SIMULATION OF QUANTUM MAGNETS

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Outline

- Motivation
  - Preliminary
  - Quantum magnets
  - Idea of quantum simulator

- Trapped ion quantum simulator
  - Coupling ions with transverse normal modes
  - Tuning spin-spin couplings for QS

- Quantum simulation of the smallest spin network
  - Phase diagram
  - Magnetic frustration

- Outlook

Qubit/spin: basic unit for QC/QS

Qubit: a quantum two-level system, e.g., a spin-1/2 particle.

We mix-use the above terms

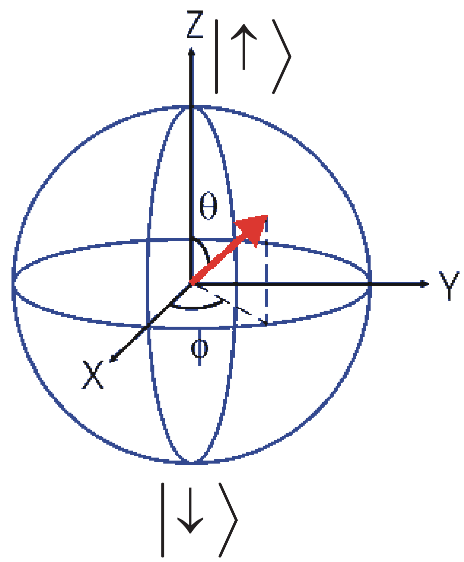
$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{|\rightarrow\rangle + |\leftarrow\rangle}{\sqrt{2}}$$

$$\sigma_z|\uparrow\rangle = |\uparrow\rangle, \qquad \sigma_z|\rightarrow\rangle = |\leftarrow\rangle$$

$$D[R_{\hat{n}}(\phi)]|\chi\rangle = \exp\left(-\frac{i\vec{\sigma}\cdot\hat{n}\phi}{2}\right)|\chi\rangle$$

$$\left[\sigma_i, \sigma_j\right] = i\varepsilon_{ijk}\sigma_k$$

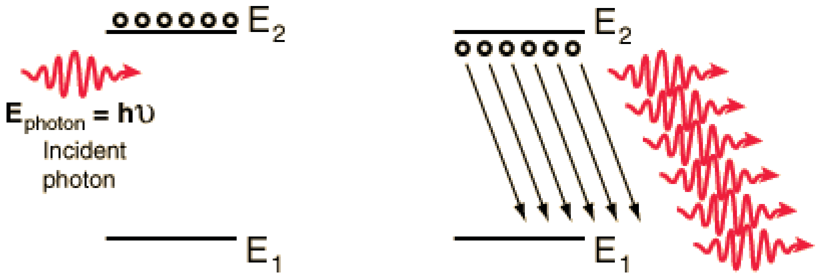


Bloch vector picture

Brief review of two-level systems

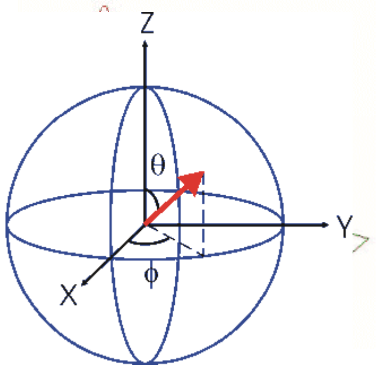
Physicists like two-level systems

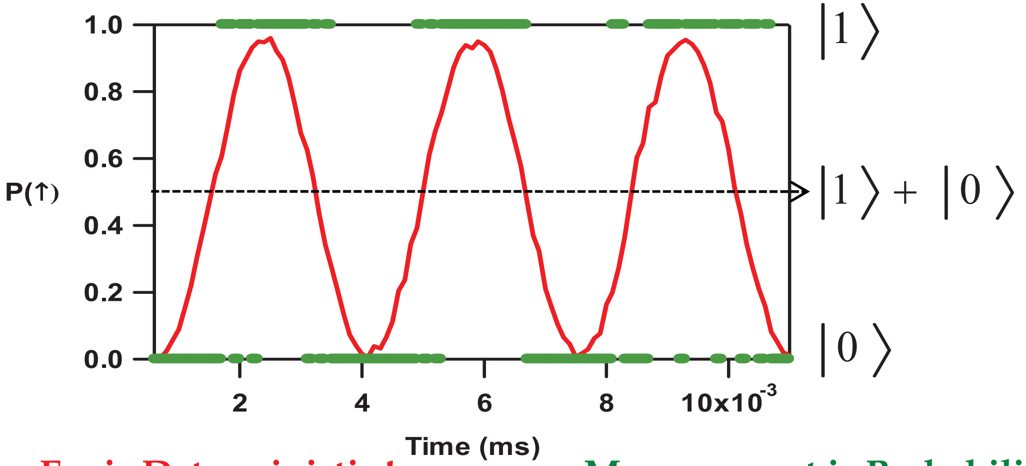
Einstein: predicted stimulated emission which led to invention of laser (population control)



A. Einstein

Rabi oscillation: coherent control of two-level atom (state control, coherent control)





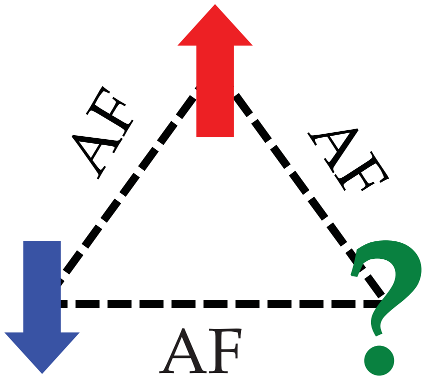
I. Rabi

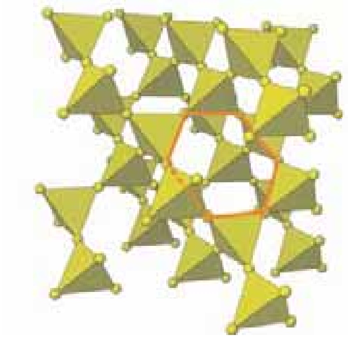
Schrodinger Eq. is Deterministic !

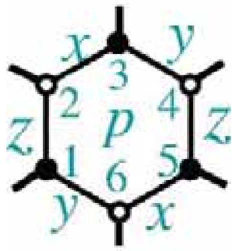
Measurement is Probabilistic !

Understand exotic material

- **Magnetic Frustration** – competing (antiferromagnetic) interactions that result in highly degenerate ground states with excess entropy and disorder even at zero temperature. QM leads to massive entangled ground states.
- **Spin Liquids** – a spin system with anti-ferromagnetic coupling, which has no long-range order, even at very low temperatures
- **Topological Order** – where particle exchange can result in "anyonic" statistics that may have applications in fault-tolerant quantum computing.







"Kitaev Lattice"

Science **294**, 1495–1501 (2001), Phys. Today **59**, 24 (2006)

Study quantum phase transition

Smooth variations in the non-thermal parameters, such as pressure or magnetic field in the Hamiltonian result in sharp boundaries between different phases near T=0.

Zero-point fluctuation prevents atoms from staying statically at lattice sites at T=0.

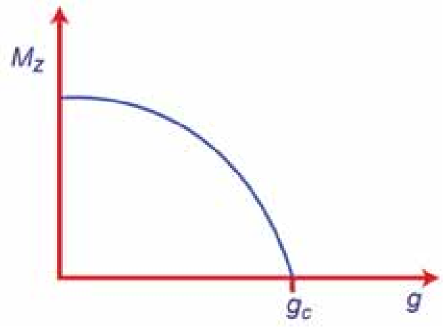
Competition between non-commutable operators can lead to different quantum phases.

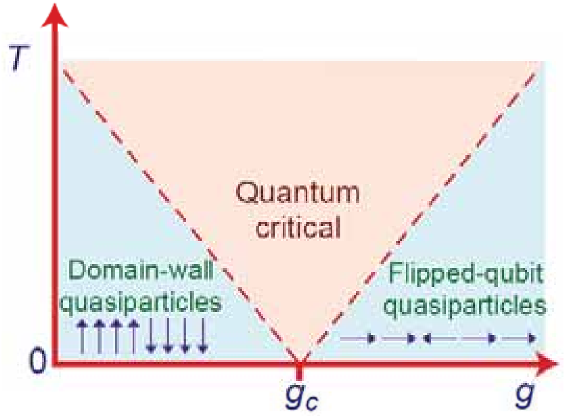
Quantum fluctuation induces transitions between quantum phases.

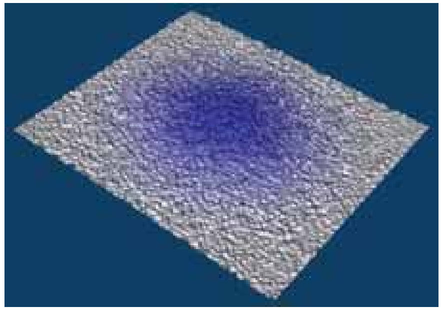
M. Greiner et al., Nature **415**, 39 (2002)

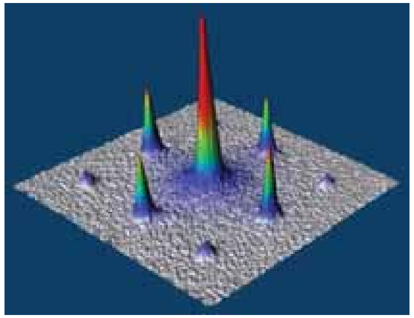
S. Sachdev, Quantum Phase Transition (1999)

$$H = H_z + gH_x$$









Modeling quantum magnets

- Ising spins in transverse B field: $\boxed{H = \sum_{i,j} J_{ij}^z \sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x}$

- XY model: $H = \sum_{i,j} J_{ij} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)$

- XXZ model : $H = \sum_{i,j} J_{ij} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + J_{ij}^z \sigma_i^z \sigma_j^z$

(~ Bose-Hubbard under Holstein-Primakoff transformation)

- Possible Observations
  - Quantum phase transition
  - Spin frustration
  - Complex entangled states

- Provided tunable spin-spin interactions:
  - Strength,
  - Sign (ferro or anti-ferro),
  - Range,
  - Coupling graph (geometry).

D. Porras & J. I. Cirac, PRL **92**, 207901(2004).
Two ion quantum simulator: Friedenauer et al., Nat. Phys., **4**, 757 (2008)

Solving a complex quantum system is difficult

$$|\psi(t_2)\rangle = \hat{U}(t_2, t_1)|\psi(t_1)\rangle,$$

where $$\hat{U}(t_2, t_1) = \hat{\mathrm{T}}\exp[-\frac{i}{\hbar}\int_{t_1}^{t_2} H(t)dt].$$

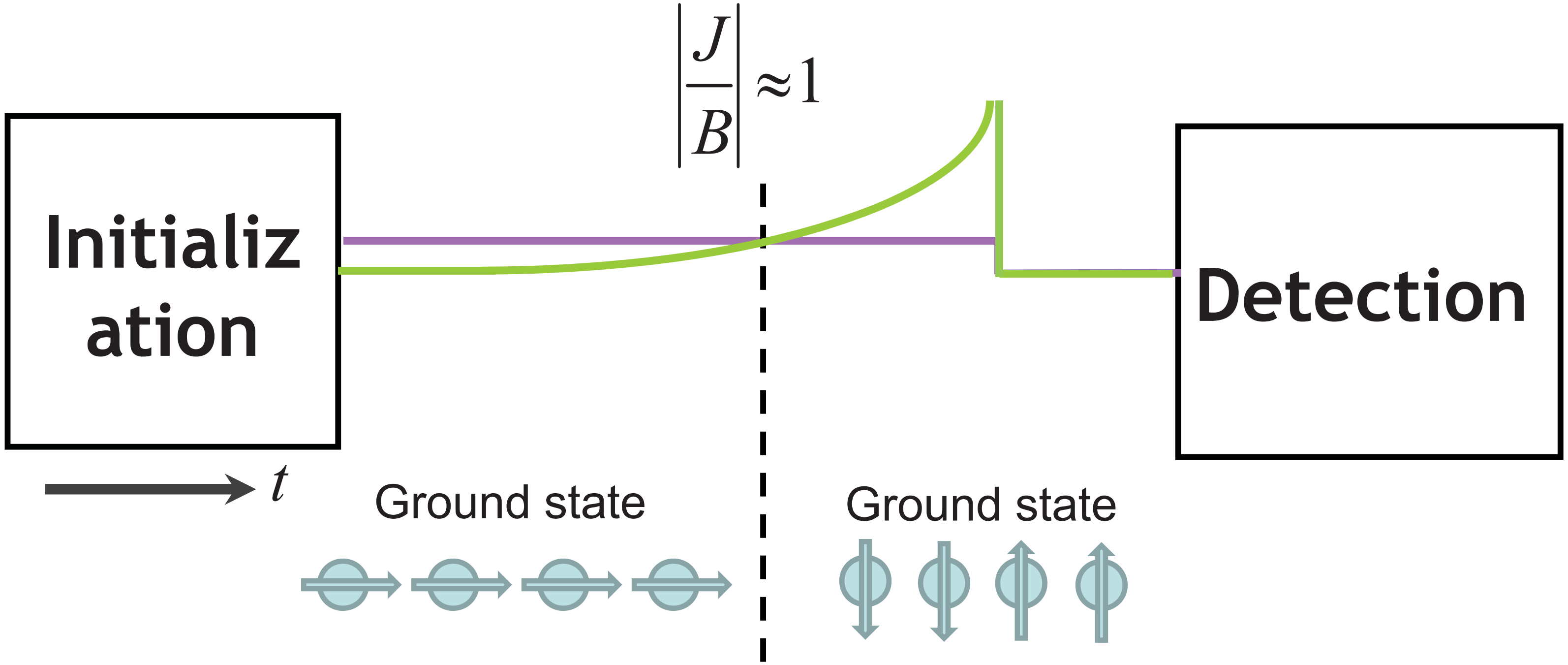
# of spins	# of c-numbers needs to be stored in a classical memory	
1 spin	$|\psi^{(1)}\rangle = \begin{bmatrix} c_{\uparrow} \\ c_{\downarrow} \end{bmatrix}$	$\hat{U}^{(1)} = [2 \times 2]$
2 spins	$|\psi^{(2)}\rangle = \begin{bmatrix} c_{\uparrow\uparrow} \\ c_{\uparrow\downarrow} \\ c_{\downarrow\uparrow} \\ c_{\downarrow\downarrow} \end{bmatrix}$	$\hat{U}^{(2)} = [4 \times 4]$
$\vdots$	$\vdots$	
30 spins	$|\psi^{(30)}\rangle = [1 \times 2^{30}]$	$\hat{U}^{(30)} = [2^{30} \times 2^{30}]$

This is classically intractable, and we need a quantum simulator. This idea was first proposed by R. Feynman (1982), and then refined by S. Lloyd (1996)

Quantum simulator: implementation

$$H = \sum_{i<j} J_{ij}\sigma_z^i\sigma_z^j + B_y\sum_i \sigma_x^i$$

$$|\psi(0)\rangle = |\downarrow_x\downarrow_x\downarrow_x\cdots\rangle \xrightarrow{\text{Adiabatically following}} |\psi(t)\rangle = \hat{\mathrm{T}}e^{-\frac{i}{\hbar}\int H(t)dt}|\psi(0)\rangle$$



$\left|\frac{J}{B}\right| \approx 1$

Initialization

Detection

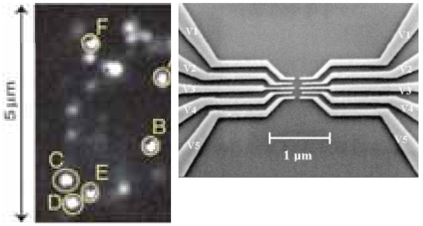
t

Ground state

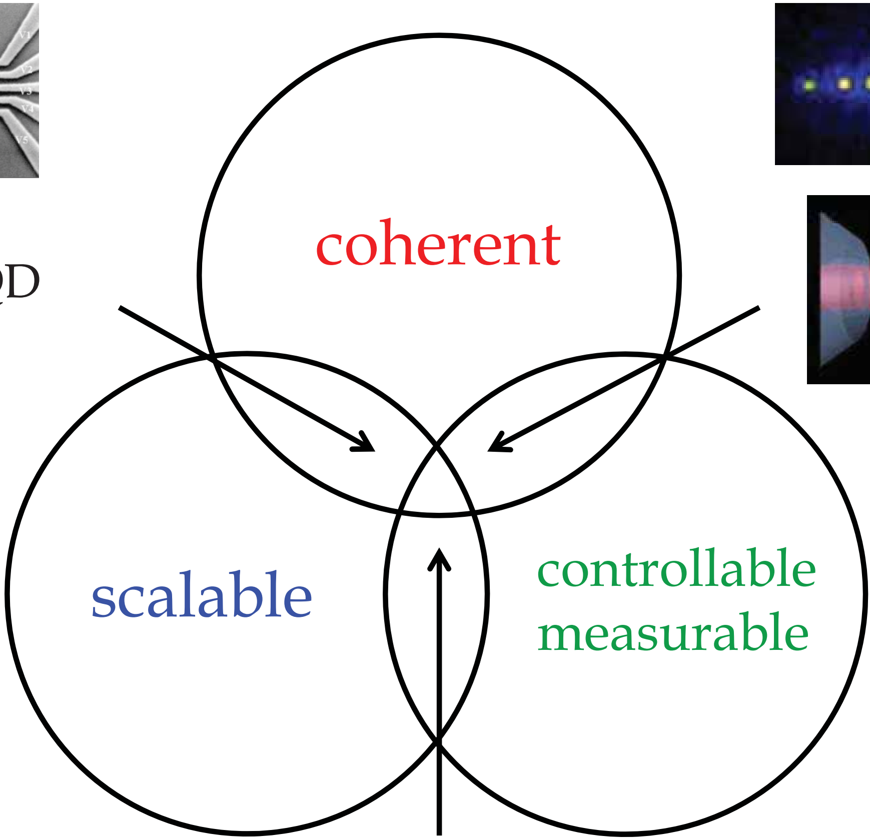
Ground state

Lloyd, Science **273**, 1073 (1996) and **319** 1209 (2008). Farhi et al., Science **292**, 472 (2001);
(2 ion simulator) Friedenauer et al., Nat. Phys., **4**, 757 (2008)

physical implementations of a QC/QS



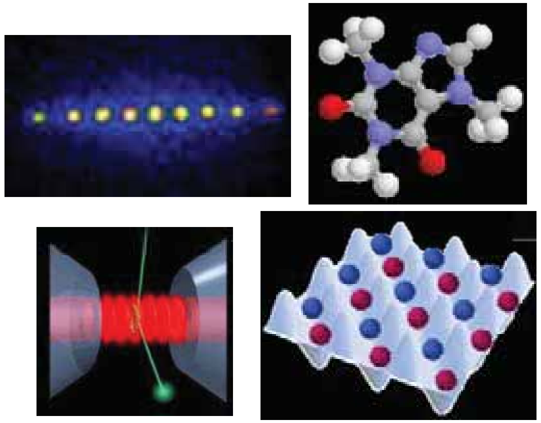
Nuclear spin in QD
Diamond NVC



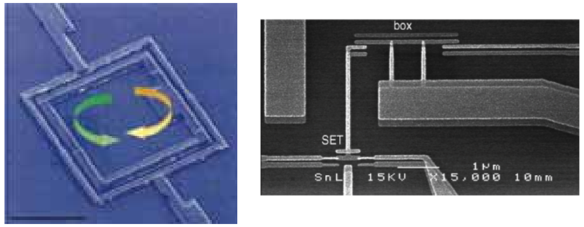
coherent

scalable

controllable
measurable



Trapped ions
Atoms in OL
Cavity QED
molecule NMR



Cooper pair box
SQUID

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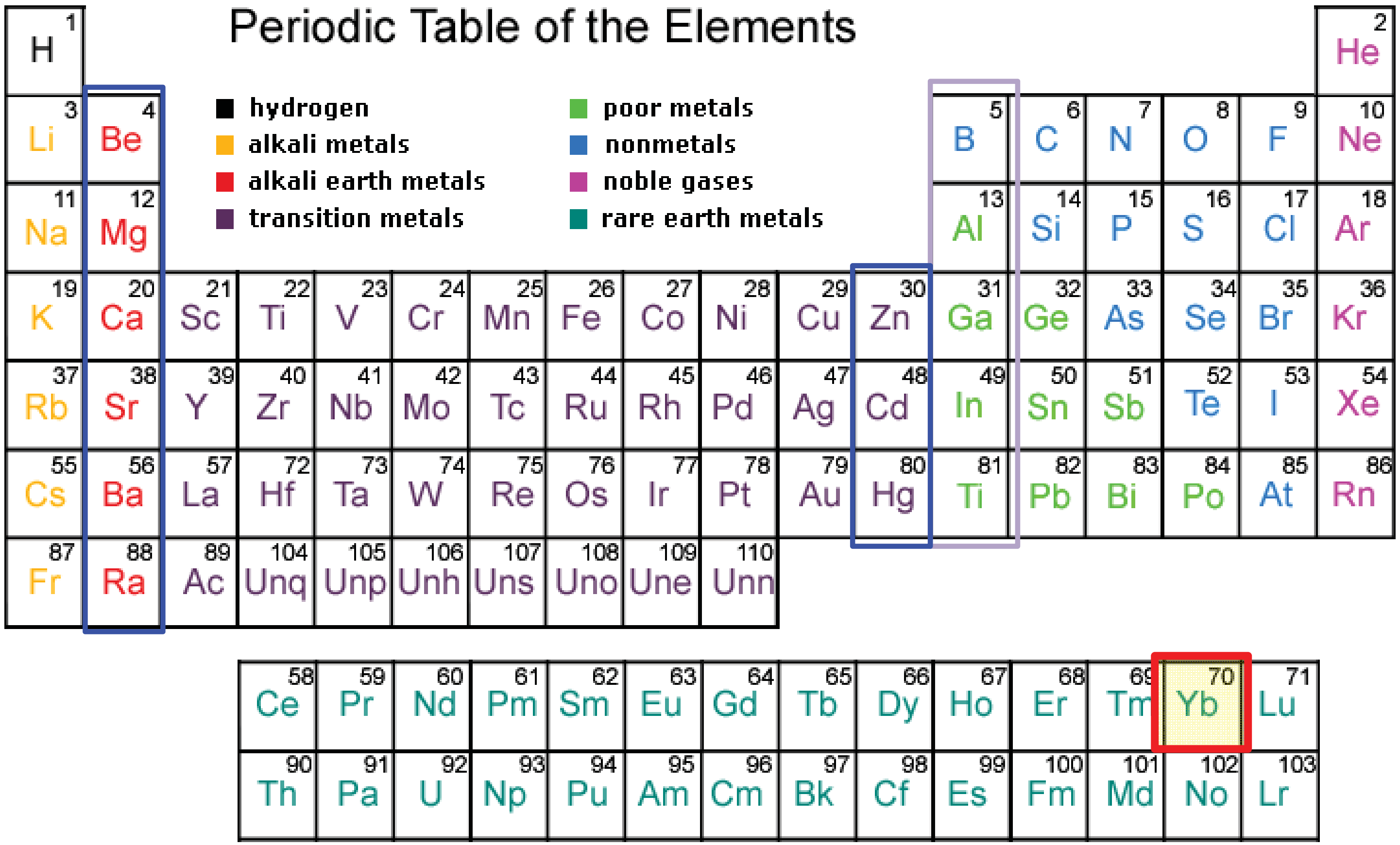
  - ▪ Magnetic frustration

- ❑ Outlook

What ion to trap?



$^{171}Yb^{+}$

Electronic: [**Xe**].**4f**^{14}.**6s**^{1}, Term $^{2}\mathbf{S}_{1/2}$ ($\boldsymbol{J}$=1/2)

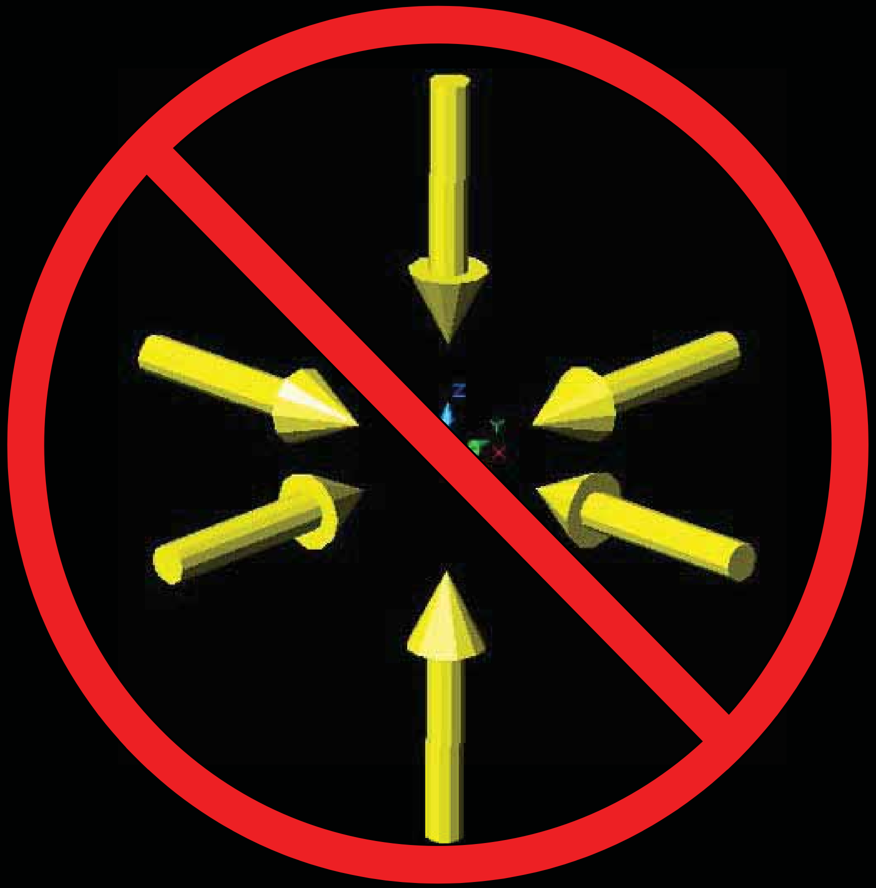
Nuclear: $\boldsymbol{I}$=1/2

Hyperfine: $\boldsymbol{F}$=1,0 (qubit / spin)

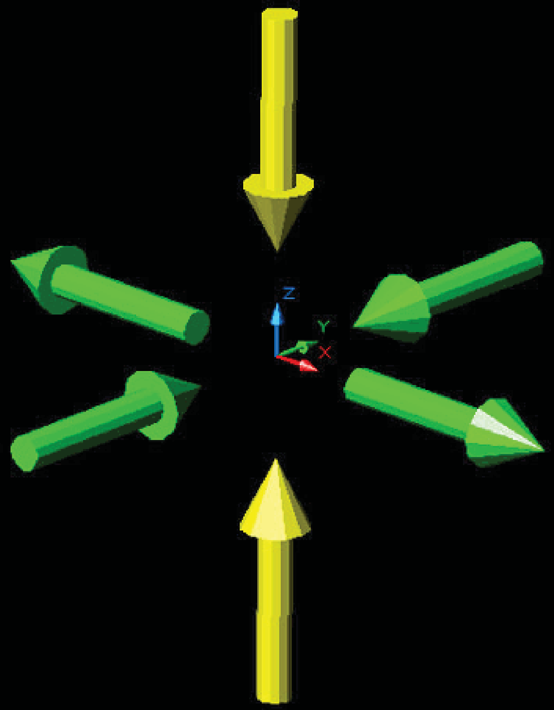
Olmschenk et al., Phys. Rev. A 76, 052314 (2007)

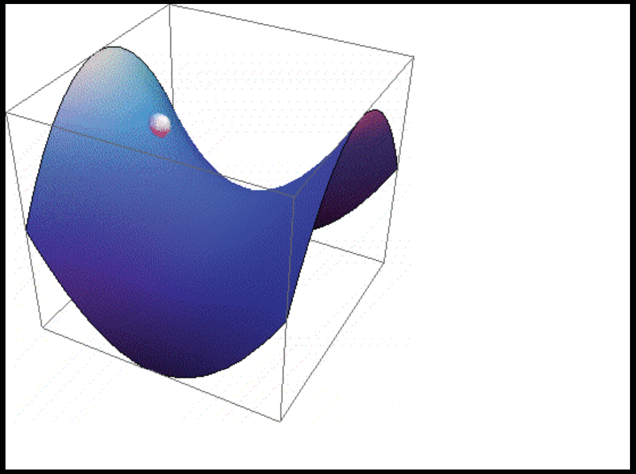
How to trap an ion?

Electric Field Vectors



$$\vec{\nabla} \bullet \vec{E} = 0$$





Linear Paul Trap

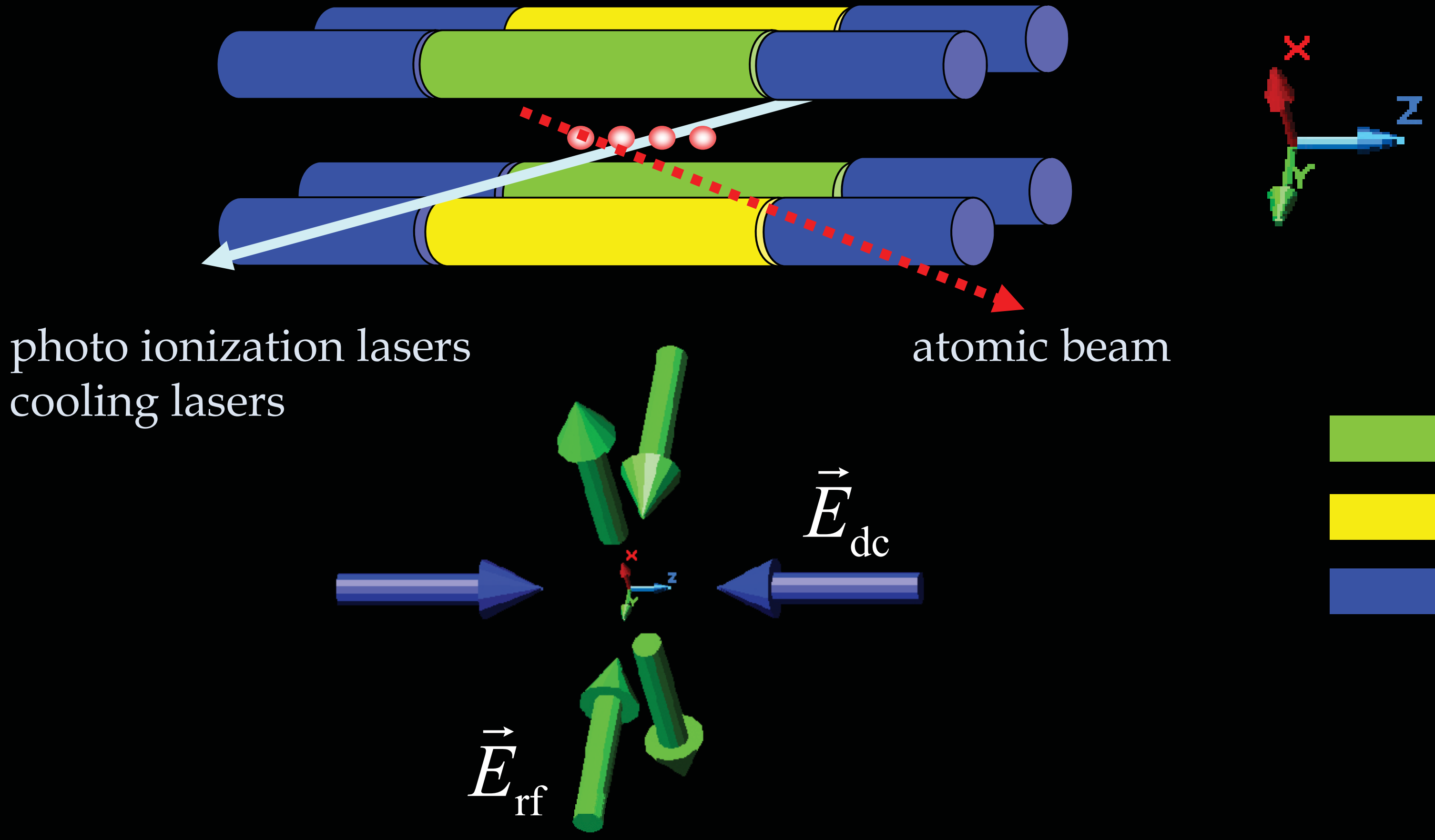


photo ionization lasers
cooling lasers

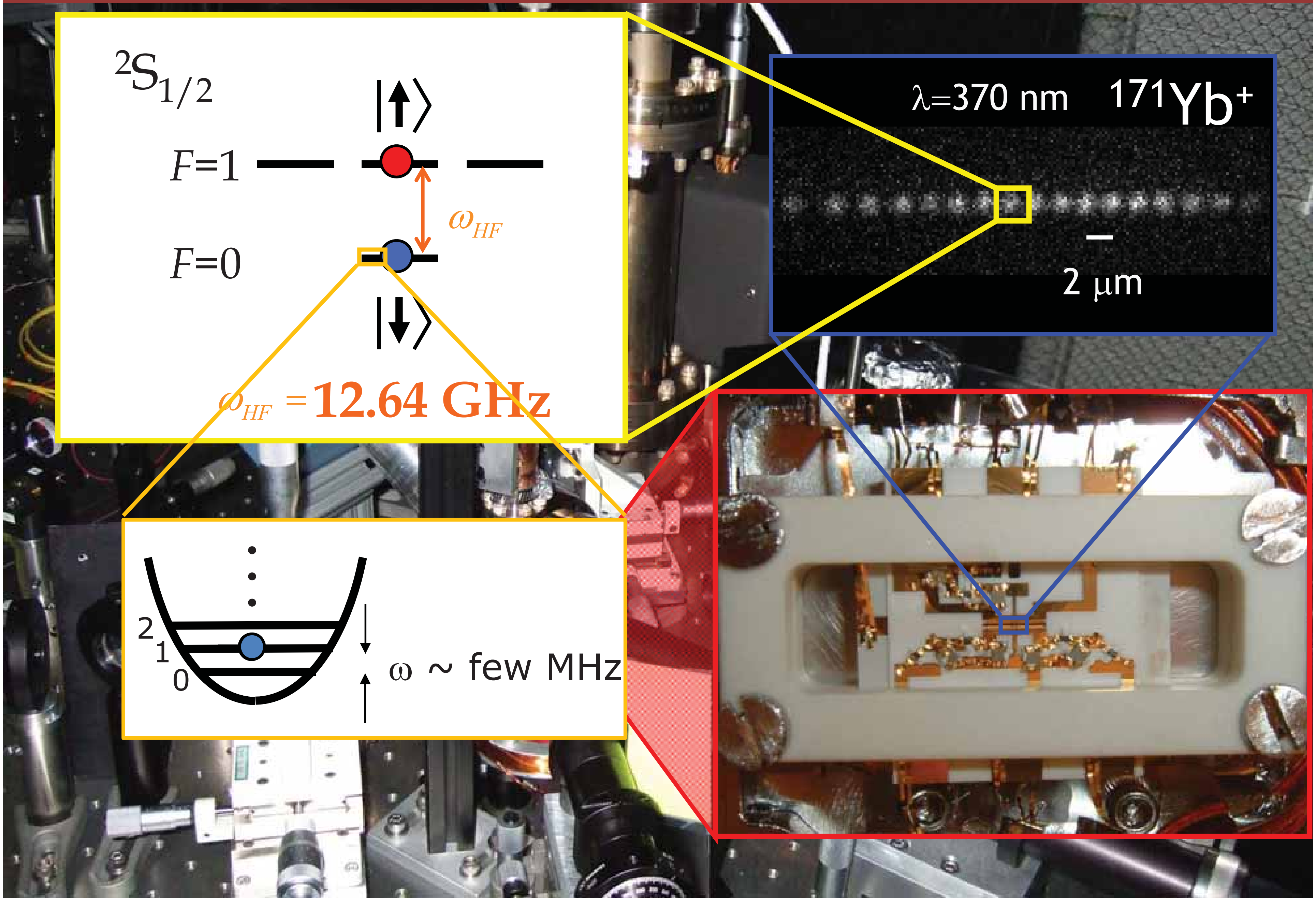
atomic beam

$\vec{E}_{\mathrm{dc}}$

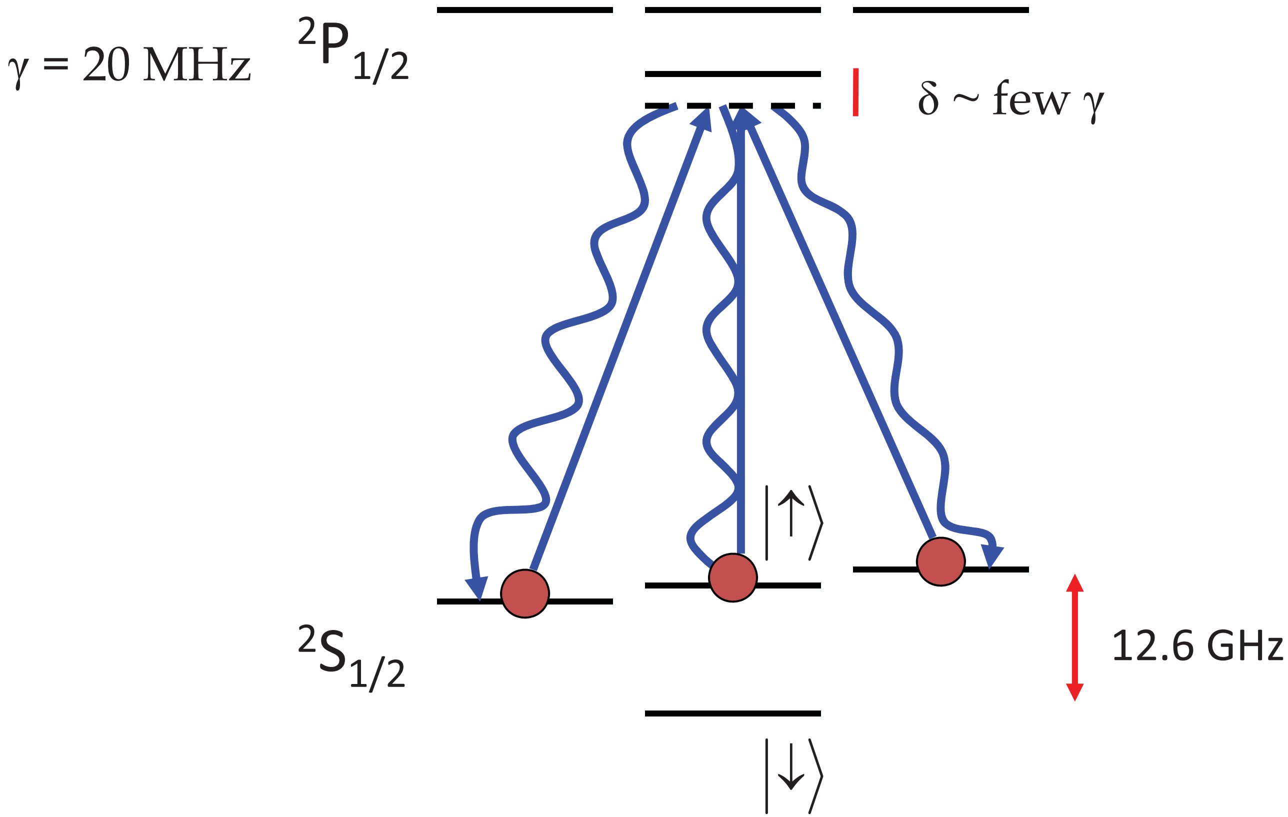
$\vec{E}_{\mathrm{rf}}$

🟩 rf
🟨 ground
🟦 static

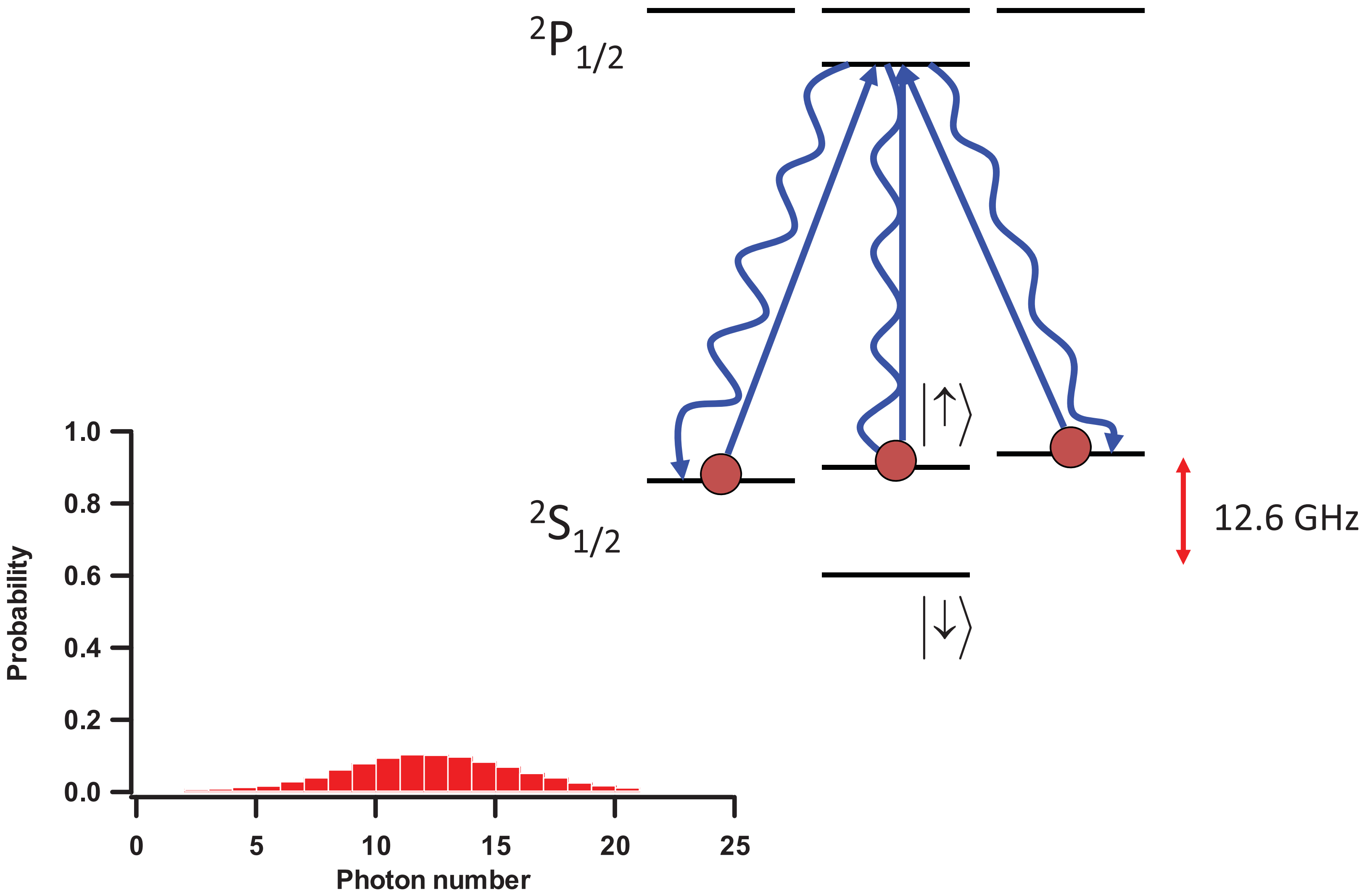
Trapped Ion QC / QS



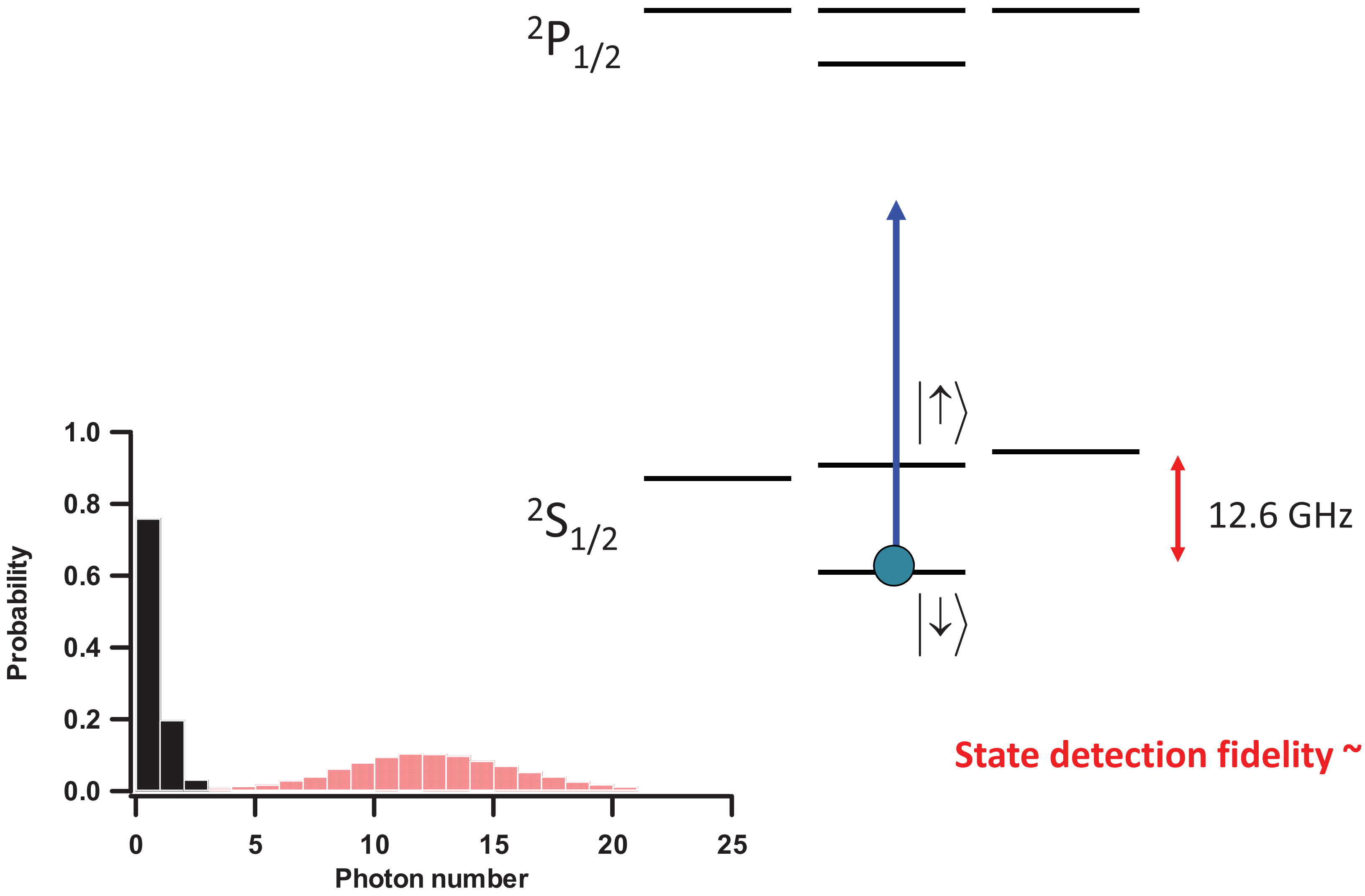
Some Atomic Physics - Laser Cooling



State Detection

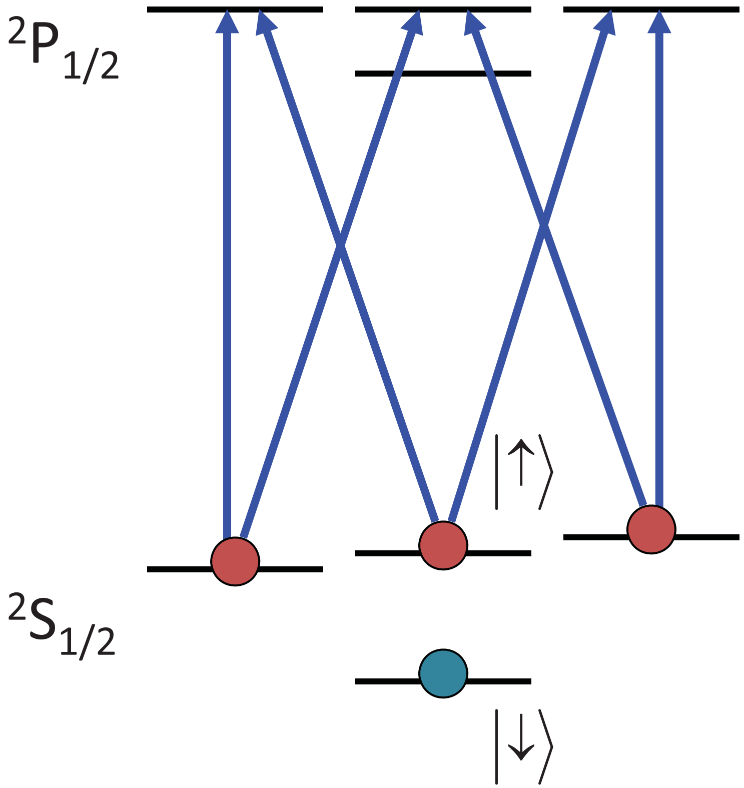


State Detection

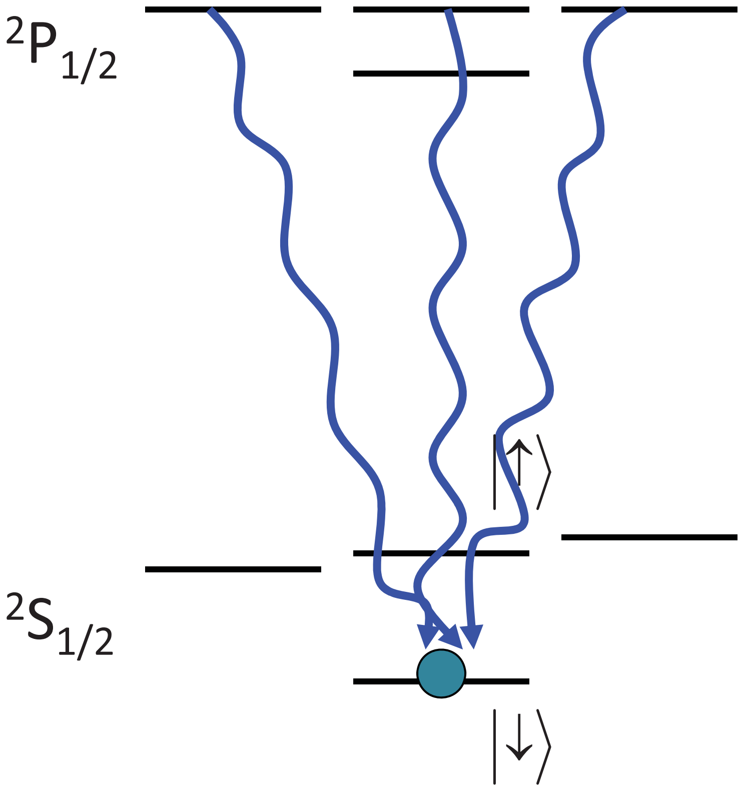


State detection fidelity ~ 97%

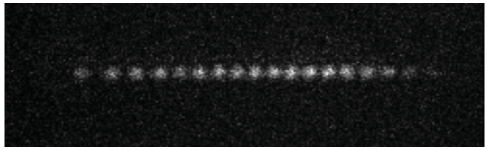
Spin (qubit) Initialization



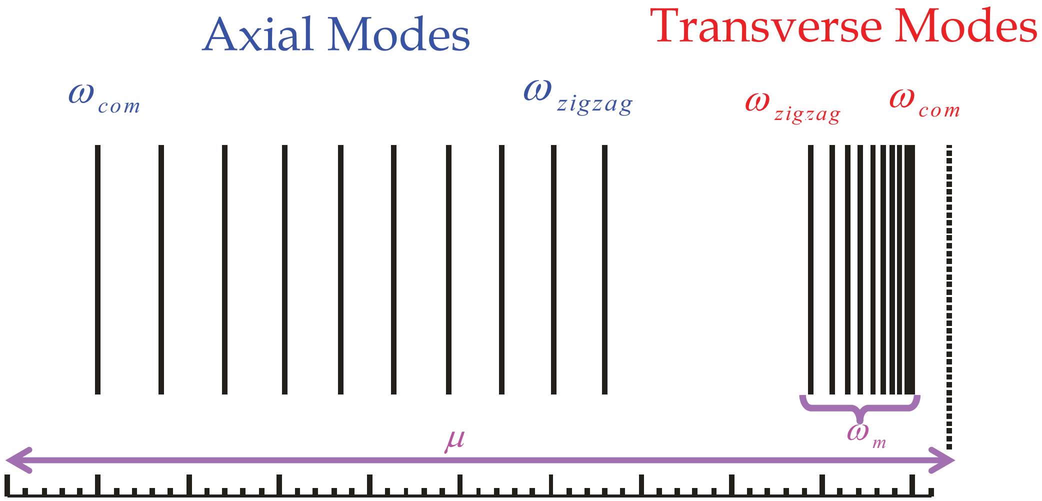
Spin (qubit) Initialization



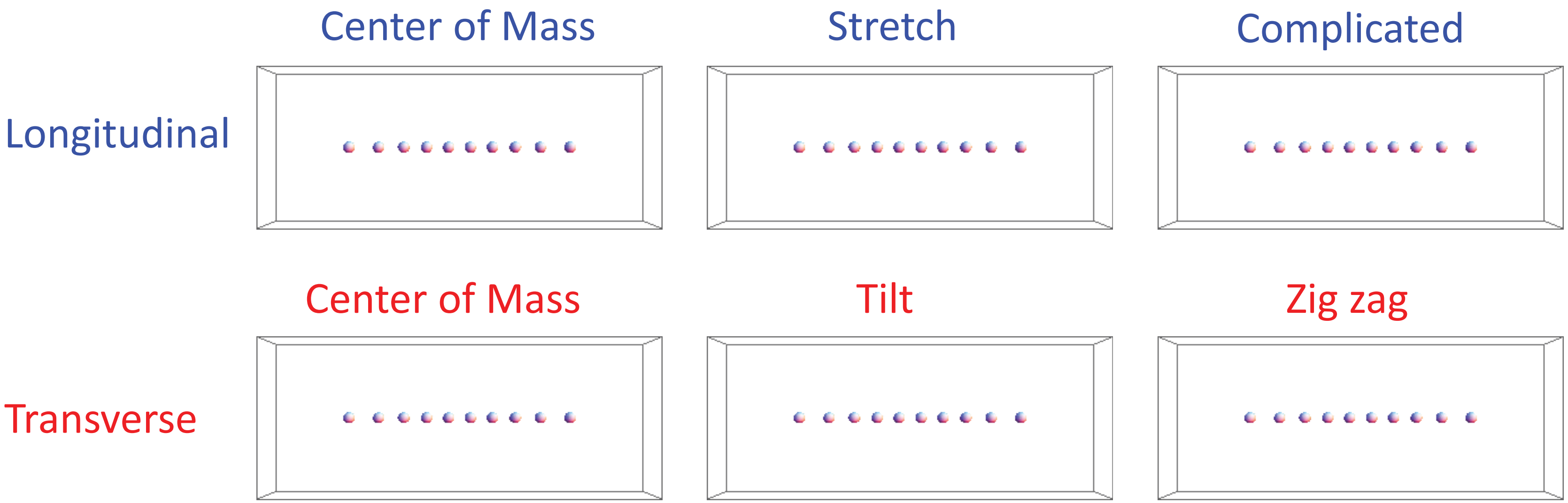
Motional normal modes



Amplitude ~ 5 nm
(1 MHz)



$$e.g.\quad \omega_{com}^{transverse} / \omega_{com}^{axial} = 10$$



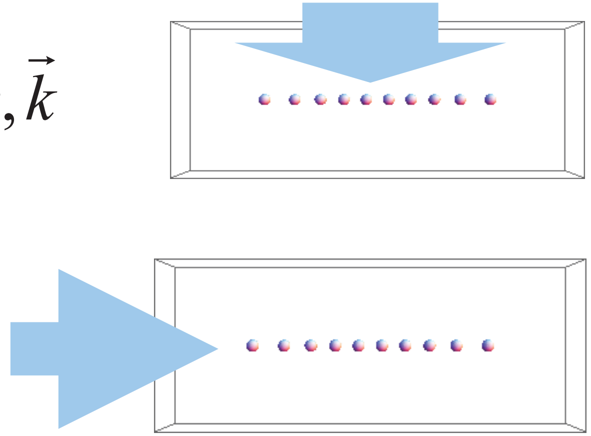
James, Appl. Phys. B 66, 181-190 (1998)

Zhu, Monroe, and Duan, PRL **97**, 050505 (2006).

The swing: control of normal modes

The Swing
by Jean-Honore Fragonard, 1766
(French painter and printmaker)

$v,\vec{k}$



Note that…

Superposition of normal modes
(classically allowed)

Superposition of atom in motion and at rest
(QM allowed)

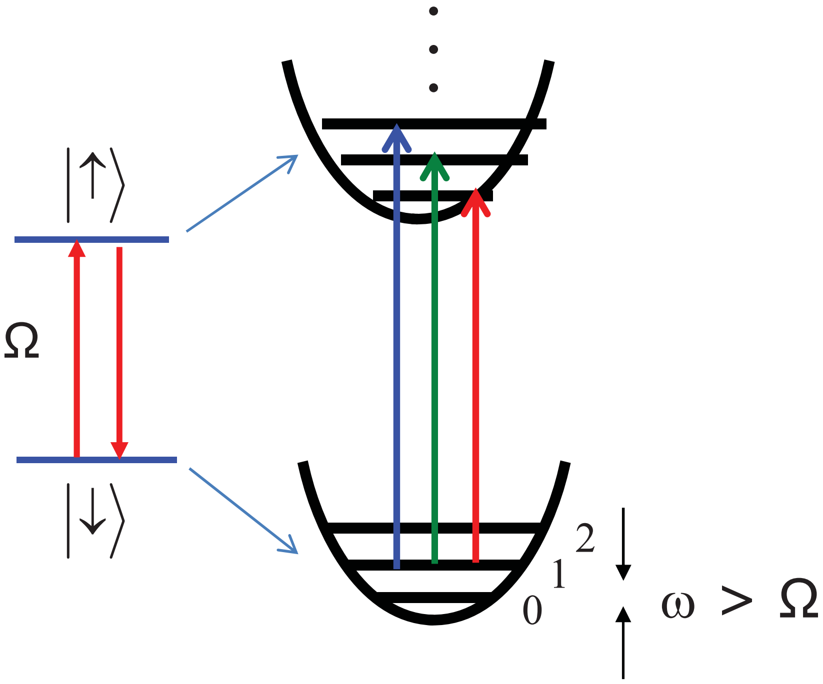
Coupling Spin and Motion with light - I

$$\hat{H}_I = \tfrac{1}{2}\hbar\Omega\hat{\sigma}^+ e^{-i\delta t} e^{ikx} + h.c.$$

$$e^{ik\hat{x}} = e^{ikx_0(a+a^+)}$$

$$= 1 + ikx_0(a+a^+) + \cancel{H.O.},$$

Lamb-Dicke regime



L-D parameter : $\eta = kx_0 \propto \dfrac{x_0}{\lambda} << 1$

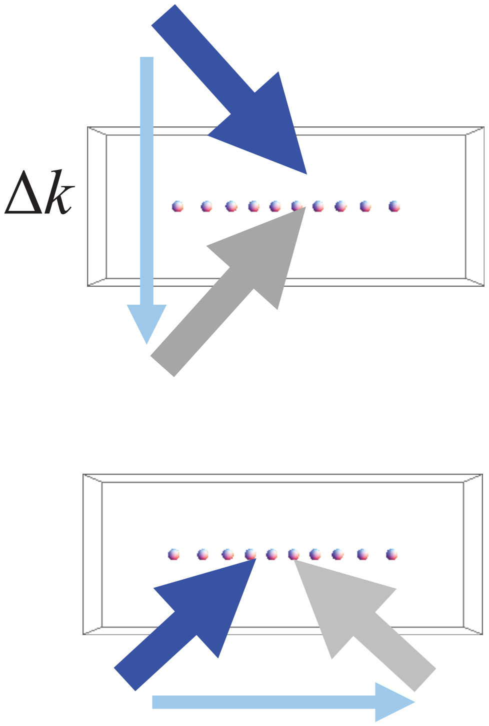
$$\hat{H}_I = \tfrac{1}{2}\hbar\Omega\hat{\sigma}^+ e^{-i\delta t}(1 + \eta a + \eta a^+) + h.c.$$

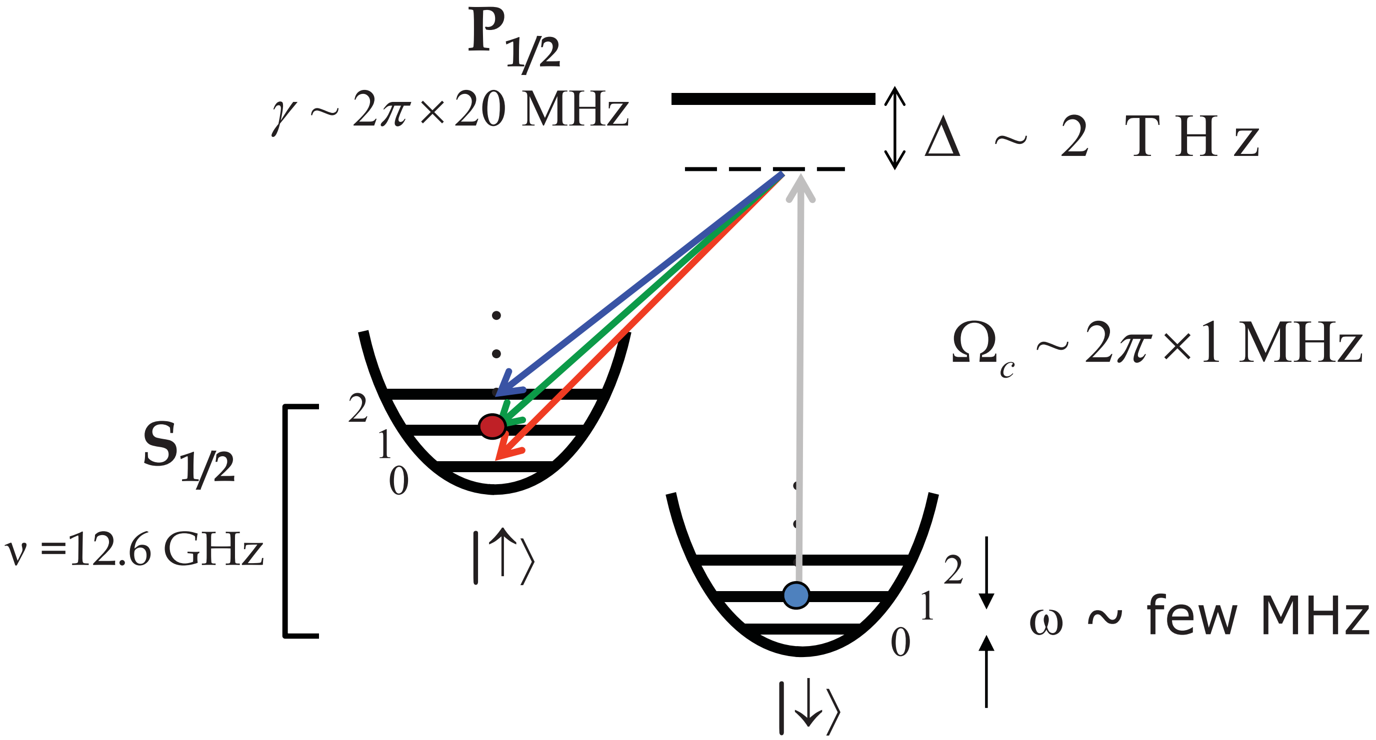
blue sideband

carrier

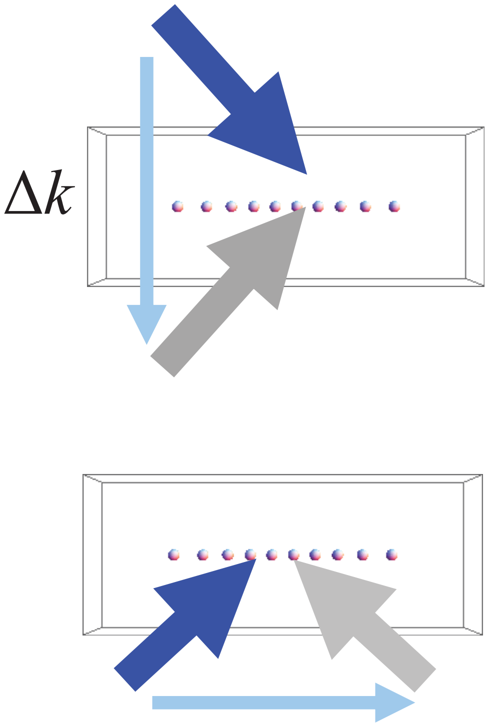
red sideband

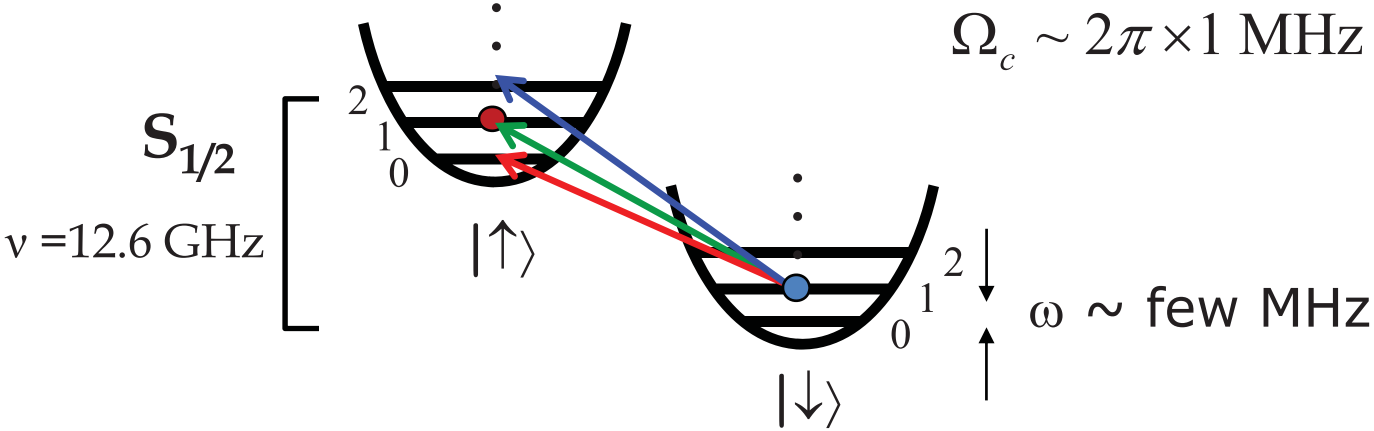
Coupling Spin and Motion - II



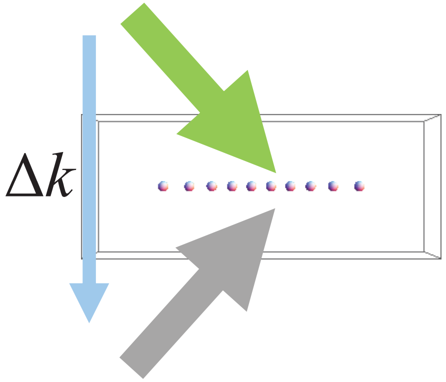


Coupling Spin and Motion - II





Ion normal mode spectrum

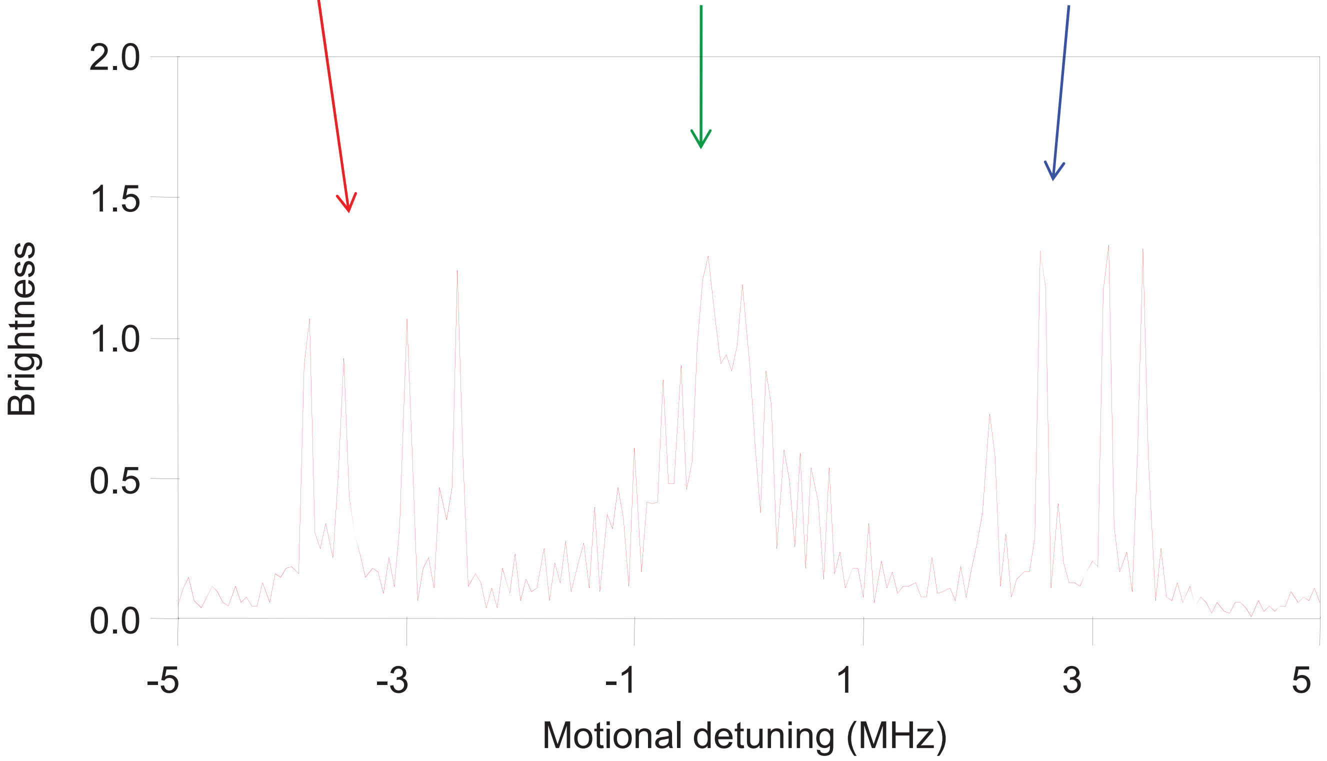


Two ion, transverse modes

RSB
Tilt x, CM x, Tilt y, CM y

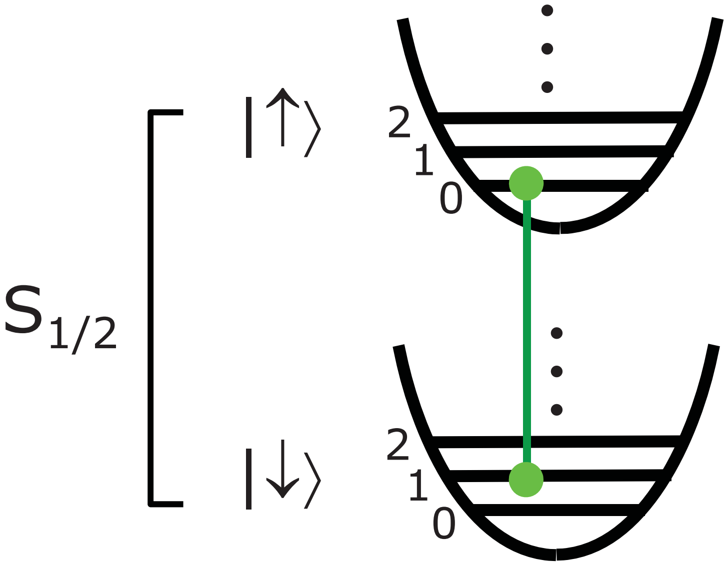
Carrier

BSB
CM y, Tilt y, CM x, Tilt x



excitation on 1st lower ("red") motional sideband (n=0)

$$H_{rsb} = \frac{\hbar\Omega_r}{2}(\sigma_+ a + \sigma_- a^+)$$



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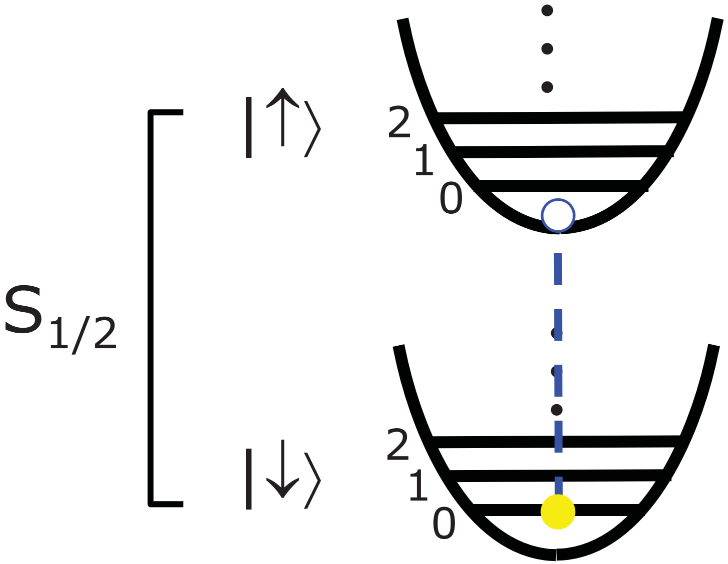

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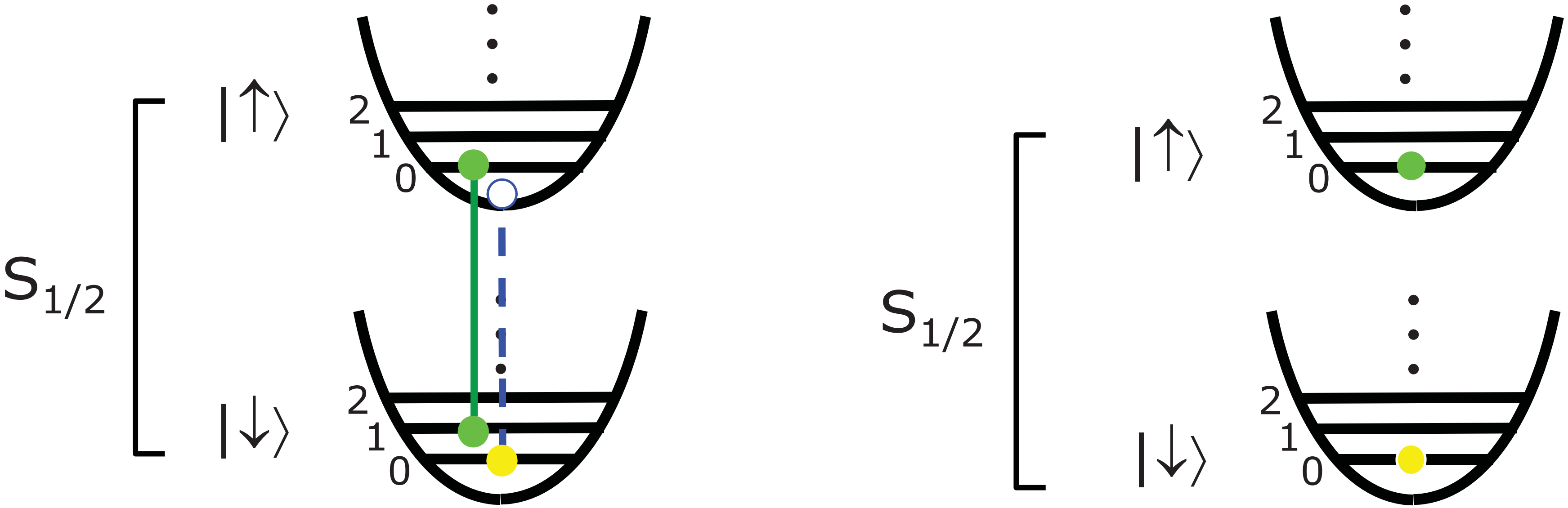
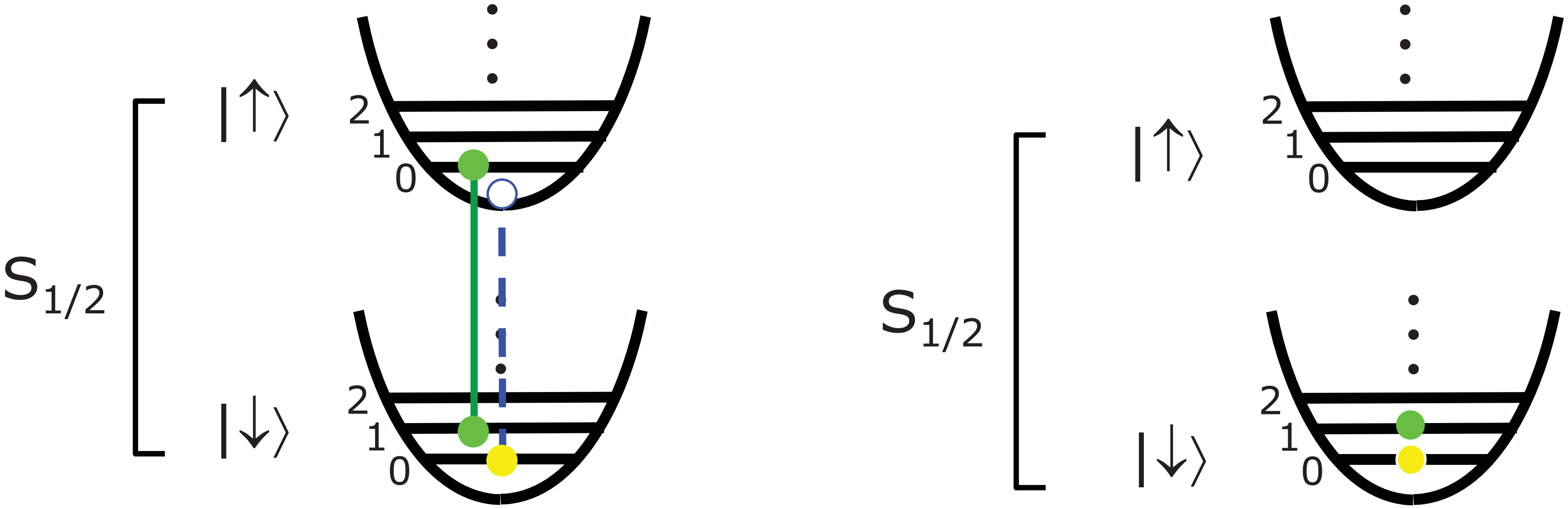


State mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m$

excitation on 1st lower ("red") motional sideband (n=0)

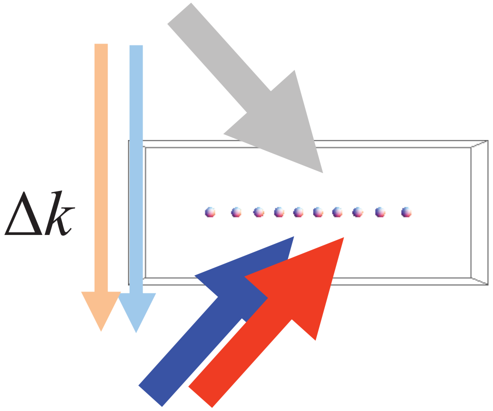
$$H_{rsb} = \frac{\hbar\Omega_r}{2}(\sigma_+ a + \sigma_- a^+)$$



State mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m \rightarrow \;|\downarrow\rangle\,(\alpha|0\rangle_m + \beta|1\rangle_m)$

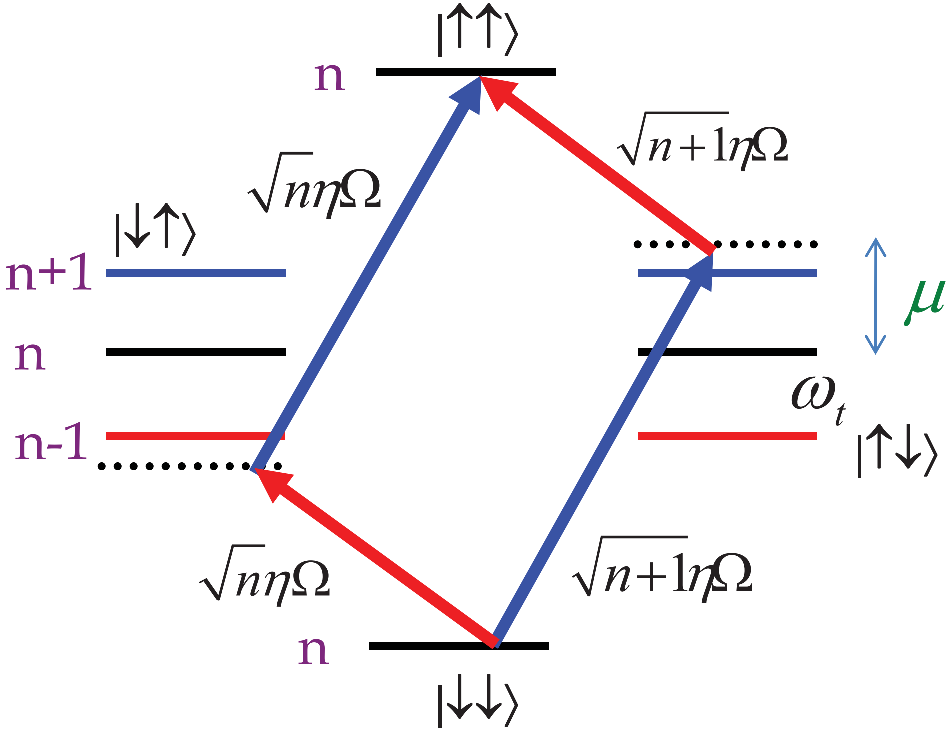
Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)

Molmer-Sorensen ($\sigma_x \otimes \sigma_x$) Gate with Two Ions



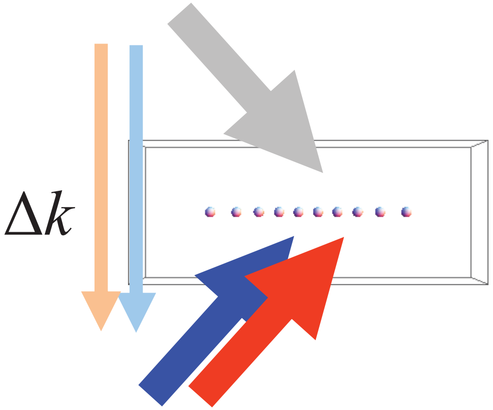
$$\omega_b = \omega_{HF} + \mu$$

$$\omega_r = \omega_{HF} - \mu$$



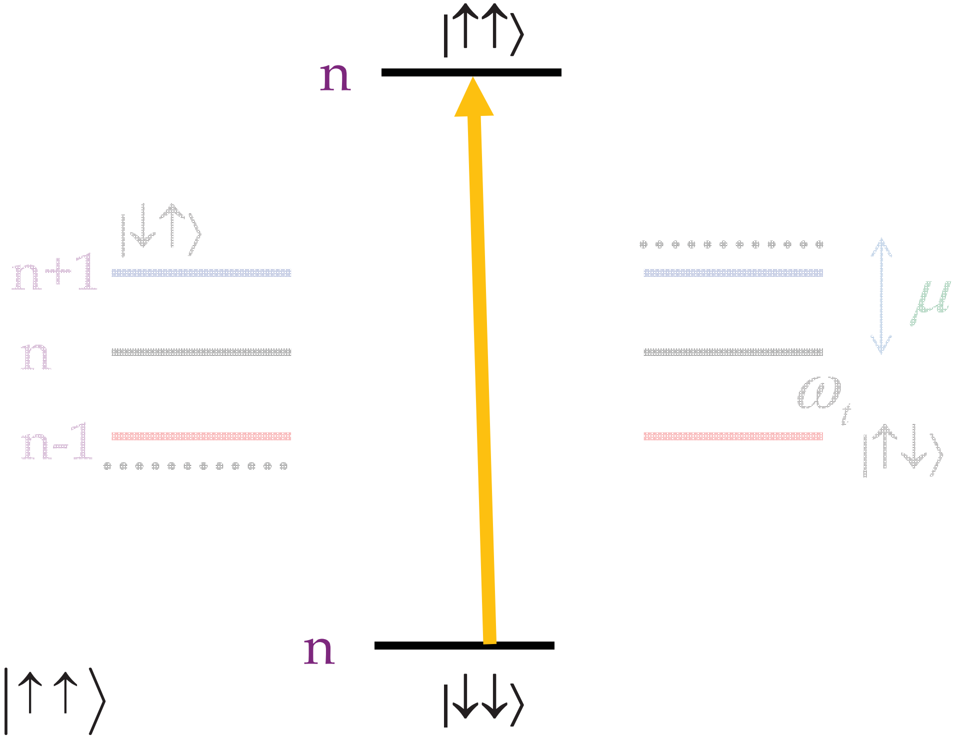
$$\tilde{\Omega} = \frac{\eta^2\Omega^2}{\mu - \omega_t}(\cancel{n} + 1 - \cancel{n})$$

Molmer-Sorensen ($\sigma_x \otimes \sigma_x$) Gate with Two Ions



$$\omega_b = \omega_{HF} + \mu$$

$$\omega_r = \omega_{HF} - \mu$$



$$|\downarrow\downarrow\rangle \Rightarrow \cos(\frac{\tilde{\Omega}T}{2})|\downarrow\downarrow\rangle + e^{i\phi}\sin(\frac{\tilde{\Omega}T}{2})|\uparrow\uparrow\rangle$$

choose $\frac{\tilde{\Omega}T}{2} = \frac{\pi}{4}$, then

$$|\downarrow\downarrow\rangle \Rightarrow \frac{1}{\sqrt{2}}|\downarrow\downarrow\rangle + e^{i\phi}\frac{1}{\sqrt{2}}|\uparrow\uparrow\rangle$$

$$\tilde{\Omega} = \frac{\eta^2\Omega^2}{\mu - \omega_t}(\cancel{n} + 1 - \cancel{n})$$

Molmer & Sorensen, PRL 82, 1971 (1999) & PRL 82, 1835 (1999)

Entangling Ions via Spin-Dependent Force

Bichromatic Raman lasers create spin-dependent force:

$$H = -F\hat{x}(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) = Fx_0(\hat{a} + \hat{a}^+)\hat{\sigma}_{z(\text{or } x,y)}$$

$$H(t) = \tfrac{1}{2}\hbar\Omega\sum_{i,m}\sigma_i^x[a_m e^{-i\delta_m t} + a_m^+ e^{i\delta_m t}]$$

$$|\psi\rangle_f = U(\tau)|\downarrow\downarrow\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + e^{i\phi}|\uparrow\uparrow\rangle)$$

$$U(\tau) = \mathbf{T}\exp\{-\frac{i}{\hbar}\int_0^\tau H(t)dt\}$$

Magnus expansion

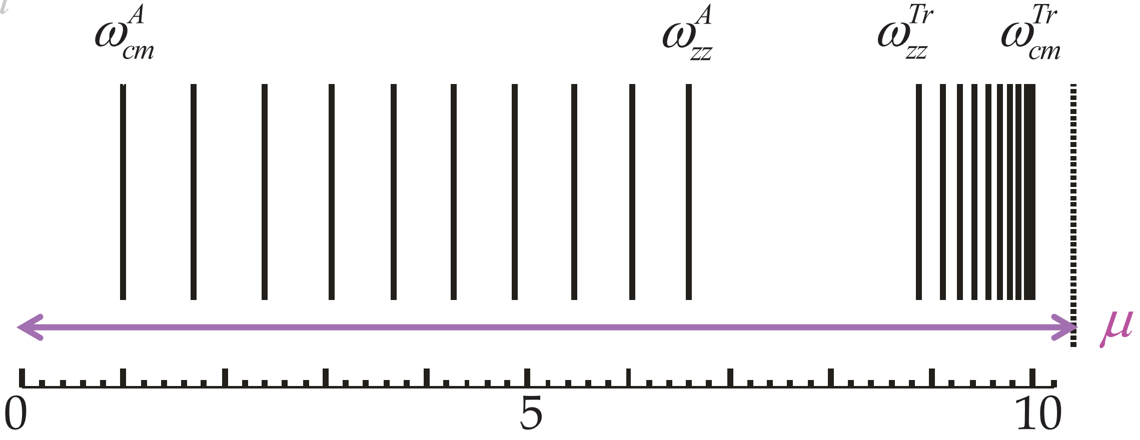
$$= \exp\{-\frac{i}{\hbar}\int_0^\tau H(t)dt - \frac{i}{2\hbar}\int_0^\tau dt_1\int_0^{t_1} dt_2[H(t_1), H(t_2)] + \cdots\}$$

$$\sim \exp\{-i\tilde{\Omega}\tau\sigma_1^x\sigma_2^x\}$$

Simulating spin-spin interactions J_{ij}

$$H = \sum_{i<j} J_{ij}\sigma_x^i\sigma_x^j + B_y \sum_i \sigma_y^i$$

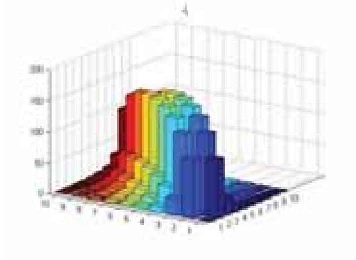
$$J_{i,j} = \sum_m \frac{\Omega_i\Omega_j\eta_i^m\eta_j^m\omega_m}{\mu^2-\omega_m^2},$$



S.-L. Zhu, et al., PRL 97, 050505 (2006)

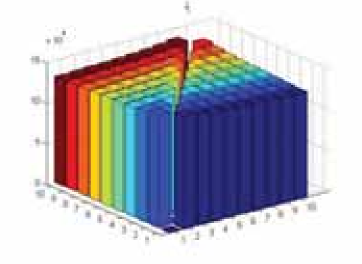
Anti-ferro, short range

$\mu = 1.1\omega_{cm}$



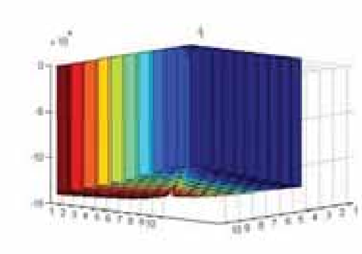
Anti-ferro, long range

$\mu = 1.0001\omega_{cm}$

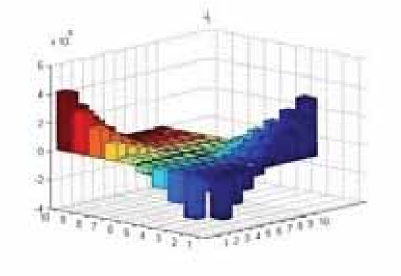


Ferro, long range

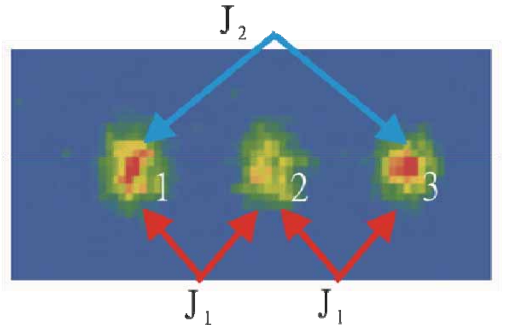
$\mu = 0.9999\omega_{cm}$

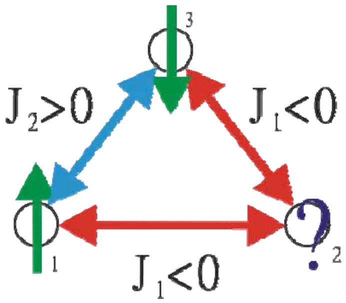


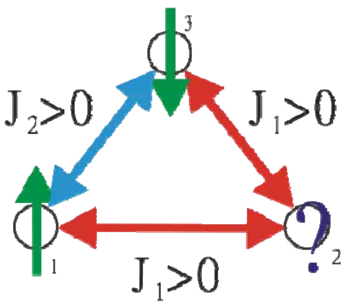
$\mu = 0.9949\omega_{cm}$

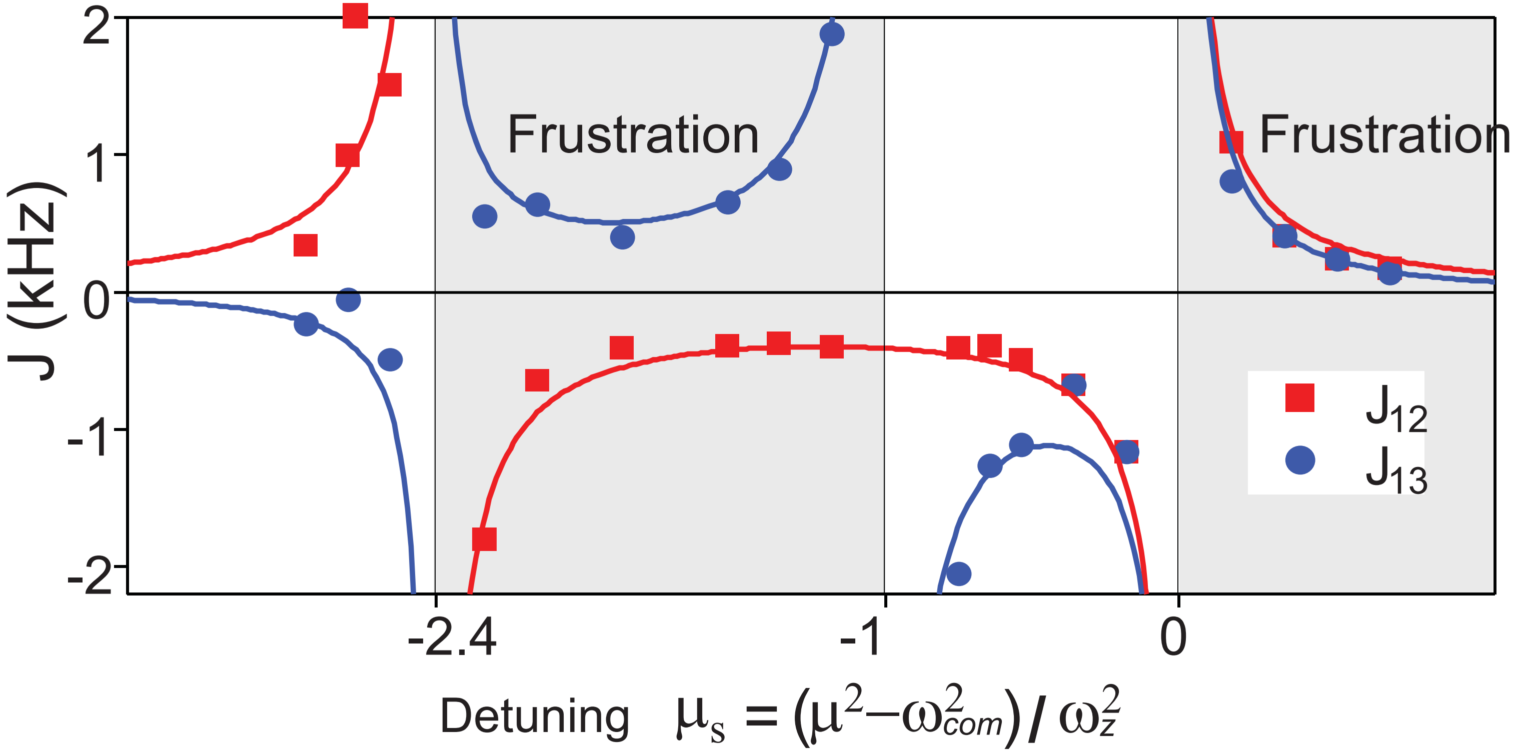


Spin frustration in triangular lattice



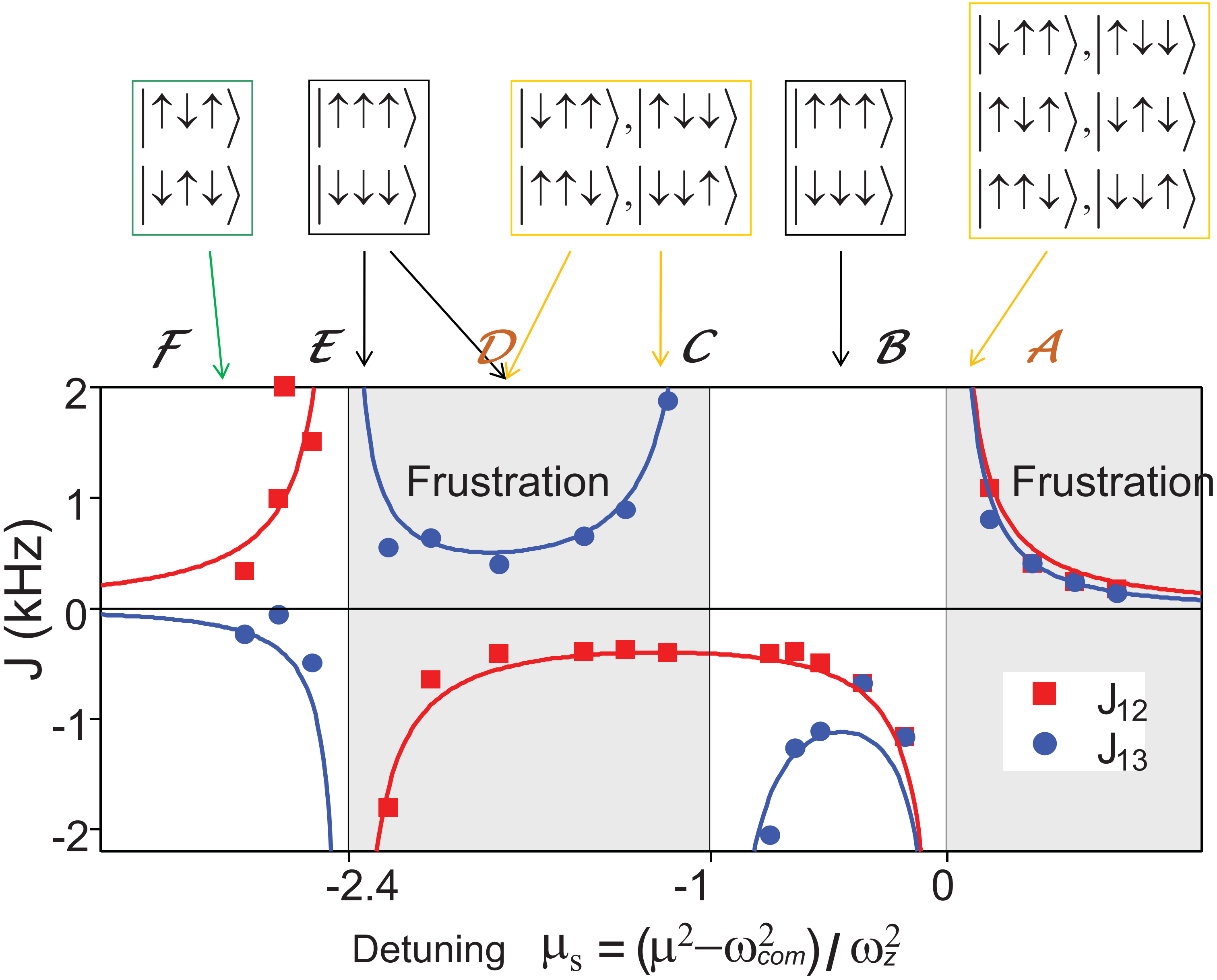






Kim et al., Nature **465**, 590 (2010)

Favorable states vs. spin-spin couplings



$|\uparrow\downarrow\uparrow\rangle$

$|\downarrow\uparrow\downarrow\rangle$

$|\uparrow\uparrow\uparrow\rangle$

$|\downarrow\downarrow\downarrow\rangle$

$|\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\rangle$

$|\uparrow\uparrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle$

$|\uparrow\uparrow\uparrow\rangle$

$|\downarrow\downarrow\downarrow\rangle$

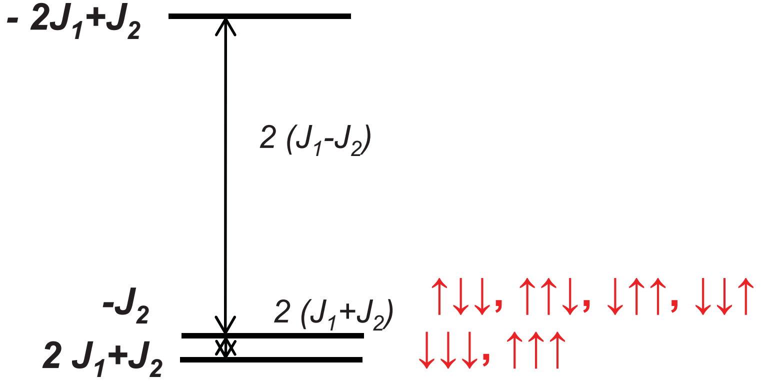
$|\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\rangle$

$|\uparrow\downarrow\uparrow\rangle, |\downarrow\uparrow\downarrow\rangle$

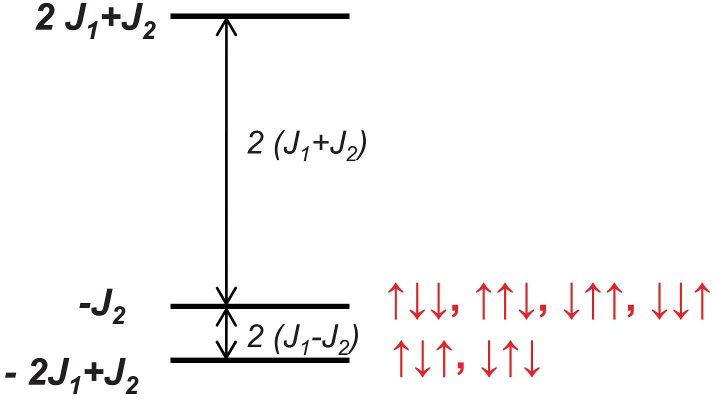
$|\uparrow\uparrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle$

Ground state degeneracy

$\mathcal{D}$ $J_1<0, J_2>0, |J_1| \sim J_2$

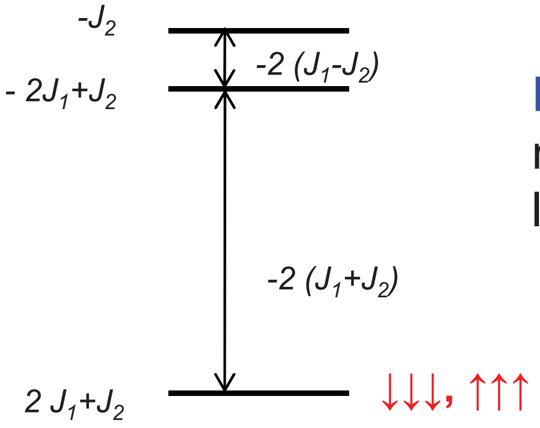


$\mathcal{A}$ $J_1>J_2>0$



Frustration leads to large degeneracy (gronund state entropy)

$\mathcal{B}$ $J_2<J_1<0$

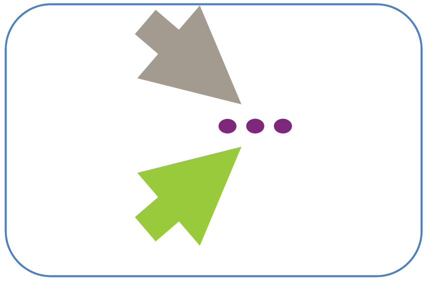


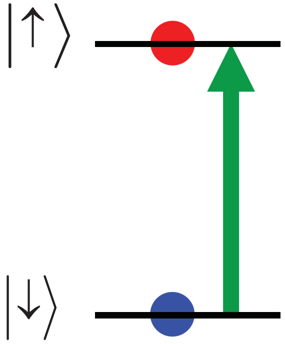
Ferromagnetic interaction
no frustration
less degeneracy

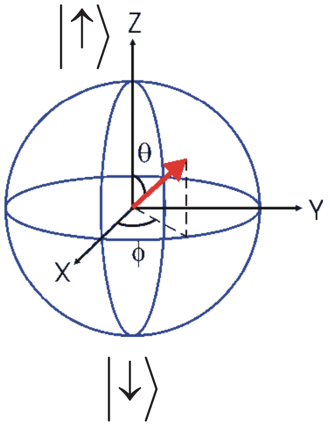
L. Pauling (1945)
Science **294**, 1495–1501 (2001),
Phys. Today **59**, 24 (2006)

Simulating a B field (single qubit rotation)

$$H_{XY} = \sum_{i<j} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} + B\sum_i \sigma_y^{(i)}$$

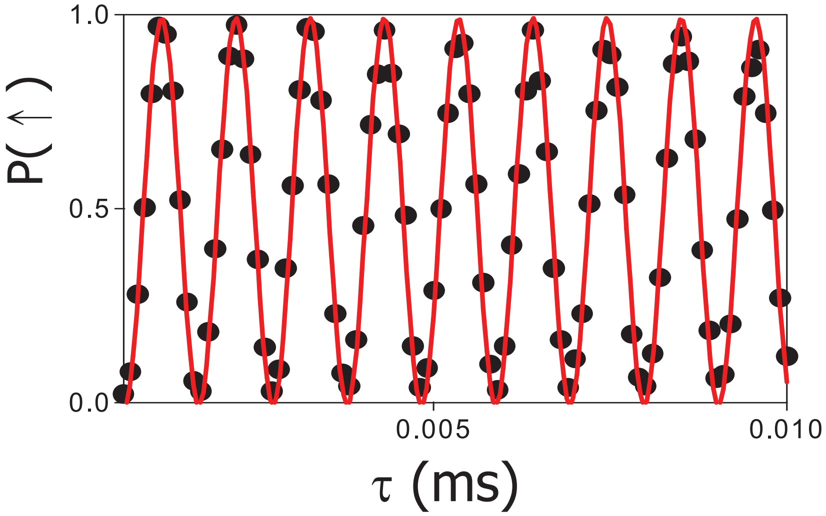




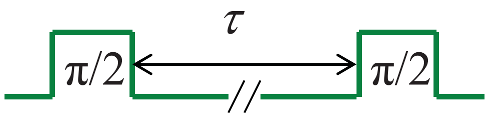


$$R_{\phi+\pi/2}(\theta)$$

Rabi oscillations



Ramsey oscillations



Coherence time > 70 ms

Outline

- ❑ Motivation
  - Preliminary
  - Quantum magnets
  - Idea of quantum simulator

- ❑ Trapped ion quantum simulator
  - Coupling ions with transverse normal modes
  - Tuning spin-spin couplings for QS

- ❑ Quantum simulation of the smallest spin network
  - Phase diagram
  - Magnetic frustration

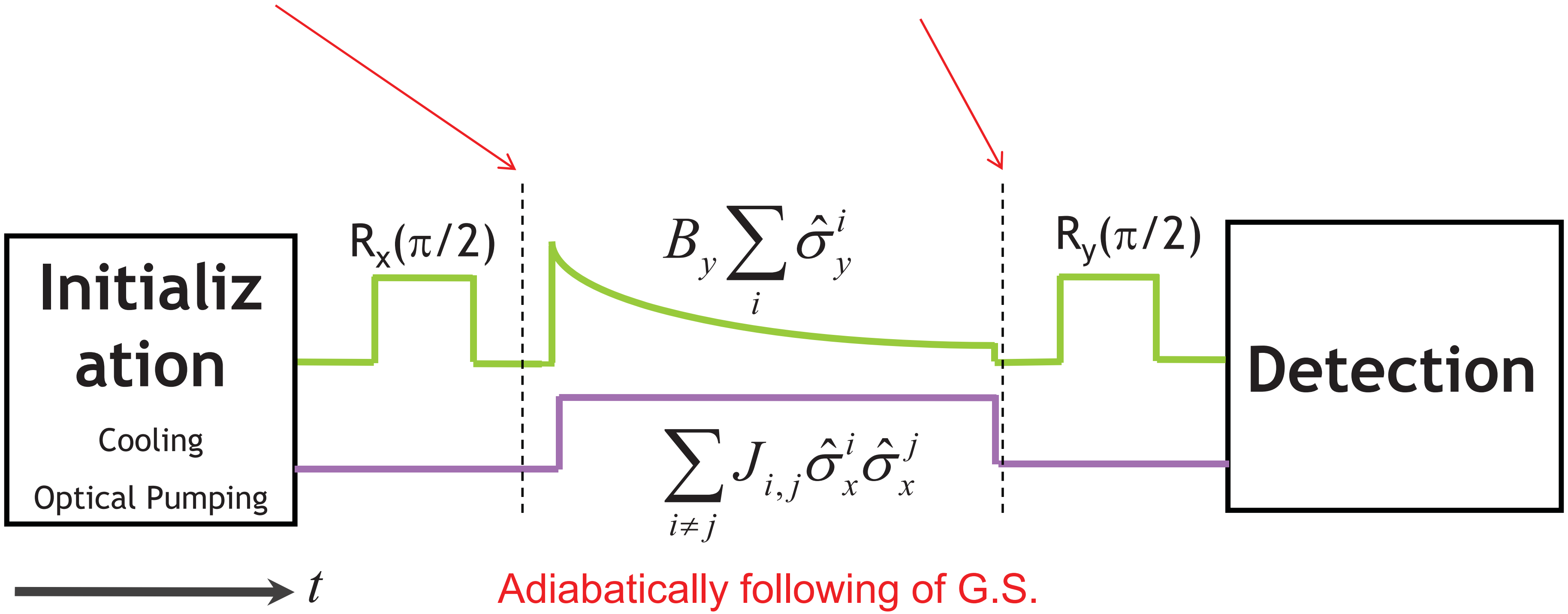
- ❑ Outlook

Quantum simulation in action

$$H\,(t) = \sum_{i<j} J_{ij}\sigma_x^i\sigma_x^j + B_y\sum_i \sigma_y^i$$

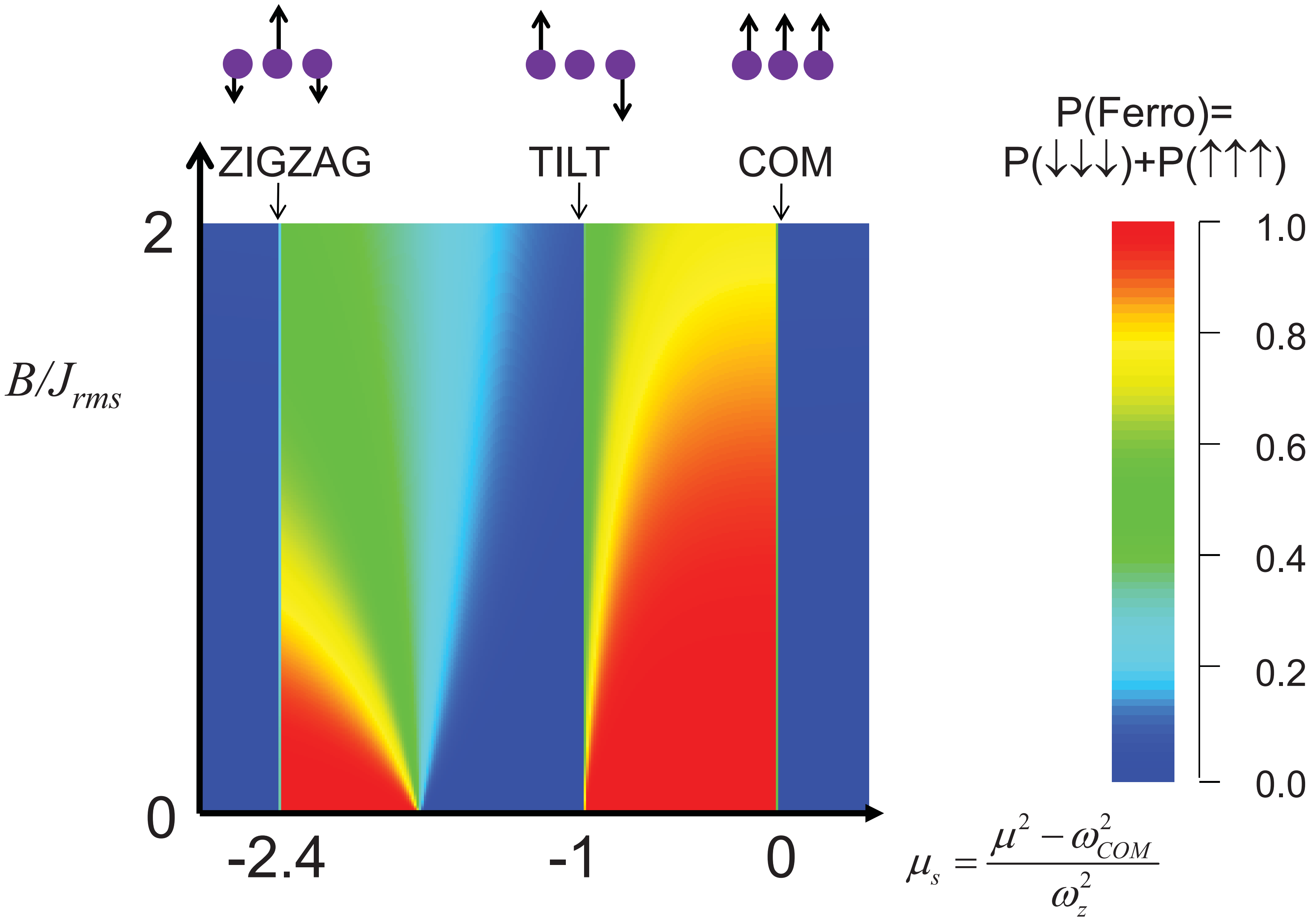
$$|\psi(0)\rangle = |\downarrow_y\downarrow_y\downarrow_y\cdots\rangle$$

$$|\psi\,(t)\rangle = \hat{\mathrm{T}}\,e^{-\frac{i}{\hbar}\int H\,(t)\,dt}\,|\psi\,(0)\rangle$$

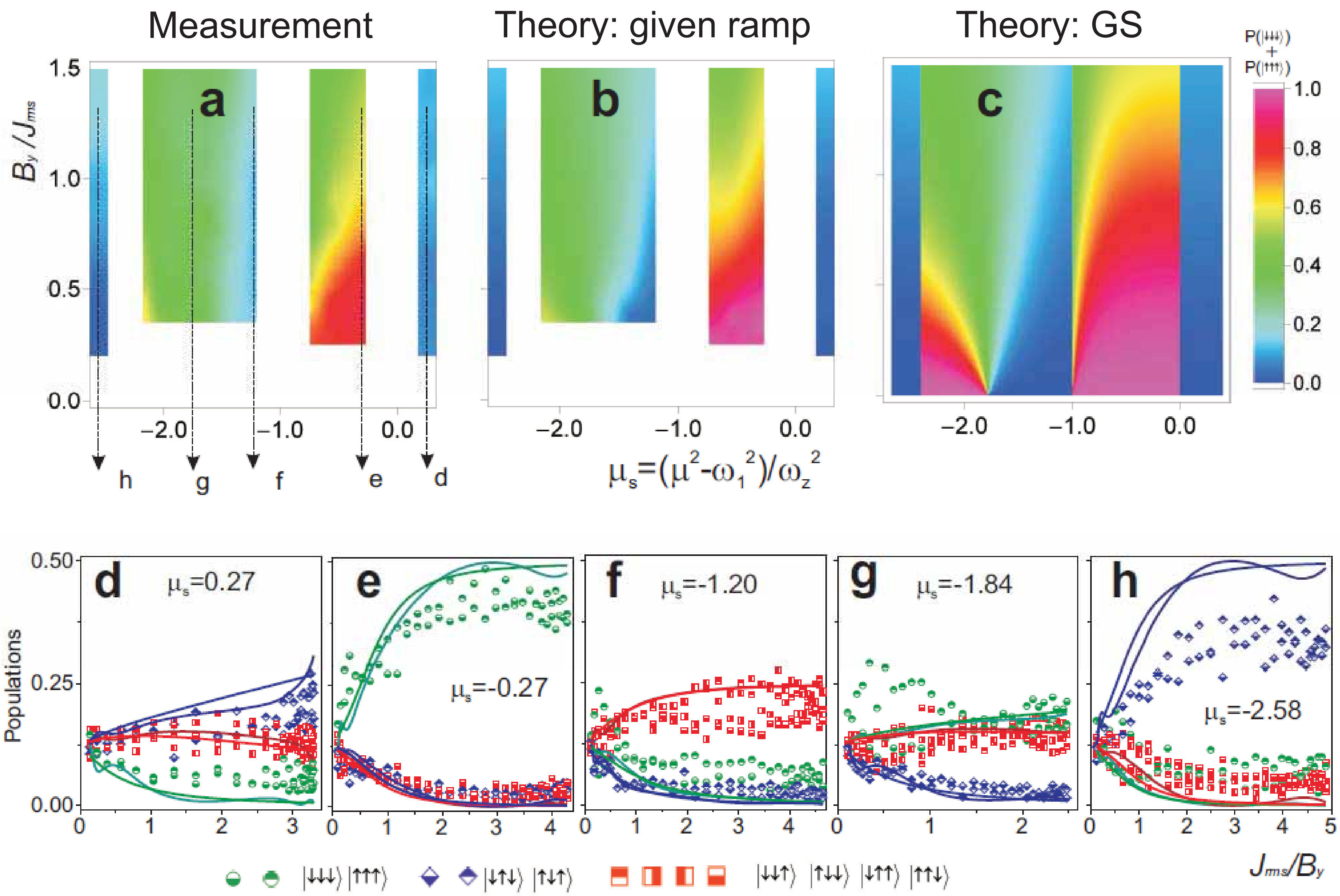


Adiabatically following of G.S.

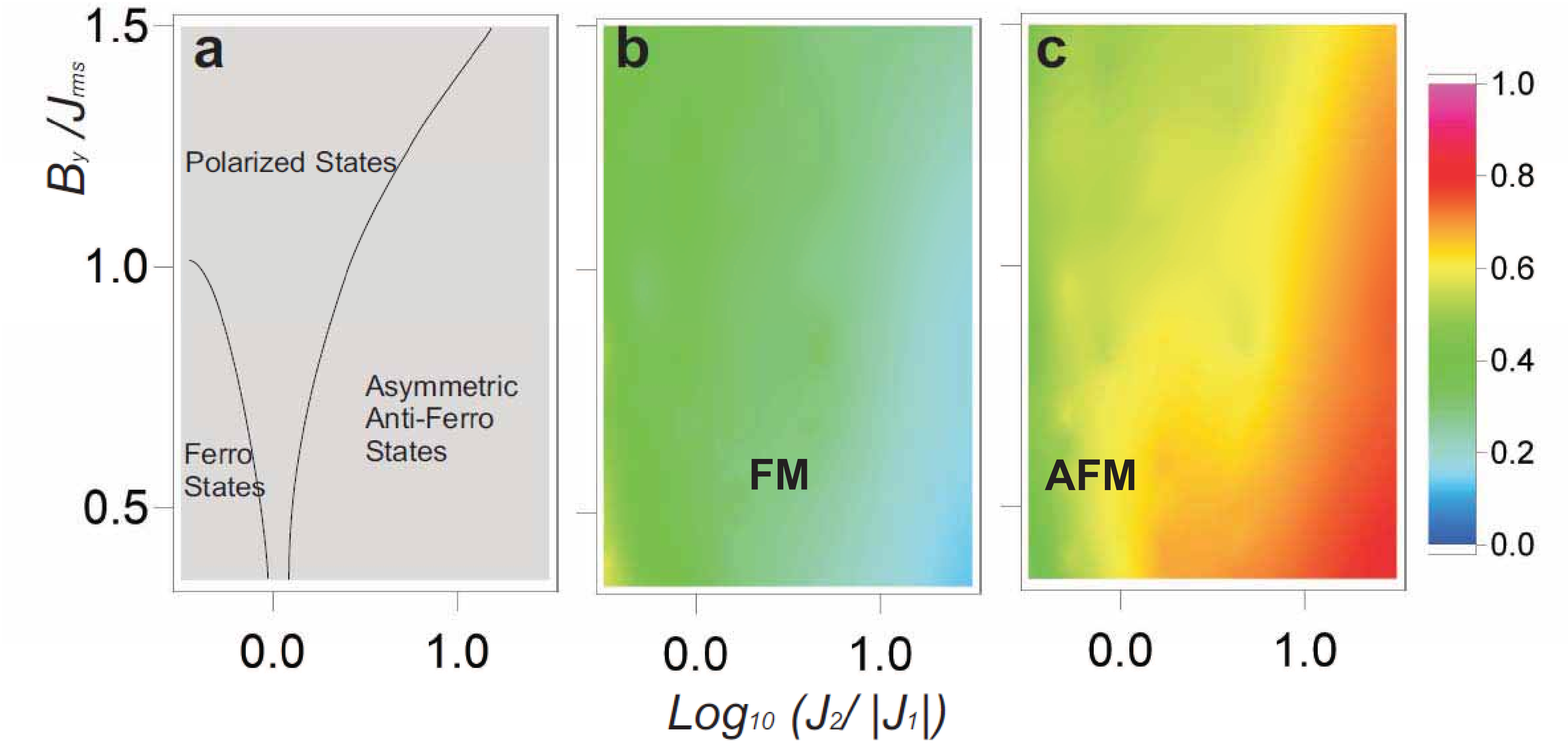
Exact Ground State (Theory) – Freericks and Duan



Phase diagram measurement



Universal phase diagram



Ground state entanglement or entropy?

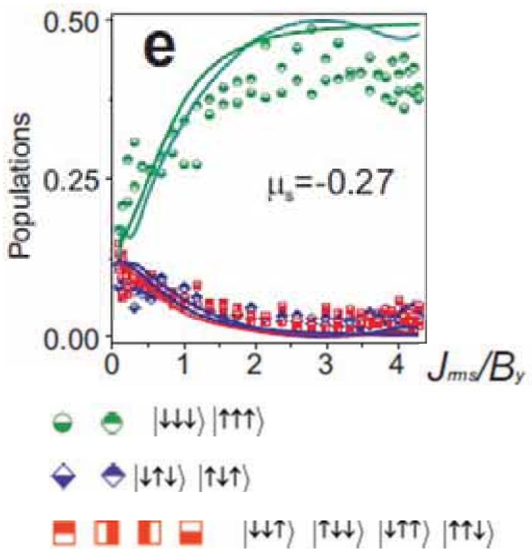
What is the FM ground state?

$$\rho_p = |\psi\rangle\langle\psi|$$

with $|\psi\rangle = \left(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\right)/\sqrt{2}$

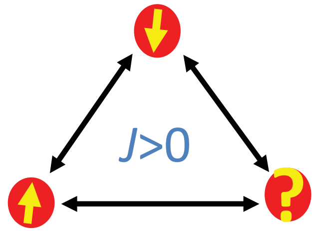
or $\rho_m = \frac{1}{2}|\uparrow\uparrow\uparrow\rangle\langle\uparrow\uparrow\uparrow| + \frac{1}{2}|\downarrow\downarrow\downarrow\rangle\langle\downarrow\downarrow\downarrow|$

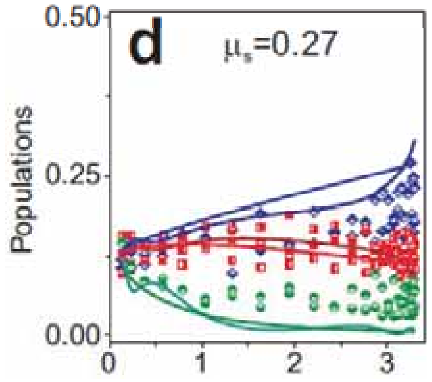
or $\rho = \alpha\rho_p + (1-\alpha)\rho_m$



How about AFM frustrated ground state?

$$|\psi\rangle = |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - |\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\rangle?$$





If we know the density matrix (8 x 8 C-numbers), we can know
the underlying state. However, density matrix is hard to reconstruct.

Short cut?

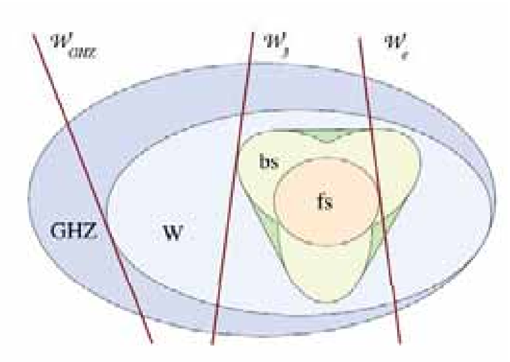
Detecting the frustrated ground state: entanglement detection

$$\left|\psi_{FM}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\uparrow\uparrow\right\rangle - \left|\downarrow\downarrow\downarrow\right\rangle\right)?$$

$$\left\langle W_{GHZ}\right\rangle = \frac{9}{4} - J_z^2 - \sigma_\phi^1\sigma_\phi^2\sigma_\phi^3, \quad J_\alpha = \frac{1}{2}\sum_i \sigma_\alpha^i, \quad \phi = y,$$

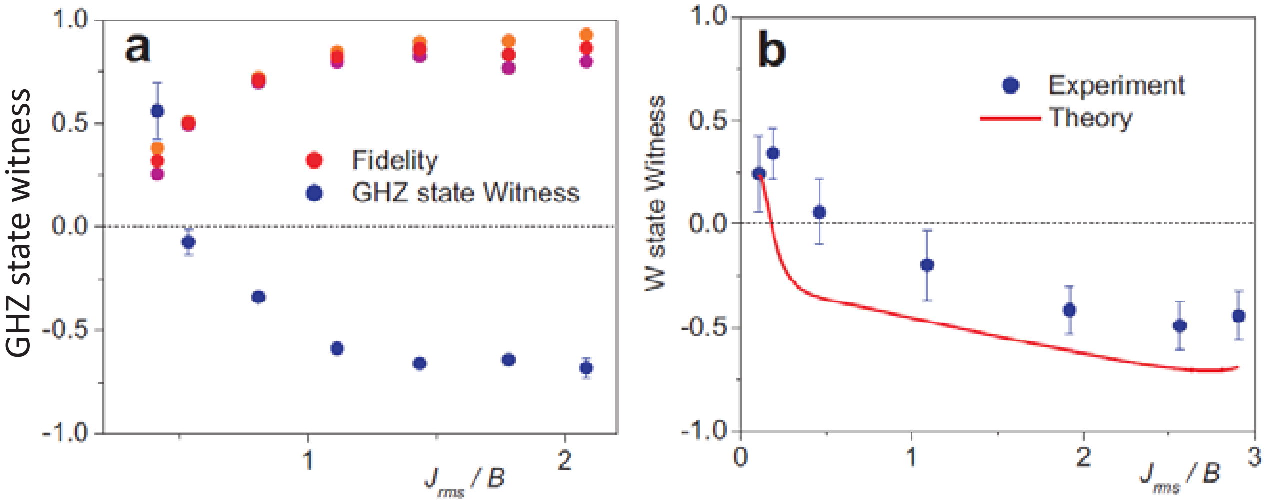
$$\left|\psi_{AFM}\right\rangle = \frac{1}{\sqrt{6}}\left(\left|\uparrow\uparrow\downarrow\right\rangle + \left|\uparrow\downarrow\uparrow\right\rangle + \left|\downarrow\uparrow\uparrow\right\rangle - \left|\downarrow\downarrow\uparrow\right\rangle - \left|\downarrow\uparrow\downarrow\right\rangle - \left|\uparrow\downarrow\downarrow\right\rangle\right)?$$

$$\left\langle W_W\right\rangle = \left(4 + \sqrt{5}\right) - 2(J_y^2 + J_z^2)$$



$$\mathrm{Tr}(\mathcal{W}\varrho_s) \geq 0 \quad \text{for all separable } \varrho_s,$$
$$\mathrm{Tr}(\mathcal{W}\varrho_e) < 0 \quad \text{for at least one entangled } \varrho_e$$

Ghne and Toth, Phys Rep **474,** 1 (2009),
Horodecki family, RMP **81**, 865 (2009).



Links frustration to ground state entanglement.

Nature **465**, 590 (2010)

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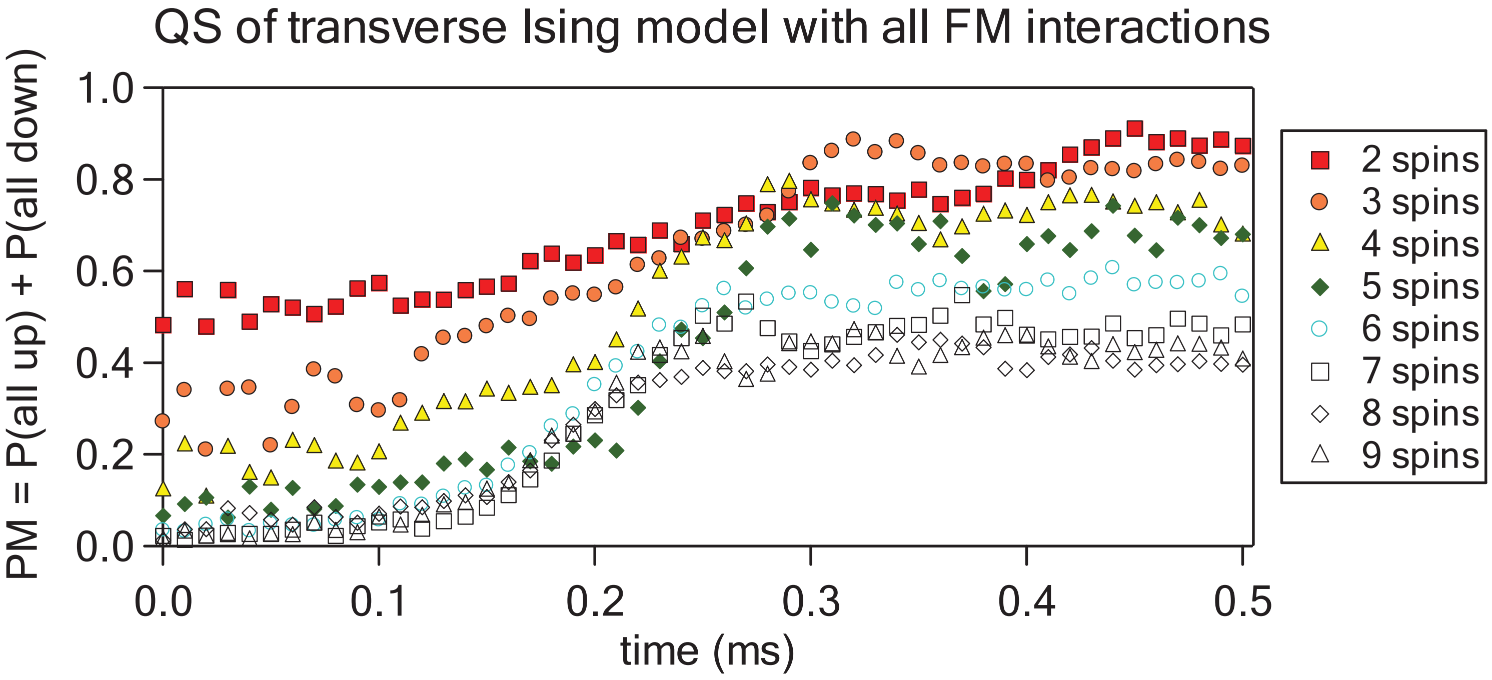
- Outlook

The more ions the better

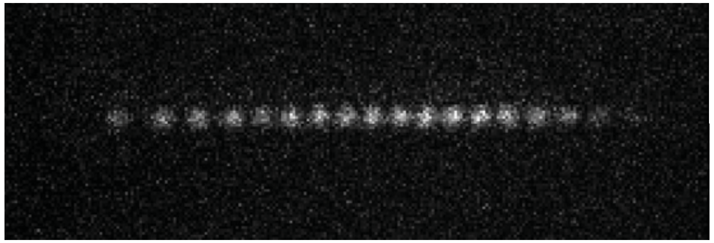


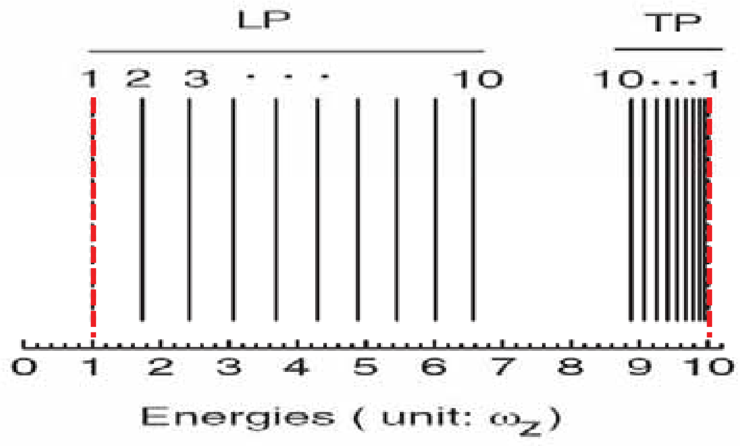
Sharper phase transition as # of spins increases.

Islam et al., arXiv:1103.2400v1 (2011)

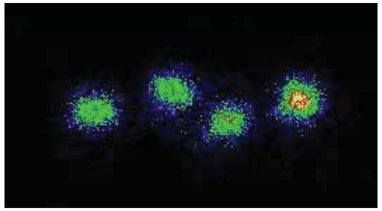
Scalability in a linear Paul trap

Harmonic external axial potential (ω_z)





linear crystal: $\frac{\omega_r}{\omega_z} > 0.73N^{0.86}$

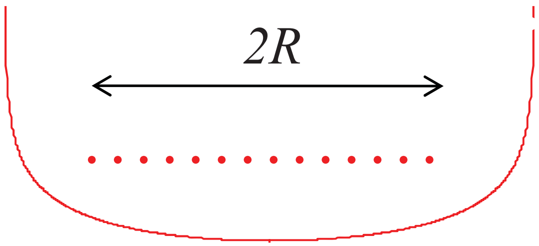


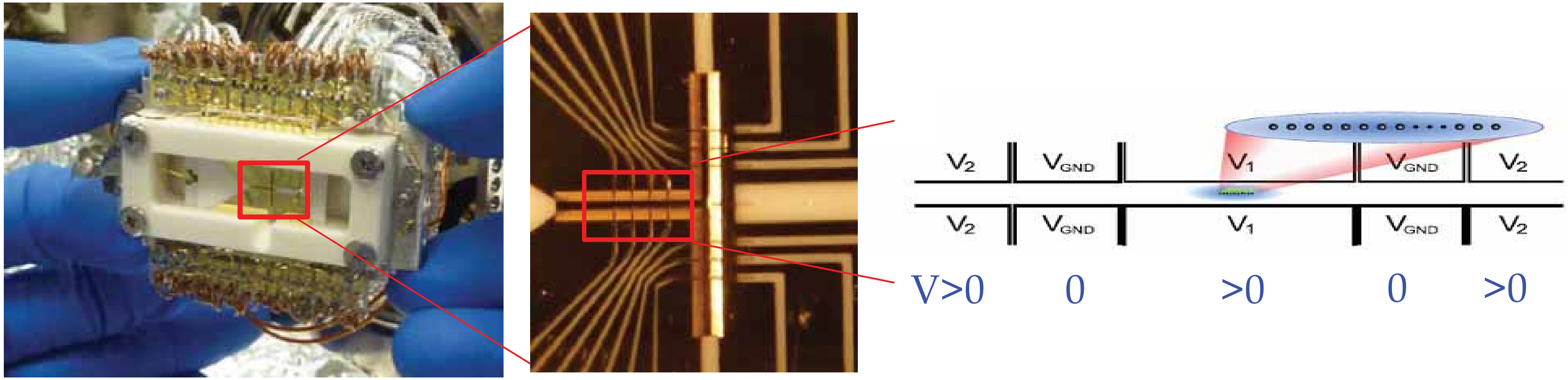
Scaling a single crystal to >> 10 ions

Uniformly-spaced ion crystal (spacing = s) $\omega_r > \sqrt{\frac{7\zeta(3)e^2}{2ms^3}}$

Requires $U(z) = U_0 \log\left(\frac{1}{1 - z^2/R^2}\right) \sim \alpha z^4$ (quartic)

Lin et al., Europhys. Lett. **86**, 60004 (2009).





V>0 0 >0 0 >0

Summary

- Trapped ion quantum simulator
  - Coupling ions with transverse normal modes
  - Engineer spin-spin interactions

- Quantum simulator of the smallest spin network
  - Phase diagram
  - Spin frustration
  - Ground state entanglement

- Outlook
  - Scalable to larger number of spins
  - XY or XXY models