Quantum dynamics in nano Josephson junctions

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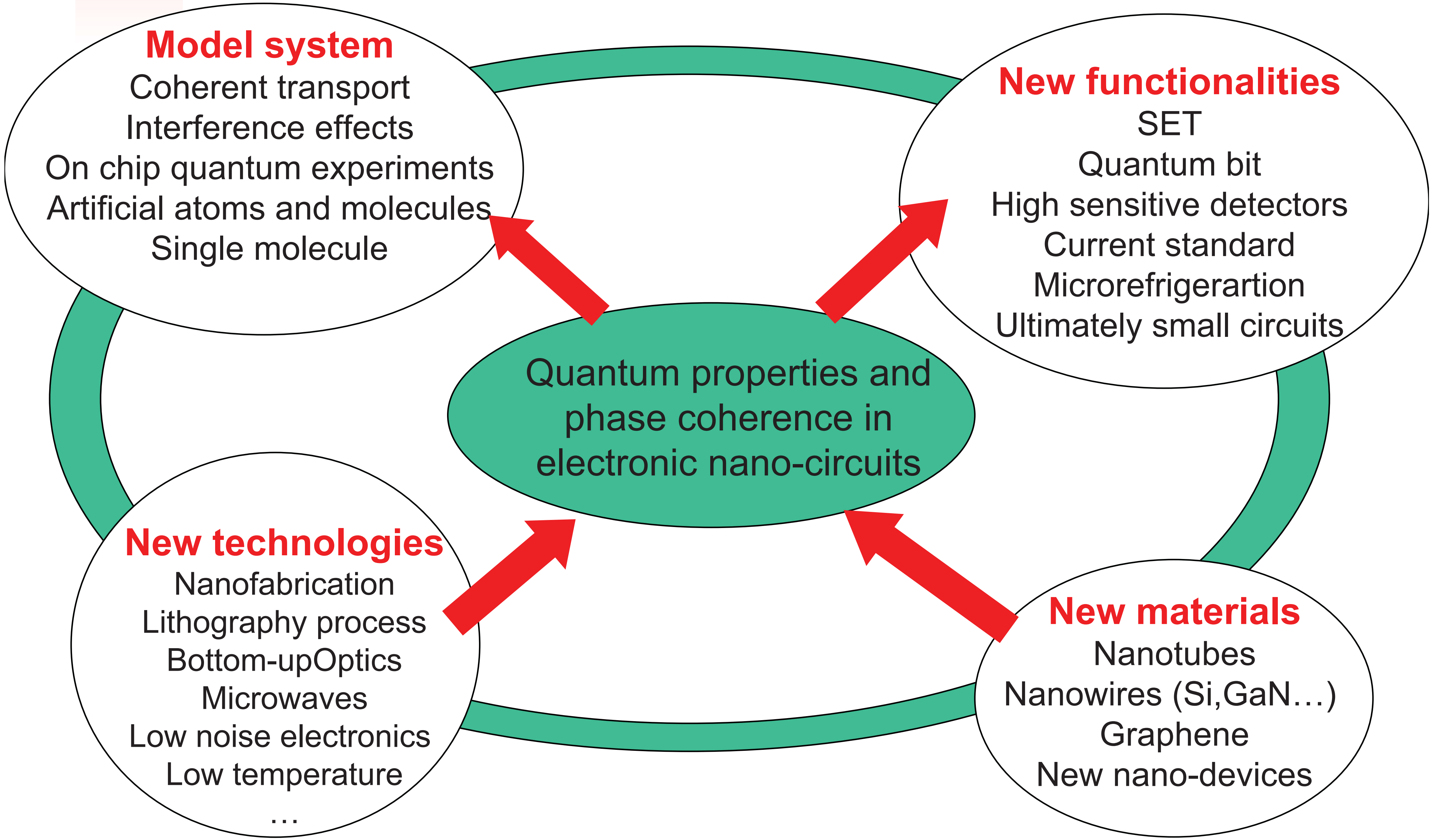
Projects: ANR QUNATJO

Quantum Nano-Electronics



Quantum experiments

Realizing quantum experiments in solid state system

Building « artificial atoms » or probing a « single atom »

- Manipulate the quantum states
- Measure the quantum states
- Long coherence time

Original properties compared to atoms:

Strong coupling to environment

(thermal noise, charge or magnetic fluctuations, microscopic defects,…)

Fast manipulations

(strong coupling with external field)

Strong and very strong coupling between qubits

Objectives: physics of these original quantum systems

Long term: - quantum information processing

Short term: - high sensitive detectors

- model system for quantum nano-electronics or nano-photonics
- experimental quantum demonstrators

Superconducting quantum circuits

Outline

Introduction to superconducting qubits

Multi-levels artificial atom

- current-biased Josephson junction and dc SQUID
- quantum measurements
- quantum dynamics in a multilevel quantum system
- quantum or classical description
- optimal control
- decoherence processes

Two-degrees of freedom artificial atom

- inductive dc SQUID
- spectroscopy measurements
- strong non-linear coupling
- coherent oscillations

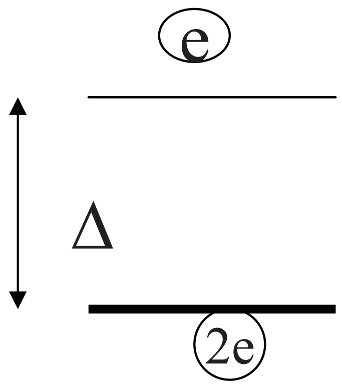
Multi-degrees of freedom system

- Josephson junction chains
- quantum phase slip
- charging effects

Introduction of the superconducting state

In many metals, T smaller than a critical temperature :

the conduction electrons condenses to electron pairs : Cooper pairs



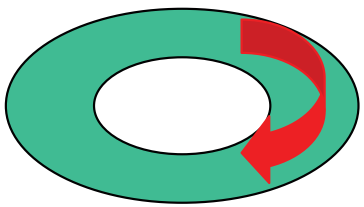
All the pairs forms a Macroscopic Quantum State: $|\Psi_G\rangle$

$|\Psi_G\rangle$ is analog to the coherent state of a laser beam

the phase is very well defined

$|\Psi_G\rangle$ is a very stable ground state

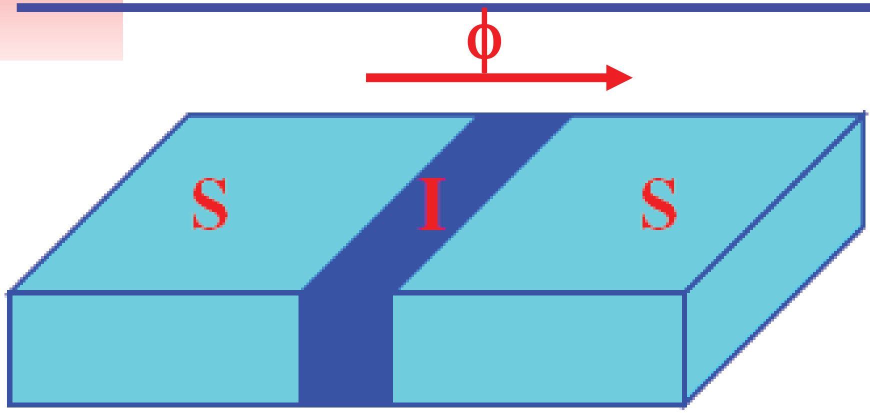
because no excitations below the gap



Persistent current without any decay!

To perform quantum experiments, we need excited levels

Basic building blocks: Josephson junction



Josephson relations:

$$I = I_c \sin\phi$$

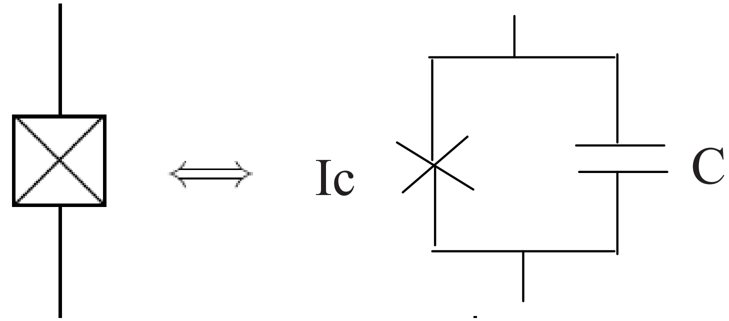
$$\dot{\phi} = 2eV/\hbar$$

Small Josephson junction: two energy scales

$$E_J = \hbar I_c/(2e)$$

$$E_C = \frac{e^2}{2C}$$

Electrical scheme



$$\hat{H} = E_C(\hat{Q}/e)^2 + E_J\cos\hat{\phi}$$

$$[Q,\phi] = -2ie$$

$$\Delta\phi\,\Delta Q \geq 2e$$

Scientific context

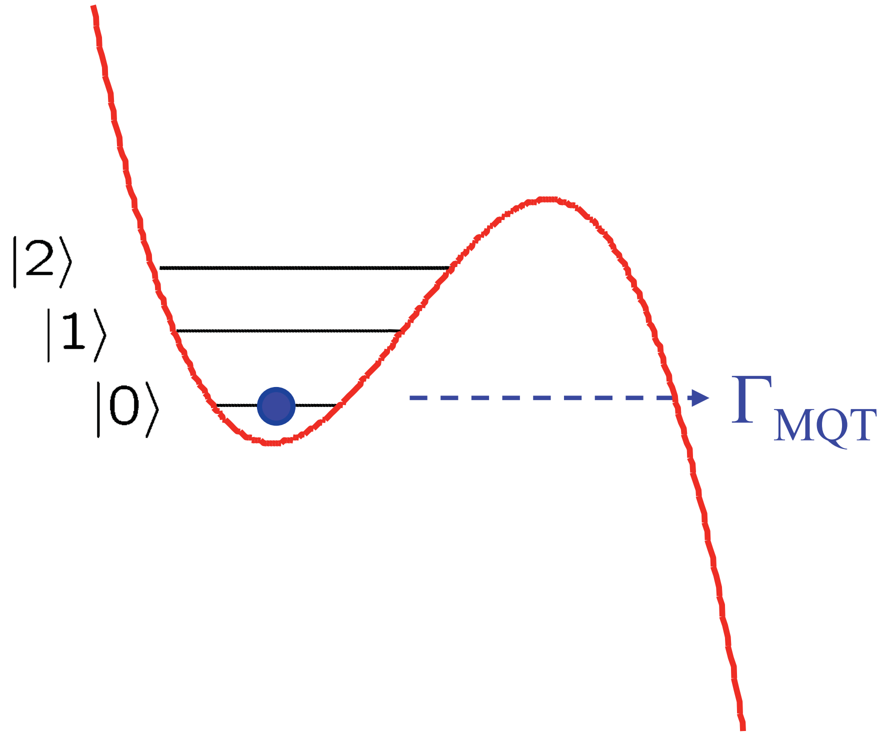
Macroscopic quantum effects…

Voss et al, PRL (1981)

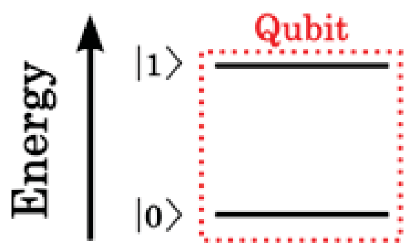
Devoret et al, PRL (1985)

… and quantized energy levels (microwave frequency regime)

Martinis et al, PRL (1985)



An artificial atom controlled by electronics signals

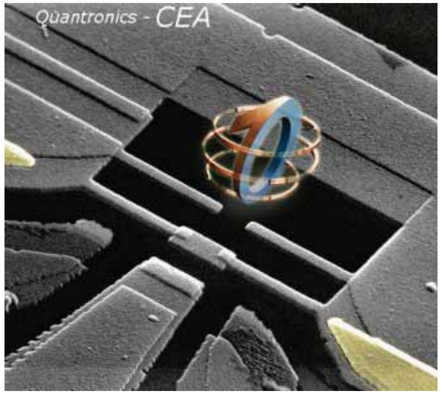


- Superconducting qubit

Nakamura et al, Nature (1999)

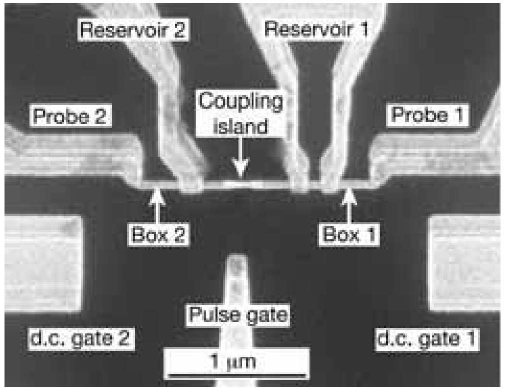
Scientific context

Engineering quantum mechanics



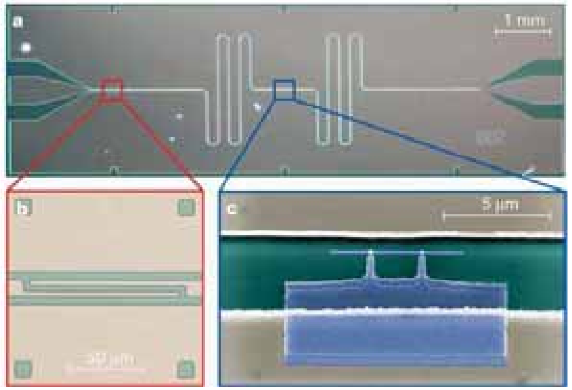
Vion et al, Science (2002)

- **Coherence**
- **Optimal point**



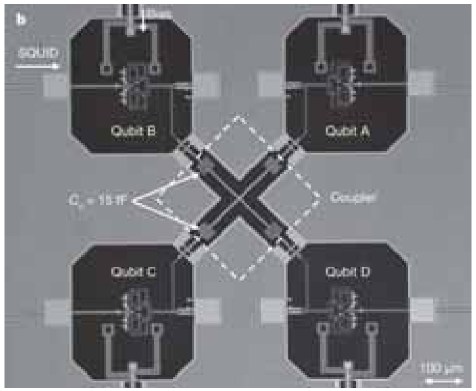
Pashkin et al, Nature (2003)

- **Coupling**
- **Gates**
- **Algorithm**

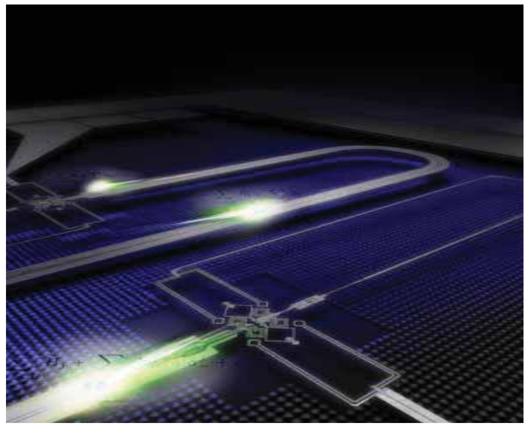


Wallraff et al, Nature (2004)

- **Readout**
- **Memory**
- **Bus**



Neelley et al, Nature (2010)
DiCarlo et al, Nature (2010)

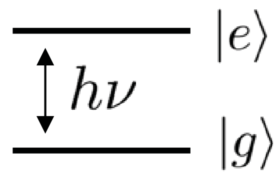


Sillanpaa et al, Nature (2007)

Motivations

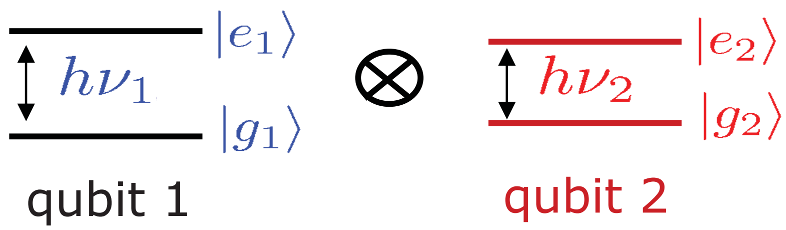
Up to now in superconducting systems:

- artificial atom with two energy level
(only one degree of freedom!)



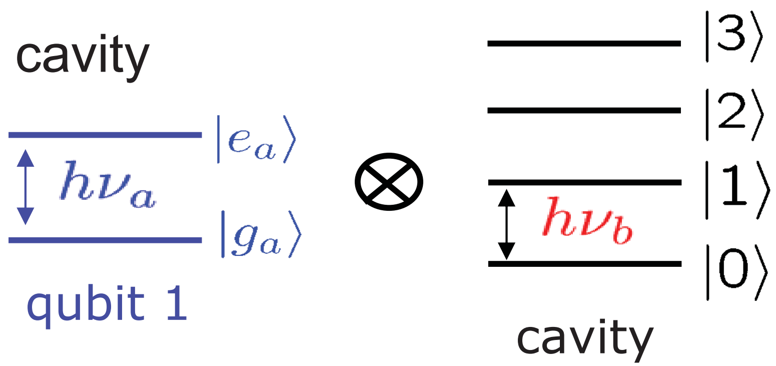
$$|\Psi\rangle = \alpha|g\rangle + \beta|e\rangle$$

- two coupled qubits



$$|\Psi\rangle = (|g_1 e_2\rangle + |e_1 g_2\rangle)/\sqrt{2}$$

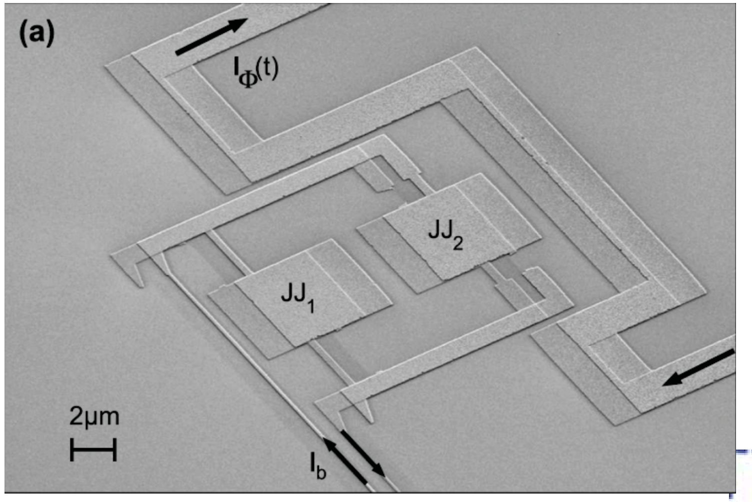
- two level system coupled to high Q cavity

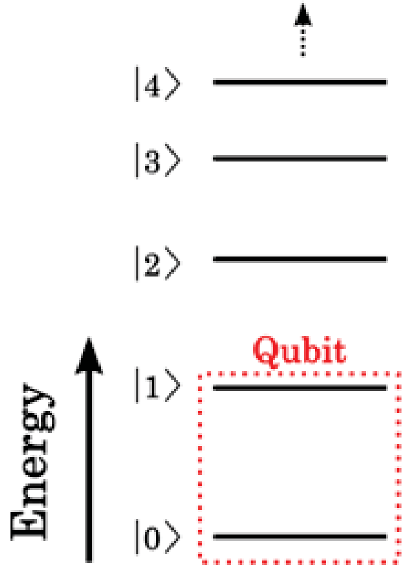


Is it possible to build and control artificial atom:
- with multi-energy levels?
- with two degree of freedom?

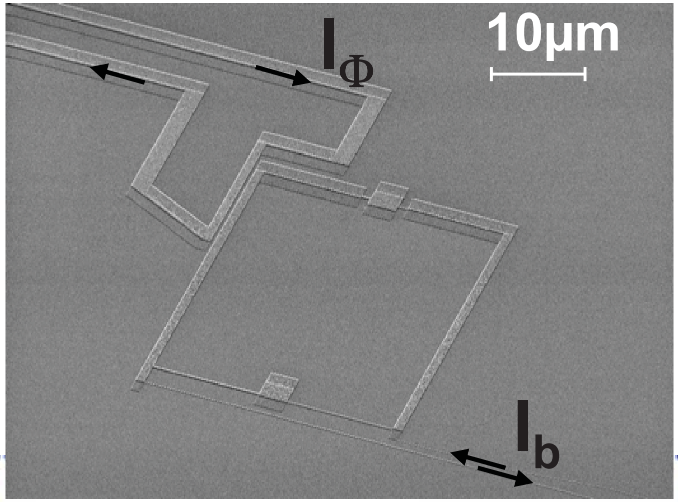
Motivations

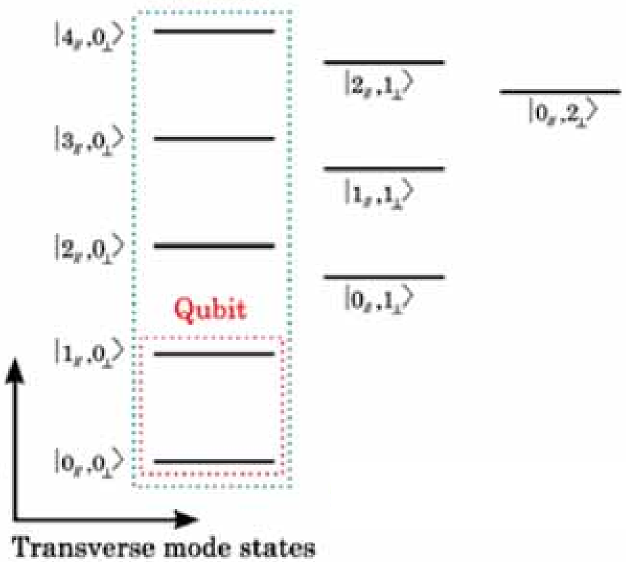
Multi-level quantum system



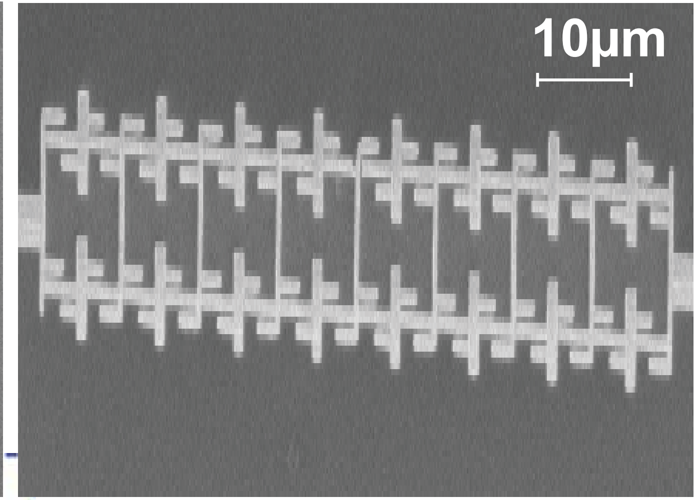


Two-degrees of freedom





Multi-degrees of freedom



New physics…

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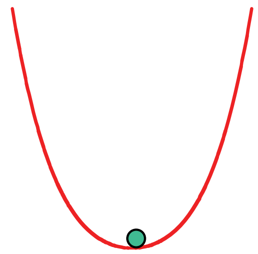
Driven anharmonic oscillator

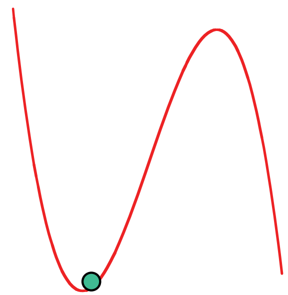
Harmonic oscillator: $$H(t)=\frac{\hat{P}^2}{2m}+\frac{1}{2}m\omega_p^2\hat{X}^2+f_{ext}\cos(2\pi\nu t)\hat{X}$$



The quantum particle follows a motion very close to the classical one

By adding anharmonic terms $-a\hat{X}^3-b\hat{X}^4$

New physics appear which was extensively studied

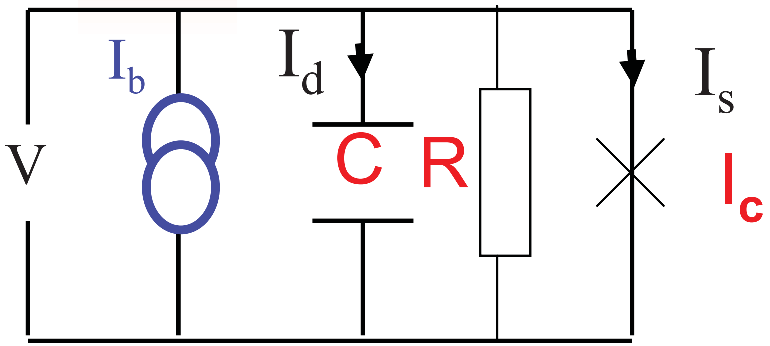


Classical mechanics:
- Landau&Lifchitz
- modification of the reonance peak
- bi stability (used as amplifier Siddiqi 04, Ithier 05)
- parametric amplifiers

Quantum mechanics: many theoretical studies (Dykman88, Milburn86, Enzer97, Katz07, etc..)

Quantum dynamics in an anharmonic quantum oscillator

Current-biased Josephson junction



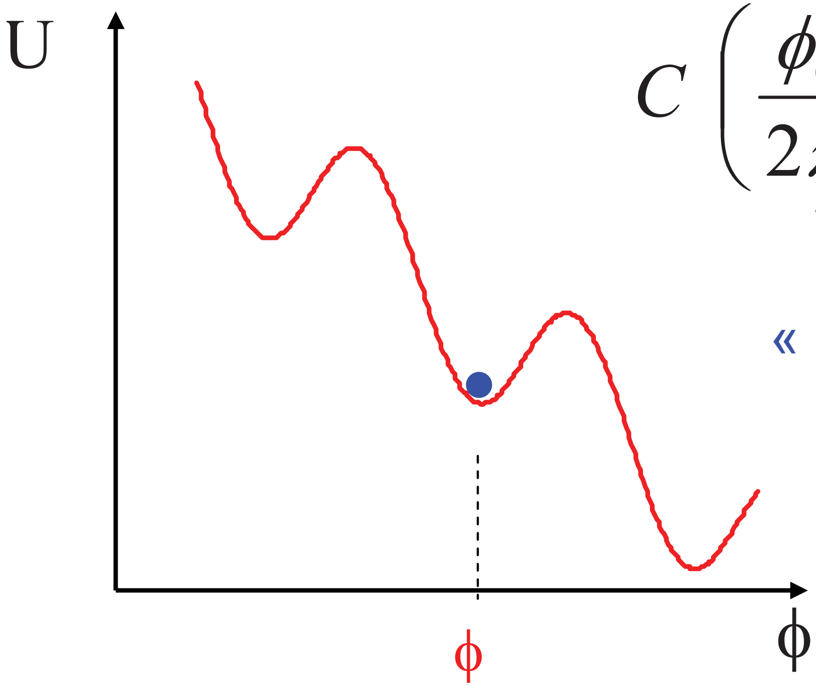
Current conservation:

$$C\frac{dV}{dt}+\frac{V}{R}=I_b-I_c\sin\phi$$

Josephson relations:

$$I=I_c\sin\phi$$

$$\dot{\phi}=2eV/\hbar$$



$$C\left(\frac{\phi_0}{2\pi}\right)\frac{d^2\phi}{dt^2}+\left(\frac{\phi_0}{2\pi R}\right)\frac{d\phi}{dt}=\frac{\phi_0 I_c}{2\pi}\left(\frac{I_p}{I_c}-\sin\phi\right)$$

« mass » m

$$-\frac{dU(\phi)}{d\phi}$$

$$U(\phi)\propto -I_b\phi-I_C\cos(\phi)$$

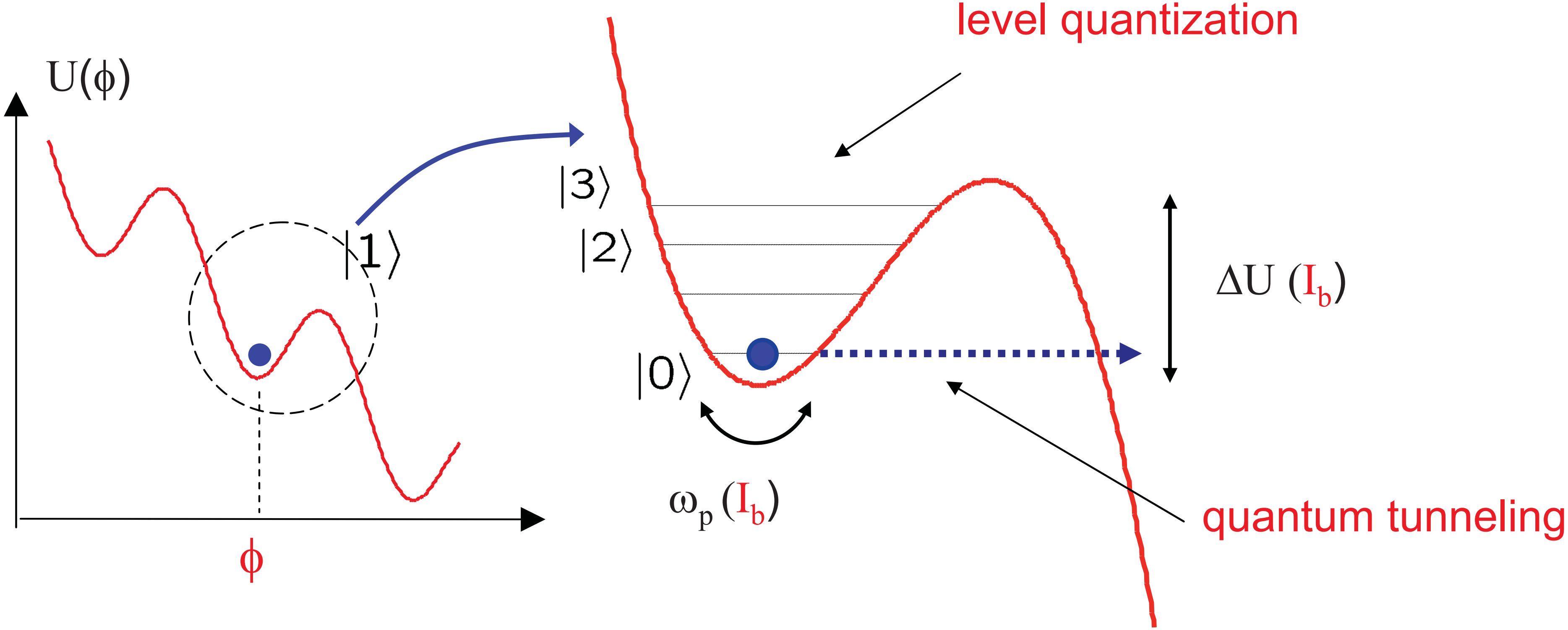
In the quantum limit…

$$\hat{H} = E_C(\hat{Q}/e)^2 + E_J(\cos\hat{\phi} - (I_b/I_c)\hat{\phi}) \qquad [Q,\phi] = -2ie$$

Charging energy

Josephson potential

$E_J >> E_C$

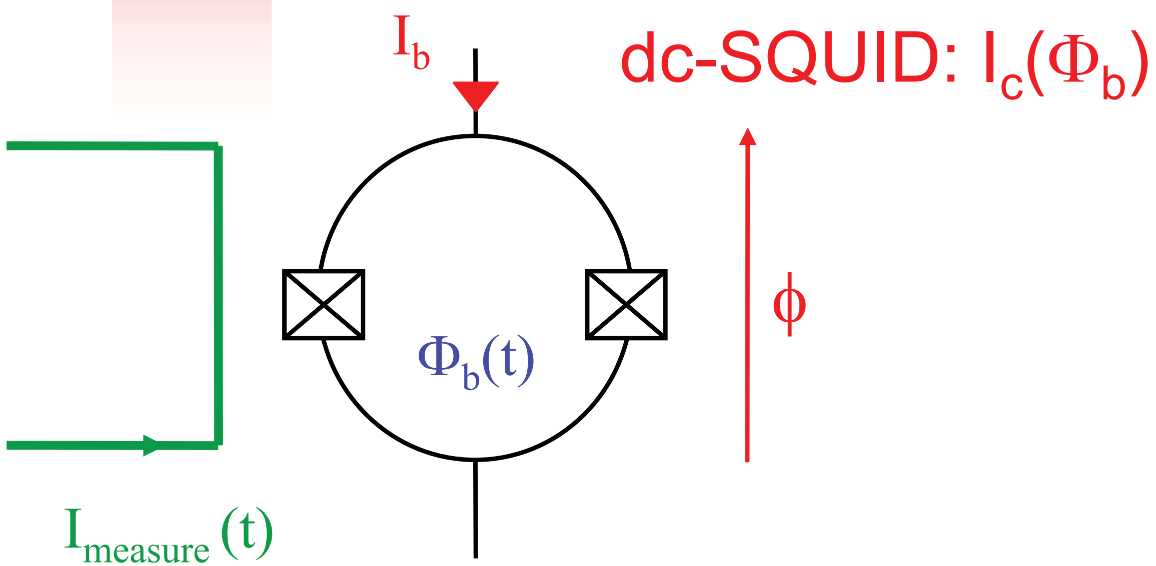


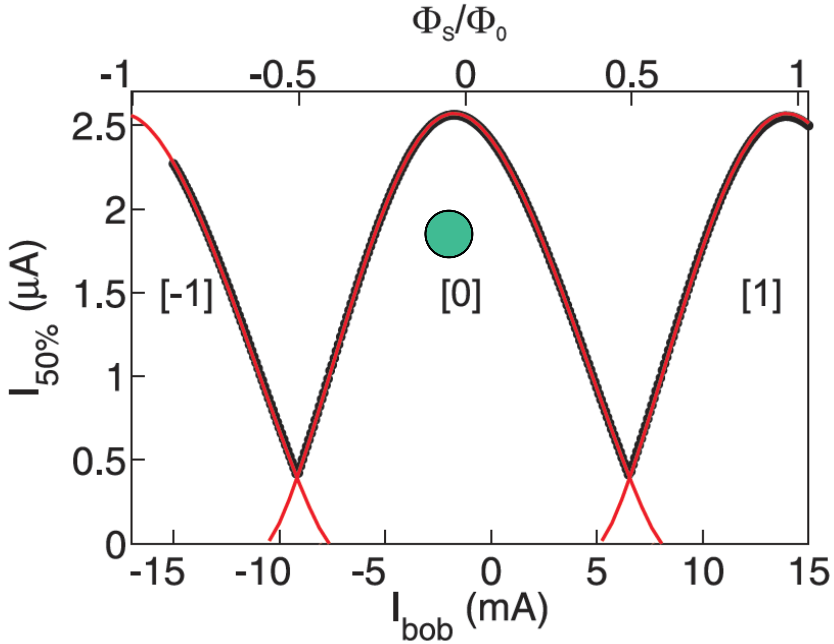
$$\frac{1}{2}\hbar\omega_p\left[\tilde{P}^2 + \tilde{X}^2\right] - \hbar\sigma\omega_p\tilde{X}^3$$

Quantum anharmonic oscillator!

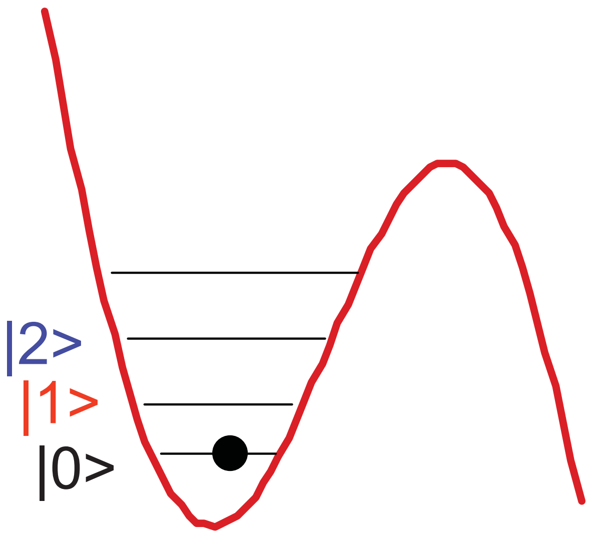
Quantum experiments with a dc-SQUID

(J. Claudon et al PRB07)



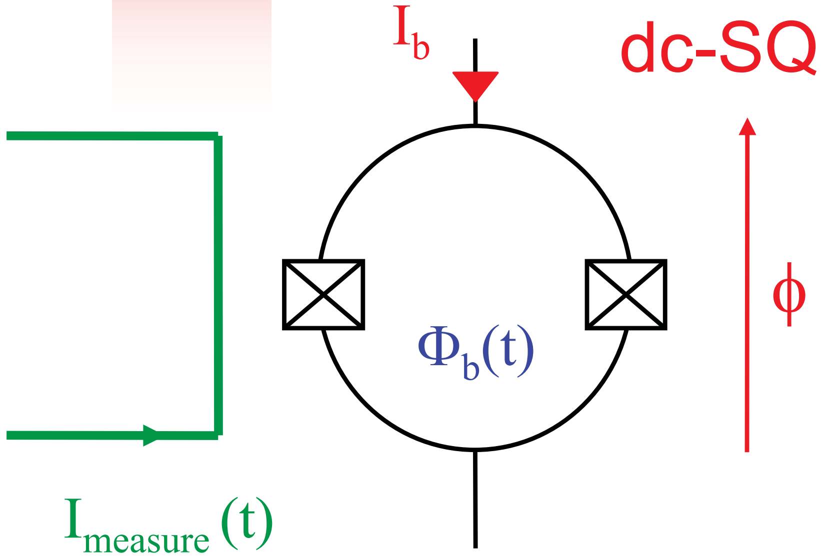


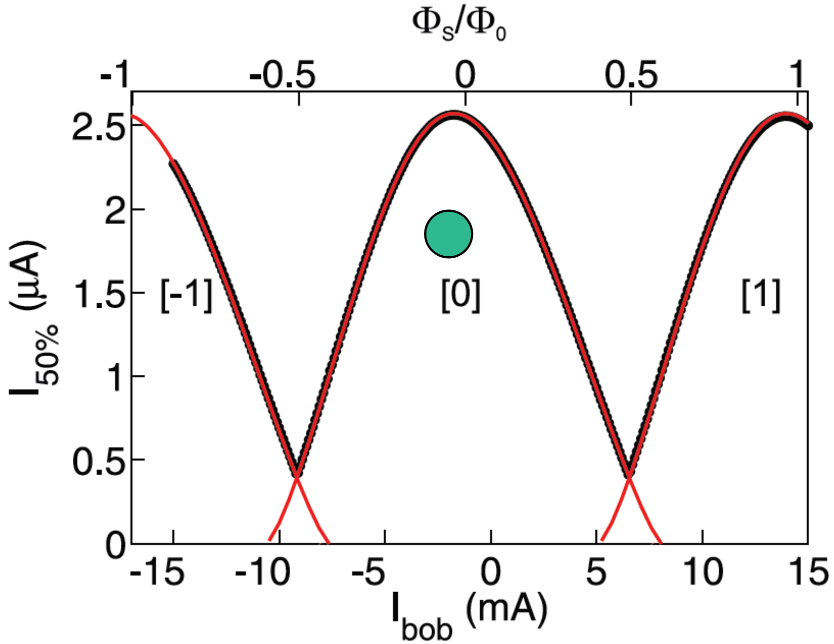
Deep well with quantized states



Quantum state manipulation

(J. Claudon et al PRB07)





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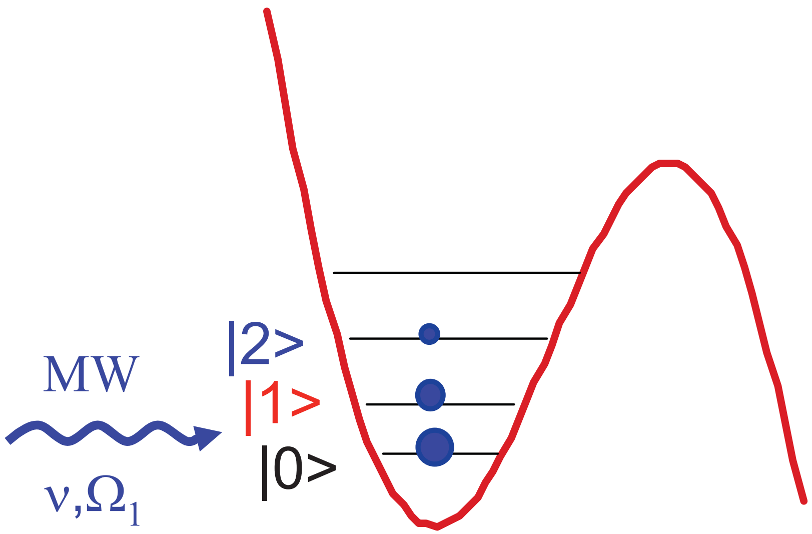


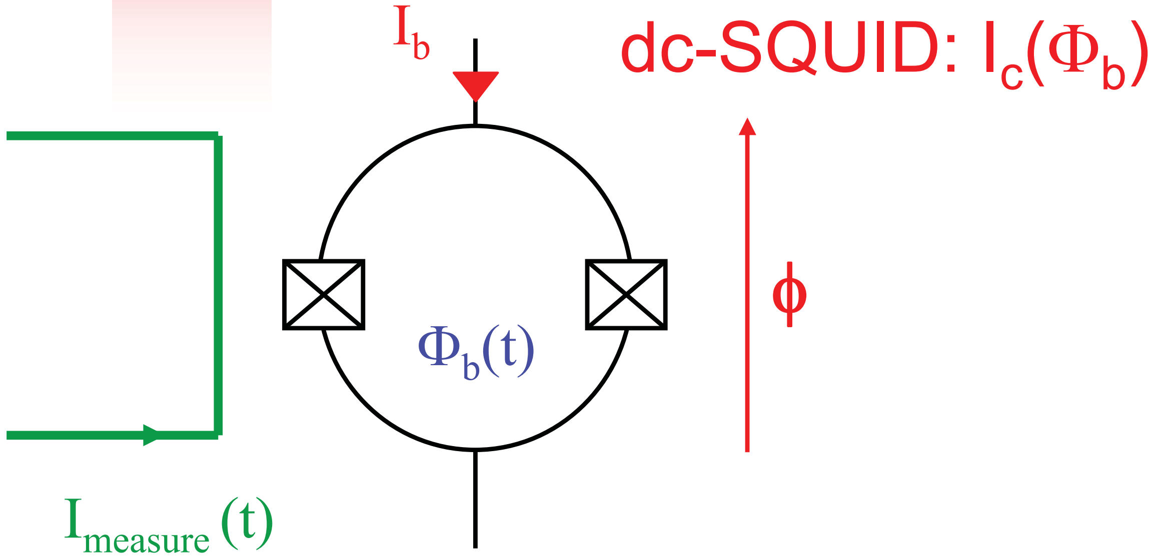
excitation ⟷ Microwave flux: $\Phi(t)$

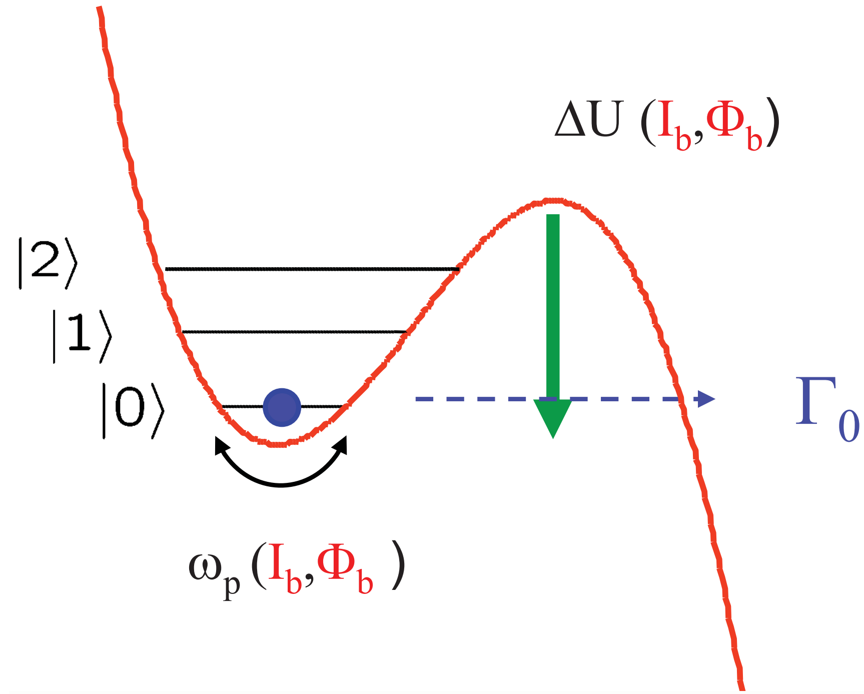
$$-\hbar\Omega_1 \cos(2\pi\nu t)\sqrt{2}\widetilde{X}$$

An external driving force!

Quantum measurements with a dc-SQUID

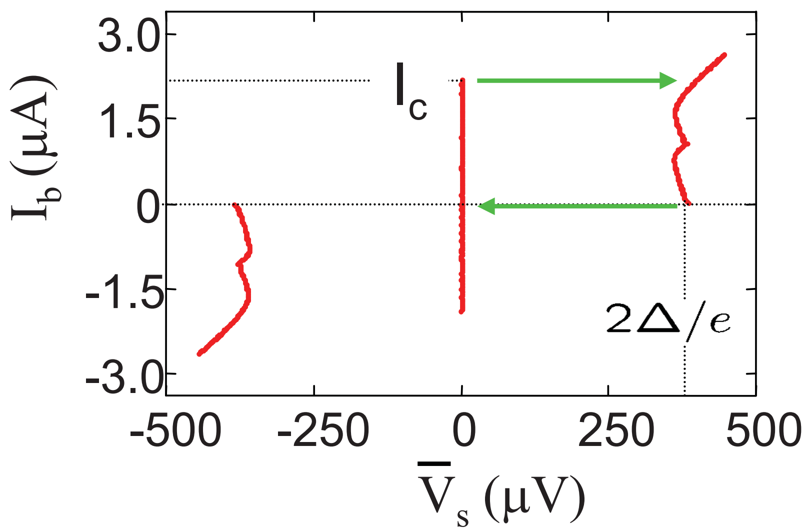
J. Claudon, et al, PRL04, PRB2007

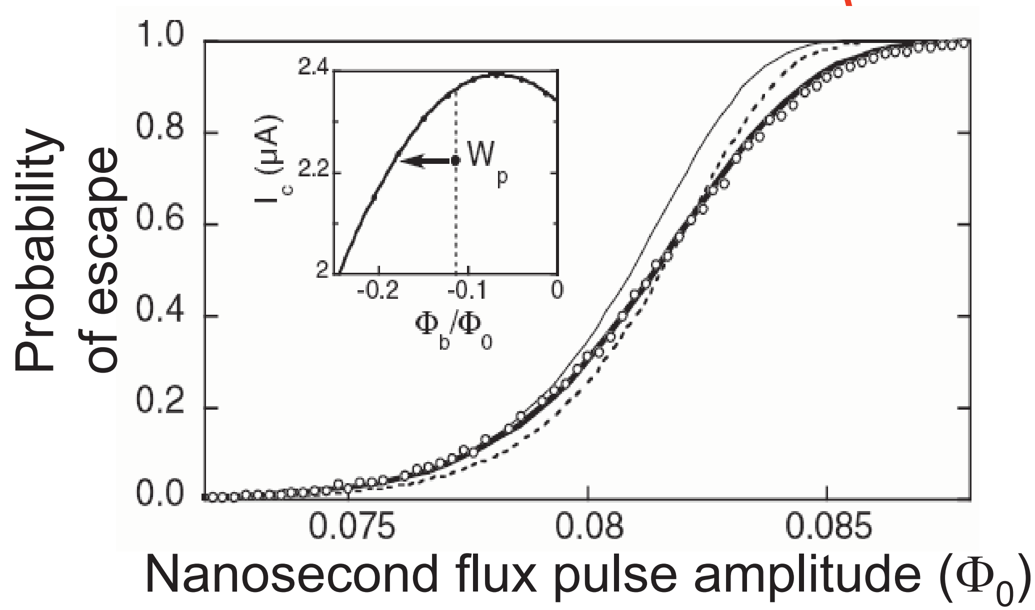




A nano-second flux pulse reduces the barrier

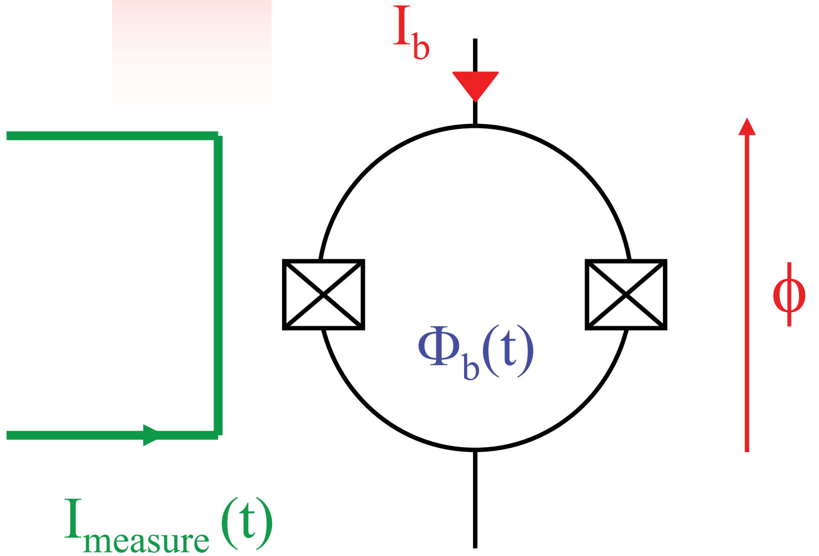
Hysteretic junction: escape leads to voltage

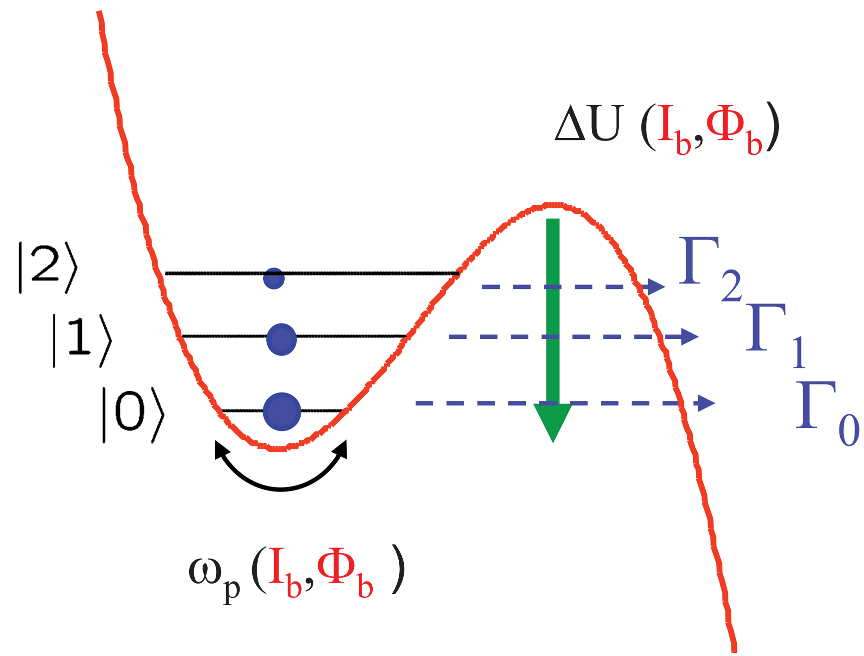




Quantum measurements

J. Claudon, et al, PRL04, PRB2007





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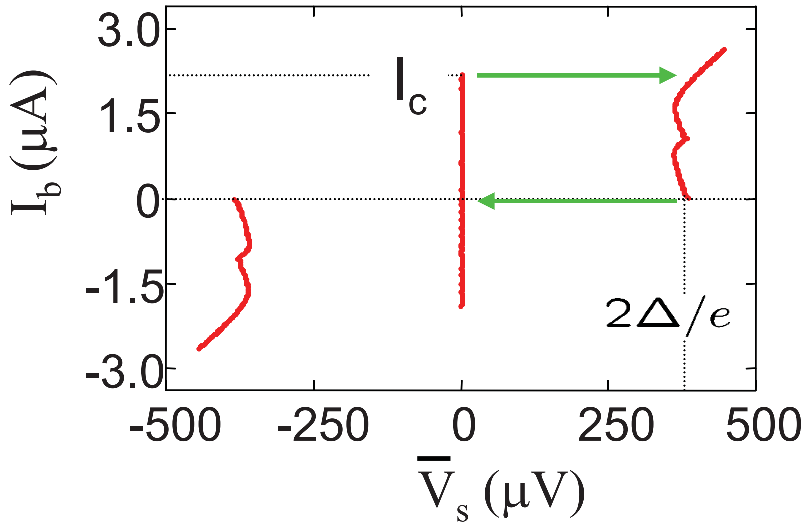
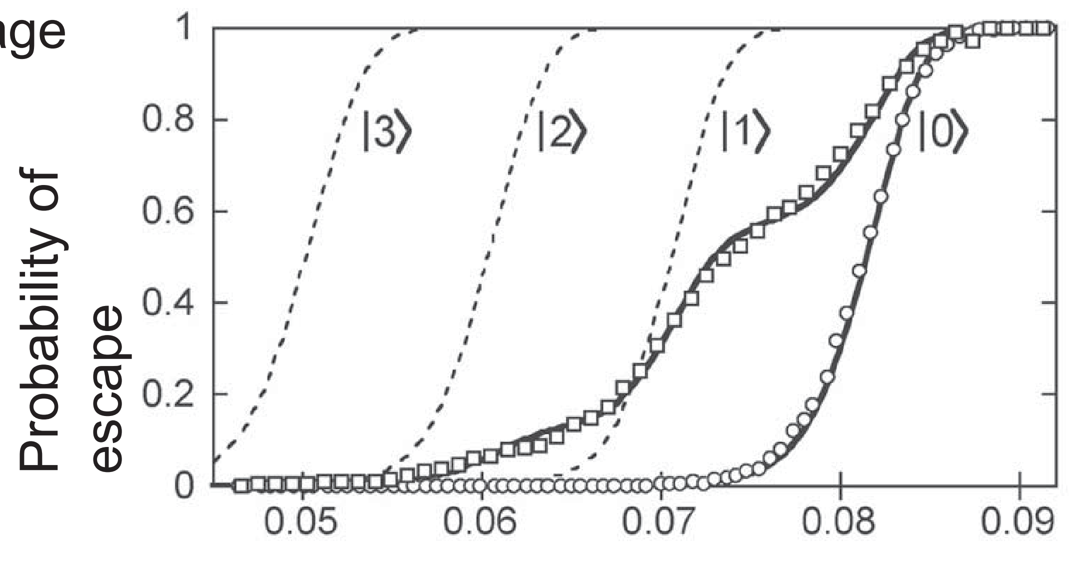
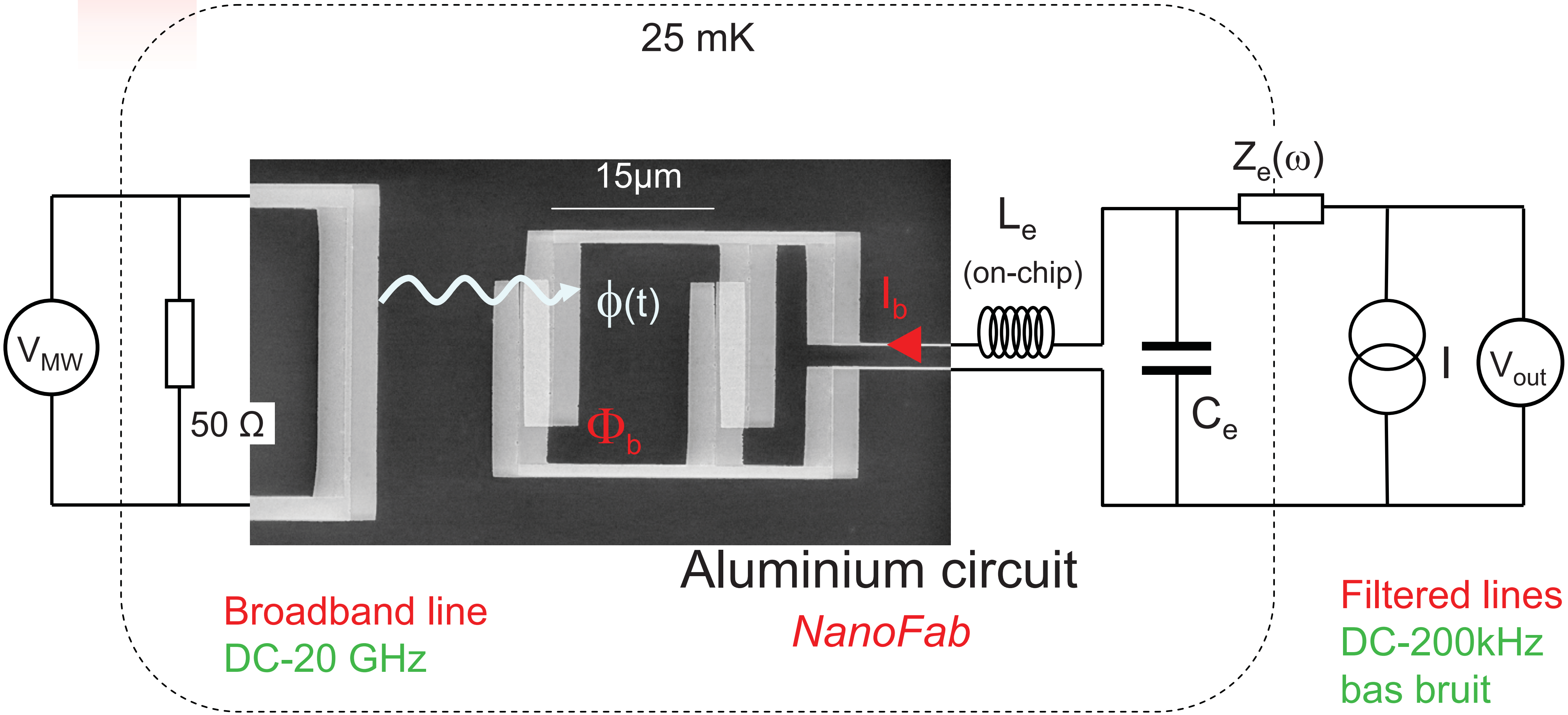




Experimental set-up



- MW manipulation
- fast measurements

- courant bias
- voltage state of SQUID

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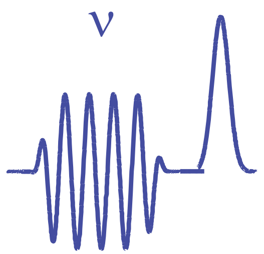
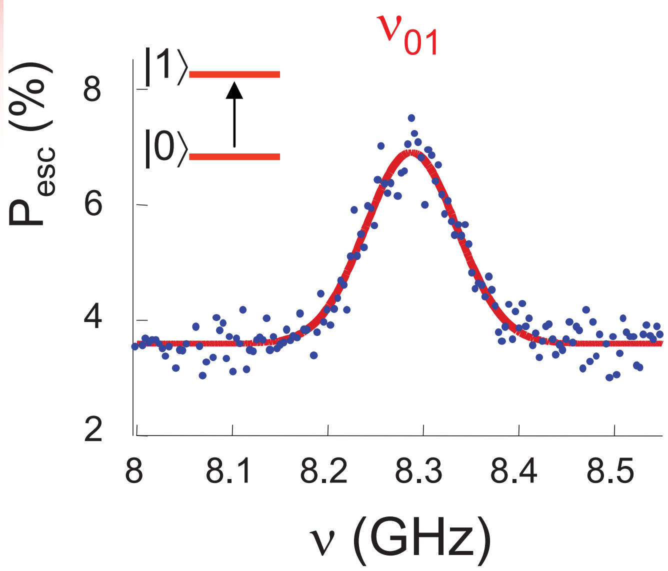
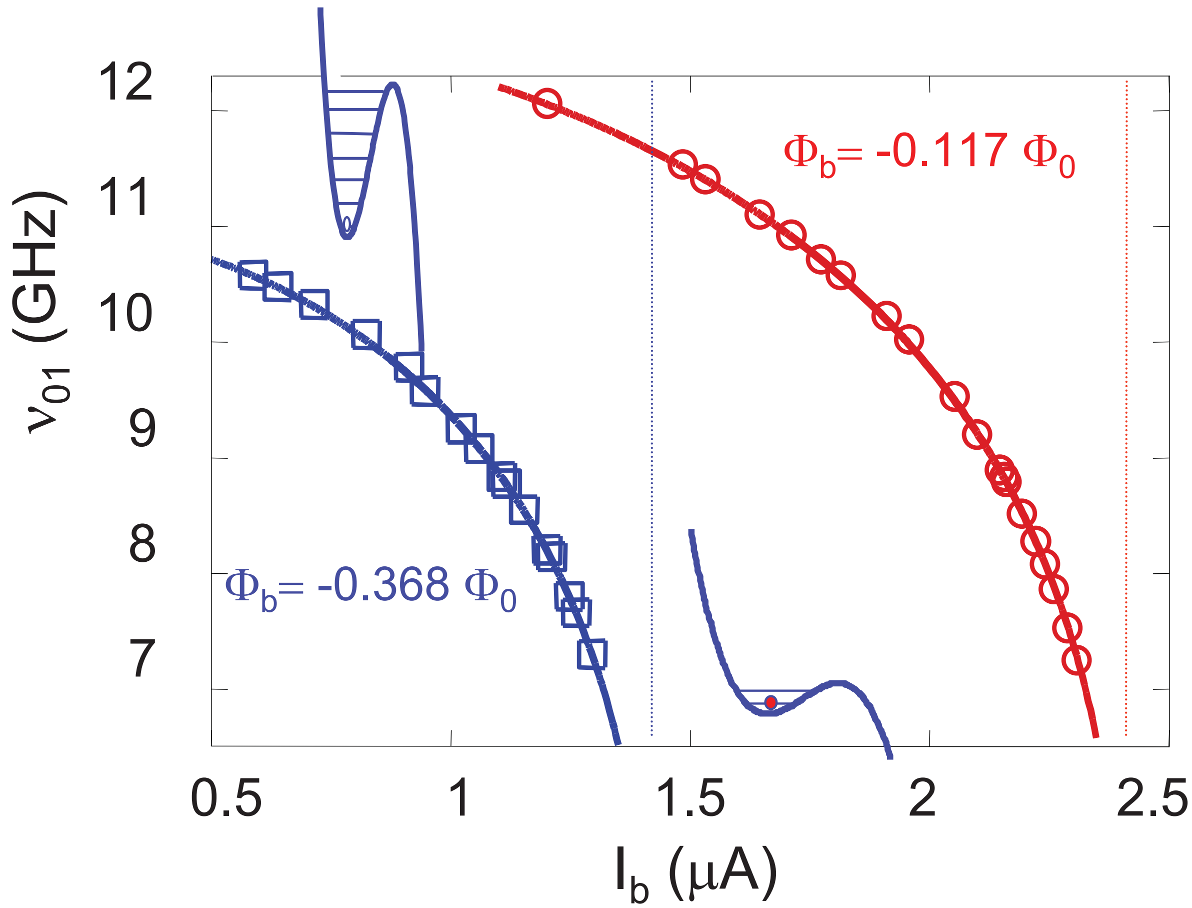
Spectroscopy measurements

J. Claudon, A. Fay, L.P. Lévy, and O. Buisson (PRB2008)





I_b = 2.222 μA, Φ_b = -0.117 Φ_0



SQUID parameters

I_0 =1.242 μA

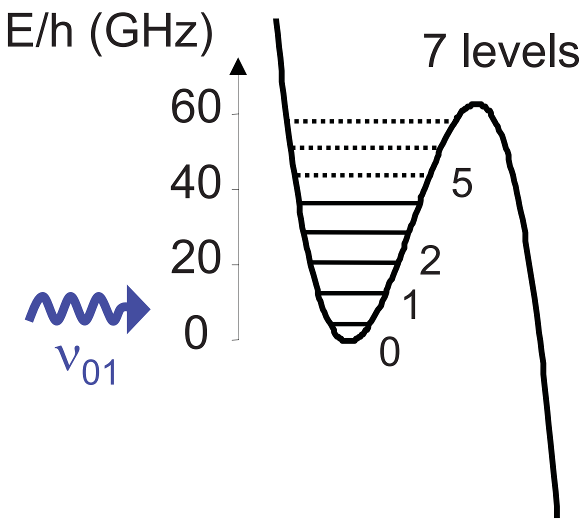
C_0= 560 fF

L_S= 280 pH

Coherent oscillations in a dc SQUID

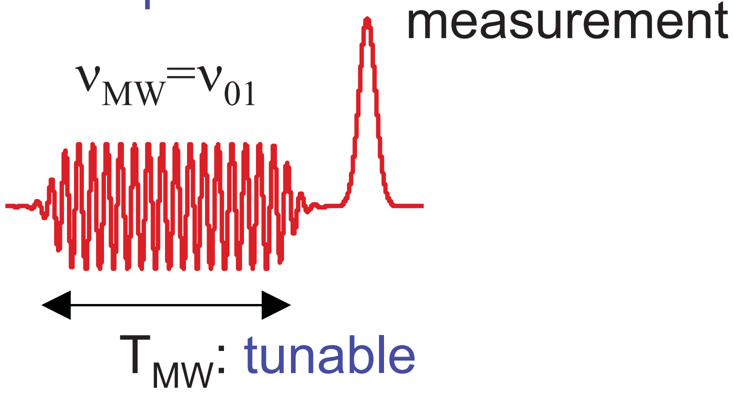
(J. Claudon F. Balestro, F. Hekking, and O. Buisson, PRL 2004)

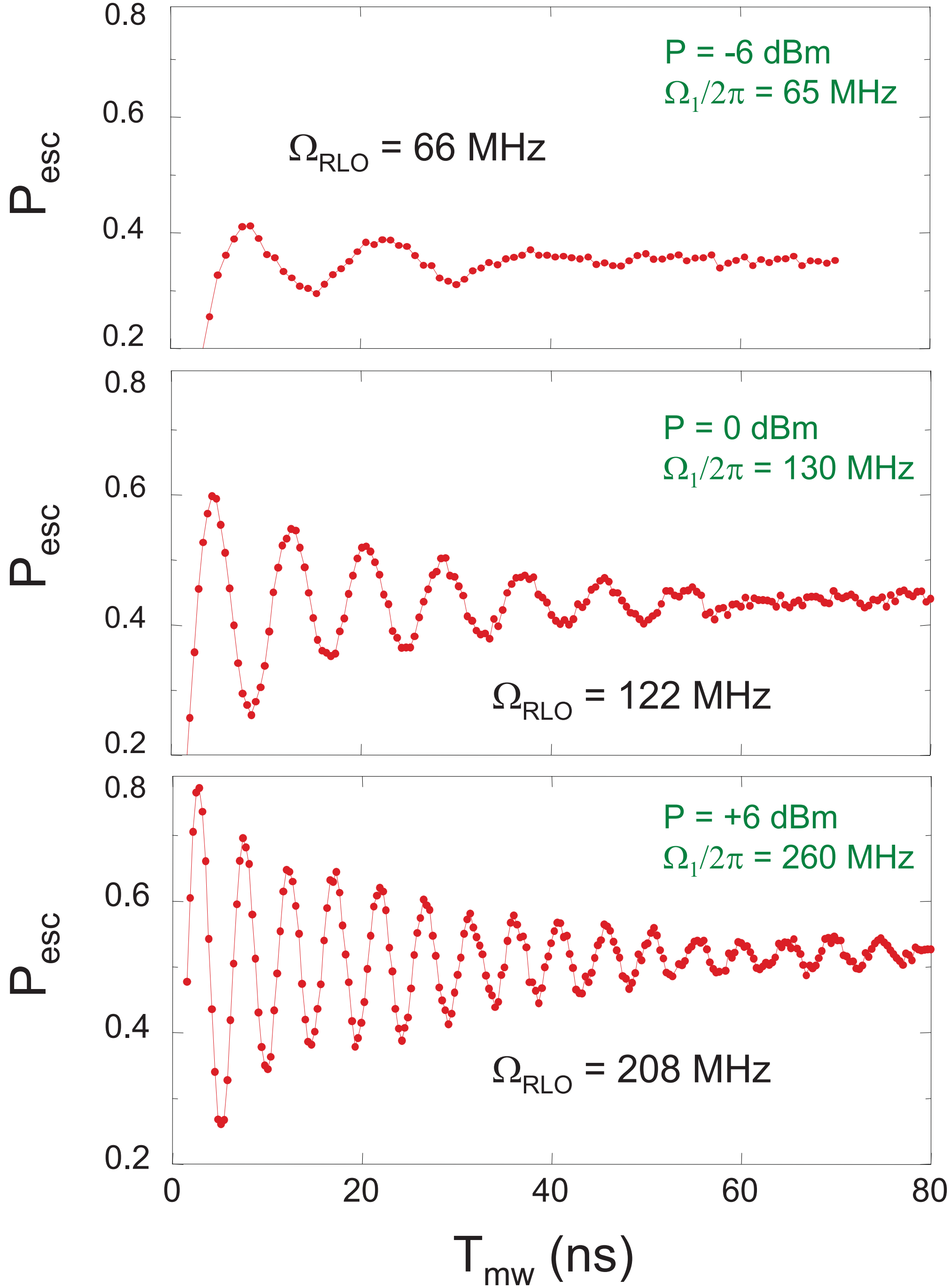
- Anharmonic oscillator:



Anharmonicity: $\nu_{01}-\nu_{12}$ = 160 MHz

- Flux-pulse sequence:

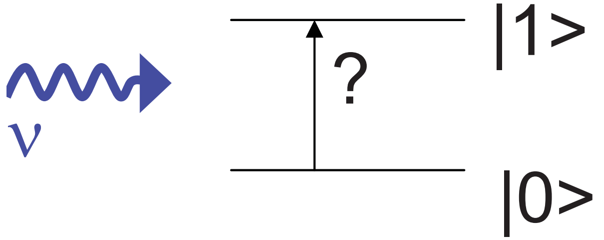




Are Rabi oscillations?

Rabi oscillations (I)

Two level system:



$$\hat{H}(t) = \frac{1}{2} h\nu_{01}\sigma_Z - \sqrt{2}\hbar\Omega_1 \cos(2\pi\nu t)\sigma_X$$

$\left|\varphi(t)\right\rangle$: temporal evolution ?

« Rotating wave » approximation

$$i\hbar\frac{d}{dt}\left|\varphi(t)\right\rangle = \hat{H}(t)\left|\varphi(t)\right\rangle \qquad \xrightarrow{\text{Rotating referential}} \qquad i\hbar\frac{d}{dt}\left|\varphi^*(t)\right\rangle = \hat{H}^*\left|\varphi^*(t)\right\rangle$$

$$\left|\varphi(0)\right\rangle = \left|0\right\rangle \qquad\qquad \left|\varphi^*(0)\right\rangle = \left|0\right\rangle$$

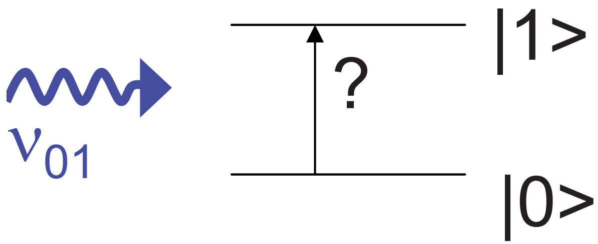
H^* : Time independent Hamiltonian

$$\hat{H}^* = \begin{pmatrix} 0 & \frac{\Omega_1}{2} \\ \frac{\Omega_1}{2} & 0 \end{pmatrix}$$

eigenvector	eigenvalues
$\left\|e_0^*\right\rangle = \frac{1}{\sqrt{2}}(\left\|0\right\rangle + \left\|1\right\rangle)$	$\lambda_0 = -\Omega_1/2$
$\left\|e_1^*\right\rangle = \frac{1}{\sqrt{2}}(\left\|0\right\rangle - \left\|1\right\rangle)$	$\lambda_1 = \Omega_1/2$

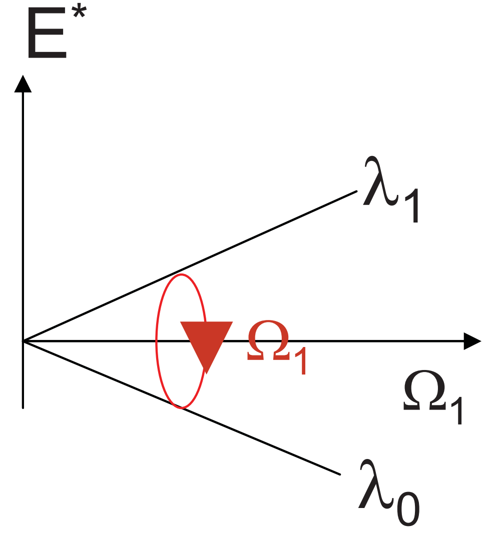
Rabi oscillations (II)

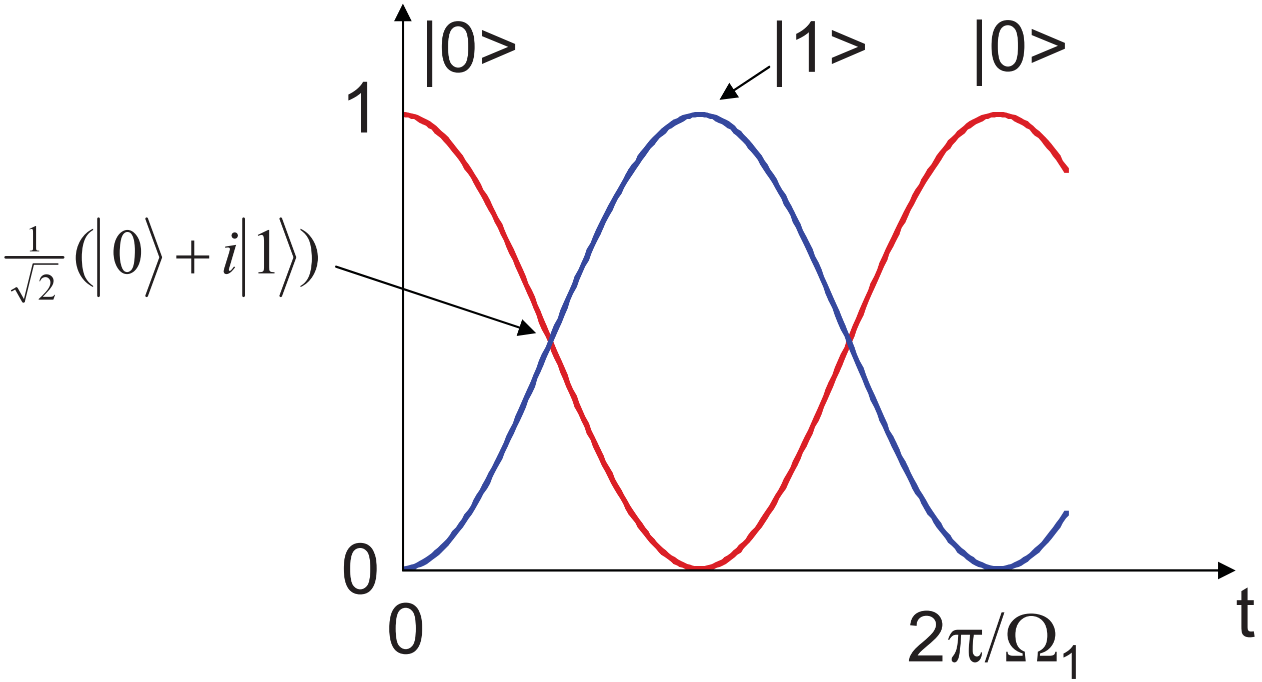
Two level system:



$$\left|\varphi^*(0)\right\rangle = \tfrac{1}{\sqrt{2}}\left(\left|e_0^*\right\rangle + \left|e_1^*\right\rangle\right)$$

$$\left|\varphi^*(t)\right\rangle = \tfrac{1}{\sqrt{2}}\left(\left|e_0^*\right\rangle + \exp(-i\Omega_1 t)\left|e_1^*\right\rangle\right)$$



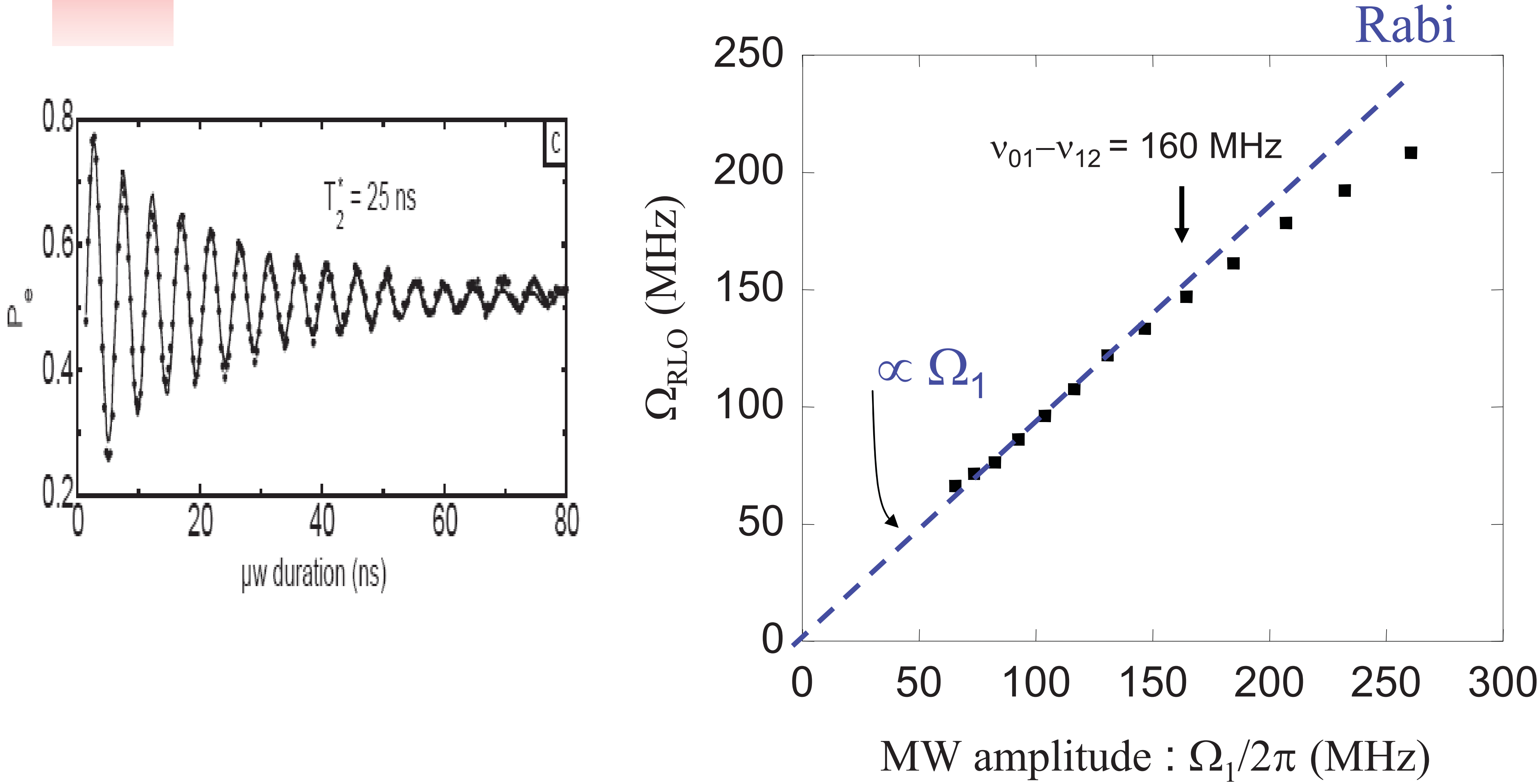


$$p_0(t) = \tfrac{1}{2}\left(1 + \cos(\Omega_1 t)\right)$$

$$p_1(t) = \tfrac{1}{2}\left(1 - \cos(\Omega_1 t)\right)$$

The duration of the microwave controls the state

Rabi oscillations of a two level system



Strong deviation compare to Rabi prediction!

→ We must take into account the multi-level dynamics

Multilevel dynamics in an harmonic oscillator

$$\hat{H}(t)=\frac{1}{2}\hbar\omega_p(\hat{P}^2+\hat{X}^2)-\cancel{\sigma\hbar\omega_P\hat{X}^3}-\sqrt{2}\hbar\Omega_1\cos(2\pi\nu t)\hat{X}$$

Harmonic oscillator with equidistant energy levels

From a coherent source ($\omega=\omega_p$),
the quantum state is descibed by the coherent states

Its energy grows as : $<E>=\hbar\omega_p(\frac{1}{2}+(\Omega_1 t/2)^2)$

Continuous growth! No oscillations!!

When its energy is very large,
its dynamics describe very well the classical motion

No way to build arbitrary states and explains oscillations

Multilevel dynamics

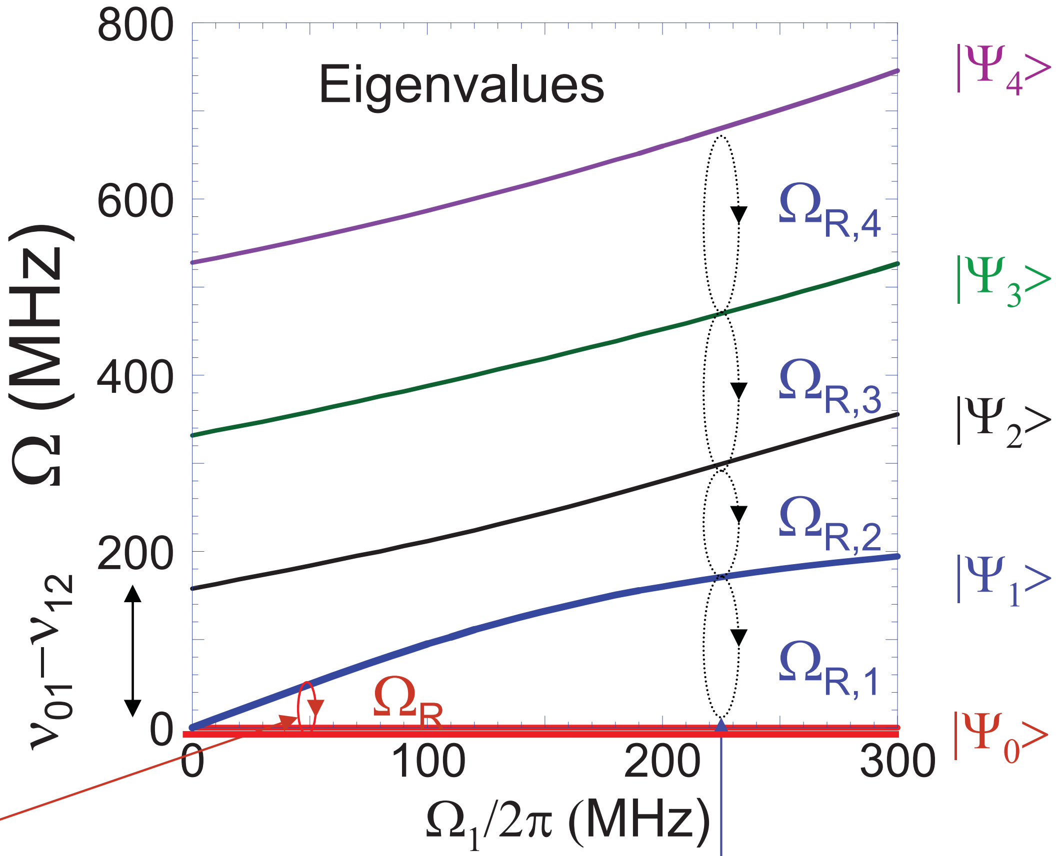
$$\hat{H}(t)=\frac{1}{2}\hbar\omega_p(\hat{P}^2+\hat{X}^2)-\sigma\hbar\omega_p\hat{X}^3-\sqrt{2}\hbar\Omega_1\cos(2\pi\nu t)\hat{X}$$ → Time independent Hamiltonian

« Rotating wave » approximation

$$\hat{H}_{\circlearrowleft,\mathrm{RWA}}=h\begin{pmatrix} 0 & \frac{\nu_1}{2} & 0 & 0 \\ \frac{\nu_1}{2} & \Delta_1(\nu) & \ddots & 0 \\ 0 & \ddots & \ddots & \sqrt{N-1}\frac{\nu_1}{2} \\ 0 & 0 & \sqrt{N-1}\frac{\nu_1}{2} & \Delta_{N-1}(\nu) \end{pmatrix}$$

Time independent Hamiltonian

$$\Delta_n(\nu)=\nu_{0n}-n\nu$$



low amplitude:

Initial state: $|0(t=0^+)\rangle=(|\Psi_0\rangle+|\Psi_1\rangle)/\sqrt{2}$

Rabi oscillations

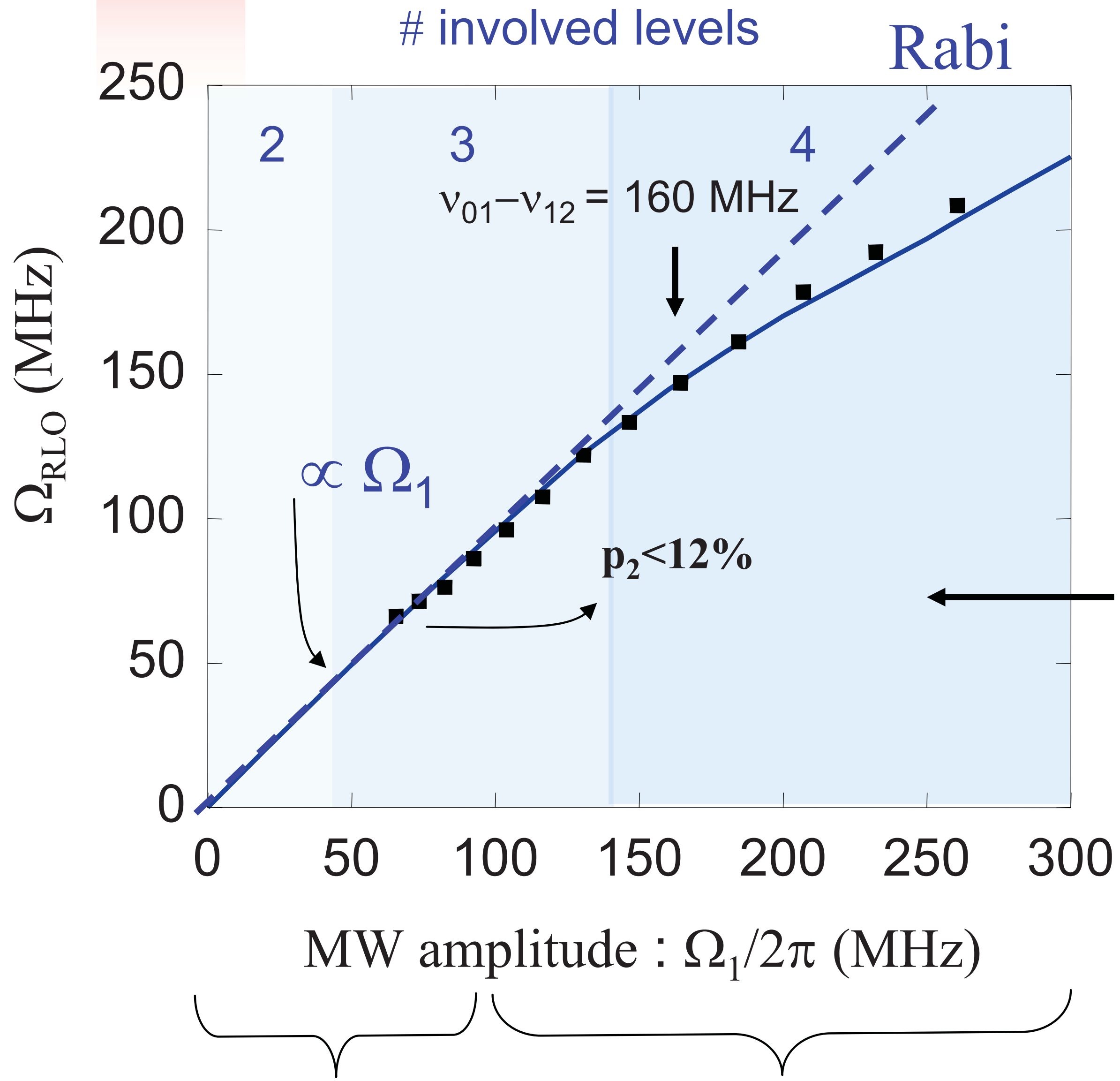
large amplitude:

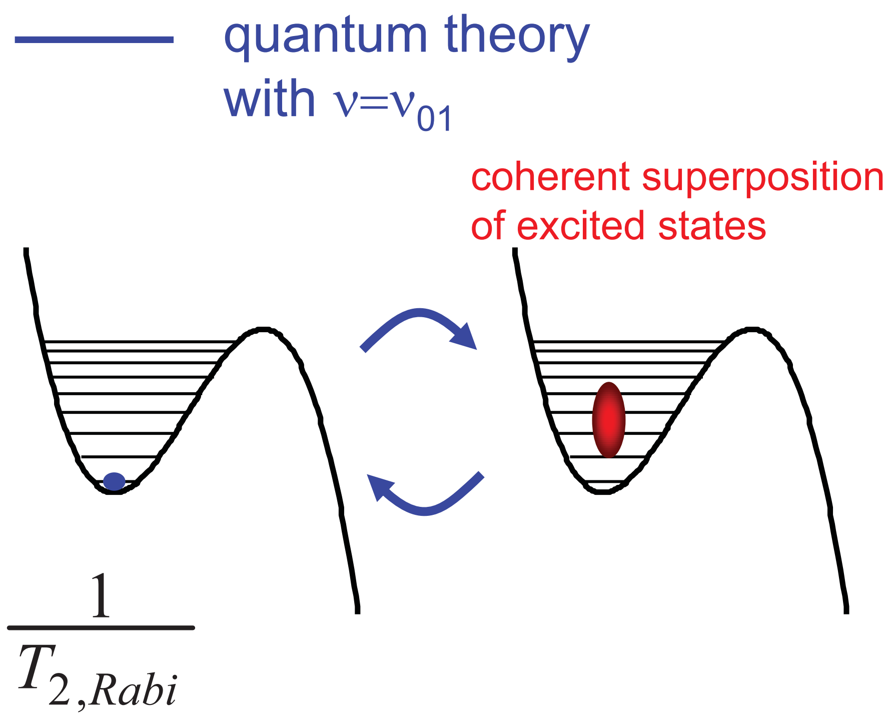
$|0(t=0^+)>=a_0|\Psi_0>+a_1|\Psi_1>+a_2|\Psi_2>+a_3|\Psi_3>$

Multi-level dynamics

Cross-over between two and multi-level

(J. Claudon, A. Zazunov, F. Hekking, and O. Buisson, arXiv:0709.3787)





• **Low excitation power :** two level description

• **Intermediate power :** multi- level description

Cross- over condition:

Anharmonicity $\nu_{01}-\nu_{12}$ ⟷ MW amplitude Ω_1

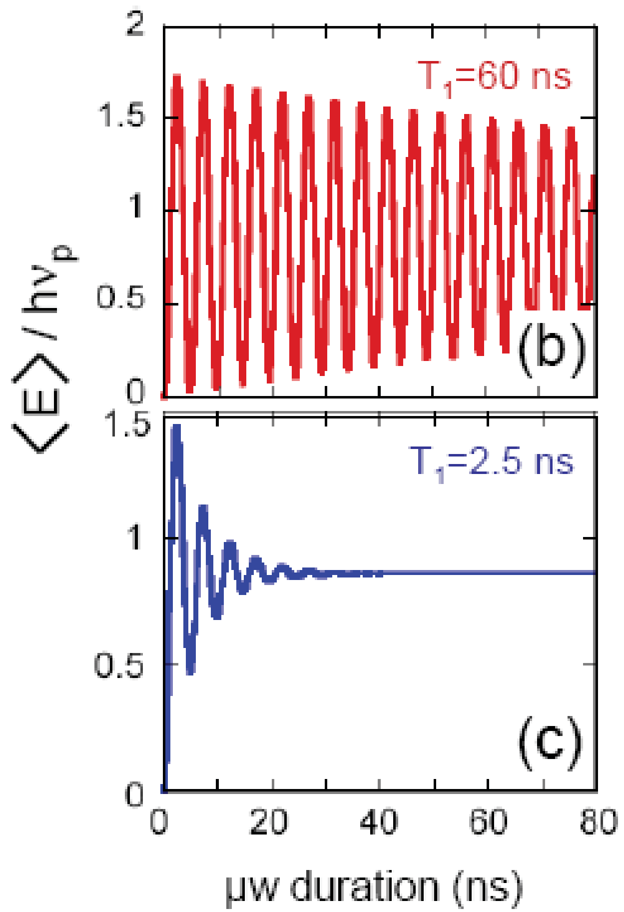
Classical description?

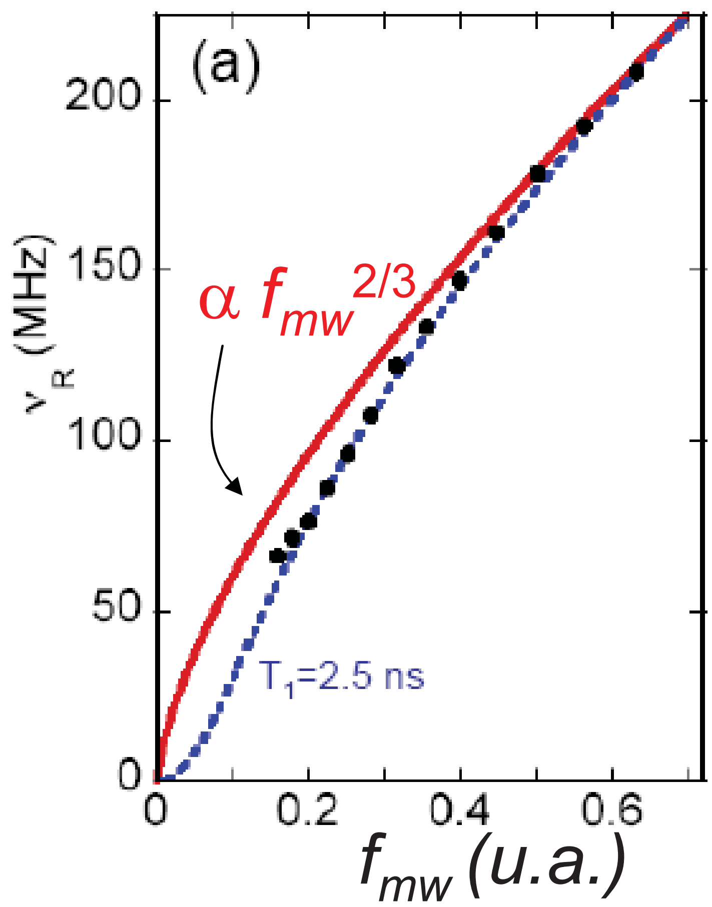
(J. Claudon A. Zazunov, FH, and O. Buisson, PRB 2008)

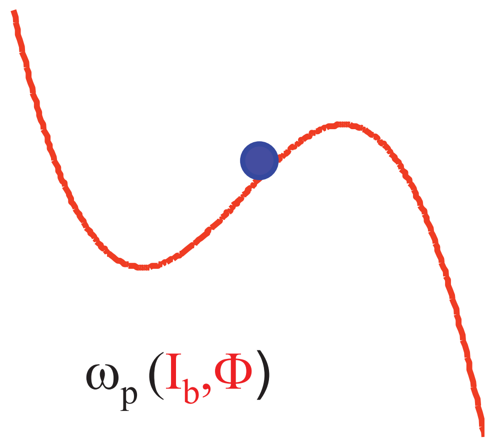
Transient regime

Integrate equations of motion

(A. Ratchov, PhD-thesis 2005
Gronbech-Jensen 2006)





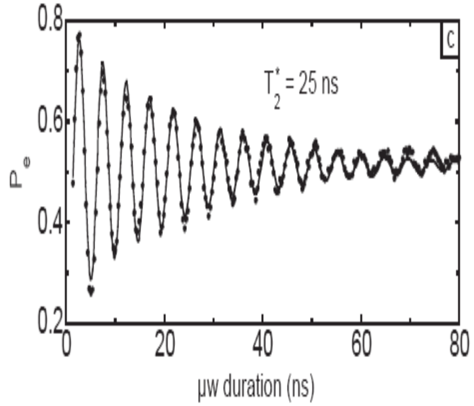


$$\nu_R = \frac{\sqrt{3}}{4}\nu_p\left(1+2A^2\right)^{1/3}\left(\frac{f_{\mu w}}{m\omega_p^2}\right)^{2/3}$$

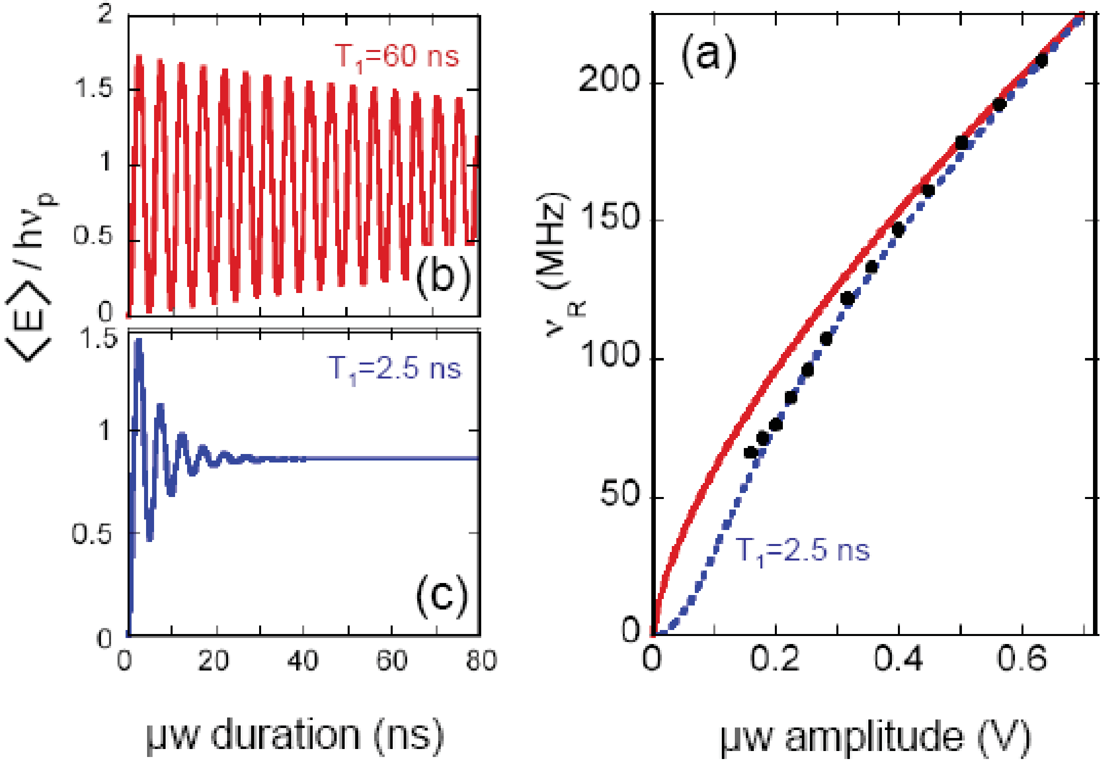
$$A = \frac{6a}{m\omega_p^2} = \left[\frac{18}{27}\frac{m\omega_p^2}{\Delta U}\right]^{1/2}$$

Classical or quantum description?

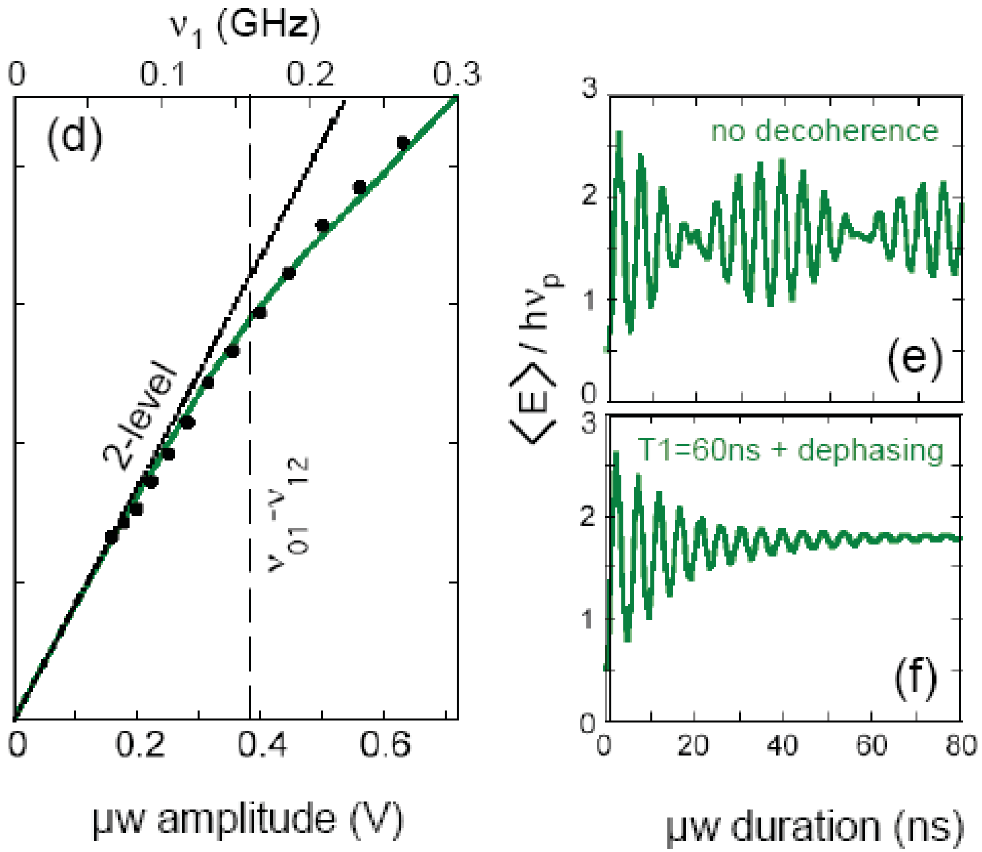
(J. Claudon A. Zazunov, F. Hekking, and O. Buisson, PRB 2008)

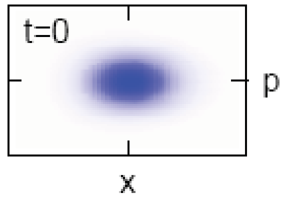


Classical model

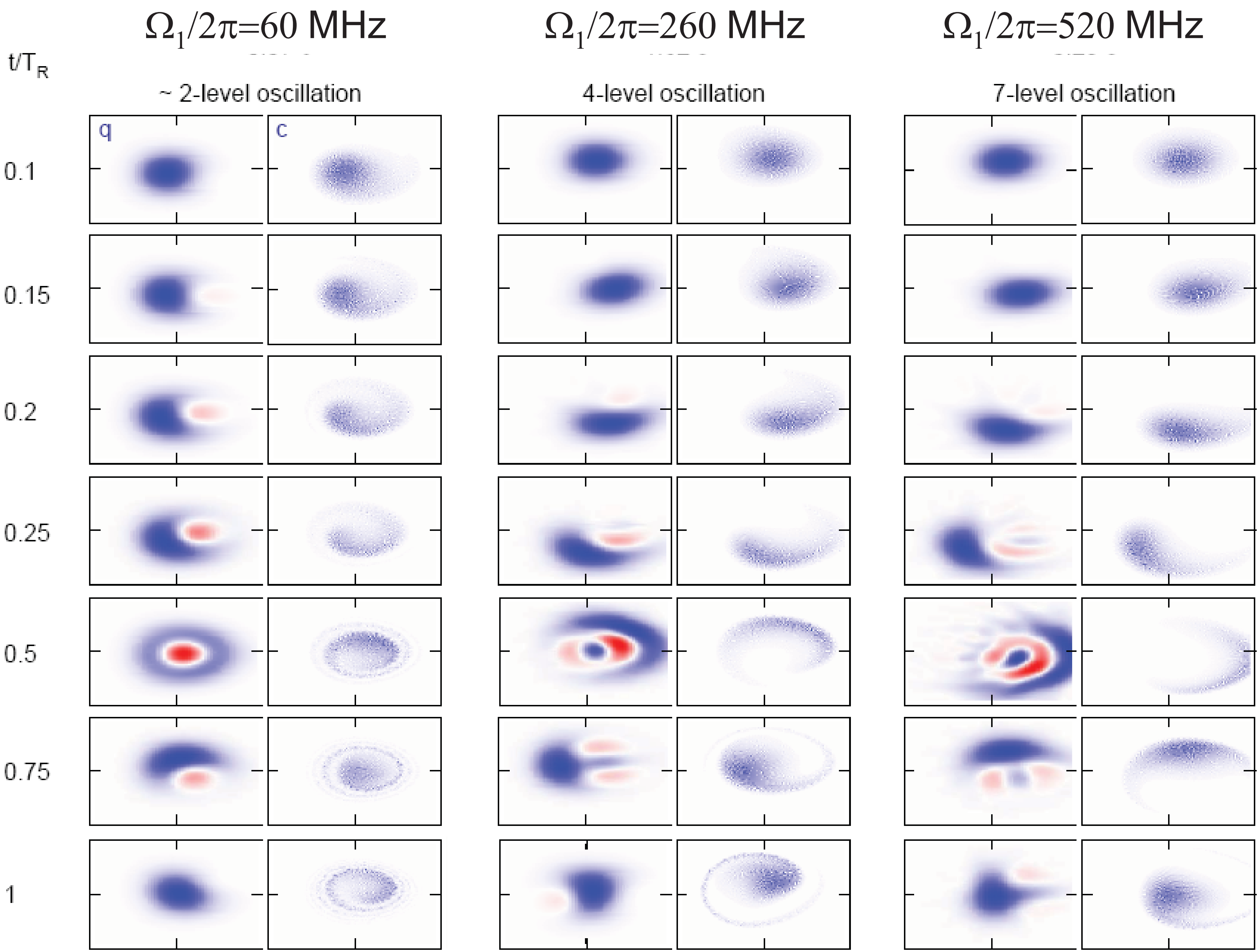


Quantum model





(J. Claudon A. Zazunov, F. Hekking, and O. Buisson, PRB 2008)



Outline

Introduction to superconducting qubits

Multi-levels artificial atom

- current-biased Josephson junction and dc SQUID
- quantum measurements
- quantum dynamics in a multilevel quantum system
- quantum or classical description
- optimal control
- decoherence processes

Two-degrees of freedom artificial atom

- inductive dc SQUID
- spectroscopy measurements
- strong non-linear coupling
- coherent oscillations

Multi-degrees of freedom system

- Josephson junction chains
- quantum phase slip
- charging effects