Dynamics of Two Detectors in a Relativistic Quantum Field: Some Remarks

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Outline

- I. Introduction
- II. Event Horizon and Unruh effect
- **III.** Entanglement Dynamics of Open Systems
- IV. Correlators vs. Density Matrix
- V. Symmetries in Field States

RQI models:

- 1. Entanglement of atoms/qubits moving in flat or curved spacetime
- 2. Entanglement of relativistic quantum fields (in curved spacetime)
- 1+2: Entanglement of atoms moving in relativistic quantum fields

Relativistic quantum fields are treated as environment.

E.g. (Cavity) QED: atoms in EM field
[Anastopoulos, Shresta, Hu 06]
Unruh-DeWitt detector theory: accelerated HO in scalar field
[Lin, Chou, Hu 08, Lin, Hu 09, 10]

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Unruh-DeWitt detector theory in (3+1)D
 [Lin PRD (03); Lin Hu PRD (06)]

$$\begin{split} S &= S_Q + S_\Phi + S_I, \\ S_Q &= \int d\tau \frac{m_0}{2} \left[(\partial_\tau Q)^2 - \Omega_0^2 Q^2 \right] \\ S_\Phi &= -\int d^4 x \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \\ S_I &= \lambda_0 \int d\tau \int d^4 x Q(\tau) \Phi(x) \frac{\delta^4 \left(x^\mu - z^\mu(\tau) \right)}{\text{Point-like object}} \end{split}$$

Internal: harmonic oscillator

Massless scalar field

bilinear interaction [DeWitt 1979]

with prescribed trajectory (UAD)

$$z^{\mu}(\tau) = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$$

We have -- quantum field theory + quantum mechanics + classical external agent

Reported in IARD 2006 @ Connecticut

- For mathematical convenience, we choose the initial conditions
- Sudden switch-on $S_I \sim \theta(\tau \tau_0)$
- Free operators before au_0
- Factorized initial state $|\tau_0\rangle = |q\rangle |0_M\rangle$

field: Minkowski vacuum $| 0_M \rangle$ detector: coherent state $| q \rangle = e^{-\alpha^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} | n \rangle$

I. Introduction Reported in IARD 2006 @ Connecticut

Determine the radiation for the Minkowski observer in (3+1)D

Find the quantum expectation values of stress-energy tensor

$$\begin{cases} \text{classical} & T_{\mu\nu}[\Phi(x)] = (1 - 2\xi) \,\Phi_{,\mu} \Phi_{,\nu} - 2\xi \Phi \Phi_{;\mu\nu} + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu} \Phi^{,\rho} \Phi_{,\rho} + \frac{\xi}{2} g_{\mu\nu} \Phi \Box \Phi \end{pmatrix} \\ \text{quantum} & \left\langle T_{\mu\nu}[\Phi(x)] \right\rangle_{\text{ren}} = \lim_{x' \to x} \left[\frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial x'^{\nu}} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \frac{\partial}{\partial x^{\rho}} \frac{\partial}{\partial x'^{\sigma}} \right] G_{\text{ren}}(x, x') \end{cases}$$

Then calculate the radiation formula for Minkowski observer at null infinity through

$$\begin{array}{ll} \textbf{(classical} & \frac{dW^{\mathrm{rad}}}{d\tau_{-}} = -\lim_{r \to \infty} \int r^2 d\Omega_{\mathrm{II}} u^{\mu} T_{\mu\nu} v^{\nu}(\tau_{-}) & \textbf{)} \\ \\ \textbf{quantum} & \left\langle \frac{dW^{\mathrm{rad}}}{d\tau_{-}} \right\rangle = -\lim_{r \to \infty} \int r^2 d\Omega_{\mathrm{II}} u^{\mu} \left\langle T_{\mu\nu} \right\rangle_{\mathrm{ren}} v^{\nu}(\tau_{-}) \end{array}$$



 Entanglement dynamics between 2 identical Unruh-DeWitt detectors in (3+1)D Minkowski space [Lin Chou Hu 08; Lin Hu 09, 10]

$$S = -\int d^{4}x \frac{1}{2} \partial_{\mu} \Phi \partial^{\mu} \Phi \qquad - \text{ massless scalar field} \\ + \int d\tau_{A} \frac{1}{2} \left[(\partial_{\tau_{A}} Q_{A})^{2} - \Omega_{0}^{2} Q_{A}^{2} \right] + \int d\tau_{B} \frac{1}{2} \left[(\partial_{\tau_{B}} Q_{B})^{2} - \Omega_{0}^{2} Q_{B}^{2} \right] \qquad - \text{ internal: HO x 2} \\ + \lambda_{0} \int d^{4}x \Phi(x) \left[\int d\tau_{A} Q_{A}(\tau_{A}) \delta^{4} \left(x^{\mu} - z^{\mu}_{A}(\tau_{A}) \right) + \int d\tau_{B} Q_{B}(\tau_{B}) \delta^{4} \left(x^{\mu} - z^{\mu}_{B}(\tau_{B}) \right) \right] \\ - \text{ bilinear interaction [DeWitt 1979]} \\ \text{ Detectors A, B are point-like objects.}$$

Features:

- 1. Linear, crystal clear;
- 2. Simplest atom-field interacting system;
- 3. In some cases there exist analytic results in the whole parameter range;
- 4. Complicated enough to give nontrivial results and insights.





A uniformly accelerated detector acts as if immersed in a bath at Unruh temperature.



Evolution of the self correlators of a detector initially in its ground state:

while $\langle P_A, Q_A \rangle = (m_0/2)(d/d\tau)\langle Q^2 \rangle$ are oscillating in small amplitude O(γ).

A uniformly accelerated detector acts as if immersed in a bath at Unruh temperature.



Evolution of the self correlators of a detector initially in its ground state:

However, the early-time behavior will be the same if the proper acceleration is nonzero only in a finite duration. In this case, no event horizon at all.



[Ostapchuk, Lin, Hu, Mann, in preparation]



III. Entanglement Dynamics of Open Systems

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Relativistic quantum fields as the environment:

- Looking at the reduced density matrix of the detectors, which is a <u>mixed</u> state in general.
- Entanglement between two detectors becomes time-dependent even for both detectors in eigenstates initially and no direct interaction between them:

sudden death, revival, creation...of entanglement.

The qualities of environment, such as spatial dependence, mutual influences, echo from the boundary, etc., will enter the entanglement dynamics of the system.

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Determine the radiation for the Minkowski observer in (3+1)D

Find the quantum expectation values of stress-energy tensor

$$\begin{array}{l} \text{(classical} \quad T_{\mu\nu}[\Phi(x)] = (1 - 2\xi) \,\Phi_{,\mu} \Phi_{,\nu} - 2\xi \Phi \Phi_{;\mu\nu} + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu} \Phi^{,\rho} \Phi_{,\rho} + \frac{\xi}{2} g_{\mu\nu} \Phi \Box \Phi \end{array} \right) \\ \text{(quantum (}\xi = 0 \text{)} \quad \left\langle T_{\mu\nu}[\Phi(x)] \right\rangle_{\text{ren}} = \lim_{x' \to x} \left[\frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial x'^{\nu}} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \frac{\partial}{\partial x^{\rho}} \frac{\partial}{\partial x'^{\sigma}} \right] G_{\text{ren}}(x, x')$$

Then calculate the radiation formula for Minkowski observer at null infinity through

(classical
$$\frac{dW^{\rm rad}}{d\tau_{-}} = -\lim_{r \to \infty} \int r^2 d\Omega_{\rm II} u^{\mu} T_{\mu\nu} v^{\nu}(\tau_{-})$$
)
quantum
$$\left\langle \frac{dW^{\rm rad}}{d\tau_{-}} \right\rangle = -\lim_{r \to \infty} \int r^2 d\Omega_{\rm II} u^{\mu} \langle T_{\mu\nu} \rangle_{\rm ren} v^{\nu}(\tau_{-})$$







III. Entanglement Dynamics of Open Systems

Entanglement can be created outside light cone by quantum fields.



III. Entanglement Dynamics of Open Systems Entanglement can be created outside light cone by quantum fields.



- 1. The detectors will be disentangled at late times.
- 2. It seems that the "entanglement time" is after the moment that the third party is able to received information from both detectors

Non-Uniform Acceleration

[Ostapchuk, Lin, Hu, Mann, in preparation]



is off resonance]

Mutual influences are small in weak coupling and large distance limit, though.

 Master equation for the reduced density matrix of two oscillator (located at the same space point) is complicated! [Chou Yu Hu 08]

$$\begin{split} i\hbar\frac{\partial\rho_r}{\partial t} &= -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial y_2^2}\right)\rho_r + \frac{1}{2}M\Omega^2(x_1^2 - y_1^2 + x_2^2 - y_2^2)\rho_r \\ &\quad + \frac{1}{2}M\delta\Omega^2(t)(x_1 - y_1 + x_2 - y_2)\frac{1}{2}(x_1 + y_1 + x_2 + y_2)\rho_r \\ &\quad -i\hbar\Gamma(t)(x_1 - y_1 + x_2 - y_2)\frac{1}{2}\left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial y_1} + \frac{\partial}{\partial x_2} - \frac{\partial}{\partial y_2}\right)\rho_r \\ &\quad -iM\Sigma(t)(x_1 - y_1 + x_2 - y_2)^2\rho_r \\ &\quad + \hbar\Delta(t)(x_1 - y_1 + x_2 - y_2)\left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial y_2}\right)\rho_r. \end{split}$$

 $\delta\Omega^2(t), \Gamma(t), \Delta(t), \Sigma(t)$ depend on spectral density of environment and time.

Sketch of our calculations

• Evolution of operators Q_A , P_A , Q_B , P_B , Φ , Π in Heisenberg picture.

$$\hat{Q}_{i}(\tau_{i}) = \sqrt{\frac{\hbar}{2\Omega_{r}}} \sum_{i} \left[q_{i}^{(j)}(\tau_{i}) \hat{a}_{j} + q_{i}^{(j)*}(\tau_{i}) \hat{a}_{j}^{\dagger} \right] + \int \frac{d^{3}k}{(2\pi)^{3}} \sqrt{\frac{\hbar}{2\omega}} \left[q_{i}^{(+)}(\tau_{i},\mathbf{k}) \hat{b}_{\mathbf{k}} + q_{i}^{(-)}(\tau_{i},\mathbf{k}) \hat{b}_{\mathbf{k}}^{\dagger} \right]$$
damped HO
damped driven HO

Sandwiched by the initial state: 10 symmetric correlators

$$V_{\mu\nu}(t,\tau) = \langle \mathcal{R}_{\mu}, \mathcal{R}_{\nu} \rangle \equiv \frac{1}{2} \langle (\mathcal{R}_{\mu}\mathcal{R}_{\nu} + \mathcal{R}_{\nu}\mathcal{R}_{\mu}) \rangle \qquad \mathcal{R}_{\mu} = (Q_{B}(\tau), P_{B}(\tau), Q_{A}(t), P_{A}(t))$$

 $\begin{array}{c}
 0 \\
 1 \\
 0
 \end{array}$

• Partial Transposition: $V^{PT} = \Lambda V \Lambda$ $\Lambda = \text{diag}(1, 1, 1, -1)$

The quantity
$$\Sigma(t, \tau = a^{-1} \sinh^{-1} at) \equiv \det \left[\mathbf{V}^{PT} + i \frac{\hbar}{2} \mathbf{M} \right]$$
 $\mathbf{M} \equiv \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

or logarithmic negativity

- degrees of entanglement for mixed Gaussian states

The covariance matrix of the detectors



Correlator dynamics vs. master equation of DM

- In the master equation approach the reduced density matrix (RDM) of two detectors at some moment is defined on a time slice where two detectors has the same time-parameters.
- RDM of the detectors in Gaussian state at some moment in some timeslicing scheme can be fully determined by their symmetric correlators,

$$V_{\mu\nu}(t,\tau) = \langle \mathcal{R}_{\mu}, \mathcal{R}_{\nu} \rangle \equiv \frac{1}{2} \langle (\mathcal{R}_{\mu}\mathcal{R}_{\nu} + \mathcal{R}_{\nu}\mathcal{R}_{\mu}) \rangle \qquad \mathcal{R}_{\mu} = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$$

- One can calculate the correlators of the detectors (located at different spatial points) at <u>different times</u>, i.e. correlations between different events in spacetime.
- RDM in a new time-slicing scheme can still be determined by the same set of correlators after transform the time parameters to the new ones, provided that the time slices where the initial state is living are the same.

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One can express the reduced density matrix $\rho^{R}(Q,Q')$ for the detector <u>at some moment</u> in terms of the two-point correlators and study the non-equilibrium statistical properties. E.g., for a detector initially in a Gaussian state $|E_0\rangle \otimes |0_M\rangle$

$$\rho^{R}(Q,Q';\tau) = Tr_{\Phi}[\rho] = \int \mathcal{D}\Phi_{k}\psi_{0}[Q,\Phi_{k};\tau]\psi_{0}^{*}[Q',\Phi_{k};\tau]$$
$$= \exp\left[-G^{ij}(\tau)Q_{i}Q_{j} - F(\tau)\right]$$
Gaussian functions

where

$$\begin{aligned} G^{11} + G^{22} + 2G^{12} &= \frac{1}{2\langle Q^2 \rangle}, \\ G^{11} + G^{22} - 2G^{12} &= \frac{2}{\hbar^2 \langle Q^2 \rangle} \left[\langle P^2 \rangle \langle Q^2 \rangle - (\langle P, Q \rangle)^2 \right], \quad \langle P, Q \rangle \equiv \frac{1}{2} \langle (PQ + QP) \rangle \\ G^{11} - G^{22} &= -\frac{i \langle P, Q \rangle}{\hbar \langle Q^2 \rangle}, \end{aligned}$$

[The three G^{ij} 's are totally determined by the three correlators.]

 In quantum field theory, a pure field state at some moment is a functional of field strength at every (real or momentum) space point

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{m^2}{2} \Phi^2 \right] = \int d\eta \frac{d^3k}{(2\pi)^3} \left[\frac{1}{2} \partial_\eta \phi_{\mathbf{k}} \partial_\eta \phi_{-\mathbf{k}} - \frac{1}{2} \Omega_{\mathbf{k}}^2(\eta) \phi_{\mathbf{k}} \phi_{-\mathbf{k}} \right]$$

Quantization $\hat{\Pi}_{\mathbf{k}} = (2\pi)^3 \frac{\hbar}{i} \frac{\delta}{\delta \phi_{\mathbf{k}}},$ $i\hbar \partial_\eta \Psi = \hat{H} \Psi$ [Lin Chou Hu PRD (10)] $\hat{H} = \int \frac{d^3k}{(2\pi)^3} \left[-\frac{\hbar^2}{2} (2\pi)^6 \frac{\delta}{\delta \phi_{\mathbf{k}}} \frac{\delta}{\delta \phi_{-\mathbf{k}}} + \frac{1}{2} \Omega_{\mathbf{k}}^2(\eta) \phi_{\mathbf{k}} \phi_{-\mathbf{k}} \right]$ $\begin{bmatrix} \chi_{\mathbf{k}}'' + \Omega_{\mathbf{k}}^2(\eta) \chi_{\mathbf{k}} = 0, \\ \chi_{\mathbf{k}} = \sqrt{1/2\Omega_{\mathbf{k}}} e^{-i\Omega_{\mathbf{k}}\eta} \text{ in } M^4 \end{bmatrix}$ Wave functional of the vacuum state (no-particle state in M^4)

$$\Psi_0 = \prod_{\mathbf{k}} \Psi_{0\mathbf{k}}, \qquad \Psi_{0\mathbf{k}} = s_{\mathbf{k}} e^{-i \int^{\eta} d\bar{\eta} \mathcal{E}_0^{\mathbf{k}}(\bar{\eta})/\hbar} \exp \frac{i}{2\hbar (2\pi)^3 \delta^3(0)} \frac{\chi_{\mathbf{k}}^{*\prime}(\eta)}{\chi_{\mathbf{k}}^{*}(\eta)} \phi_{\mathbf{k}} \phi_{-\mathbf{k}}$$

which is defined over the whole time-slice (3-space) at that moment.

$$\hat{Q}_{i}(\tau_{i}) = \sqrt{\frac{\hbar}{2\Omega_{r}}} \sum_{i} \left[q_{i}^{(j)}(\tau_{i})\hat{a}_{j} + q_{i}^{(j)*}(\tau_{i})\hat{a}_{j}^{\dagger} \right] + \int \frac{d^{3}k}{(2\pi)^{3}} \sqrt{\frac{\hbar}{2\omega}} \left[q_{i}^{(+)}(\tau_{i},\mathbf{k})\hat{b}_{\mathbf{k}} + q_{i}^{(-)}(\tau_{i},\mathbf{k})\hat{b}_{\mathbf{k}}^{\dagger} \right]$$



Time-Slicing dependence?



Reduced density matrix

 $\rho^R(Q_A, Q_B; Q'_A, Q'_B) = \int d^2 \vec{Q}_A (Q_B, Q'_B) d^2 \vec{Q}_A (Q_B, Q'_B) = 0$

$$\int_{S} d\vec{\phi} \rho(Q_A, Q_B, \vec{\phi}; Q'_A, Q'_B, \vec{\phi})$$

is independent of time-slicing (once it is consistent with *t*=0 hypersurface) for <u>Gaussian</u> states in our <u>linear</u> system (UD detectors).

In general??

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Minkowski vacuum (field state), and wave functions of free particles are Lorentz invariant in Minkowski space Minkowski vacuum + atoms (bound states localized in space) + back reactions to each other = temporal and spatial dependence of the configuration

- In a relativistic field theory, the <u>action</u>, and the <u>field equations</u> and <u>physical quantities</u> derived from the action should respect Lorentz symmetry.
 - But the boundary or initial conditions and thus <u>solutions</u> of the equations can break the full symmetry.
 - e.g. presence of bound states (atoms, hadrons), which are localized in space and produce retarded field breaking the full Lorentz symmetry.

 With the presence of matter, one can use matter distribution as a natural reference frame, e.g. CMBR (This is why Einstein needed a hole in his hole argument.)

Even the vacuum (no-particle state of the field in some coordinate) could act like a medium, the "new aether" [DeWitt 79].

Simultaneity and time order







Minkowski vacuum + detectors + back reactions to each other: temporal and spatial dependence of the configuration

 Fiducial time: one have to choose a time slice (3-space) where the initial state is living.



Fiducial time restricts the choice of coordinates in spacetime.



Reduced density matrix

 $\rho^R(Q_A,Q_B;Q_A',Q_B') =$

$$\int_{S} d\vec{\phi} \rho(Q_A, Q_B, \vec{\phi}; Q'_A, Q'_B, \vec{\phi})$$

is independent of time-slicing (once it is consistent with *t*=0 hypersurface) for <u>Gaussian</u> states in our <u>linear</u> system (UD detectors).

In general??

Initial state of the field



Summary

- Event horizon is not essential for the excitations of the detector in Unruh effect.
- Entanglement of two detectors in quantum fields is time-dependent.
- Entanglement dynamics depend on coordinate but are independent of the time-slicing scheme.
- The field state in a dynamical atom-field interacting system started at some initial moment does not respect the full Lorentz symmetry.

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