4th Relativistic Quantum Information Workshop

RQI-N Hualien 28-30 May 2010

How statistics rules fermion correlations in Rindler and Schwarzschild spacetimes

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How statistics rules fermion correlations in Rindler and Schwarzschild spacetimes

and a ?bit more

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Works done in collaboration with

Eduardo Martín –Martínez
and
Luis Garay

Phys Revs A 2009, 2010
and next week in the arXiv

Bipartite systems with two observers

$$|\phi_A, \phi_R\rangle \equiv \underbrace{|\phi_A\rangle}_{\text{Alice's}} \otimes \underbrace{|\phi_R\rangle}_{\text{Rob's}}$$

$$|1_{\hat{\omega}_1} 1_{\hat{\omega}_2}\rangle = |1_{\hat{\omega}_1}\rangle \otimes |1_{\hat{\omega}_2}\rangle$$

The case when Rob is a non inertial observer

With uniform acceleration … (Rindler)

Near a black hole … (Schwarzschid)

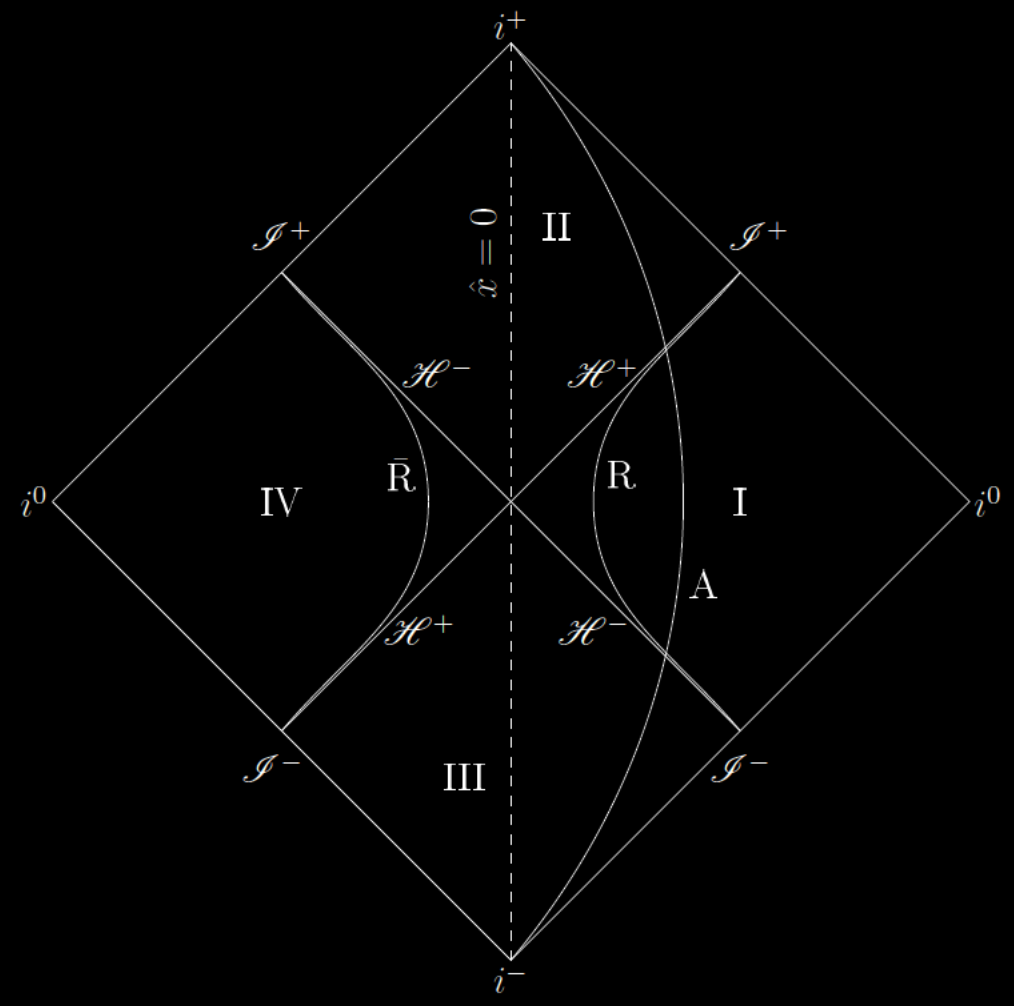


Figure 1: Flat space-time conformal diagram showing Alice, Rob and AntiRob trajectories

Minkowski and Rindler modes

$$u^{\mathrm{M}}_{\omega_j} = \sum_i \left(\alpha^{I}_{ij} u^{\mathrm{I}}_{\omega_i} + \beta^{IV}_{ij} u^{\mathrm{IV}*}_{\omega_i} + \alpha^{IV}_{ij} u^{\mathrm{IV}}_{\omega_i} + \beta^{I}_{ij} u^{\mathrm{I}*}_{\omega_i} \right)$$

$$a_{\omega_j} = \sum_i \left(\alpha^{\mathrm{I}}_{ij} a_{\omega_i, I} + \beta^{\mathrm{IV}}_{ij} a^{\dagger}_{-\omega_i, IV} + \alpha^{\mathrm{IV}}_{ij} a_{\omega_i, IV} + \beta^{\mathrm{I}}_{ij} a^{\dagger}_{-\omega_i, I} \right)$$

$$\alpha^{R}_{ij} = \left(u^{\mathrm{M}}_{\omega_j}, u^{R}_{\omega_i} \right) \qquad \beta^{R}_{ij} = -\left(u^{\mathrm{M}}_{\omega_j}, u^{R*}_{\omega_i} \right) \qquad R = I, IV$$

Bogoliubov transformations for scalar fields

$$\psi_j^{\mathrm{M}} = \sum_i C_{ij}\, u_{\omega_i}$$

$$\hat{\alpha}^{\mathrm{I}}_{ij} = \left(\psi_j^{\mathrm{M}}, u^{I}_{\omega_i}\right) = \cosh r_i\, \delta_{ij}\,, \qquad \hat{\alpha}^{\mathrm{IV}}_{ij} = \left(\psi_j^{\mathrm{M}}, u^{\mathrm{IV}}_{\omega_i}\right) = 0$$

$$\hat{\beta}^{\mathrm{I}}_{ij} = -\left(\psi_j^{\mathrm{M}}, u^{II*}_{\omega_i}\right) = -\sinh r_i\, \delta_{ij}\,, \quad \hat{\beta}^{\mathrm{I}}_{ij} = -\left(\psi_j^{\mathrm{M}}, u^{\mathrm{I}*}_{\omega_i}\right) = 0$$

$$a'_{\omega_R} = \cosh r_{\mathrm{s}}\, a_{\omega_R, I} - \sinh r_{\mathrm{s}}\, a^{\dagger}_{\omega_R, IV}$$

$$\tanh r_i = \exp\left[-\pi \frac{\omega_i c}{a}\right]$$

Bogoliubov transformations for Fermion fields

$$\psi^{\mathrm{M}}_{j,\sigma} = \sum_i D_{ij}\, u_{\omega_i,\sigma}, \qquad \bar{\psi}^{\mathrm{M}}_{j,\sigma} = \sum_i E_{ij}\, v_{\omega_i,\sigma}$$

$$\begin{aligned} c'_{\omega_R,s} &= \cos r_{\mathrm{d}}\, c_{I,\omega_R,s} - \sin r_{\mathrm{d}}\, d^{\dagger}_{IV,\omega_R,-s} \\ {d'_{\omega_R,s}}^{\dagger} &= \cos r_{\mathrm{d}}\, d^{\dagger}_{IV,\omega_R,s} + \sin r_{\mathrm{d}}\, c_{I,\omega_R,-s} \end{aligned}$$

$$\tan r_{\mathrm{d}} = \exp\left[-\pi \frac{\omega_R c}{a}\right]$$

How a Rindler observer would tell the Minkowski vacuum
and
its first excitation?

Scalar states

$$|0\rangle_{\mathrm{M}} = \bigotimes_{\omega_R} |0_{\omega_R}\rangle_{\mathrm{M}} \qquad a'_{wR}\,|0> = 0$$

$$|0_{\omega_R}\rangle_{\mathrm{M}} = \frac{1}{\cosh r_{\mathrm{s}}} \sum_{n=0}^{\infty} \tanh^n r_{\mathrm{s}}\, |n_{\omega_R}\rangle_I\, |n_{\omega_R}\rangle_{IV}$$

$$|1_{\omega_R}\rangle_{\mathrm{M}} = (a'_{\omega_R})^{\dagger}\, |0\rangle_{\mathrm{M}}$$

$$|1_{\omega_{\mathrm{R}}}\rangle_{\mathrm{M}} = \frac{1}{\cosh^2 r_{\mathrm{s}}} \sum_{n=0}^{\infty} \tanh^n r_{\mathrm{s}}\, \sqrt{n+1}\, |n+1_{\omega_R}\rangle_I |n_{\omega_R}\rangle_{IV}$$

Fermion states

$$|s_{\omega_R}\rangle_{\mathrm{M}} = (c'_{\omega_R,s})^\dagger\,|0\rangle_{\mathrm{M}}$$

$$|s_{\omega_R}\rangle_I = c^\dagger_{I,\omega_R,s}\,|0\rangle_I\,, \qquad |s_{\omega_R}\rangle_{IV} = d^\dagger_{IV,\omega_R,s}\,|0\rangle_{IV}$$

$$|p_{\omega_R}\rangle_I = c^\dagger_{I,\omega_R,\uparrow}c^\dagger_{I,\omega_R,\downarrow}\,|0\rangle_I \qquad |p_{\omega_R}\rangle_{IV} = d^\dagger_{IV,\omega_R,\uparrow}d^\dagger_{IV,\omega_R,\downarrow}|0\rangle_{IV}$$

$$\begin{aligned}|0_{\omega_R}\rangle &= \cos^2 r_{\mathrm{d}}\;|0\rangle_I\,|0\rangle_{IV} + \sin r_{\mathrm{d}}\;\cos r_{\mathrm{d}}\,(|{\uparrow}_{\omega_R}\rangle_I\,|{\downarrow}_{\omega_R}\rangle_{IV} \\ &+ |{\downarrow}_{\omega_R}\rangle_I\,|{\uparrow}_{\omega_R}\rangle_{IV}) + \sin^2 r_{\mathrm{d}}\;|p_{\omega_R}\rangle_I\,|p_{\omega_R}\rangle_{IV}\end{aligned}$$

$$\begin{aligned}|{\uparrow}_{\omega_R}\rangle &= \cos r_{\mathrm{d}}\,|{\uparrow}_{\omega_R}\rangle_I\,|0\rangle_{IV} + \sin r_{\mathrm{d}}\,|p_{\omega_R}\rangle_I\,|{\uparrow}_{\omega_R}\rangle_{IV}\,, \\ |{\downarrow}_{\omega_R}\rangle &= \cos r_{\mathrm{d}}\,|{\downarrow}_{\omega_R}\rangle_I\,|0\rangle_{IV} - \sin r_{\mathrm{d}}\,|p_{\omega_R}\rangle_I\,|{\downarrow}_{\omega_R}\rangle_{IV}\,.\end{aligned}$$

What if one bipartite state is observed by Alice and Rob?

Bipartite states, scalar field

Example

$$\boxed{|\Psi\rangle_s = \frac{1}{\sqrt{2}}\left(|0\rangle_A |0\rangle_R + |1\rangle_A |1\rangle_R\right)}$$

$$\begin{aligned}|\psi\rangle_s \; &= \sum_{n=0}^{\infty} \frac{\tanh^n r_\mathrm{s}}{\sqrt{2}\cosh r_\mathrm{s}} \Bigg(|0_{\omega_1}\rangle_\mathrm{M} \otimes |n_{\omega_2}\rangle_I \, |n_{-\omega_2}\rangle_{IV} \\ &+ \; \frac{\sqrt{n+1}}{\cosh r_\mathrm{s}} |1_{\omega_1}\rangle_\mathrm{M} \otimes |n+1_{\omega_2}\rangle_I \, |n_{-\omega_2}\rangle_{IV} \Bigg)\end{aligned}$$

$$\begin{aligned}|\psi\rangle_s \; = \sum_{n=0}^{\infty} \frac{\tanh^n r_\mathrm{s}}{\sqrt{2}\cosh r_\mathrm{s}} \Bigg(& |0_{\omega_1}\rangle_A \, |n_{\omega_2}\rangle_R \, |n_{-\omega_2}\rangle_{\bar{R}} + \frac{\sqrt{n+1}}{\cosh r_\mathrm{s}} \\ & \times |1_{\omega_1}\rangle_A \, |n+1_{\omega_2}\rangle_R \, |n_{-\omega_2}\rangle_{\bar{R}} \Bigg)\end{aligned}$$

Bipartite states Fermion field

Example

$$\boxed{|\Psi\rangle_d = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_R + |\uparrow\rangle_A|\downarrow\rangle_R\right)}$$

The triple
(There are no two without a third)

$$\rho^{s}_{AR\bar{R}} = |\Psi_s\rangle\langle\Psi_s|\,, \qquad \rho^{d}_{AR\bar{R}} = |\Psi_d\rangle\langle\Psi_d|$$

1. Alice-Rob (AR)
2. Alice-AntiRob $(A\bar{R})$
3. Rob-AntiRob $(R\bar{R})$

$$\rho^{AR} = \mathrm{Tr}_{IV}\,\rho^{AR\bar{R}}$$
$$\rho^{A\bar{R}} = \mathrm{Tr}_{I}\,\rho^{AR\bar{R}}$$
$$\rho^{R\bar{R}} = \mathrm{Tr}_{M}\,\rho^{AR\bar{R}}.$$

A two character drama with three actors

Compute

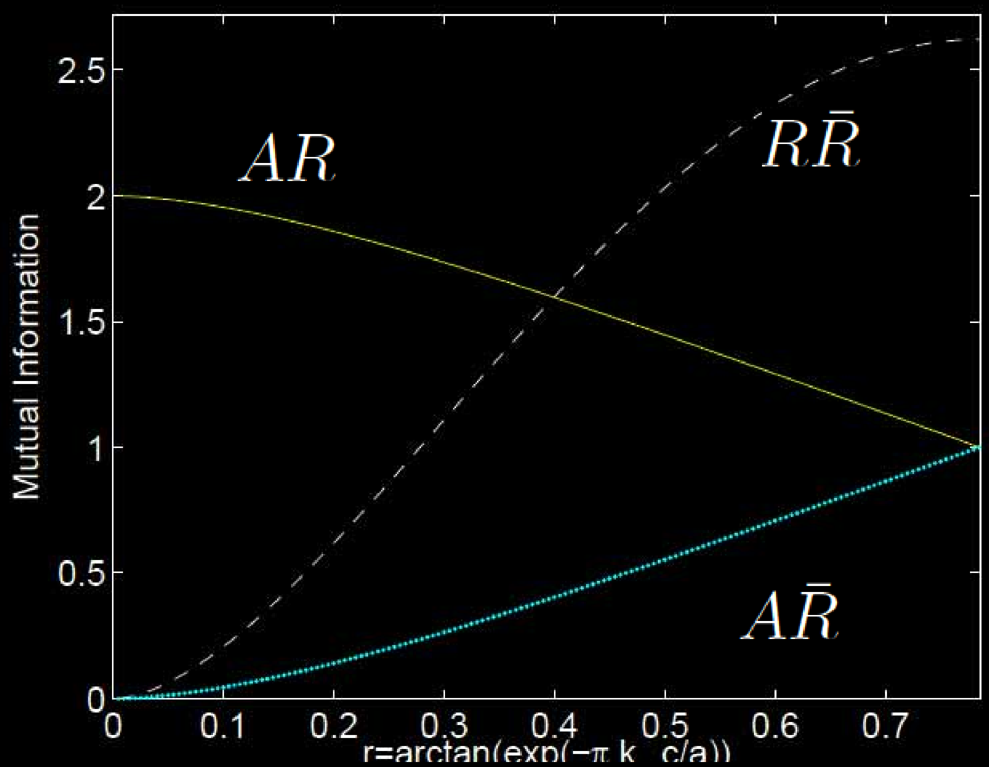
density matrices, partial transposes, eigenvalues ....

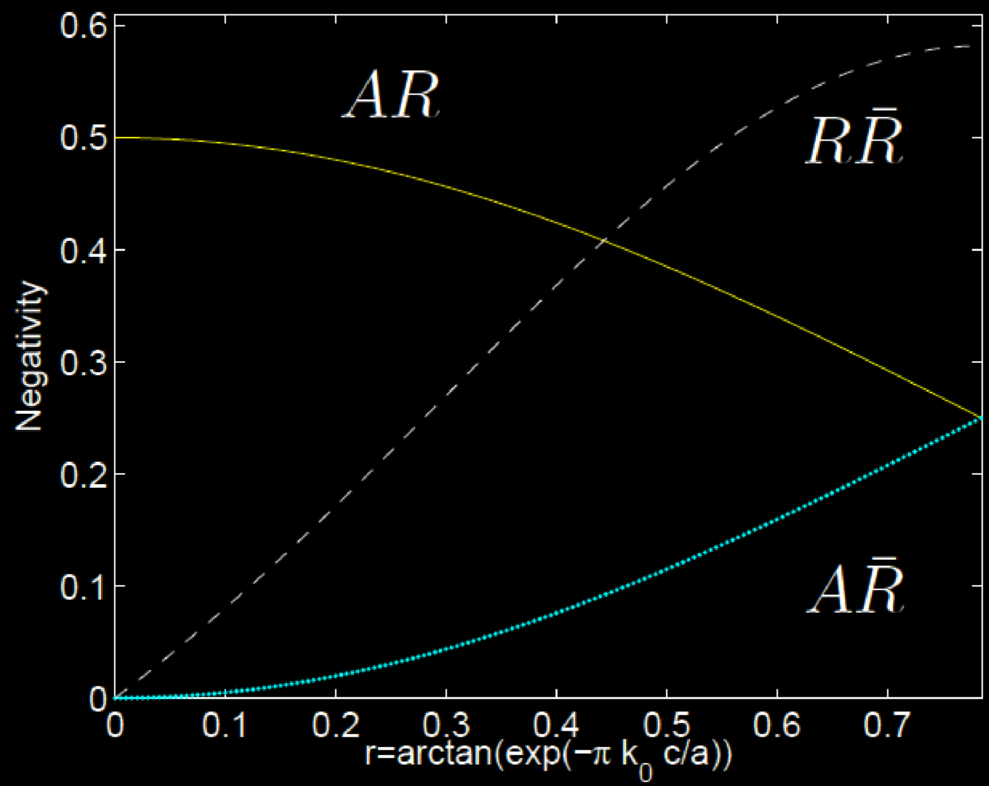
......

entropies, negativities, mutuals, discords...

A personal summary

Correlations $\longrightarrow$ Dirac field



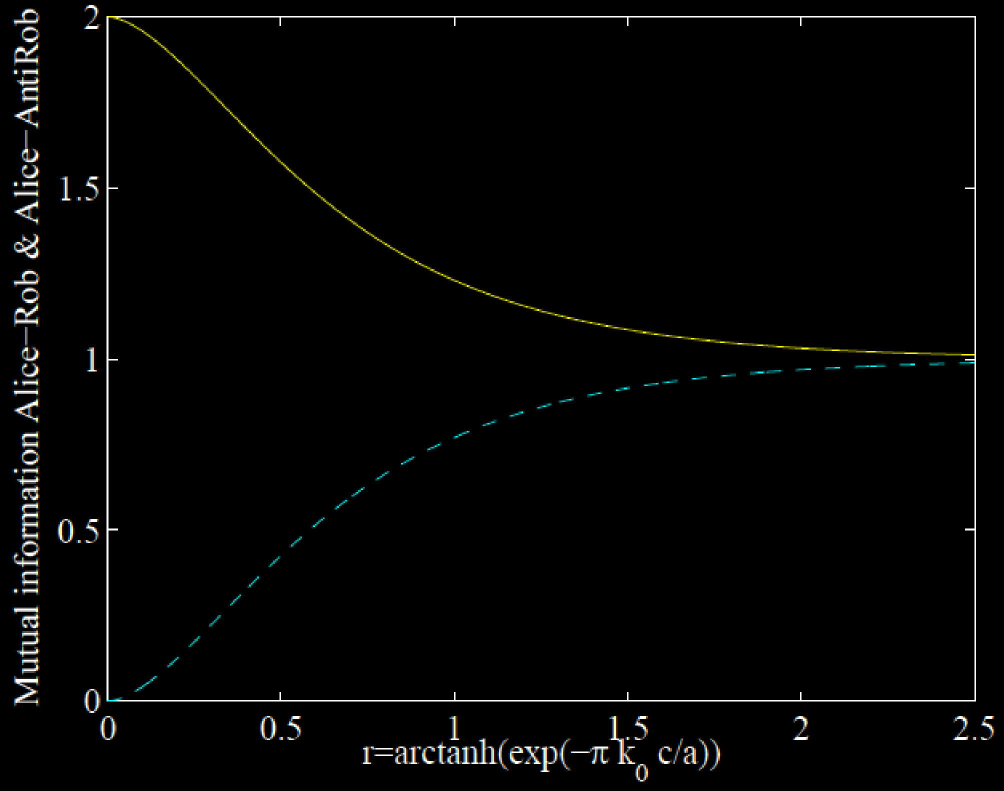


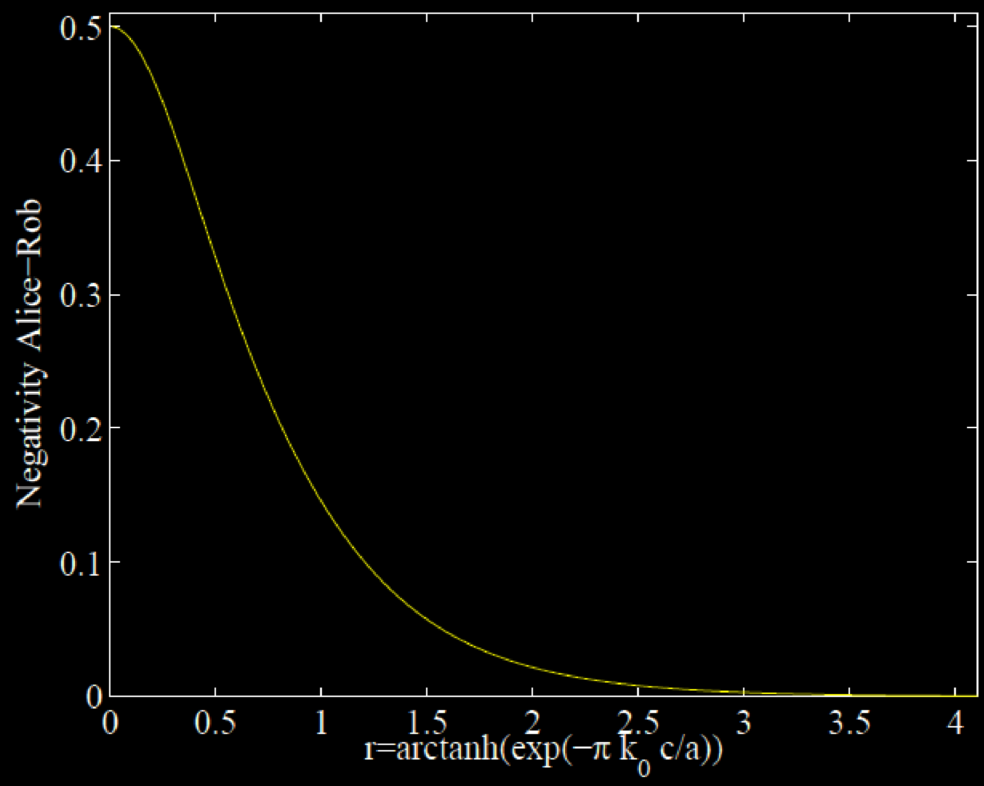
$$I_{AR} + I_{A\bar{R}} = 2$$

$$\mathcal{N}_d^{A\bar{R}} + \mathcal{N}_d^{AR} = \frac{1}{2}$$

N.B. correlations are quantum

Correlations $\longrightarrow$ Scalar field





$$I_{AR} + I_{A\bar{R}} = 2$$

$N_{AR} \rightarrow 0, \quad N_{A\,antiR} = 0\ \forall\ a$

Quantum correlations degrade fast, only classical remain

Correlations $\longrightarrow$ Fermion field

- Preserved as Rob accelerates
- Independent of the state (maximally entangled) chosen
- Independent of the kind of Fermion field

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Why?

Correlations $\longrightarrow$ Fermion field

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- Independent of the state (maximally entangled) chosen
- Independent of the kind of Fermion field

Why?

It's statistics!

Correlations preservation

statistical origin

~~Hilbert space dimensionality~~

Bound to ocupation number

Finite dimension - *N* - analogs to vacuum and one particle states

$$|0_N\rangle_M = \frac{1}{\cosh r}\sum_{n=0}^{N} \tanh^n r\,|n\rangle_I\,|n\rangle_{IV}$$

$$|1_N\rangle_M = \frac{1}{\cosh^2 r}\sum_{n=0}^{N-1} \tanh^n r\,\sqrt{n+1}\,|n+1\rangle_I\,|n\rangle_{IV}$$

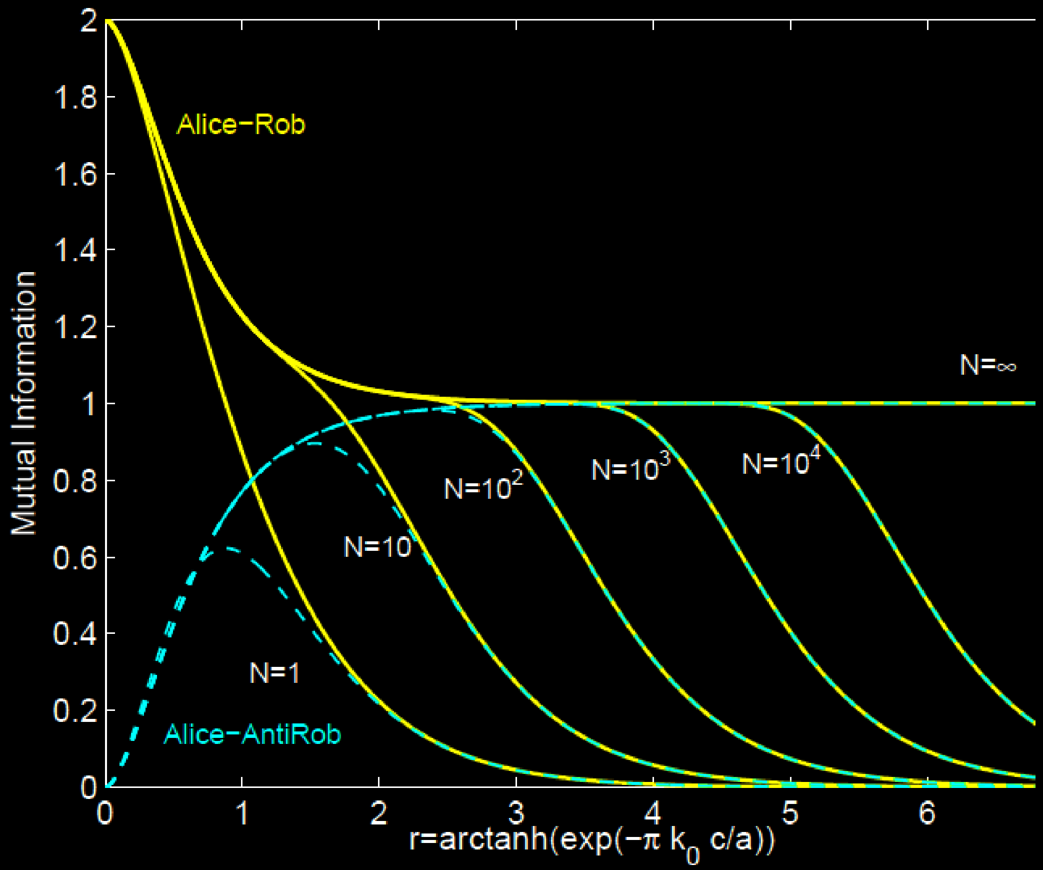
$$|\Psi\rangle = \frac{1}{C_N(r)}\left(|0\rangle_M\,|0_N\rangle_M + |1\rangle_M\,|1_N\rangle_M\right)$$

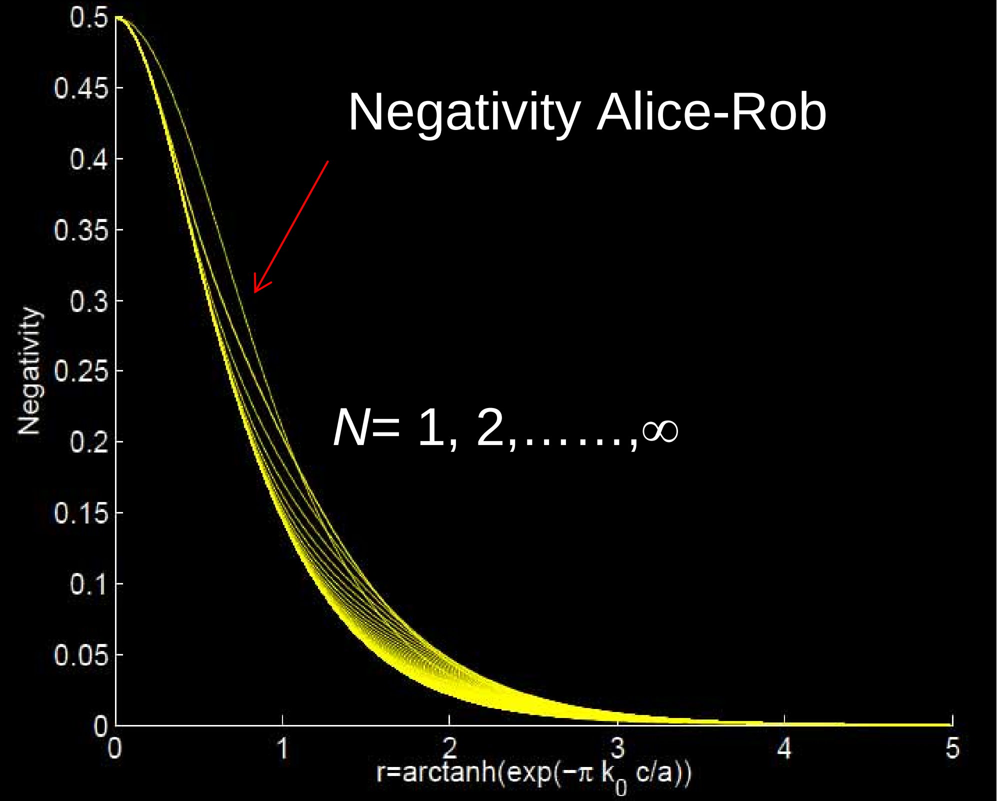
Bipartite state

$$C_N(r) = \sqrt{\langle 0_N\,|0_N\,\rangle_M + \langle 1_N\,|1_N\,\rangle_M}$$

Normalized

Mutual information and Negativity in front of *N*





Only mutual information is preserved and only for $N \to \infty$

Conclusion.

Reason for
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It is Fermion statistics
In the form of

Pauli Exclusion Principle

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Operating in the density matrices

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Operating in the density matrices

Also responsible for universality

Conclusion.

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In the form of

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Operating in the density matrices

TECHNICAL

Also responsible for universality

Unveiling entanglement decoherence near a black hole

Martin-Martinez, Garay, Leon

arXiv next week

Schwarzschild & Rindler

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega$$

$$ds^2 = -f\,dt^2 + f^{-1} dr^2 \qquad f = 1 - \frac{2m}{r}$$

In terms of the proper time t_0 of an observer placed at r_0

$$ds^2 = -\frac{f}{f_0} dt_0^2 + f^{-1} dr^2$$

$\xi = \partial_t$ at $r = r_0$ $\Longrightarrow$ $t_0 = \sqrt{f_0}\, t$

Where z is introduced

$$r - 2m = \frac{z^2}{8m} \Rightarrow f = \frac{(\kappa z)^2}{1 + (\kappa z)^2} \qquad \kappa = 1/4m$$

$$ds^2 = -\frac{1}{f_0}\frac{(\kappa z)^2}{1 + (\kappa z)^2} dt_0^2 + \left[1 + (\kappa z)^2\right] dz^2$$

Near the event horizon ($z = 0$)

$$ds^2 = -\left(\frac{\kappa z}{\sqrt{f_0}}\right)^2 dt_0^2 + dz^2$$

Schwarzschild $\longrightarrow$ Rindler BUT Meaning?

Proper acceleration *a* of an accelerated observer at an arbitrary fixed position *r*

$$a = \frac{\kappa}{\sqrt{f}}(1 - f)^2$$

At $r = r_0$, $$a_0 = \frac{\kappa}{\sqrt{f_0}}(1 - f_0)^2$$

For r_0 near the event horizon

$$(r_0 \approx r_s) \longrightarrow 1 + (\kappa z_0)^2 \approx 1 \longrightarrow a_0 \approx \frac{\kappa}{\sqrt{f_0}}$$

$$ds^2 = -(a_0 z)^2 dt_0^2 + dz^2$$

i.e.. the connection expelled out!

Schwarzschild - Kruskal connection

Alice, Rob, and AntiRob again

Introduce Kruskal Szeckeres coordinates

$$u = -\kappa^{-1}\exp[-\kappa(t - r^*)], \quad v = \kappa^{-1}\exp[\kappa(t + r^*)]$$

$$r^* = r + 2m\log|1 - r/2m|$$

Near the horizon $$uv = -(\kappa z)^2$$

and $$ds^2 = dudv + r^2 d\Omega^2$$

?

Introduce Kruskal Szeckeres coordinates

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Near the horizon $$uv = -(\kappa z)^2$$

and $$ds^2 = dudv + r^2 d\Omega^2$$

three regions in which we can define physical timelike vectors with respect to which classify possitive ad negative frequencies.

Hartle-Hawking vacuum $|0\rangle_H$ associated to the positive frequency modes of $\partial_{\hat{t}} \propto (\partial_u + \partial_v)$

Boulware vacuum $|0\rangle_B$ associated to the positive frequencies of $\partial_t \propto (u\partial_u - v\partial_v)$,i.e. an observer with proper acceleration $a \approx \kappa/\sqrt{f}$ Close to the horizon.

The Killing for $-\partial_t$ defines a Boulware vacuum in region IV; this we call AntiBoulware vacuum $|0\rangle_{\bar{B}}$

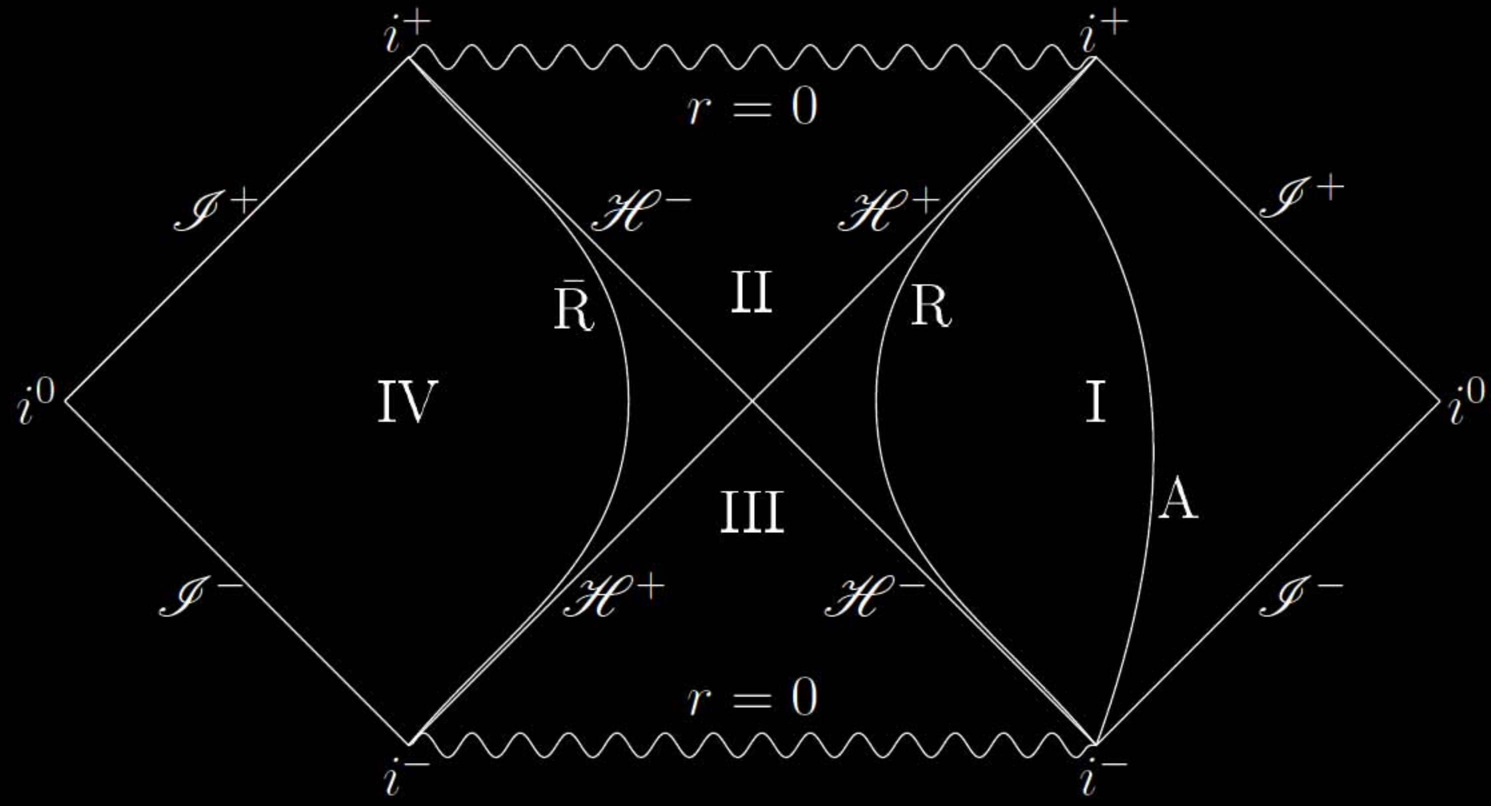

Figure 2: Kruskal space-time conformal diagram showing trajectories for Alice, Rob and AntiRob

Field excitations

$$|1_\omega\rangle_{\mathrm{H}} \equiv u_\omega^{\mathrm{H}} \propto \frac{1}{\sqrt{2\omega}} e^{-i\omega\hat{t}}$$

$$|1_\omega\rangle_{\mathrm{B}} = a^\dagger_{\omega,B} |0_\omega\rangle_B \equiv u_\omega^{\mathrm{B}} \propto \frac{1}{\sqrt{2\omega}} e^{-i\omega t}$$

$$|1_\omega\rangle_{\bar{\mathrm{B}}} = a^\dagger_{\omega,\bar{\mathrm{B}}} |0_\omega\rangle_{\bar{\mathrm{B}}} \equiv u_\omega^{\bar{\mathrm{B}}} \propto \frac{1}{\sqrt{2\omega}} e^{-i\omega t}.$$

$$\begin{array}{ccccc}
|0\rangle_{\mathrm{R}} & \rightarrow & |0\rangle_{\mathrm{I}} & \rightarrow & |0\rangle_{\mathrm{B}} \\
|0\rangle_{\bar{\mathrm{B}}} & \rightarrow & |0\rangle_{\mathrm{IV}} & \rightarrow & |0\rangle_{\bar{\mathrm{B}}} \\
|0\rangle_{\mathrm{A}} & \rightarrow & |0\rangle_{\mathrm{M}} & \rightarrow & |0\rangle_{\mathrm{H}}
\end{array}$$

Scalar field

$$\psi_j^{\mathrm{H}} = \sum_i C_{ij}\, u_{\omega_i}^{\mathrm{H}}$$

$$|0_{\omega_R}\rangle_{\mathrm{H}} = \frac{1}{\cosh q_{\mathrm{s}}} \sum_{n=0}^{\infty} \tanh^n q_{\mathrm{s}}\, |n_{\omega_R}\rangle_{\mathrm{B}}\, |n_{\omega_R}\rangle_{\bar{\mathrm{B}}}$$

$$|1_{\omega_R}\rangle_{\mathrm{H}} = \frac{1}{\cosh^2 q_{\mathrm{s}}} \sum_{n=0}^{\infty} \tanh^n q_{\mathrm{s}}\, \sqrt{n+1}\, |n+1_{\omega_R}\rangle_{\mathrm{B}} |n_{\omega_R}\rangle_{\bar{\mathrm{B}}}$$

$$\tanh q_{\mathrm{s}} = \exp\left[-\pi\omega_R \frac{\sqrt{f_0}}{\kappa}\right] = \exp\left[-4\pi\omega_R \sqrt{m^2 - \frac{2m^3}{r_0}}\right]$$

Fermion fields

$$\psi^{\mathrm{H}}_{j,s} = \sum_i D_{ij}\, u^{\mathrm{H}}_{\omega_i,s} \qquad \bar{\psi}^{\mathrm{H}}_{j,s} = \sum_i E_{ij}\, v^{\mathrm{H}}_{\omega_i,s}$$

$$\begin{aligned}|0_{\omega_R}\rangle_{\mathrm{H}} = \cos^2 q_{\mathrm{d}}\ |0\rangle_{\mathrm{B}}\,|0\rangle_{\bar{\mathrm{B}}} + \sin q_{\mathrm{d}}\ \cos q_{\mathrm{d}}\,(|{\uparrow}_{\omega_R}\rangle_{\mathrm{B}}\,|{\downarrow}_{\omega_R}\rangle_{\bar{\mathrm{B}}} \\ + |{\downarrow}_{\omega_R}\rangle_{\mathrm{B}}\,|{\uparrow}_{\omega_R}\rangle_{\bar{\mathrm{B}}}) + \sin^2 q_{\mathrm{d}}\ |p_{\omega_R}\rangle_{\mathrm{B}}\,|p_{\omega_R}\rangle_{\bar{\mathrm{B}}} \qquad (63)\end{aligned}$$

$$\begin{aligned}|{\uparrow}_{\omega_R}\rangle_{\mathrm{H}} &= \cos q_{\mathrm{d}}\,|{\uparrow}_{\omega_R}\rangle_{\mathrm{B}}\,|0\rangle_{\bar{\mathrm{B}}} + \sin q_{\mathrm{d}}\,|p_{\omega_R}\rangle_{\mathrm{B}}\,|{\uparrow}_{\omega_R}\rangle_{\bar{\mathrm{B}}} \\ |{\downarrow}_{\omega_R}\rangle_{\mathrm{H}} &= \cos q_{\mathrm{d}}\,|{\downarrow}_{\omega_R}\rangle_{\mathrm{B}}\,|0\rangle_{\bar{\mathrm{B}}} - \sin q_{\mathrm{d}}\,|p_{\omega_R}\rangle_{\mathrm{B}}\,|{\downarrow}_{\omega_R}\rangle_{\bar{\mathrm{B}}}\end{aligned}$$

$$\tan q_{\mathrm{d}} = \exp\left[-\pi\omega_R\frac{\sqrt{f_0}}{\kappa}\right] = \exp\left[-4\pi\omega_R\sqrt{m^2 - \frac{2m^3}{r_0}}\right]$$

Bipartite states

$$|\Psi\rangle_{\mathrm{s}} = \frac{1}{\sqrt{2}}\left(|0_{\omega_1}\rangle_{\mathrm{H}}\,|0_{\omega_R}\rangle_{\mathrm{H}} + |1_{\omega_1}\rangle_{\mathrm{H}}\,|1_{\omega_R}\rangle_{\mathrm{H}}\right)$$

$$|\Psi\rangle_{\mathrm{d}} = \frac{1}{\sqrt{2}}\left(|0_{\omega_1}\rangle_{\mathrm{H}}\,|0_{\omega_R}\rangle_{\mathrm{H}} + |{\uparrow}_{\omega_R}\rangle_{\mathrm{H}}\,|{\downarrow}_{\omega_R}\rangle_{\mathrm{H}}\right)$$

$$\rho^{\mathrm{AR}} = \mathrm{Tr}_{\bar{\mathrm{B}}}\,\rho^{\mathrm{AR\bar{R}}}$$
$$\rho^{\mathrm{A\bar{R}}} = \mathrm{Tr}_{\mathrm{B}}\,\rho^{\mathrm{AR\bar{R}}}$$
$$\rho^{\mathrm{R\bar{R}}} = \mathrm{Tr}_{\mathrm{H}}\,\rho^{\mathrm{AR\bar{R}}}$$

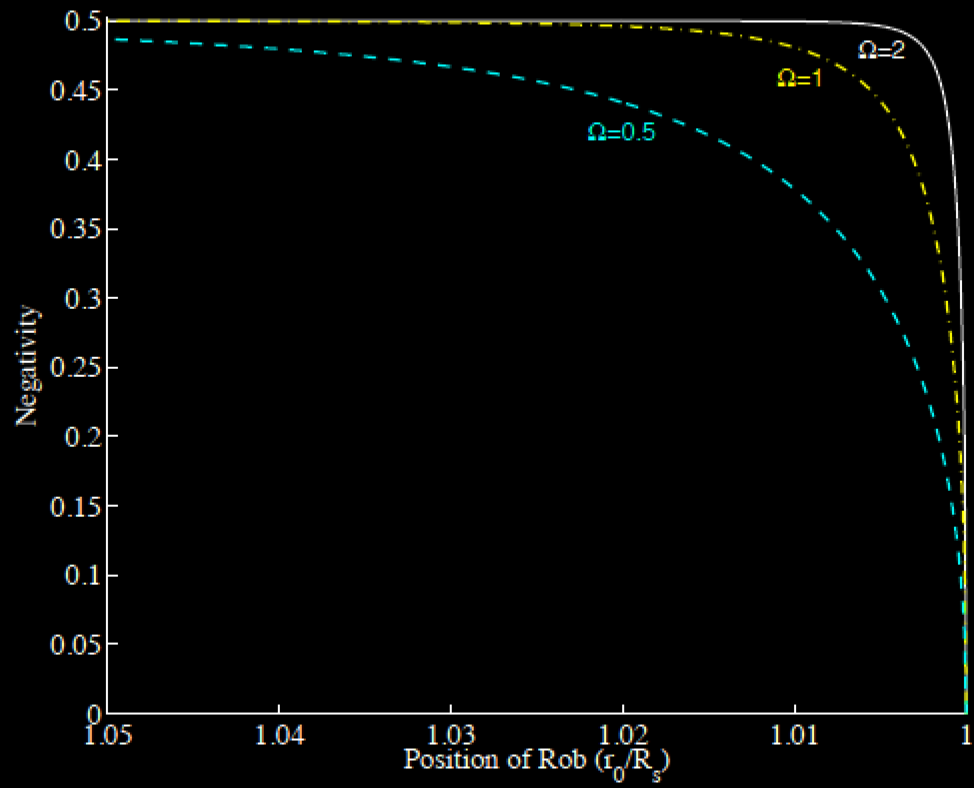

Figure 3: Scalar field: Entanglement of the system Alice-Rob as a function of the position of Rob for different values of Ω. Entanglement vanishes as Rob approaches to the Schwarzschild radius while no entanglement is created between Alice and AntiRob. The smaller the value of Ω the more decoherence is produced by the black hole.

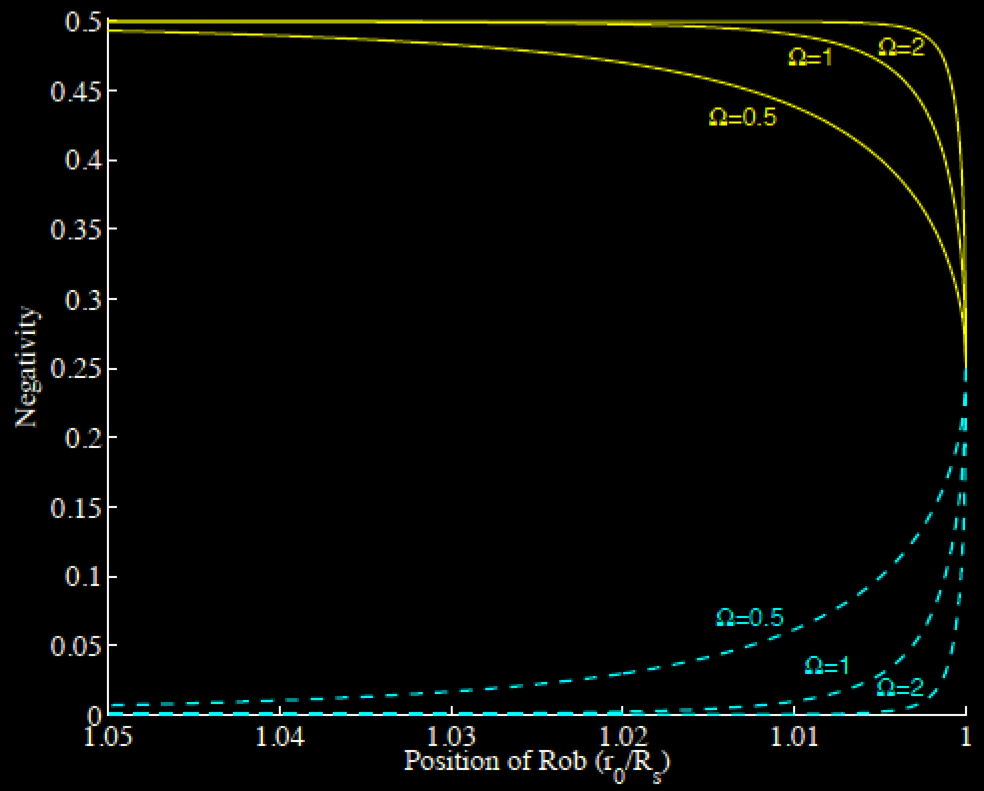

Figure 4: Dirac field: Entanglement Alice-Rob (blue solid line) and Alice-AntiRob (red dashed line). Universal conservation law for fermions is shown for different values of Ω. The decoherence is quicker when Ω is smaller

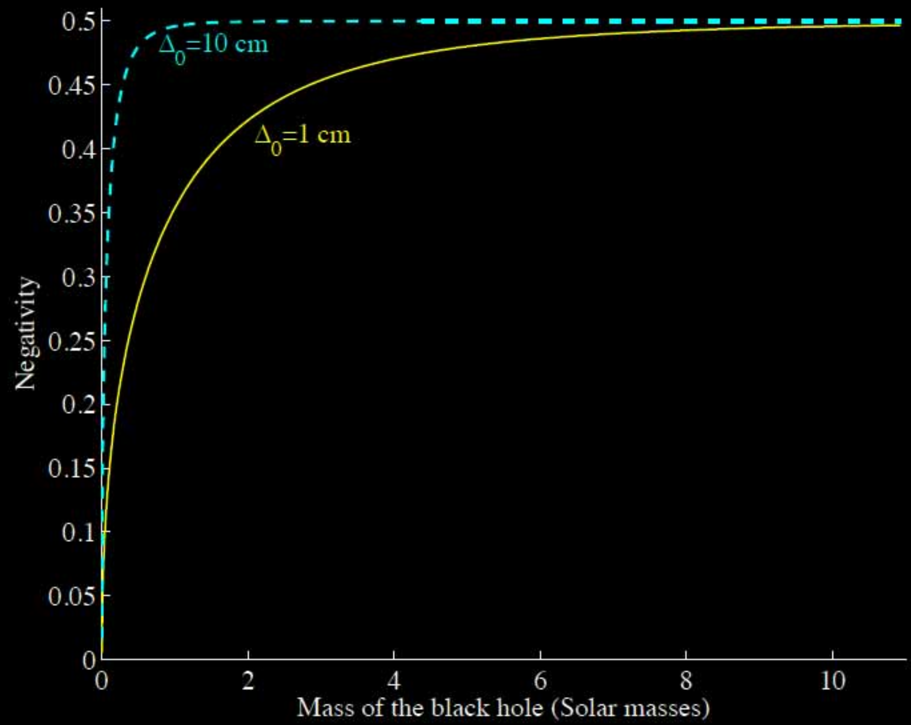

Figure 9: Scalar field: Entanglement Alice-Rob when Rob stands at a distance of 1 cm and 10 cm from the event horizon for a fixed frequency $\omega_R = 1.5$ Mhz as a function of the black hole mass. Notice that, for these values of Δ_0 the approximation holds perfectly for any mass $m > 10^{-5}$ solar masses.

$$\Omega = 2\pi\omega_R/\kappa = 8\pi m\omega_R \quad R_0 = 2r_0\kappa = r_0/2m$$

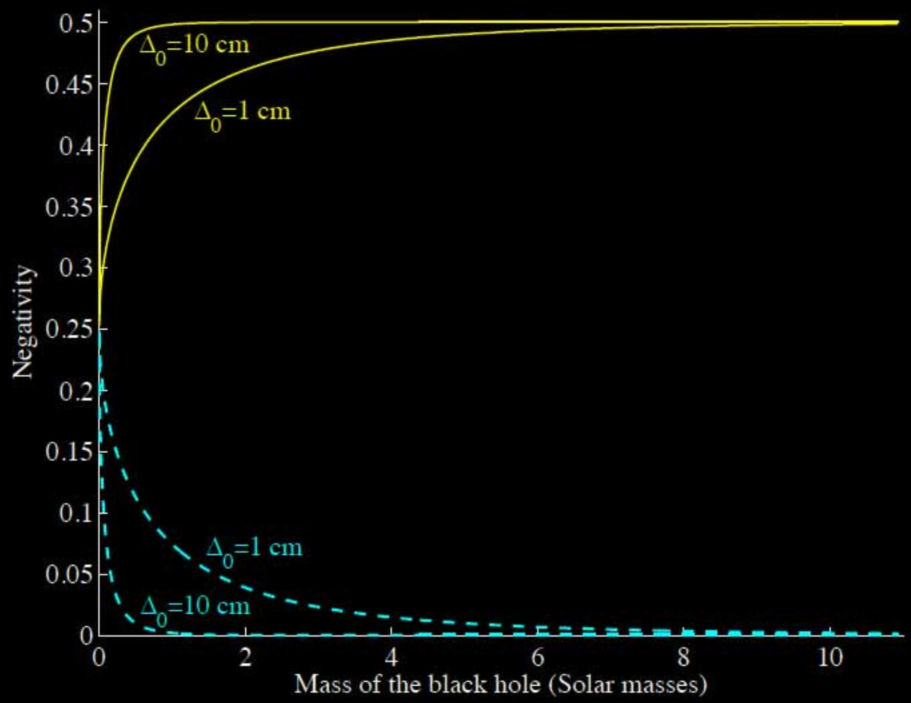

Figure 10: Dirac field: Entanglement AR (Blue continuous line) and A$\bar{\text{R}}$ (Red dashed line) when Rob stands at a distance of 1 cm and 10 cm from the event horizon for a fixed frequency $\omega_R = 1.5$ Mhz as a function of the black hole mass. Notice that, for these values of Δ_0 the approximation holds perfectly for any mass $m > 10^{-5}$ solar masses.

All the interesting decoherence effects due Hawking effect are produced very close to the event horizon of Scwarzschild BH

No residual effects when Rob is far away from the horizon

For fixed Rob's frequency and fixed distance, entanglement decoherence is larger for less massive BHs (higher T)

Hawking entanglement decoherence universal, independent of the BH mass in the natural adapted system of units

Fermionic correlations ruled by statistics also in this case