Quantum information processing with individual neutral atoms in optical tweezers

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The single atom(s) project(s)


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Quantum Information Processing with Individual Neutral Atoms in Optical Tweezers.

Lecture 1 : trapping and moving single atoms

Lecture 2 : using single atoms
as controlled single-photon sources

Lecture 3 : encoding qubits on single atoms,
and entangling them using Rydberg blockade.

Lecture 1 : trapping and moving single atoms

1. Basics of neutral atoms cooling and trapping

2. Manipulating single atoms in optical tweezers

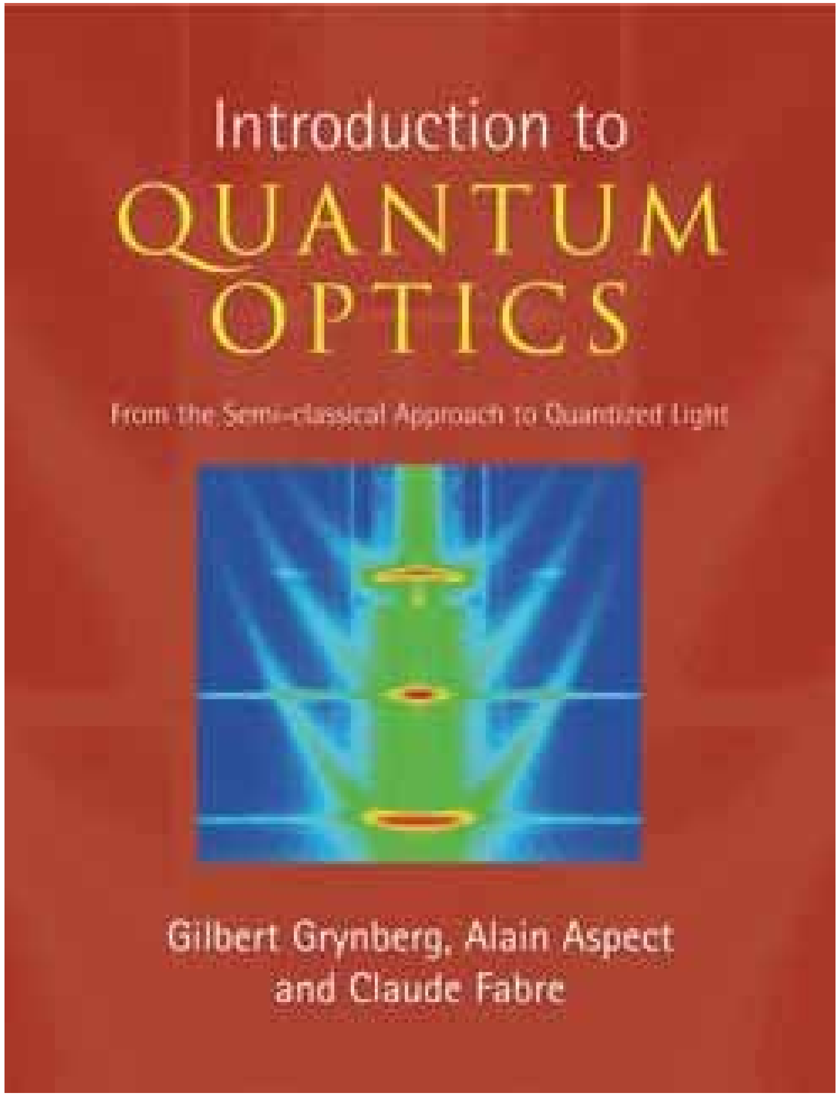
Laser manipulation of atoms

A. Radiative forces
   - Momentum exchanges between atoms and photons
   - Atomic polarisability
   - Two types of force

B. Resonant radiation pressure
   - Properties
   - Stopping atoms with light

C. Dipole force
   - Properties
   - Optical tweezers and lattices…

D. Cooling: optical molasses
   - Doppler cooling
   - Limit temperature
   - Below the Doppler limit



Laser manipulation of atoms.

A remarkable illustration of laser-atom interaction

From the early 1980's, from basic research (Nobel Prize 1997 for laser cooling) to applications (atom interferometers sensitive to inertial and gravitational effects; ultra-cold atom clocks)

A case study showing the interest of diversified theoretical treatments:

- Semi-classical approach: only the atom is quantized
- Fully quantum approach: light also is quantized

Lead to the emergence, in 1995, of a new research field : Bose-Einstein condensation of atomic gases (Nobel Prize 2001)…

… including atom lasers: concepts similar to the ones developed in the case of photon lasers!

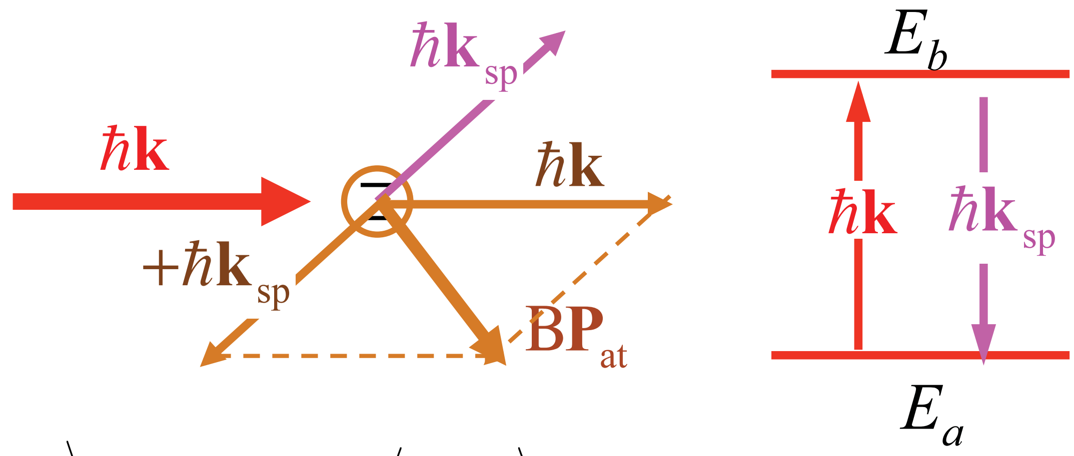
Radiation pressure and linear momentum exchange between photons and atoms

Linear momentum balance sheet in a fluorescence cycle

Photon in a running wave with **k**-vector : $\mathbf{p} = \hbar \mathbf{k}$

Absorption-Reemission

$$\Delta \mathbf{P}_{\text{at}} = \hbar \mathbf{k} - \hbar \mathbf{k}_{\text{sp}}$$



Average over many cycles $\langle \mathbf{k}_{\text{sp}} \rangle = 0 \quad \Rightarrow \quad \langle \Delta \mathbf{P}_{\text{at}} \rangle_{1\ \text{cycle}} = \hbar \mathbf{k}$

Average force :

$$\mathbf{F}_{\text{res}} = \mathcal{N} \langle \Delta \mathbf{P}_{\text{at}} \rangle_{1\ \text{cycle}} = \mathcal{N} \hbar \mathbf{k} = \hbar \mathbf{k} \frac{\Gamma}{2} \frac{s}{1+s}$$

$$s = \frac{\Omega_1^2 / 2}{(\omega_0 - \omega)^2 + \Gamma^2 / 4}$$

Orders of magnitude

Rb $\begin{cases} \Gamma / 2\pi = 6\ \text{MHz} \\ \text{I}_{\text{sat}} = 1.6\ \text{mW} / \text{cm}^2 \end{cases}$

$$\frac{F_{\text{max}}^{\text{max}}}{M} = \frac{\hbar k}{M} \frac{\Gamma}{2} \approx 10^5\ \text{m} / \text{s}^{-2} \approx 10^4\ \text{g}$$

Application: stopping an atomic beam with a laser

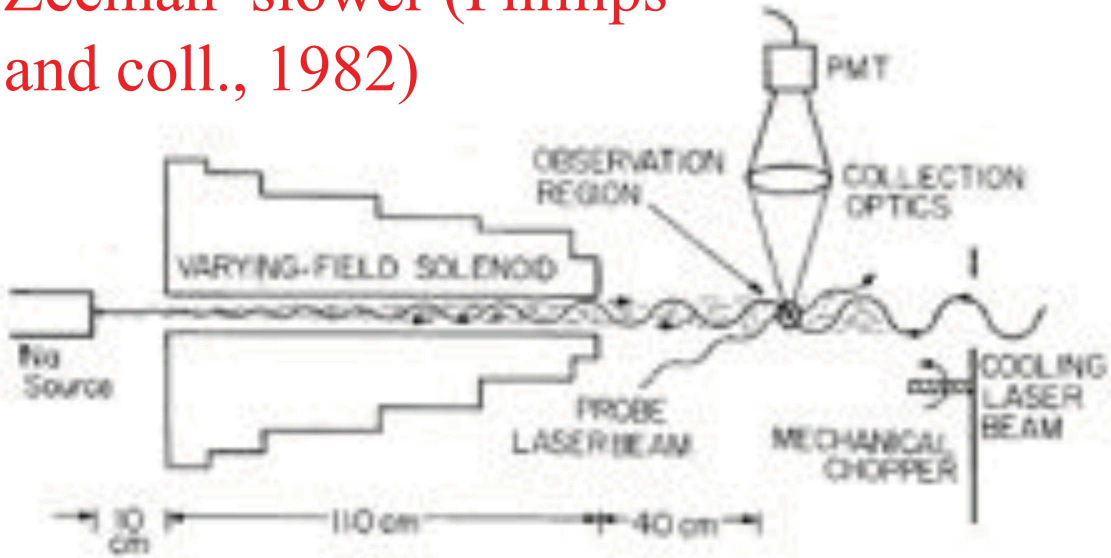
Sodium atomic beam

$$V \approx \sqrt{\frac{2k_B T}{M}} \approx 600 \text{ m / s}$$

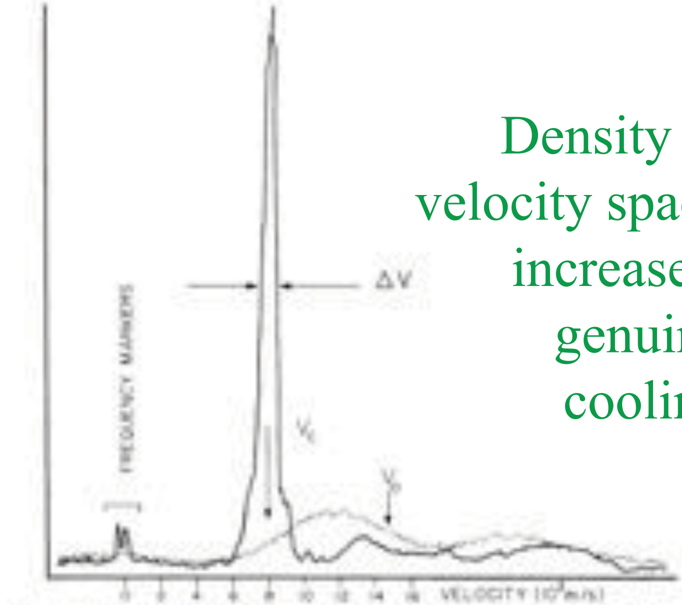
$$\text{stopping distance} = \frac{V^2}{2\gamma} \approx \frac{36\times 10^4}{2\times 10^5} = 1.8 \text{ m}$$

Atoms must be kept in resonance during slowing (Doppler effect compensation): position depending Zeeman shifts

Zeeman slower (Phillips and coll., 1982)

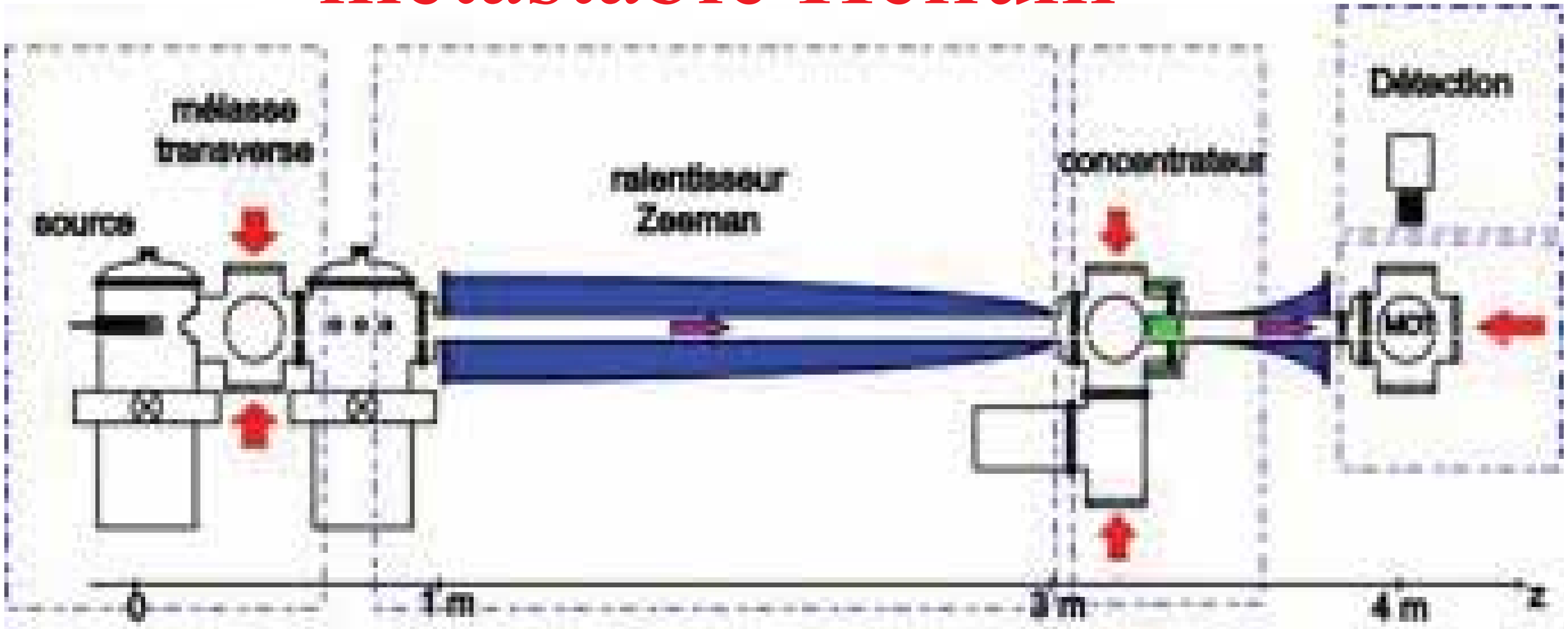


Velocity distribution



Density in velocity space increases: genuine cooling

Zeeman slowing of metastable Hélium



Radiative forces : general approach

Forced oscillation of the atomic dipole, under the effect of the field

$$\mathbf{E}(\mathbf{r},t) = \boldsymbol{\varepsilon}\, E_0(\mathbf{r}) \cos\left(\omega t - \varphi(\mathbf{r})\right) = \boldsymbol{\varepsilon}\, \mathcal{E}(\mathbf{r},t) + \text{c.c.}$$

$$\textit{with} \quad \mathcal{E}(\mathbf{r},t) = \frac{E_0(\mathbf{r})}{2} \exp\left\{i\varphi(\mathbf{r})\right\} \exp\left\{-i\omega t\right\}$$

$$\left\langle \hat{D}_{\boldsymbol{\varepsilon}} \right\rangle = \varepsilon_0 \alpha\, \mathcal{E}(\mathbf{r},t) + \varepsilon_0 \alpha^* \mathcal{E}^*(\mathbf{r},t) \qquad \text{after transient damping}$$

Complex polarisability $\alpha = \alpha' + i\alpha''$ (*cf.* next slide)

Force due to the interaction between the field and the induced atomic dipole

$$\mathbf{F} = \left\langle \hat{D}_{\boldsymbol{\varepsilon}} \right\rangle \left[\vec{\nabla} \left\{ E(\mathbf{r},t) \right\} \right]_{\mathbf{r}_{\text{at}}} = \varepsilon_0 \left(\alpha \mathcal{E} + \alpha^* \mathcal{E}^* \right) \vec{\nabla} \left\{ \mathcal{E} + \mathcal{E}^* \right\}$$

Keeping only the non oscillating terms (time average)

$$\mathbf{F} = \varepsilon_0 \alpha\, \mathcal{E} \left[\vec{\nabla} \mathcal{E}^* \right]_{\mathbf{r}_{\text{at}}} + \text{ c.c.}$$

Radiation pressure and dipole force

$$\mathbf{F} = \varepsilon_0 \alpha \mathcal{E}\left[\vec{\nabla}\mathcal{E}^*\right]_{\mathbf{r}_{\mathrm{at}}} + \text{ c.c.} \quad \text{with} \quad \mathcal{E}(\mathbf{r},t) = \frac{E_0(\mathbf{r})}{2}\exp\{i\varphi(\mathbf{r})\}\exp\{-i\omega t\}$$

$$\mathbf{F} = \frac{\varepsilon_0\alpha}{4} E_0(\mathbf{r})\left(\vec{\nabla}\left[E_0(\mathbf{r})\right] - i\vec{\nabla}\left[\varphi(\mathbf{r})\right]E_0(\mathbf{r})\right) + \text{ c.c.}$$

$$= \frac{\varepsilon_0\alpha'}{2} E_0(\mathbf{r})\vec{\nabla}\left[E_0(\mathbf{r})\right] + \frac{\varepsilon_0\alpha''}{2}\left(E_0(\mathbf{r})\right)^2 \vec{\nabla}\left[\varphi(\mathbf{r})\right]$$

the expression is evaluated at $\mathbf{r} = \mathbf{r}_{\mathrm{at}}$

$$\mathbf{F}_{\mathrm{res}} = \frac{\varepsilon_0\alpha''}{2}\left(E_0(\mathbf{r})\right)^2 \vec{\nabla}\left[\varphi(\mathbf{r})\right]$$

Resonant radiation pressure

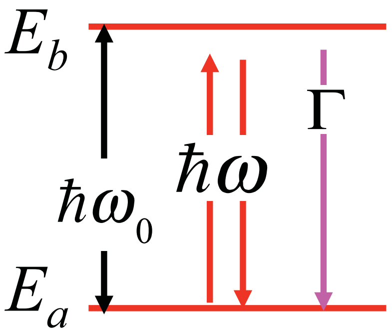
- Dissipative part (imaginary) of polarisability
- Phase gradient

$$\mathbf{F}_{\mathrm{dip}} = \frac{\varepsilon_0\alpha'}{2} E_0(\mathbf{r})\vec{\nabla}\left[E_0(\mathbf{r})\right]$$

Dipole force

- Reactive part (real) of polarisability
- Amplitude gradient

Atomic polarisability for a resonance transition



Resonance line: a = ground state

- $\Gamma_a = 0$
- Line width $\Gamma = \Gamma_b$
- Closed transition

Forced dipole oscillation, by $\mathbf{E} = \boldsymbol{\varepsilon}\, E_0 \cos\left(\omega t - \varphi\right) = \boldsymbol{\varepsilon}\left(\mathcal{E} + \mathcal{E}^*\right)$

Density matrix formalism and Optical Bloch equations (dissipation…)

$\Rightarrow$ complex polarizability α

$$\langle \mathbf{D} \rangle = \boldsymbol{\varepsilon} \langle \hat{D}_{\boldsymbol{\varepsilon}} \rangle$$

$$\hat{D}_{\boldsymbol{\varepsilon}} = -\begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix}$$

$$\langle \hat{D}_{\boldsymbol{\varepsilon}} \rangle = \varepsilon_0 \alpha \mathcal{E} + \varepsilon_0 \alpha^* \mathcal{E}^*(\mathbf{r},t)$$

$$\alpha = \underbrace{\frac{d^2}{\varepsilon_0 \hbar} \frac{1}{(\omega_0 - \omega) - i\frac{\Gamma}{2}}}_{\text{linear response}} \frac{1}{1+s}$$

saturation

$$\hbar\Omega_1 = -d\, E_0$$

$$s = \frac{\Omega_1^2 / 2}{(\omega_0 - \omega)^2 + \Gamma^2 / 4}$$

Radiative forces for a two level atom submitted to a quasi-resonnant laser

$$\mathcal{E}(\mathbf{r},t) = \frac{E_0(\mathbf{r})}{2}\exp\{i\varphi(\mathbf{r})\}\exp\{-i\omega t\} \qquad \alpha = \frac{d^2}{\varepsilon_0\hbar}\frac{1}{(\omega_0-\omega) - \mathrm{i}\Gamma/2}\frac{1}{1+s} = \alpha' + \mathrm{i}\alpha''$$

$$\mathbf{F} = \frac{\varepsilon_0\alpha'}{2}E_0(\mathbf{r})\vec{\nabla}\left[E_0(\mathbf{r})\right] + \frac{\varepsilon_0\alpha''}{2}\left(E_0(\mathbf{r})\right)^2\vec{\nabla}\left[\varphi(\mathbf{r})\right] \qquad \mathbf{r} = \mathbf{r}_{\mathrm{at}}$$

$$\mathbf{F}_{\mathrm{dip}} = \frac{\varepsilon_0\alpha'}{2}E_0(\mathbf{r})\vec{\nabla}\left[E_0(\mathbf{r})\right]$$

Dipole force

- Reactive part (real) of polarisability
- Amplitude gradient

$$\mathbf{F}_{\mathrm{res}} = \frac{\varepsilon_0\alpha''}{2}\left(E_0(\mathbf{r})\right)^2\vec{\nabla}\left[\varphi(\mathbf{r})\right]$$

Resonant radiation pressure

- Dissipative part (imaginary) of polarisability
- Phase gradient

Substituting α' et α'' by their expressions, we will find properties (and applications) very different for these two forces.

Resonnant radiation pressure

Running wave $\mathcal{E}(\mathbf{r},t)=\frac{E_0}{2}\exp\left\{i\mathbf{k}\cdot\mathbf{r}\right\}\exp\left\{-i\omega t\right\}$

constant amplitude E_0 $\Rightarrow$ $\mathbf{F}_{\mathrm{dip}}=\frac{\varepsilon_0\alpha'}{2}E_0(\mathbf{r})\vec{\nabla}\left[E_0(\mathbf{r})\right]=0$

$$\mathbf{F}_{\mathrm{res}}=\frac{\varepsilon_0\alpha''}{2}\left(E_0(\mathbf{r})\right)^2\vec{\nabla}\left[\varphi(\mathbf{r})\right]=\mathbf{k}\frac{d^2E_0^2}{2\hbar}\frac{\Gamma/2}{(\omega_0-\omega)^2+\Gamma^2/4}\frac{1}{1+s}$$

$$\mathbf{F}_{\mathrm{res}}=\hbar\mathbf{k}\frac{\Omega_1^2}{2}\frac{\Gamma/2}{(\omega_0-\omega)^2+\Gamma^2/4}\frac{1}{1+s}=\hbar\mathbf{k}\frac{\Gamma}{2}\frac{s}{1+s}$$

$$s=\frac{\Omega_1^2/2}{(\omega_0-\omega)^2+\Gamma^2/4}=\frac{I}{I_{\mathrm{sat}}}\frac{1}{1+4(\omega_0-\omega)^2/\Gamma^2}$$ saturation parameter

Dipole force

$$\mathbf{F}_{\text{dip}} = \frac{\varepsilon_0 \alpha'}{2} E_0(\mathbf{r}) \vec{\nabla}\left[E_0(\mathbf{r})\right] \qquad \textit{with} \quad \mathcal{E}(\mathbf{r},t) = \frac{E_0(\mathbf{r})}{2} \exp\left\{i\varphi(\mathbf{r})\right\} \exp\left\{-i\omega t\right\}$$

$\mathbf{F}_{\text{dip}} \neq 0$ for an inhomogeneous wave: $\vec{\nabla} E_0 \neq 0$

$$\mathbf{F}_{\text{dip}} = \frac{d^2}{\hbar} \frac{\omega_0 - \omega}{(\omega_0 - \omega)^2 + \frac{\Gamma^2}{4}} \frac{\vec{\nabla}\left[E_0(\mathbf{r})\right]^2}{1 + s(\mathbf{r})} = \frac{\hbar(\omega_0 - \omega)}{2} \frac{\vec{\nabla}\left[s(\mathbf{r})\right]}{1 + s(\mathbf{r})}$$

$$\mathbf{F}_{\text{dip}} = -\vec{\nabla}\left[U(\mathbf{r})\right] \quad \text{avec} \quad U(\mathbf{r}) = \frac{\hbar(\omega - \omega_0)}{2} \log\left[1 + s(\mathbf{r})\right]$$

Derives from a potential (reactive part of polarisability) varying as intensity

$$s(\mathbf{r}) = \frac{I(\mathbf{r})}{I_{\text{sat}}} \frac{1}{1 + 4(\omega_0 - \omega)^2 / \Gamma^2}$$

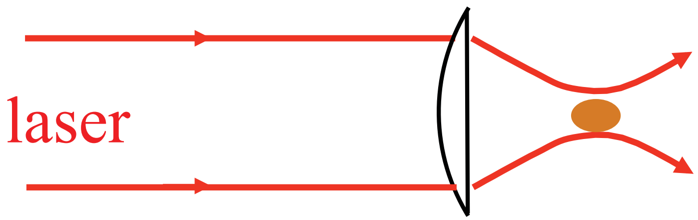
Applications

atom $\begin{cases} \text{attracted towards high intensity if } \omega < \omega_0 \\ \text{repelled from high intensity if } \omega > \omega_0 \end{cases}$

Dipole trap and optical tweezer

$$U_{\text{dip}}(\mathbf{r}) = \frac{\hbar(\omega-\omega_0)}{2}\log\left[1+s(\mathbf{r})\right] \qquad \textit{with} \;\; s(\mathbf{r}) = \frac{I(\mathbf{r})}{I_{\text{sat}}}\frac{1}{1+4(\omega_0-\omega)^2/\Gamma^2}$$

Trapping at the focus of a "red detuned" laser beam $(\omega < \omega_0)$: optical tweezer



Shallow trap: demands ultra-cold atoms ($T < 1$ mK)

Manipulation of atom clouds, individual atoms, Bose-Einstein atomic condensates, but also of biological objects

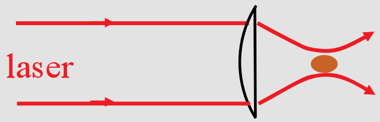
Dipole force: a very useful tool

Derives from a potential "proportionnal" to the light intensity (saturation)

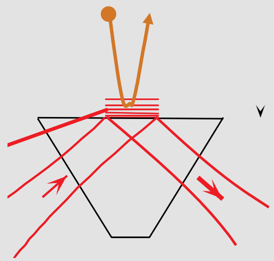
$$U_{\text{dip}}(\mathbf{r}) = \frac{\hbar(\omega-\omega_0)}{2}\log\left[1+s(\mathbf{r})\right]$$

A remarkable tool to manipulate ultra-cold atoms

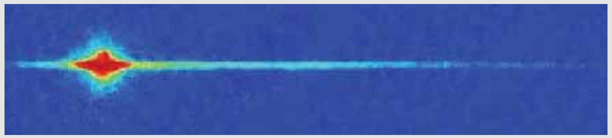
- Optical tweezer



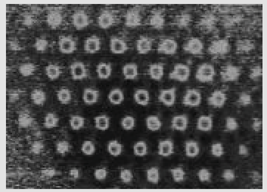
- Atomic mirror



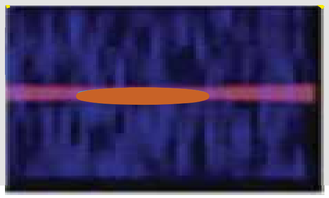
- Atomic wave guide



- Optical lattice (~ electrons in a crystal)

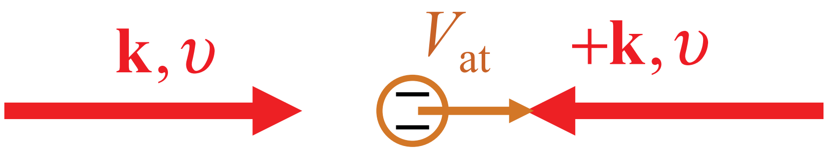


- Disordered medium (~ electrons in an amorphous medium)

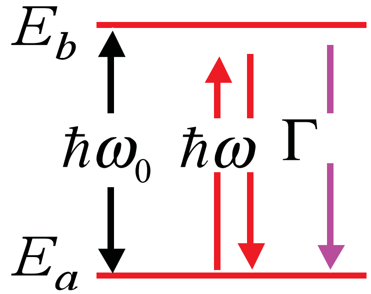


Doppler cooling

Resonant radiation pressure from two opposite running waves



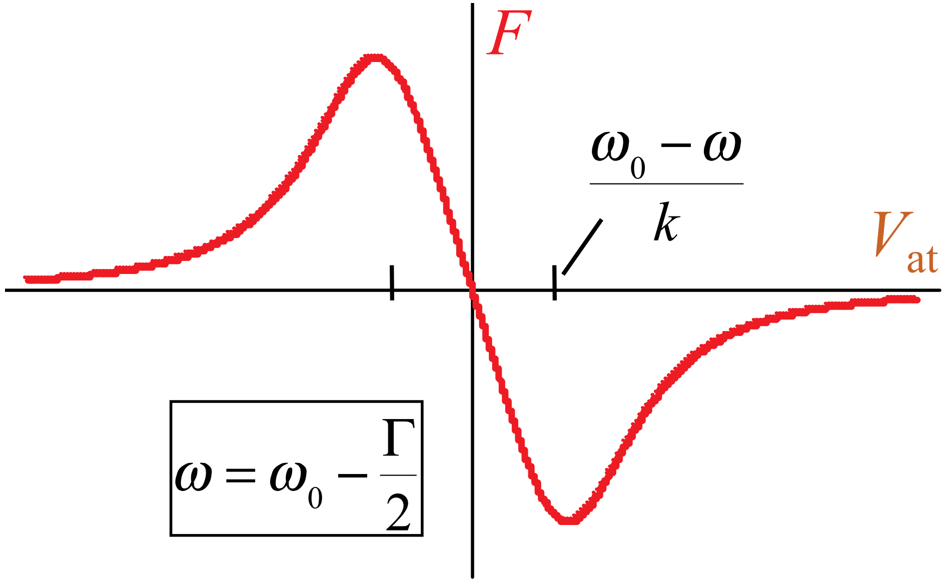
$$\omega < \omega_0$$



Non saturating lasers, detuned below resonance

Atomic velocity: different Doppler effect for each wave

$$\mathbf{F}_{res} = \mathbf{F}_0 \left\{ \frac{\Gamma^2/4}{(\omega - \mathbf{k}\cdot\mathbf{V}_{at} - \omega_0)^2 + \Gamma^2/4} - \frac{\Gamma^2/4}{(\omega + \mathbf{k}\cdot\mathbf{V}_{at} - \omega_0)^2 + \Gamma^2/4} \right\}$$



Force opposite to velocity:
friction ⇒ cooling

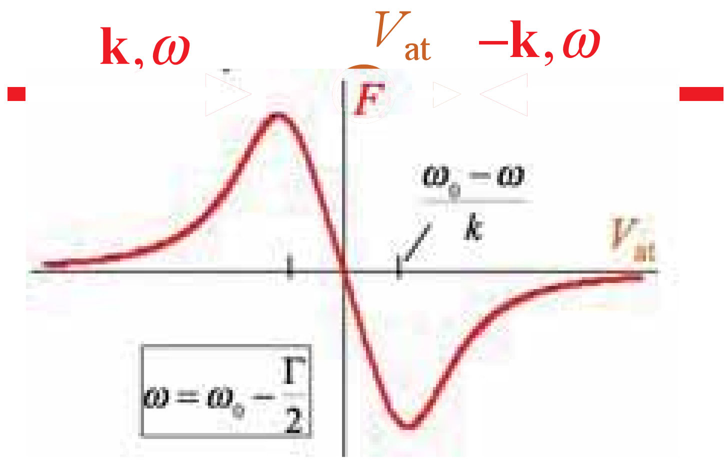
Generalization to 3-D (6 waves)

Doppler molasses

Near $V = 0$: viscous friction

$$F = -\kappa M V \quad \text{or} \quad \frac{dV}{dt} = -\kappa V$$

$$\kappa = \frac{\hbar k^2}{M} s_0 \quad \text{for} \quad \omega = \omega_0 - \frac{\Gamma}{2}$$



Order of magnitude (rubidium, $s_0 = 0.5$)

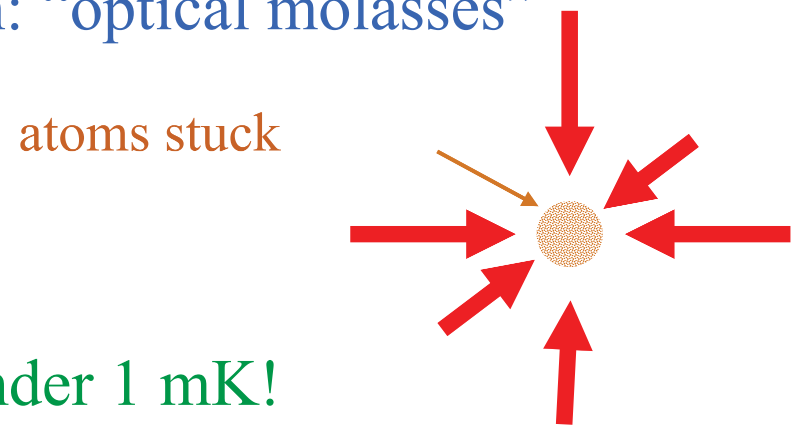
$$\kappa \simeq 2.5 \times 10^{-4} \ \text{s}^{-1}$$

Damping time: 40 µs !

Highly viscous medium: "optical molasses"

Exponential cooling!

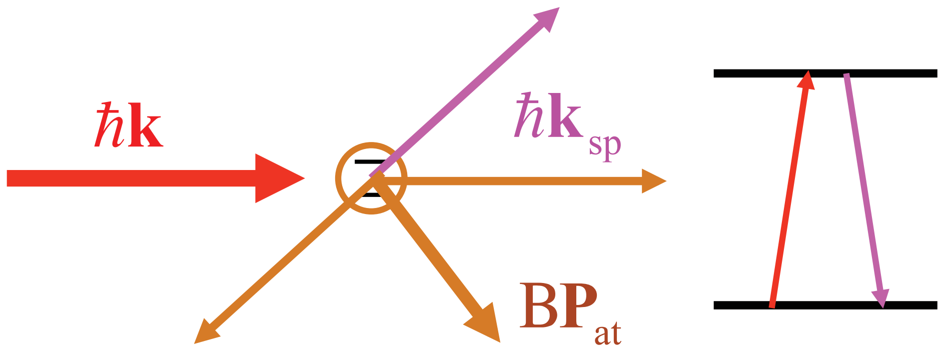
$$\frac{d}{dt}\langle V^2 \rangle = -2\kappa \langle V^2 \rangle$$



Temperature decreases: where to? Under 1 mK!

Fluctuations of the resonant radiation pressure

Fluctuations of linear momentum **P** due to spontaneous emission role in resonant radiation pressure.



Random jump in **P**, with magnitude $\hbar k$ and random direction, at each fluorescence cycle.

$$2D_{\mathbf{P}} = (\hbar k)^2 \mathcal{N}_{\text{fluo}}$$

coeff. de diffusion (Einstein)

On top of the mean force effect, one has a random walk in the **P** space:

$$\frac{d}{dt}\langle \mathbf{P}^2 \rangle = 2D$$

Heating

Limits of Doppler cooling

Cooling $$\frac{d}{dt}\langle \mathbf{P}^2 \rangle = -2\kappa \langle \mathbf{P}^2 \rangle$$

Heating $$\frac{d}{dt}\langle \mathbf{P}^2 \rangle = 2D$$

Steady state $$\langle \mathbf{P}_{\mathrm{at}}^2 \rangle = \frac{D}{\kappa}$$

Einstein relation

Analogous to the evolution of the amplitude of a laser: random walk with elastic force towards equilibrium

Langevin equation $$\frac{d}{dt}\mathbf{P}_{\mathrm{at}} = -\kappa \mathbf{P}_{\mathrm{at}} + \vec{\mathcal{F}}_{\mathrm{heat}}$$

Equilibrium temperature

$$\frac{3}{2}k_{\mathrm{B}}T = \frac{\langle \mathbf{P}_{\mathrm{at}}^2 \rangle}{2M} = \frac{3}{4}\hbar\Gamma$$

Order of magnitude: 100 µK

Experimentally observed in 1985 (S. Chu)

Below the Doppler limit temperature

Lowest predicted Doppler temperature: 100 µK range (observed in 1985)

Sisyphus cooling :

•observation (1988) of temperatures in the 10 µK range

•Interpretation: "Sisyphus" effect. Takes into account the internal atomic structure (several ground state sublevels) and light polarization.

Sub-recoil cooling :

•1988: method allowing one to reach residual velocities below the "one photon recoil velocity" $V_{\mathrm{R}} = \frac{\hbar k}{M}$

•On reaches the nK range (< 1 µK) : $\lambda_{\text{de Broglie}} > \lambda_{\text{laser}}$

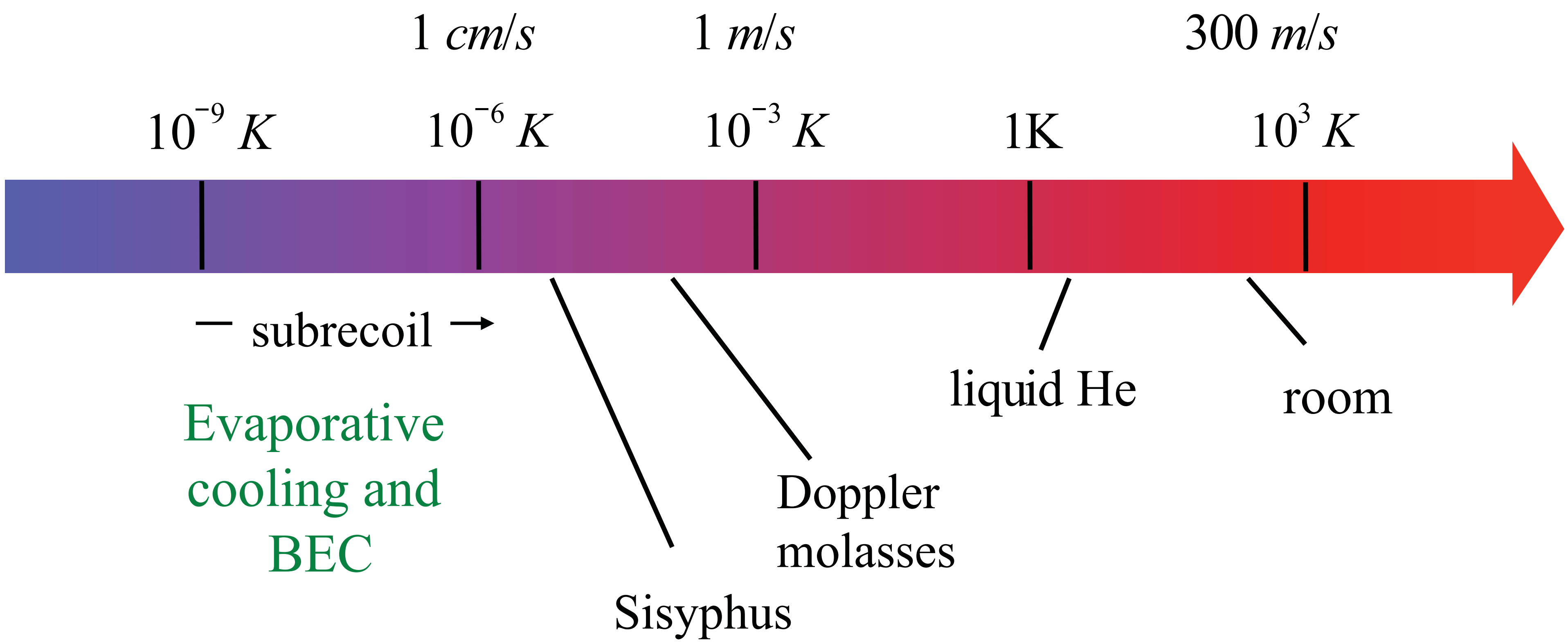
quantum description $\psi(\mathbf{r},t)$ of the atomic motion

Forced evaporative cooling :

In a non-dissipative trap (magnetic or dipole), eliminate the atoms with energy > $\eta k_{\mathrm{B}} T$ (typically $\eta \approx 6$) then rethermalize by elastic collisions

The density in phase space increases => Bose-Einstein condensation !

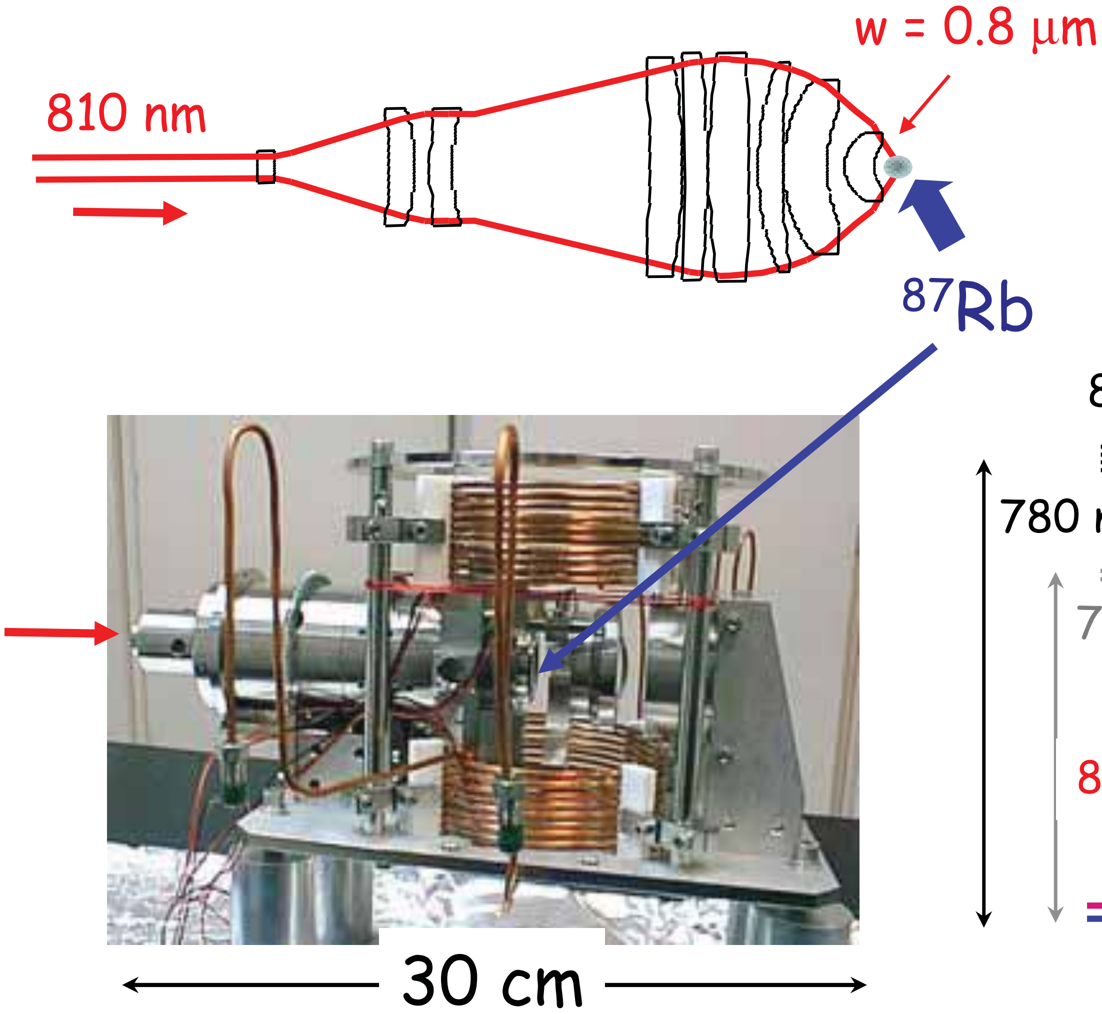
Steps of atom cooling



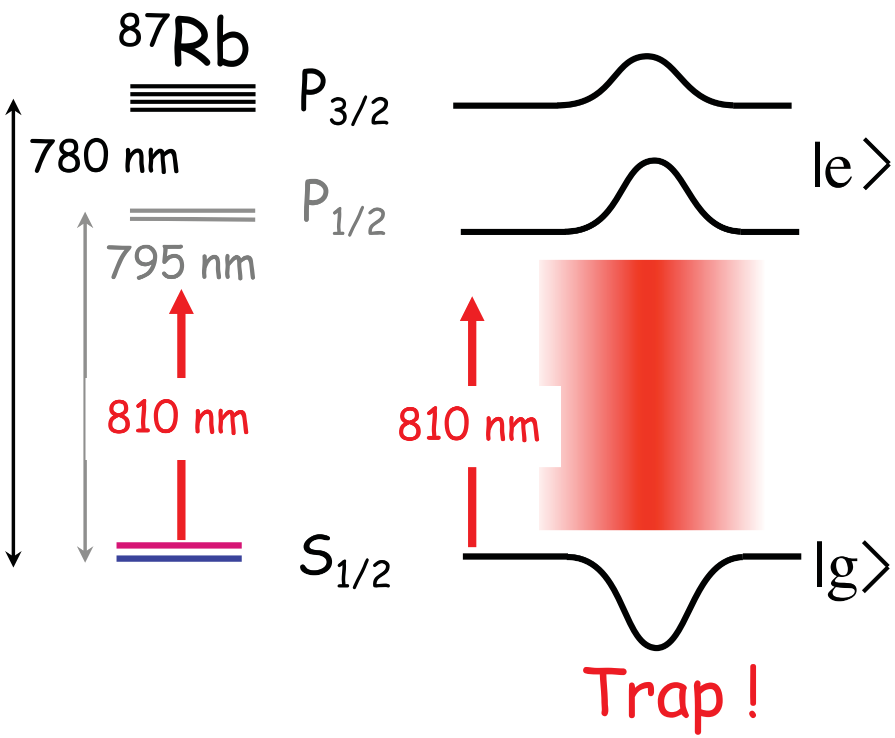
The logarithmic scale emphasizes the magnitude of cooling, characterized by the ratio of the initial to the final temperature, not by the difference.

Optical tweezer with large numerical aperture lenses

Very tightly confined optical dipole trap



Large Aperture NA = 0.7
Working distance ~ 10 mm
9 spherical lenses
Vacuum compatible

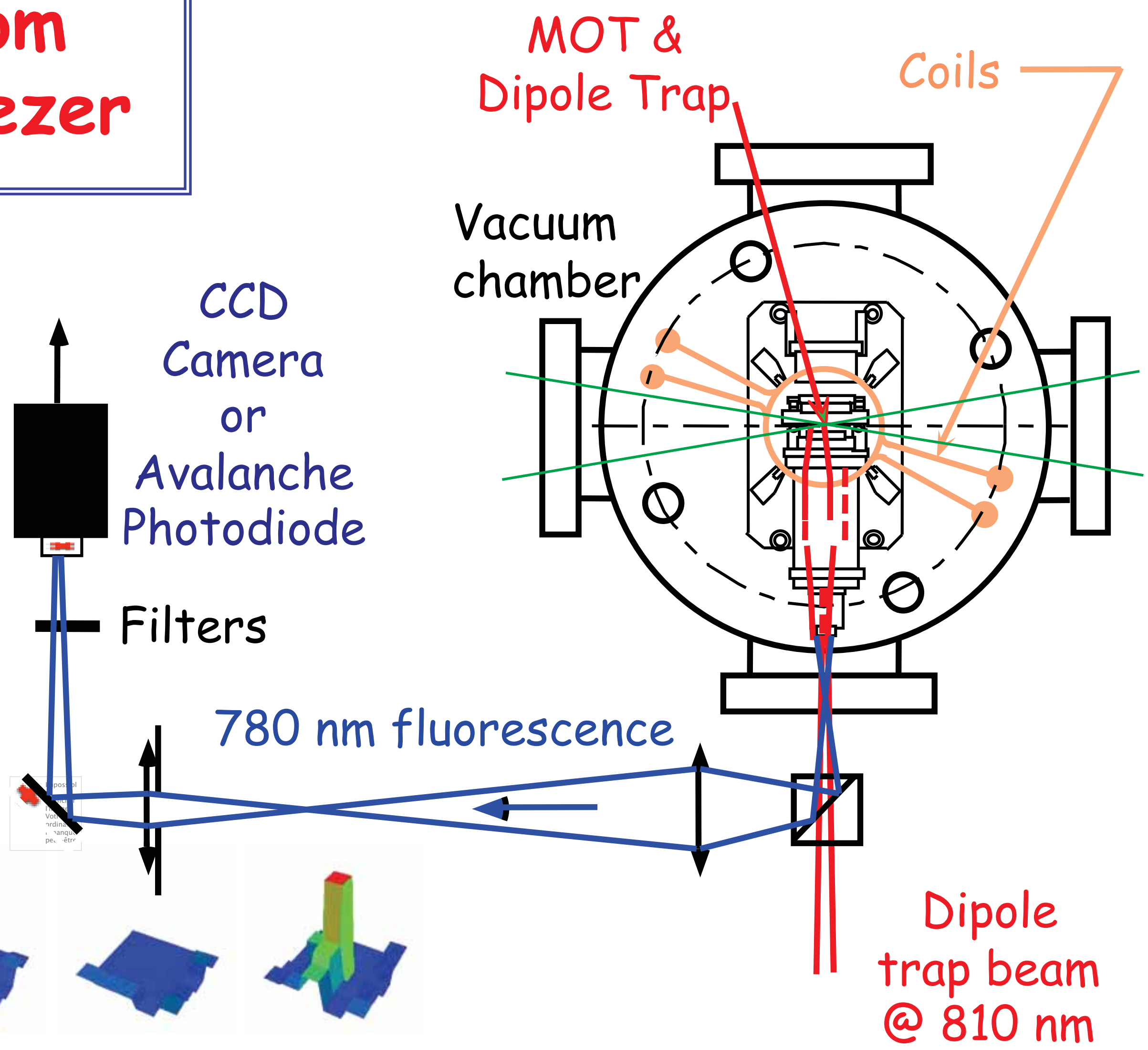


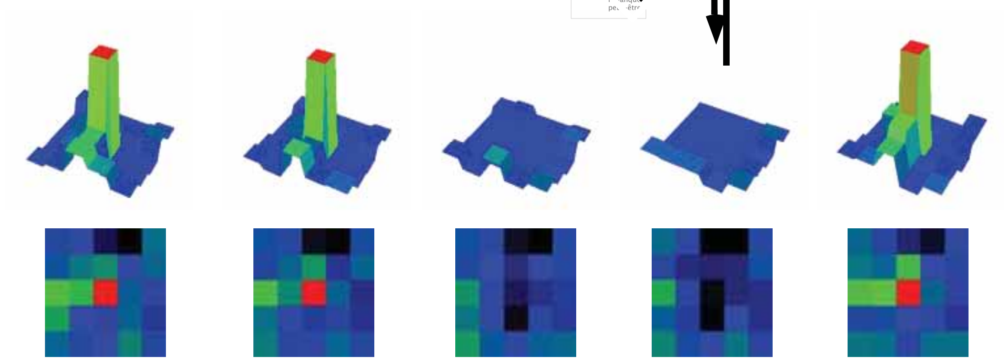
Schlosser **et al.**, Nature **411**, 1024 (2001)

Single atom optical tweezer

Microscopic dipole trap :
Spot size < 1 μm
Power < 10 mW
Trapping time > 1s

Imaging :1μm/pixel

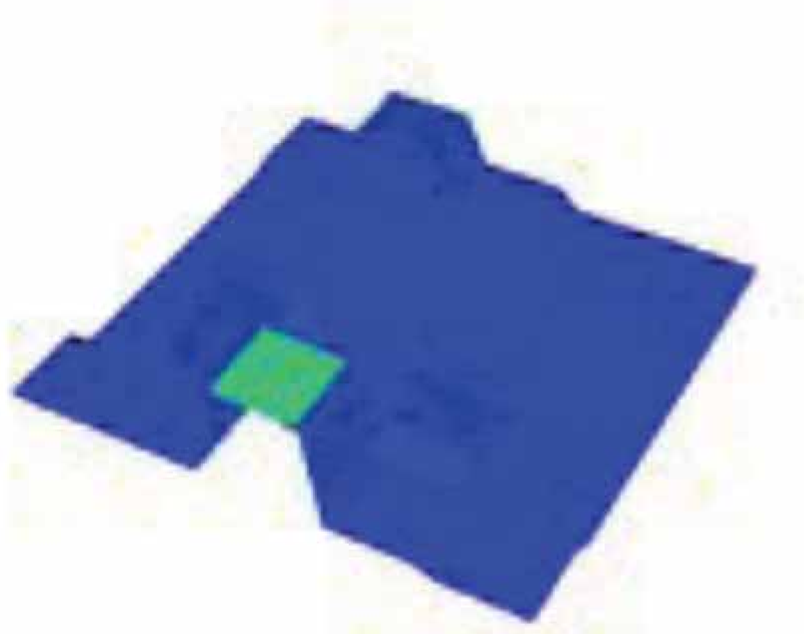




Individual atoms « jumping » in the trap
« Collisionnal blockade » : only one atom !

N. Schlosser et al,
Nature 411, 1024 (2001)
PRL 89, 023005 (2002)

Looking at single atoms

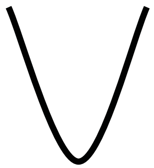


« Real-time blinking » of individual atoms
going in and out of the microscopic dipole trap
Fluorescence rate : 8000 cnts/at/s
Trapping time : a few seconds

« Collisional blockade »

N. Schlosser et al, PRL 89, 023005 (2002)

FET
QIPC



Main effect : light - assisted collision loss (due to the MOT light) is huge as soon as there are two atoms in the trap

When an atom enters the trap with one atom inside, both atoms escape : either zero or one atom only !

Some measured trapped atom parameters :

Trap frequencies (P_{trap} = 2 mW) : $w_{radial} / 2\pi = 130$ kHz
$w_{axial} / 2\pi = 30$ kHz

Trap depth (lightshift) : 1 mK / mW

Trap beam waist (P_{trap} = 2 mW) : $w_0 = 0.9\ \mu m$

Temperature : < 100 μK

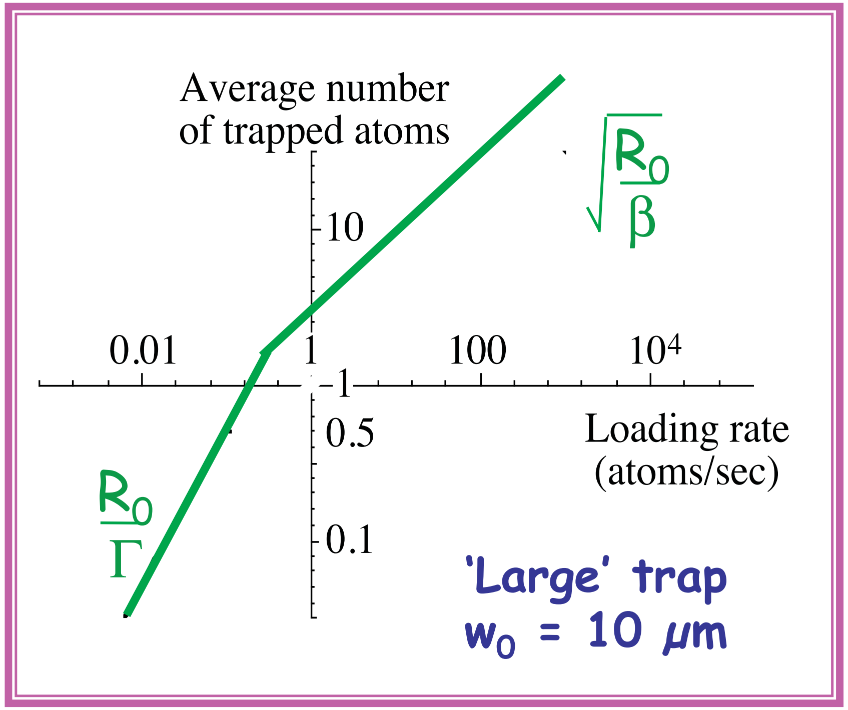
Collisional Blockade and Beyond

N. Schlosser et al, PRL 89, 023005 (2002)

Behaviour of the number N of trapped atoms :

$$dN/dt = R_0 - \Gamma N - \beta N(N-1)$$

R_0 : loading rate, Γ : one-atom loss (0.2 s^{-1}), β : two-atom loss



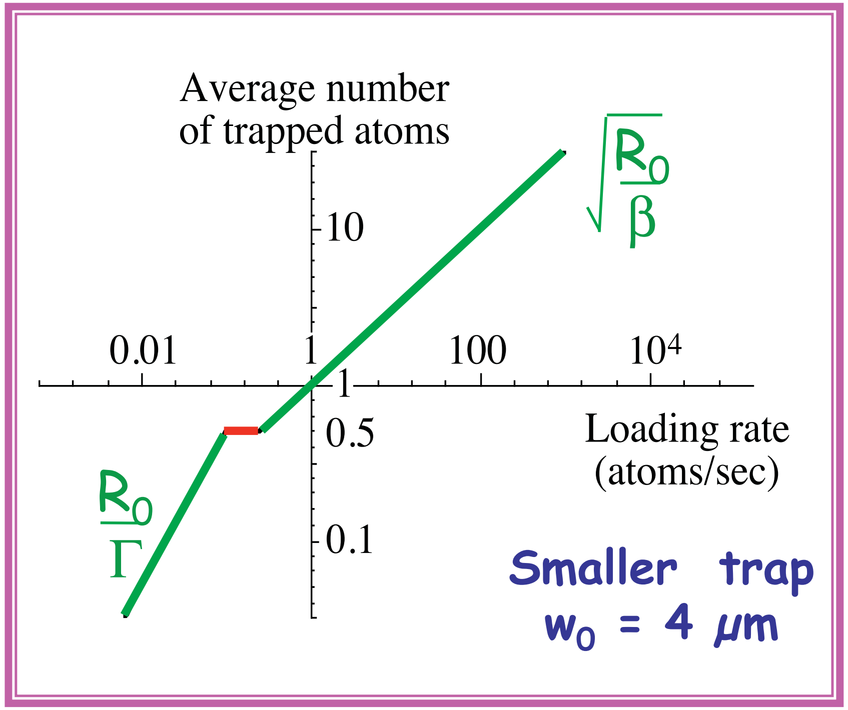
Collisional Blockade and Beyond

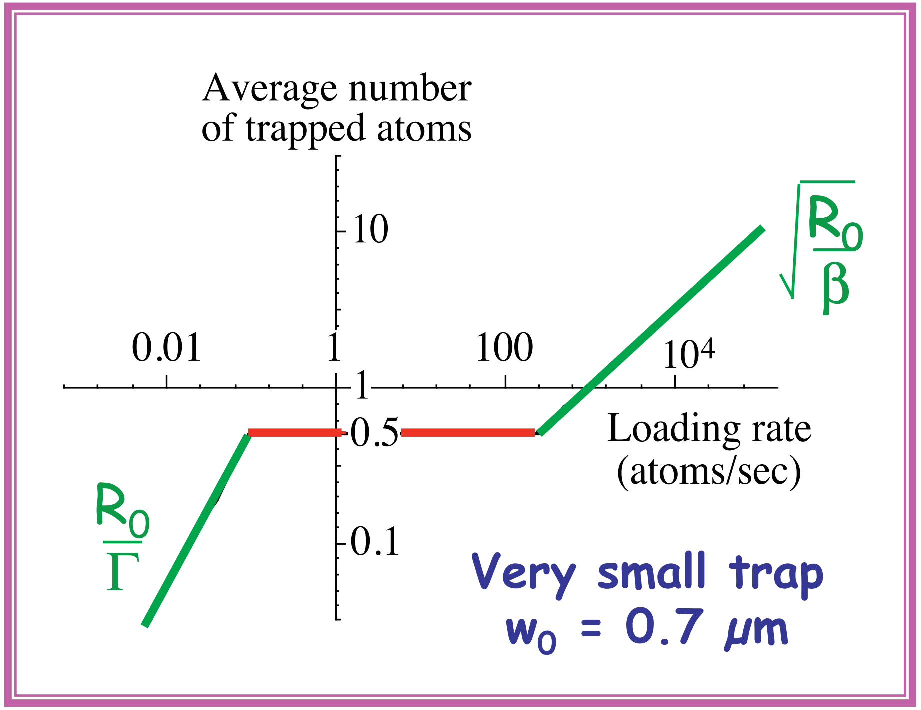
N. Schlosser et al, PRL 89, 023005 (2002)

Behaviour of the number N of trapped atoms :

$$dN/dt = R_0 - \Gamma N - \beta N(N-1)$$

R_0 : loading rate, Γ : one-atom loss (0.2 s^{-1}), β : two-atom loss

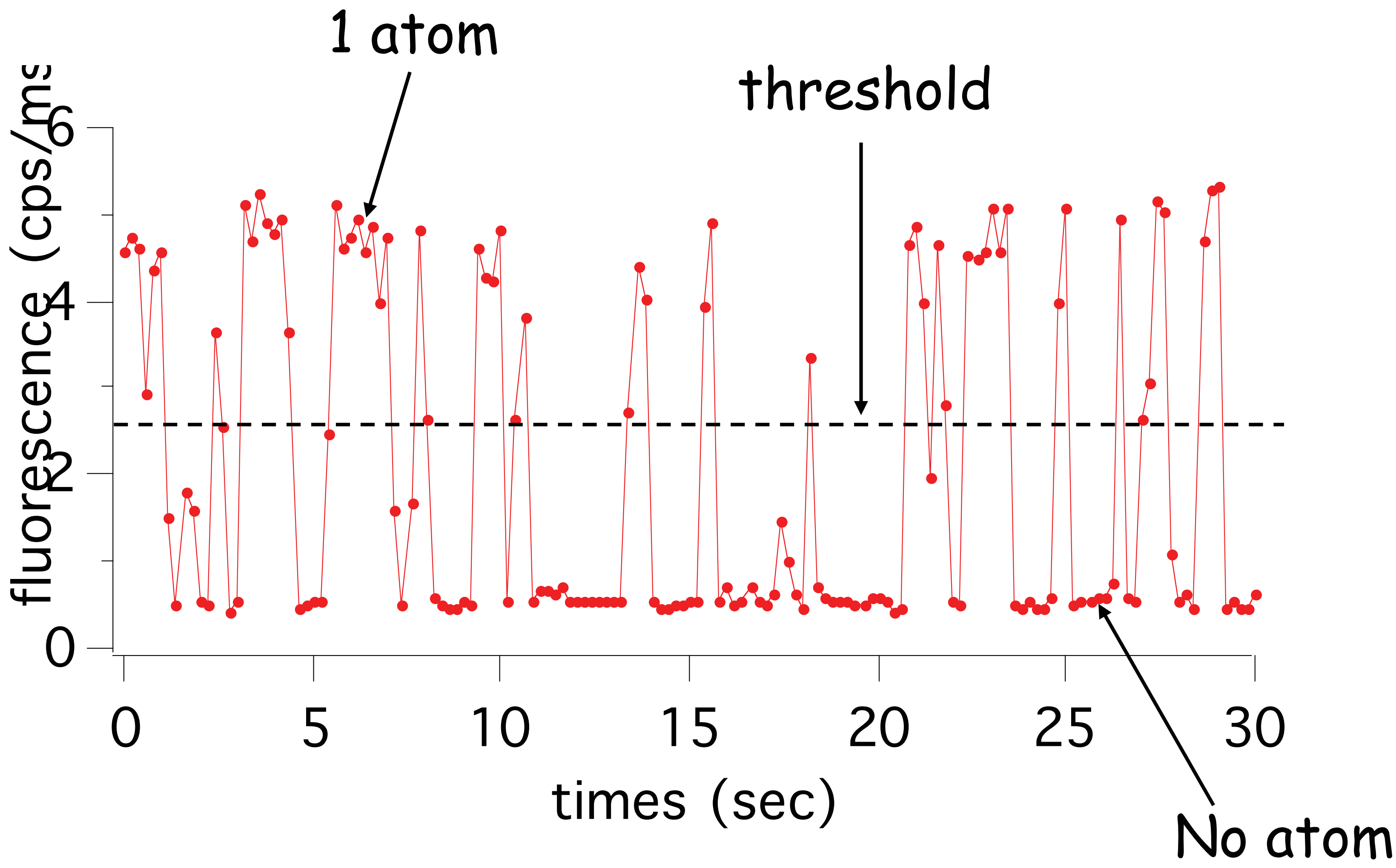




Collisional blockade : specific behaviour of small (< 4μm) dipole trap !

Detecting and "heralding" a single trapped atom

Fluorescence induced by the cooling lasers (780 nm)



Trapping time in the dark ~ 1 - 3 s

A more compact optical setup : aspherical lens

Molded aspherical lenses for laser diode objective

Working distance 6 mm,

N.A. = 0.5

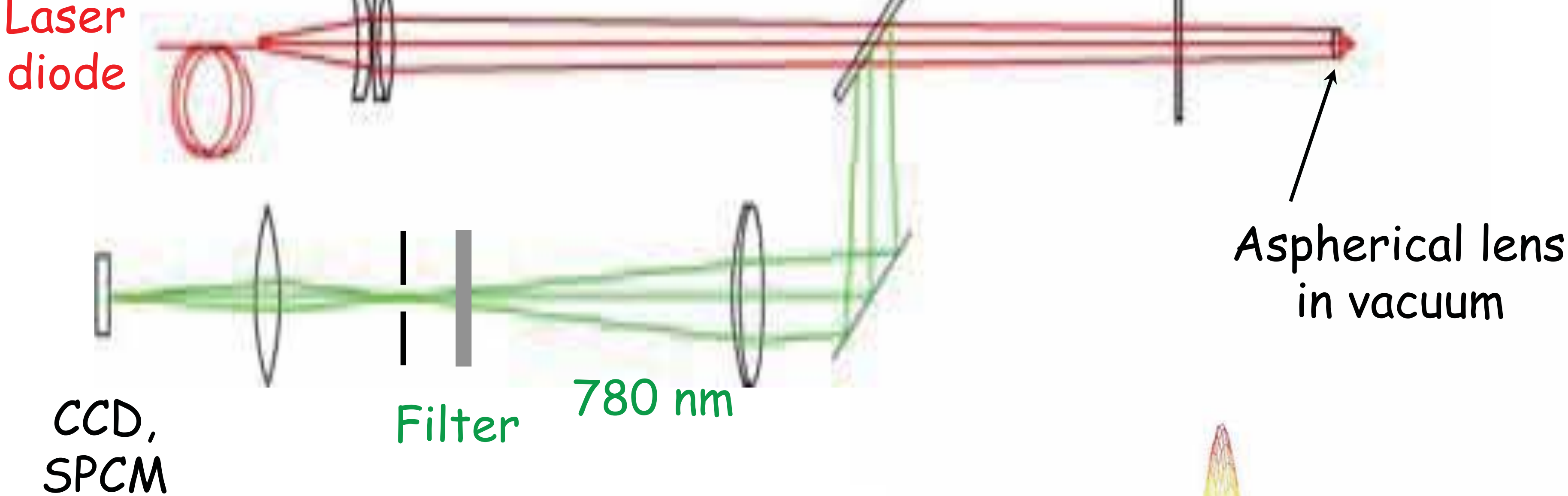
700 nm < λ < 900 nm

Typical trap parameters for 850 nm, 5 mW :

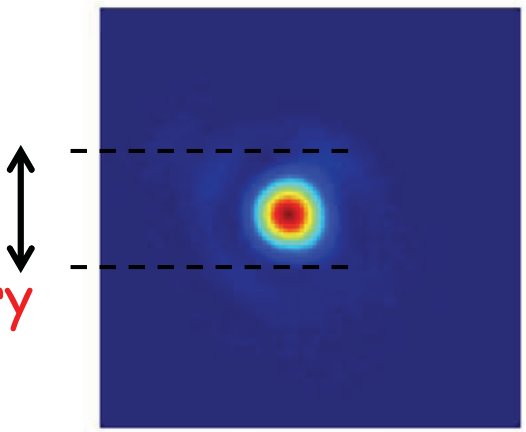
Depth = 2 mK

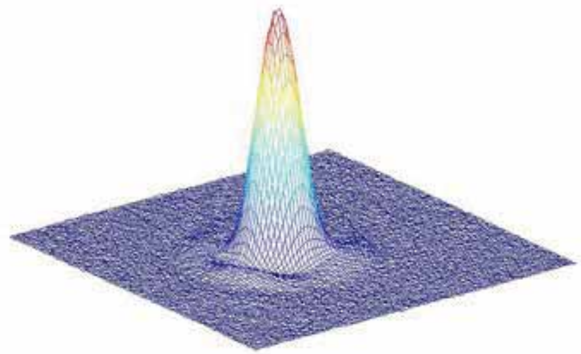
Transverse oscillation frequency = 135 kHz

Optical tweezer setup



2.15 µm (Airy function diameter @ 850 nm, theory = 2.09 µm)





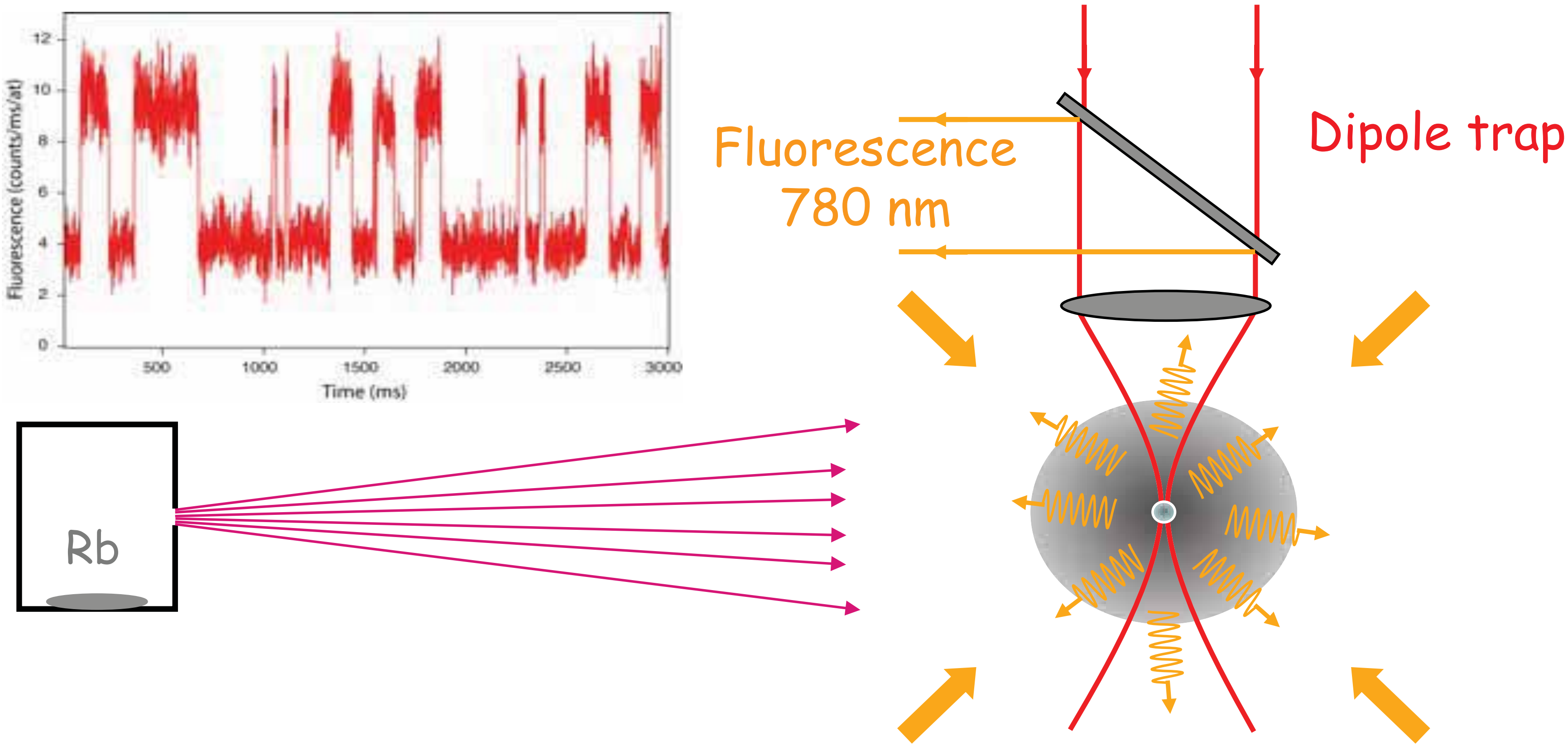
Transverse field = ± 30 µm

Catching single atoms from an slow atomic beam

Sortais **et al.**, PRA **75**, 013406 (2007)

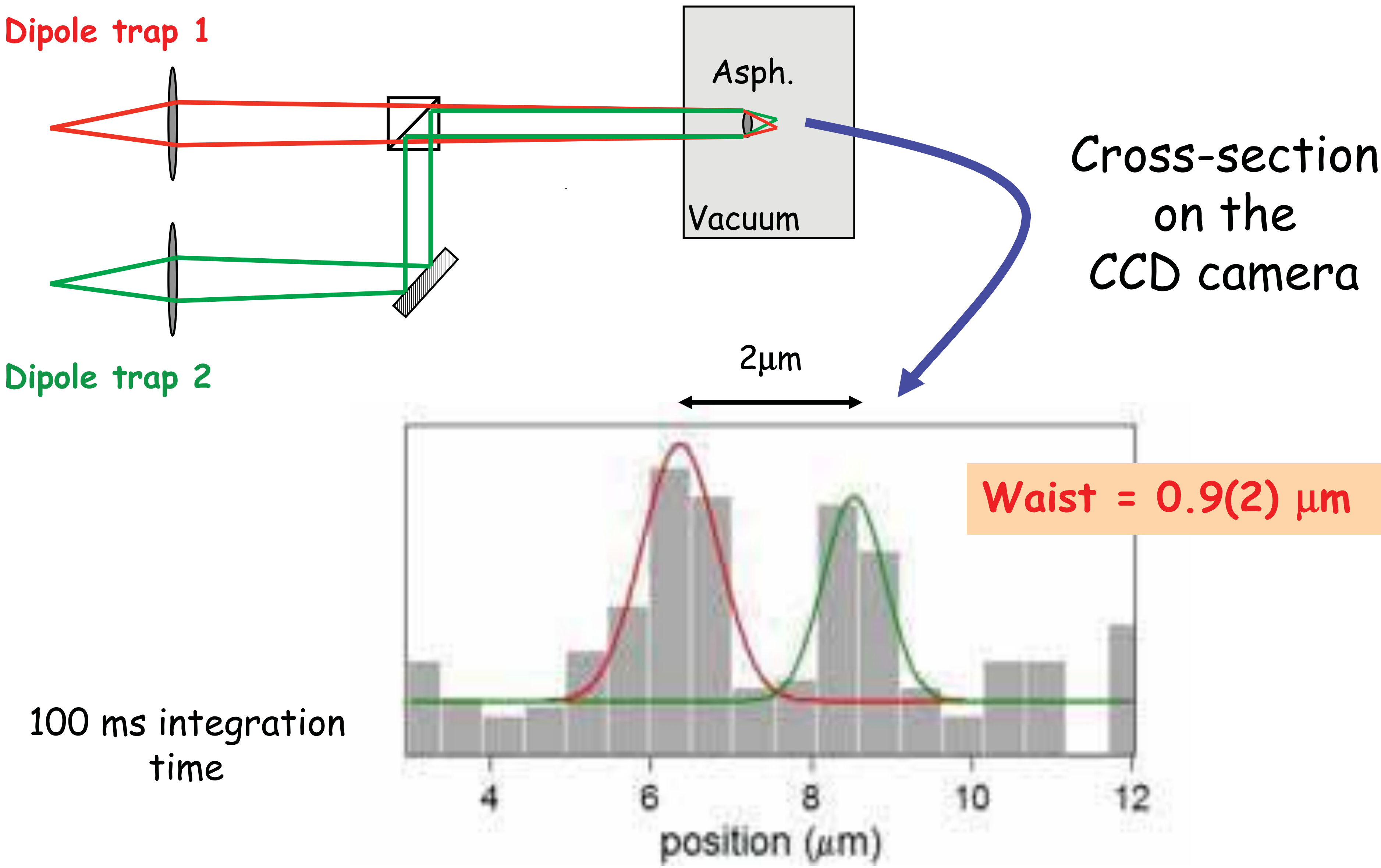
≈ 10000 counts/sec/atom on the APD

Trapping time in the dark ~ 10 s



Oven	Slowing	Dilute atomic cloud
200 m/s	200 m / s → ~ 1 m / s	T ~ 100 μK - 10 μK

Trapping 2 single atoms

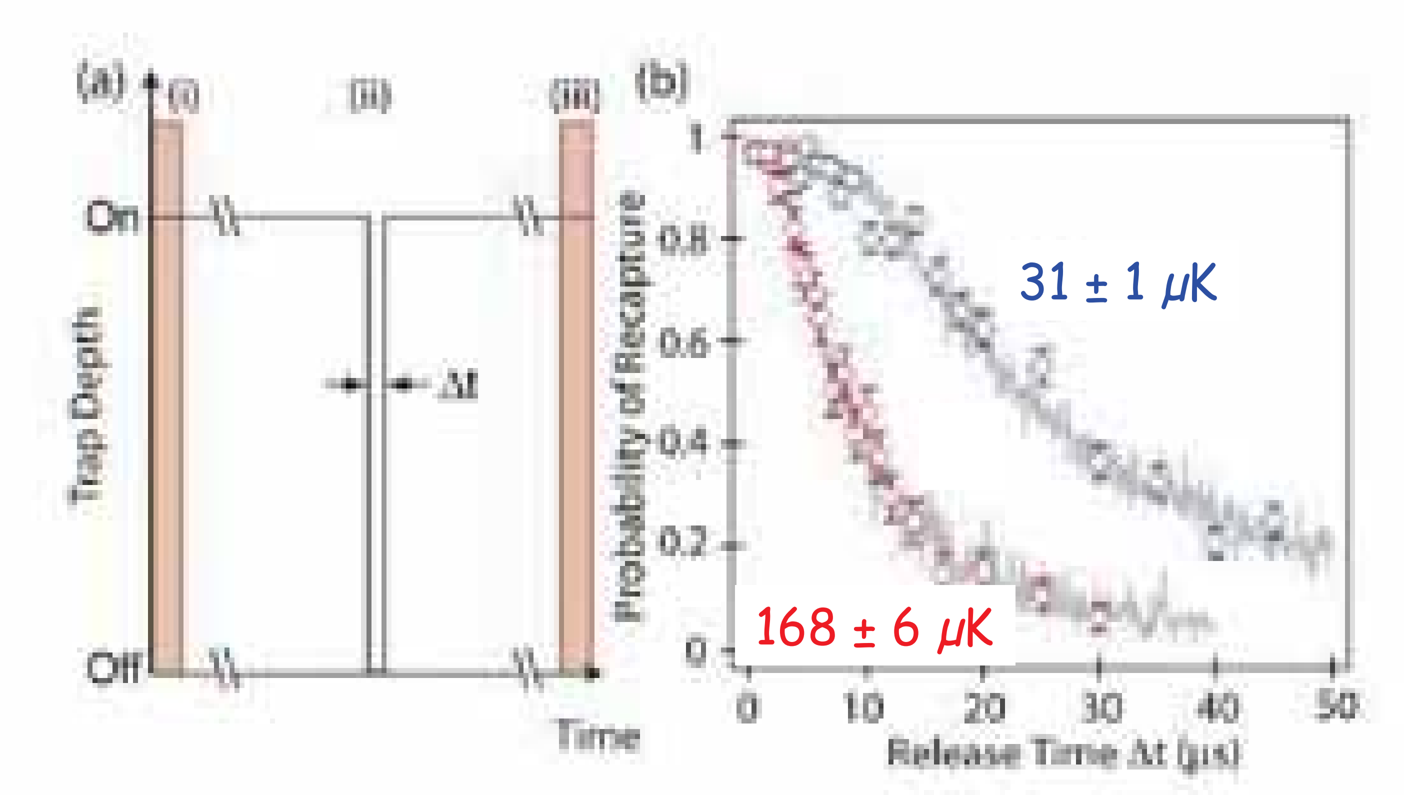


Single atom temperature measurements

C. Tuchendler et al, Phys. Rev. A 78, 033425 (2008)

* Turn off the trap for a short time Δt, then measure the probability to recapture the atom (average on 100 shots)

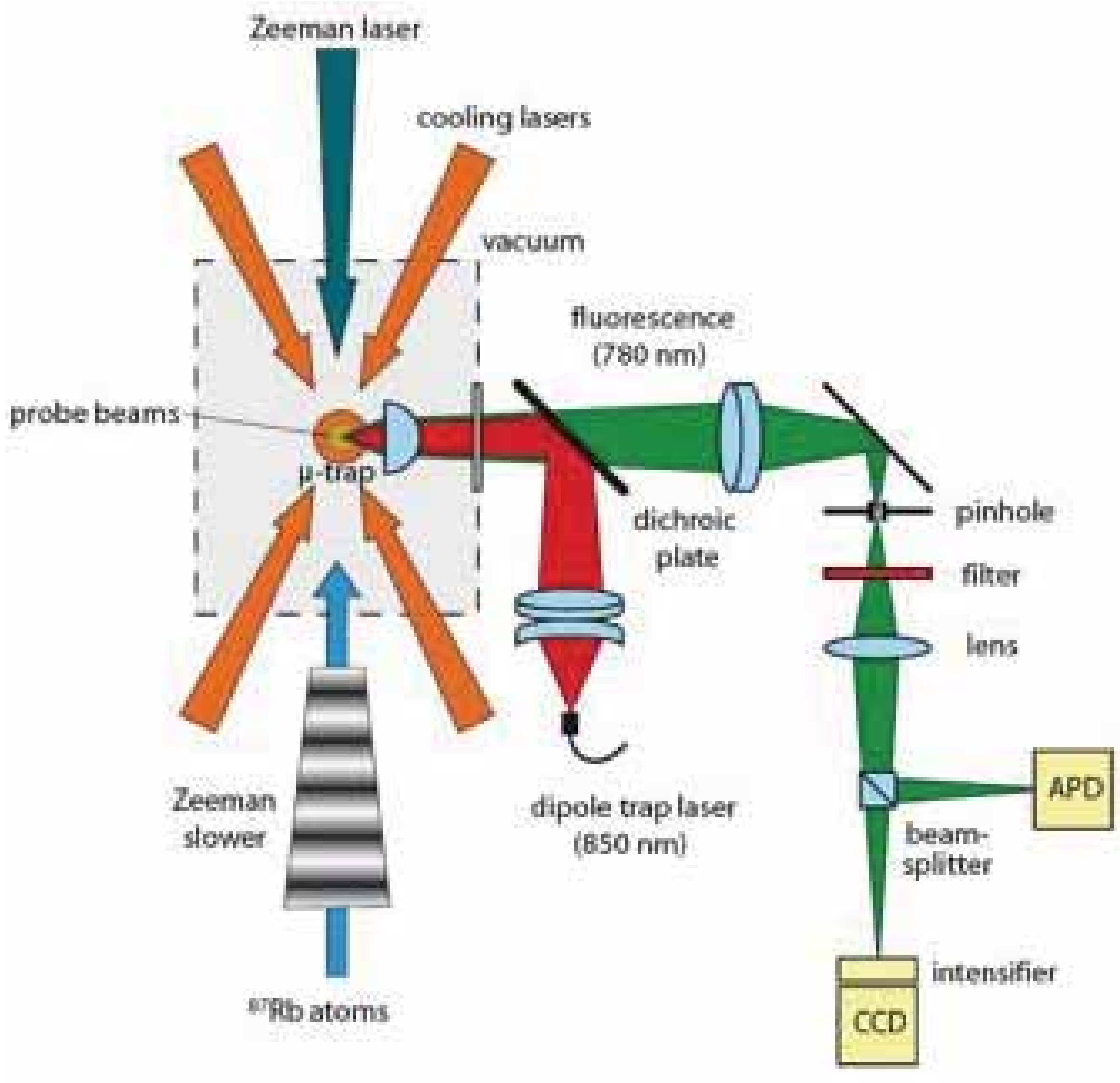
* Fit with a Monte-Carlo calculation for a given temperature T and trap depth (2.5 mK), assuming a thermal distribution.

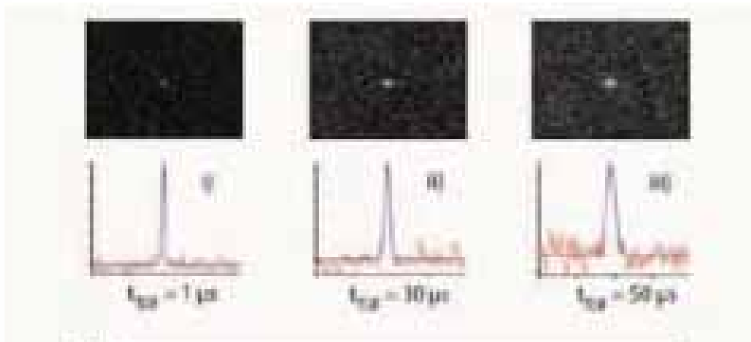


Two examples with and without molasse cooling : excellent fit !

Another method : Single Atom Time Of Flight

A. Fuhrmanek et al, New Journal of Physics 12, 053028 (2010)





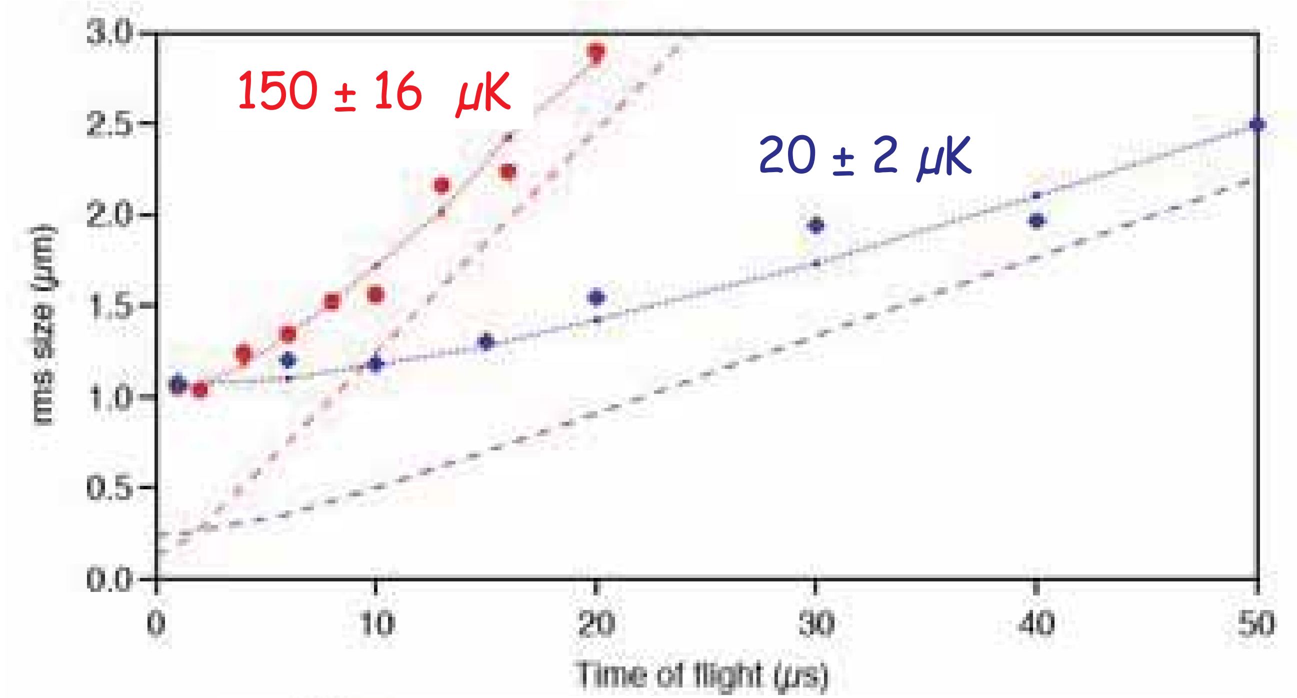
* Imaging the released atom on an intensified CCD (2 stages MCP + fluo)

* Free flight then probe pulse 2 μs, probability to detect the atom : 4.4 %

* Averaging !
1 μs TOF : 3400 shots, 150 photons
50 μs TOF : 12000 shots, 520 photons

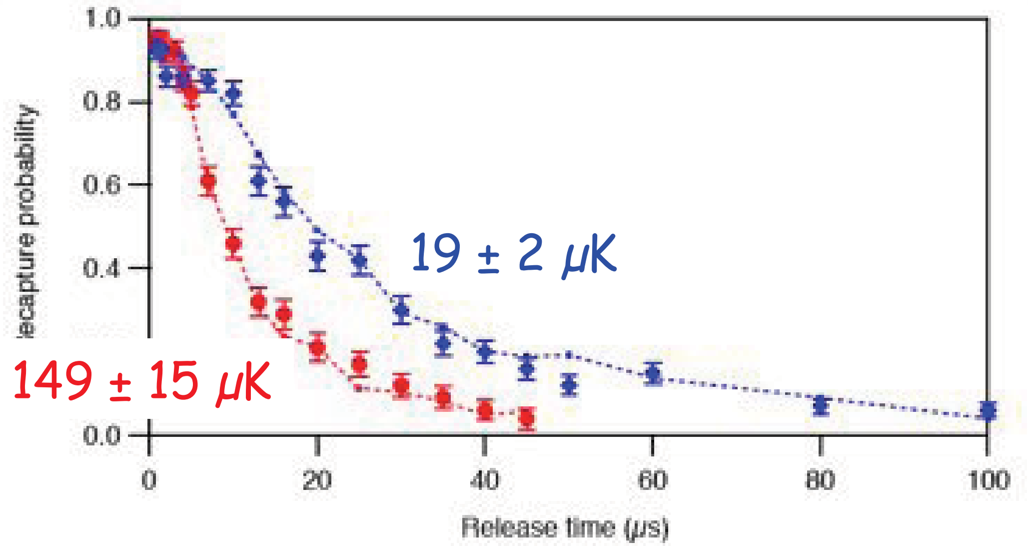
Single Atom Time Of Flight : results

A. Fuhrmanek et al, New Journal of Physics 12, 053028 (2010)



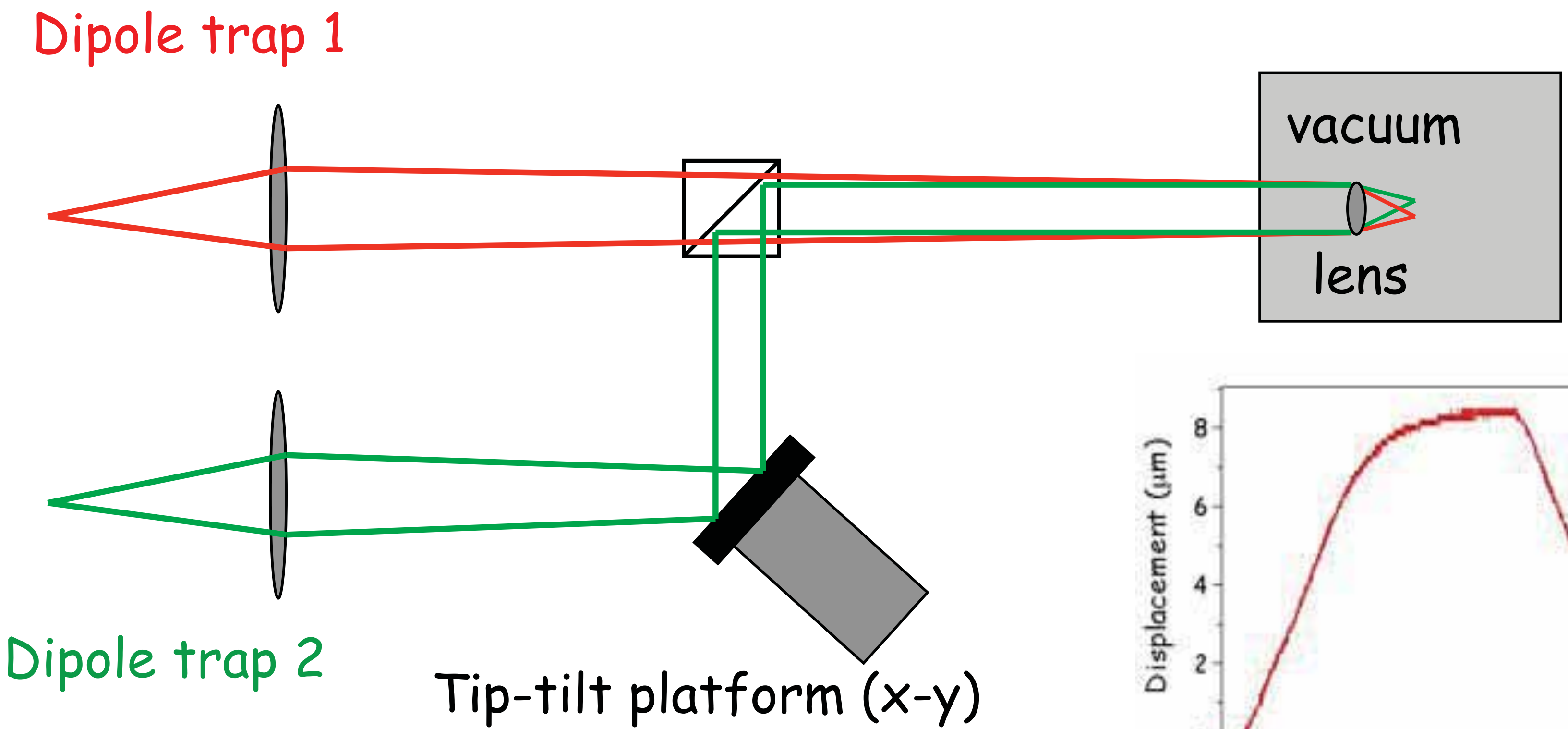
Full line : simulation including instrumental resolution (1 μm)

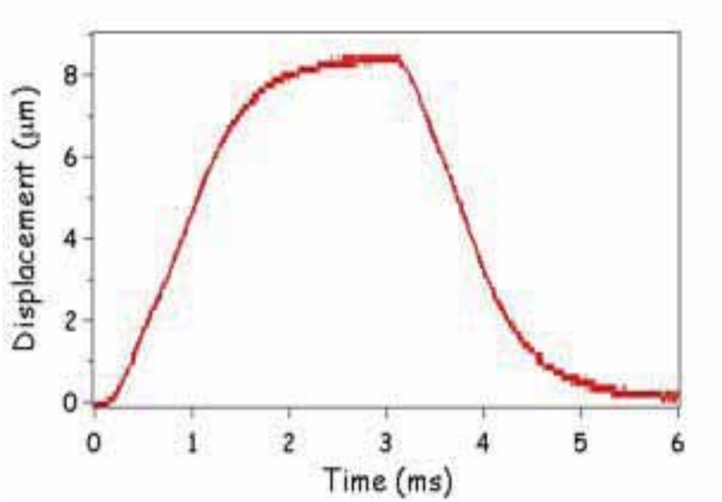
Dashed line : calculated size of the atom cloud



Comparison with the release and recapture method : excellent agreement !

Moving the atom





- Scale of the motion :
  a few µm in a few ms
- OK for a quantum register
  (coherence time : tens of ms)

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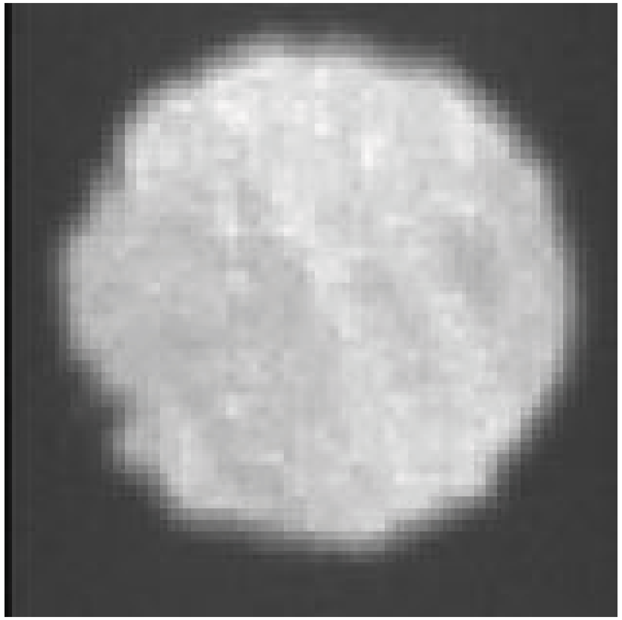
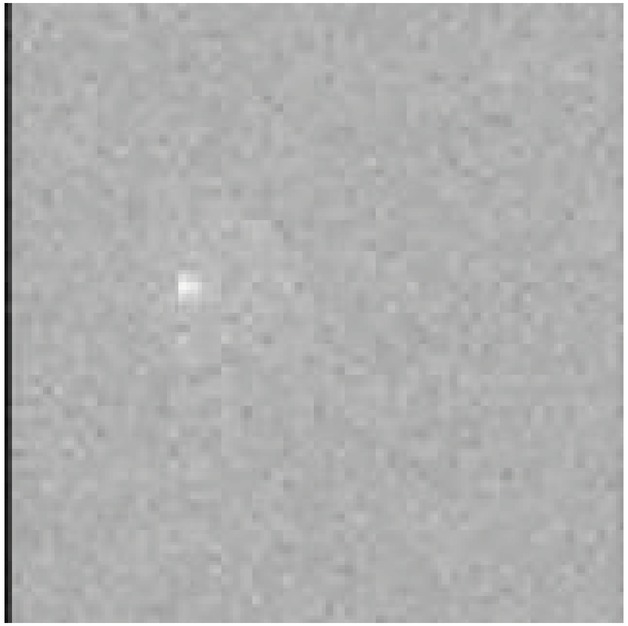
Motion of a tweezer

(background light subtracted)



IO Institut d' Optique



Motion length ≈ 40 μm

Motion radius ≈ 40 μm

Movie is 100 stills of 100 ms integration. Atoms are being laser cooled while being trapped & moved

Thank you for your attention...

(and see you soon !)