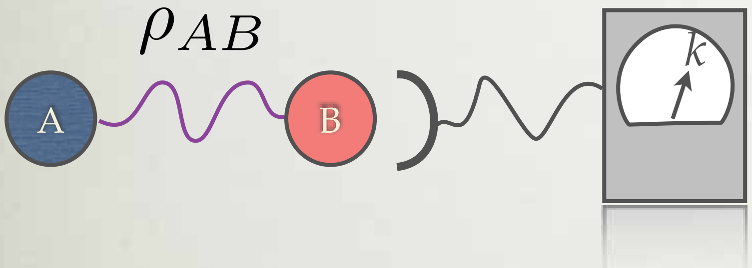


$$\rho_{AB}^k = \frac{\Pi_k \rho_{AB} \Pi_k}{p_k} \qquad \Pi_k = |\phi_k\rangle\langle\phi_k|_B$$

$$p_k = Tr\{E_k^\dagger E_k \rho_{AB}\}$$

Post and pre-selected states

Measurement on B with outcome k



$$\rho^k_{AB} = \frac{\Pi_k \rho_{AB} \Pi_k}{p_k} \qquad \Pi_k = |\phi_k\rangle\langle\phi_k|_B$$

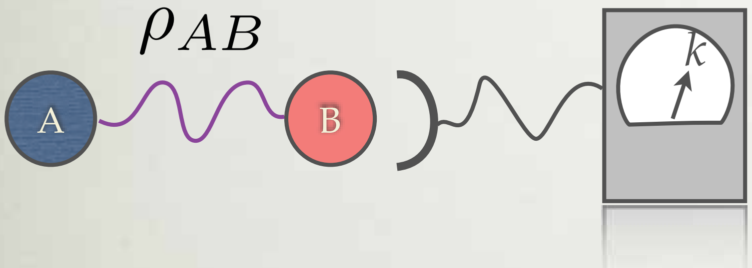
$$p_k = Tr\{E_k^\dagger E_k \rho_{AB}\}$$

$$\rho^k_A = \frac{Tr_B\{\Pi_k \rho_{AB} \Pi_k\}}{p_k}$$

Post-selected state

Post and pre-selected states

Measurement on B with outcome k



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$$p_k = Tr\{E_k^\dagger E_k \rho_{AB}\}$$

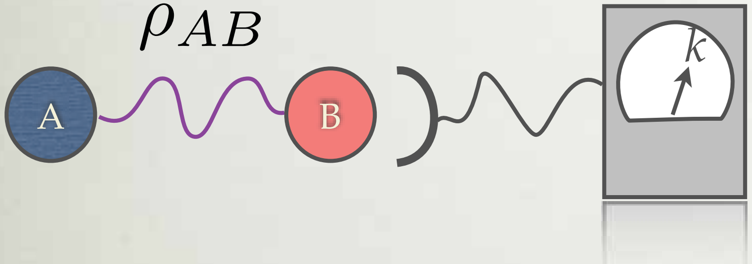
$$\rho^k_A = \frac{Tr_B\{\Pi_k \rho_{AB} \Pi_k\}}{p_k}$$

Post-selected state

$$\rho_A = \sum_k p_k \rho^k_A = \sum_k Tr_B\{\Pi_k \rho_{AB} \Pi_k\}$$

Post and pre-selected states

Measurement on B with outcome k



$$\rho_{AB}^{k} = \frac{\Pi_k \rho_{AB} \Pi_k}{p_k} \qquad \Pi_k = |\phi_k\rangle\langle\phi_k|_B$$

$$p_k = Tr\{E_k^{\dagger} E_k \rho_{AB}\}$$

$$\rho_{A}^{k} = \frac{Tr_B\{\Pi_k \rho_{AB} \Pi_k\}}{p_k}$$

Post-selected state

$$\rho_A = \sum_k p_k \rho_A^k = Tr_B\{\rho_{AB}\}$$

Pre-selected state

(QUANTUM) MUTUAL INFORMATION

(Quantum) Mutual Information

$$S(A:B) = S_A - S(A|B)$$

(QUANTUM) MUTUAL INFORMATION

$$S(A:B) = S_A - S(A|B)$$

$$J_{A|B} = S(\rho_A) - \sum_j p_j S(\rho_A^j)$$

$$p_j = \mathrm{Tr}_{AB}\left\{\Pi_j^B \rho_{AB} \Pi_j^B\right\}, \;\; \rho_A^j = \frac{\mathrm{Tr}_B\{\Pi_j^B \rho_{AB} \Pi_j^B\}}{p_j}$$

(Quantum) Mutual Information

$$S(A:B) = S_A - S(A|B)$$

$$J_{A|B} = S(\rho_A) - \sum_j p_j S(\rho_A^j)$$

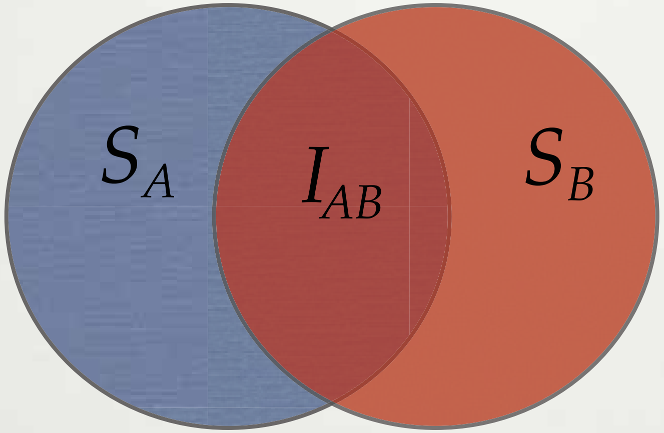
$$p_j = \mathrm{Tr}_{AB}\left\{\Pi_j^B \rho_{AB} \Pi_j^B\right\}, \quad \rho_A^j = \frac{\mathrm{Tr}_B\{\Pi_j^B \rho_{AB} \Pi_j^B\}}{p_j}$$

$$J_{AB}^{\leftarrow} = \max_{\{\Pi_j^B\}} \left[S(\rho_A) - \sum_j p_j S(\rho_A^j) \right]$$

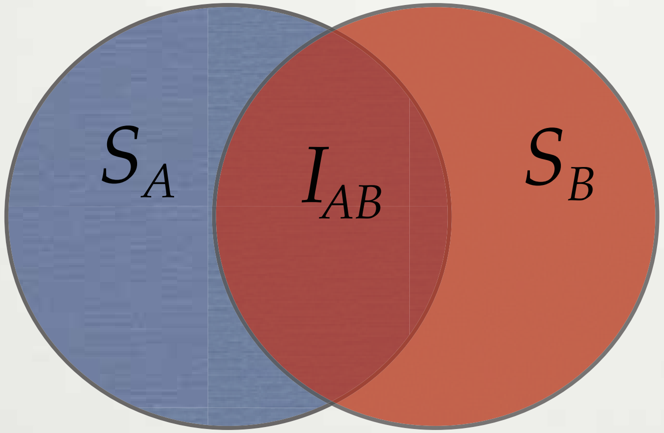
Classical Correlation

L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

Local accessible and Inaccessible information

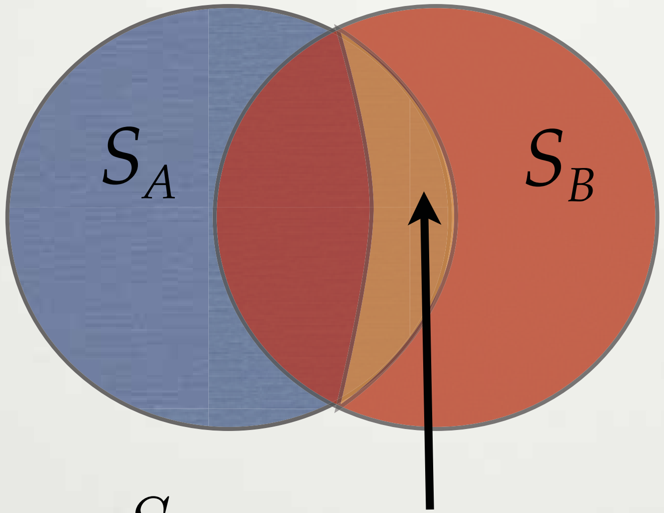


Local accessible and Inaccessible information



$$I_{AB} = S_A + S_B - S_{AB}$$

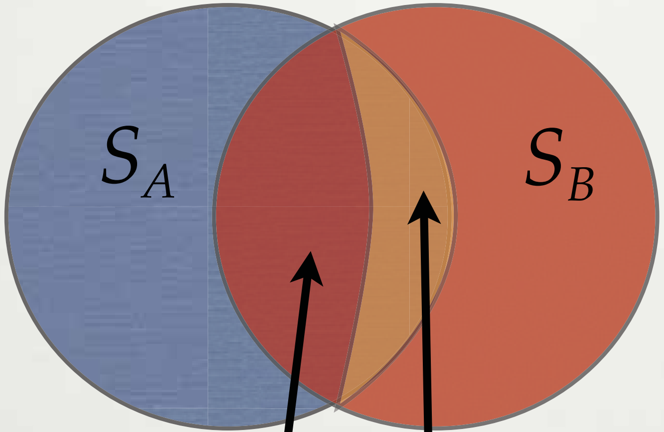
Local accessible and Inaccessible information



$$I_{AB} = S_A + S_B - S_{AB}$$

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right],$$

Local accessible and Inaccessible information



$$I_{AB} = S_A + S_B - S_{AB}$$

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right],$$

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$

(Quantum Discord)

$$E_{AB} = 0 \Leftrightarrow \rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

$$\delta_{AB}^{\leftarrow} = 0 \Leftrightarrow \rho_{AB} = \sum_i p_i \rho_i^A \otimes \Pi_i^B$$

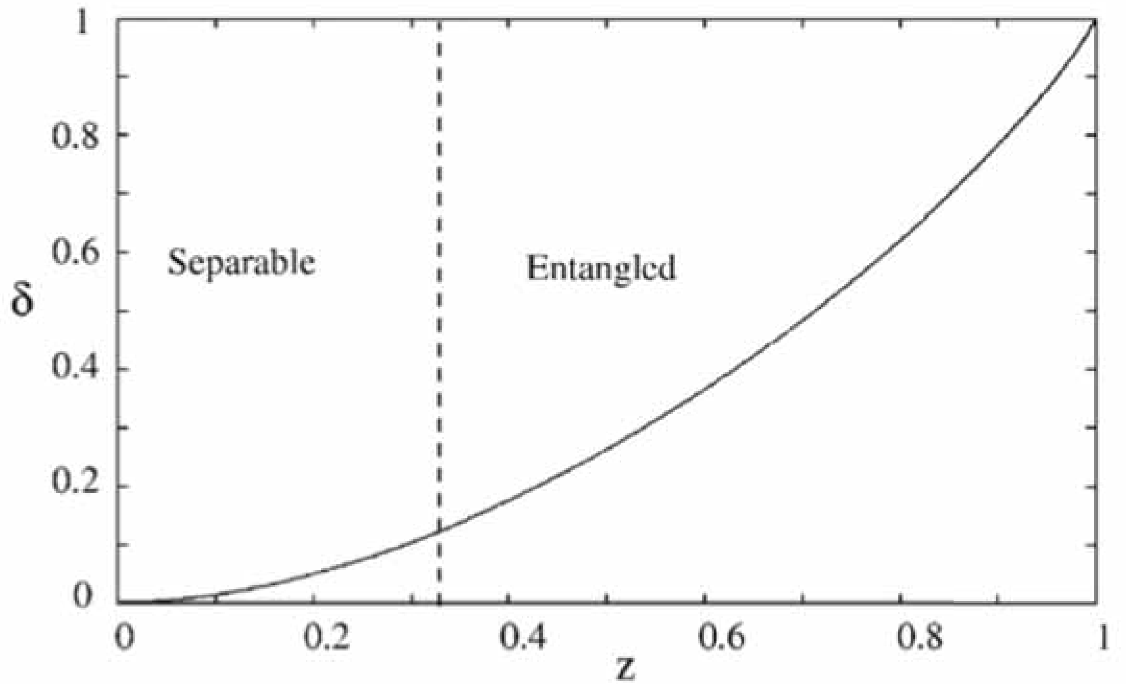


FIG. 2. Value of the discord for Werner states $\frac{1-z}{4}\mathbf{1} + z|\psi\rangle\langle\psi|$, with $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Discord does not depend on the basis of measurement in this case because both $\mathbf{1}$ and $|\psi\rangle$ are invariant under local rotations.

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001)

Preliminary Remark

Trade-off between the bipartite entanglement of A with B and the entanglement of A with C.

If $$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0_A, 0_B\rangle + |1_A, 1_B\rangle\right)$$

There is no way to A get entangled to C without decreasing entanglement with B.

Preliminary Remark

Trade-off between the bipartite entanglement of A with B and the entanglement of A with C.

$$\mathcal{E}_{A|BC} = \mathcal{E}_{A|B} + \mathcal{E}_{A|C} + \tau_{ABC}$$

Preliminary Remark

Trade-off between the bipartite entanglement of *A* with *B* and the entanglement of *A* with *C*.

$$\mathcal{E}_{A|BC} = \mathcal{E}_{A|B} + \mathcal{E}_{A|C} + \tau_{ABC}$$

$$C^2_{A(BC)} = C^2_{AB} + C^2_{AC} + \tau_{ABC}$$

V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A 61, 052306 (2000)

Quantum Systems

- Extension of classical form

$$S(A:B) \equiv I_{AB} = S_A + S_B - S_{AB}$$

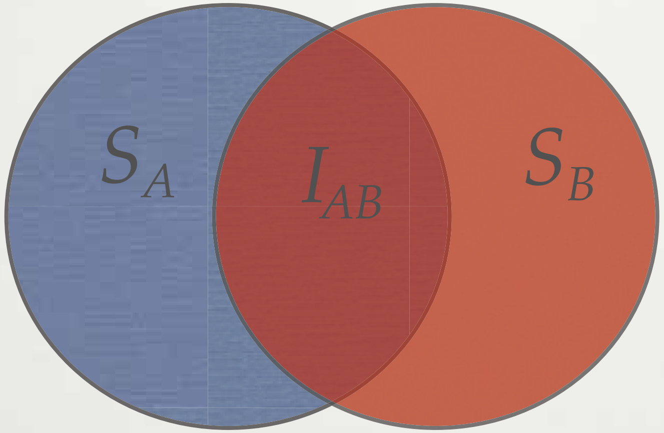
Not always subadditive

$$S(A:B,C) \not\leq S(A:B) + S(A:C)$$

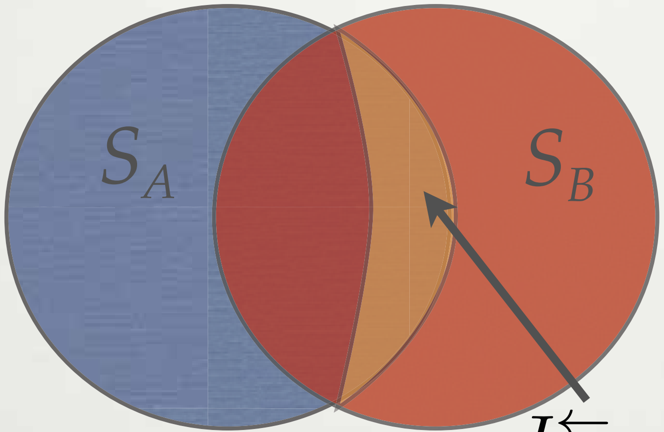
Proper form

$$S(A:B) = S_A - S(A|B)$$

Local accessible and Inaccessible information



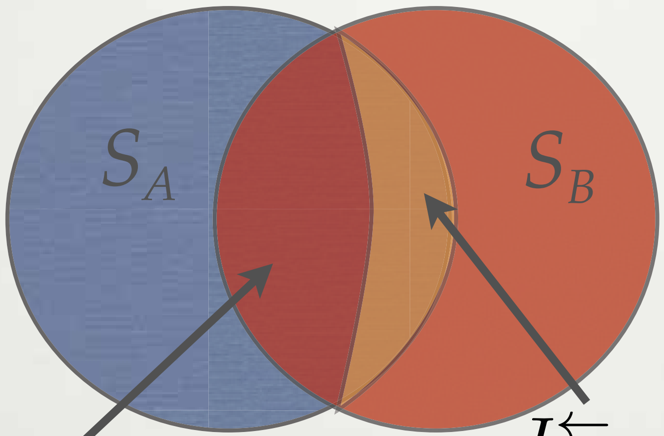
Local accessible and Inaccessible information



(CC)

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right]$$

Local accessible and Inaccessible information



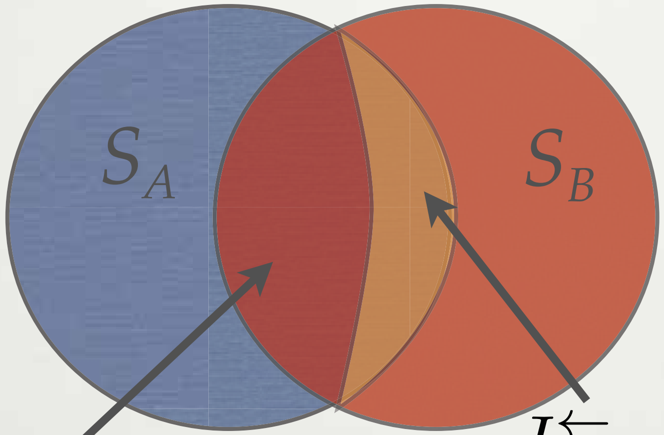
(QD)

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$

(CC)

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right]$$

Local accessible and Inaccessible information



(CC)

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right]$$

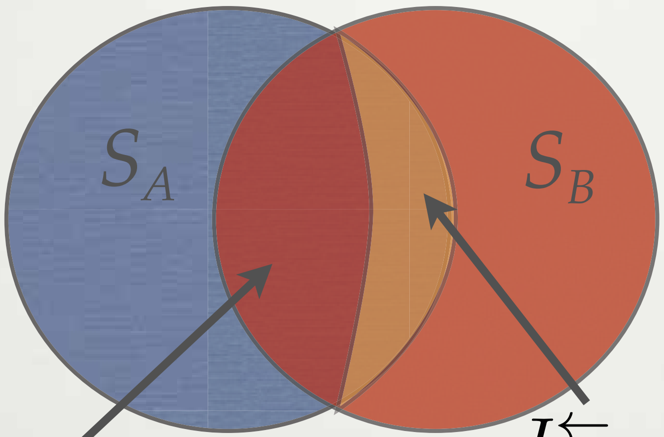
(QD)

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$

Discrepancy: $\Delta^{\leftarrow}_{AB} \equiv J^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AB}$

$$-I_{AB} \leq \Delta^{\leftarrow}_{AB} \leq I_{AB}$$

Local accessible and Inaccessible information



(CC)

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right]$$

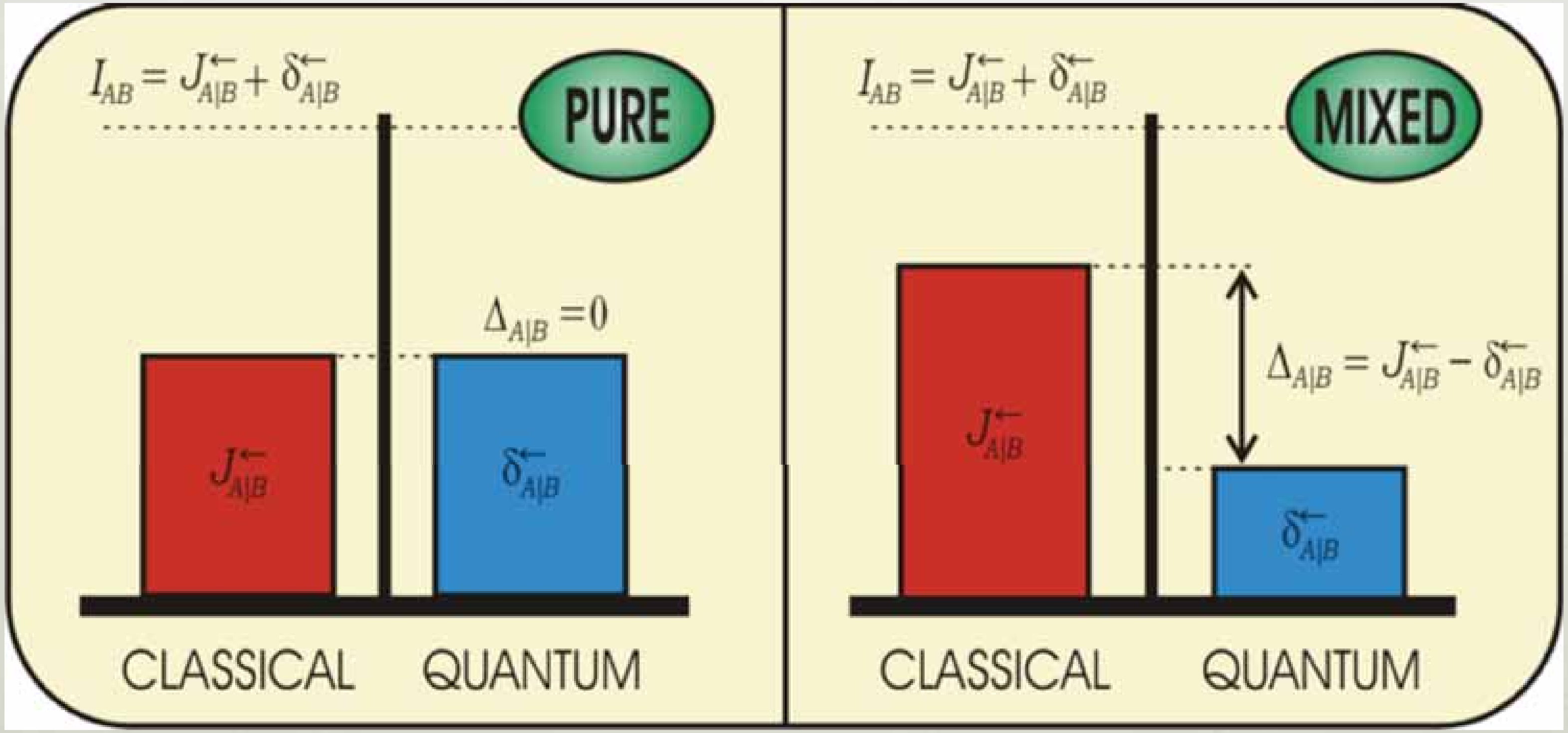
(QD)

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$

Discrepancy: $\Delta^{\leftarrow}_{AB} \equiv J^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AB}$ $\quad -I_{AB} \leq \Delta^{\leftarrow}_{AB} \leq I_{AB}$

Balance between the gain in work extraction by the use of global operations over local ones, and the work extracted locally only.

Correlation Discrepancy



EOF Monogamy

ρ_{ABC} pure:

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} = \delta^{\leftarrow}_{AC} + S_{A|C}$$

$$E_{AB} = E_F\left(\rho_{ab}\right) = \min_{\mathcal{E}} \left\{ \sum_i p_i E_F\left(|\varphi_i\rangle\right) \right\}$$

M. Koashi and A. Winter, PRA 69, 022309 (2004)

F. F. Fanchini, M. F. Cornelio, MCO, and A. O.Caldeira, PRA 84, 012313 (2011).

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} = \delta^{\leftarrow}_{AC} + S_{A|C}$$
$$E_{AC} = \delta^{\leftarrow}_{AB} + S_{A|B}$$

$$E_{AB} = E_F\left(\rho_{ab}\right) = \min_{\mathcal{E}} \left\{ \sum_i p_i E_F\left(|\varphi_i\rangle\right) \right\}$$

M. Koashi and A. Winter, PRA 69, 022309 (2004)

F. F. Fanchini, M. F. Cornelio, MCO, and A. O.Caldeira, PRA 84, 012313 (2011).

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} + E_{AC} = \delta^{\leftarrow}_{AB} + \delta^{\leftarrow}_{AC}$$

F. F. Fanchini, M. F. Cornelio, MCO, and A. O.Caldeira, PRA 84, 012313 (2011).

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} + E_{AC} + S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC} = S_A$$

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} + E_{AC} + (S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC}) = E_{A(BC)}$$

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} + E_{AC} + \tau_A = E_{A(BC)}$$

$$\tau_A = (S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC}) = \frac{1}{2}[\Delta^{\leftarrow}_{AB} + \Delta^{\leftarrow}_{AC}]$$

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} + E_{AC} + \tau_A = E_{A(BC)}$$

$$\tau_A = (S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC}) = \frac{1}{2}[\Delta^{\leftarrow}_{AB} + \Delta^{\leftarrow}_{AC}]$$

$$\tau_A \geq 0 \quad \leftrightarrow \quad J^{\leftarrow}_{AB} + J^{\leftarrow}_{AC} \geq \delta^{\leftarrow}_{AB} + \delta^{\leftarrow}_{AC}$$

EOF Monogamy

ρ_{ABC} pure:

$$E_{AB} + E_{AC} + \tau_A = E_{A(BC)}$$

$$\tau_A = (S_A - \delta^{\leftarrow}_{AB} - \delta^{\leftarrow}_{AC}) = \frac{1}{2}[\Delta^{\leftarrow}_{AB} + \Delta^{\leftarrow}_{AC}]$$

$$\tau_A \geq 0 \qquad \leftrightarrow \qquad J^{\leftarrow}_{AB} + J^{\leftarrow}_{AC} \geq \delta^{\leftarrow}_{AB} + \delta^{\leftarrow}_{AC}$$

EOF <u>not</u> monogamous if

$$S_A < S_q(A|B) + S_q(A|C) \leq 2S_A$$

$$S_q(A|i) = \min_{\{\Pi_k\}} \sum_k p_k S(\rho_{A|k}), \qquad \rho_{A|k} = \frac{\mathrm{Tr}_i(\Pi^i_k \rho_{Ai} \Pi^i_k)}{\mathrm{Tr}_{Ai}(\Pi^i_k \rho_{Ai} \Pi^i_k)}, \quad i = B, C$$

Example

Example

$$\tau_{ABC} \equiv \tau_A + \tau_B + \tau_C = \Delta_{\circlearrowright} \equiv \Delta^{\leftarrow}_{BA} + \Delta^{\leftarrow}_{CB} + \Delta^{\leftarrow}_{AC}$$

Example

$$\tau_{ABC} \equiv \tau_A + \tau_B + \tau_C = \Delta_{\circlearrowright} \equiv \Delta^{\leftarrow}_{BA} + \Delta^{\leftarrow}_{CB} + \Delta^{\leftarrow}_{AC}$$

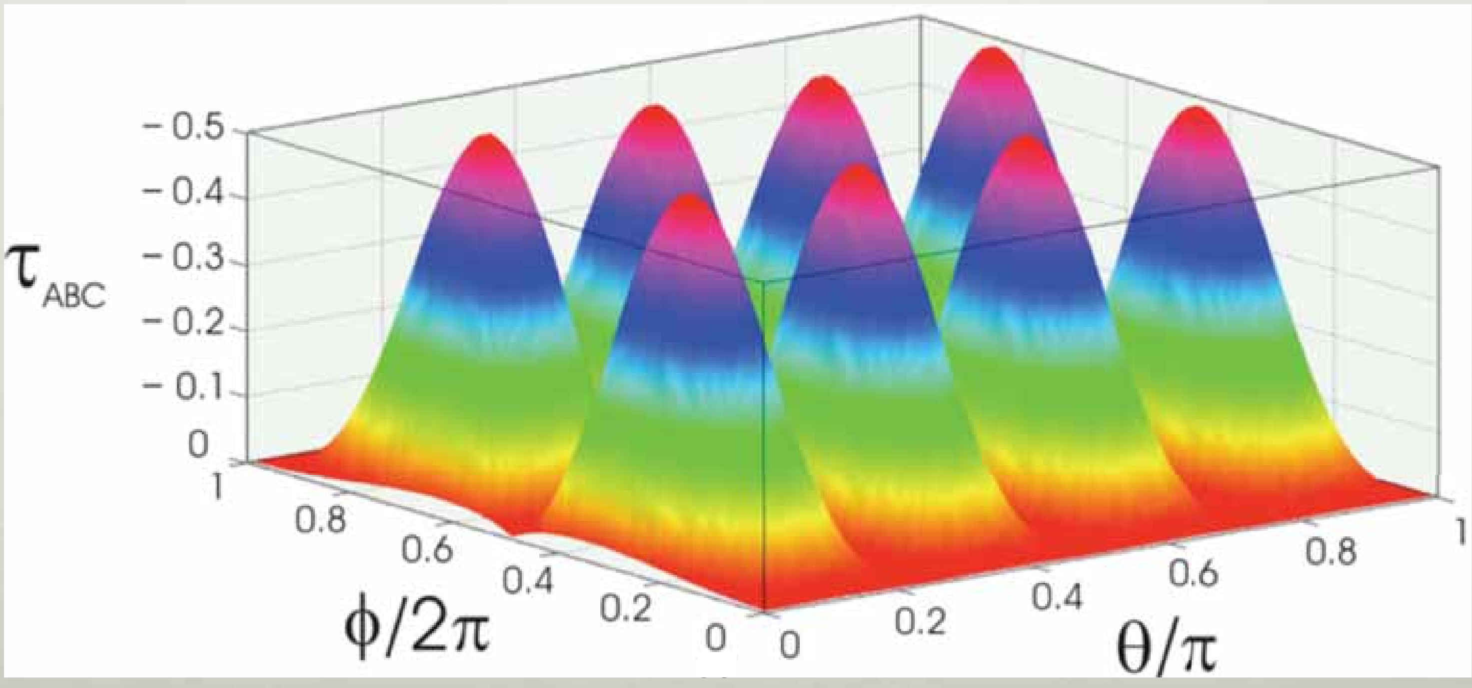
$$|GHZ\rangle = \theta|\uparrow\uparrow\uparrow\rangle + \phi|\downarrow\downarrow\downarrow\rangle \quad \Rightarrow \quad \tau_{ABC} > 0$$

Example

$$\tau_{ABC} \equiv \tau_A + \tau_B + \tau_C = \Delta_{\circlearrowright} \equiv \Delta^{\leftarrow}_{BA} + \Delta^{\leftarrow}_{CB} + \Delta^{\leftarrow}_{AC}$$

$$|GHZ\rangle = \theta|\uparrow\uparrow\uparrow\rangle + \phi|\downarrow\downarrow\downarrow\rangle \quad \Rightarrow \quad \tau_{ABC} > 0$$

$$|W\rangle = \alpha|\uparrow\uparrow\downarrow\rangle + \beta|\uparrow\downarrow\uparrow\rangle + \gamma|\downarrow\uparrow\uparrow\rangle \quad \Rightarrow \quad \tau_{ABC} < 0$$



Conclusions

- Quantum correlation exists whenever a quantum feature is present
- Resource for information processing
- Two forms of correlation: local accessible and non-local accessible
- Exists states that although with zero entanglement are still quantum correlated