RQI-N / IARD 2010

Quantum Information in Relativistic Dynamics

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*Thanks to **all invited speakers** of these two conferences for coming this far and*
*To Professors **Da Shin Lee** and **Shih-Yuin Lin** for making this possible*

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Quantum Coherence and Information Theory

Relativistic Quantum Information:

Scope, Issues, and Prospects

Relativistic

-- Starting with Special Relativity

as a reminder to people now working on QIS (mainly from AMO, CMP) that some basic invariance principles in physical laws such as Lorentz invariance, if ignored, could undermine the basis of QIS (such as different measured quantities of quantum entanglement for different Lorentz observers) or even render it meaningless or non-viable. As we shall see it is not restricted to just special relativistic effects in QI.

Quantum Field Theory (QM + SR) enters QI in a fundamental way, but is largely ignored in common practice: "My atoms move slowly so no need for QFT." Little do they know that quantum field which is always around has relativistic properties – not just **retardation** (which is there in classical field theory), but also **virtual particle processes** which show up in e. g., **causally disconnected (outside of light cone) quantum entanglement** (Lin's talk) from **induced properties of quantum fields**

Should address **issues of nonlocality in quantum mechanics** (e.g., EPR) from the more complete perspective of QFT. **QFT is local.**

Quantum Information

- Behavior and dynamics of **Bits** (computer science, classical information theory) obeying laws of **QM.**

  Entanglement, the unique and most important quantum feature absent in classical.

- **Classical information** theory is based on **probability theory**, but when applied to physical phenomena should obey physical laws. One can use classical information theory to describe the classical universe, but is the result identical or completely equivalent to physics?

- More specifically: Is it just a translation of physics in **a different language**? Is it a faithful one? Does it offer us a useful different perspective to physics? (hopefully yes)

What about **Quantum Information?**: Is there a **quantum probability theory** which it is based on? **Quantum Logic**? (Chris Isham). Can it reproduce quantum physics and go beyond? Can QI give a faithful description of the quantum universe? (Lloyd says yes, but I don't even know what the quantum universe is like)

Does it have more? Or less? If equivalent, what good does it bring us speaking this new language in understanding Nature?

Quantum Information obeys physical laws. It cannot be outside of the realm of physics. **Information is physics** (Landauer): Agree? Disagree? (Fuchs)

Scope

I. Kinematics: physics in different inertial frames (in relativistic quantum mechanics) Spin - momentum differ in different Lorentz frames. (Peres and Terno)

Find covariant definition of Q entanglement. (Prof C P Soo's work)

Inertial motion: Special Relativity + Quantum Mechanics.

Quantum entanglement has many subtle features, such as outside-light cone entanglement (Prof Lin's talk)

⇒ **QI carried out in the full context of Quantum Field Theory.**

⇒ **QI features in QFT:** particle-particle-field entanglement in moving detectors, black holes and cosmology

Non-inertial motion: uniform acceleration, Unruh effect, QI consequences

General Relativity. Q Entanglement between different observers in black hole and cosmological spacetimes

(Prof Chou and Lin's work)

QFT in curved spacetimes where **geometry and topology** enter.

Black holes and cosmology. Information, dark energy

Quantum Gravity: microscopic structures of spacetime

Helpful perspective and ideas from QI?

Geometry and topology in QI

(e.g., Geometry of quantum states; Decoherence-free space; Topological quantum computing)

More contents and Issues than just kinematics.
RQI should encompass QFT & QI, QG & QI

Non-relativistic QI

QI applied to AMO and CMP vivid and rich arena where we could glean off new and interesting physics

- Quantum phase transition characterized by quantum entanglement (Latore et al, Sahdev)
- Topological order / Quantum Order (Wen)

Features of CMP from QFT, **geometry and topology**, and **NEq Statistical Mechanical**

New subject: **Quantum CMP**

QI and Nonequilibrium QSM

- Probability theory
- Stochastic processes, noise and fluctuations
- Open systems, stochastic equations,
- Approach to equilibrium
- Quantum - classical correspondence /transition

- Relation to information theory, classical and quantum
- NEq SM + QFT: Can we recover QI fully from this?

Extended RQI: QI at the Foundations and Frontiers of Physics

Now add in our consideration **Spacetime Structure**.

Macroscopic structure: GR Classical.
What are the key Information theoretical features?
Looking for the microscopic structures: Task of **Quantum Gravity**
(**Not equal to** quantizing GR, collective variables, effective theory]

What about Macro Quantum Phenomena?
---Spacetime condensate, dark energy, Λ
Construct spacetime from the basic constituents (loops, strings, graphs):
Tasks of Emergent Gravity
How could QI help for both of these tasks going in both directions?

- Measurement-based quantum computation and quantum graph theory.
- Spin network (Penrose) Spin foams (Freidel). String nets (Wen), etc

RQI: A potent subject bearing on foundational issues in **Relativity** (S & G), **Quantum** mechanics and **Q Information**

- Extending to issues in **geometry and topology** of spacetimes and in gauge fields, but from the angle of quantum information.

- Could provide important hints to **Quantum Gravity**, including **black hole information** loss issues

  (e.g., my talk and others' at QG&QI wksp, PI, Dec. 2007)

- And how **Laws of Physics and the Universe** take shape

  (Wheeler, Landauer, Deutsch, Lloyd etc)

Scope and Issues in **our Program in RQI**: **Entanglement Dynamics** of Qubits (in Relativistic motion) interacting with a Quantum Field (relativistic!)

- Relativistic treatment is needed/desirable even though the atoms are stationary or moving slowly,
- **Quantum field is intrinsically relativistic** and relativistic treatments reveal many new physics of fundamental importance hitherto ignored, such as

- **Birth, death and revival of q entanglement** in full display even in the simplest possible configurations, e.g., 2 Inertial HOs (Lin and Hu, PRD 09)
- **Meaning of `Q nonlocality'**, **entanglement generation from 2 objects with no causal contact** in their whole history (S Y Lin's talk)
- **Entanglement and Entropy in Cosmological Particle Creation**

  Lin, Chou and Hu, PRD81, **084018** (2010), Yvette's talk.
- **Quantum Twin Paradox:** No paradox**.** History, not Symmetry!
- **Quantum Narcissism:** Gets fully entangled with oneself (go see a shrink!)

Detectors/atoms in a quantum field:

- Field states
  - infinite degrees of freedom
  - back-reaction from the field / Interplay with the field

I. Entanglement Dynamics of Stationary Atoms: Qubits and Oscillators

- generation, revival, sudden death of entanglement and residual entanglement
- causality, retardation of mutual influences
- spatial separation of atoms
- Non-Markovian regime

II. Entanglement Dynamics of Detectors in Relativistic Motion

- Coordinate (time-slicing) dependence?
- time dilation
- Accelerated detectors, Unruh effect
- role of event horizon

Entanglement of Two Detectors via an Electromagnetic Field

- Each Qubit interacting with its own EMF (cavity field):

  "Sudden death" of entanglement [Yu, Eberly PRL93, 140404 (2004)]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

Initial state:

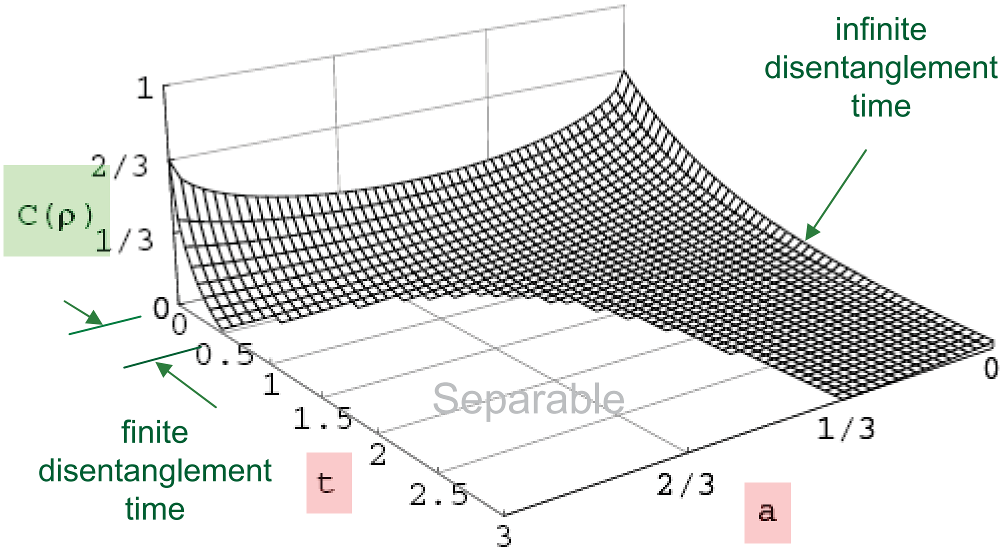
$$\rho_{\rm in} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

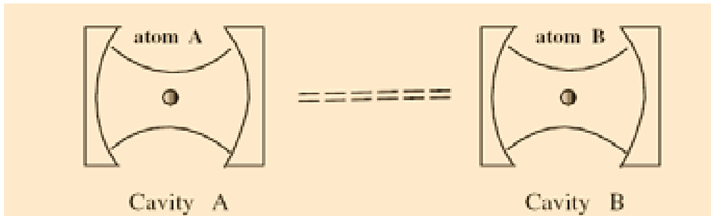
$$f(t) = 1 - \sqrt{a(1-a+2\omega^2+\omega^4 a)}$$

$$\omega = \sqrt{1-\exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$

In Markovian regime



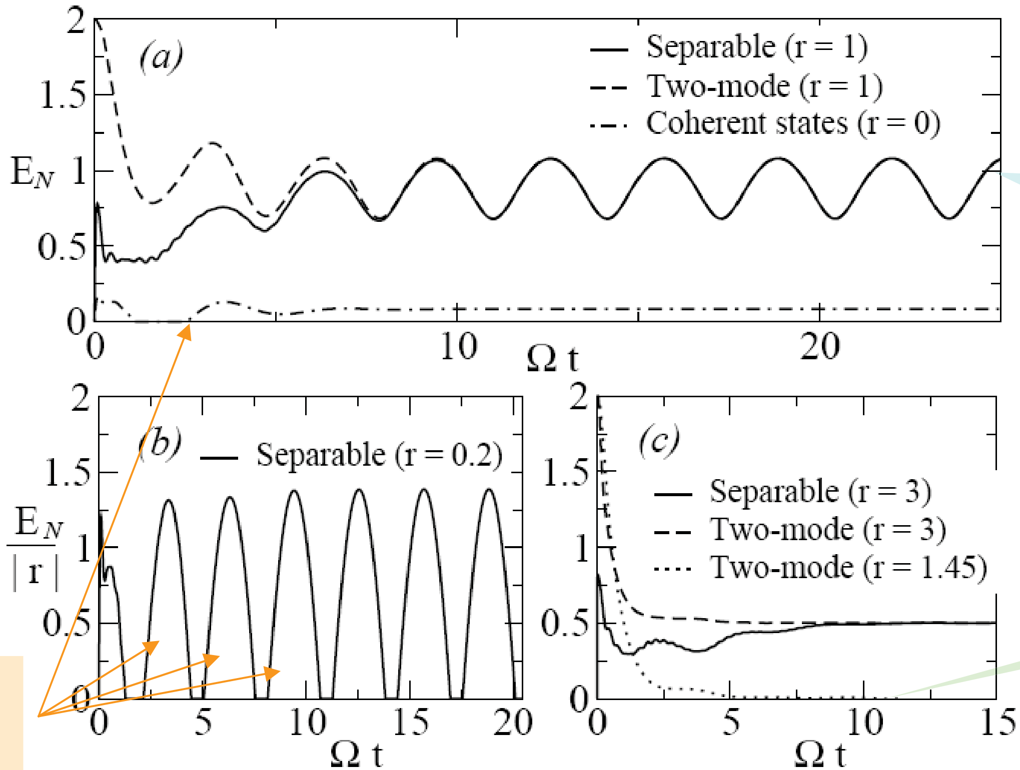


Two independent environments (cavities)

Residual entanglement, entanglement revival

2 harmonic oscillators located at the same point in space interacting via a quantum field [Paz, Roncaglia, arXiv:0801.0464]

Entanglement Via Negativity



residual entanglement at late times

Initial squeezed state parametrized by r

sudden death

entanglement revival

Entanglement of Two Qubits in the same EM field

- Needed for logical gates in quantum computers.
- More intricate because coupling is via induction effect in the field

Under **Born-Markov Approximation** does not see distance dependence -- Ficek and Tanas: PRA 06: *`Darkness and Revivals'*

Full treatment: **nonMarkovian (Memory) effects** arise, and relativistic effects of fields implicit, like outside-of- light cone entanglement

C. Anastopoulos, Sanjiv Shresta and B. L. Hu, [quant-ph/0610007]

- Entanglement dynamics for different spatial separation of two atoms
- Exact results for general cases, analytic expressions under weak coupling assumption
- At close separation (gate conditions): Enhanced Entanglement and Diminished decoherence.

Entanglement Evolution under NonMarkovian Dynamics in Atom-Field Systems

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S Y Lin, Charis Anastopoulos, K. Shiokawa (oscillators)
C.A., Nick Cummings, Kanu Sinha (qubits)

1) 2 Qubit interaction with common EM field modes [Anastopoulos, Cummings, Sinha]
2) 2 Harmonic Oscillators interacting through a common quantum field *via Relativistic Quantum Field Theory* [Lin, Chou, Shiokawa]

2 Two level atoms in Same EMF $H_0 = \hbar \sum_{\mathbf{k}} \omega_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + \hbar\omega_o S^{(1)}_+ S^{(1)}_- + \hbar\omega_o S^{(2)}_+ S^{(2)}_-$

$$H_I = \hbar \sum_{\mathbf{k}} g_{\mathbf{k}} \left(b^\dagger_{\mathbf{k}} (e^{-i\mathbf{k}\cdot\mathbf{r}/2} S^{(1)}_- + e^{i\mathbf{k}\cdot\mathbf{r}/2} S^{(2)}_-) + b_{\mathbf{k}} (e^{i\mathbf{k}\cdot\mathbf{r}/2} S^{(1)}_+ + e^{-i\mathbf{k}\cdot\mathbf{r}/2} S^{(2)}_+) \right) \quad g_{\mathbf{k}} = \frac{\lambda}{\sqrt{\omega_{\mathbf{k}}}}$$

Master Equation in Operator Form

$$\Gamma_0 := -Im\,\alpha(\omega_o)$$

$$-\sigma(r) + i\Gamma_r := \beta(\omega_o, r).$$

Dipole Interaction

$$\hat{H}_i = -\sigma(\hat{S}^- \otimes \hat{S}_+ + \hat{S}^+ \otimes \hat{S}^-).$$

$$\dot{\hat{\rho}} = -i[\hat{H}_0 + \hat{H}_i, \hat{\rho}] + \sum_{i,j=1}^{2} \Gamma_{ij} (\hat{S}^+_i \hat{S}^-_j \hat{\rho} + \hat{\rho}\hat{S}^+_i \hat{S}^-_j - 2\hat{S}^-_i \hat{\rho} \hat{S}^+_j)$$

$$+ \ (\Gamma_0 + \Gamma_r)\mathbf{F}_{\mathbf{t}}[\hat{\rho}] + (\Gamma_0 - \Gamma_r)\mathbf{G}_{\mathbf{t}}[\hat{\rho}].$$

^ Off-diagonal, **Non-Markovian $\mathbf{F}_t$, $\mathbf{G}_t$** are
trace-preserving linear operators on the space of density matrices

we defined $\Gamma_{11} = \Gamma_{22} = \Gamma_0$ and $\Gamma_{12} = \Gamma_{21} = \Gamma_r$

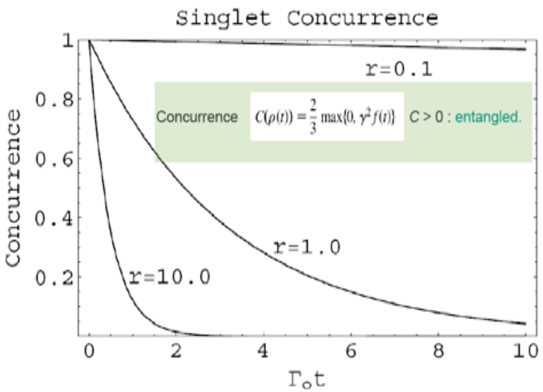
The term $\sigma(r)$ (see Appendix B) is a frequency shift caused by the vacuum fluctuations. It breaks the degeneracy of the two-qubit system and generates an effective dipole coupling

Entanglement Dynamics: Distance dependence

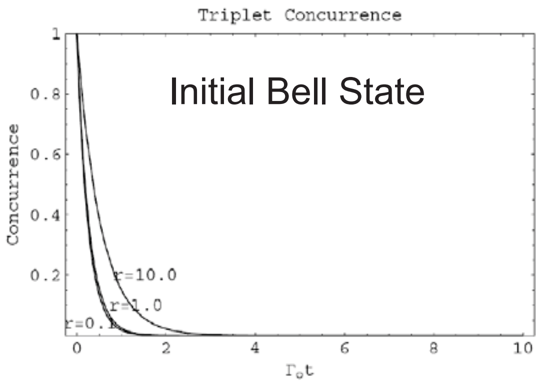
2 Qubit interaction with common EM field modes

Rigorous treatment of entanglement dynamics of two qubits coupled to a common quantized radiation field: How it depends on the initial state as well as interatomic separation: Entanglement can be protected if r<λ.

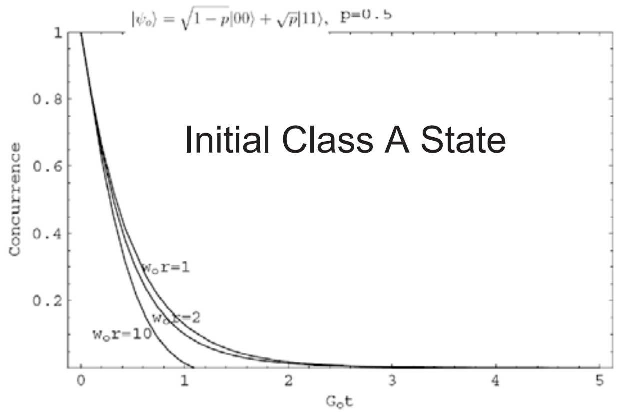
- **Disentanglement**
  - Initial Bell states: the decay of concurrence is slower when the qubits are closer. Opposite qualitative behavior for triplet states. Class A very different. **Experience ESD for $r \gg \lambda$ with no revival, but for r<λ there is no ESD.**
- **Decoherence**
  - For initial product states with the other atom in the ground state decoherence occurs at the single-atom rate for $r \gg \lambda$ while for r<λ decoherence is suppressed, decays much slower
  - With initial product states with the other atom in the excited state the late time decoherence is not suppressed for r<λ and remains modified from the single atom rate at $r \gg \lambda$



A plot of the concurrence as a function of time for the initial singlet state and for different values of the inter-qubit distance r (in units of ω_0^{-1}).The decay of concurrence are slower when the qubits are closer together.



Plot of the concurrence as a function of time for initial triplet state and for different values of the inter-qubit distance r (in units of ω_0^{-1})



A plot of the concurrence as a function of time for an initial Class A state with p = 0.5 for different values of interqubit distance $\omega_0 r$

Publications:

C. Anastopoulos, S. Shresta and B. L. Hu , quant-ph/0610007 Quant. Inf. Processing, 8, 594 (2009)

Entanglement Dynamics: 2 two-level atoms and 2 EM field modes

K. Sinha, N. Cummings and B. L. Hu, Protecting and Dynamically Generating Entanglement in a Two atom-Two Field Mode Model [arXiv1004xxx]

$$\hat{H}_I = \hbar g \left[\hat{\sigma}_1^{+}\hat{a}_1 + \hat{\sigma}_1^{+}\hat{a}_2 + \hat{\sigma}_2^{+}\hat{a}_1 + e^{i\phi}\hat{\sigma}_2^{+}\hat{a}_2 + h.c.\right]$$

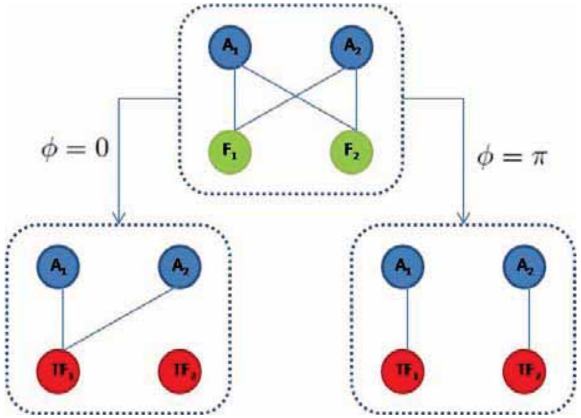
Distance effects is encapsulated in $\phi = (\mathbf{k}_2 - \mathbf{k}_1)\cdot(\mathbf{R}_2 - \mathbf{R}_1)$

$$|\Phi\rangle \equiv (|ee\rangle + |gg\rangle)/\sqrt{2}$$
$$|\Psi\rangle \equiv (|eg\rangle + |ge\rangle)/\sqrt{2}$$

Fields: $\hat{a}_1, \hat{a}_2$

Transformed Fields TF$_{1,2}$:

$$\hat{A}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)$$
$$\hat{A}_2 = \frac{1}{\sqrt{2}}(\hat{a}_1 - \hat{a}_2)$$



Initial field State		Atomic State				
TMSC	SMSC	A. $\|ee\rangle$	B. $\|eg\rangle$	C. $\|gg\rangle$	D. $\|\Phi\rangle$	E. $\|\Psi\rangle$
1. $\|00\rangle$ Vacuum	$\|0\rangle$	No	Yes, DI	Yes, DI	AL	DI
2. $\|n_N, m_N\rangle$ Fock	$\hat{\rho}_{nm}$	No	Yes, DI	Yes^a, DI/SD	SD	SD
3. $\|\eta_{nm}\rangle$	$\|n_N\rangle$	No	Yes, DI	Yes, SD	SD	SD
4. $\hat{\rho}_{th}$ Thermal	$\hat{\rho}_{th}$	No	Yes, DI	Yes, SD	AL/SD	SD
5. $\left\|\frac{1}{\sqrt{2}}(\alpha_c+\beta_c), \frac{1}{\sqrt{2}}(\alpha_c-\beta_c)\right\rangle$ Coherent	$\|\alpha_c\rangle$	Yes, SD/AL	Yes, SD	Yes, SD/AL	-	-
6. $\|\xi_{sq}, -\xi_{sq}\rangle$ Squeezed	$\hat{\rho}_{th}$	No	Yes, DI	Yes, SD	AL/SD	SD
7. $\|\xi, 0, 0\rangle$ TMSS	$\|\xi_{sq}\rangle$	Yes,SD/AL	Yes,SD	Yes,SD/AL	AL	SD

Initial field State		Atomic State				
TMAC	DJC	A. $\|ee\rangle$	B. $\|eg\rangle$	C. $\|gg\rangle$	D. $\|\Phi\rangle$	E. $\|\Psi\rangle$
1. $\|00\rangle$	$\|00\rangle$	No	No	No	SD	DI
2. $\|n_N, m_N\rangle$	$\|\eta_{nm}\rangle$	Yes^a, SD	Yes^a, SD	Yes^a, SD/DI	SD	SD/AL
3. $\|\eta_{nm}\rangle$	$\|n_N, m_N\rangle$	No	No	No	SD	SD
4. $\hat{\rho}_{th}$	$\hat{\rho}_{th}$	No	No	No	SD	SD
5. $\|\alpha_c, \beta_c\rangle$	$\left\|\frac{1}{\sqrt{2}}(\alpha_c+\beta_c), \frac{1}{\sqrt{2}}(\alpha_c-\beta_c)\right\rangle$	No	No	No	SD	SD
6. $\|\xi_{sq}, -\xi_{sq}\rangle$	$\|\xi, 0, 0\rangle$	Yes,SD	Yes,SD	Yes,SD	SD	SD
7. $\|\xi, 0, 0\rangle$	$\|\xi_{sq}, -\xi_{sq}\rangle$	No	No	No	SD	SD

^{a}No entanglement for $n_N = m_N$

Table I: Entanglement dynamics for two modes symmetrical coupling (TMSC) with $\phi = 0$ and anti-symmetrical coupling with $\phi = \pi$. Columns A-C list whether there is an entanglement generation in an initially separable atomic state (yes or no). The dynamical phenomena observed in columns A-E are listed as entanglement sudden death (SD), entanglement dies for only an instant (DI), entanglement remains non-zero at all times and so is "always living" (AL). A '/' denotes that both kinds of dynamics are present depending on the particular initial state chosen from within the class indicated. The initial states have been explained in Sect.III.

$$|\eta_{nm}\rangle = 2^{-(n+m)/2}(\hat{a}_1 + \hat{a}_2)^n(\hat{a}_1 - \hat{a}_2)^m|00\rangle \qquad \hat{\rho}_{nm} \equiv \mathrm{Tr}_{TF_2}\left[|\eta_{nm}\rangle\langle\eta_{nm}|\right]$$

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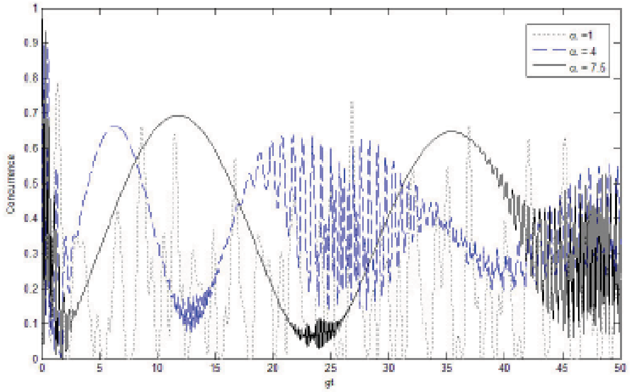
Entanglement Dynamics:
2 two-level atoms and 2 EM field modes

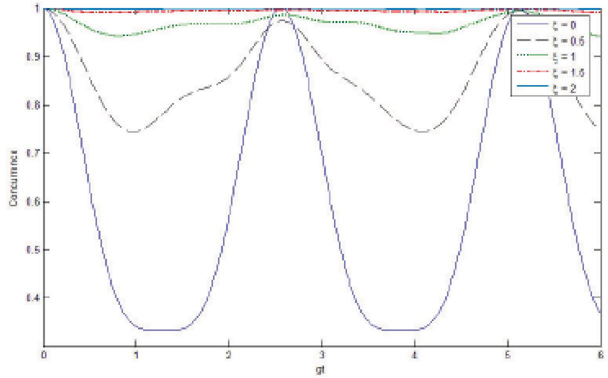
Entanglement generation and protection
For two atoms with symmetric coupling entanglement stays "always alive" (AL)

Symmetrically coupled coherent state



(a) $|ee\rangle$ - No sudden death for a range of values of α

Symmetrically coupled two-mode squeezed state



$|\Phi\rangle$ - State moves towards being maximally entangled at all times as squeezing is increased

Threshold temperature for entanglement protection

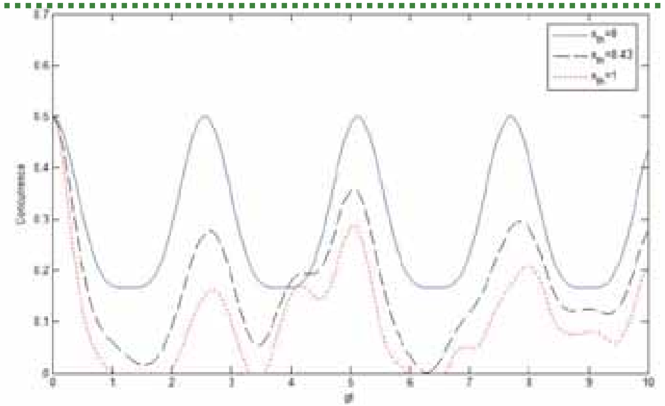
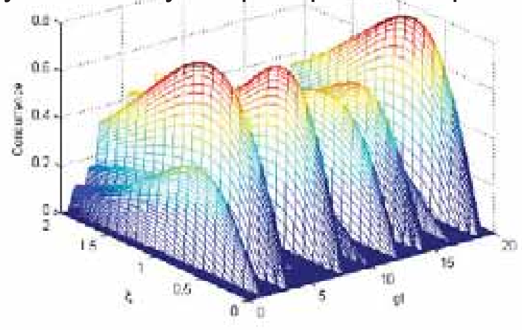


Figure 10: Single mode thermal field interacting with $|\Phi\rangle$

Entanglement generation from transfer of field mode correlations

Anti-symmetrically coupled product squeezed state



(a) $|ee\rangle$ - Entanglement generation from transfer of correlations from the field, optimal squeezing can be seen $\xi \approx 1$

Entanglement Dynamics:
2 two-level atoms and 2 EM field modes

Main Features

Symmetric and anti-symmetric couplings lead to very different entanglement dynamics

– **symmetric coupling being more conducive to entanglement generation and protection**

- "**Always alive" (AL) behavior** arises in separable atomic states for symmetrically coupled coherent and two-mode squeezed (TMSS) states

– **maximal entanglement protection** for initially entangled state $|\Phi\rangle$ nteracting with a TMSS

- **Entanglement generation from correlated field modes in Dbl Jaynes-Cummings via transfer**

– no entanglement for uncorrelated modes even though the original modes may be correlated

- Many to one mapping between initial field states for TMSC and SMSC shows that a set of states with same partially traced transformed mode have identical atom-atom entanglement dynamics

- Transition between AL and SD for a thermal field: determines condition for entanglement protection

Entanglement between two Relativistic Detectors

No causal contact: S. Y. Lin and B. L. Hu, Phys. Rev. D 81, 045019 (2010)
Two inertial detectors: S. Y. Lin and B. L. Hu, Phys. Rev. D 79, 085020 (2009),
Alice-Rob: S. Y. Lin, C. H. Chou and B. L. Hu, Phys. Rev. D78, 125025 (2008)

- Quantum entanglement plays a crucial role in EPR paradox, violation of Bell's inequality, quantum teleportation, etc.

- Entangled states are essentially "non-local" (cannot be modeled by any local hidden-variable theory.)

- Measure of entanglement for mixed states is known only for limited cases.

  - We need to build up our intuition on entanglement. Study simple models in depth.

2 identical Unruh-DeWitt detectors in (3+1)D Minkowski space

- bilinear interaction [DeWitt 1979] Detectors A, B are point-like objects.

$$S = -\int d^4x \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi$$

- massless scalar field

$$+\int d\tau_A \frac{1}{2}\left[(\partial_{\tau_A}Q_A)^2 - \Omega_0^2 Q_A^2\right] + \int d\tau_B \frac{1}{2}\left[(\partial_{\tau_B}Q_B)^2 - \Omega_0^2 Q_B^2\right]$$

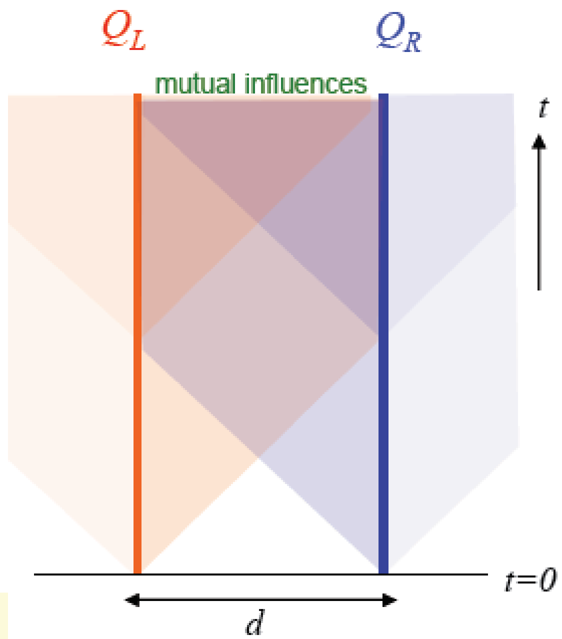
- internal: HO

$$+\lambda_0 \int d^4x \Phi(x)\left[\int d\tau_A Q_A(\tau_A)\delta^4\left(x^\mu - z_A^\mu(\tau_A)\right) + \int d\tau_B Q_B(\tau_B)\delta^4\left(x^\mu - z_B^\mu(\tau_B)\right)\right]$$

Both detectors are at rest at all times:

Q_L : $\quad z_L^\mu(t) = (t, -d/2, 0, 0)$

Q_R : $\quad z_R^\mu(t) = (t, d/2, 0, 0)$



Initial State: two-mode squeezed state:

$$|\,\psi(0)\,\rangle = |\,q_L, q_R\,\rangle \otimes |\,0_M\,\rangle$$

Minkowski vacuum

~ represented by Wigner function, is a Gaussian state.

$$\rho(Q_L, P_L, Q_R, P_R) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_L+Q_R)^2 + \frac{1}{\alpha^2}(Q_L-Q_R)^2 + \frac{\alpha^2}{\hbar^2}(P_L-P_R)^2 + \frac{1}{\beta^2}(P_L+P_R)^2\right]$$

- Criterion for separability / entanglement of A and B:

- Define

$$\Sigma(t, \tau = a^{-1}\sinh^{-1} at) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathbf{M}\right]$$

$$\mathbf{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

- For Gaussian states, $\Sigma < 0$ $\Leftrightarrow$ entangled, otherwise separable [Simon 00]

$\mathbf{V}^{PT} = \mathbf{\Lambda V \Lambda}$: Partial Transposition of $\mathbf{V}$, $\quad \mathbf{\Lambda} = \mathrm{diag}(1,1,1,-1)$

$$V_{\mu\nu}(t,\tau) = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu\, \rangle \equiv \frac{1}{2}\langle\, (\mathcal{R}_\mu\mathcal{R}_\nu + \mathcal{R}_\nu\mathcal{R}_\mu)\,\rangle$$

: 10 symmetric two-point functions (variances) of two detectors.

$$\mathcal{R}_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$$

Note: This criterion is testing the property of the PT Wigner functions, thus the reduced density matrix, of the detectors:

$$\rho^R(Q, Q'; \tau) = \int \mathcal{D}\Phi_k \psi_0[Q, \Phi_k; \tau]\psi_0^*[Q', \Phi_k; \tau]$$

So t and τ in Σ must be on the same time-slice as the field's.

Here, behavior of Σ ~ (logarithmic) negativity.

Sketch of calculation scheme:

- Evolution of operators $Q_A\,, P_A\,, Q_B\,, P_B\,, \Phi\,, \Pi$ in Heisenberg picture.
-

damped HO　　　　　　damped driven HO

$$\hat{Q}_i(\tau_i) = \sqrt{\frac{\hbar}{2\Omega_r}}\sum_j\left[q_i^{(j)}(\tau_i)\hat{a}_j + q_i^{(j)*}(\tau_i)\hat{a}_j^\dagger\right] + \int\frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}\left[q_i^{(+)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k} + q_i^{(-)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k}^\dagger\right]$$

- 10 symmetric 2-pt functions
-
- $$V_{\mu\nu}(t,\tau) = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu\,\rangle \equiv \frac{1}{2}\langle\, (\mathcal{R}_\mu\mathcal{R}_\nu + \mathcal{R}_\nu\mathcal{R}_\mu)\,\rangle$$

$$\mathcal{R}_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$$

$$\mathbf{\Lambda} = \mathrm{diag}(1,1,1,-1)$$

- Find entanglement dynamics with parametric depen
separation

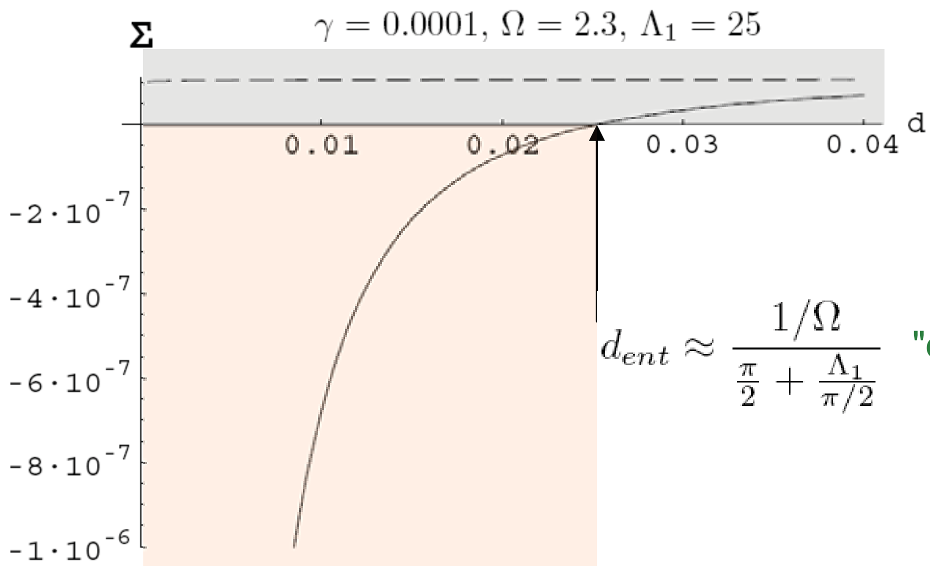
$$\mathrm{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\mathbf{V}^{PT} = \mathbf{\Lambda}\mathbf{V}\mathbf{\Lambda}$$

$$\Sigma(t, \tau = a^{-1}\sinh^{-1}at) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathrm{M}\right]$$

At short distances: late time behavior: steady state

Summing up
all orders of
mutual influences
and wait until all
transient responses
decay out -



$$d_{ent} \approx \frac{1/\Omega}{\frac{\pi}{2} + \frac{\Lambda_1}{\pi/2}}$$

"entanglement distance"

Initial state of detectors	$d < d_{ent}$: entangled	$d \geq d_{ent}$: separable
entangled	residual entanglement	sudden death
separable	entanglement creation	?

Non-Markovian dynamics: Entanglement generation, death and revival from a relativistic quantum field theory treatment

Instability: Two detectors with low frequencies get unstable when they are sufficiently close to each other.

In the stable regime:

Outside the light cone

The dependence of the relative "strength" of entanglement on the spatial separation oscillates in time.

$$d_{ent} \approx \frac{1/\Omega}{\frac{\pi}{2} + \frac{\Lambda_1}{\pi/2}}$$

Inside the light cone

At early times,

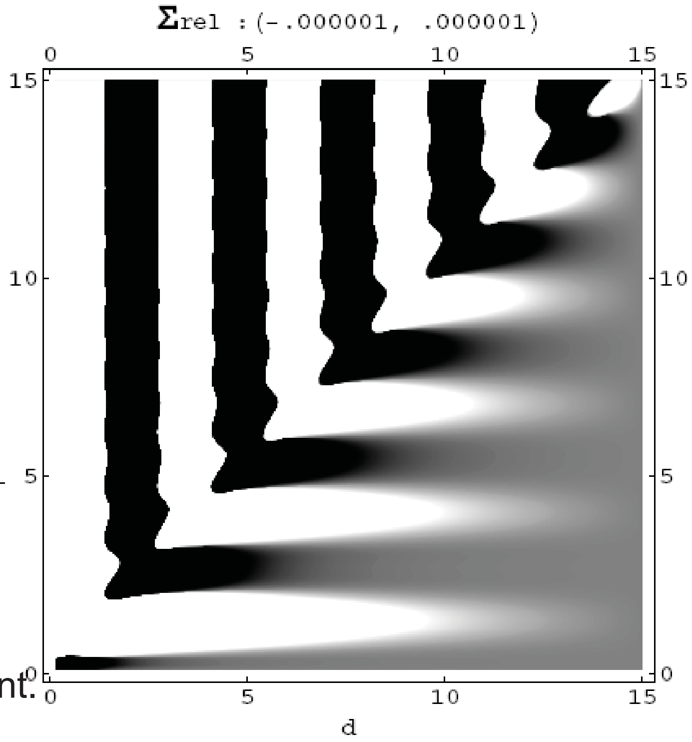
Interference pattern in entanglement develops for all d.

mid session,

Sudden death of entanglement if d> the entanglement distance *d*ent.

At **late times,**

Initial separable state will get entangled if the separation is less than the entanglement distance *d*ent.



"Degree of entanglement" (indicated by Σ) vs. separation d: **oscillates in time**. At some fixed time (bight region) increasing distance enhances entanglement. At some other times (dark region) the opposite is true.

Publications

1. S.Y. Lin and B.L. Hu, Phys. Rev D 79, 085020 (2009)
2. S.Y. Lin and B.L. Hu, Phys. Rev. D81, 045019 (2010)

Autangle: A case of Quantum Narcissism

- Evil Queen admiring herself in front of a mirror: getting fully entangled with herself

- Question 1: Is this configuration identical to that of a charge interacting with its image charge, as inelectrostatics. Can one use image charge method to calculate quantum entanglement?

- **Question 2:** How is this situation related to that of entanglement between two inertial detectors above?

Autangle: a new case in Freud's quantum repertoire

- Answer:

Q2: No. different from entanglement between 2 physical detectors mediated by a q field

Q1: Can work out the entanglement between the Evil Queen and the q field around her as modified by the presence of a mirror

Work in progress with Behunin, Lin and Zhou

Quantum Twin Paradox:
History, Not Symmetry!

A Quiz for experts of Unruh effect

(we have a lot here)

- **Question**: Aimed at `existence of event horizon' as an explanation or even precondition for Unruh - Hawking radiation. (e.g., view of GR community)

Is there radiation when the detector is accelerating but not uniformly?

1) **Mathematically,** there is no event horizon – it is a global concept. **NO**
2) **Physically,** this is a continuous extension of uniform acceleration**. YES, but not thermal**

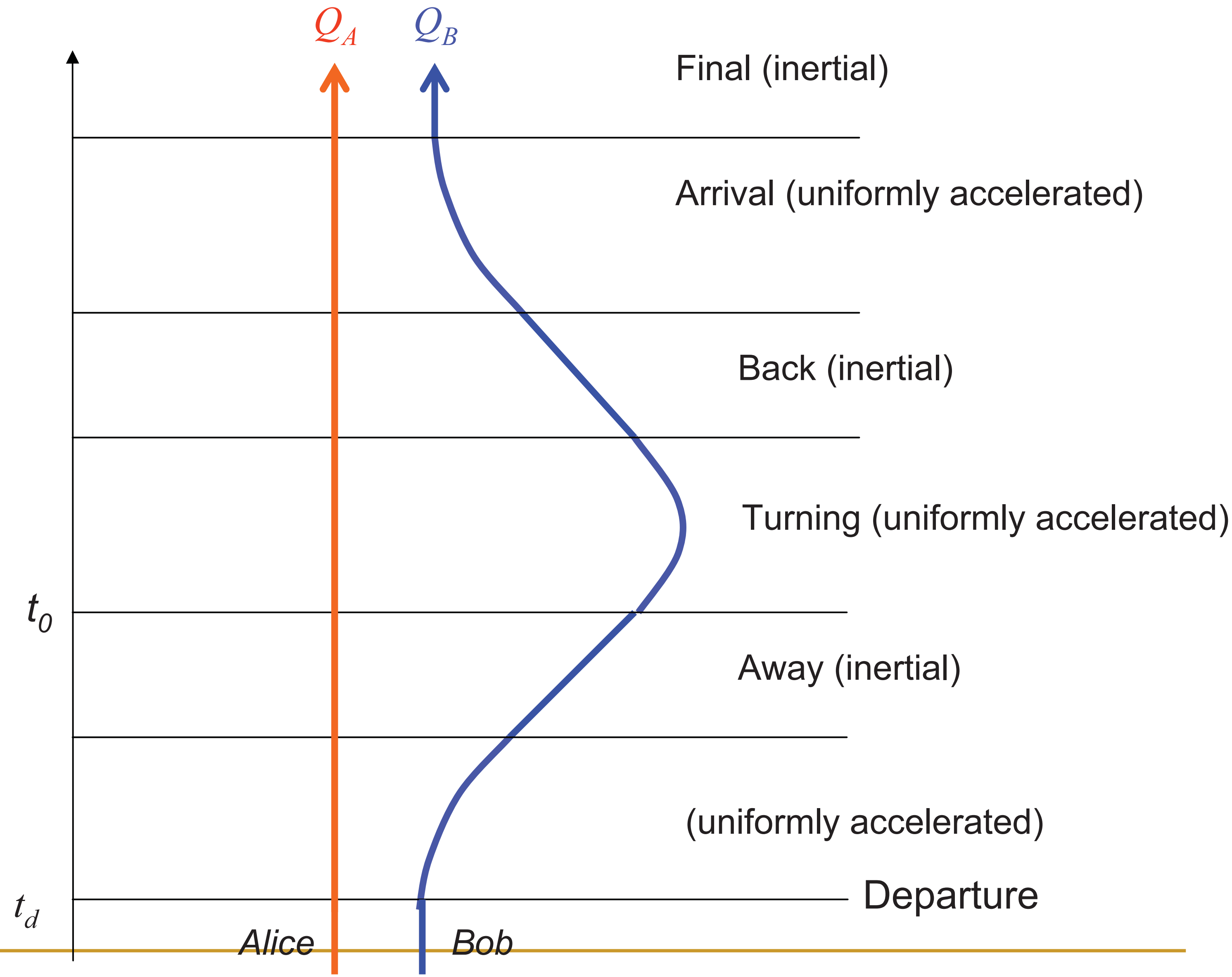
- **An Example**: *Passing a slow driver, or making a U turn*

  Uniform velocity to uniform acceleration then to uniform velocity.

  The traditional way of thinking using global concepts is too restrictive / rigid. A new way of thinking with a new method are better suited for treating the problem of a moving charge or detector interacting with a quantum field.

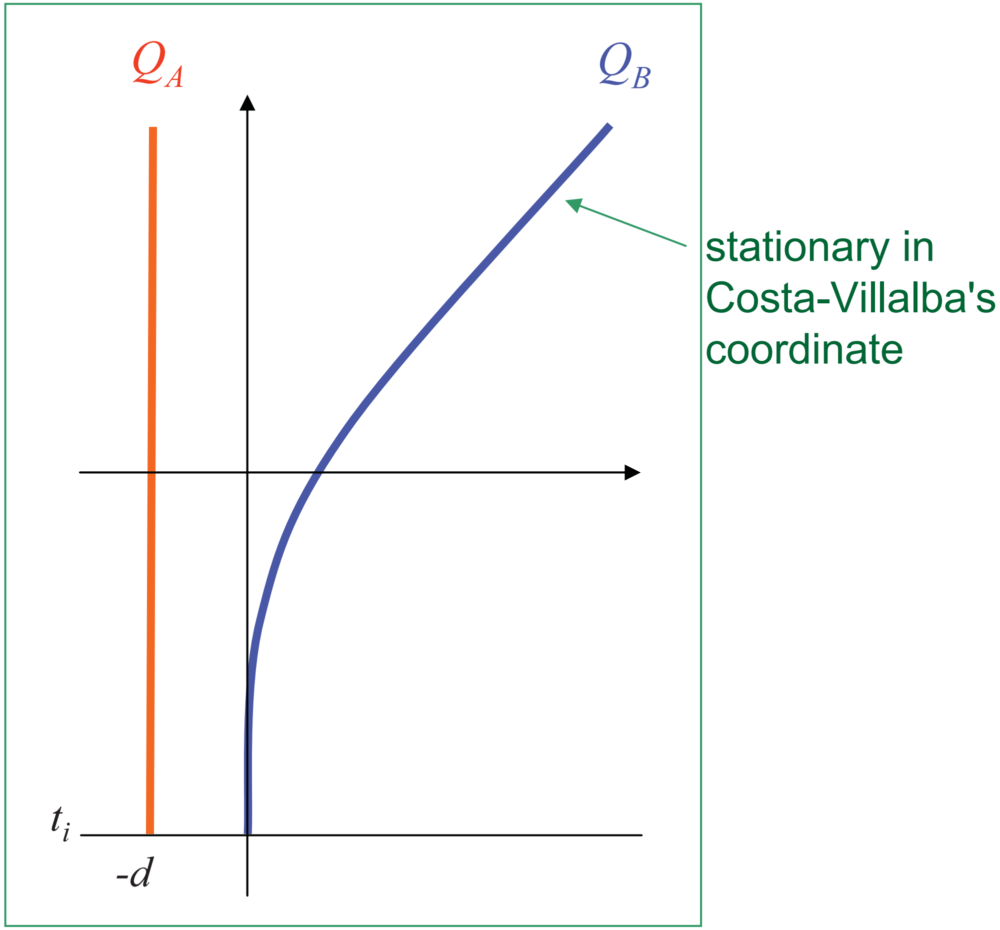
- We can show using **stochastic field theory** that 2) is correct and deduce the spectrum of radiation. NO temperature concept because this is under **nonequilibrium condition**.

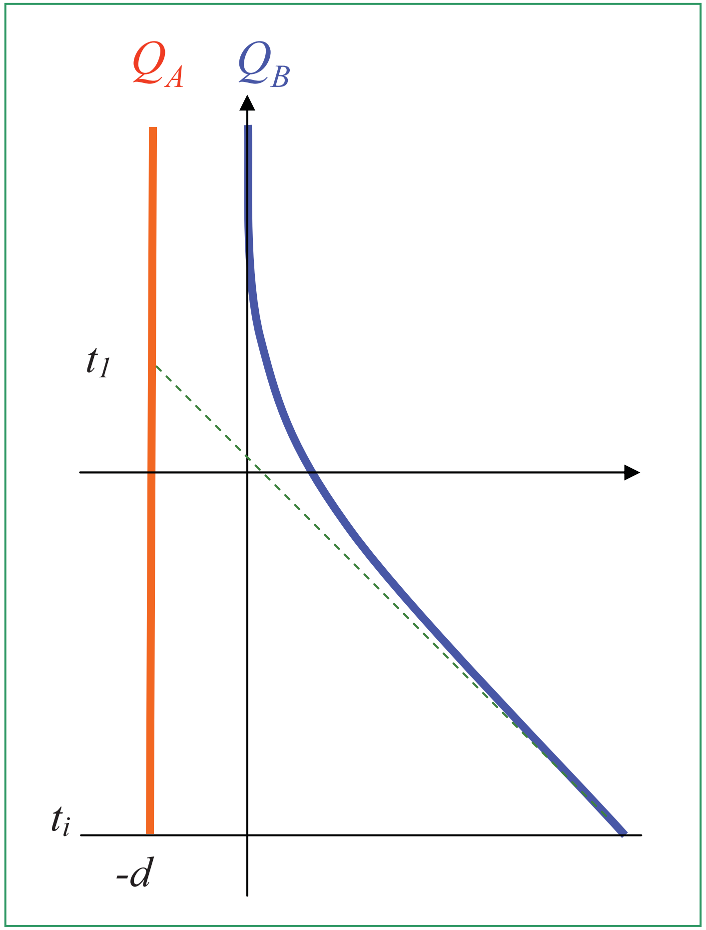
Quantum Twin Paradox: setup



Non-Uniform Acceleration

[Ostapchuk, Lin, Hu, Mann, in preparation]





Initial state (Gaussian)

$$| \psi(0) \rangle = | q_A, q_B \rangle \otimes | 0_M \rangle$$

Most of the retarded influences from B is off resonance for A at t_1.

Accumulated high frequency, components corresponding to classical shock wave are of little concern. Thus perturbative treatment OK

Stochastic Field Theory based on **Influence functional method** can treat m**ore general conditions**

- **An Example**: *Finite time* uniform accelerated detector

A. Raval, B. L. Hu and Don Koks, Phys. Rev. D55, 4795 NEAR-THERMAL RADIATION IN DETECTORS, MIRRORS AND BLACK HOLES: A Stochastic Approach"

$$x(t)=x_0^{-1}(a^{-2}-t_0t) \quad (t<-t_0)$$

$$=(a^{-2}+t^2)^{1/2} \quad (-t_0<t<t_0)$$

$$=x_0^{-1}(a^{-2}+t_0t) \quad (t>t_0).$$

Before : uniform

velocity is $-c^2t_0/x_0$

Turnaround:
Uniform acceleration

After: same velocity, opposite direction

Set up is the same as the **Twin Paradox:** Quantum version

We may also define null coordinates $u=t-x$ and $v=t+x$. In terms of these, the time at which the trajectory crosses the future horizon $u=0$ of the uniformly accelerated interval is $t_H=-(a^2u_0)^{-1}$.

If we choose to parametrize the trajectory by the proper time τ, it can be expressed as (with the zero of proper time chosen at $t=0$)

$$u(\tau)=\theta(-\tau_0-\tau)v_0\{a(\tau+\tau_0)-1\}-a^{-1}\theta(\tau_0+\tau)\times\theta(\tau_0-\tau)e^{-a\tau}+\theta(\tau-\tau_0)u_0\{1+a(\tau_0-\tau)\}, \tag{3.2}$$

$$v(\tau)=-\theta(-\tau_0-\tau)u_0\{a(\tau+\tau_0)+1\}+a^{-1}\theta(\tau_0+\tau)\times\theta(\tau_0-\tau)e^{a\tau}+\theta(\tau-\tau_0)v_0\{1-a(\tau_0-\tau)\}, \tag{3.3}$$

where $\pm\tau_0$ is the proper time of the trajectory when it exits (enters) the uniformly accelerated phase. It satisfies the relations

$$v_0\equiv t_0+x_0=a^{-1}e^{a\tau_0},$$

$$u_0\equiv t_0-x_0=-a^{-1}e^{-a\tau_0}. \tag{3.4}$$

Quantum Twin Paradox [Lin, Behunin and Hu, work in progress]

We expect...

- Information of the initial state of the detectors will be dispersed into the field.
- Initial entanglement between the detectors, if any, would disappear (sudden death) if the traveling time of Bob is long enough.
- During the acceleration and deceleration stages, Bob will experience something similar to the Unruh effect.
- If Bob and Alice are sitting close enough to each other in the final stage, then the field induced quantum entanglement would take over, namely, entanglement will be created (revived) by mutual influences mediated by the field.
- **No time reversal symmetry in the history of the system.**
- Finite time acceleration. No event horizon
- Time dilation of Bob should be considered.
- Mutual influence due to the deceleration of Bob does not have an impact on Alice.

Influence Functional Approach to Stochastic Dynamics of Moving Atom- Field Interaction

Earlier work with **Alpan Raval, James Anglin and Don Koks** (1996-97) on

Moving Atom – Quantum Field Interaction via the Influence Functional Method

1. Atom - Field Interaction, Moving Detectors (harmonic oscillators)
2. Dissipation and Noise kernels
3. Coherence, Fluctuations, Correlations
4. Entanglement and Teleportation

Summary (second part):

Interaction between atoms (or detectors) and a quantum field

1. Models / Methods: Quantum field in terms of parametric oscillators
   - QBM model of one or many oscillators in a common q. field
   - Influence Functional Formalism: Dissipation and Noise kernels
2. Many atoms coupled through a quantum field
   - Quantum Coherence, Fluctuations, Correlations: (RHA 96)
   - Fluctuation-Dissipation / Correlation- Propagation relations
3. Quantum Entanglement:
   - Q Twin (RHK 97),
   - Autangle
4. Quantum Teleportation
5. Applications to:
   - Atoms in a cavity, atom-array, ensemble
   - spin chain, strongly correlated systems
   - Uniformly accelerated detector: beyond Unruh Effect
   - Black hole fluctuations, entanglement and backreaction

A. Influence functional for N arbitrarily moving detectors

Consider N detectors $i=1,\ldots,N$ in $1+1$ dimensions with internal oscillator coordinates $Q_i(\tau_i)$, and trajectories $(x_i(\tau_i),t_i(\tau_i))$, τ_i being a parameter along the trajectory of detector i. In the following analysis, we do not need to assume that τ_i is the proper time, although this is, in most cases, a convenient choice. However, we will assume hereafter that the trajectories $(t_i(\tau_i),x_i(\tau_i))$ are smooth and that the parameters τ_i are chosen such that $t_i(\tau_i)$ is a strictly increasing function of τ_i .

The detectors are coupled to a massless scalar field $\phi(x,t)$ via the interaction action

$$S_{\mathrm{int}}=\sum_i e_i\int_{-\infty}^{t_i^{-1}(T)} d\tau_i s_i(\tau_i)\frac{dQ_i}{d\tau_i}\phi(x_i(\tau_i),t_i(\tau_i)). \qquad (2.1)$$

Here, T is a global Minkowski time coordinate which defines a spacelike hypersurface, e_i denotes the coupling constant of detector i to the field, $s_i(\tau_i)$ is the switching function for detector i (typically a step function), and t_i^{-1} is the inverse function of t_i. $t_i^{-1}(T)$ is, therefore, the value of τ_i at the point of intersection of the spacelike hypersurface defined by T with the trajectory of detector i. Note that the strictly increasing property of $t_i(\tau_i)$ implies that the inverse, if it exists, is unique.

The action of the system of detectors is

$$S_{\text{osc}} = \frac{1}{2}\sum_i \int_{-\infty}^{t_i^{-1}(T)} d\tau_i [(\partial_{\tau_i} Q_i)^2 - \Omega_i^2 Q_i^2]. \qquad (2.2)$$

The scalar field action is given by

$$S_{\text{field}} = \frac{1}{2}\int_{-\infty}^{T} dt \int dx [(\partial_t \phi)^2 - (\partial_x \phi)^2] \qquad (2.3)$$

and the complete action

$$S = S_{\text{field}} + S_{\text{osc}} + S_{\text{int}}. \qquad (2.4)$$

Expanding the field in normal modes,

$$\phi(x,t)=\sqrt{\frac{2}{L}}\sum_{k}{}' \, [q_k^+(t)\cos kx+q_k^-(t)\sin kx], \quad (2.5)$$

where Σ_k' denotes that the summation is restricted to the upper half k space, $k>0$. Then the action for the scalar field is given by $(\sigma=+,-)$

$$S_{\text{field}}=\frac{1}{2}\sum_{k,\sigma}{}' \, [(\dot{q}_k^\sigma)^2-\omega_k^2 q_k^2] \qquad (2.6)$$

$$S_{\text{int}} = -\sum_{k,\sigma}{}' \int_{-\infty}^{T} dt J_k^\sigma(t) q_k^\sigma(t), \tag{2.8}$$

where

$$J_k^\sigma(t) = -\sum_i e_i \sqrt{\frac{2}{L}} \int_{-\infty}^{t_i^{-1}(T)} d\tau_i \delta(t - t_i(\tau_i)) \frac{dQ_i}{d\tau_i} u_k^\sigma(\tau_i) s_i(\tau_i) \tag{2.9}$$

and

$$u_k^+(\tau_i) = \cos k x_i(\tau_i), \quad u_k^-(\tau_i) = \sin k x_i(\tau_i). \tag{2.10}$$

The action $S_{\text{field}}+S_{\text{int}}$, therefore, describes a system of decoupled harmonic oscillators each driven by separate source terms. The zero temperature influence functional (corresponding to the initial state of the field being the Minkowski vacuum) for this system has the form [22]

$$\mathcal{F}[J,J'] = \exp\Bigg\{ -\frac{1}{\hbar}\sum_{k,\sigma}{}' \int_{-\infty}^{T} ds \int_{-\infty}^{s} ds' [J_k^{\sigma}(s) - J_k'^{\sigma}(s)]$$

$$\times [\zeta_k(s,s')J_k^{\sigma}(s') - \zeta_k^*(s,s')J_k'^{\sigma}(s')]\Bigg\}, \quad (2.11)$$

where for temperature T=0: $$\zeta_k \equiv \nu_k + i\mu_k = \frac{1}{2\omega_k}\exp[-i\omega_k(s-s')]$$

And for finite T = 1/ β $$\zeta_k = \frac{1}{2\omega_k}\left[\coth\left(\frac{\beta\omega_k\hbar}{2}\right)\cos\omega_k(s-s') - i\sin\omega_k(s-s')\right]$$

Influence Functional

$$\mathcal{F}[\{Q\};\{Q'\}] = \exp\left\{ -\frac{1}{\hbar}\left[\sum_{i,j=1}^{N} \int_{-\infty}^{t_i^{-1}(T)} \right.\right.$$

$$\times d\tau_i s_i(\tau_i) \int_{-\infty}^{t_j^{-1}(t_i(\tau_i))} d\tau_j' s_j(\tau_j')$$

$$\times \left(\frac{dQ_i}{d\tau_i} - \frac{dQ_i'}{d\tau_i} \right) \left(Z_{ij}(\tau_i, \tau_j') \frac{dQ_j}{d\tau_j'} \right.$$

$$\left.\left.\left. - Z_{ij}^{*}(\tau_i, \tau_j') \frac{dQ_j'}{d\tau_j'} \right) \right] \right\}, \qquad (2.14)$$

$$Z_{ij}(\tau_i, \tau_j') = \frac{2}{L} e_i e_j \sum_{k,\sigma}{}' \; \zeta_k(t_i(\tau_i), t_j(\tau_j')) u_k^{\sigma}(\tau_i) u_k^{\sigma}(\tau_j')$$

In the above, the continuum limit in the mode sum is recovered through the replacement $\sum_k' \to \frac{L}{2\pi}\int_0^\infty dk$. We then obtain, after substituting for u_k^σ and ζ_k,

$$Z_{ij}(\tau_i,\tau_j') = \frac{e_i e_j}{2\pi}\int_0^\infty \frac{dk}{k} e^{-ik(t_i(\tau_i)-t_j(\tau_j'))}\cos k(x_i(\tau_i)-x_j(\tau_j')). \tag{2.16}$$

In this form, Z_{ij} is proportional to the two point function of the free scalar field in the Minkowski vacuum, evaluated for the two points lying on trajectories i and j of the detector system. It obeys the symmetry relation

$$Z_{ij}(\tau_i,\tau_j') = Z_{ji}^*(\tau_j',\tau_i) \tag{2.17}$$

Corresponding to (2.12), we may also split Z_{ij} into its real and imaginary parts. Thus we define

$$Z_{ij}(\tau_i,\tau_j') = \tilde{\nu}_{ij}(\tau_i,\tau_j') + i\tilde{\mu}_{ij}(\tau_i,\tau_j') \tag{2.18}$$

where

$$\begin{aligned}
\tilde{\nu}_{ij}(\tau_i,\tau_j') &= \frac{e_i e_j}{2\pi}\int_0^\infty \frac{dk}{k}\cos k(t_i(\tau_i)-t_j(\tau_j'))\cos k(x_i(\tau_i)-x_j(\tau_j')) \\
\tilde{\mu}_{ij}(\tau_i,\tau_j') &= -\frac{e_i e_j}{2\pi}\int_0^\infty \frac{dk}{k}\sin k(t_i(\tau_i)-t_j(\tau_j'))\cos k(x_i(\tau_i)-x_j(\tau_j')).
\end{aligned} \tag{2.19}$$

$\tilde{\nu}$ and $\tilde{\mu}$ are proportional to the anticommutator and the commutator of the field in the Minkowski vacuum, respectively.

Two Inertial Detectors:

We now consider the case of two detectors moving on the inertial trajectories $x_1(\tau_1) = -x_0/2$, $x_2(\tau_2) = x_0/2$, and $t_1(\tau_1) = t_2(\tau_2) = \tau$, coupled to a scalar field initially in the Minkowski vacuum state, with coupling constants $e_{1,2}$. They are separated by a fixed coordinate distance x_0. As before, we will assume that both detectors have been forever switched on, i.e., $s_i(\tau) = 1$, $i = 1,2$.

$$Z_{11}(\tau,\tau') = \frac{e_1^2}{2\pi}\int_0^\infty \frac{dk}{k}\exp[-ik(\tau-\tau')],$$

$$Z_{22}(\tau,\tau') = \frac{e_2^2}{2\pi}\int_0^\infty \frac{dk}{k}\exp[-ik(\tau-\tau')],$$

$$Z_{12}(\tau,\tau') = Z_{21}(\tau,\tau')$$

$$= \frac{e_1 e_2}{2\pi}\int_0^\infty \frac{dk}{k}\exp[-ik(\tau-\tau')]\cos kx_0$$

The coupled Langevin equations for the system are

$$\frac{d^2 Q_1}{d\tau^2}+\frac{e_1^2}{2}\frac{dQ_1}{d\tau}+\frac{e_1 e_2}{2}\frac{dQ_2}{d\tau}\Big|_{\tau-x_0}+\Omega_1^2 Q_1=\frac{d\eta_1}{d\tau}, \tag{3.27}$$

$$\frac{d^2 Q_2}{d\tau^2}+\frac{e_2^2}{2}\frac{dQ_2}{d\tau}+\frac{e_1 e_2}{2}\frac{dQ_1}{d\tau}\Big|_{\tau-x_0}+\Omega_2^2 Q_2=\frac{d\eta_2}{d\tau}, \tag{3.28}$$

where $\tau - x_0$ is the retarded time between the two trajectories, and

$$\langle\{\eta_i(\tau),\eta_j(\tau')\}\rangle=\hbar\,\tilde{\nu}_{ij}(\tau-\tau'). \tag{3.29}$$

$$\widetilde{Q}_2(\omega) = L_{22}(\omega)\,\widetilde{\eta}_2(\omega) + L_{21}(\omega)\,\widetilde{\eta}_1(\omega), \qquad (3.33)$$

where L_{22} is the modified self-impedance of detector two because of the presence of detector one, and L_{21} is the mutual impedance:

$$L_{22}(\omega) = \chi_\omega^{(2)}(1 - 4\gamma_1\gamma_2 e^{-2i\omega x_0}\chi_\omega^{(1)}\chi_\omega^{(2)})^{-1},$$

$$L_{21}(\omega) = -2\sqrt{\gamma_1\gamma_2}e^{-i\omega x_0}\chi_\omega^{(2)}\chi_\omega^{(1)} \times (1 - 4\gamma_1\gamma_2 e^{-2i\omega x_0}\chi_\omega^{(1)}\chi_\omega^{(2)})^{-1}. \qquad (3.34)$$

Correlation Function

$$\langle\{Q_i(\omega), Q_j(\omega')\}\rangle = \sum_{\alpha=1}^{2}\sum_{\beta=1}^{2} L_{i\alpha}(\omega)L_{j\beta}(\omega') \times \langle\{\widetilde{\eta}_\alpha(\omega), \widetilde{\eta}_\beta(\omega')\}\rangle.$$