Quantum Stress Tensor Fluctuation Effects in Inflationary Cosmology

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Some Properties of Stress Tensor Fluctuations:

Anticorrelations

Skewed Probability Distributions

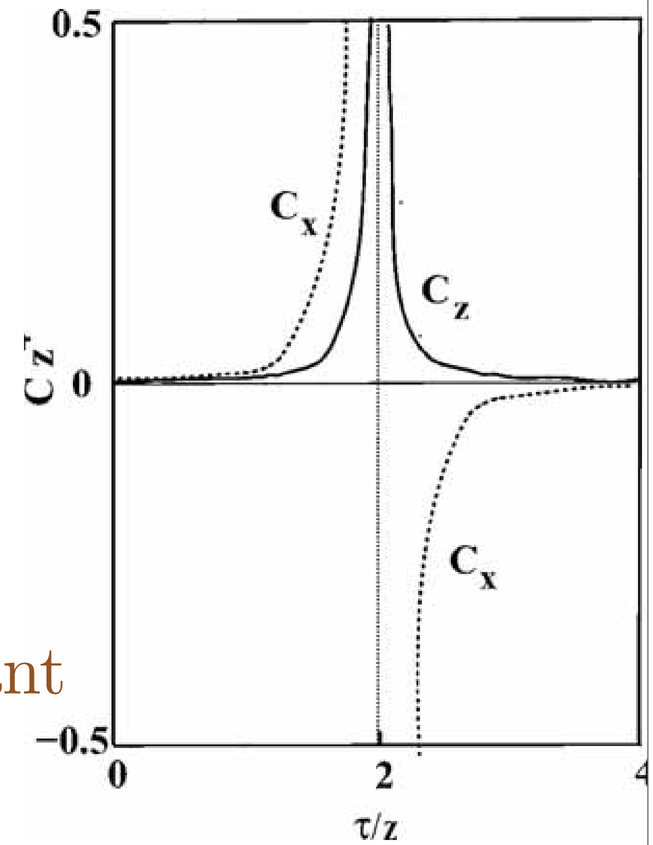
Anticorrelations

Electromagnetic analogy: electric field fluctuations near a perfectly reflecting plate

H.Yu & LF

electric field correlation function:

$$C_i(t - t') = \langle E_i(t)\, E_i(t') \rangle$$



mean squared velocity of a particle of charge q and mass m:

$$\langle v_i^2 \rangle = \frac{q^2}{m^2} \int_0^{t_0} dt \int_0^{t_0} dt'\, C_i(t - t') = \text{constant}$$

Anticorrelations prevent $\langle v_i^2 \rangle$ from growing in time.

(Required by energy conservation.)

Brownian Motion of Charged Particles in an Expanding Universe

Bessa, Bezerra &LH

Now the equation for $\langle v^2 \rangle$ contains powers of the scale factor multiplying the correlation function.

Result: $\langle v^2 \rangle$ can now grow due to non-cancellation of anti-correlations.

Energy conservation no longer required.

Stress tensor correlations and anticorrelations in flat space

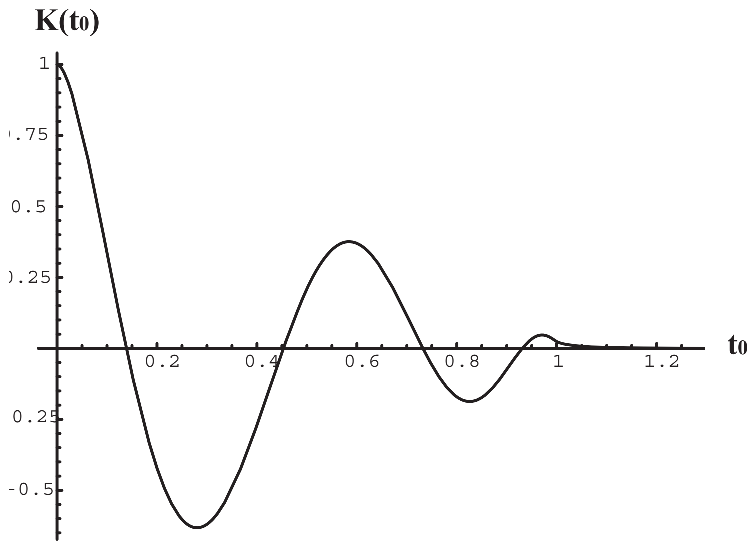
(T. Roman and LF)

$$\langle S(t_0)S(0)\rangle = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt'\, g(t-t_0)\, g(t')\, C(t,t')$$

$$g(t) = \frac{630}{a^9}\,(t-a/2)^4\,(t+a/2)^4\,, \quad \text{for } |t| \leq a/2, \text{ and } g(t) = 0 \text{ for } |t| \geq a/2.$$

$$C(t,t') = \langle : T_{tt}(t) :: T_{tt} : (t')\rangle = \text{energy density correlation function}$$

$$K(t_0) = \frac{\langle S(t_0)S(0)\rangle}{\langle S^2(0)\rangle}$$



Probability distribution for quantum stress tensor fluctuations

Must be a skewed, non-Gaussian distribution

In general $\langle (T_{\mu\nu})^3 \rangle \neq 0$

Expect the probability distribution to have a lower cutoff at the quantum inequality bound.

Quantum Inequalities

Lower bounds on averaged expectation values in an arbitrary quantum state:

$$\int \langle T_{\mu\nu} \rangle \, u^\mu \, u^\nu \, g(\tau, \tau_0) \, d\tau \geq -\frac{C}{\tau_0^d}$$

$g(\tau, \tau_0)$ = sampling function

C = positive constant

τ_0 = sampling time

d = spacetime dimension

Let $u = \int T_{tt}\, g(t,\tau)\, dt$ averaged energy density

A result for vacuum fluctuations in conformal field theory (2 spacetime dimensions)

C. Fewster, T. Roman & LF

$$u = \frac{1}{\sqrt{\pi}\tau} \int_{-\infty}^{\infty} T_{tt}(x,t)\, e^{-t^2/\tau^2}\, dt$$

$$P(x) = \frac{\pi^{c/24}}{\Gamma(c/24)}\, (x + x0)^{\frac{c}{24}-1}\, e^{-\pi(x+x_0)}$$

$x = u\,\tau^2$ $\qquad x \geq x_0$

$P(x) = 0 \quad x < x_0$ $\qquad x_0$ = quantum inequality bound

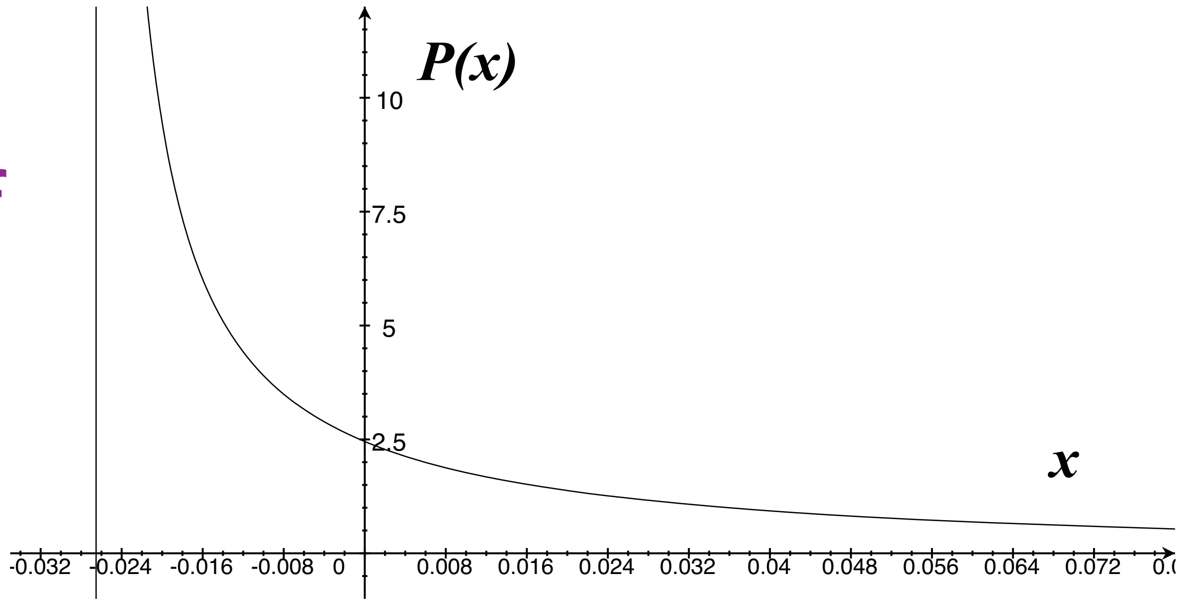
c = central charge

A massless scalar field in two dimensions (c = 1):

$$P(x) = \frac{\pi^{1/24}}{\Gamma(1/24)} \left(x + \frac{1}{24\pi}\right)^{-23/24} e^{-\pi(x+1/24\pi)}$$

$$P(x) = 0 \quad x < -\frac{1}{24\pi}$$

84% chance of finding u < 0



$$x = \tau^2 u$$

P(u) is a shifted Gamma distribution

Negative energy is more likely than positive energy

Positive fluctuations tend to be larger in magnitude

Possible applications to anthropic arguments?

“Boltzmann Brains”

$\langle T_{\mu\nu} \rangle$ in and near de Sitter spacetime

in de Sitter: $\langle T_{\mu\nu} \rangle \propto g_{\mu\nu}$

shifts the cosmological constant

near de Sitter:

$\langle T_{\mu\nu} \rangle$ contains both local geometrical terms and a nonlocal term

Starobinski, Horowitz & Wald

Does not lead to instability (within the limits of semiclassical theory) but does seem to alter to propagation of gravity waves

JT Hsiang, DS Lee, HL Yu & LF

Possible role of stress tensor fluctuations in cosmological models

C.H. Wu, K.W. Ng , S.P. Miao, R. Woodard & LF

I. A Kinematic Model

Basic idea: Look at the effects of fluctuations of the vacuum electromagnetic field stress tensor on the local expansion rate.

Treat the Raychaudhuri equation as a Langvin equation.

Stress tensor and expansion fluctuations

u^α = 4-velocity of a congruence of timelike geodesics

$\theta = u^\alpha_{;\alpha}$ = expansion of the congruence

Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu}u^\mu u^\nu - \frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}$$

$$R_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$$

Ordinary matter: focussing

θ fluctuations (in flat RW models)

Assume $\sigma_{\mu\nu} = \omega_{\mu\nu} = 0$, so that

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu}u^\mu u^\nu - \frac{1}{3}\theta^2$$

Let $\theta = \theta_0 + \theta_1$, where $\theta_0 = 3\dot{a}/a$, and

$$\frac{d\theta_1}{dt} = -\left(R_{\mu\nu}u^\mu u^\nu\right)_q - \frac{2}{3}\theta_0\theta_1$$

$$\Longrightarrow \quad \theta_1(t) = -a^{-2}(t)\int_{t_0}^{t} dt'\, a^2(t')\,\left(R_{\mu\nu}u^\mu u^\nu\right)_q$$

Inflationary expansion followed by reheating and a radiation dominated universe

$$a(\eta) = \frac{1}{1 - H\eta}, \qquad \eta_0 < \eta < 0, \quad \eta_0 = \text{conformal time when inflation begins}$$

$$a(\eta) = 1 + H\,\eta, \qquad \eta > 0,$$

$t_R =$ reheating time in comoving time

$$a(t) = \mathrm{e}^{H(t - t_R)}, \quad t \leq t_R,$$

$$a(t) = \sqrt{1 + 2H(t - t_R)}, \qquad t \geq t_R.$$

Expansion correlation function:

$$\langle\theta(\eta_1)\,\theta(\eta_2)\rangle - \langle\theta(\eta_1)\rangle\langle\theta(\eta_2)\rangle =$$
$$a^{-2}(\eta_1)\,a^{-2}(\eta_2)\int_{\eta_0}^{\eta_1} d\eta\,a^{-1}(\eta)\int_{\eta_0}^{\eta_2} d\eta'\,a^{-1}(\eta')\,\mathcal{E}(\Delta\eta, r)$$

$\mathcal{E}(\Delta\eta, r)$ = flat space energy density correlation function

Conformally invariant fields:

$$C^{RW}_{\mu\nu\alpha\beta}(x,x') = a^{-4}(\eta)\,a^{-4}(\eta')\,C^{flat}_{\mu\nu\alpha\beta}(x,x')$$

Stress tensor correlation function

$$C_{\mu\nu\alpha\beta}(x,x') = \langle T_{\mu\nu}(x)\,T_{\alpha\beta}(x')\rangle - \langle T_{\mu\nu}(x)\rangle\langle T_{\alpha\beta}(x')\rangle$$

(conformal anomaly cancels)

Effect of expansion fluctuations on redshifting after reheating

Conservation law for a perfect fluid:

$$\dot{\rho} + \theta(\rho + p) = 0$$

Expansion fluctuations imply density fluctuations

(Hawking-Olson approach to density perturbations)

Let $p = w\rho$ and integrate the conservation law to find the density correlation function.

$$C_\rho(x_1, x_2) = \left\langle \frac{\delta\rho(x_1)}{\rho}\ \frac{\delta\rho(x_2)}{\rho} \right\rangle$$

$$= (8\,\pi)^2\,\ell_p^4\,(1+w)^2 \int_0^{\eta_s} \frac{d\eta_1}{a(\eta_1)} \int_0^{\eta_s} \frac{d\eta_2}{a(\eta_2)} \int_{\eta_0}^{\eta_1} \frac{d\eta}{a(\eta)} \int_{\eta_0}^{\eta_2} \frac{d\eta'}{a(\eta')}\,\mathcal{E}(\Delta\eta, r)$$

ℓ_p = Planck length

η_s = conformal time of last scattering

upsets cancellation of anticorrelations!

(Non-Gaussian fluctuations)

Power spectrum of the density fluctuations: $\mathcal{P}_k = 4\pi k^3\,P_k$

$$P_k = \frac{1}{(2\pi)^3} \int d^3r\,\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\,C_\rho(\mathbf{r}, \eta_s)$$

Result:

$$\mathcal{P}_k \sim \ell_p\, k^6\, H^{-3}(-k\, S^3 + H\, S^2)$$

$S \gg 1$

$\ell_p =$ Planck length

$S =$ scale factor increase during inflation

Leading term gives an unobservable δ -function in C_ρ , so ignore it.

Then $$\mathcal{P}_k \sim \ell_p^4\, k^6\, H^{-2}\, S^2$$

Not scale invariant and increases as S increases.

II. A Dynamical Model

Consider a single scalar inflaton field φ and a fluctuating conformal field stress tensor.

Basic idea:

Solve the coupled Einstein-φ equations to find $\Delta\varphi$, the fluctuation in φ induced by the stress tensor fluctuations.

$$\Box\varphi + V'(\varphi) = 0$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{\rm total} = 8\pi(T_{\mu\nu}^{\rm infl} + T_{\mu\nu}^{\rm conf})$$

Use the resulting inflaton fluctuations to compute a power spectrum of density perturbations.

Result:

$$\mathcal{P}_k \sim \ell_p^4\, k^2\, H\, (\cancel{-k} S^3 + H\, S^2) \sim \ell_p^4\, k^2\, H\, S^2$$

ignore, as before

Again not scale invariant and increases as S increases.

Interpret the increase as due to the non-cancellation of the anticorrelated fluctuations.

(Non-Gaussian fluctuations)

This non-Gaussian, non-scale invariant contribution must be a small part of the total power spectrum.

Constraint on the duration of inflation:

$$S < 10^{42}\left(\frac{10^{12}\ \mathrm{GeV}}{T_R}\right)^3$$

reheat energy

Allows enough inflation to solve the horizon and flatness problems $(S > 10^{23})$.

Opens the possibility of observing quantum gravity effects as a non-Gaussian, non-scale invariant component in the large scale structure.

Use of transplanckian modes.

Dominant contribution comes from modes above the Planck scale.

Possibility of an observational test, and a possible probe of transplanckian physics and quantum gravity.

Summary

Quantum stress tensor fluctuations exhibit subtle correlations and anticorrelations.

Their probability distribution is highly skewed.

In an expanding universe, stress tensor fluctuations lead to expansion and density fluctuations.

Leads to a constraint on the duration of inflation.

Possibility of an observational test? Search for a non-scale invariant, non-Gaussian component in the cosmic microwave background.