Adiabatic Dynamics in Open Critical Systems

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In collaboration with

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Alessandro Silva *ICTP - Trieste*

Giuseppe Santoro *SISSA - Trieste*

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- Phys. Rev. Lett. **101**, 175701 (2008)
- Phys. Rev. B **80**, 024302 (2009)

Non-equilibrium quantum many-body systems

Artificial many-body systems (nanoscience, cold atoms)

Driven system

Quantum quenches

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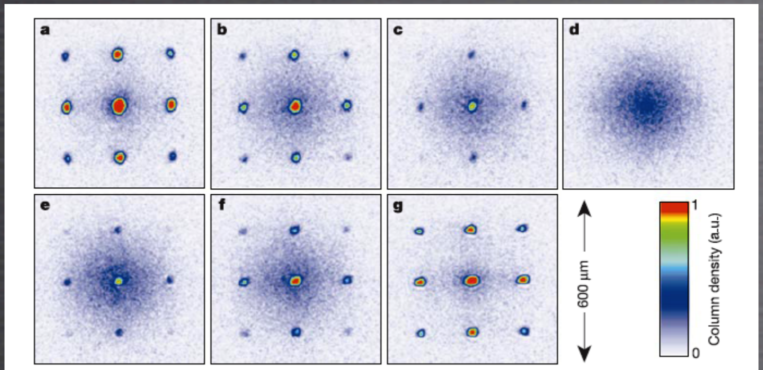
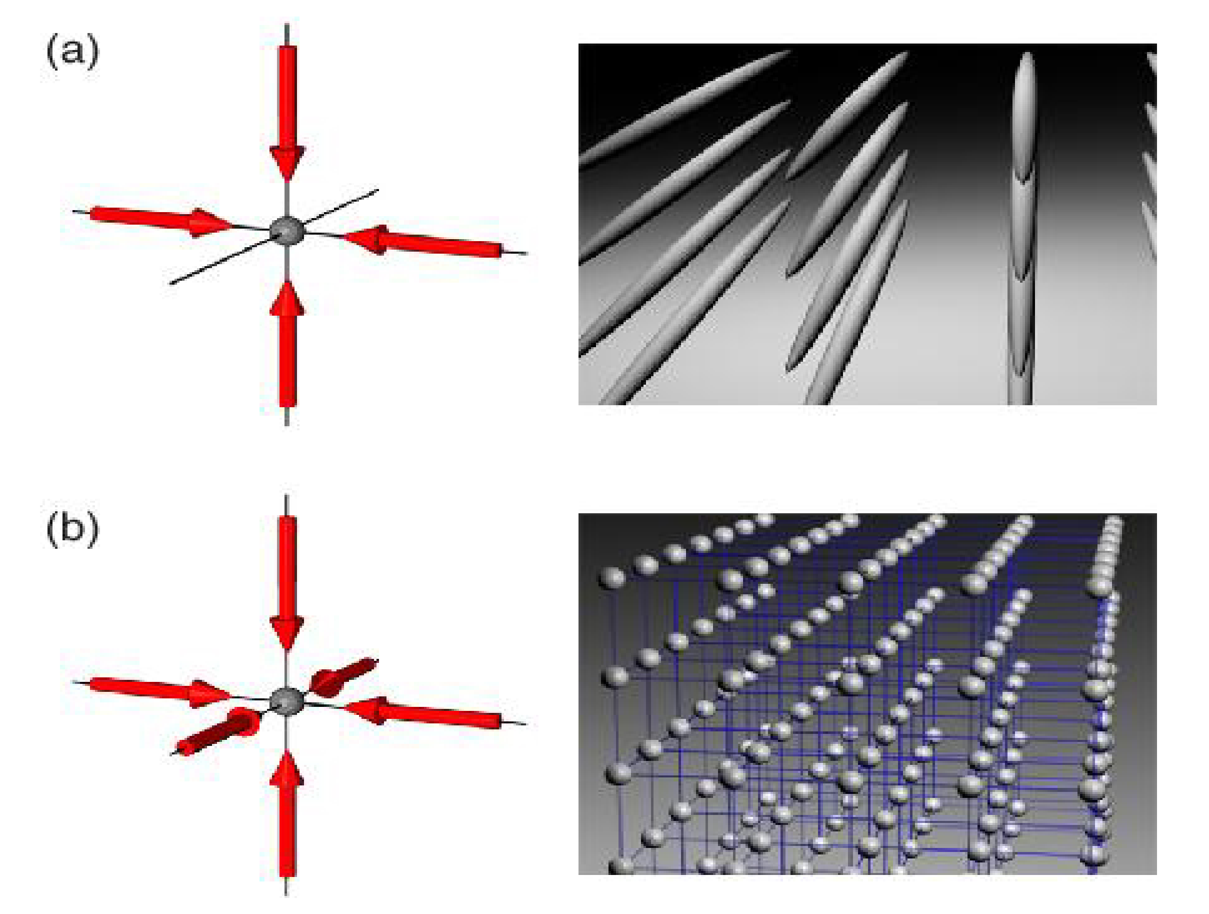
Optical lattices





Fisher et al, Phys Rev B 40, 546 (1989).
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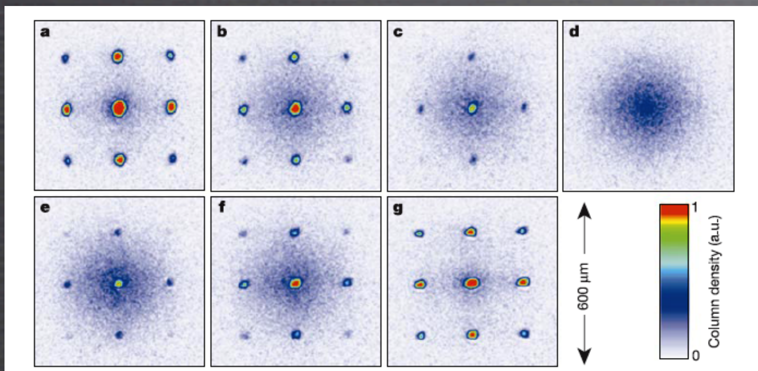

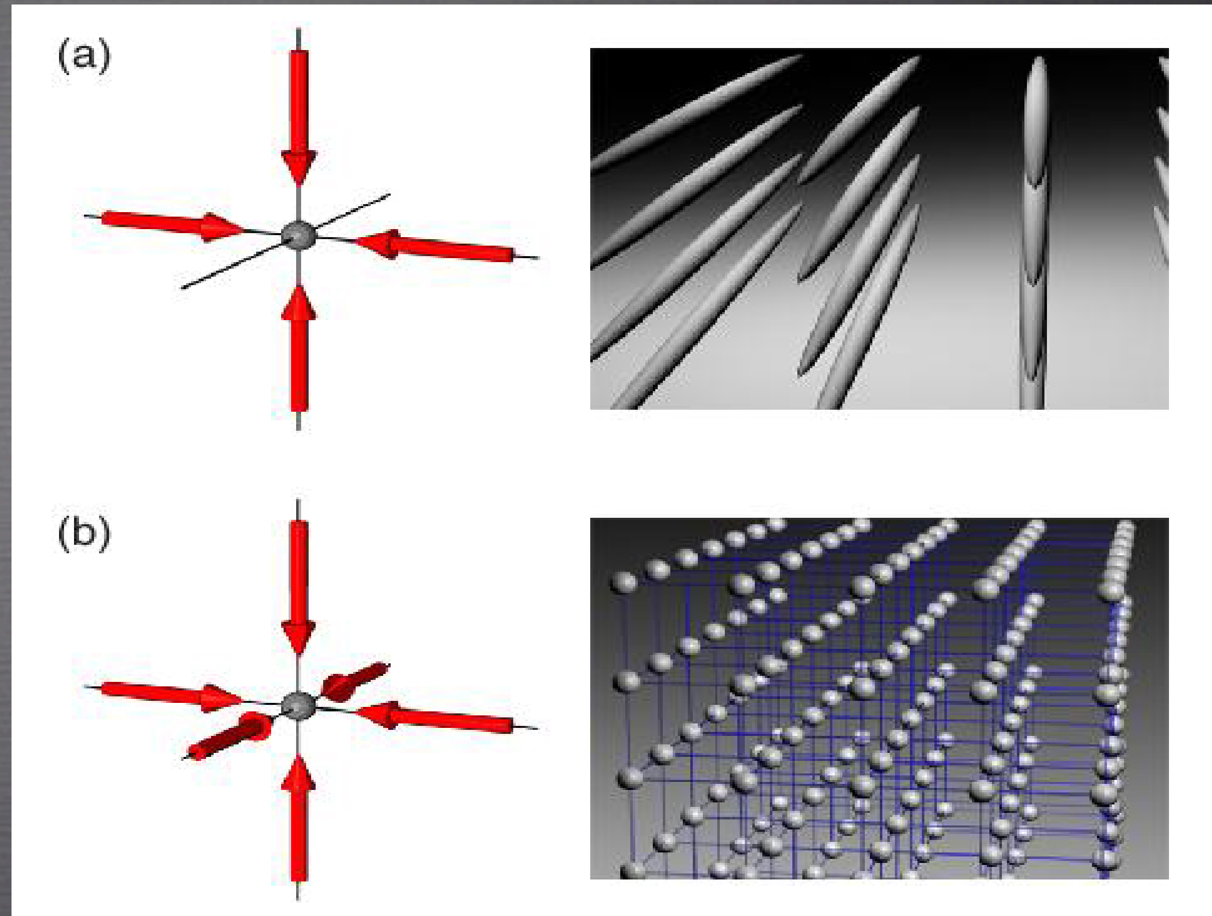

$$H = \frac{1}{2}\sum_{ij} n_i U_{ij} n_j - \mu \sum_i n_i - \frac{t}{2}\sum_{\langle ij \rangle} b_i^\dagger b_j + \text{h.c.}$$

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$$\mathcal{H} = \mathcal{H}_0 + \lambda(t)\mathcal{H}_1$$

Early works by Baruch, McCoy, Dresden, Mazur, Girardeau ('70)

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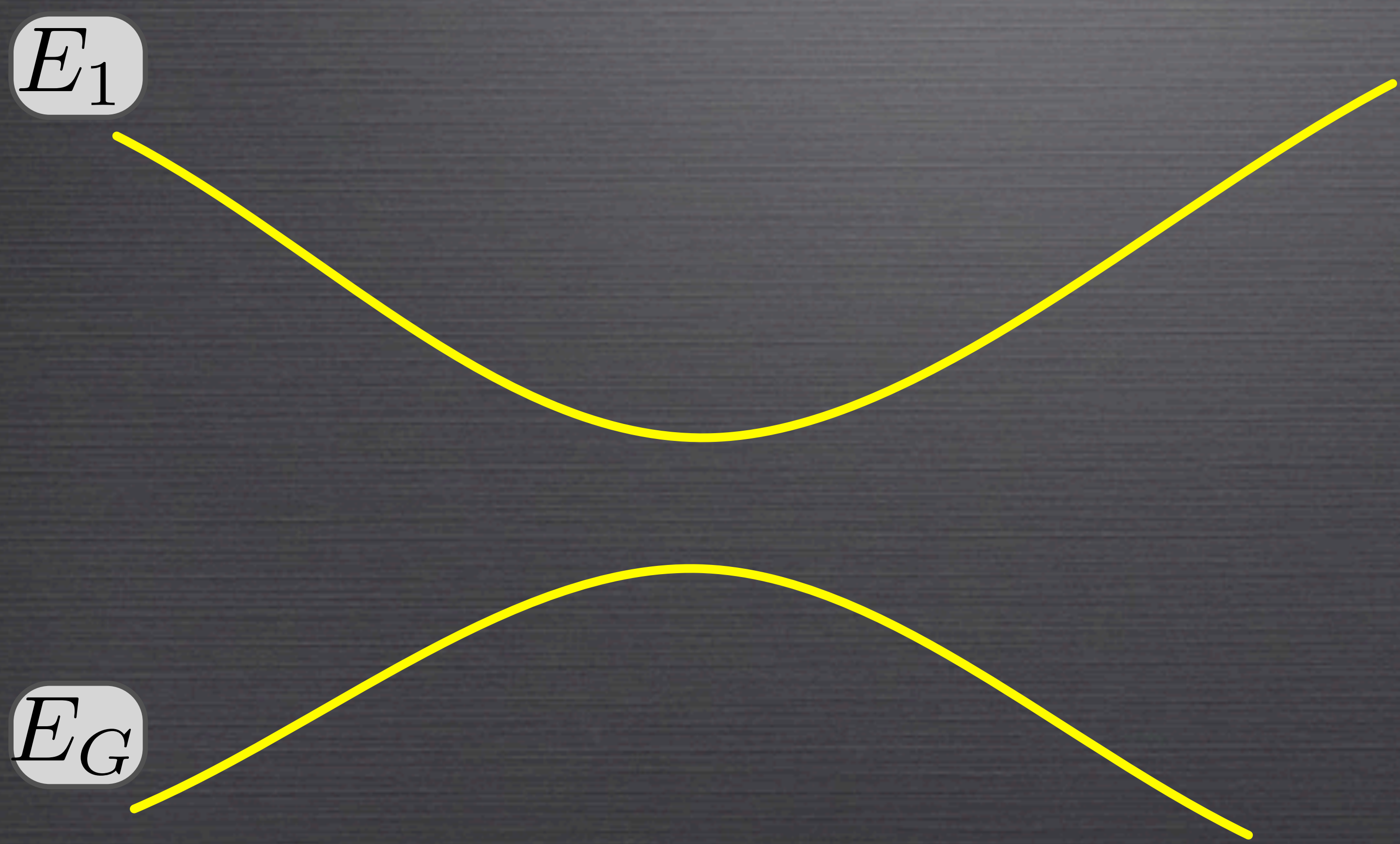
$$\mathcal{H}(t) = \frac{T-t}{T}\mathcal{H}_i + \frac{t}{T}\mathcal{H}_f$$

Adiabatic Quantum Computation



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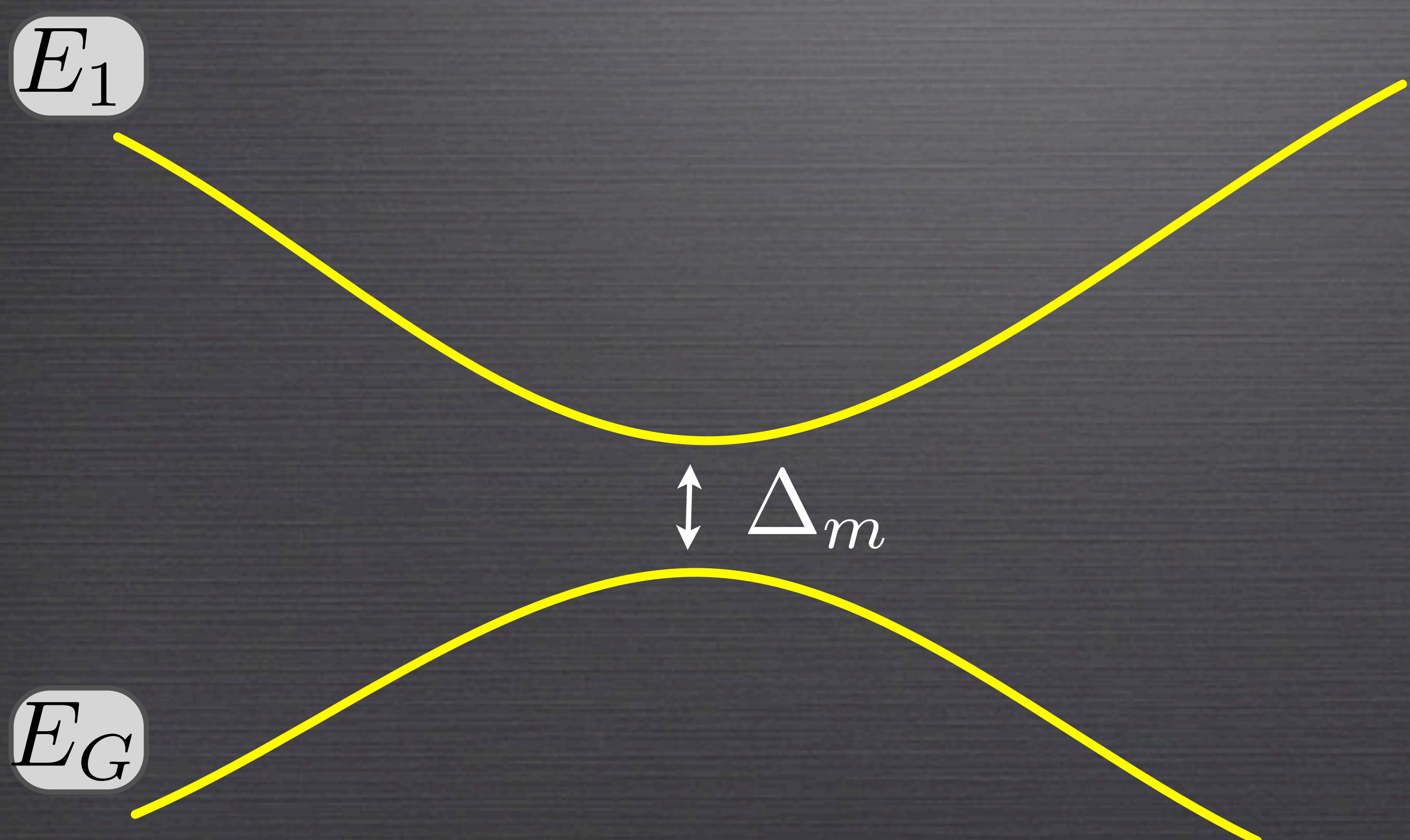


$$T \sim \frac{1}{\Delta_m^2}$$

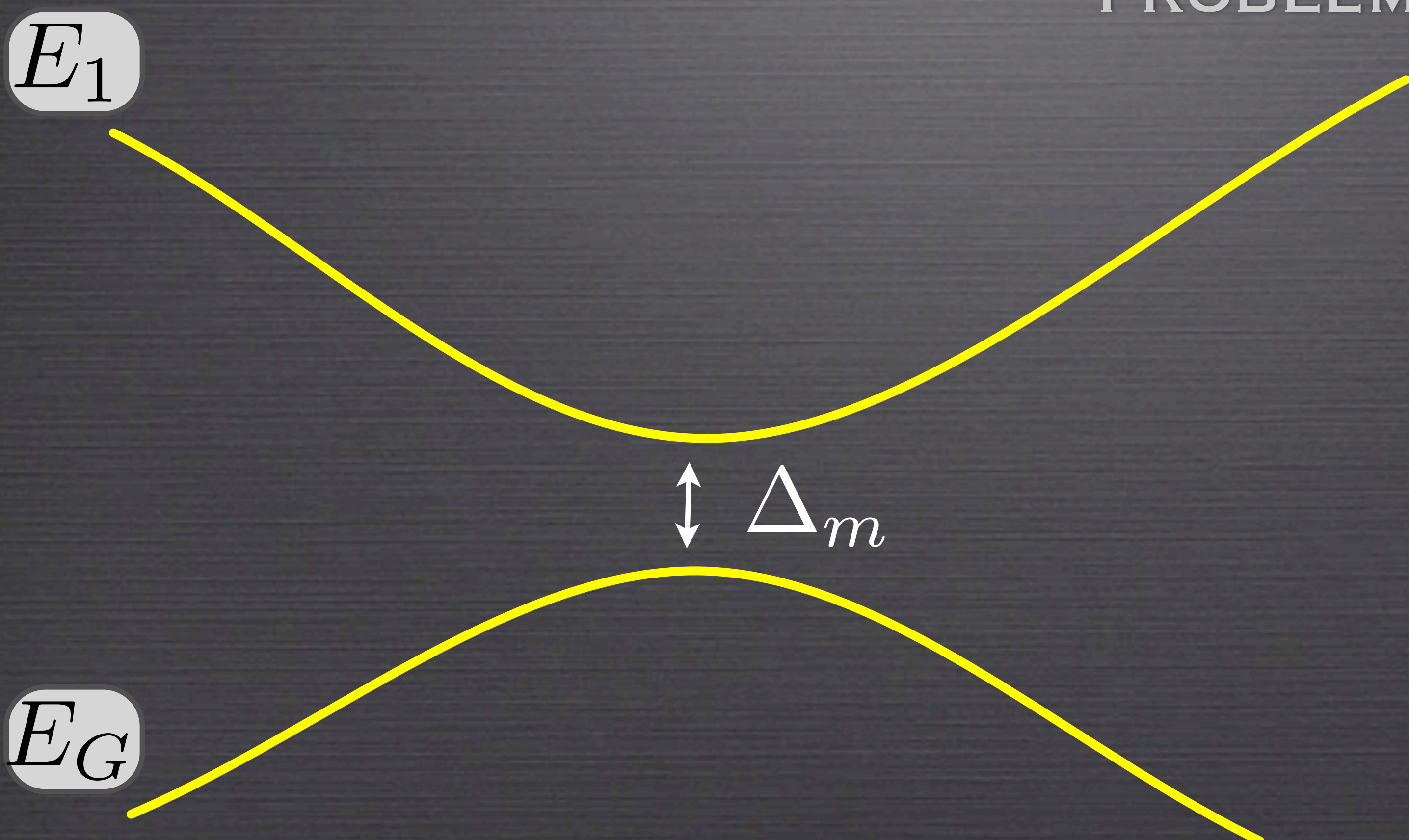
Adiabatic Quantum Computation

$$\Delta_m \sim \frac{1}{N^\eta}$$

Easy problem

$$\Delta_m \sim e^{-N^\eta}$$

Difficult problem



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Topological Defect Formation

Signatures of phase transitions which have occurred in the early universe by determining the density of defects left in the broken symmetry phase as a function of the rate of quench.

Kibble '76, Zurek '85

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Topological Defect Formation

Simulation of phase transitions in the early universe in condensed matter systems (superfluids and Josephson junctions)

Th: Zurek '85-'88

Exps:Bauerle et al '96,Ruutu et al'96}

Extension to quantum phase transitions

Zurek, Dorner, Zoller '05

Polkovnikov '05

...

Adiabatic Dynamics Close to a Critical Point

Adiabatic Dynamics Close to a Critical Point

- How effective is it to execute a given computational task by slowly varying in time the Hamiltonian of a quantum system?
- Is it possible to find the ground state of a classical system by slowly annealing away its quantum fluctuations?
- What is the density of defects left over after a passage through a continuous (quantum) phase transition?

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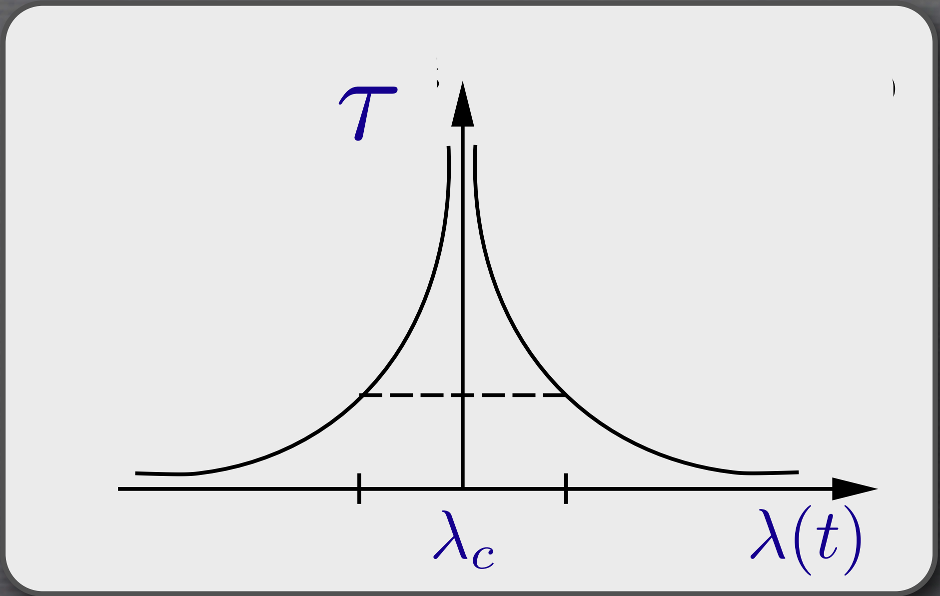

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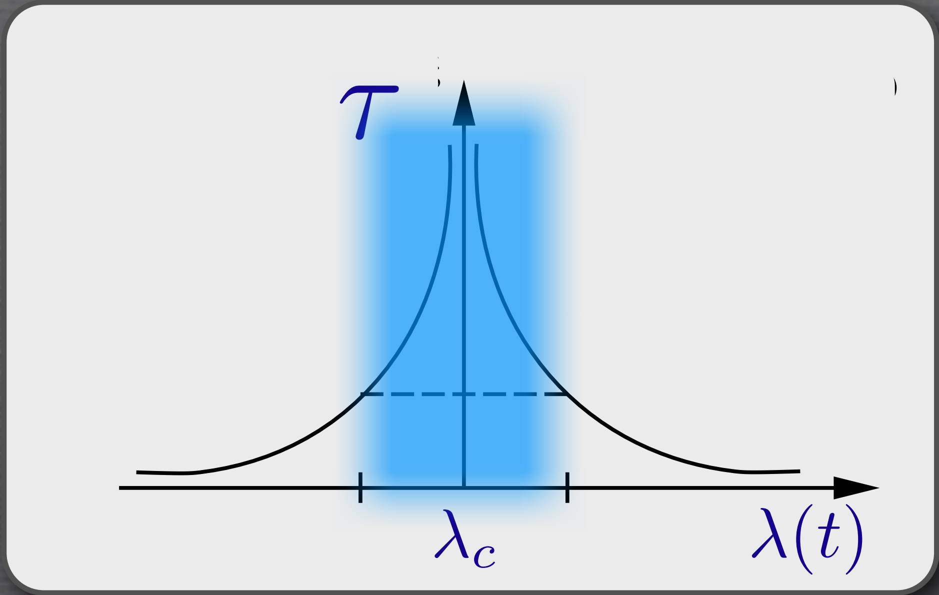

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Defect density

W. Zurek '85
W. Zurek, U. Dorner and P. Zoller '05
A. Polkovnikov '05

$$\lambda - \lambda_c = vt$$

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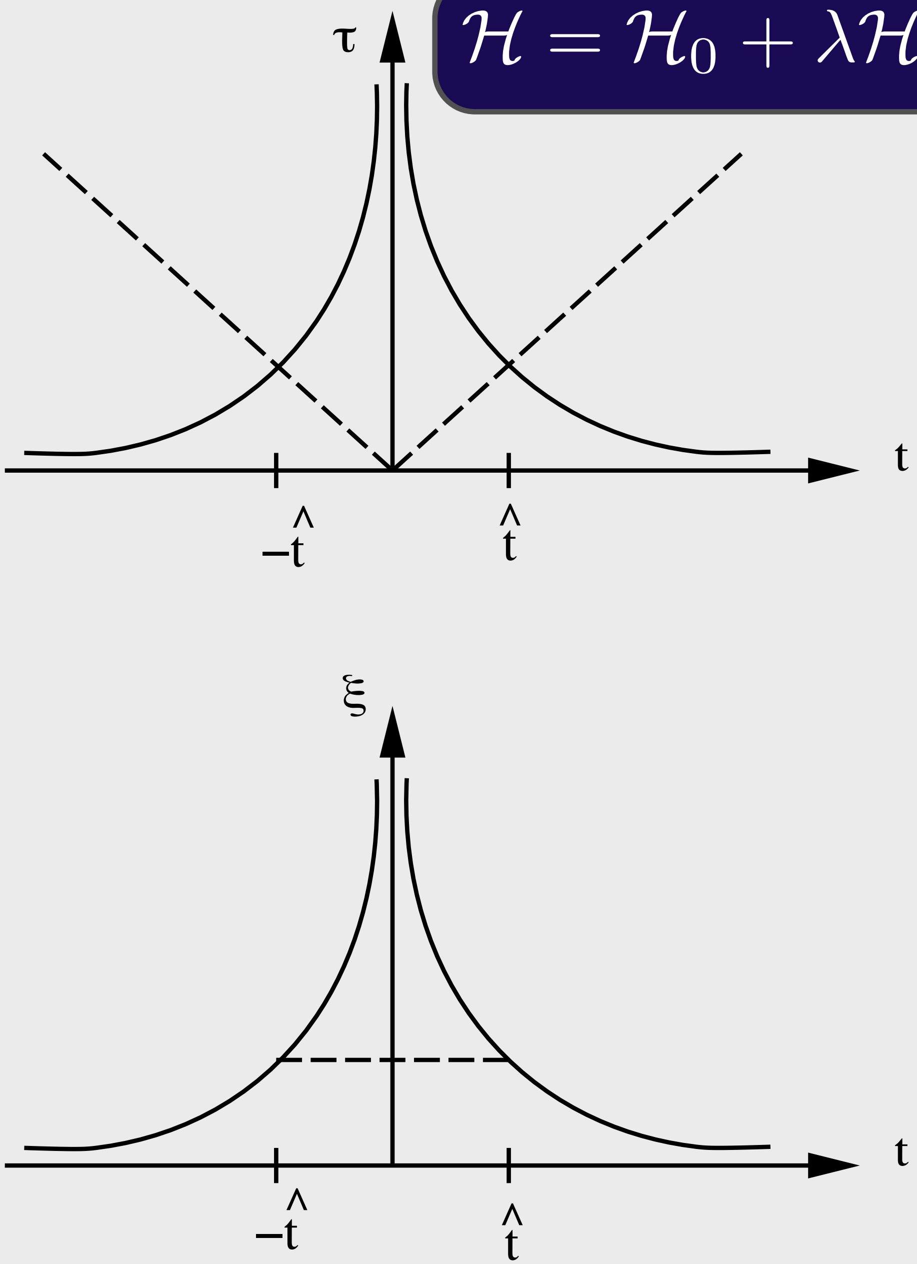

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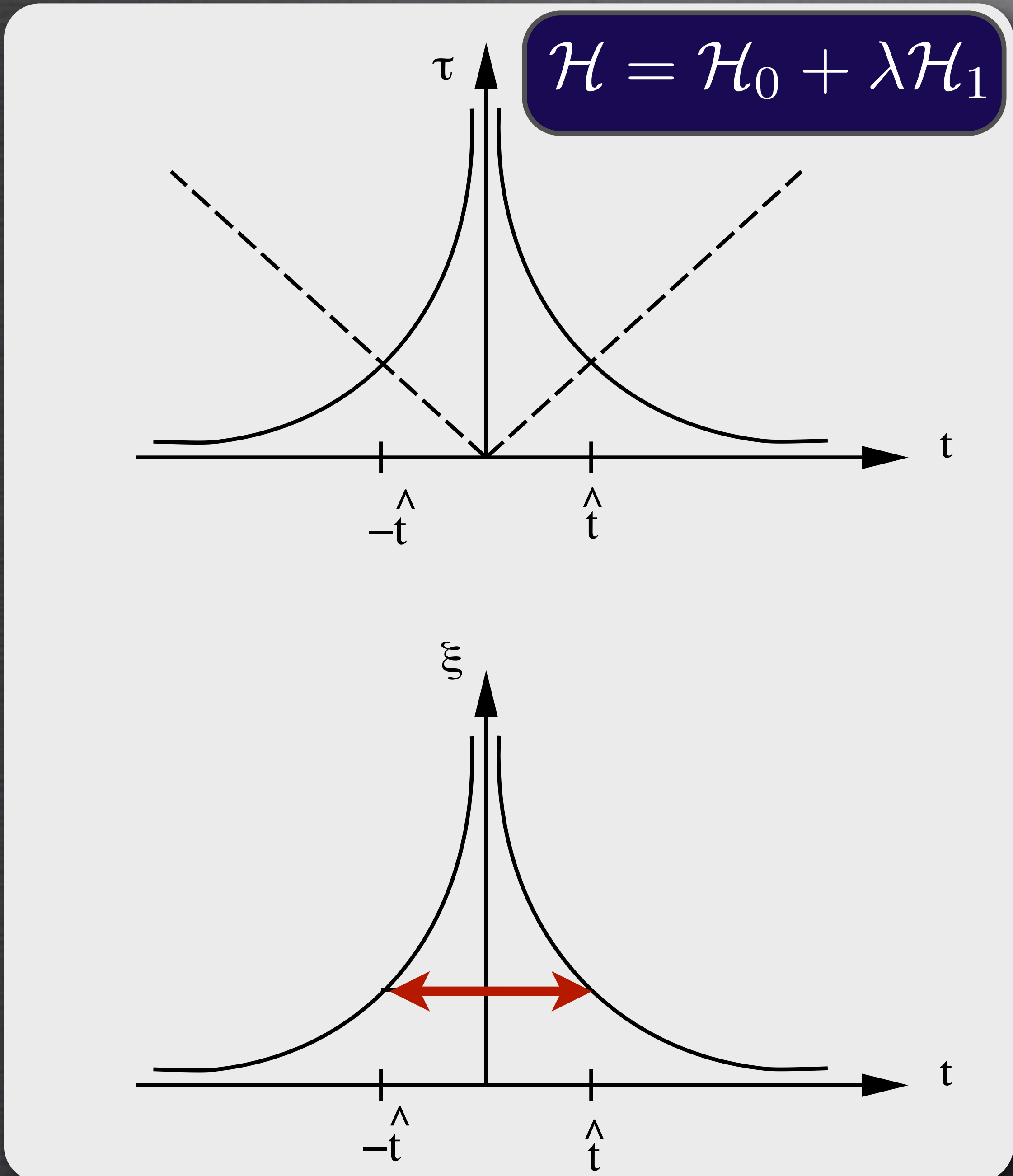

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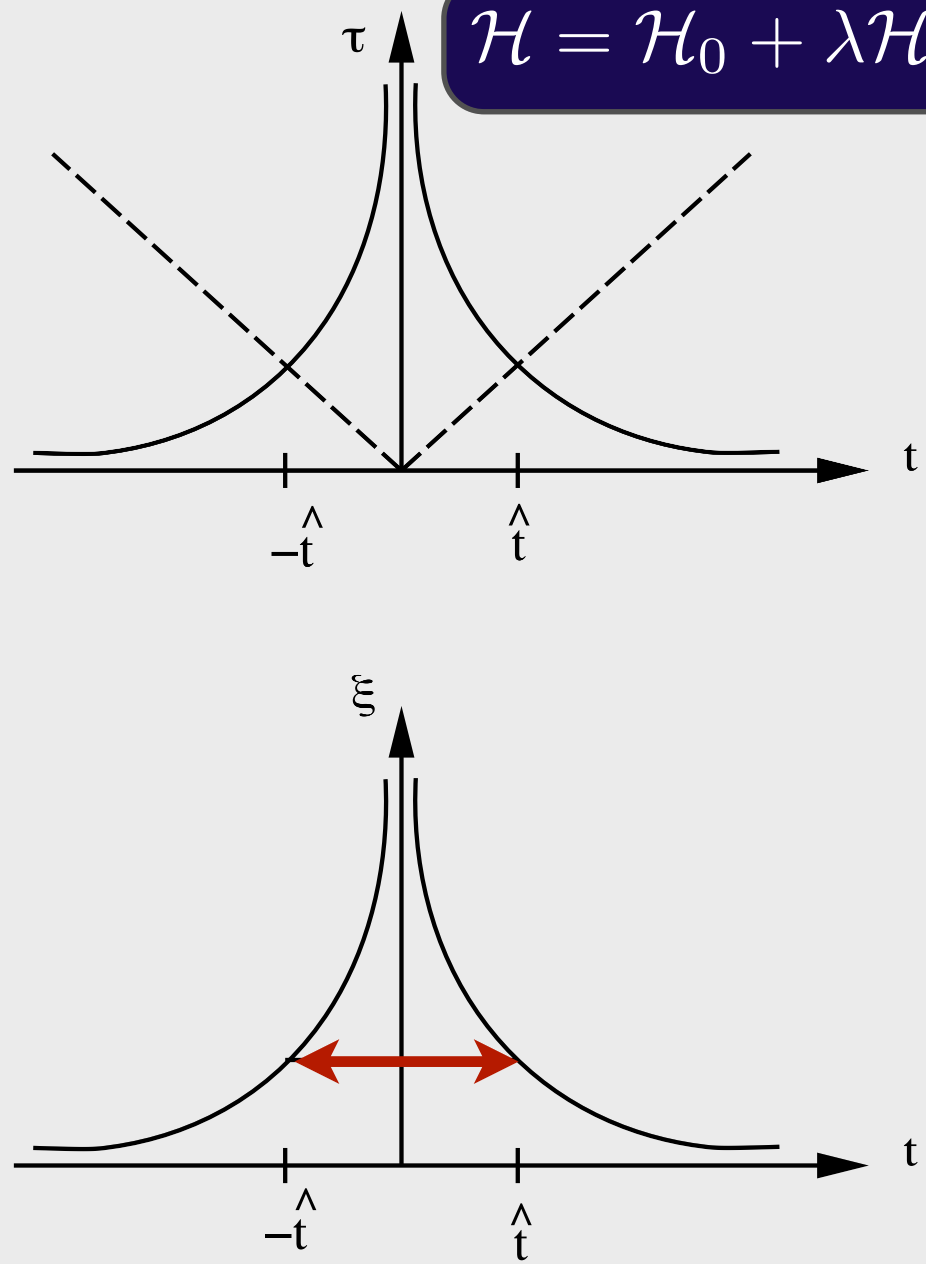

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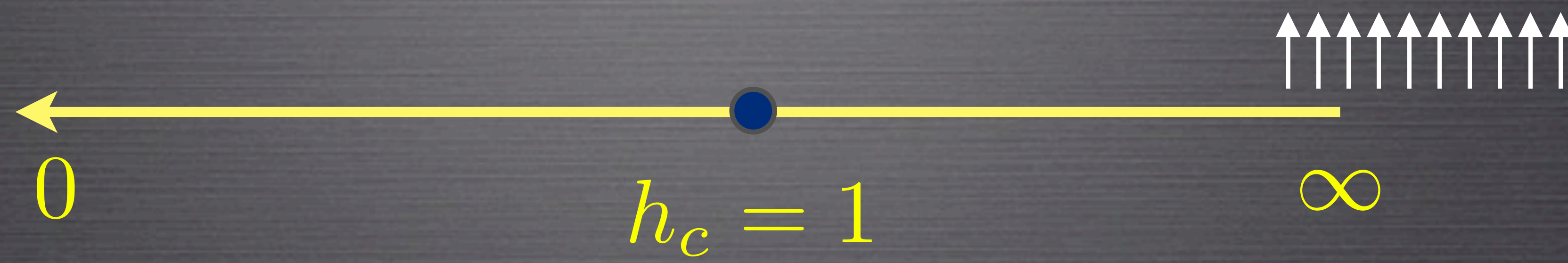
$$\rho_{def} \sim \hat{\xi}^{-d} \sim v^{\frac{d\nu}{z\nu+1}}$$

$$\mathcal{E}_{res} \sim J\rho_{def}$$

1D Ising Model

$$H = -\frac{J}{2}\sum_{j}^{N}\{\sigma_j^x\sigma_{j+1}^x + h(t)\sigma_j^z\}$$



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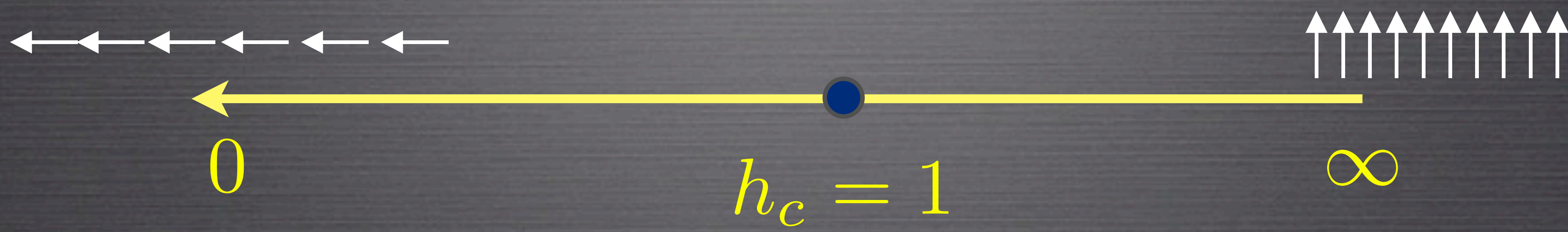

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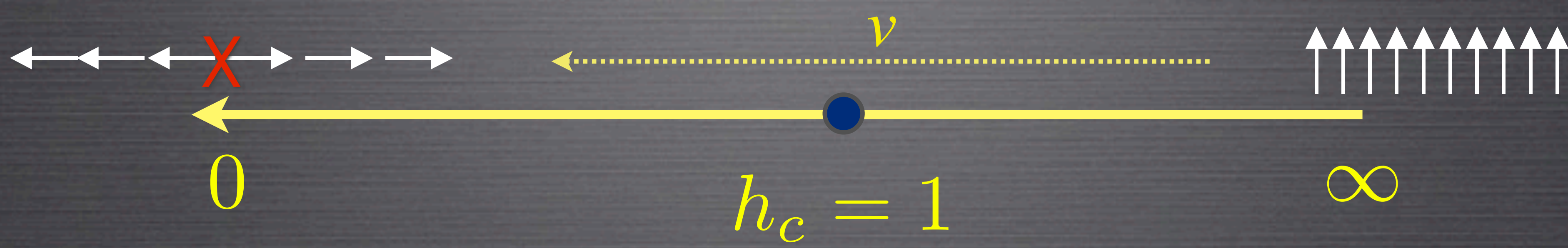

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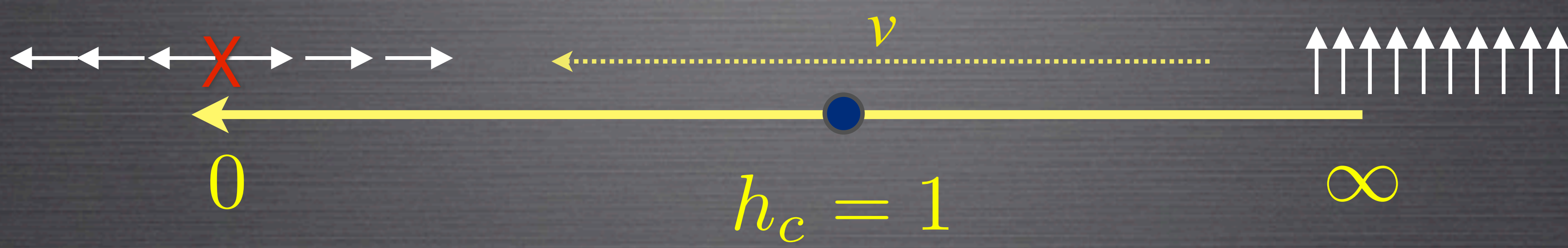

$$\mathcal{E}_{res} \sim \sqrt{v}$$

Consider a quantum system coupled to an environment at a temperature T

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Is it possible in the presence of dissipation and dephasing to describe universally the production of defects in an adiabatic quench ?

Motivations

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- Coherent vs incoherent defect production

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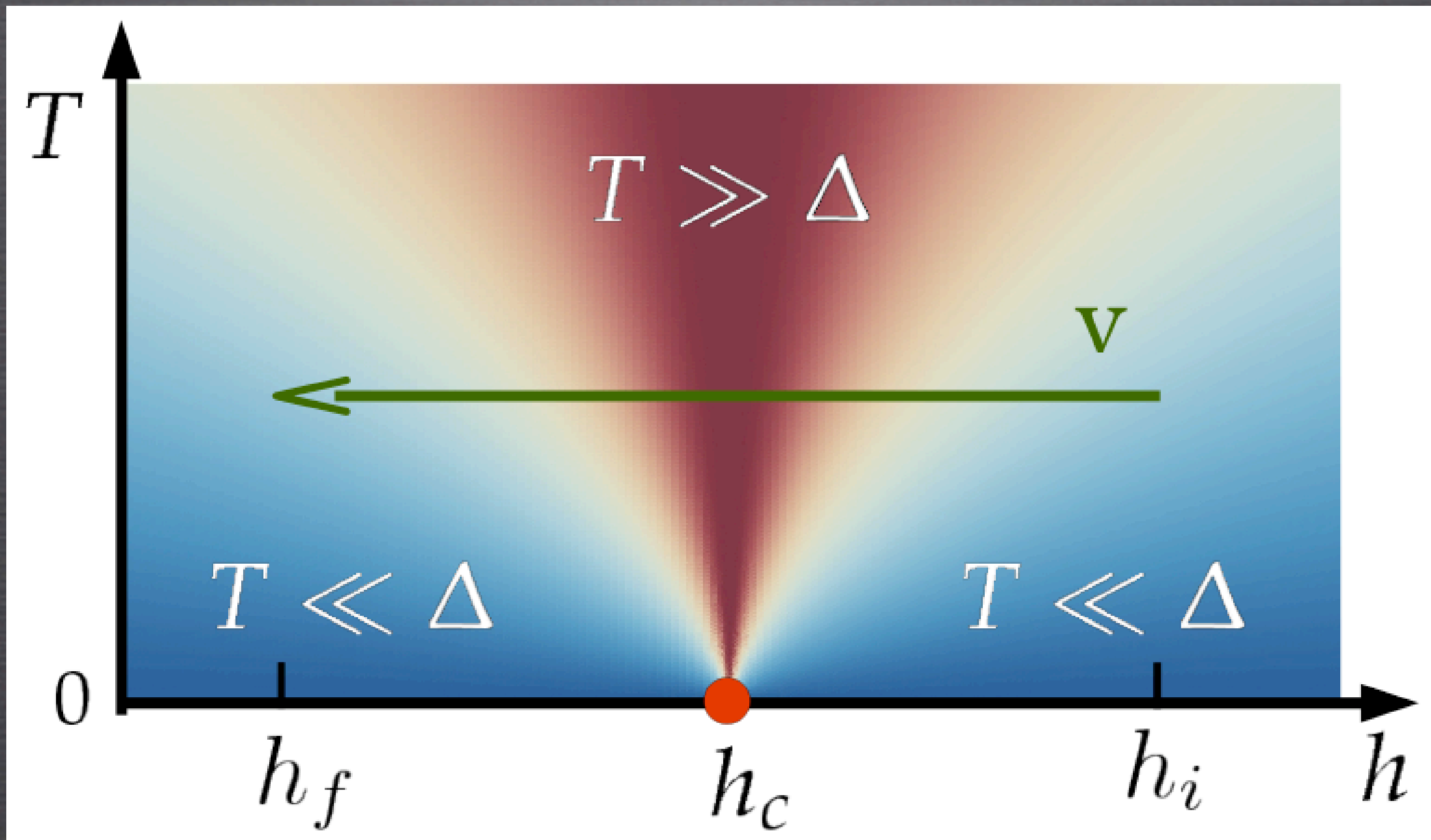
- Coherent vs incoherent defect production
- Adiabatic approximation for open systems
- Adiabatic quantum computation
- Classical vs quantum annealing
- Experimental comparison

Quantum Critical Region



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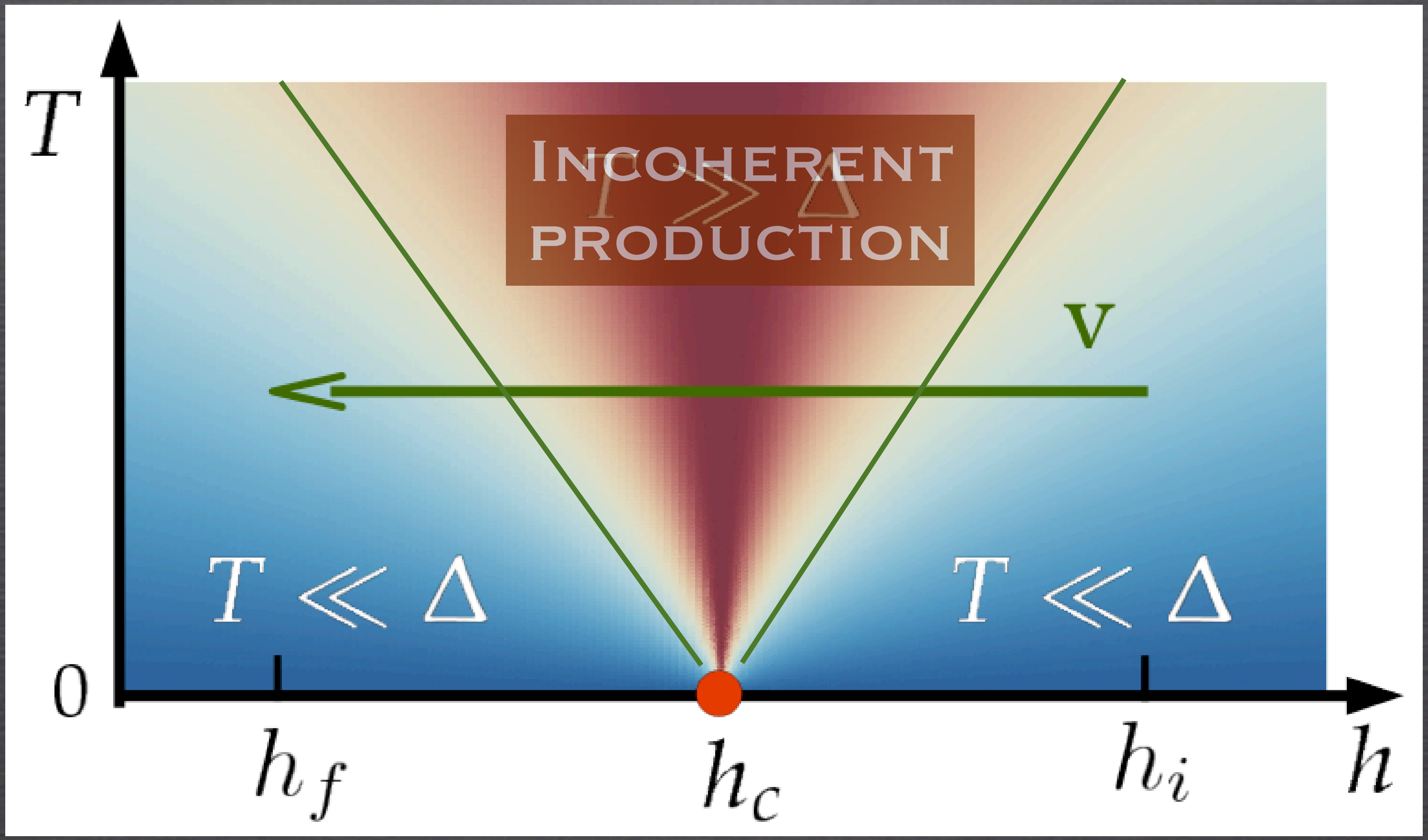

"Incoherent" Defects

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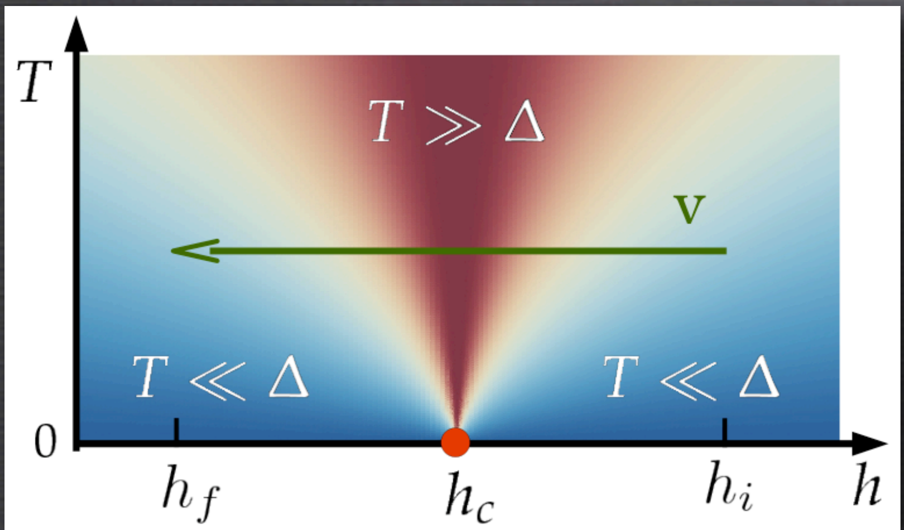
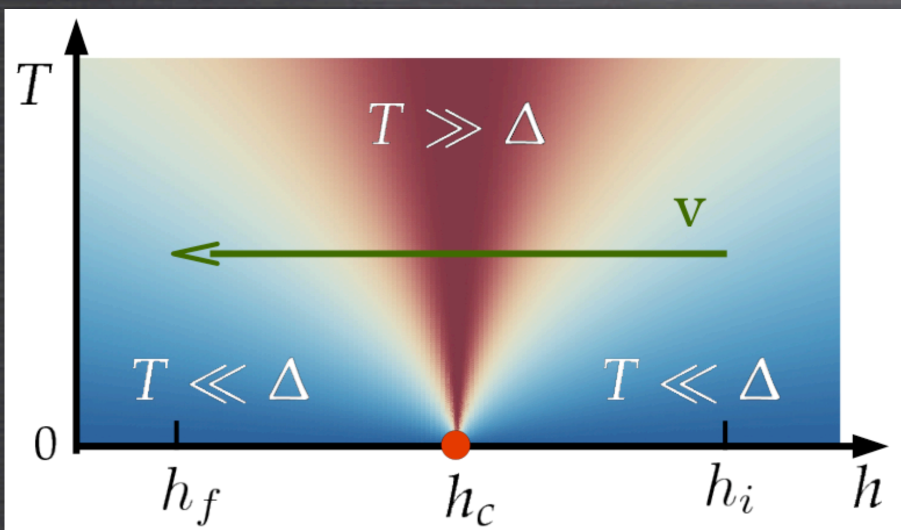


“Incoherent” Defects

- ✓ Density of defects $\mathcal{E} \simeq \mathcal{E}_{KZ} + \mathcal{E}_{inc}$
- ✓ The bath does not influence the system for $T \ll \Delta$
- ✓ Relaxation in the critical region $\tau_r^{-1} \propto \alpha T^{\theta}$



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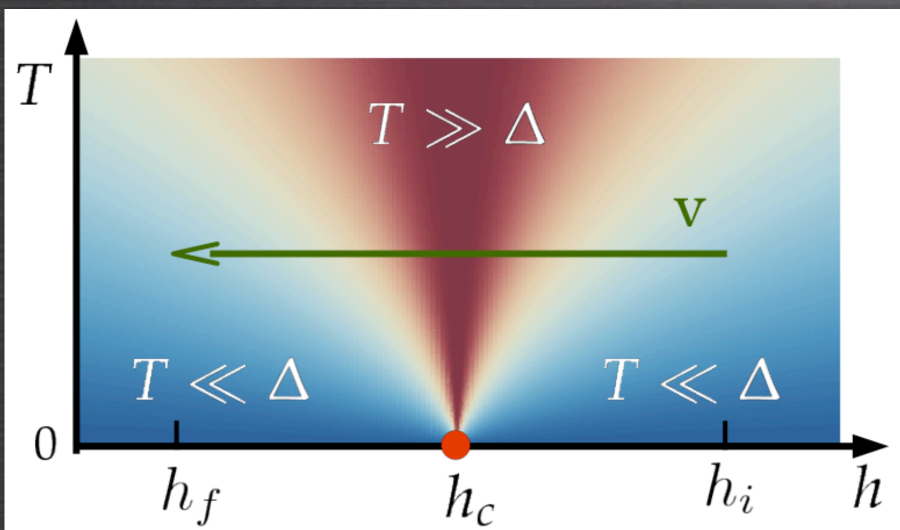

$$\mathcal{E} = \int \frac{d^d k}{(2\pi)^d} \mathcal{P}_k$$

$$\frac{d}{dt}\mathcal{P}_k = -\frac{1}{\tau}\left[\mathcal{P}_k - \mathcal{P}_k^{th}(h_c)\right]$$

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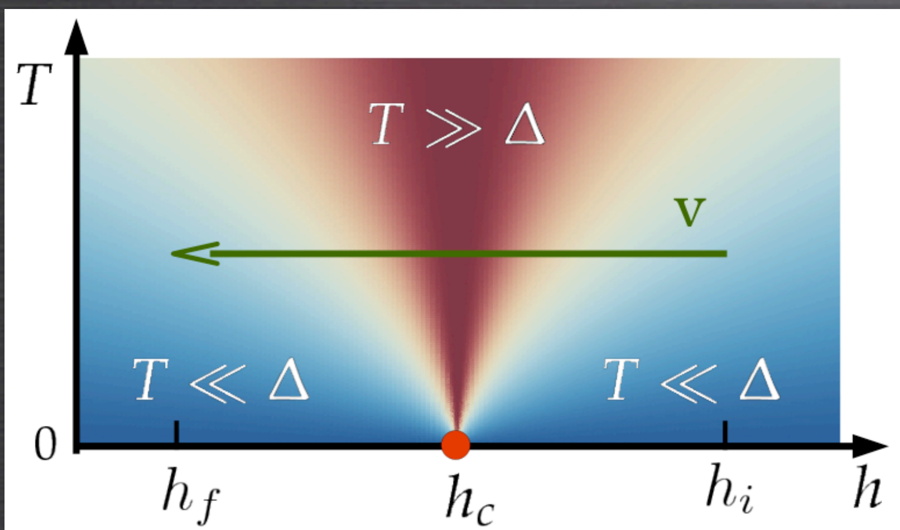

$$t_{QC} = 2T^{1/\nu z} v^{-1}$$

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$$\mathcal{E}_{inc} \propto \alpha v^{-1} T^{\theta + \frac{d\nu + 1}{\nu z}}$$

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$$\mathcal{E}_{inc} \propto \alpha v^{-1} T^{\theta + \frac{d\nu+1}{\nu z}}$$

$$v_{cross} \propto \alpha^{\frac{\nu z+1}{\nu(z+d)+1}} T^{\left(1+\frac{(\theta-1)\nu z}{\nu(z+d)+1}\right)\left(1+\frac{1}{\nu z}\right)}$$

1D Ising Model coupled to a Bath

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$$H = -\frac{J}{2}\sum_{j}^{N}\{\sigma_j^x\sigma_{j+1}^x + [h(t) + X_j]\sigma_j^z\} + H_B$$

$$H_B = \sum_{j,\beta}\omega_\beta b_{\beta j}^\dagger b_{\beta j} \qquad X_j = \sum_\beta \lambda_\beta (b_{\beta,j}^\dagger + b_{\beta,j})$$

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Ohmic bath

$$\sum_\beta \lambda_\beta^2 \delta(\omega - \omega_\beta) = 2\alpha\omega\exp(-\omega/\omega_c)$$

Kinetic Equations

Jordan-Wigner transformation

$$H = \sum_{k>0} \Psi_k^\dagger \hat{\mathcal{H}}_k \Psi_k + \frac{1}{\sqrt{N}} \sum_{k,q} \Psi_k^\dagger \hat{\tau}^z \Psi_{k+q} X_q + H_B$$

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$$-i[G_k^<(t,t)]_{i,j} \equiv \langle \Psi_{k,j}^\dagger(t) \Psi_{k,i}(t) \rangle$$

Kinetic Equations

self-consistent Born
+ Markov approximation

$$\partial_t \hat{G}_k^< + i\left[\hat{\mathcal{H}}_k,\, \hat{G}_k^<\right] =$$
$$\frac{1}{N}\sum_q \hat{\tau}^z(\hat{1} + i\hat{G}_q^<)\hat{D}_{qk}\hat{G}_k^< + \hat{\tau}^z\hat{G}_q^<\hat{D}_{kq}^\dagger(\hat{1} + i\hat{G}_k^<)$$

$$\hat{D}_{qk} = i\int_0^\infty ds\, g^>(s)\hat{\mathcal{U}}_q^\dagger(t, t-s)\hat{\tau}^z\hat{\mathcal{U}}_k(t, t-s)$$

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$$\partial_t \hat{G}^<_k + i\left[\hat{\mathcal{H}}_k,\, \hat{G}^<_k\right] = \frac{1}{N}\sum_q \hat{\tau}^z(\hat{1} + i\hat{G}^<_q)\hat{D}_{qk}\hat{G}^<_k + \hat{\tau}^z\hat{G}^<_q\hat{D}^\dagger_{kq}(\hat{1} + i\hat{G}^<_k)$$

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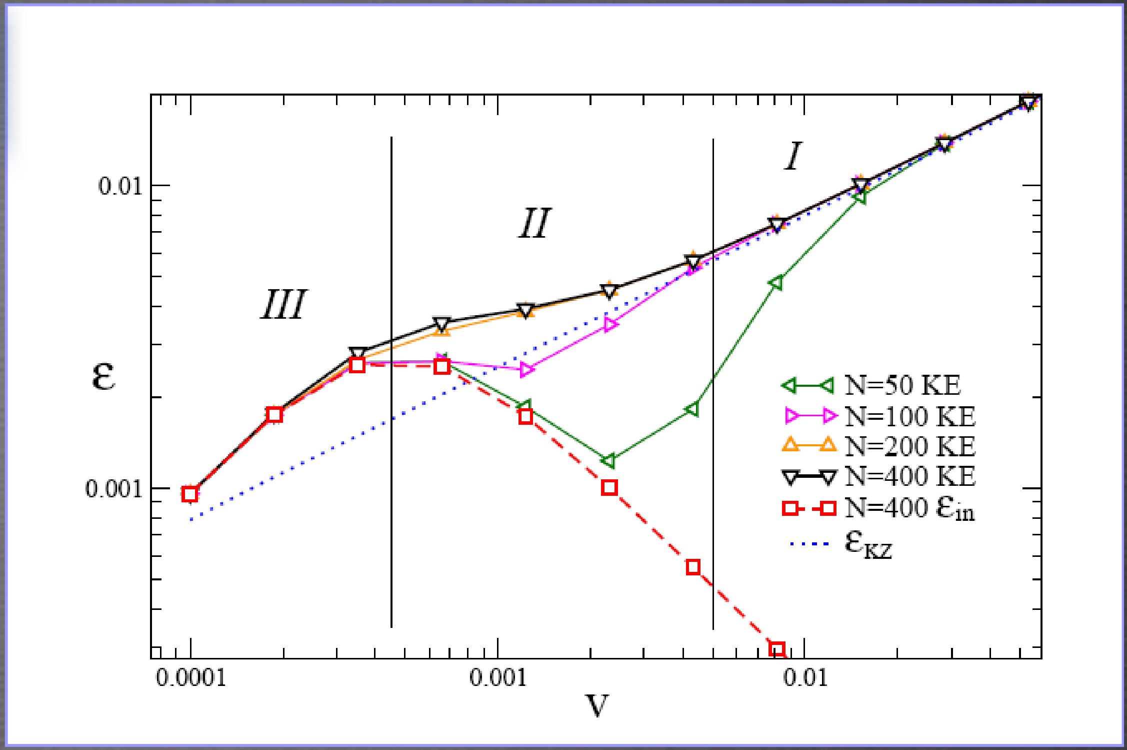
Density of Defects

The density of defects (or the residual energy) is obtained by evaluating the average number of excitations after the quench

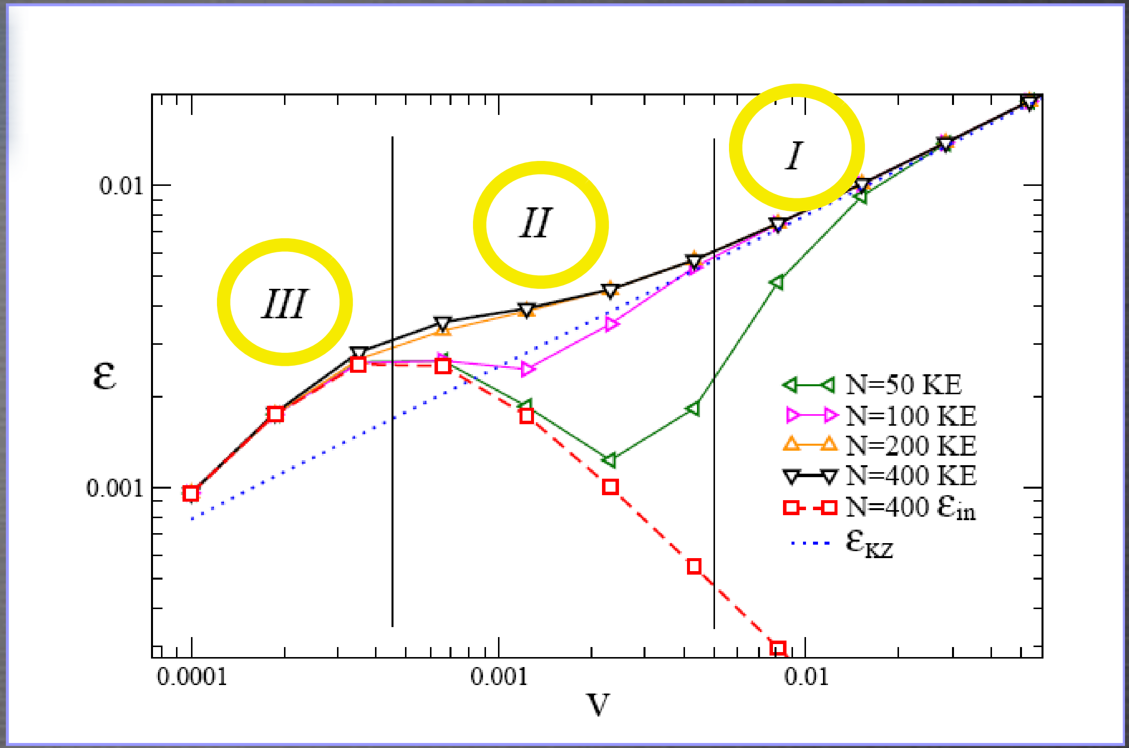
$$\mathcal{E} = \frac{-i}{2N} \sum_{k>0} \mathrm{Tr}\left[(\hat{1} + \hat{\tau}^z) \hat{G}_k^< \right]$$

Density of Defects



Density of Defects

Three regimes



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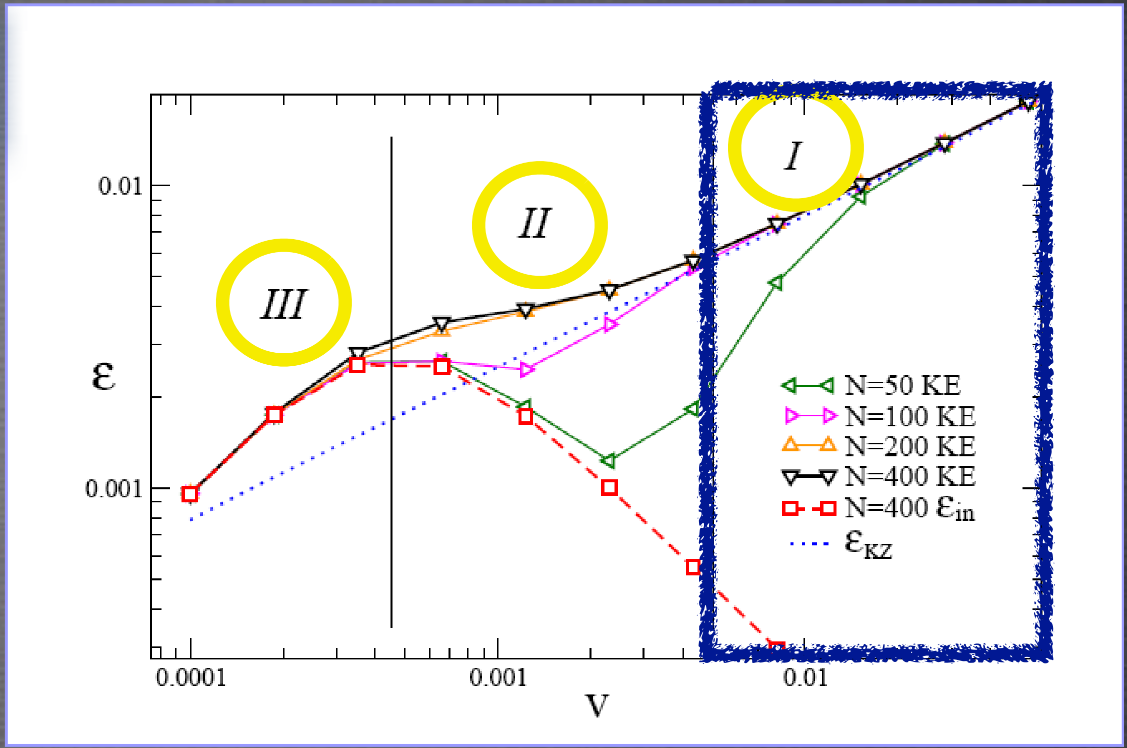
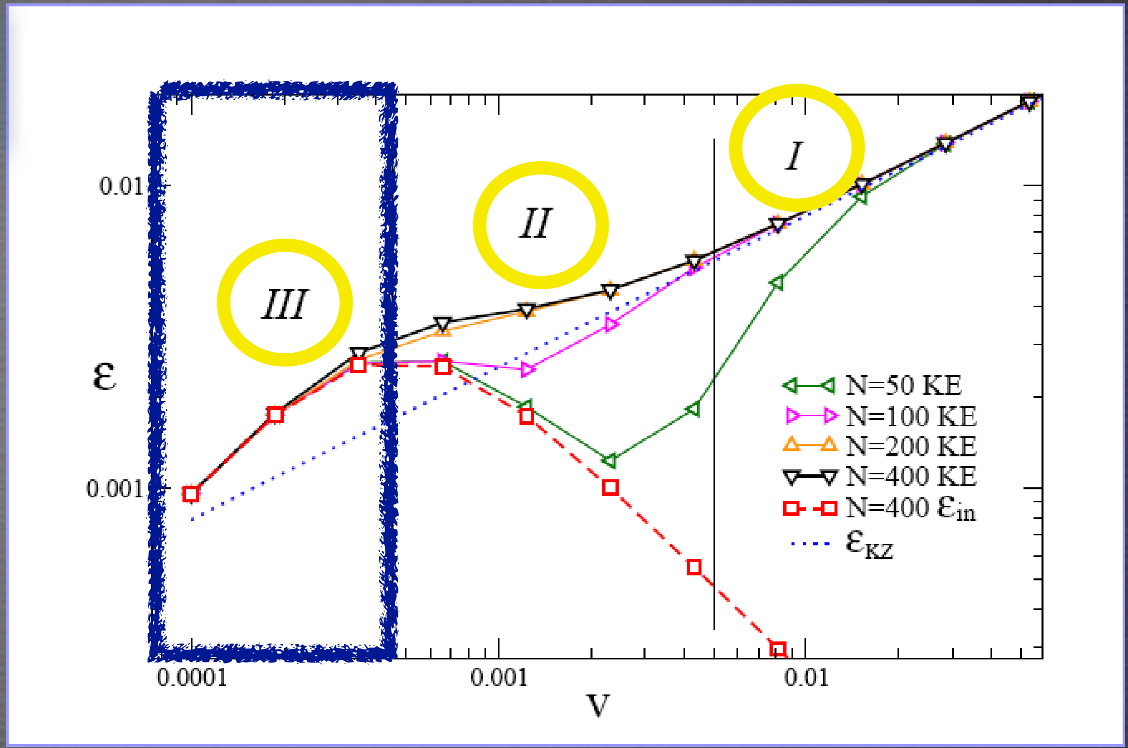


Three regimes

- For fast quenches the bath is unable to affect the system and the KZ scaling is preserved.

Density of Defects



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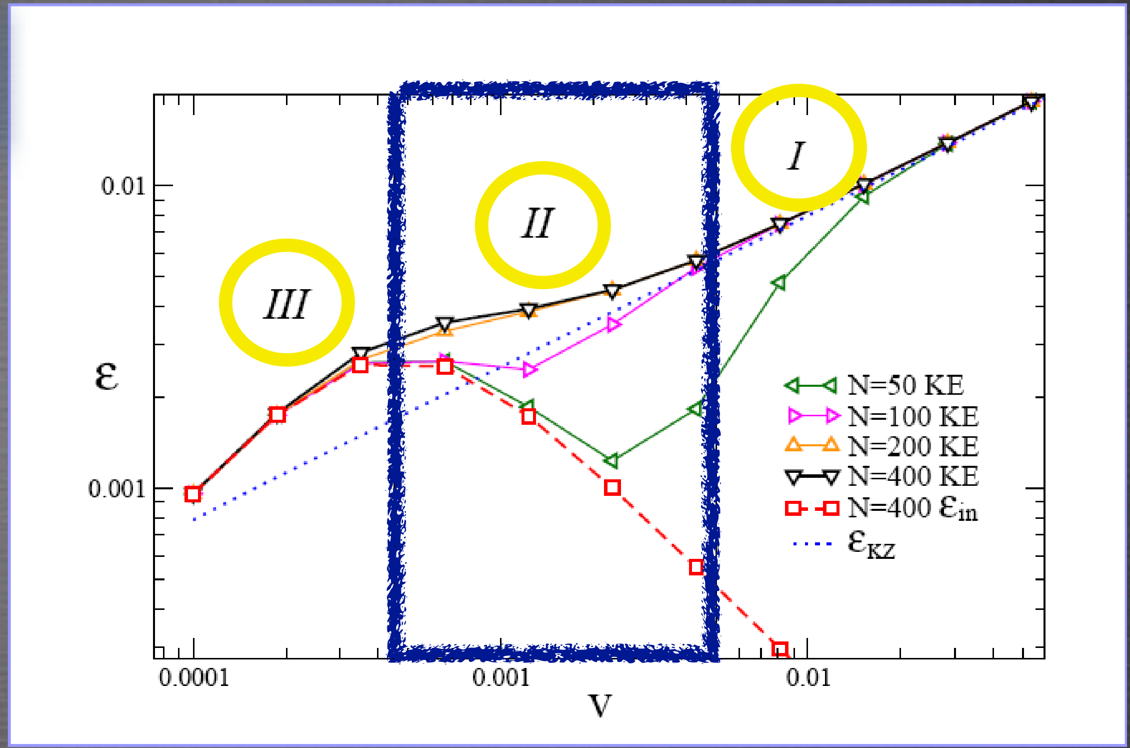

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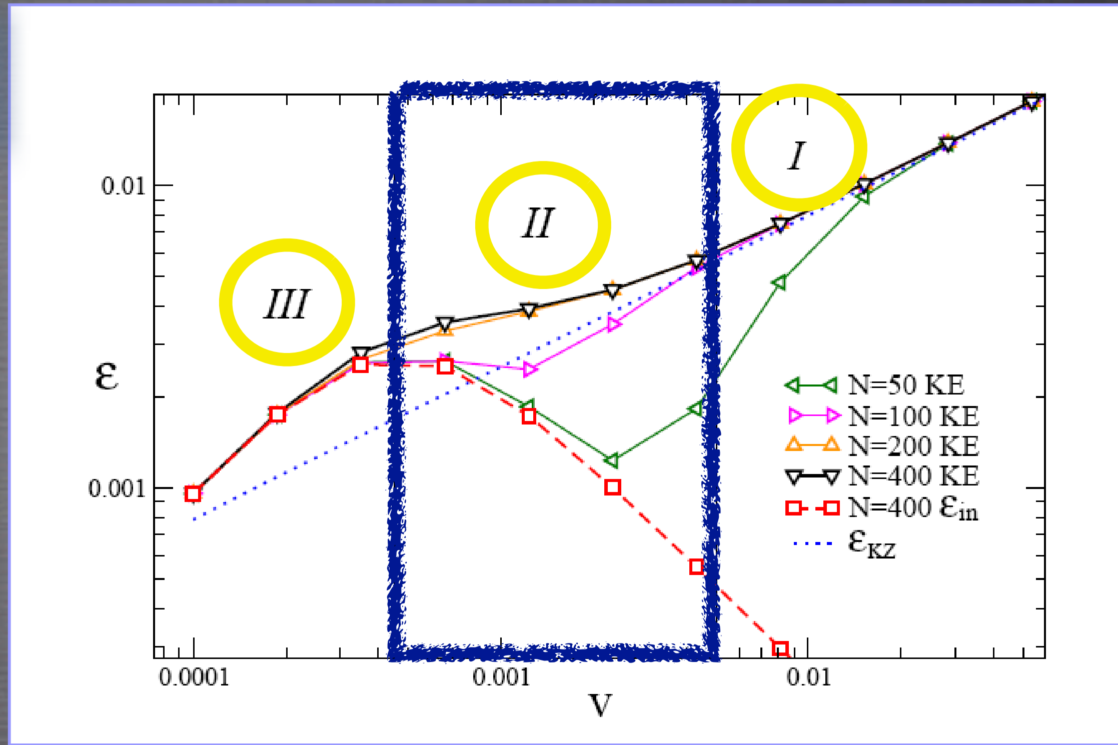
- In the crossover region both thermal and non-adiabatic excitations contribute.

Density of Defects

The bath has two effects:
- It creates excitations near the critical point
- It relaxes the system to its ground state after leaving the quantum critical region



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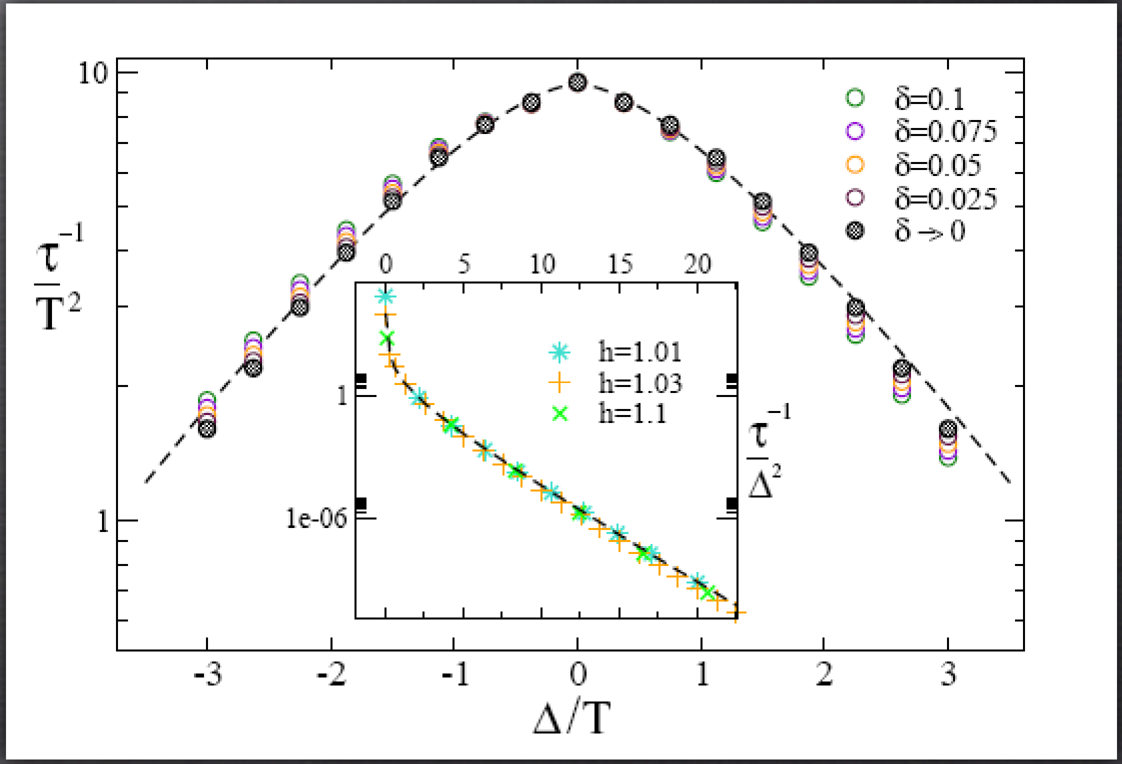
Relaxation Time

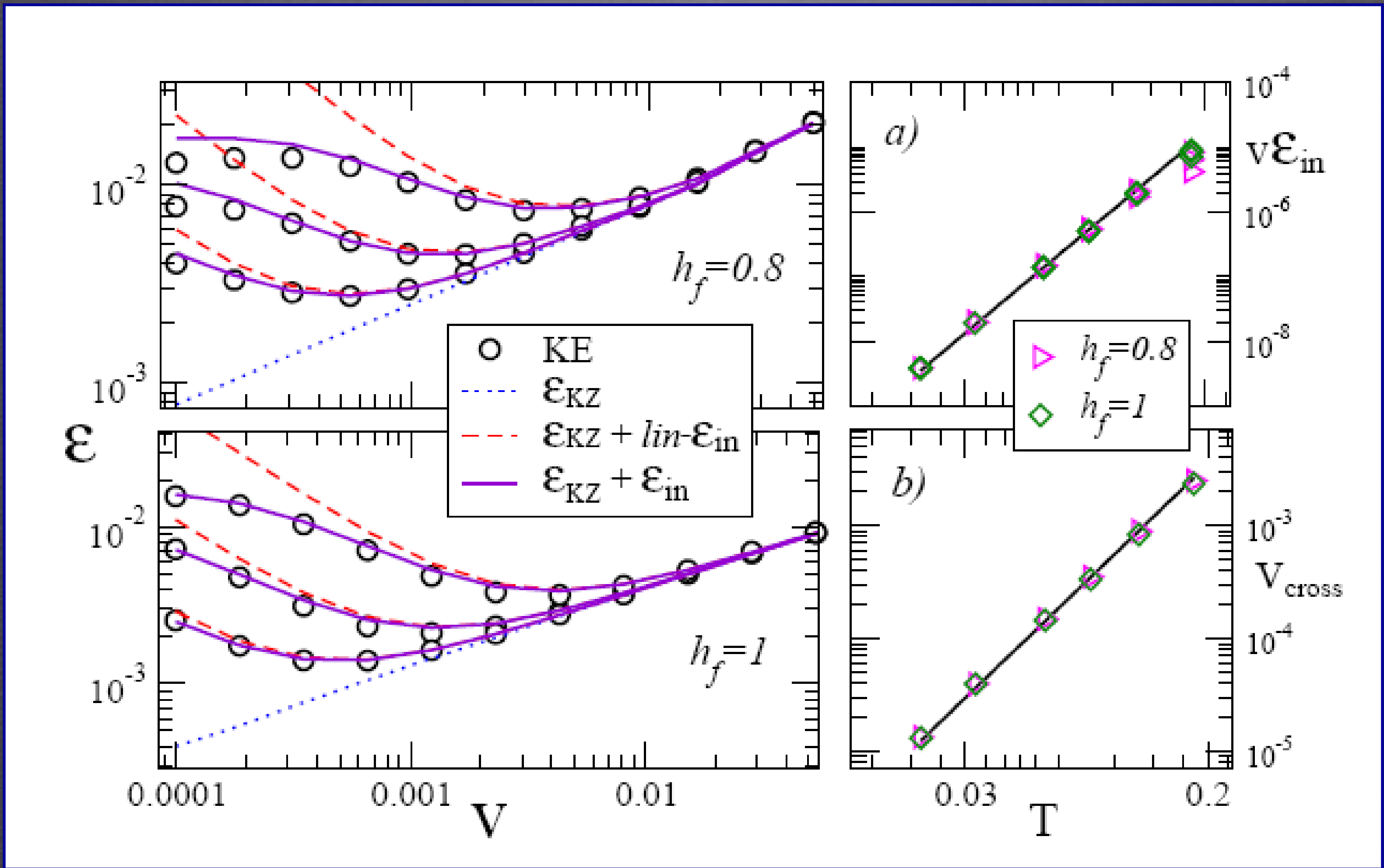
$$\delta = \sqrt{T^2 + (h - h_c)^2}$$



Close to the critical point curves collapse into a unique scaling function

$$f(\Delta/T) = a(1 + b\Delta/T)\exp\{-\Delta/T\}$$

Comparison of The Kinetic Equations with The Scaling analysis



Conclusions

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