Quantum measurement and control in circuit QED systems

Xin-Qi Li（李新奇）

Dept. of Physics, Beijing Normal University

Outline

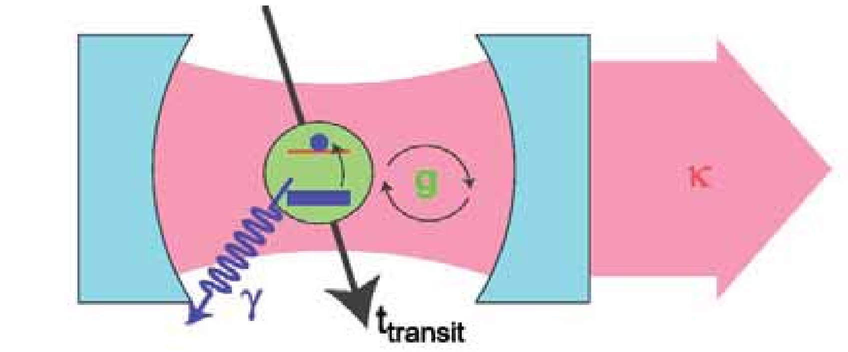
- **Part (I): Quantum Measurements**
  - **Simplest version: transmission measurement**
  - **Qubit-cavity state correlation: S-cat paradox**
  - **Homodyne measurement: SNR**
  - **Single microwave photon measurement**

- **Part (II): Quantum Control**
  - **Entanglement**
  - **2-bit Bell state control**
  - **3-bit GHZ state control**

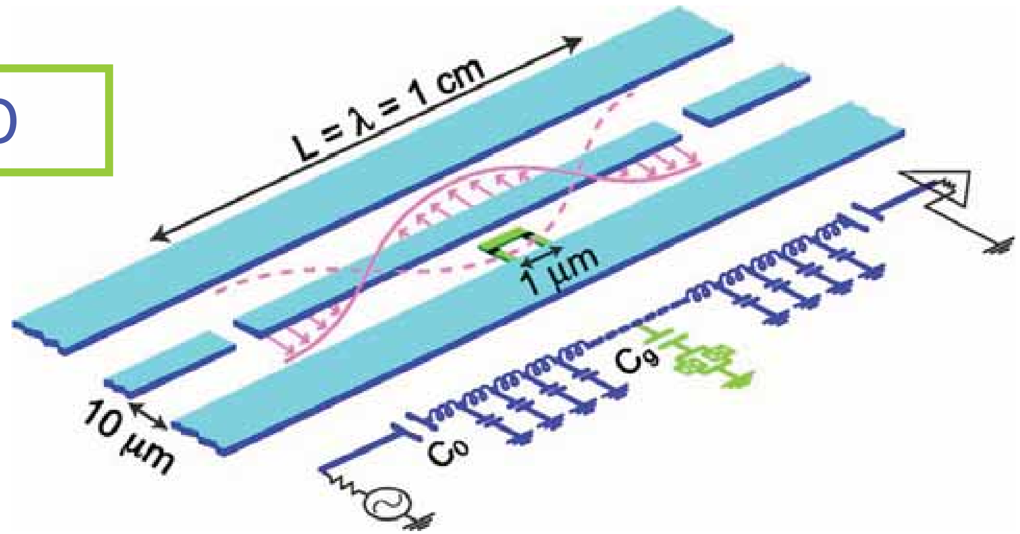
Part （I）

Quantum Measurements

Cavity QED



circuit QED



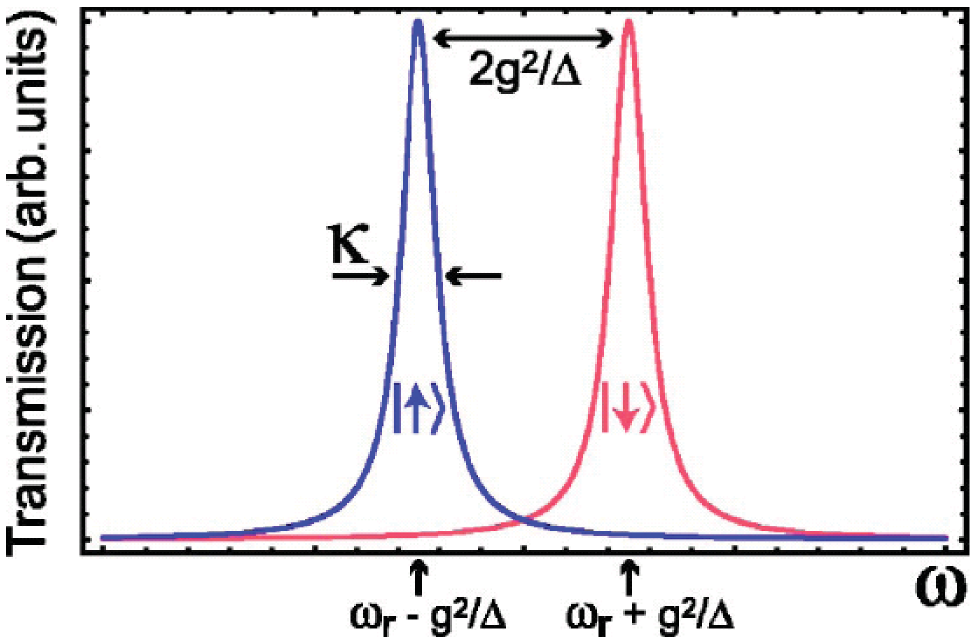
Blais, R.-S. Huang, A. Wallraff, S.M.Girvin, and R.J. Schoelkopf, Phys. Rev. A 69, 062320 (2004).

Wallraff *et al*, Nature 431, 162 (2004).

(1) Measurement Principles

J-C model: $H = \hbar\omega_{\mathrm{r}}\left(a^{\dagger}a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^{z} + \hbar g(a^{\dagger}\sigma^{-} + \sigma^{+}a)$

Dispersive regime: $UHU^{\dagger} \approx \hbar\left[\omega_{\mathrm{r}} + \frac{g^{2}}{\Delta}\sigma^{z}\right]a^{\dagger}a + \frac{\hbar}{2}\left[\Omega + \frac{g^{2}}{\Delta}\right]\sigma^{z}$



Initial state $|0\rangle \otimes (|e\rangle + |g\rangle)/\sqrt{2}$

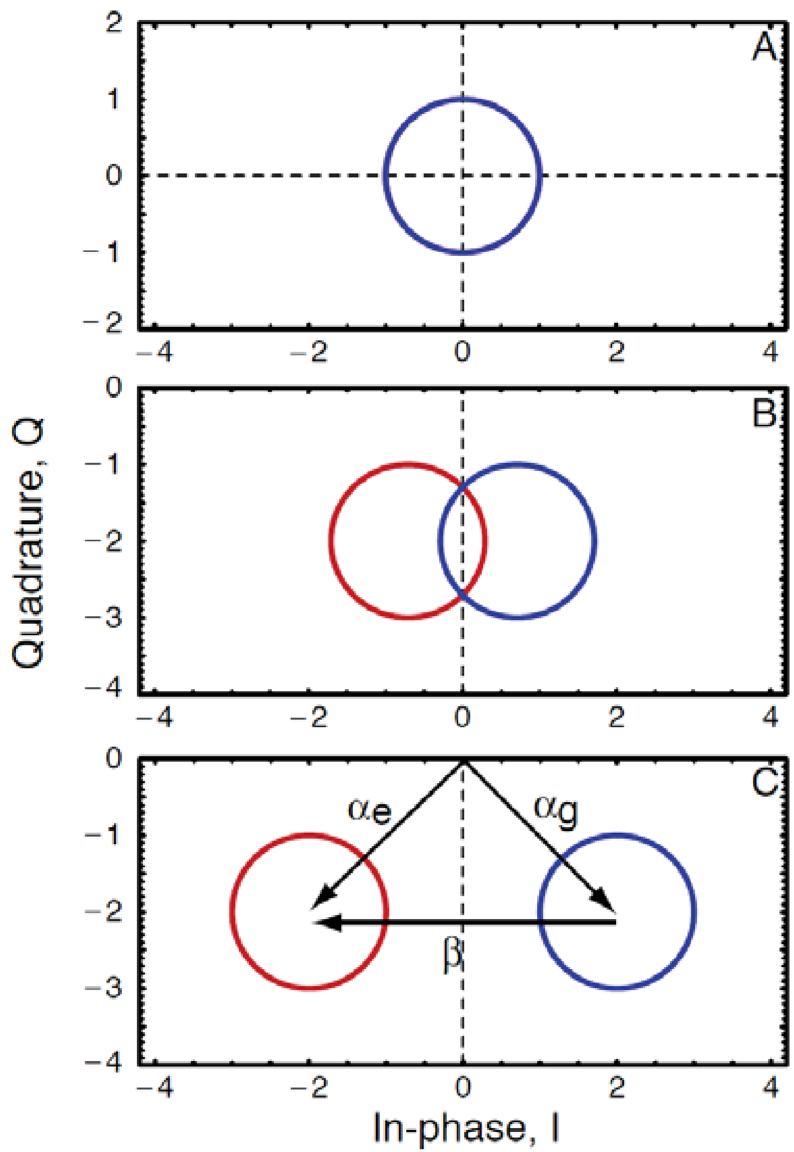
$$H = \frac{\hbar\omega_a}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar g\left(a^\dagger\sigma_- + a\sigma_+\right) + \hbar\left[\varepsilon_d(t)a^\dagger e^{-i\omega_d t} + \varepsilon_d^*(t)a e^{i\omega_d t}\right].$$

$$H_{\text{eff}} = \frac{\hbar\tilde{\omega}_a}{2}\sigma_z + \hbar\Delta_r a^\dagger a + \hbar\chi a^\dagger a\sigma_z + \hbar\left[\varepsilon_d(t)a^\dagger + \varepsilon_d^*(t)a\right]$$

$$\boxed{\begin{aligned}\dot{\alpha}_e(t) &= -i\varepsilon_d(t) - i(\Delta_r + \chi)\alpha_e(t) - \kappa\alpha_e(t)/2\\ \dot{\alpha}_g(t) &= -i\varepsilon_d(t) - i(\Delta_r - \chi)\alpha_g(t) - \kappa\alpha_g(t)/2\end{aligned}}$$

In-phase $I = \mathrm{Re}[\langle a\rangle] = \langle a + a^\dagger\rangle/2$

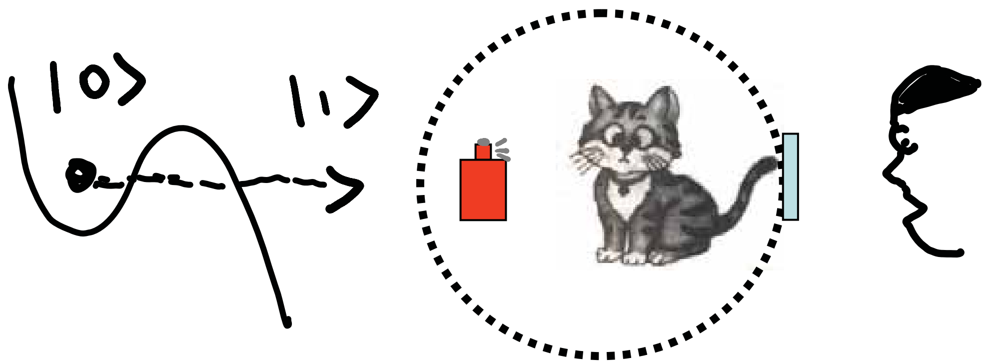
quadrature $Q = \mathrm{Im}[\langle a\rangle] = \langle ia^\dagger - ia\rangle/2$



量子力学从刚刚诞生起，关于如何理解它的基本概念和图象，就一直有深刻剧烈的争论，特别著名的包括**Einstein** 和 **Bohr** 之间长达几十年的争论。

美国著名物理学家**Feynman**曾说：“我确信没有一个人理解量子力学”

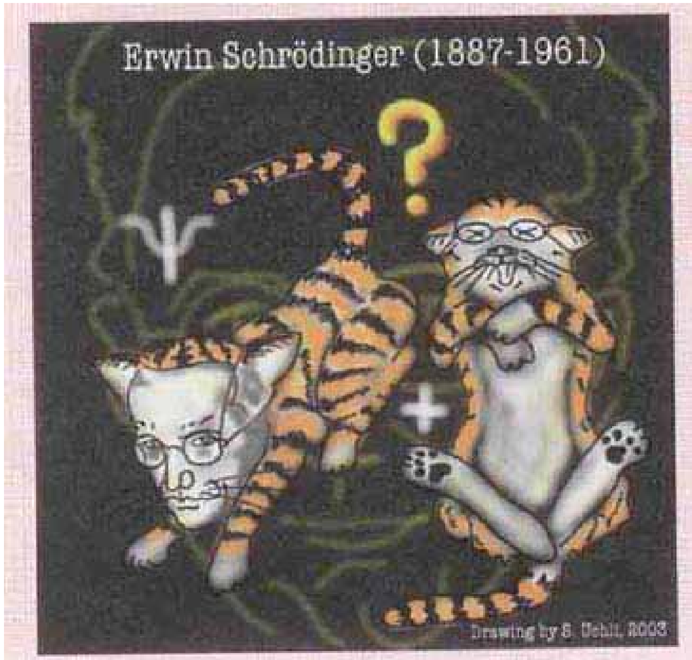
Schrodinger’s cat (1935):



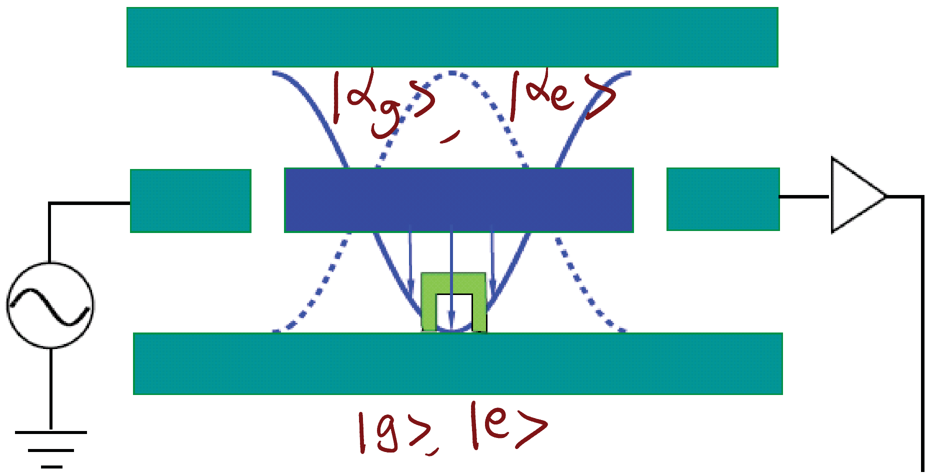
$$|\Psi\rangle = 1/\sqrt{2}\left(|0\rangle \otimes |\text{alive}\rangle + |1\rangle \otimes |\text{dead}\rangle\right)$$

各种形象有趣的薛定谔猫。1935 年薛定谔写下了《量子力学的现状》一文，在这篇文章中出现了著名的薛定谔猫的悖论。这是科学史上的一个著名的思想实验。

如何解释和理解量子力学的成果是学界，尤其是科学哲学上的热门话题。爱因斯坦和玻尔为之争论了一辈子，“薛定谔猫”则被爱因斯坦认为是最好地揭示了量子力学的通用解释的悖谬性。“薛定谔猫”佯谬假设了这样一种情况：将一只猫关在装有少量镭和氰化物的密闭容器里。镭的衰变存在概率，如果镭发生衰变，会触发机关打碎装有氰化物的瓶子，猫就会死；如果镭不发生衰变，猫就存活。根据量子力学理论，由于放射性的镭处于衰变和没有衰变两种状态的叠加，猫就理应处于死猫和活猫的叠加状态。显然，既死又活的猫是荒谬的。薛定谔提出这一悖论，想要阐述的物理问题是：宏观世界是否也遵从适用于微观尺度的量子叠加原理。“薛定谔猫”佯谬巧妙地把微观放射源和宏观的猫联系起来，旨在否定宏观世界存在量子叠加态。然而随着量子力学的发展，科学家已先后通过各种方案获得了宏观量子叠加态。



关于 Schrodinger 猫佯谬的一点理解



Δ_g , Δ_e

$|\alpha_g\rangle$, $|\alpha_e\rangle$

$$\rho(t) = c_{ee}(0)|e\rangle\langle e| \otimes |\alpha_e(t)\rangle\langle\alpha_e(t)|$$
$$+c_{gg}(0)|g\rangle\langle g| \otimes |\alpha_g(t)\rangle\langle\alpha_g(t)|$$
$$+c_{eg}(t)|e\rangle\langle g| \otimes |\alpha_e(t)\rangle\langle\alpha_g(t)|$$
$$+c_{ge}(t)|g\rangle\langle e| \otimes |\alpha_g(t)\rangle\langle\alpha_e(t)|$$

$$\alpha_0 = -\frac{i\epsilon_m}{i\Delta_r + \kappa/2}, \quad |\alpha_0\rangle$$

$$(c_g|g\rangle + c_e|e\rangle) \otimes |\alpha_0\rangle$$

1) $\xrightarrow{?} c_g|g\rangle \otimes |\alpha_g\rangle + c_e|e\rangle \otimes |\alpha_e\rangle$

2) $\xrightarrow{\checkmark} |c_g|^2 |g\rangle\langle g| \otimes |\alpha_g\rangle\langle\alpha_g|$
$+ |c_e|^2 |e\rangle\langle e| \otimes |\alpha_e\rangle\langle\alpha_e|$

$$c_{eg}(t) = \frac{c_{eg}(0)e^{-i(\tilde{\omega}_a - i\gamma_2)t - i2\chi\int_0^t \alpha_e(s)\alpha_g^*(s)ds}}{\langle\alpha_g(t)|\alpha_e(t)\rangle}$$

(2) Quadrature Measurement: Signal-to-Noise Ratio (SNR)

Short Summary:

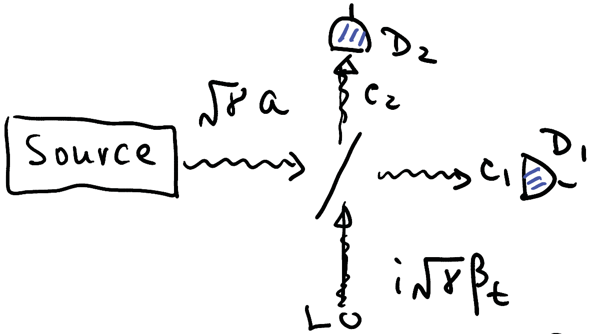
- ☐ (Strength) transmission measurement: frequency dependent transmission

- ☐ Measurement using single-frequency microwave photons:

1. Qubit-cavity “entanglement” dynamics; cavity state

2. Cavity-photon state measurement: homodyne/heterodyne quadrature measurements

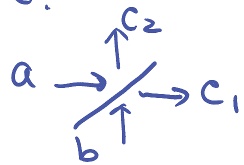
(Balanced) Homodyne 测量



经典 coherent 场

$$c_t^1 = \sqrt{\frac{\gamma}{2}}\,(a-\beta_t)\,e^{-i\omega t} \qquad c_t^2 = \sqrt{\frac{\gamma}{2}}\; i(a+\beta_t)\,e^{-i\omega t}$$

Note: BS 原理（半透半反，反射产生 $\frac{\pi}{2}$ 相移）



$$c_1 = \frac{1}{\sqrt{2}}(a+ib)$$

$$c_2 = \frac{1}{\sqrt{2}}(ia+b)$$

结果：
$$I(t) = I_2(t) - I_1(t)$$
$$= 2\gamma\langle X_\varphi\rangle_c + \sqrt{\gamma}\,\xi(t)$$

$\beta = |\beta| e^{i\varphi}$

$$X_\varphi = \frac{1}{2}\left(e^{-i\varphi}a + e^{i\varphi}a^{+}\right)$$

（某个发射光子的 quadrature）

$$d\rho_c(t) = dt\left\{-i\mathcal{L}\rho_c + \gamma\mathcal{D}[a]\rho_c\right\}$$
$$+ \sqrt{\gamma}\,dW(t)\,\mathcal{H}\left[e^{-i\varphi}a\right]\rho_c$$

在 jump 方案中取 $a/|\beta|$ 小量展开

推导要点：

Q-jump（Point 过程）→ diffusive 过程

dt 时间内，D_1, D_2 光子计数

$$m_1 = E\left[dN_c^1(t)\right] + \delta m_1(t)$$
$$= \frac{\gamma}{2}|\beta|^2\left[1-\left\langle\frac{\hat{a}}{\beta}+\frac{\hat{a}^{+}}{\beta^*}\right\rangle_c\right]dt + \sqrt{\frac{\gamma}{2}}|\beta|\,dW_1(t)$$

$$m_2 = E\left[dN_c^2(t)\right] + \delta m_2(t)$$
$$= \frac{\gamma}{2}|\beta|^2\left[1+\left\langle\frac{\hat{a}}{\beta}+\frac{\hat{a}^{+}}{\beta^*}\right\rangle_c\right]dt + \sqrt{\frac{\gamma}{2}}|\beta|\,dW_2(t)$$

(t, t+dt) 内测量结果偏离"t 时刻平均的平均结果"，携带有测量信息…

jump → diffusive Regime

$$|\beta|^2 \gg \langle a^+a\rangle$$

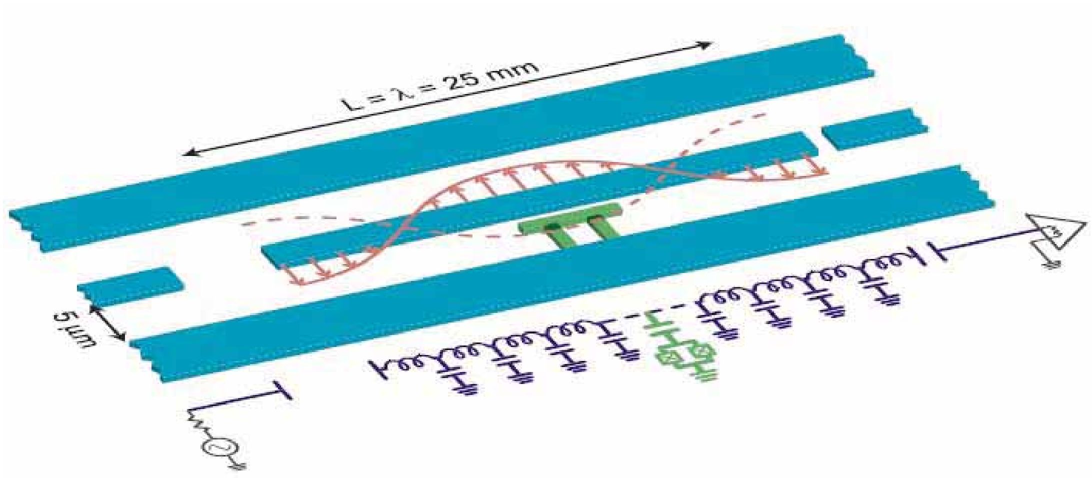
$$dN_c^k = P_c^k(t)\,dt + \sqrt{\frac{\gamma}{2}}|\beta|\,dW_k(t)$$

$$I(t) = \frac{m_2-m_1}{|\beta|\cdot dt}$$

$$\boxed{m_2-m_1} = 2\gamma|\beta|\langle X_\varphi\rangle_c dt + \sqrt{\gamma}|\beta|\,dW(t)$$

$$dW(t) = \frac{1}{\sqrt{2}}\left[dW_2(t) - dW_1(t)\right]$$

$\boxed{\text{Gambetta et al: PRA 77, 012112 (2008)}}$

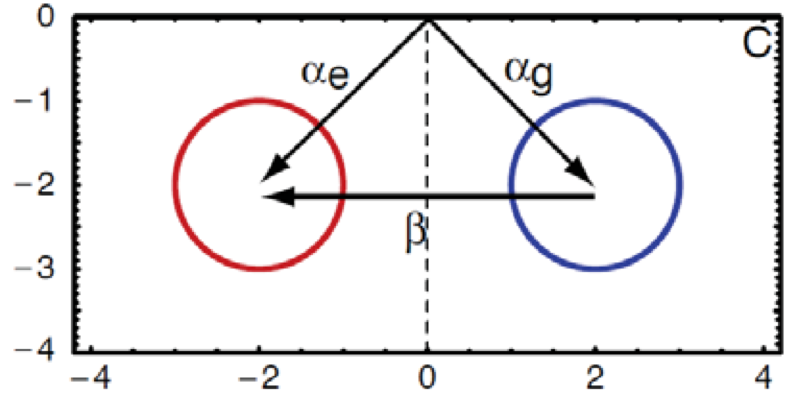


$$H_{\text{eff}} = \frac{\hbar\tilde{\omega}_a}{2}\sigma_z + \hbar\Delta_r a^\dagger a + \hbar\chi a^\dagger a\sigma_z + \hbar\left[\varepsilon_d(t)a^\dagger + \varepsilon_d^*(t)a\right]$$

$$\boxed{\begin{aligned}\dot{\alpha}_e(t) &= -i\varepsilon_d(t) - i(\Delta_r + \chi)\alpha_e(t) - \kappa\alpha_e(t)/2 \\ \dot{\alpha}_g(t) &= -i\varepsilon_d(t) - i(\Delta_r - \chi)\alpha_g(t) - \kappa\alpha_g(t)/2\end{aligned}}$$

$$\boxed{\beta(t) = \alpha_e(t) - \alpha_g(t)} \qquad \boxed{\theta_\beta = \arg(\beta)}$$

$$\boxed{\beta(t) = \alpha_e(t) - \alpha_g(t)} \quad \boxed{\theta_\beta = \arg(\beta)}$$



Polaron transformation: Eliminate cavity-photon states

$$\dot{\rho}(t) = -i\frac{\omega_{\mathrm{ac}}(t)}{2}[\sigma_z, \rho(t)] + \gamma_1 \mathcal{D}[\sigma_-]\rho(t) + [\gamma_\phi + \Gamma_{\mathrm{d}}(t)]\,\mathcal{D}[\sigma_z]\rho(t)/2 = \mathcal{L}\rho(t),$$

$$\boxed{\Gamma_{\mathrm{d}}(t) = 2\chi \mathrm{Im}[\alpha_g(t)\alpha_e^*(t)]}$$

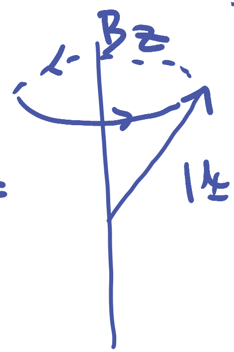
coherent –information gain rate

$$\dot{\rho}_{\bar{J}}(t) = \mathcal{L}\rho_{\bar{J}}(t) + \sqrt{\Gamma_{\mathrm{ci}}(t)}\mathcal{M}[\sigma_z]\rho_{\bar{J}}(t)(\bar{J}(t) - \sqrt{\Gamma_{\mathrm{ci}}(t)}\langle\sigma_z\rangle_t) - i\frac{\sqrt{\Gamma_{\mathrm{ba}}(t)}}{2}[\sigma_z, \rho_{\bar{J}}(t)](\bar{J}(t) - \sqrt{\Gamma_{\mathrm{ci}}(t)}\langle\sigma_z\rangle_t).$$

(Stochastic) extra unitary backaction

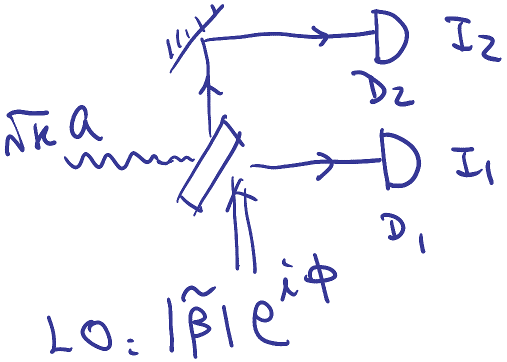
$$\boxed{\begin{aligned}\Gamma_{\mathrm{ci}}(t) &= \eta\kappa|\beta(t)|^2\cos^2(\phi - \theta_\beta)\\ \Gamma_{\mathrm{ba}}(t) &= \eta\kappa|\beta(t)|^2\sin^2(\phi - \theta_\beta)\end{aligned}}$$

$H = B_z\sigma_z$



$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

Understanding: \phi-dependent information-gain rate



$$I(t) = I_2(t) - I_1(t) = 2\kappa \langle X_\phi \rangle + \sqrt{\kappa}\, \xi(t)$$

$$X_\phi = \frac{1}{2}\left(e^{-i\phi} a + e^{i\phi} a^\dagger \right)$$

qubit state: $|e\rangle \rightarrow |\alpha_e\rangle \rightarrow I_e \sim \frac{1}{2}\left(\alpha_e e^{-i\phi} + c.c.\right)$

$|g\rangle \rightarrow |\alpha_g\rangle \rightarrow I_g \sim \frac{1}{2}\left(\alpha_g e^{-i\phi} + c.c.\right)$

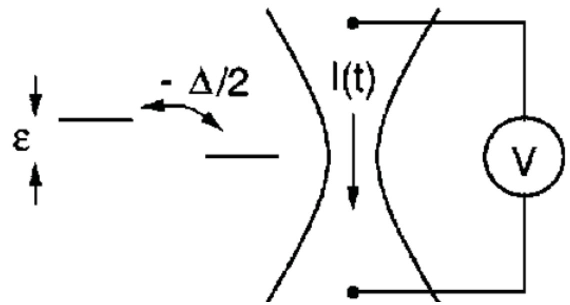
measurement signal: $\Delta I = I_e - I_g = \frac{1}{2}\left[\underbrace{(\alpha_e - \alpha_g)}_{\beta(t) = |\beta| e^{i\theta_\beta}} e^{-i\phi} + c.c. \right]$

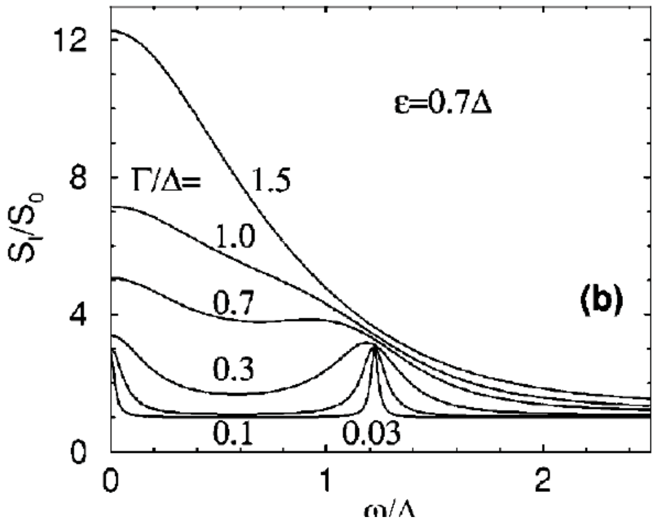
Therefore, as $\phi = \theta_\beta$, strongest signal

maximum information gain rate

Signal-to-noise ratio: $\mathrm{SNR} = \frac{\Gamma_{\mathrm{ci}}}{\gamma_1} = \frac{\eta \Gamma_{\mathrm{m}} \cos^2(\theta_\beta - \phi)}{\gamma_1}$

where $\Gamma_{\mathrm{m}}(t) = \kappa |\beta(t)|^2$ is the maximum measurement rate





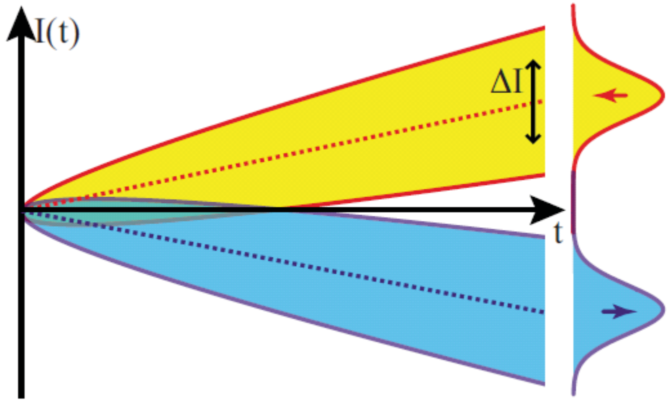
Compare: qubit + QPC/SET measurements

Korotkov & Averin: PRB 64, 165310 (2001)
Jordan & Buttiker: PRL 95, 220401 (2005)
Jiao, Li, Wang & Li: PRB 79, 075320 (2009)

$$S(\omega) = S_I + \frac{\lambda^2 \Gamma}{2} \frac{\Omega^2}{(\omega^2 - \Omega^2)^2 + \omega^2 \Gamma^2}$$

SNR: $\mathcal{R} = S_{\mathrm{max}}/S_I \leq 4$

$$\hat{H} = \frac{1}{2}\hbar\omega_{01}\hat{\sigma}_z + \hbar\omega_c(1 + A\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \hat{H}_{\mathrm{envt}}$$



$$I = \theta_0 t + \int_0^t d\tau\,\delta\theta(\tau)$$

$$\theta = I/t$$

$$\langle\theta\rangle = \theta_0$$

$$S_{\theta\theta} = 1/4\dot{\bar{N}}$$

$$\sqrt{S_{\dot{N}\dot{N}}S_{\theta\theta}} = \frac{1}{2}$$

$$|\alpha\rangle = \exp\left\{-\frac{|\alpha|^2}{2}\right\}\exp\{\alpha\hat{a}^\dagger\}|0\rangle$$

$$(\Delta N)^2 = \langle(\hat{N} - \bar{N})^2\rangle = |\alpha|^2\langle 0|\hat{d}\hat{d}^\dagger|0\rangle = \bar{N}$$

$$\hat{X} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger)$$

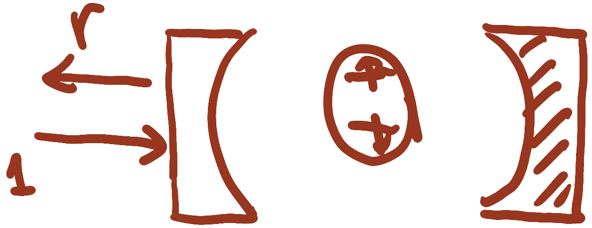
$$\hat{Y} = \frac{i}{\sqrt{2}}(\hat{a}^\dagger - \hat{a})$$

$$\langle\theta\rangle = \frac{\langle\hat{Y}\rangle}{\langle\hat{X}\rangle}$$

$$(\Delta\theta)^2 = \frac{\langle\hat{Y}^2\rangle}{(\langle\hat{X}\rangle)^2} = \frac{\frac{1}{2}\langle 0|\hat{d}\hat{d}^\dagger|0\rangle}{2\bar{N}} = \frac{1}{4\bar{N}}$$

$$\Delta\theta\Delta N = \frac{1}{2}$$

Reflection Measurements



$$r = -(1 + 2iAQ_c\hat{z})/(1 - 2iAQ_c\hat{z})$$

$$r = -e^{i\theta}$$

$$\theta \approx 4Q_c A\hat{z} = (A\omega_c \hat{z})t_{\mathrm{WD}}$$

$$\theta_0 = A\omega_c t_{\mathrm{WD}}$$

$$\langle e^{-i\varphi}\rangle = \left\langle \exp\left(-i\int_0^t d\tau \Delta\omega_{01}(\tau)\right)\right\rangle$$

$$\Delta\omega_{01} = 2\hat{F}_z/\hbar$$

$$\langle e^{-i\varphi}\rangle = \exp\left(-\frac{1}{2}\left\langle\left[\int_0^t d\tau\Delta\omega_{01}(\tau)\right]^2\right\rangle\right)$$

$$= \exp\left(-\frac{2}{\hbar^2}S_{F_zF_z}t\right).$$

$$\Gamma_\varphi = (2/\hbar^2)S_{F_zF_z} = 2\theta_0^2 S_{\dot{N}\dot{N}}$$

$$\langle I\rangle = \pm\theta_0 t, \qquad \Delta I = \sqrt{S_{\theta\theta}t}$$

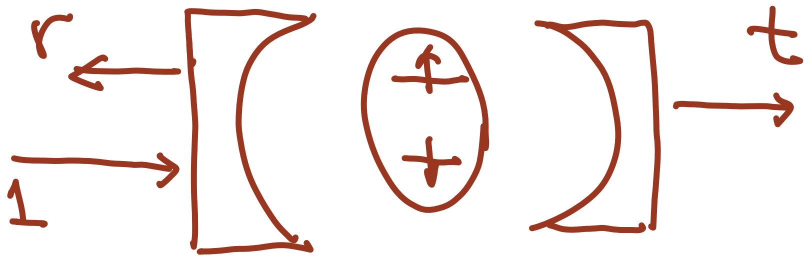
$$\mathrm{SNR} = \langle I\rangle^2/(\Delta I)^2 = \theta_0^2/S_{\theta\theta}t$$

$$\Gamma_{\mathrm{meas}} \equiv \mathrm{SNR}/2 = \theta_0^2/2S_{\theta\theta} = 1/2S^{\mathrm{I}}_{zz}$$

Quantum Limited Measurements:

$$\boxed{\Gamma_\varphi/\Gamma_{\mathrm{meas}} = (4/\hbar^2)S^{\mathrm{I}}_{zz}S_{F_zF_z} = 4S_{\dot{N}\dot{N}}S_{\theta\theta} = 1}$$

Transmission Measurements



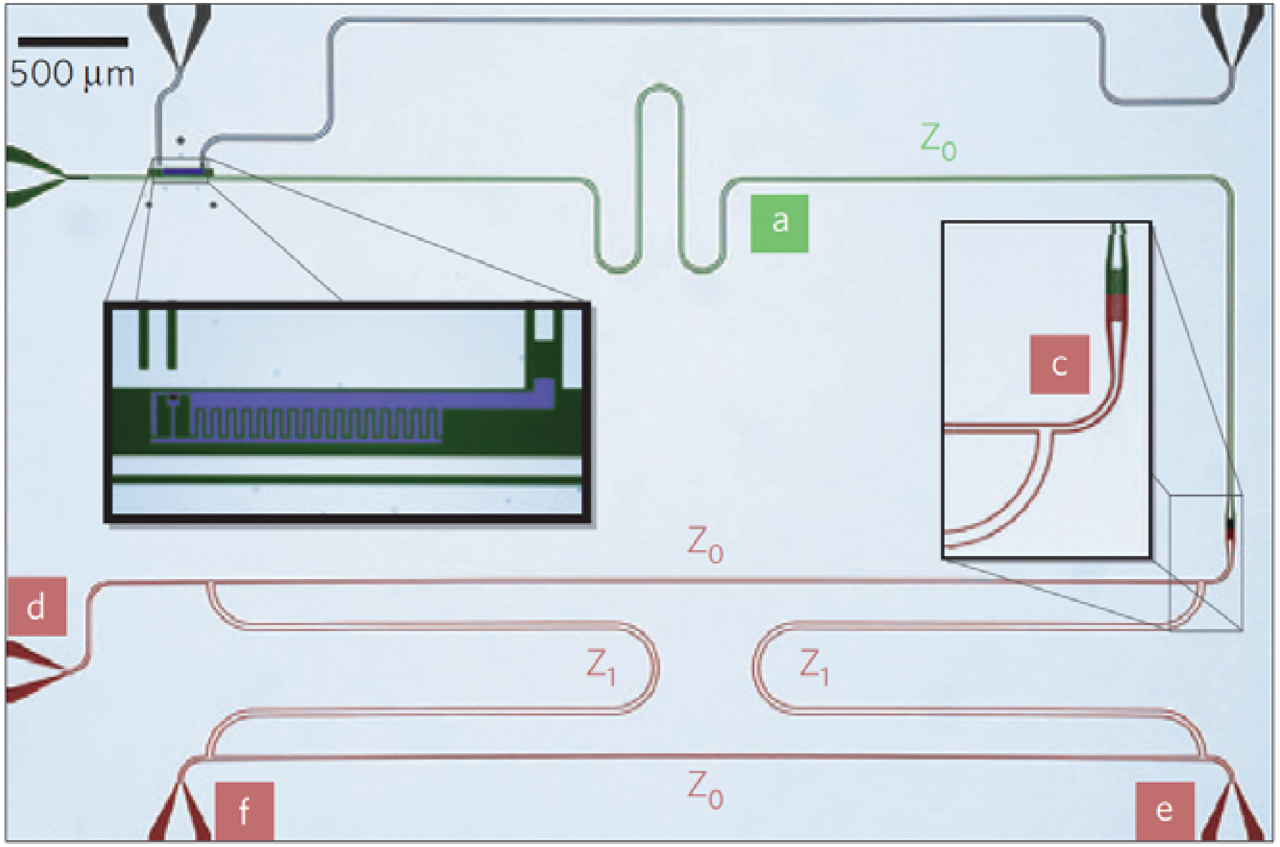
$$t_{\downarrow} = 1/(1 + 2iAQ_c)$$

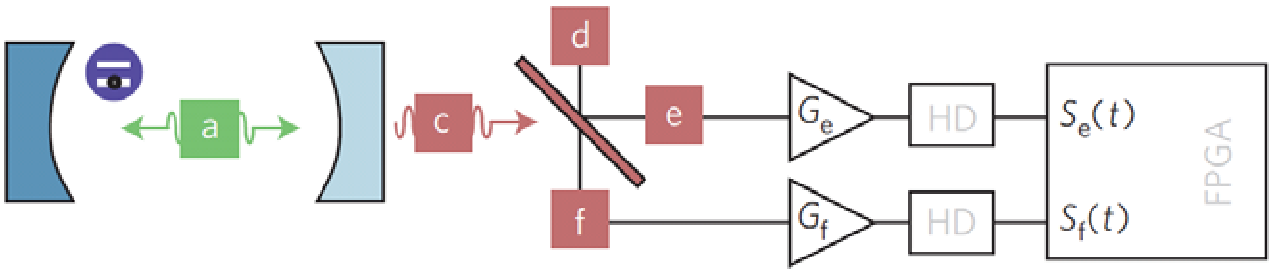
$$\tilde{\theta}_{\uparrow/\downarrow} = \pm\, \tilde{\theta}_0 = \pm\, 2AQ_c$$

$$\boxed{\Gamma_{\text{meas}}/\Gamma_{\varphi} = 2S_{\dot{N}\dot{N}}S_{\theta\theta} = \frac{1}{2}}$$

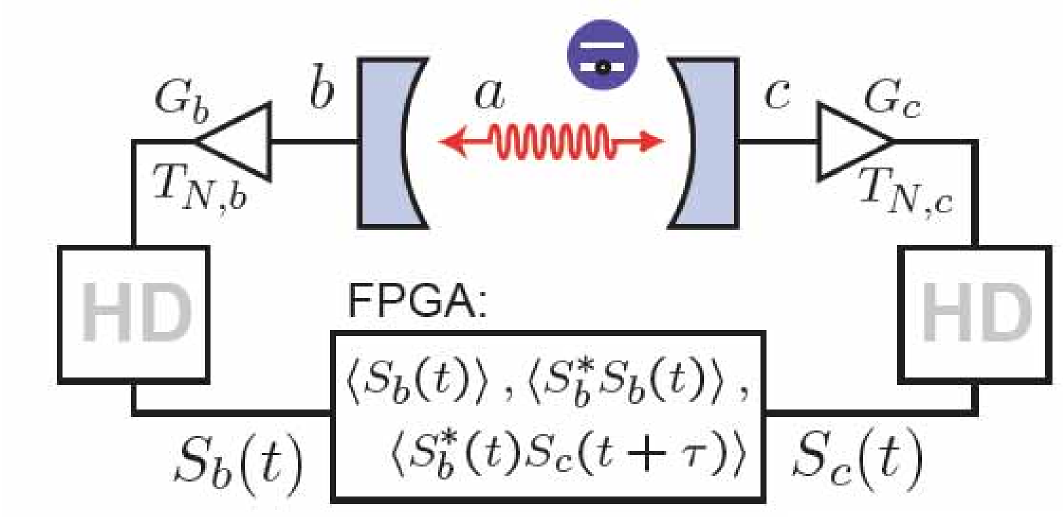
(3) Quadrature Measurement: a single microwave photon

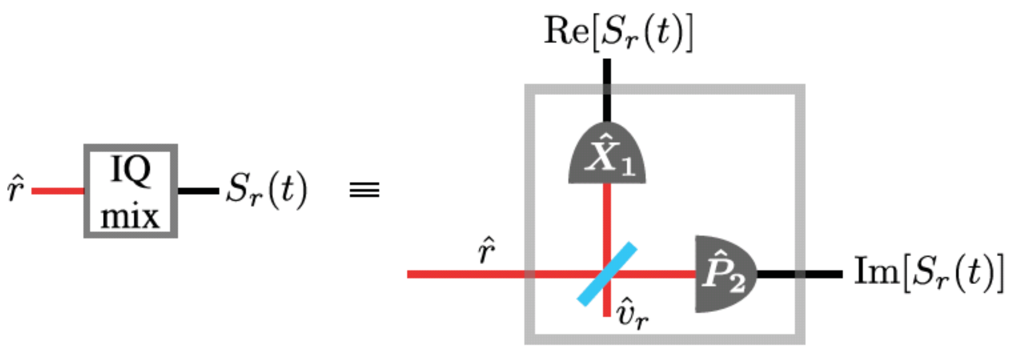
Blais & Wallraff: Nature Physics 7, 154-158 (2011)

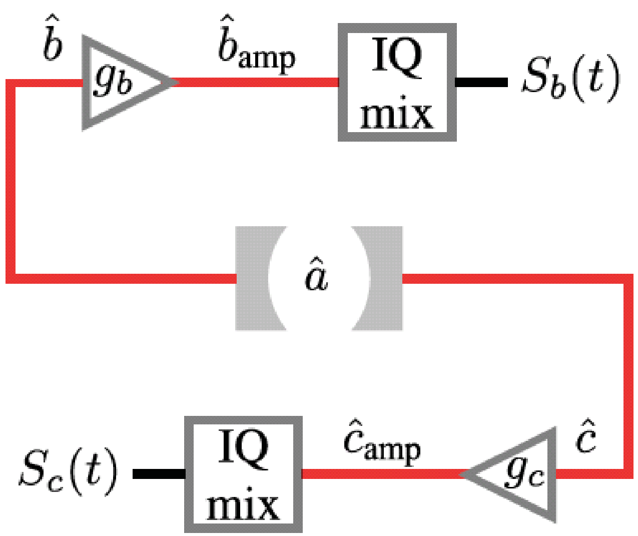




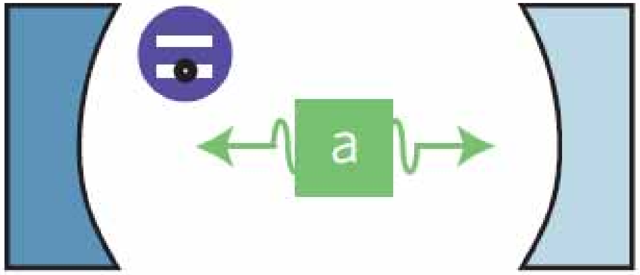
da Silva, M. P., Bozyigit, D., Wallraff, A. & Blais, A. Schemes for the observation of photon correlation functions in circuit QED with linear detectors. *Phys. Rev. A* **82**, 043804 (2010).







$$\hat{c}_{\mathrm{amp}} = \sqrt{g_c}\ \hat{c} + \sqrt{g_c - 1}\hat{h}_c^\dagger$$

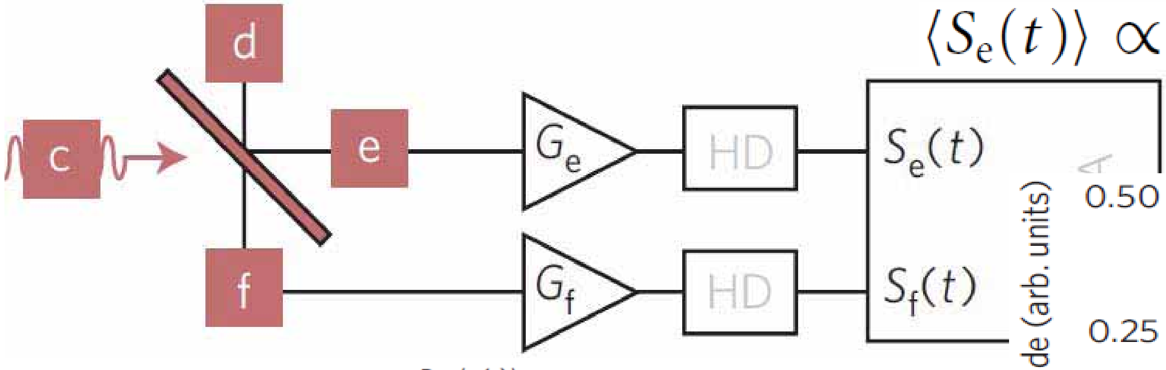


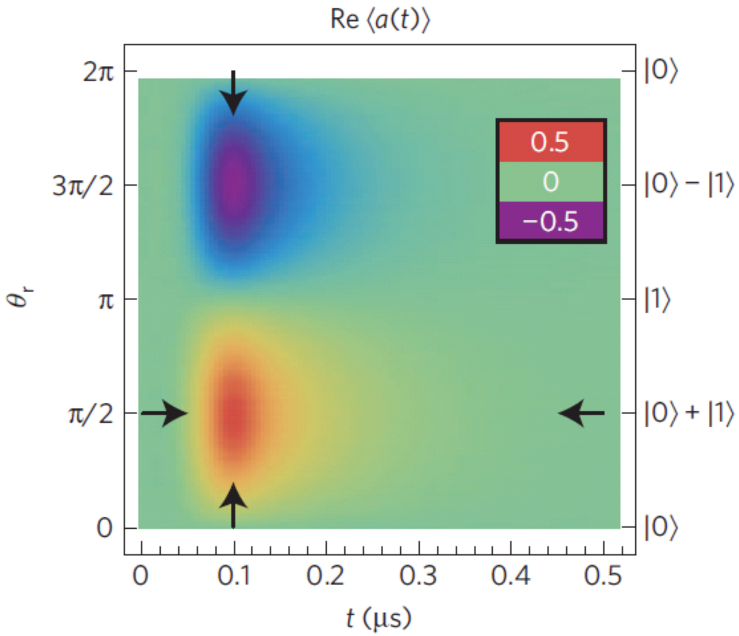
$$|\psi_q\rangle = \alpha|g\rangle + \beta|e\rangle$$

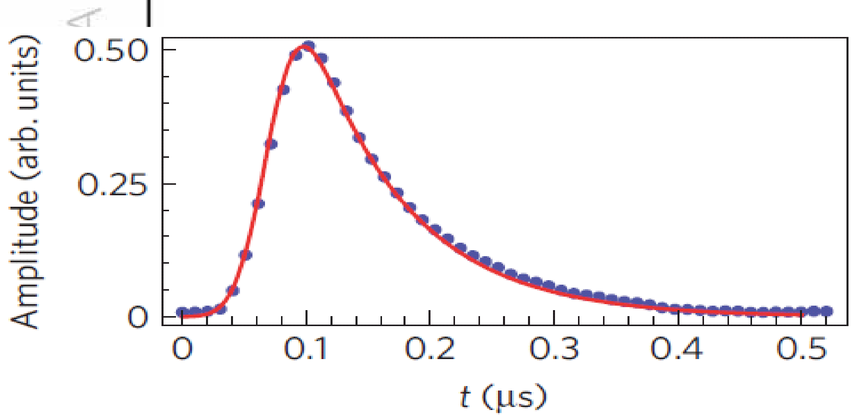
$$\alpha = \cos(\theta_r/2) \quad \beta = \sin(\theta_r/2)e^{i\phi}$$

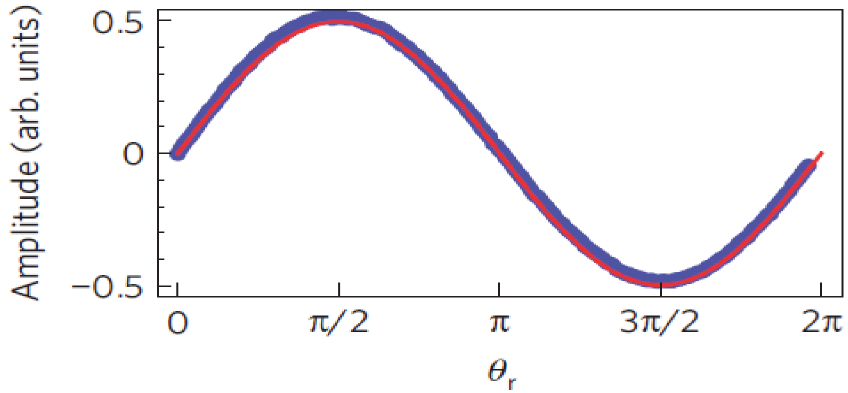
$$|\psi_c^+\rangle = (|0\rangle + |1\rangle)/\sqrt{2} \quad |\psi_c^-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$$

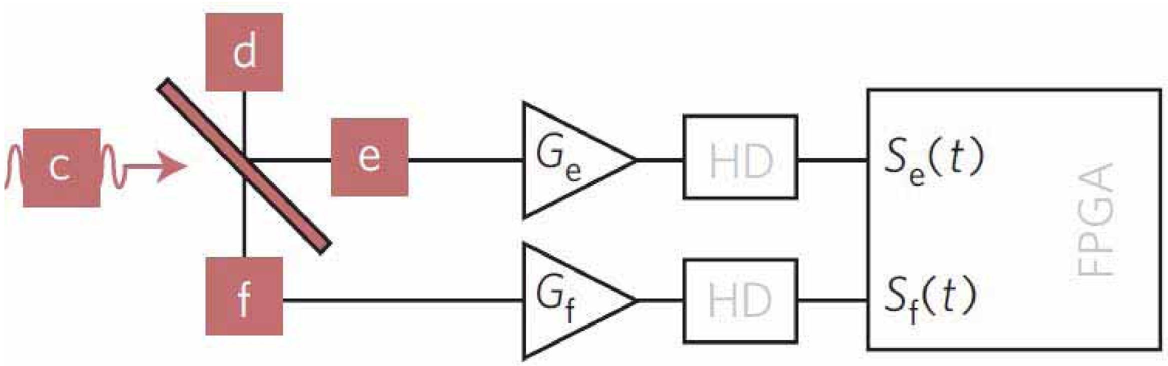
$$\langle S_e(t)\rangle \propto \langle a(t)\rangle \qquad \langle a\rangle \propto \sin(\theta_r)/2$$











$$\langle S_e^*(t) S_f(t) \rangle \propto \langle a^\dagger(t) a(t) \rangle + P(N_{ef})$$

$$\Gamma^{(1)}(\tau) = \int \langle S_e^*(t) S_f(t+\tau) \rangle \mathrm{d}t$$

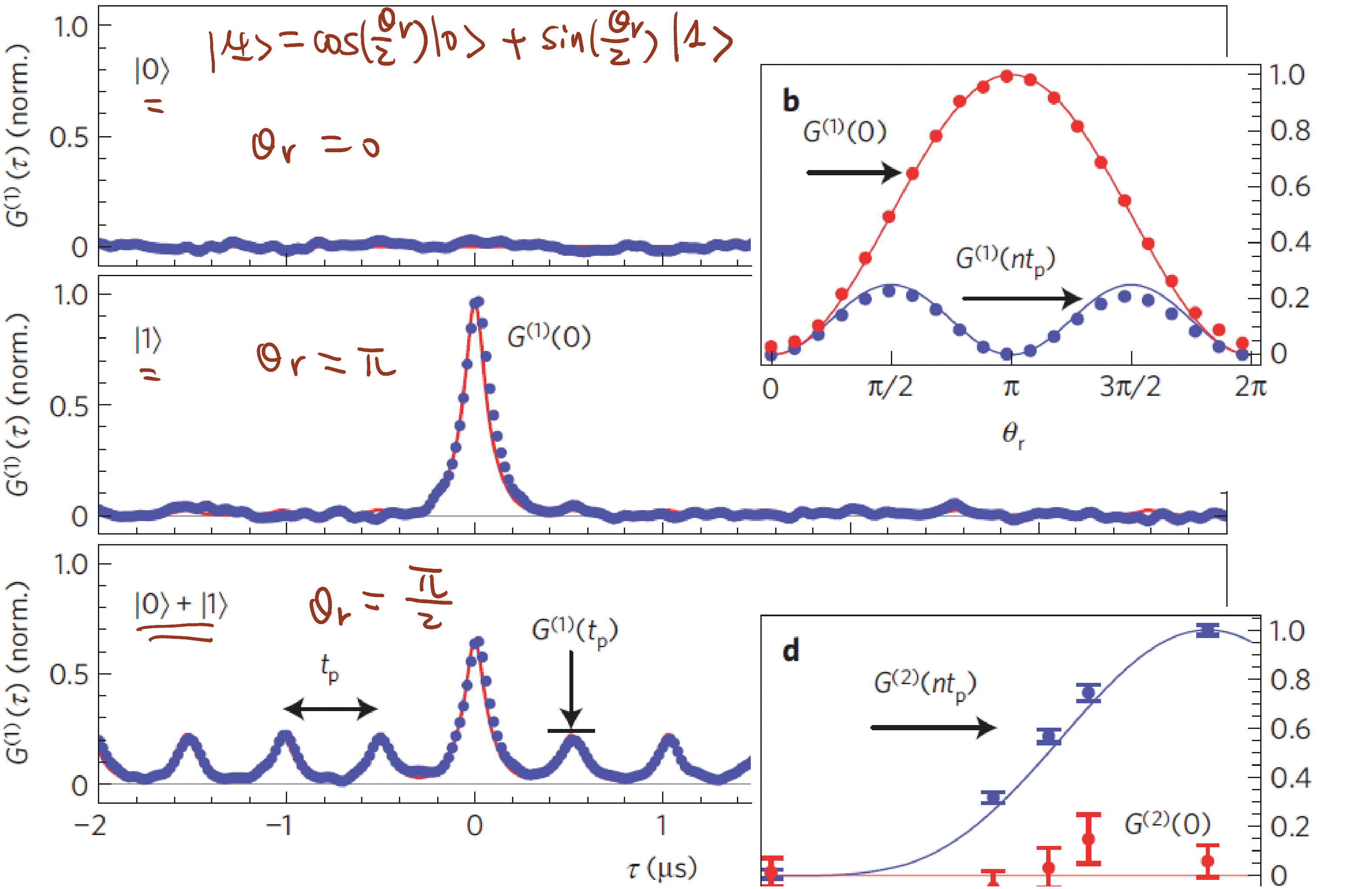
$$\Gamma^{(1)}(\tau) - \Gamma_{ss}^{(1)}(\tau) \propto G^{(1)}(\tau)$$

$$G^{(1)}(\tau) = \int \langle a^\dagger(t) a(t+\tau) \rangle \mathrm{d}t$$

$$\Gamma^{(2)}(\tau) = \int \langle S_e^*(t) S_e^*(t+\tau) S_f(t+\tau) S_f(t) \rangle \mathrm{d}t$$

$$\boxed{G^{(1)}(0) \propto \langle a^\dagger a\rangle \propto |\beta|^2 = \sin^2(\theta_\mathrm{r}/2)}$$

$$\boxed{G^{(1)}(nt_\mathrm{p}) \propto \langle a^\dagger\rangle\langle a\rangle} \qquad \boxed{G^{(1)}(nt_\mathrm{p}) \propto |\alpha\beta|^2 = \sin^2(\theta_\mathrm{r})/4}$$



Part （ II ）

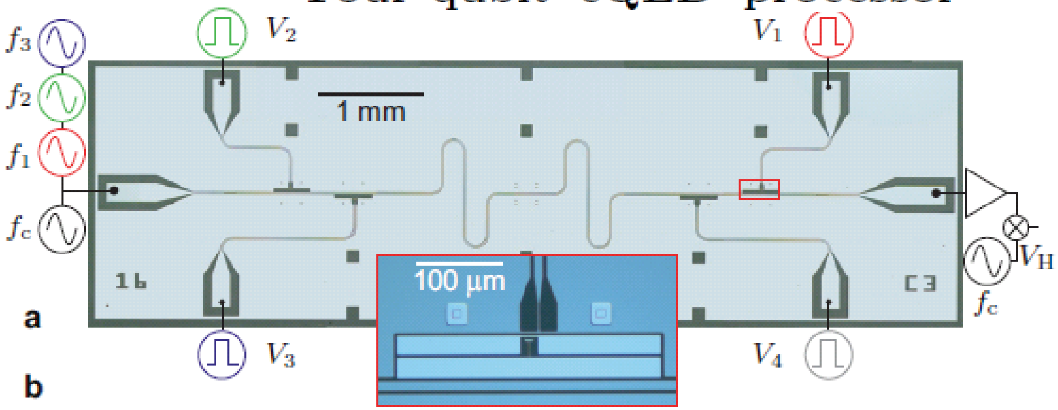
Quantum Entanglement: generation and control

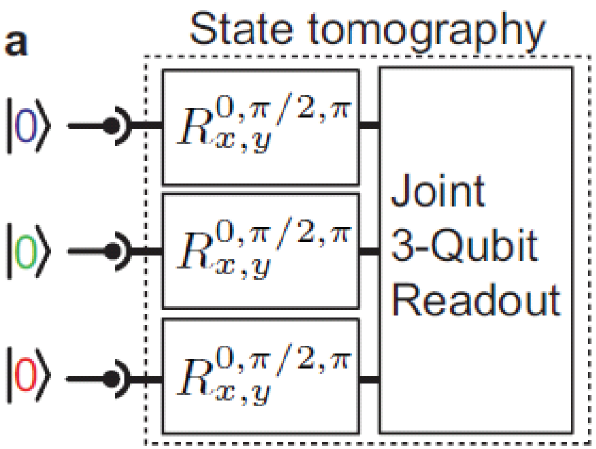
Preparation and Measurement of Three-Qubit Entanglement in a Superconducting Circuit

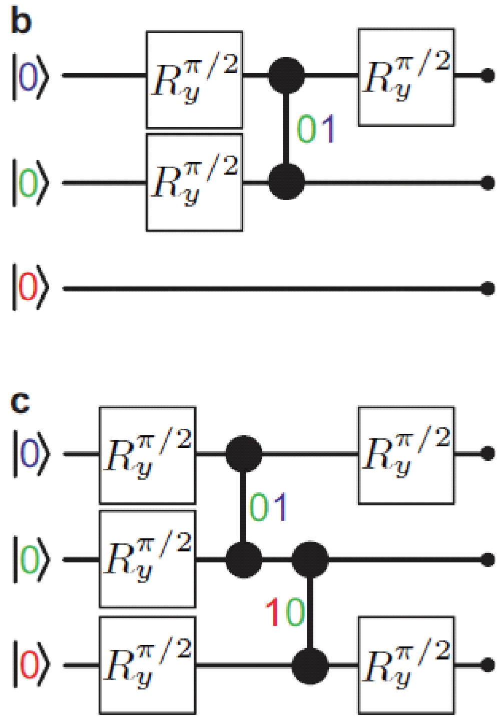
L. DiCarlo,[1] M. D. Reed,[1] L. Sun,[1] B. R. Johnson,[1] J. M. Chow,[1] J. M. Gambetta,[2] L. Frunzio,[1] S. M. Girvin,[1] M. H. Devoret,[1] and R. J. Schoelkopf[1]

arXiv: 1004.4324

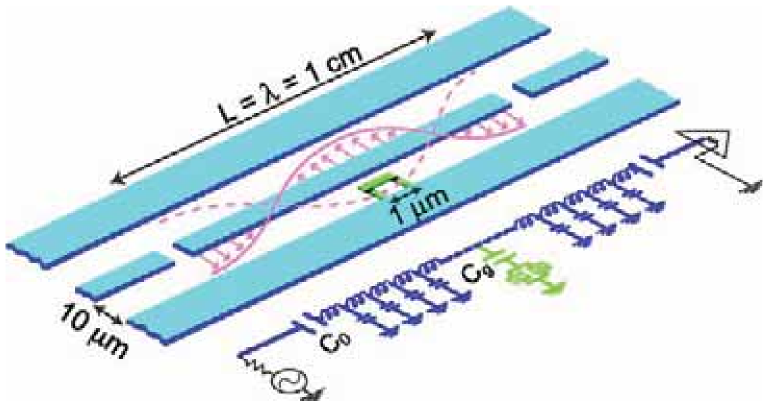
Four-qubit cQED processor

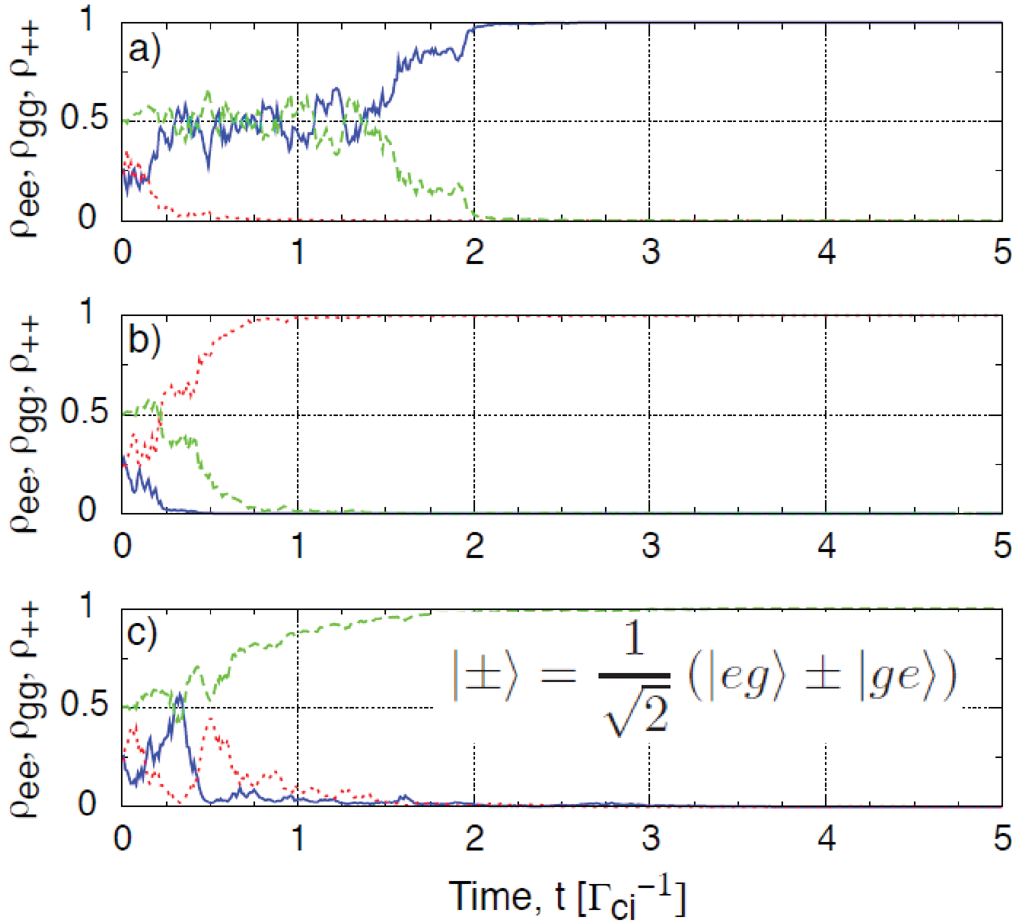






C.L. Hutchison, J.M. Gambetta, A. Blais, and F.K. Wilhelm, arXiv:0812.0218; Can. J. Phys. **87**, 225 (2009).



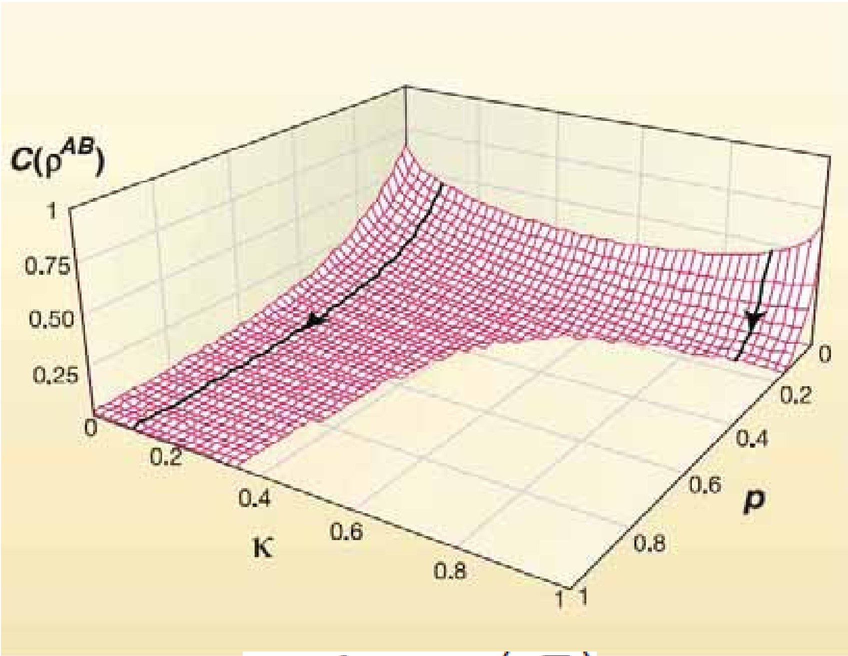


30 JANUARY 2009 VOL 323 SCIENCE 598

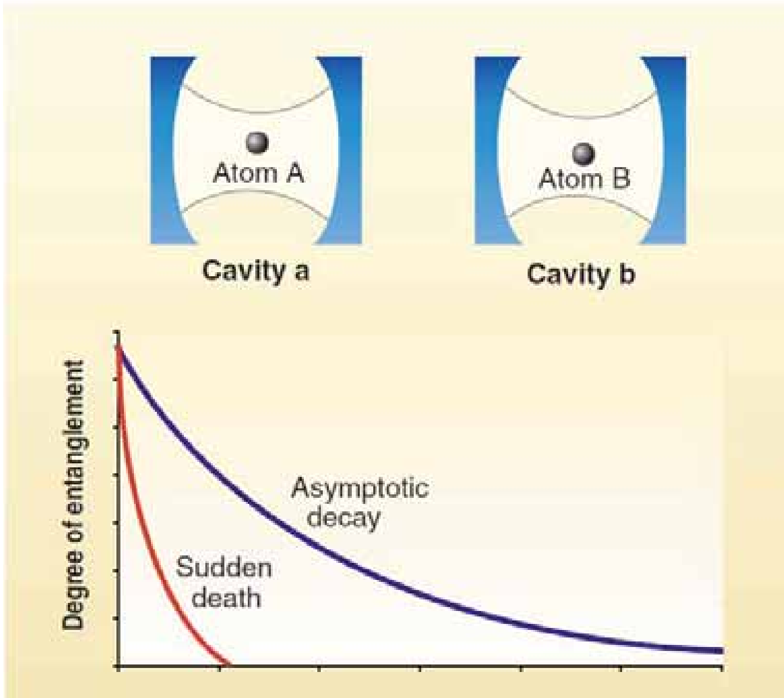
Sudden Death of Entanglement

Ting Yu[1*] and J. H. Eberly[2*]

$$C(\rho) = \max[0, Q(t)]$$



$p = 1 - \exp(-\Gamma_t)$



2-bit Bell state control

PHYSICAL REVIEW A **82**, 032335 (2010)

Deterministic creation and stabilization of entanglement in circuit QED by homodyne-mediated feedback control

Zhuo Liu, Lülin Kuang, Kai Hu, Luting Xu, Suhua Wei, Lingzhen Guo,* and Xin-Qi Li†
Department of Physics, Beijing Normal University, Beijing 100875, China
(Received 17 May 2010; published 29 September 2010)

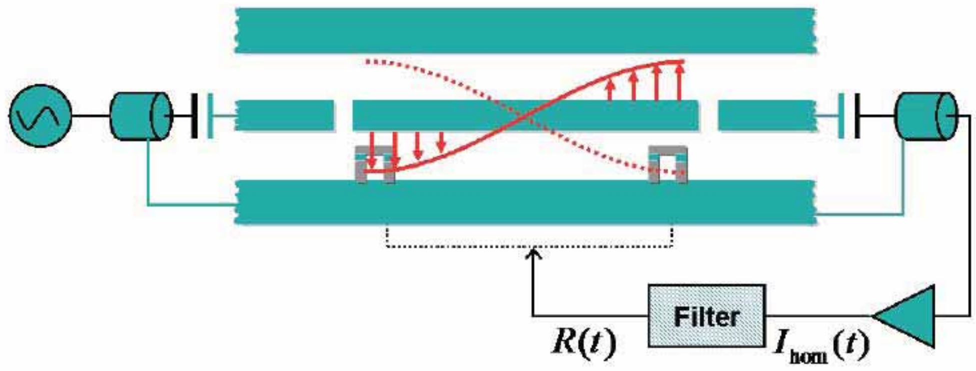
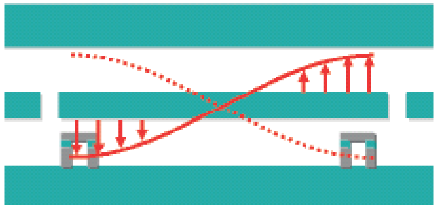


FIG. 1: Schematic diagram of circuit QED together with a microwave transmission measurement and a measurement-current-based feedback loop. The Cooper-pair box qubits are fabricated inside a superconducting transmission-line resonator and are capacitively coupled to the voltage standing wave.

$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \qquad |\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$



$$\Omega_1 = \Omega_2 = \Omega \qquad g_1 = -g_2 = g$$

$$\mathcal{E} = \epsilon e^{-i\omega_m t} + \text{c.c,} \qquad \Delta_r \equiv \omega_r - \omega_m \neq 0$$

$$\Delta = \omega_r - \Omega$$

1)

$$H = \omega_r a^\dagger a + \mathcal{E}(a^\dagger + a) + \sum_{j=1,2}\left[\frac{\Omega_j}{2}\sigma_j^z + g_j(\sigma_j^- a^\dagger + \sigma_j^+ a)\right]$$

$$H_{\text{eff}} \simeq U^\dagger H U \qquad U = \exp[\textstyle\sum_j \lambda_j (a\sigma_j^+ - a^\dagger \sigma_j^+)]$$

$$\lambda_j = g_j/\Delta$$

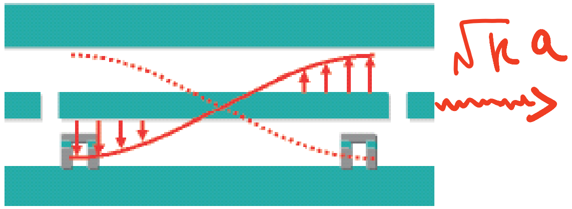
2)

Dispersive Regime,

Effective Hamiltonian

$$H_{\text{eff}} \simeq \Delta_r a^\dagger a + \epsilon(a + a^\dagger) + (\Omega + \chi)J_z/2$$
$$+\chi a^\dagger a J_z + \chi(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+).$$

$$\chi = g^2/\Delta \text{ and } J_z = \sigma_1^z + \sigma_2^z.$$



1)

$$\kappa \mathcal{D}[a]\rho \qquad \mathcal{D}[a]\rho = a\rho a^\dagger - \tfrac{1}{2}\{a^\dagger a, \rho\}$$

$$I_{\text{hom}}(t) = \kappa \langle a + a^\dagger \rangle_c(t) + \sqrt{\kappa}\xi(t)$$

$$E[\xi(t)] = 0 \qquad E[\xi(t)\xi(t')] = \delta(t - t')$$

$$\mathcal{H}[a]\rho_c\xi(t)$$

$$\mathcal{H}[a]\rho_c \equiv a\rho_c + \rho_c a^\dagger - \mathrm{Tr}[(a + a^\dagger)\rho_c]\rho_c$$

2) Adiabatic Elimination of Cavity Photon

the effective coupling $\chi a^\dagger a J_z$

$$\sim \mathcal{D}[J_z]\rho_c$$

$$\sim \mathcal{H}[J_z]\rho_c\xi(t)$$

$$I_{\text{hom}}(t) \sim \langle J_z \rangle_c(t) + \xi(t)$$

3)

$$\boxed{\begin{aligned}\dot{\rho}_c = -i\chi|\alpha|^2[J_z, \rho_c] + \sum_{j=1,2} \gamma_j \mathcal{D}[\sigma_j^-]\rho_c + \sum_{j=1,2} \frac{\gamma_{\phi j}}{2}\mathcal{D}[\sigma_j^z]\rho_c \\ + \gamma_p \mathcal{D}[\sigma_1^- - \sigma_2^-]\rho_c + \frac{\Gamma_d}{2}\mathcal{D}[J_z]\rho_c + \frac{\sqrt{\Gamma_m}}{2}\mathcal{H}[J_z]\rho_c\xi(t). \quad (3)\end{aligned}}$$

$$\Gamma_d = 8|\alpha|^2\chi^2/\kappa \qquad \alpha = -2i\epsilon/\kappa \qquad \eta = \Gamma_m/(2\Gamma_d)$$

补充 CQED/ bad cavity: cavity states 的 "绝热消除"

Milburn, Walls: PRA (94)

is done as follows [17]. In the absence of the coupling to the cantilever, the field reaches a steady state with coherent amplitude α_0 given by

$$\alpha_0 = -\frac{2iE}{\gamma} \; . \quad 1) \tag{32}$$

We then transform the total state of the system in the steady state by

$$\tilde{W} = D^\dagger(\alpha_0) W D(\alpha_0) \; . \quad 2) \tag{33}$$

In this "displacement" picture the steady state of the field is close to the vacuum state and we can try an approximate solution of the form

$$\tilde{W} = \rho_0|0\rangle_a\langle 0| + (\rho_1|1\rangle_a\langle 0| + \text{H.c}) \quad 3)$$
$$+ \rho_2|1\rangle_a\langle 1| + (\rho_2{}'|2\rangle_a\langle 0| + \text{H.c.}). \tag{34}$$

The cantilever density operator is then given by

$$\rho_M = \text{tr}W \quad 4)$$
$$= \rho_0 + \rho_2 \; . \tag{35}$$

1)

cavity 受驱动　　cavity 光子 decay

可证明: 稳态为相干态

2) 位移变换

$D[\alpha_0] = \exp[\alpha_0 a^\dagger - \alpha_0^* a]$

3) 在"位移"后的表象中

cavity 光子数 $\to 0$

小量(ϵ)展开 到 二阶 $O(\epsilon^2)$

4) $\rho_s = \mathrm{Tr}_{cav}[W]$
$= \langle 0|W|0\rangle + \langle 1|W|1\rangle$

Homodyne-Mediated Feedback

$$H_{\mathrm{fb}}(t) = I_{\mathrm{hom}}(t-\tau)\hat{F} \qquad I_{\mathrm{hom}}(t) = \sqrt{\Gamma_m}\langle J_z\rangle_c(t) + \xi(t)$$

$$[\dot{\rho}_c(t)]_{\mathrm{fb}} = I_{\mathrm{hom}}(t-\tau)\mathcal{K}\rho_c(t) \qquad \mathcal{K}\rho_c(t) \equiv -i[\hat{F}, \rho_c(t)]$$

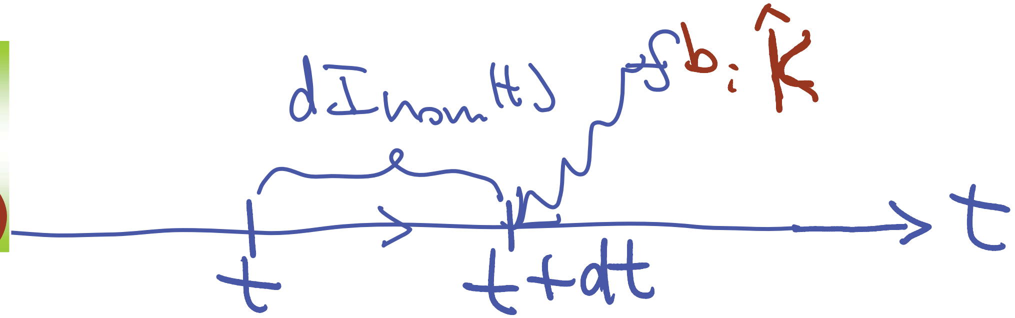
$$\begin{aligned}
\dot{\rho}_c = \; & -i\chi|\alpha|^2\,[J_z, \rho_c] + \frac{\Gamma_d}{2}\mathcal{D}[J_z]\rho_c \\
& + \sum_{j=1,2}\gamma_j\mathcal{D}[\sigma_j^-]\rho_c + \sum_{j=1,2}\frac{\gamma_{\phi j}}{2}\mathcal{D}[\sigma_j^z]\rho_c \\
& + \gamma_p\mathcal{D}[\sigma_1^- - \sigma_2^-]\rho_c + \frac{\sqrt{\Gamma_m}}{2}\mathcal{K}(J_z\rho_c + \rho_c J_z) \\
& + \frac{1}{2}\mathcal{K}^2\rho_c + \left(\frac{\sqrt{\Gamma_m}}{2}\mathcal{H}\,[J_z] + \mathcal{K}\right)\rho_c\xi(t).
\end{aligned}$$

Wiseman - Milburn Feedback Equation

PRL (1993)

补充

An alternative derivation (understanding)



dW(t)

①

$$\rho_c(t+dt) = e^{d\mathcal{K}(t)}\tilde{\rho}_c(t+dt)$$

$$d\mathcal{K}(t) = dI_{\mathrm{hom}}(t)\mathcal{K}$$

②

$$dI_{\mathrm{hom}}(t) = \sqrt{\Gamma_m}\langle J_z\rangle_c(t)dt + dW(t)$$

$$dW(t) = \xi(t)dt \qquad E[dW(t)] = 0$$

$$[dW(t)]^2 = dt \qquad E[dW(t)dW(s)] = \delta(t-s)dt$$

Feedback Design: Basic Consideration

Target States

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \qquad |\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Initial State

$$(|0\rangle + |1\rangle)_1 \otimes (|0\rangle + |1\rangle)_2 = (|00\rangle + |11\rangle) + (|10\rangle + |01\rangle)$$

Measurement and Feedback

2

$$H_{\text{fb}}(t) = uI_{\text{hom}}(t)J_x$$

$J_x = \sigma_1^x + \sigma_2^x$

$J_x|00\rangle \to |01\rangle + |10\rangle$

$J_x|11\rangle \to |01\rangle + |10\rangle$

1

$$\sim \mathcal{D}[J_z]\rho_c$$

$$\sim \mathcal{H}[J_z]\rho_c\xi(t)$$

$$I_{\text{hom}}(t) \sim \langle J_z\rangle_c(t) + \xi(t)$$

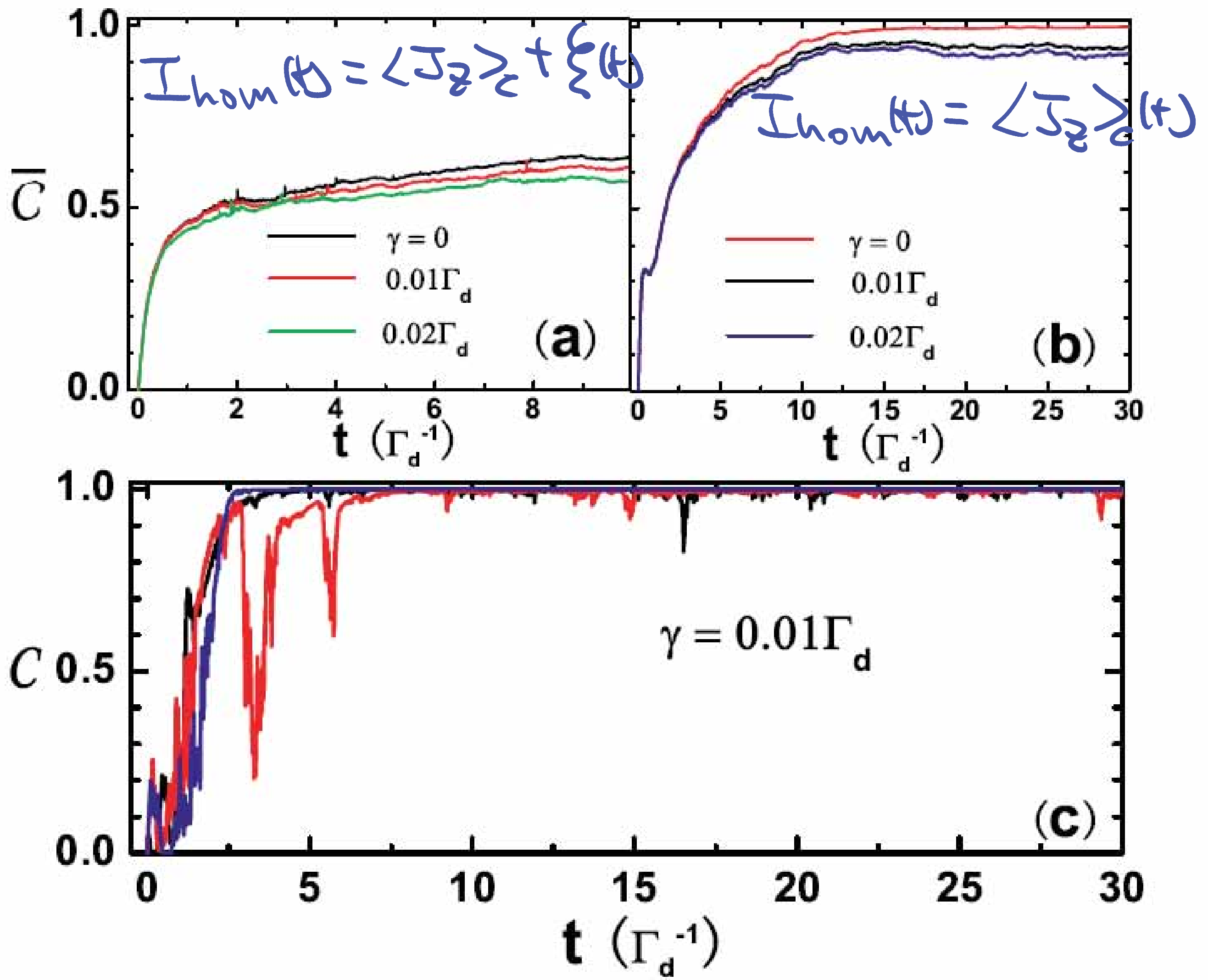
Project out $|00\rangle$, $|11\rangle$, $|10\rangle + |01\rangle$

Preliminary Result

Concurrence :

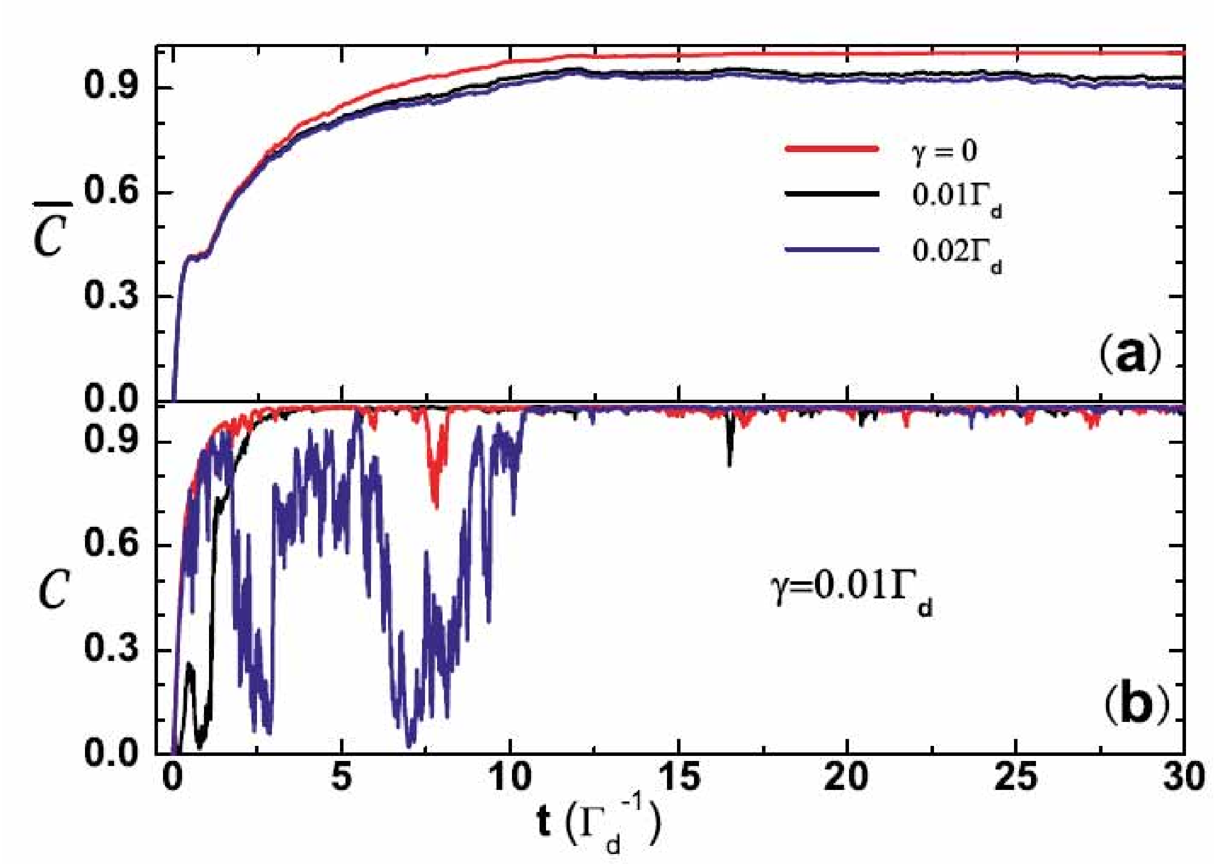
$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\overline{\rho}(\sigma_y \otimes \sigma_y)$$

$$C(\rho) = \max(\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0)$$



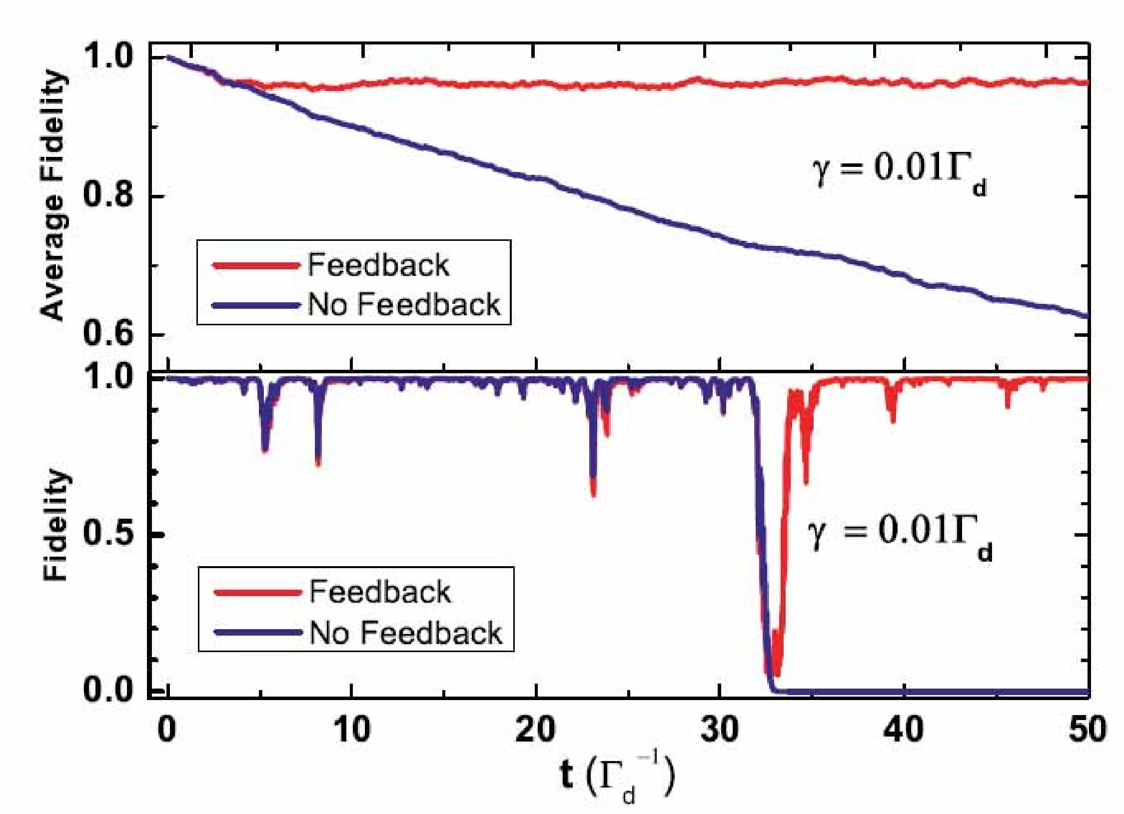
Filtered-Current-Based Feedback: Improved Result

$$R(t) \; = \; \frac{1}{N}\int_{t-T}^{t} e^{-\gamma_{\rm ft}(t-\tau)} dI_{\rm hom}(\tau) \qquad H_{fb}(t) = uR(t)^{P} J_x$$



ENTANGLEMENT DEGRADATION IN THE ABSENCE OF FEEDBACK

Fidelity: $$F(t) = \mathrm{Tr}[|\Phi_+\rangle\langle\Phi_+|\rho(t)]$$



Discussion: Feedback Implementation and other Bell States

Feedback implementation:

1) Cavity driving induced: $H_{\mathrm{dr}} = \lambda \epsilon_c J_x$

2) Qubit gating technique …

Other Bell States:

1) $|\Phi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ $\quad \bar{J}_x \equiv \sigma_1^x - \sigma_2^x$

2) σ_2^x-π-pulse: $|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$

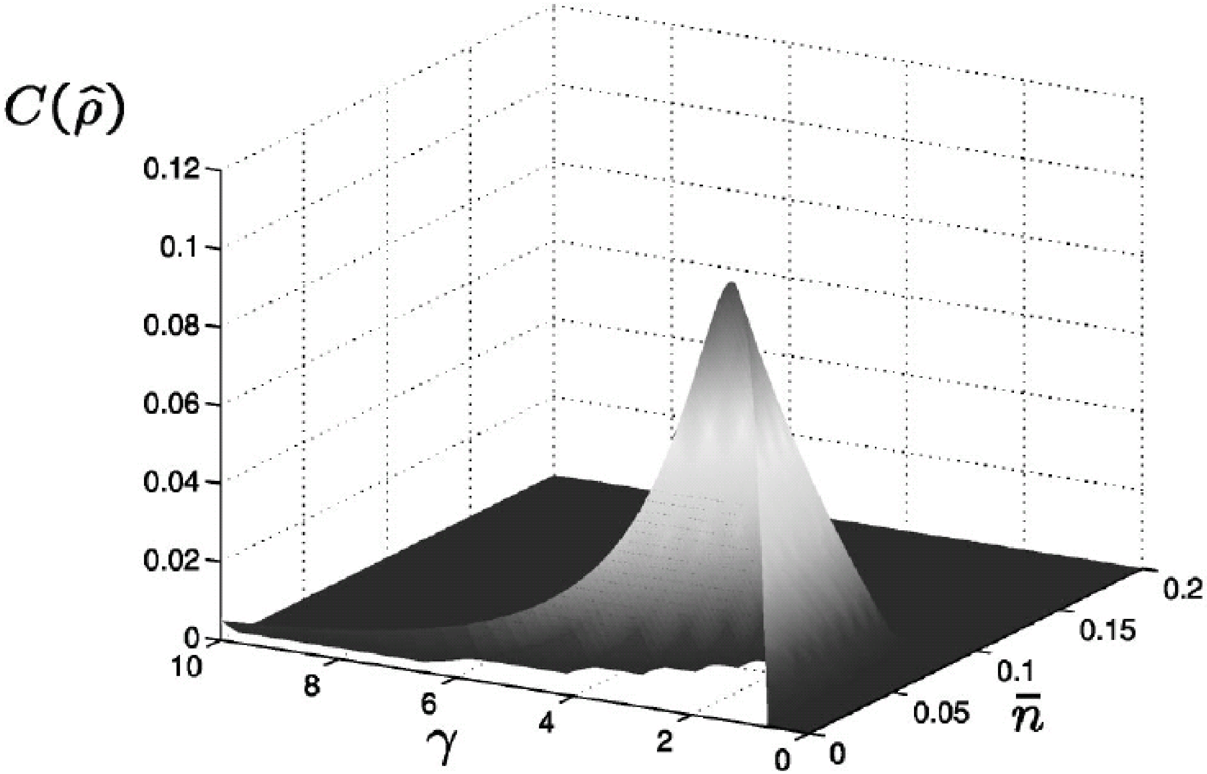
3) $\underset{\omega_r - \Omega_1}{\delta_1} = -\underset{\Omega_2 - \omega_r}{\delta_2} = \delta \rightarrow \bar{J}_z = \sigma_1^z - \sigma_2^z$

Previous Studies

S. Schneider, and G.J. Milburn,
Phys. Rev. A 65, 042107 (2002).

Entanglement in the steady state of a collective-angular-momentum (Dicke) model

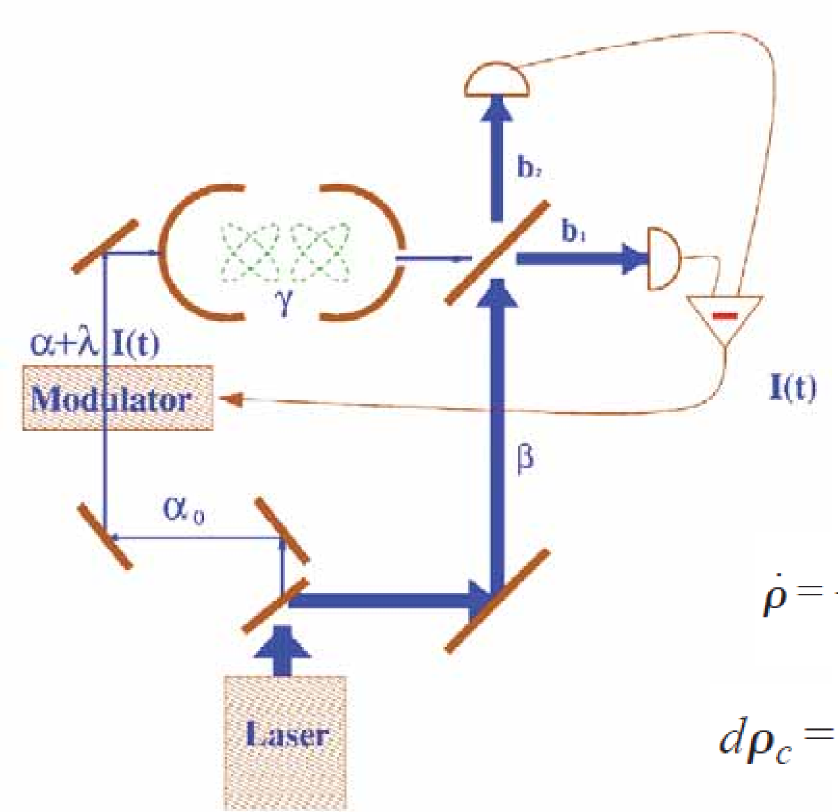
$$\frac{\partial\hat{\rho}}{\partial t}=-i\frac{\Omega}{2}[\hat{J}_{+}+\hat{J}_{-},\hat{\rho}]+\frac{\gamma_A}{2}(2\hat{J}_{-}\hat{\rho}\hat{J}_{+}-\hat{J}_{+}\hat{J}_{-}\hat{\rho}-\hat{\rho}\hat{J}_{+}\hat{J}_{-})$$



PHYSICAL REVIEW A **71**, 042309 (2005)

Dynamical creation of entanglement by homodyne-mediated feedback

Jin Wang,[1,2] H. M. Wiseman,[3] and G. J. Milburn[2]



$$\hat{H} = \frac{g}{2}[\hat{J}^- b^+ + \hat{J}^+ b]$$

$$I(t) = \frac{I_1(t) - I_2(t)}{|\beta|} = \sqrt{\gamma}\langle e^{-i\Phi}\sigma^\dagger + e^{i\Phi}\sigma\rangle_c(t) + \xi(t)$$

$$\dot{\omega} = -\frac{1}{2}\alpha[b - b^\dagger, \omega] - i\frac{g}{2}[J^- b^+ + J^+ b, \omega] + \gamma_1 \mathcal{D}[\sigma_1]\omega + \gamma_2 \mathcal{D}[\sigma_2]\omega + \gamma_p \mathcal{D}[b]\omega.$$

$$\dot{\rho} = -i\frac{g\alpha}{2\gamma_p}[(J^+ + J^-), \rho] + \gamma_1 \mathcal{D}[\sigma_1]\rho + \gamma_2 \mathcal{D}[\sigma_2]\rho + \gamma \mathcal{D}[J^-]\rho$$

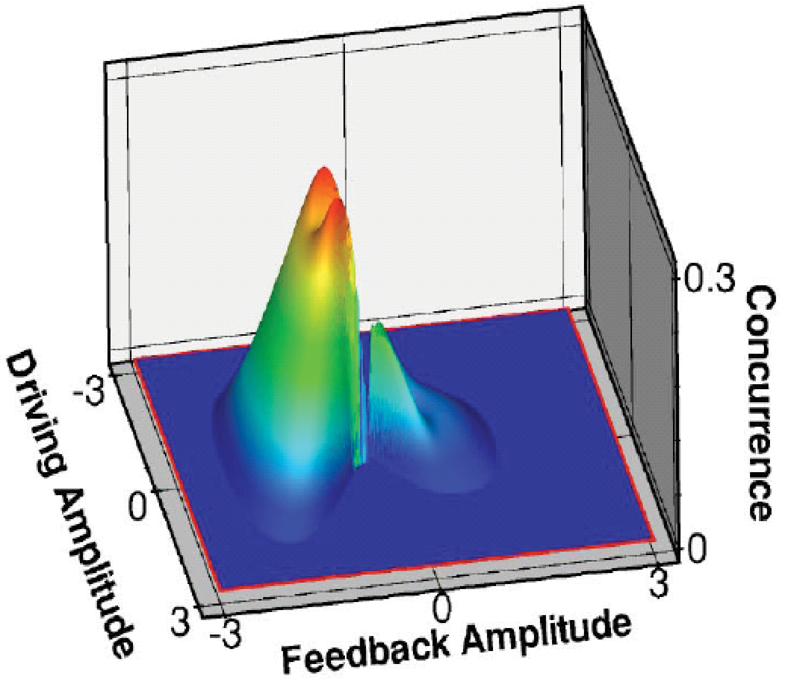
$$d\rho_c = -i[H, \rho_c(t)] + dt\gamma \mathcal{D}[J^-]\rho_c(t) + \sqrt{\gamma}dW(t)\mathcal{H}[J^-]\rho_c$$

$$I(t) = \sqrt{\gamma}\langle J_x\rangle_c(t) + \xi(t)/\sqrt{\eta},$$

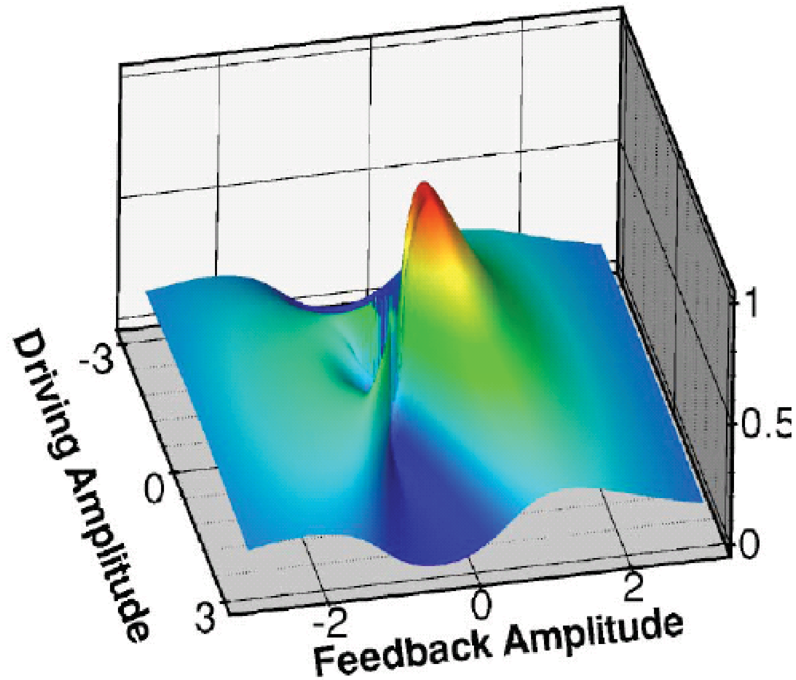
$$H_{fb} = I(t)F$$

$$d\rho_c = dt\,\gamma\mathcal{D}[J^-]\rho_c - i\,dt[H_\alpha, \rho_c] - i\,dt[F, -iJ^-\rho_c + i\rho_c J^+] + dt\frac{1}{\gamma}\mathcal{D}[F]\rho_c + dW(t)\mathcal{H}[-i\sqrt{\gamma}J^- - i\lambda J_x]\rho_c.$$

Concurrence for different drivng and feedback



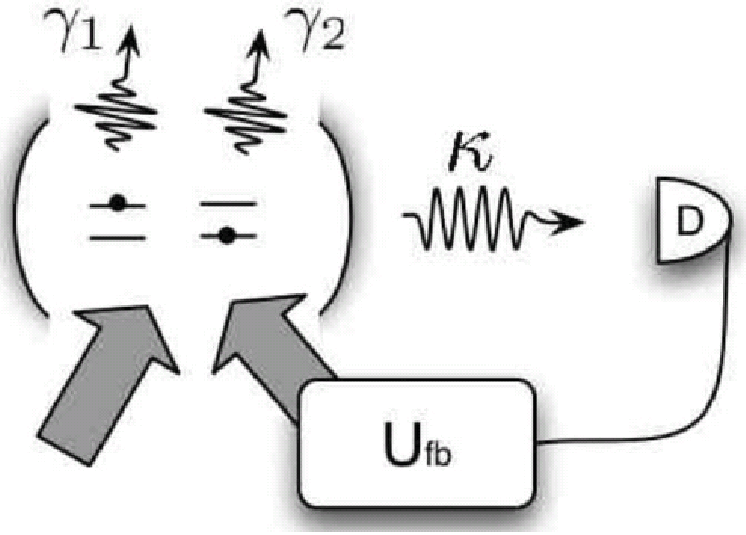
Purity for different driving and feedback



PHYSICAL REVIEW A **76**, 010301(R) (2007)

Stabilizing entanglement by quantum-jump-based feedback

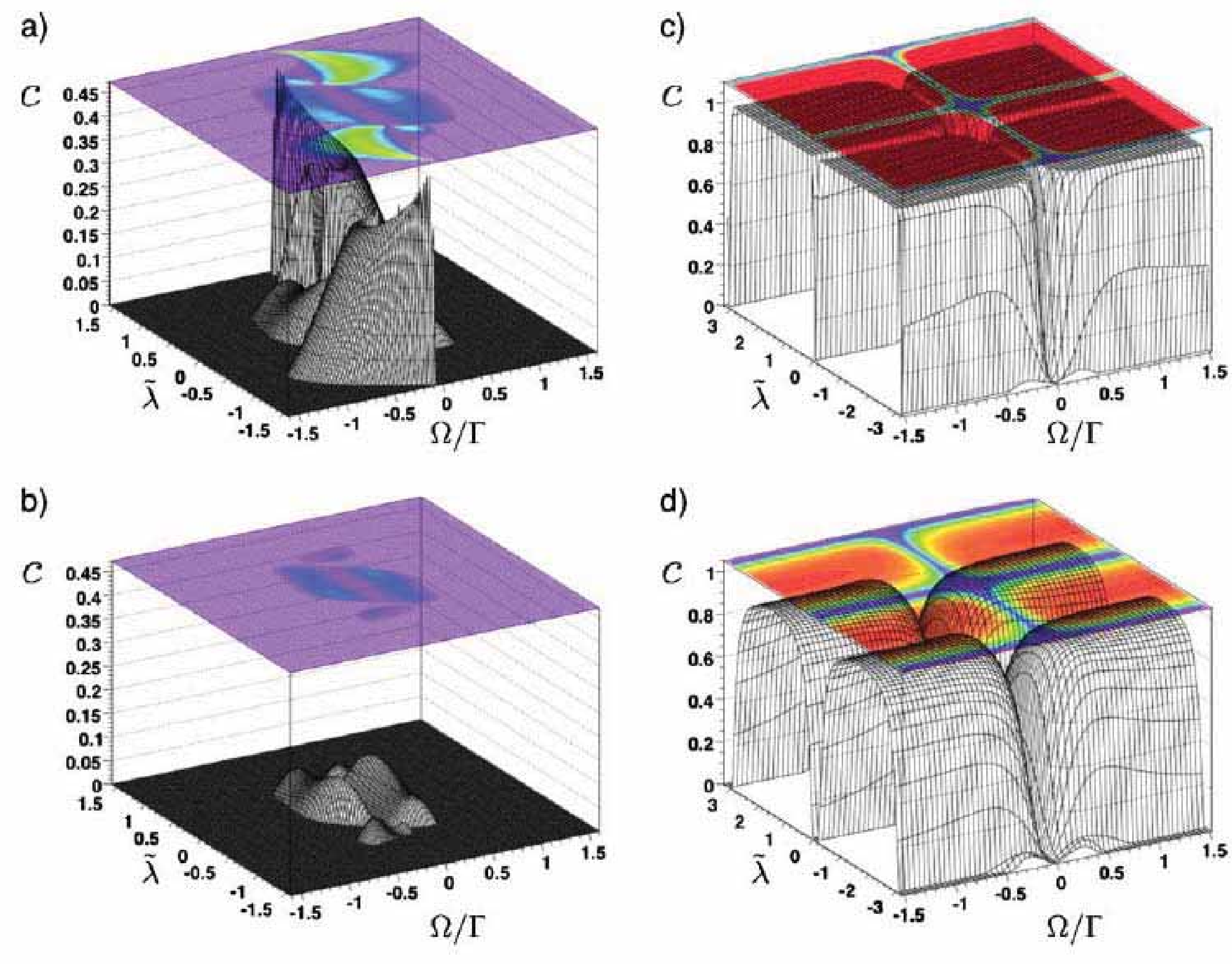
A. R. R. Carvalho
Department of Physics, Faculty of Science, The Australian National University, Australian Capital Territory 0200, Australia



$$\dot{\rho} = -i\Omega[(J_+ + J_-), \rho] + \Gamma\mathcal{D}[J_-]\rho + \gamma_1\mathcal{D}[\sigma_1]\rho + \gamma_2\mathcal{D}[\sigma_2]\rho$$

$$\dot{\rho} = -i\Omega[(J_+ + J_-), \rho] + \Gamma\mathcal{D}[U_{\text{fb}}J_-]\rho$$

$U_{\mathrm{fb}} = \exp(-i\tilde{\lambda} J_x)$ $\qquad$ $U_{\mathrm{fb}} = U_1 \otimes \mathbb{I} \quad U_1 = \exp(-i\tilde{\lambda}\sigma_x)$

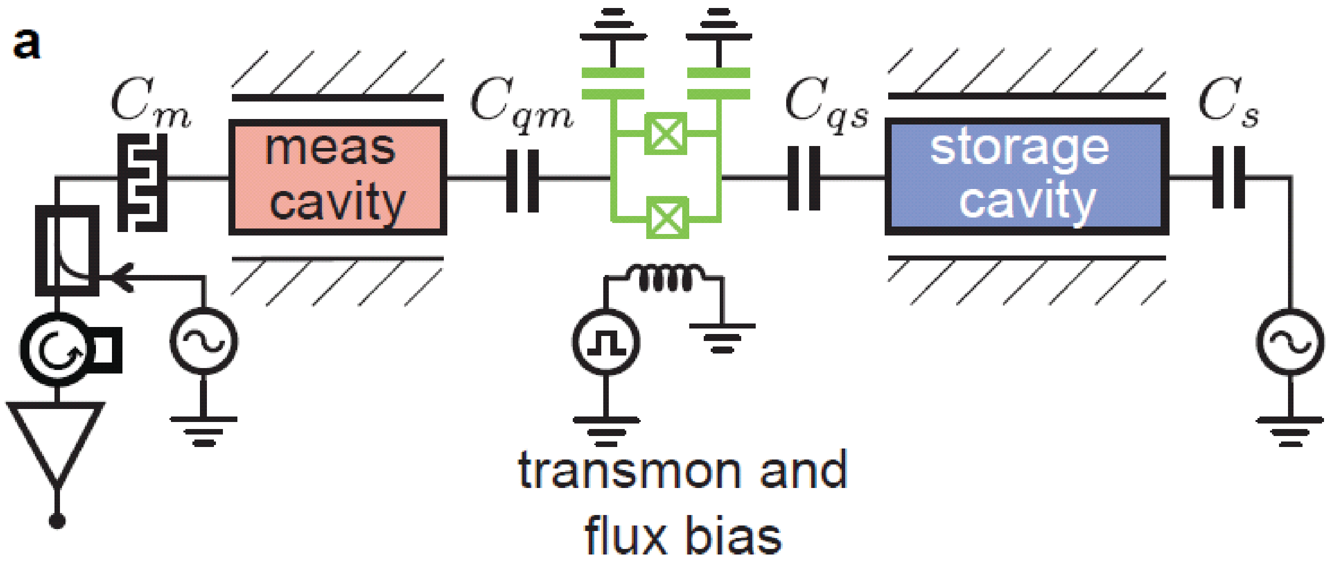


arXiv: 1003.2734

About the single photon detection …
i.e., the jump-based feedback ?

Quantum Non-demolition Detection of Single Microwave Photons in a Circuit

B. R. Johnson,[1] M. D. Reed,[1] A. A. Houck,[2] D. I. Schuster,[1] Lev S. Bishop,[1] E. Ginossar,[1]
J. M. Gambetta,[3] L. DiCarlo,[1] L. Frunzio,[1] S. M. Girvin,[1] and R. J. Schoelkopf[1]

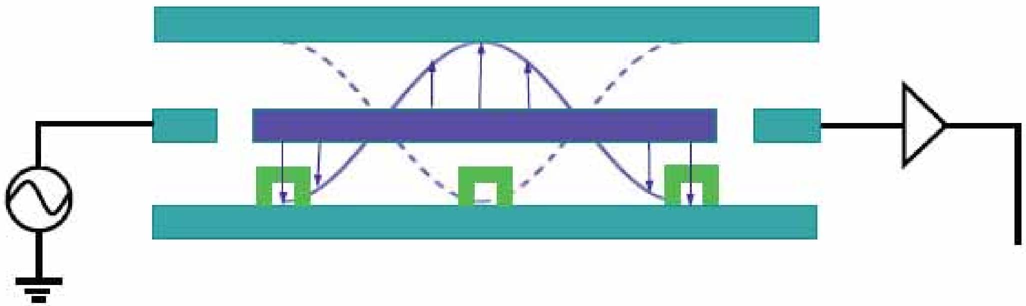


3-bit GHZ state control

PHYSICAL REVIEW A **83**, 042313 (2011)

Generating and stabilizing the Greenberger-Horne-Zeilinger state in circuit QED: Joint measurement, Zeno effect, and feedback

Wei Feng, Peiyue Wang, Xinmei Ding, Luting Xu, and Xin-Qi Li*
Department of Physics, Beijing Normal University, Beijing 100875, China
(Received 22 January 2011; published 11 April 2011)



$$H_{\rm eff} \simeq \Delta_r a^\dagger a + (\epsilon^* a + \epsilon a^\dagger) + \sum_{j=1}^{3} (\omega_j + \chi_j)\frac{\sigma_j^z}{2} + \sum_{j=1}^{3} \chi_j a^\dagger a \sigma_j^z$$

$$\chi_j = g_j^2/\Delta_j$$

$$|\Psi_i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_1 \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_2 \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_3$$

Dipsersive shift:

$$\chi_1 : \chi_2 : \chi_3 = 1:1:2$$

Target state $|000\rangle + |111\rangle$

pre-GHZ state $|001\rangle + |110\rangle$

Effective meas. operator:

$$J_z = \sigma_1^z + \sigma_2^z + 2\sigma_3^z$$

Algorithm: Deterministic generation of the pre-GHZ state

(i) If the result is $J_z = 2$, which indicates the state $|011\rangle + |101\rangle$ projected out, we perform a σ_x flip on the first qubit and a $\pi/2 - \sigma_y$ rotation on the third qubit. Noting that $|011\rangle + |101\rangle$ can be rewritten as $(|01\rangle + |10\rangle) \otimes |1\rangle$, it is clear that the above rotations will transform it to $|000\rangle + |111\rangle + |001\rangle + |110\rangle$, which then has a new probability of $1/2$ in the successive measurement to be collapsed onto the pre-GHZ state $|001\rangle + |110\rangle$. (ii) Similarly, for the result $J_z = -2$, a σ_x flip on the first qubit and a $3\pi/2 - \sigma_y$ rotation on the third one can be performed to achieve the same goal as described in (i). (iii) If the measurement result is $J_z = 4$ or -4, which indicates the state $|111\rangle$ or $|000\rangle$ obtained, we then apply a $\pi/2 - \sigma_y$ or a $3\pi/2 - \sigma_y$ rotation on each qubit, making the state return back to the initial one $(|0\rangle + |1\rangle)_1 \otimes (|0\rangle + |1\rangle)_2 \otimes (|0\rangle + |1\rangle)_3$, which allows us to rerun the generating procedures.

Efficiency Assessment:

(compare to a naïve restart scheme)

Reduce single-bit rotations

and/or

Enhance success probabilities

and/or

Avoid "data clearing"

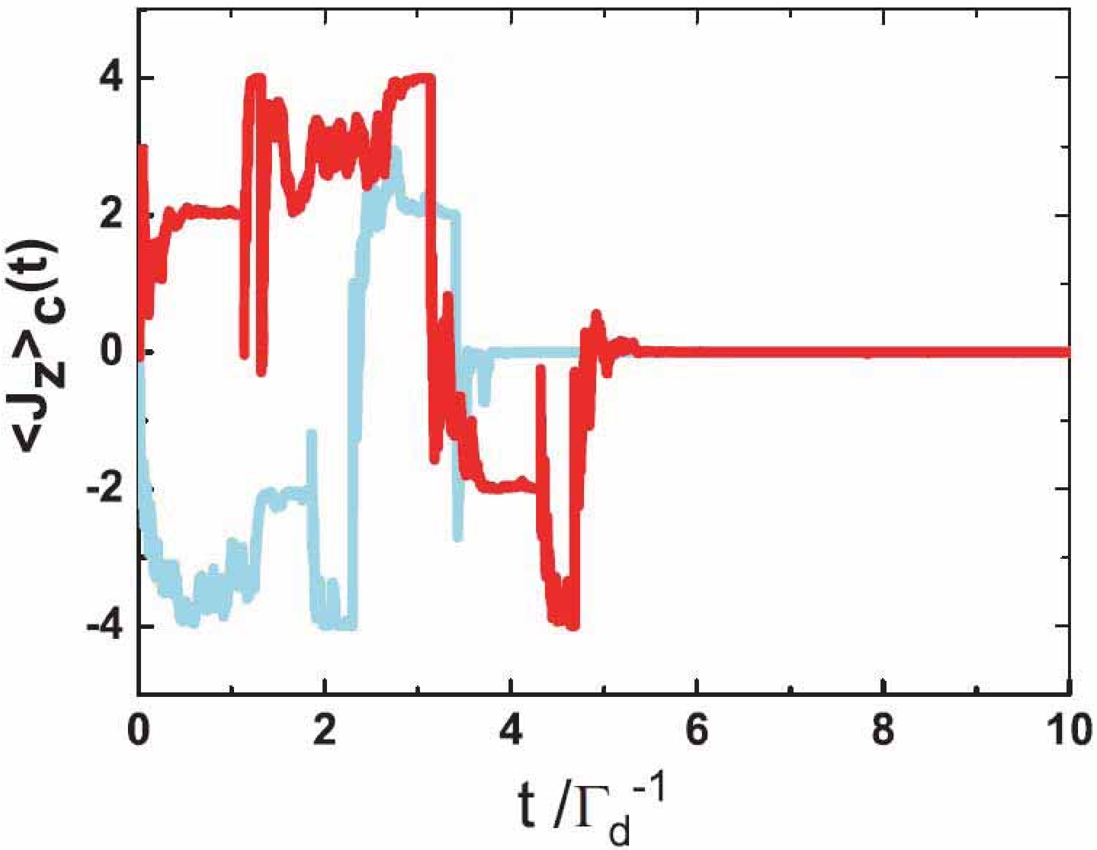


FIG. 2. (Color online) Two representative quantum trajectories showing the deterministic generation of the pre-GHZ state.

Naïve Zeno Stabilization

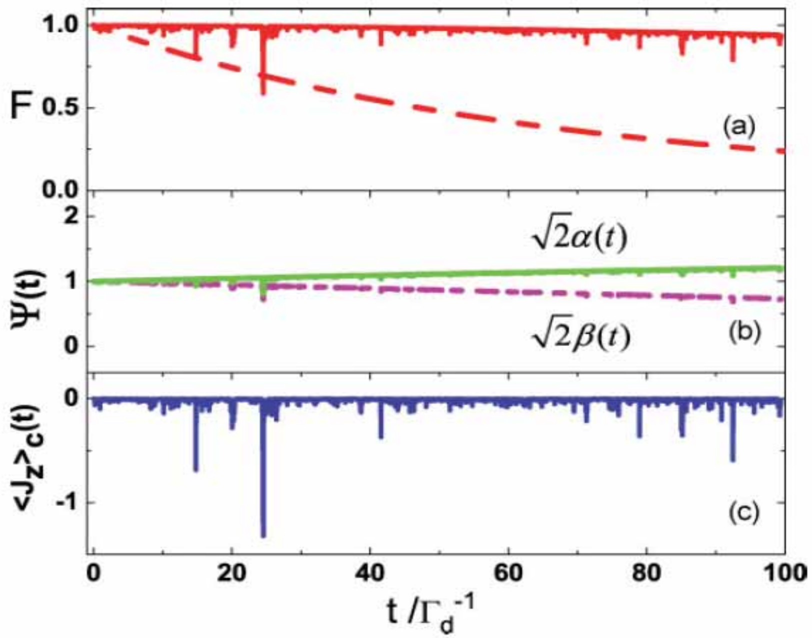


FIG. 3. (Color online) (a) State fidelity under the conventional quantum Zeno (not the AFIZ) stabilization for the pre-GHZ state, showing the result (solid line) much better than the uncontrolled one (dashed line). (b) Detailed inspection for the Zeno pulled-back state in (a), $|\Psi(t)\rangle = \alpha(t)|001\rangle + \beta(t)|110\rangle$, showing a gradual deviation from the target state $|\Psi_T\rangle = (|001\rangle + |110\rangle)/\sqrt{2}$. (c) Unconscious output current for the changing state $|\Psi(t)\rangle$. Single-qubit decoherence rate: $\gamma = 0.01\Gamma_d$.

AFIZ Scheme

Qubit relaxation: $$\sum_j (\gamma_j + \gamma_{pj})\mathcal{D}[\sigma_j^-]\rho_c$$

(coupling with environment, entangling evolution)

"Jz=0" is equivalent to

a "null-result" of environmental measurement

$$\tilde{H}_{\rm qu} = H_{\rm qu} - i\frac{\gamma}{2}\sum_{j=1}^{3}\sigma_j^+\sigma_j$$

$$|\Psi\rangle = \alpha|001\rangle + \beta|110\rangle$$

$$|\Psi(t)\rangle = (\alpha e^{-\gamma t/2}|001\rangle + \beta e^{-\gamma t}|110\rangle)/||\cdot||$$

imbalance of "0" and "1" in |001> and |110>

Strategy: AFIZ (Alternate-flip-interrupted Zeno) scheme

(Very efficient provided $\gamma \ll \Gamma_d$)

Complementary Strategy:

During the process of AFIZ stabilization, there exists very small (but nonzero) probabilities being projected to : |000>, and a mixture of |100> and |010>

The J_z = - 4 and J_z = - 2 output currents will trigger the "deterministic generation scheme"

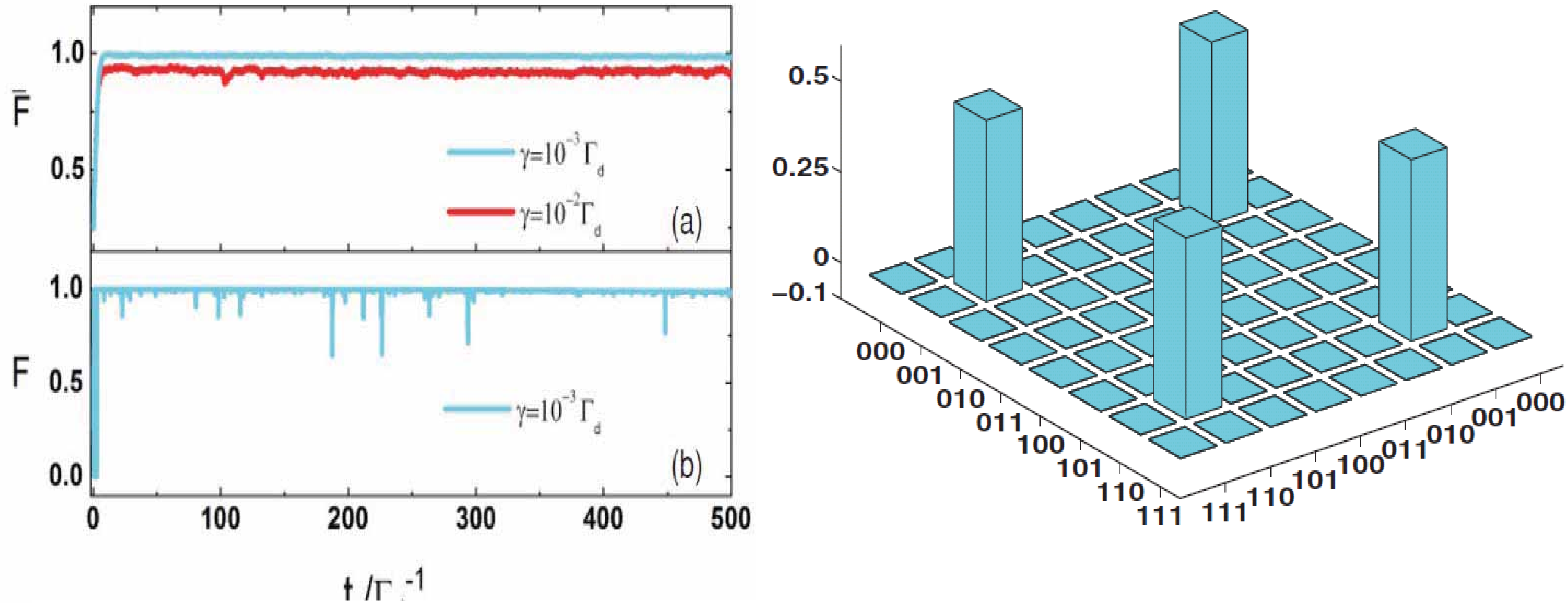


FIG. 5. (Color online) (a) Average fidelity of the pre-GHZ state over 1000 quantum trajectories. (b) Fidelity of an individual realization with $\gamma = 0.001\Gamma_d$, showing perfect control result under this even weaker decoherence when compared to $\gamma = 0.01\Gamma_d$ in Fig. 4. (c) The full state density matrix at a specific time in (b).

Summary

- **Part (I): Quantum Measurements**
  - **Simplest version: transmission measurement**
  - **Qubit-cavity state correlation: S-cat paradox**
  - **Homodyne measurement: SNR**
  - **Single microwave photon measurement**

- **Part (II): Quantum Control**
  - **Entanglement**
  - **2-bit Bell state control**
  - **3-bit GHZ state control**

谢 谢 大 家 ！

欢迎来北京做客！