Dynamics of Two Detectors in a Relativistic Quantum Field: Some Remarks

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Outline

I. Introduction

II. Event Horizon and Unruh effect

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I. Introduction
I. Introduction

RQI models:
1. Entanglement of atoms/qubits moving in flat or curved spacetime
2. Entanglement of relativistic quantum fields (in curved spacetime)

1+2: Entanglement of atoms moving in relativistic quantum fields

Relativistic quantum fields are treated as environment.
E.g. (Cavity) QED: atoms in EM field
[Anastopoulos, Shresta, Hu 06]
Unruh-DeWitt detector theory: accelerated HO in scalar field
[Lin, Chou, Hu 08, Lin, Hu 09, 10]
I. Introduction

- Unruh-DeWitt detector theory in (3+1)D
  [Lin PRD (03); Lin Hu PRD (06)]

\[ S = S_Q + S_\Phi + S_I, \]
\[ S_Q = \int d\tau \frac{m_0}{2} \left[ (\partial_\tau Q)^2 - \Omega_0^2 Q^2 \right] \]
\[ S_\Phi = -\int d^4x \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \]
\[ S_I = \lambda_0 \int d\tau \int d^4x Q(\tau) \Phi(x) \Phi_0^4 (x^\mu - z^\mu(\tau)) \]

with prescribed trajectory (UAD)

\[ z^\mu(\tau) = (a^{-1} \sinh a\tau, a^{-1} \cosh a\tau, 0, 0) \]

We have -- quantum field theory +
quantum mechanics + classical external agent
I. Introduction

- For mathematical convenience, we choose the initial conditions

- Sudden switch-on: $S_I \sim \theta(\tau - \tau_0)$

- Free operators before $\tau_0$

- Factorized initial state: $|\tau_0\rangle = |q\rangle |0_M\rangle$

  field: Minkowski vacuum $|0_M\rangle$

  detector: coherent state $|q\rangle = e^{-\alpha^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

\[ \alpha = g_0 \sqrt{\Omega_r/2\hbar} \]
I. Introduction

- Determine the radiation for the Minkowski observer in (3+1)D

Find the quantum expectation values of stress-energy tensor

\[
T_{\mu \nu}[\Phi(x)] = (1 - 2\xi) \Phi_{,\mu} \Phi_{,\nu} - 2\xi \Phi \Phi_{,\mu \nu} + \left(2\xi - \frac{1}{2}\right) g_{\mu \nu} \Phi^\rho \Phi_{,\rho} + \frac{\xi}{2} g_{\mu \nu} \Phi_{,\rho} \Phi^{\rho}
\]

Reported in IARD 2006 @ Connecticut

Then calculate the radiation formula for Minkowski observer at null infinity through

\[
\frac{dW_{\text{rad}}}{d\tau_-} = - \lim_{r \to \infty} \int r^2 d\Omega_{11} w^\mu T_{\mu \nu} v^\nu(\tau_-)
\]

\[
\left\langle \frac{dW_{\text{rad}}}{d\tau_-} \right\rangle = - \lim_{r \to \infty} \int r^2 d\Omega_{11} w^\mu \left\langle T_{\mu \nu} \right\rangle_{\text{ren}} v^\nu(\tau_-)
\]
I. Introduction

Entanglement dynamics between 2 identical Unruh-DeWitt detectors in (3+1)D Minkowski space [Lin Chou Hu 08; Lin Hu 09, 10]

\[ S = -\int d^4x \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \]
\[ + \int d\tau_A \frac{1}{2} \left[ (\partial_\tau Q_A)^2 - \Omega_0^2 Q_A^2 \right] + \int d\tau_B \frac{1}{2} \left[ (\partial_\tau Q_B)^2 - \Omega_0^2 Q_B^2 \right] \]
\[ + \lambda_0 \int d^4x \Phi(x) \left[ \int d\tau_A Q_A(\tau_A) \delta^4(x^\mu - z_\mu^A(\tau_A)) + \int d\tau_B Q_B(\tau_B) \delta^4(x^\mu - z_\mu^B(\tau_B)) \right] \]

- massless scalar field
- internal: HO x 2
- bilinear interaction [DeWitt 1979]

Detectors A, B are point-like objects.

Features:

1. Linear, crystal clear;
2. Simplest atom-field interacting system;
3. In some cases there exist analytic results in the whole parameter range;
4. Complicated enough to give nontrivial results and insights.
I. Introduction: Setups

Initial state (Gaussian)

\[ |\psi(0)\rangle = |a_A, q_B \rangle \otimes |0_M \rangle \]
II. Event Horizon and Unruh Effect
II. Event Horizon and Unruh Effect

Event horizons can be sharply defined since the detectors are always point-like and non-spreading.

\[ t = \frac{\sinh a \tau_0}{a} \]

[Lin, Chou, Hu PRD 2008]

[Lin, Hu PRD 2010]
II. Event Horizon and Unruh Effect

A uniformly accelerated detector acts as if immersed in a bath at Unruh temperature.

Evolution of the self correlators of a detector initially in its ground state:

\[ \langle Q^2 \rangle \]

\[ \approx \frac{\hbar}{2\Omega m_0} \coth \frac{\pi \Omega}{a} \]

in weak coupling limit

while \( \langle P_A, Q_A \rangle = (m_0/2)(d/d\tau)\langle Q^2 \rangle \) are oscillating in small amplitude \( O(\gamma) \).
II. Event Horizon and Unruh Effect

A uniformly accelerated detector acts as if immersed in a bath at Unruh temperature.

Evolution of the self correlators of a detector initially in its ground state:

\[ \langle Q^2 \rangle \quad \text{Unruh effect} \]

\[ \langle Q(\eta)^2 \rangle_v \approx \frac{\hbar}{2\Omega m_0} \coth \frac{\pi \Omega}{a} \]

in weak coupling limit

Time dependent perturbation theory regime

\[ \langle P^2 \rangle (m_0 = 1) \quad \Lambda_0 = \Lambda_1 = 20 \]

However, the early-time behavior will be the same if the proper acceleration is nonzero only in a finite duration. In this case, no event horizon at all.
II. Event Horizon and Unruh Effect

- Uniformly accelerated
- Turnaround (uniformly accelerated)
- Inertial
- Arrival
- Final (inertial)
- Departure

Diagram showing the paths of Alice and Bob with milestones and labels.
II. Event Horizon and Unruh Effect

[Ostapchuk, Lin, Hu, Mann, in preparation]

Non-uniform acceleration

\[ \delta <Q^2>_v \]

\[ \delta \left( \frac{Q_B^2(\tau)}{v} \right) \equiv \lim_{\tau' \to \tau} \left[ \left( Q_B(\tau)Q_B(\tau') \right)_v - \left( Q_B(\tau)Q_B(\tau') \right)_{v,T_U=0} \right] \]

Detector B acts like an oscillator immersed in a bath at a time-varying "temperature".

Event horizon is not essential for the evolution such as the excitations in TDPT regime, while acceleration is.
III. Entanglement Dynamics of Open Systems
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Relativistic quantum fields as the environment:

- Looking at the reduced density matrix of the detectors, which is a mixed state in general.

- Entanglement between two detectors becomes time-dependent even for both detectors in eigenstates initially and no direct interaction between them: sudden death, revival, creation...of entanglement.

- The qualities of environment, such as spatial dependence, mutual influences, echo from the boundary, etc., will enter the entanglement dynamics of the system.
I. Introduction

- Determine the radiation for the Minkowski observer in (3+1)D

Find the quantum expectation values of stress-energy tensor

\[
\langle T_{\mu\nu}[\Phi(x)] \rangle_{\text{ren}} = \lim_{x' \to x} \left[ \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x'^\nu} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \frac{\partial}{\partial x^\rho} \frac{\partial}{\partial x'^\sigma} \right] G_{\text{ren}}(x, x')
\]

Then calculate the radiation formula for Minkowski observer at null infinity through

\[
\left\langle \frac{dW^{\text{rad}}}{d\tau_-} \right\rangle = -\lim_{r \to \infty} \int r^2 d\Omega_{11} w^\mu T_{\mu\nu} v^\nu(\tau_-)
\]

\[
\left\langle \frac{dW^{\text{rad}}}{d\tau_-} \right\rangle = -\lim_{\tau \to \infty} \int r^2 d\Omega_{11} w^\mu \langle T_{\mu\nu} \rangle_{\text{ren}} v^\nu(\tau_-)
\]

Reported in IARD 2006 @ Connecticut

Backreaction from detector
III. Entanglement Dynamics of Open Systems

Initial state (Gaussian)

$$|\psi(0)\rangle = |q_A, q_B\rangle \otimes |0_M\rangle$$

Retarded Mutual Influences (Causal)
III. Entanglement Dynamics of Open Systems

Entanglement dynamics is highly non-Markovian.

Sudden Death

Entanglement revival

Spatial dependence

Lin, Chou, Hu 2008

Lin, Hu 2009

Lin, Hu 2010
III. Entanglement Dynamics of Open Systems

Entanglement can be created outside light cone by quantum fields.

Entanglement is created at $t, \tau > 0$ ($\tau \sim 1000$ here).

$\gamma = 10^{-5}$, $\Omega = 2.3$, $\hbar = a = 1$, $\Lambda_0 = \Lambda_1 = 20$

$c_- < \hbar/2$ : Entangled

$\Sigma = \left( c_+ - \frac{\hbar^2}{4} \right) \left( c_- - \frac{\hbar^2}{4} \right)$

$E_N = \max\{0, -\log_2 2c_-\}$

$z_A^\mu = (a^{-1} \sinh a\tau, a^{-1} \cosh a\tau, 0, 0)$

$z_B^\mu = (a^{-1} \sinh a\tau, -a^{-1} \cosh a\tau, 0, 0)$
III. Entanglement Dynamics of Open Systems

Entanglement can be created outside light cone by quantum fields.

Entanglement is created at $t$, $\tau > 0$ ($\tau \sim 1000$ here).

More observations:
1. The detectors will be disentangled at late times.
2. It seems that the "entanglement time" is after the moment that the third party is able to received information from both detectors.
Non-Uniform Acceleration

Stationary in Costa-Villalba's coordinate

[Most of the impact at $t_1$ on $Q_A$ is off resonance]

Mutual influences are small in weak coupling and large distance limit, though.
IV. Correlators vs. Density Matrix
IV. Correlators vs. Density Matrix

- Master equation for the reduced density matrix of two oscillator (located at the same space point) is complicated! [Chou Yu Hu 08]

\[
i\hbar \frac{\partial \rho_r}{\partial t} = -\frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial y_2^2} \right) \rho_r + \frac{1}{2} M \Omega^2 (x_1^2 - y_1^2 + x_2^2 - y_2^2) \rho_r \\
+ \frac{1}{2} M \delta \Omega^2 (t) (x_1 - y_1 + x_2 - y_2) \left( x_1 + y_1 + x_2 + y_2 \right) \rho_r \\
- i\hbar \Gamma(t) (x_1 - y_1 + x_2 - y_2) \frac{1}{2} \left( \frac{\partial}{\partial x_1} - \frac{\partial}{\partial y_1} + \frac{\partial}{\partial x_2} - \frac{\partial}{\partial y_2} \right) \rho_r \\
- iM \Sigma(t) (x_1 - y_1 + x_2 - y_2)^2 \rho_r \\
+ \hbar \Delta(t) (x_1 - y_1 + x_2 - y_2) \left( \frac{\partial}{\partial x_1} + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial y_2} \right) \rho_r.
\]

\(\delta \Omega^2 (t), \Gamma(t), \Delta(t), \Sigma(t)\) depend on spectral density of environment and time.
IV. Correlators vs. Density Matrix

Sketch of our calculations

- Evolution of operators $Q_A, P_A, Q_B, P_B, \Phi, \Pi$ in Heisenberg picture.

$$\hat{Q}_i(\tau) = \sqrt{\frac{\hbar}{2\Omega_\tau}} \sum_j \left[ q_i^{(j)}(\tau) \hat{a}_j + q_i^{(j)*}(\tau) \hat{a}_j^* \right] + \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\hbar}{2\omega}} \left[ q_i^{(+)}(\tau, k) \hat{b}_k + q_i^{(-)}(\tau, k) \hat{b}_k^* \right]$$

- Sandwiched by the initial state: 10 symmetric correlators

$$V_{\mu\nu}(t, \tau) = \langle [R_\mu, R_\nu] \rangle \equiv \frac{1}{2} \langle [R_\mu R_\nu + R_\nu R_\mu] \rangle \quad R_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$$

- Partial Transposition: $V_{PT} = \Lambda V \Lambda$

$$\Lambda = \text{diag}(1, 1, 1, -1)$$

The quantity

$$\Sigma(t, \tau = a^{-1} \sinh^{-1}(a t)) \equiv \det \left[ V_{PT} + i \frac{\hbar}{2} M \right]$$

or logarithmic negativity

- degrees of entanglement for mixed Gaussian states
IV. Correlators vs. Density Matrix

- The covariance matrix of the detectors

\[
V \equiv \begin{pmatrix}
\langle Q_A^2 \rangle & \langle Q_A, P_A \rangle & \langle Q_A, Q_B \rangle & \langle Q_A, P_B \rangle \\
\langle Q_A, P_A \rangle & \langle P_A^2 \rangle & \langle P_A, Q_B \rangle & \langle P_A, P_B \rangle \\
\langle Q_A, Q_B \rangle & \langle P_A, Q_B \rangle & \langle Q_B^2 \rangle & \langle Q_B, P_B \rangle \\
\langle Q_A, P_B \rangle & \langle P_A, P_B \rangle & \langle Q_B, P_B \rangle & \langle P_B^2 \rangle 
\end{pmatrix}
\]
IV. Correlators vs. Density Matrix

Correlator dynamics vs. master equation of DM

- In the master equation approach the reduced density matrix (RDM) of two detectors at some moment is defined on a time slice where two detectors have the same time-parameters.

- RDM of the detectors in Gaussian state at some moment in some time-slicing scheme can be fully determined by their symmetric correlators,

\[ V_{\mu\nu}(t, \tau) = \langle R_\mu, R_\nu \rangle = \frac{1}{2} \langle (R_\mu R_\nu + R_\nu R_\mu) \rangle \quad R_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t)) \]

- One can calculate the correlators of the detectors (located at different spatial points) at different times, i.e. correlations between different events in spacetime.

- RDM in a new time-slicing scheme can still be determined by the same set of correlators after transform the time parameters to the new ones, provided that the time slices where the initial state is living are the same.
IV. Correlators vs. Density Matrix

One can express the reduced density matrix $\rho^R(Q,Q')$ for the detector at some moment in terms of the two-point correlators and study the non-equilibrium statistical properties. E.g., for a detector initially in a Gaussian state $|E_0\rangle \otimes |0_M\rangle$

$$\rho^R(Q,Q';\tau) = Tr_\Phi [\rho] = \int D\Phi_k \psi_0^*[Q,\Phi_k;\tau] \psi_0^*[Q',\Phi_k;\tau] = \exp[-G^{ij}(\tau)Q_iQ_j - F(\tau)]$$

where

$$G^{11} + G^{22} + 2G^{12} = \frac{1}{2\langle Q^2 \rangle},$$

$$G^{11} + G^{22} - 2G^{12} = \frac{2}{\hbar^2 \langle Q^2 \rangle} \left[ \langle P^2 \rangle \langle Q^2 \rangle - \langle (P,Q) \rangle^2 \right],$$

$$G^{11} - G^{22} = -\frac{i}{\hbar} \langle P,Q \rangle \, \langle Q^2 \rangle,$$

[The three $G^{ij}$'s are totally determined by the three correlators.]
IV. Correlators vs. Density Matrix

- In quantum field theory, a pure field state at some moment is a functional of field strength at every (real or momentum) space point

\[
S = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{m^2}{2} \Phi^2 \right] = \int d\eta \frac{d^3 k}{(2\pi)^3} \left[ \frac{1}{2} \partial_\eta \phi_k \partial_\eta \phi_{-k} - \frac{1}{2} \Omega^2_k(\eta) \phi_k \phi_{-k} \right]
\]

Quantization

\[
\hat{\Pi}_k = (2\pi)^3 \frac{\hbar}{i} \frac{\delta}{\delta \phi_k}, \quad \hat{H} = \int \frac{d^3 k}{(2\pi)^3} \left[ -\frac{\hbar^2}{2} \frac{\delta}{\delta \phi_k} \frac{\delta}{\delta \phi_{-k}} + \frac{1}{2} \Omega^2_k(\eta) \phi_k \phi_{-k} \right]
\]

Wave functional of the vacuum state (no-particle state in \( M^4 \))

\[
\Psi_0 = \Pi_k \Psi_{0k}, \quad \Psi_{0k} = s_k e^{-i \int'' d\eta \varepsilon^k_\eta / \hbar} \exp \frac{i}{2\hbar (2\pi)^3} \frac{\Omega^*(\eta)}{\Omega_k(\eta)} \phi_k \phi_{-k}
\]

which is defined over the whole time-slice (3-space) at that moment.

\[
Q_i(\tau_i) = \sqrt{\frac{\hbar}{2\Omega_i}} \sum_j \left[ q_i^{(j)}(\tau_i) a_j + q_i^{(j)*}(\tau_i) a_j^* \right] + \int \frac{d^3 k}{(2\pi)^3} \sqrt{\frac{\hbar}{2\omega}} \left[ q_i^{(+)}(\tau_i, k) b_k + q_i^{(-)}(\tau_i, k) b_k^* \right]
\]
IV. Correlators vs. Density Matrix

Coordinate dependence

$V_{\mu\nu}(t, \tau) = \langle R_{\mu} R_{\nu} \rangle \equiv \frac{1}{2} \langle (R_{\mu} R_{\nu} + R_{\nu} R_{\mu}) \rangle$
Reduced density matrix

\[ \rho^R(Q_A, Q_B; Q'_A, Q'_B) = \int_S d\vec{\phi} \rho(Q_A, Q_B, \vec{\phi}; Q'_A, Q'_B, \vec{\phi}) \]

is independent of time-slicing (once it is consistent with \( t=0 \) hypersurface) for Gaussian states in our linear system (UD detectors).

In general??
V. Symmetries in Field States
V. Symmetries in Field States

Minkowski vacuum (field state), and wave functions of free particles are Lorentz invariant in Minkowski space.

Minkowski vacuum + atoms (bound states localized in space) + back reactions to each other = temporal and spatial dependence of the configuration.
V. Symmetries in Field States

- In a relativistic field theory, the action, and the field equations and physical quantities derived from the action should respect Lorentz symmetry.

But the boundary or initial conditions and thus solutions of the equations can break the full symmetry.

e.g. presence of bound states (atoms, hadrons), which are localized in space and produce retarded field breaking the full Lorentz symmetry.
V. Symmetries in Field States

- With the presence of matter, one can use matter distribution as a natural reference frame, e.g. CMBR (This is why Einstein needed a hole in his hole argument.)

  Even the vacuum (no-particle state of the field in some coordinate) could act like a medium, the "new aether" [DeWitt 79].
V. Symmetries in Field States

- Simultaneity and time order
V. Symmetries in Field States

Minkowski vacuum + detectors + back reactions to each other: temporal and spatial dependence of the configuration
V. Symmetries in Field States

- Fiducial time: one have to choose a time slice (3-space) where the initial state is living.
Fiducial time restricts the choice of coordinates in spacetime.

Reduced density matrix

\[
\rho^R(Q_A, Q_B; Q'_A, Q'_B) = \int_{S} d\phi \rho(Q_A, Q_B, \phi; Q'_A, Q'_B, \phi)
\]

is independent of time-slicing (once it is consistent with \( t=0 \) hypersurface) for Gaussian states in our linear system (UD detectors).

In general??
V. Symmetries in Field States

Initial state of the field

- Initial state? vacuum state
- Initial state? squeezed state?
- Initial moment
- $t$
- $x$
Summary

- Event horizon is not essential for the excitations of the detector in Unruh effect.

- Entanglement of two detectors in quantum fields is time-dependent.

- Entanglement dynamics depend on coordinate but are independent of the time-slicing scheme.

- The field state in a dynamical atom-field interacting system started at some initial moment does not respect the full Lorentz symmetry.
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