Entanglement Creation Outside Light Cone

Shih-Yuin Lin 林 世 昀
Department of Physics
National Cheng Kung University

in collaboration with Bei-Lok Hu

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Outline

I. Motivation

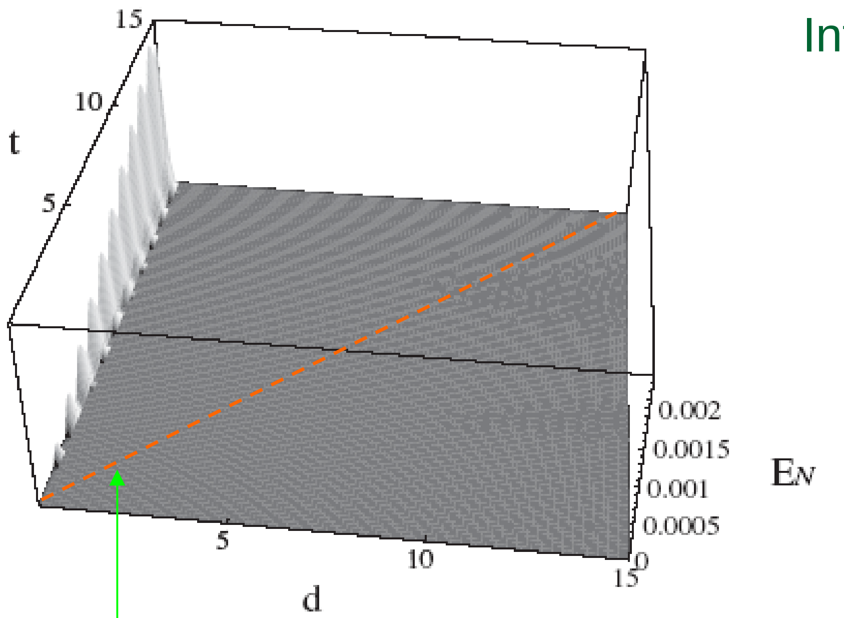
II. The Model

III. Concluding Remarks

I. Motivation

I. Motivation

Two HO separated in distance d in vacuum state of a massless scalar field
[S-Y Lin and BL Hu, PRD79, 085020 (2009)]



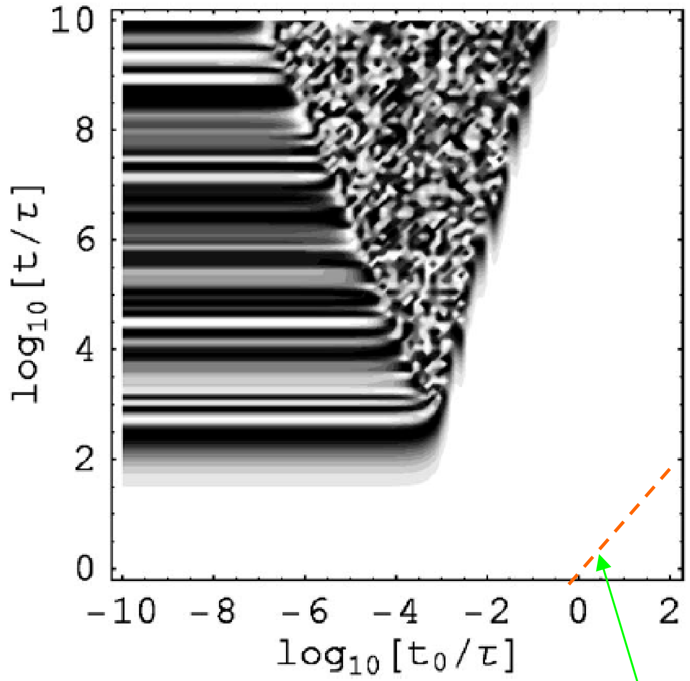
Interaction is switched on at $t=0$.

Entanglement is created
long after one detector
enters the other's light cone
(started at the initial moment).

$c_0 t = d$, the "light cone"

I. Motivation

Two double-quantum-dots separated in distance R in thermal EM field
[D. Braun, PRA72, 062324 (2005)]



$c_0 t = R$, the "light cone"

Interaction is switched on at t=0.

FIG. 1. Entanglement of formation E for two initially not entangled DQDs with d=10 nm coupled to the CMB at T=2.73 K as a function of $\log_{10}(t_0/\tau)$ and $\log_{10}(t/\tau)$, $\tau=\beta\hbar$. Black means perfect entanglement, E=1, white no entanglement E=0. Entanglement is created only for $t/\tau \gtrsim 10^{12}(t_0/\tau)^3$.

$\boxed{t_0 = R/c_0}$

Entanglement is created deeply in the light cone, too.

I. Motivation

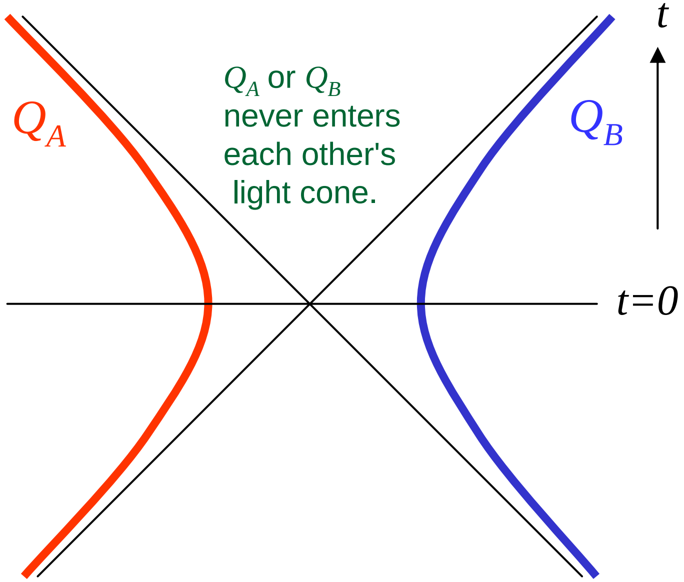
Two initially separated two-level atoms are back-to-back, uniformly accelerated

[B. Reznik, Found. Phys.33, 167 (2003)]

$$|\langle 0 | X_{AB} \rangle|^2 > |E_A|^2 \, |E_B|^2 \quad : \text{Entangled}$$

Now $$\frac{|\langle 0 | X_{AB} \rangle|}{|E_A|^2} = \frac{e^{-\pi\Omega L/2} \sum_{n=0}^{\infty} e^{-\pi n \Omega L}}{\sum_{n=1}^{\infty} e^{-\pi n \Omega L}} = e^{\pi \Omega L/2}$$

> 1 for all L>0 !!



$$x_A = -L/2 \cosh(2\tau/L), \qquad t_A = L/2 \sinh(2\tau/l),$$
$$x_B = L/2 \cosh(2\tau'/L), \qquad t_B = L/2 \sinh(2\tau'/L).$$

Entanglement can be created outside light cone !

I. Motivation

$$x_A = -L/2\cosh(2\tau/L), \qquad t_A = L/2\sinh(2\tau/l),$$
$$x_B = L/2\cosh(2\tau'/L), \qquad t_B = L/2\sinh(2\tau'/L).$$

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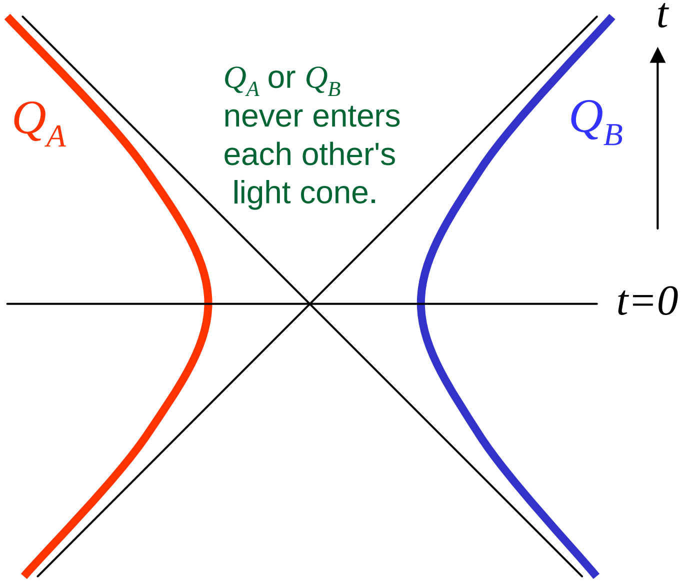

$$\rho = \begin{pmatrix} 1-C & -\langle X_{AB}|0\rangle & 0 & 0 \\ -\langle 0|X_{AB}\rangle & |X_{AB}|^2 & 0 & 0 \\ 0 & 0 & |E_A|^2 & \langle E_B|E_A\rangle \\ 0 & 0 & \langle E_A|E_B\rangle & |E_B|^2 \end{pmatrix}$$

the basis $\{|i\rangle, |j\rangle\} = \{\downarrow\downarrow, \uparrow\uparrow, \downarrow\uparrow, \uparrow\downarrow\}$

$|\langle 0|X_{AB}\rangle|^2 > |E_A|^2|E_B|^2$: Entangled

$$\langle 0|X_{AB}\rangle = \int d\tau_A \int d\tau_B\, e^{i\Omega(\tau_A+\tau_B)} D^+(A,B).$$
$$|E_A|^2 = \int d\tau_A \int d\tau'_A\, e^{-i\Omega(\tau'_A-\tau_A)} D^+(A',A)$$

$$D^+(x',x) = \langle 0|\,\phi(x',t')\,\phi(x,t))\,|0\rangle = \frac{\hbar/4\pi^2}{|\mathbf{x}-\mathbf{x}'|^2 - (t-t'-i\epsilon)^2}$$

Now $\boxed{\dfrac{|\langle 0|X_{AB}\rangle|}{|E_A|^2} = \dfrac{e^{-\pi\Omega L/2}\sum_{n=0}^{\infty} e^{-\pi n\Omega L}}{\sum_{n=1}^{\infty} e^{-\pi n\Omega L}} = e^{\pi\Omega L/2} \quad > 1 \text{ for all } L>0}$!!

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Two initially separated two-level atoms are back-to-back, uniformly accelerated

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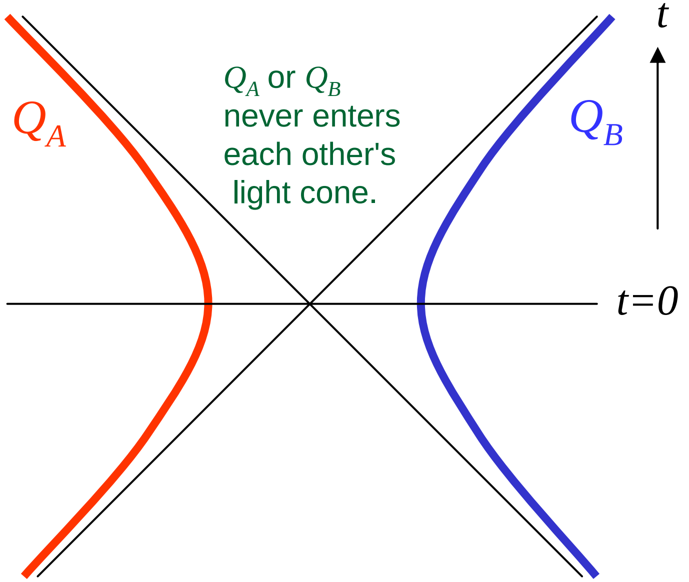


$$|\langle 0 | X_{AB} \rangle|^2 > |E_A|^2 \, |E_B|^2$$: Entangled

Now
$$\boxed{\frac{|\langle 0 | X_{AB} \rangle|}{|E_A|^2} = \frac{e^{-\pi\Omega L/2} \sum_{n=0}^{\infty} e^{-\pi n \Omega L}}{\sum_{n=1}^{\infty} e^{-\pi n \Omega L}} = e^{\pi\Omega L/2}}$$

> 1 for all L>0 !!

→ Entanglement can be created outside light cone.

Shortcomings of the argument:

- Time-dependent perturbation theory: range of validity
- Comparing two infinities
- No time evolution

II. The Model

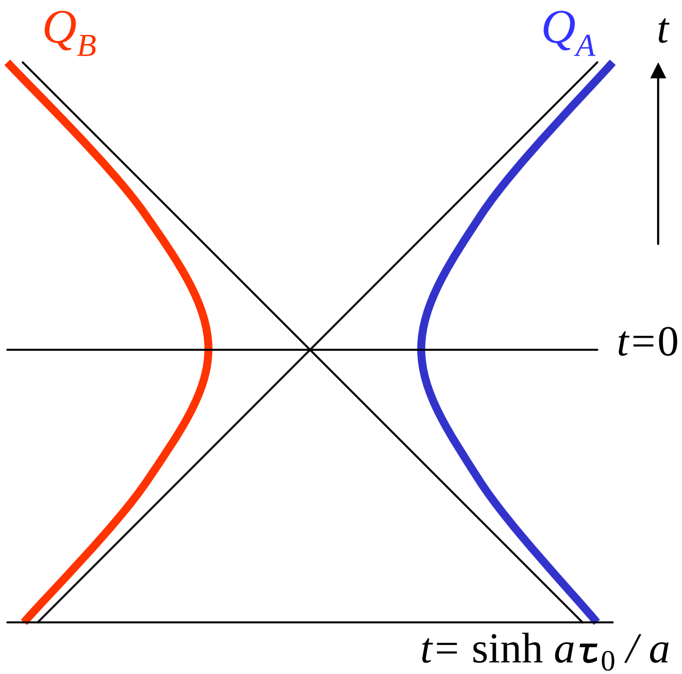
II. The Model

- Two Unruh-DeWitt detectors (HO) are back-to-back, uniformly accelerated in a massless scalar field:

Q_A @ $z_A^\mu = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$

Q_B @ $z_B^\mu = (a^{-1}\sinh a\tau, -a^{-1}\cosh a\tau, 0, 0)$

One never enters the other's light cone.



- Initial state at $\tau = \tau_0$ (Gaussian)

$$|\,\psi(0)\,\rangle = |Q_A, Q_B\rangle\rangle \otimes |\,0_M\,\rangle$$

Free detectors' ground states　　Minkowski vacuum

$$\text{Action}\quad S = -\int d^4x \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \sum_{j=A,B}\Big\{\int d\tau_j \frac{1}{2}[(\partial_{\tau_j}Q_j)^2 - \Omega_0^2 Q_j^2] + \lambda_0 \int d^4x\Phi(x)\int d\tau_j Q_j(\tau_j)\delta^4(x^\mu - z_j^\mu(\tau_j))\Big\}$$

II. The Model

Sketch of calculation

- Evolution of operators Q_L , P_L , Q_R , P_R , Φ , Π in Heisenberg picture.

$$\hat{Q}_i(\tau_i) = \sqrt{\frac{\hbar}{2\Omega_r}}\sum_j\left[q_i^{(j)}(\tau_i)\hat{a}_j + q_i^{(j)*}(\tau_i)\hat{a}_j^\dagger\right] + \int\frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}\left[q_i^{(+)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k} + q_i^{(-)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k}^\dagger\right]$$

damped HO **damped driven HO**

- Sandwiched by the initial state: 10 symmetric correlators as elements of the covariance matrix

$$V_{\mu\nu}(t) = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle \equiv \frac{1}{2}\langle\, (\mathcal{R}_\mu\mathcal{R}_\nu + \mathcal{R}_\nu\mathcal{R}_\mu)\,\rangle$$

$$\mathcal{R}_\mu = (Q_L(t), P_L(t), Q_R(t), P_R(t))$$

- Partial Transposition: $\mathbf{V}^{PT} = \mathbf{\Lambda}\mathbf{V}\mathbf{\Lambda}$

$$\mathbf{\Lambda} = \mathrm{diag}(1,1,1,-1)$$

$$\mathrm{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

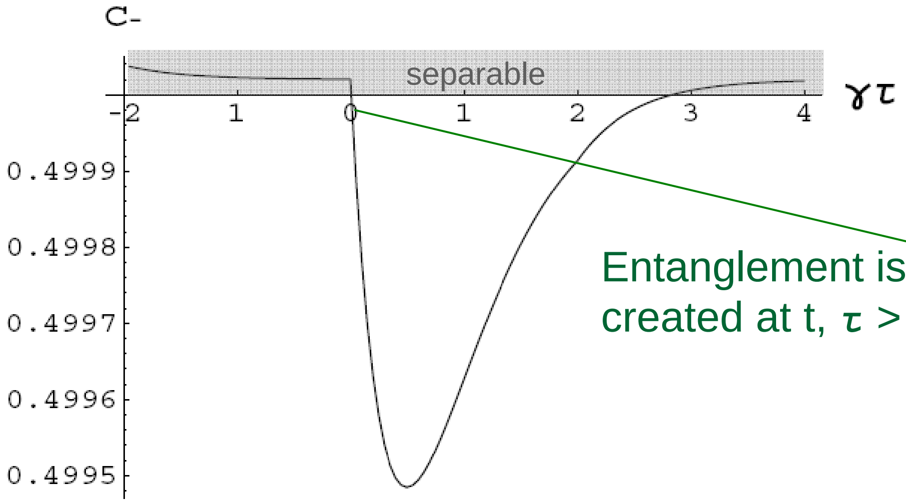
The quantity $\Sigma(t) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathbf{M}\right]$

< 0 iff entangled [Simon 2000].

or logarithmic negativity $E_\mathcal{N} \equiv \max\{0, -\log_2 2c_-\}$

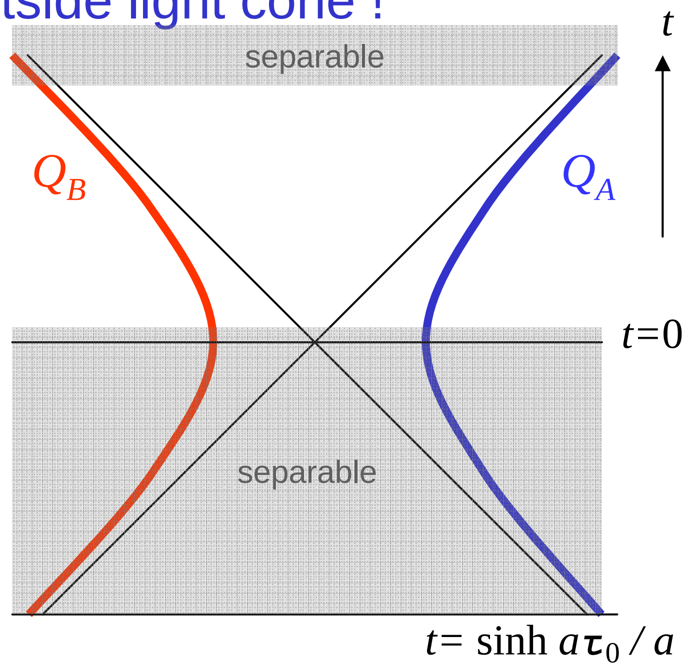
II. The Model

Result: Entanglement can be created outside light cone !



$$\gamma = 10^{-5},\ \Omega = 2.3,\ \hbar = a = 1,\ \Lambda_0 = \Lambda_1 = 20.$$

Entanglement is created at t, $\tau > 0$.



$$\boxed{c_- < \hbar/2 \quad : \text{Entangled}}$$

$$\Sigma = \left(c_+^2 - \frac{\hbar^2}{4}\right)\left(c_-^2 - \frac{\hbar^2}{4}\right)$$

$$E_{\mathcal{N}} \equiv \max\{0, -\log_2 2c_-\}$$

$$z_A^\mu = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$$

$$z_B^\mu = (a^{-1}\sinh a\tau, -a^{-1}\cosh a\tau, 0, 0)$$

II. The Model

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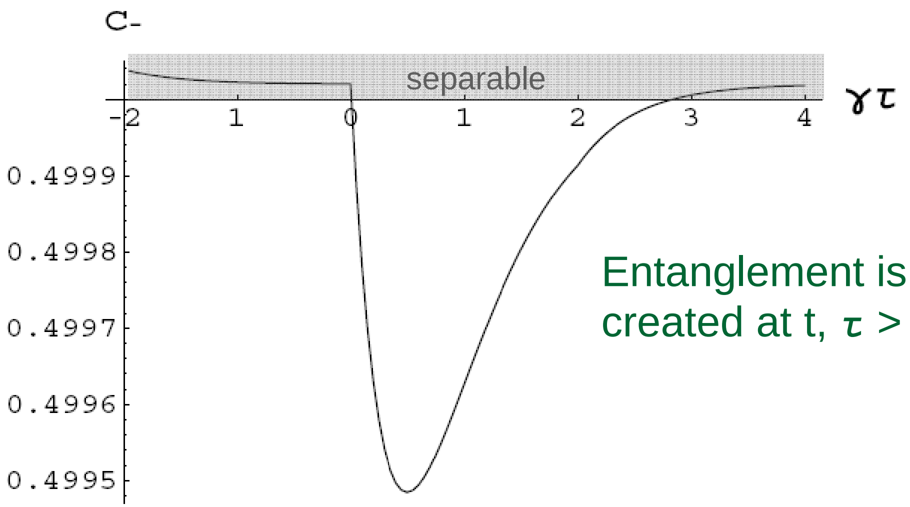

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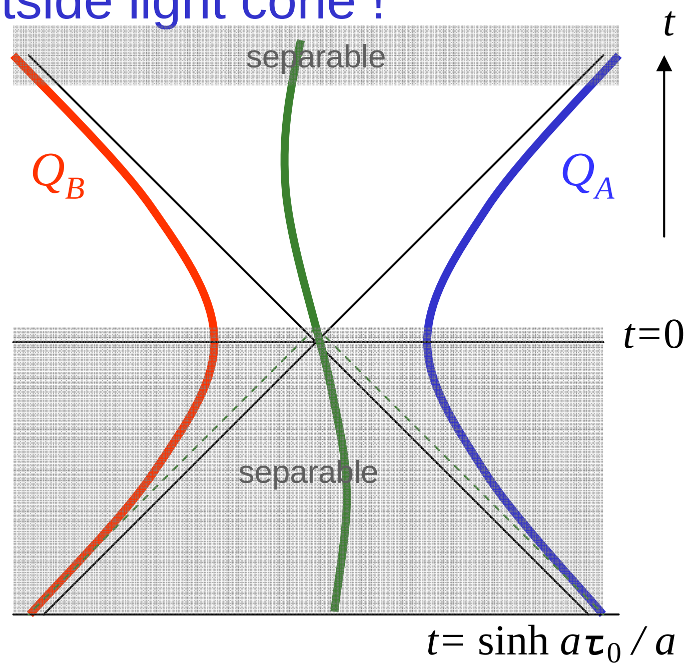


$\boxed{c_- < \hbar/2 \quad : \text{ Entangled}}$

$$z_A^\mu = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$$
$$z_B^\mu = (a^{-1}\sinh a\tau, -a^{-1}\cosh a\tau, 0, 0)$$

More observations:

1. The detectors will be disentangled at late times.
2. It seems that the entanglement time is after the moment that the third party can have causal contact with both detectors

II. The Model

- Covariance matrix of the detectors

Self Correlators | Cross Correlators

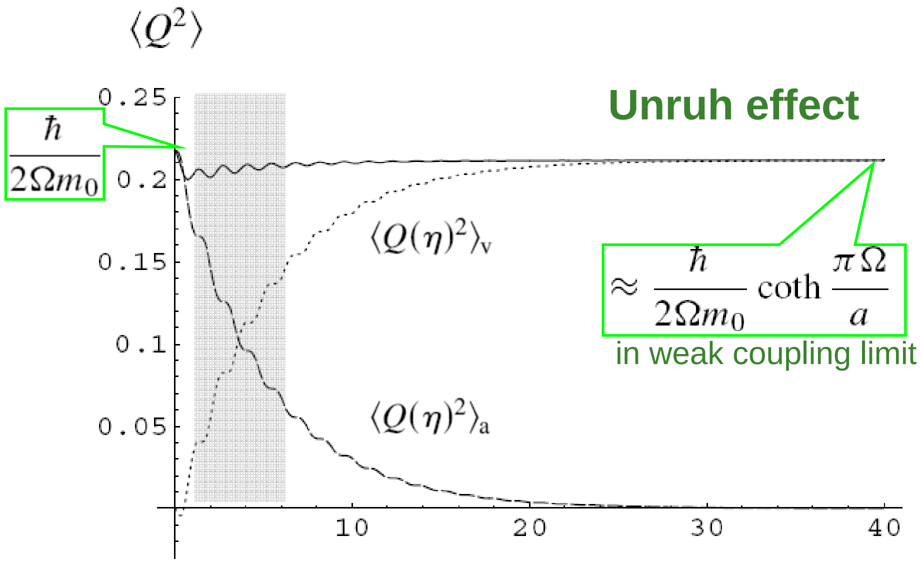
$$V \equiv \begin{pmatrix} \langle Q_A^2 \rangle & \langle Q_A, P_A \rangle & \langle Q_A, Q_B \rangle & \langle Q_A, P_B \rangle \\ \langle Q_A, P_A \rangle & \langle P_A^2 \rangle & \langle P_A, Q_B \rangle & \langle P_A, P_B \rangle \\ \langle Q_A, Q_B \rangle & \langle P_A, Q_B \rangle & \langle Q_B^2 \rangle & \langle Q_B, P_B \rangle \\ \langle Q_A, P_B \rangle & \langle P_A, P_B \rangle & \langle Q_B, P_B \rangle & \langle P_B^2 \rangle \end{pmatrix}$$

Cross Correlators | Self Correlators

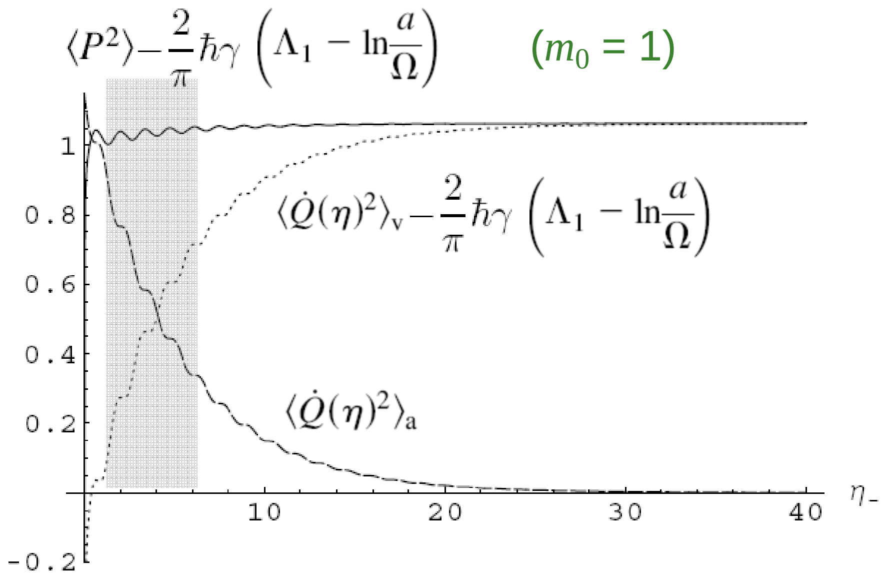
II. The Model

Self correlators $\langle Q_A^2 \rangle = \langle Q_B^2 \rangle$, $\langle P_A^2 \rangle = \langle P_B^2 \rangle$ are always positive and "large",

$a = 1,\ \gamma = 0.1,\ \Omega = 2.3$ $\boxed{\gamma \equiv \lambda_0^2/8\pi}$: coupling strength



$\langle Q_A^2 \rangle = \langle Q_B^2 \rangle$



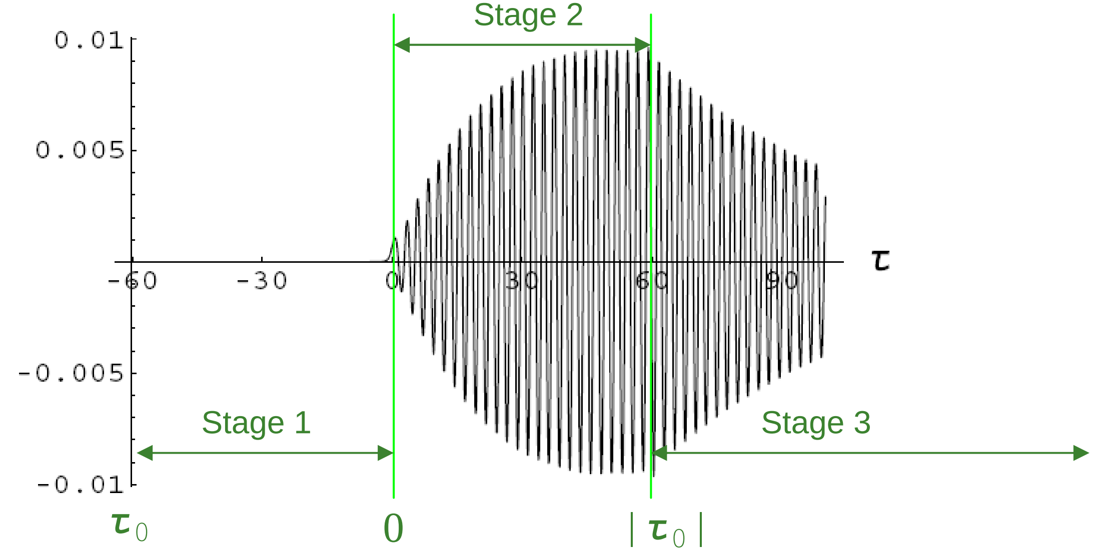
$\langle P_A^2 \rangle = \langle P_B^2 \rangle$

while $\langle P_A, Q_A \rangle, \langle P_B, Q_B \rangle = (m_0/2)(d/d\tau)\langle Q^2 \rangle$ are oscillating in small amplitude.

II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$.

$\langle Q_A, Q_B \rangle$ with $\gamma = 0.01,\ \Omega = 1.3,\ a = 1,\ \hbar = 1,\ \ \tau_0 = -60.$



$$\langle Q_A, Q_B \rangle \approx \frac{\hbar\gamma \boxed{e^{-2\gamma\tau}}}{\Omega \sinh\frac{\pi\Omega}{a}}\left[-\boxed{2\tau\cos 2\Omega\tau} + \frac{\pi}{a}\coth\frac{\pi\Omega}{a}\sin 2\Omega\tau\right]$$

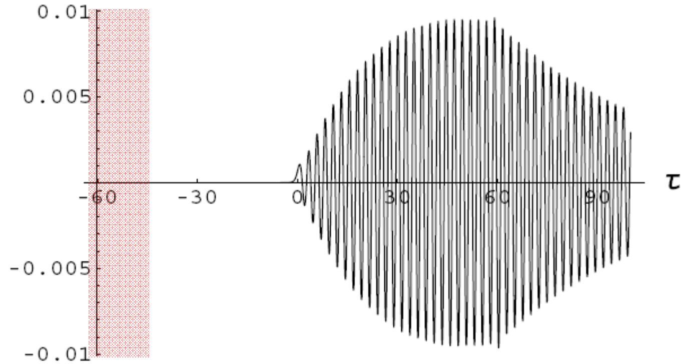
in Stage 2

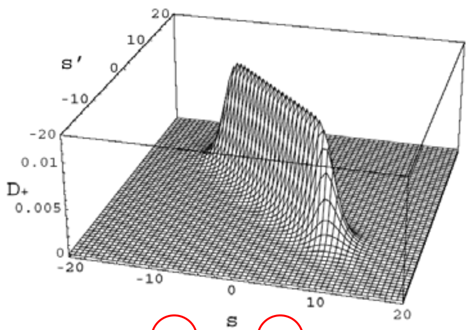
$$\approx \frac{\hbar\gamma e^{-2\gamma\tau}}{\Omega^2 \sinh\frac{\pi\Omega}{a}}\left[2\Omega\tau_0\cos 2\Omega\tau + \ldots\right]$$

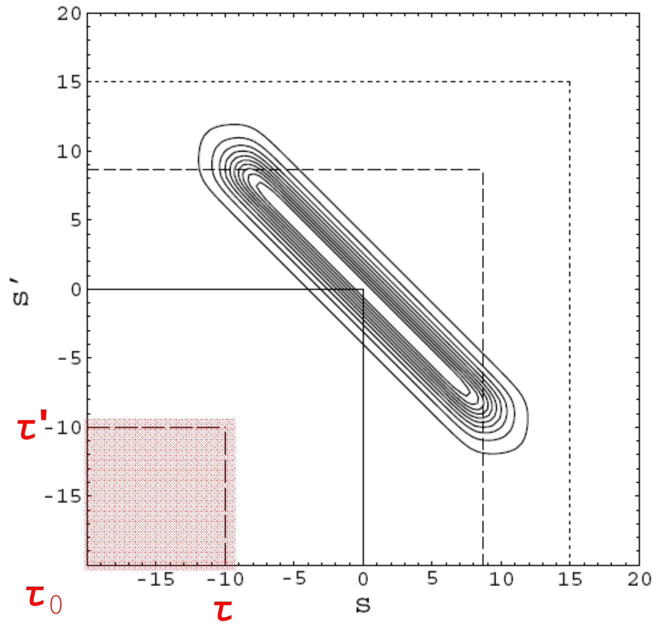
in Stage 3

II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$.







$$|D^{+}(z_A^{\mu}(s), z_B^{\nu}(s'))|$$

Positive frequency Wightman function
of the massless scalar field

$$\langle\, Q_A(\eta), Q_B(\eta')\,\rangle = \frac{\lambda_0^2 \hbar}{\Omega^2} \mathrm{Re} \int_{\tau_0}^{\tau} ds \int_{\tau_0'}^{\tau} ds' e^{-\gamma(\tau - s) - \gamma(\tau' - s')} \sin\Omega(\tau - s) \sin\Omega(\tau' - s') D^{+}(z_A^{\mu}(s), z_B^{\nu}(s'))$$

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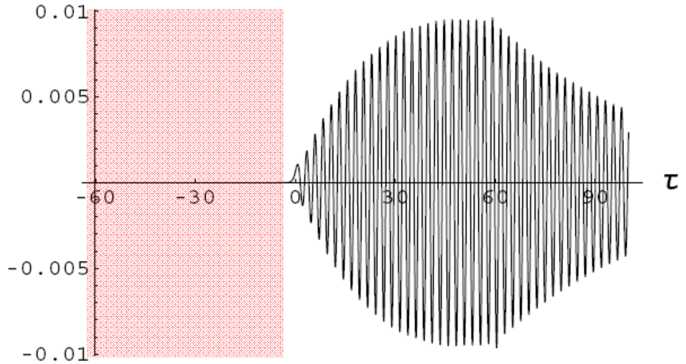
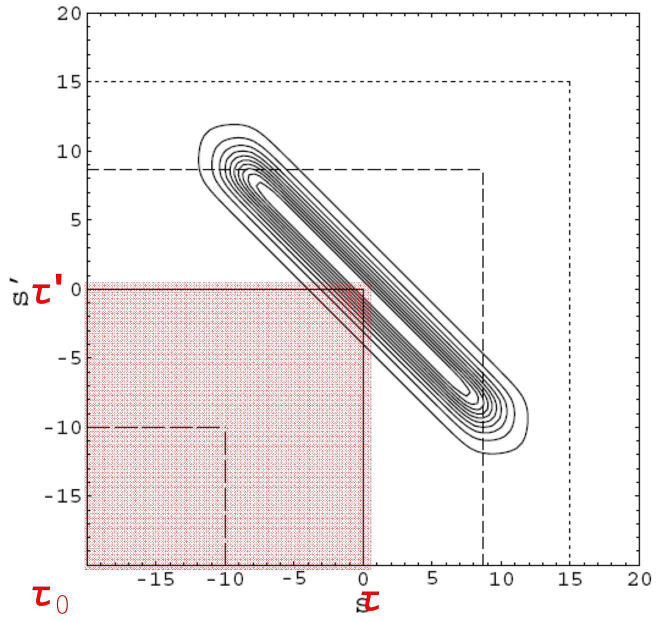
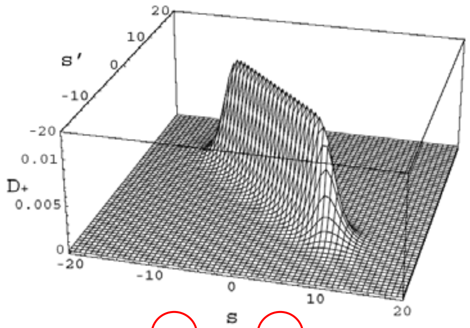




$$|D^+(z_A^\mu(s), z_B^\nu(s'))|$$

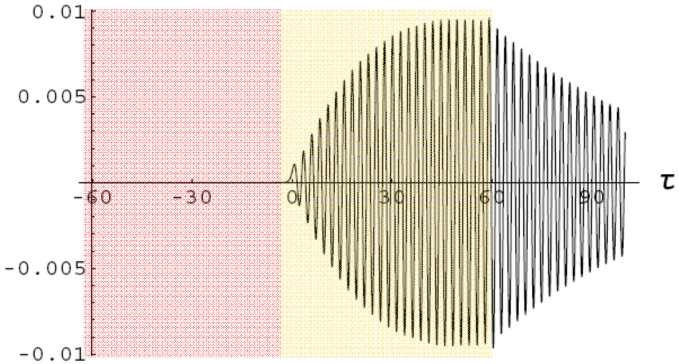


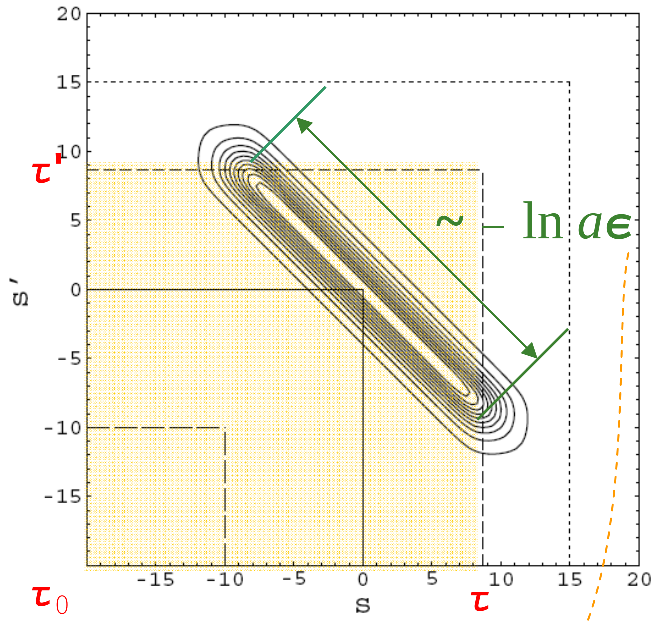
Positive frequency Wightman function of the massless scalar field

$$\langle\, Q_A(\eta), Q_B(\eta')\,\rangle = \frac{\lambda_0^2\hbar}{\Omega^2}\mathrm{Re}\int_{\tau_0}^{\tau} ds \int_{\tau_0'}^{\tau} ds' e^{-\gamma(\tau-s)-\gamma(\tau'-s')} \sin\Omega(\tau-s)\sin\Omega(\tau'-s') D^+(z_A^\mu(s), z_B^\nu(s'))$$

II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$.





$$D^+(z_j^\mu, z_{j'}^\nu) = \frac{\hbar/4\pi^2}{|\mathbf{z}_j - \mathbf{z}_{j'}|^2 - \left(z_j^0 - z_{j'}^0 - i\epsilon\right)^2}$$

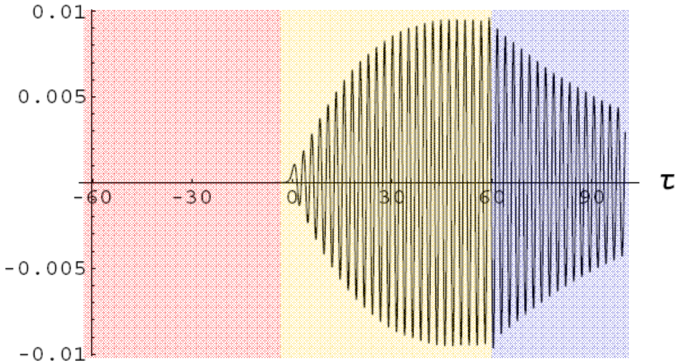
$$|D^+(z_A^\mu(s), z_B^\nu(s'))|$$

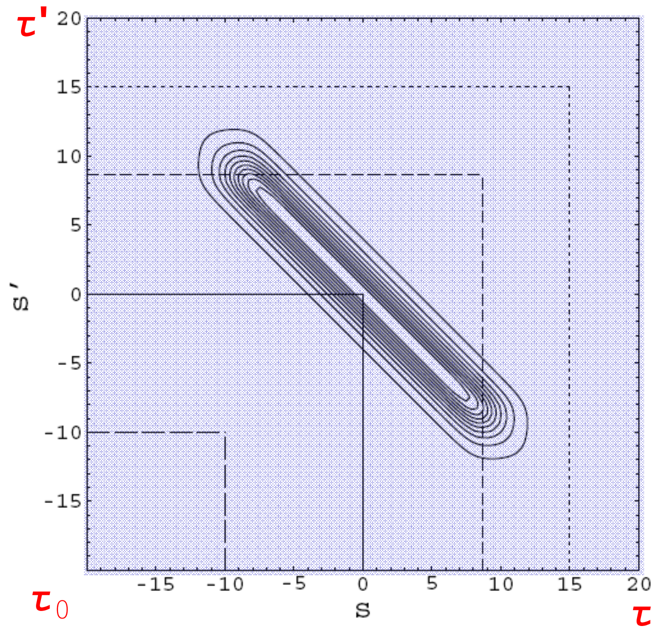
Positive frequency Wightman function
of the massless scalar field

$$\langle\, Q_A(\eta), Q_B(\eta')\,\rangle = \frac{\lambda_0^2\hbar}{\Omega^2}\mathrm{Re}\int_{\tau_0}^{\tau} ds \int_{\tau_0'}^{\tau'} ds' e^{-\gamma(\tau-s)-\gamma(\tau'-s')}\sin\Omega(\tau-s)\sin\Omega(\tau'-s') D^+(z_A^\mu(s), z_B^\nu(s'))$$

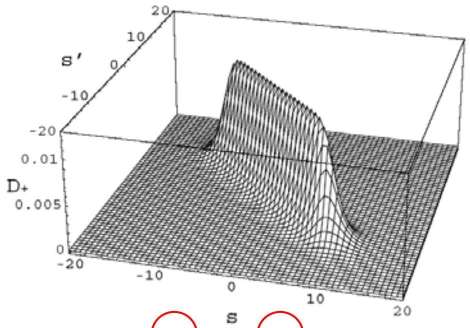
II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$.





$$|D^{+}(z_A^{\mu}(s), z_B^{\nu}(s'))|$$



Positive frequency Wightman function
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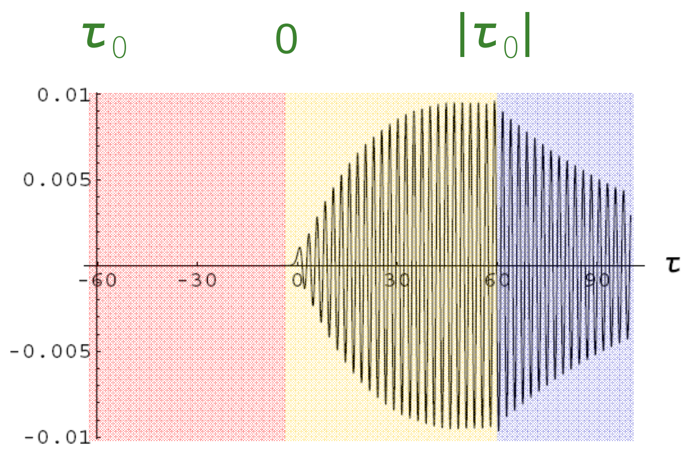
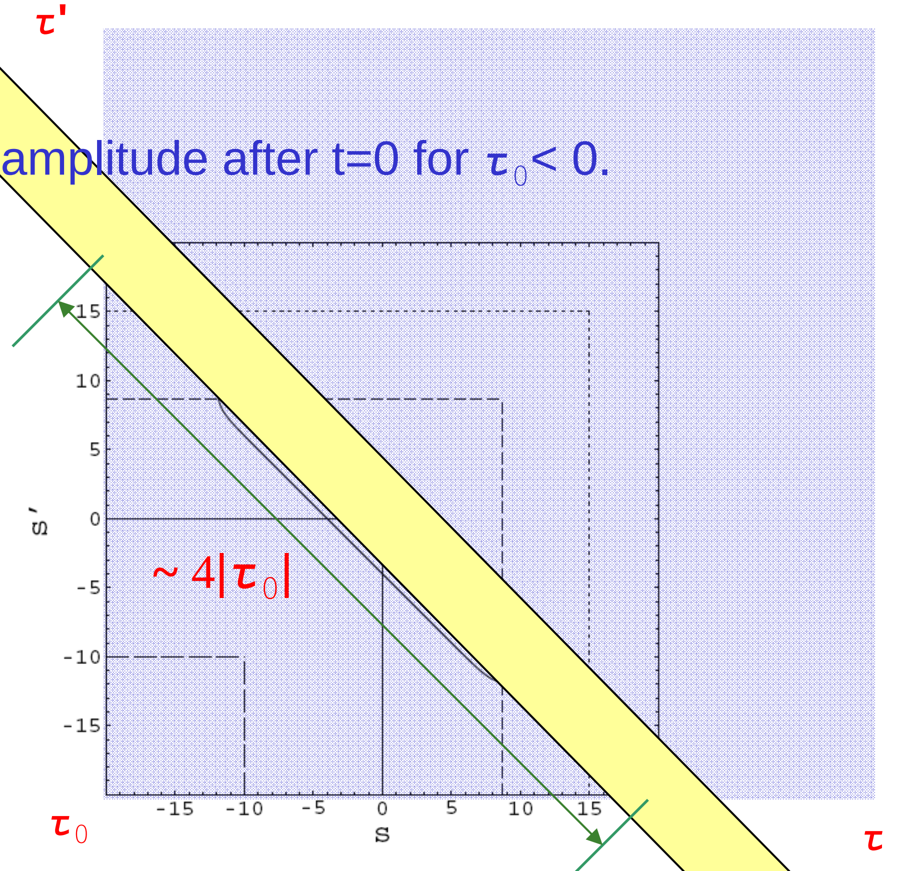
$$\langle\, Q_A(\eta), Q_B(\eta')\,\rangle = \frac{\lambda_0^2 \hbar}{\Omega^2} \mathrm{Re} \int_{\tau_0}^{\tau} ds \int_{\tau_0'}^{\tau} ds' e^{-\gamma(\tau-s)-\gamma(\tau'-s')} \sin\Omega(\tau-s) \sin\Omega(\tau'-s') D^{+}(z_A^{\mu}(s), z_B^{\nu}(s'))$$

II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$.





$$D^+(z_j^\mu, z_{j'}^\nu) = \frac{\hbar/4\pi^2}{|\mathbf{z}_j - \mathbf{z}_{j'}|^2 - \left(z_j^0 - z_{j'}^0 - i\cancel{\epsilon}\right)^2}$$

0

$$|D^+(z_A^\mu(s), z_B^\nu(s'))|$$

Positive frequency Wightman function
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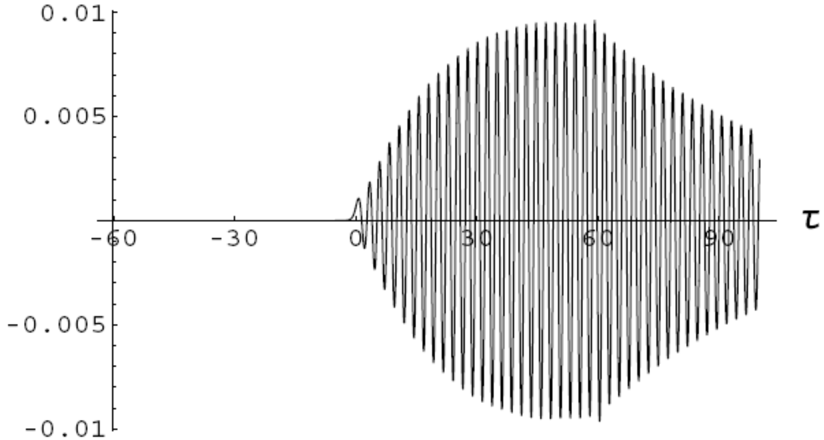
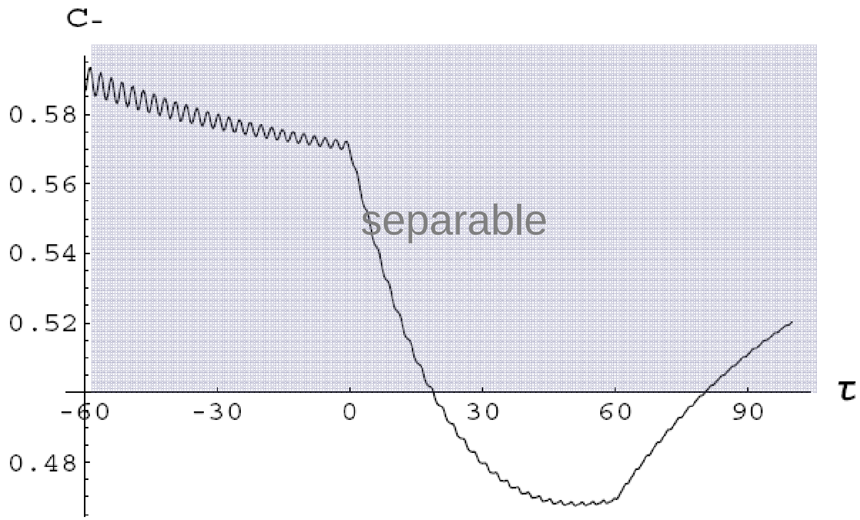
$$\langle\, Q_A(\eta), Q_B(\eta')\,\rangle = \frac{\lambda_0^2\hbar}{\Omega^2}\mathrm{Re}\int_{\tau_0}^{\tau} ds \int_{\tau_0'}^{\tau'} ds' e^{-\gamma(\tau-s)-\gamma(\tau'-s')}\sin\Omega(\tau-s)\sin\Omega(\tau'-s') D^+(z_A^\mu(s), z_B^\nu(s'))$$

II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$. .
This could generate entanglement.

$\langle Q_A, Q_B \rangle$ with $\gamma = 0.01,\ \Omega = 1.3,\ a = 1,\ \hbar = 1,\ \ \tau_0 = -60.$



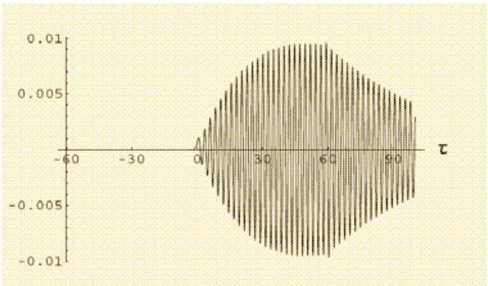


$$\langle Q_A, Q_B \rangle \approx \frac{\hbar\gamma \boxed{e^{-2\gamma\tau}}}{\Omega \sinh\frac{\pi\Omega}{a}}\left[-2\boxed{\tau\cos 2\Omega\tau} + \frac{\pi}{a}\coth\frac{\pi\Omega}{a}\sin 2\Omega\tau\right]$$ in Stage 2

$$\approx \frac{\hbar\gamma e^{-2\gamma\tau}}{\Omega^2 \sinh\frac{\pi\Omega}{a}}\left[2\Omega\tau_0\cos 2\Omega\tau + \ldots\right]$$ in Stage 3

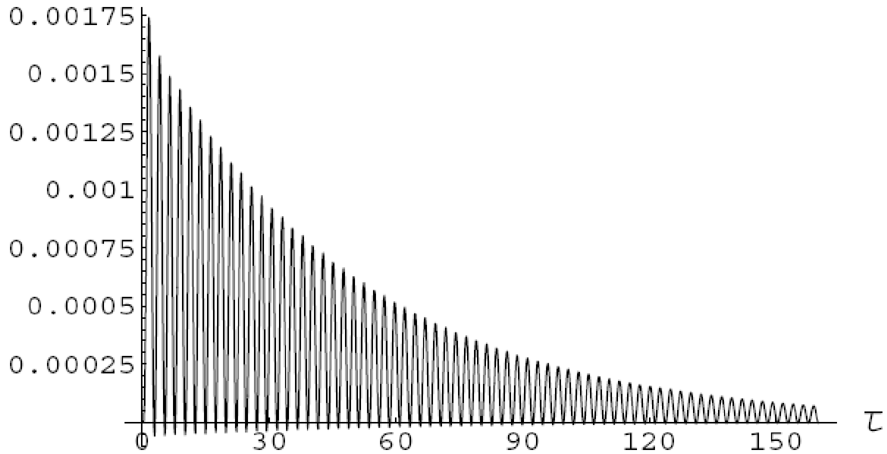
II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$.
This could generate entanglement.



$\tau_0 = -60.$

$\langle Q_A, Q_B \rangle$ with $\gamma = 0.01,\ \Omega = 1.3,\ a = 1,\ \hbar = 1,\ \boxed{\tau_0 = 0}$

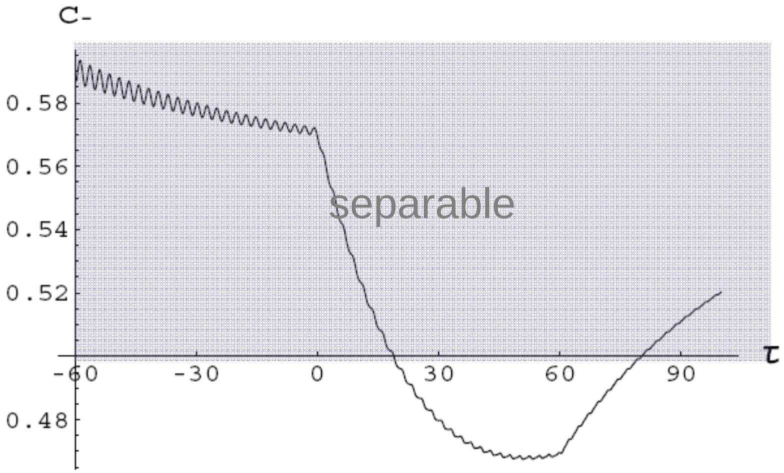


If $\tau_0=0$, $<Q_A,Q_B>$ has no Stage 1 or 2 and is always small.

→ No entanglement creation outside light cone if $\tau_0=0$
- similar to [Lin, Hu 2009] and [Braun 2005].

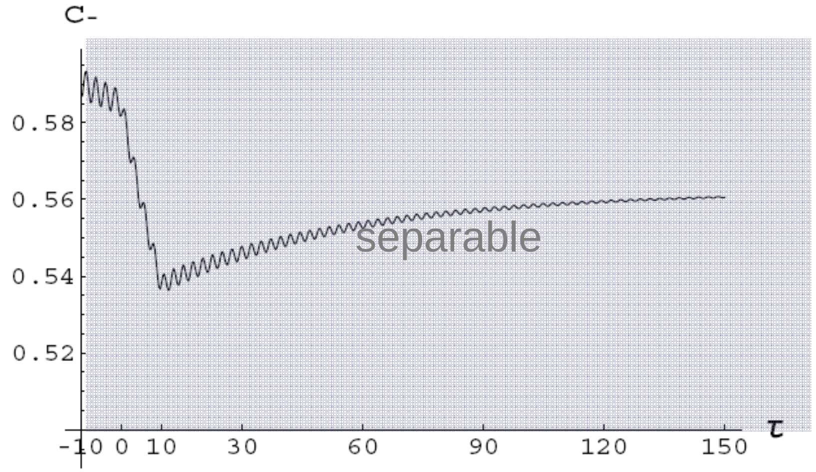
II. The Model

Cross correlators oscillate with growing amplitude after t=0 for $\tau_0 < 0$.
This could and could not generate entanglement.



$\tau_0 = -60.$

entanglement creation



$\tau_0 = -10$

No entanglement creation

No entanglement creation outside light cone if τ_0=0

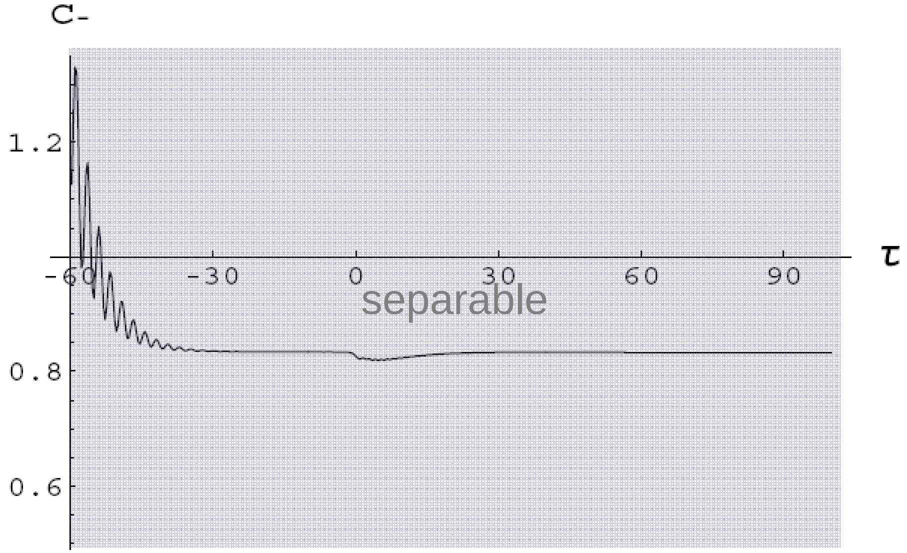
Entanglement Dynamics depend on τ_0 : Non-Markovian.

II. The Model

- In <u>strong-coupling</u> regime and <u>ultrahigh-acceleration</u> regime,

the **self correlators** always dominate over the **cross correlators....**

$\Omega = 1.3 \quad a = 1,\ \Lambda_0 = \Lambda_1 = 20,$ and $\tau_0 = -60$



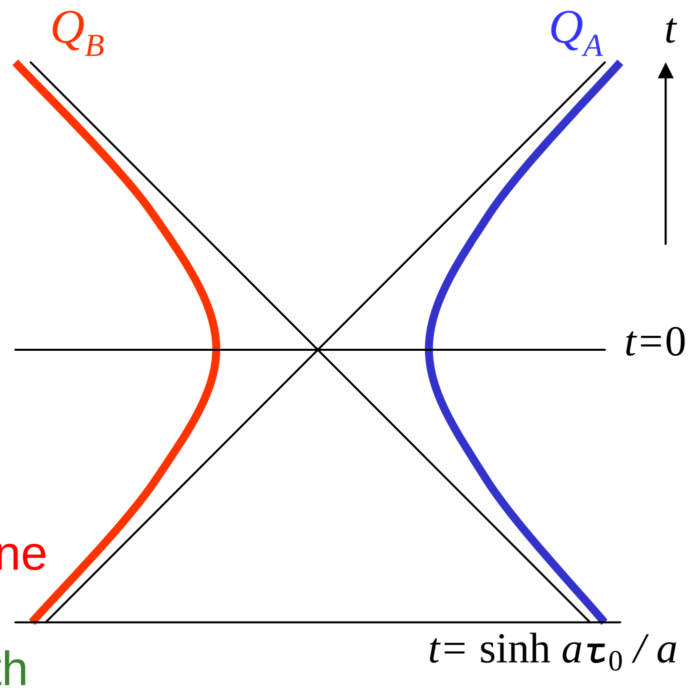
$\boxed{\gamma = 0.1}$

No entanglement creation.

II. The Model

Summary

For two initially separable,
local-in-space quantum objects
without direct interaction between them,
a long-lifetime quantum entanglement
between the two quantum objects
could be generated by the environment
before one object enters the other's light cone
under some conditions
(e.g. back-to-back accelerated uniformly with
proper acceleration, natural frequency, coupling,
and initial moment τ_0 in certain parameter range).



Entanglement created in this way will disappear in a finite time
if the two detectors are sufficiently far from each other at late times.

II. The Model

Two ions separated by 1m

[Maunz, Olmschenk, Hayes, Matsukevich, Duan, Monroe, PRL102, 250502 (2009)]

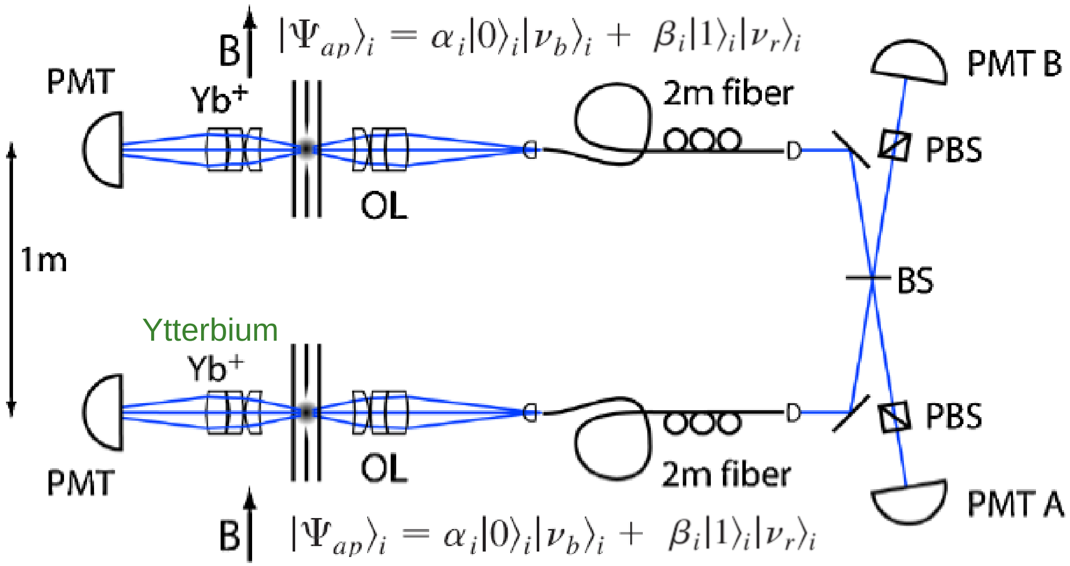
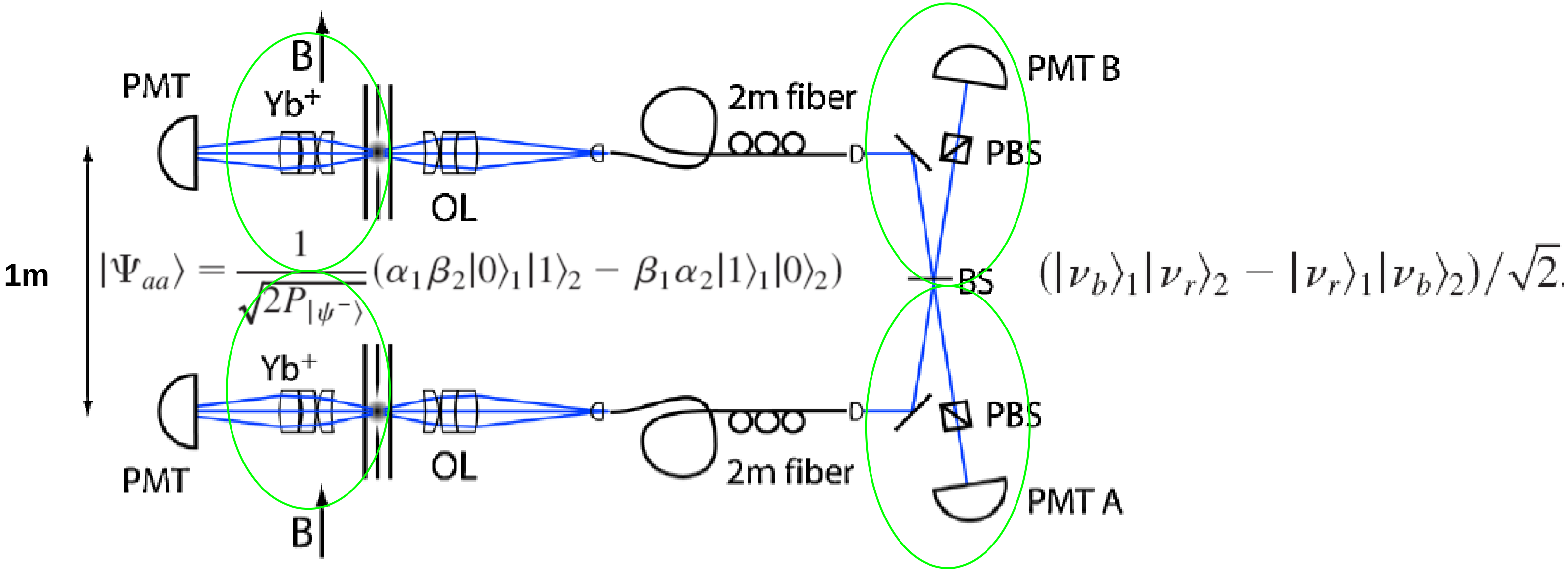


FIG. 1 (color online). The experimental apparatus. Two $^{171}Yb^+$ ions are trapped in identically constructed ion traps separated by 1 m. A magnetic field B is applied perpendicular to the excitation and observation axes to define the quantization axis. About 2% of the emitted light from each ion is collected by an imaging system (OL) with numerical aperture of about 0.3 and coupled into single-mode fibers. Polarization control paddles are used to adjust the fibers to maintain linear polarization. The output of these fibers is directed to interfere on a polarization-independent 50% beam splitter (BS). Polarizers (PBS) transmit only the π-polarized light from the ions. The photons are detected by single-photon counting photomultiplier tubes (PMT A and PMT B). Detection of the atomic state is done independently for the two traps with dedicated photomultiplier tubes (PMTs).

II. The Model

Two ions separated by 1m

[Maunz, Olmschenk, Hayes, Matsukevich, Duan, Monroe, PRL102, 250502 (2009)]

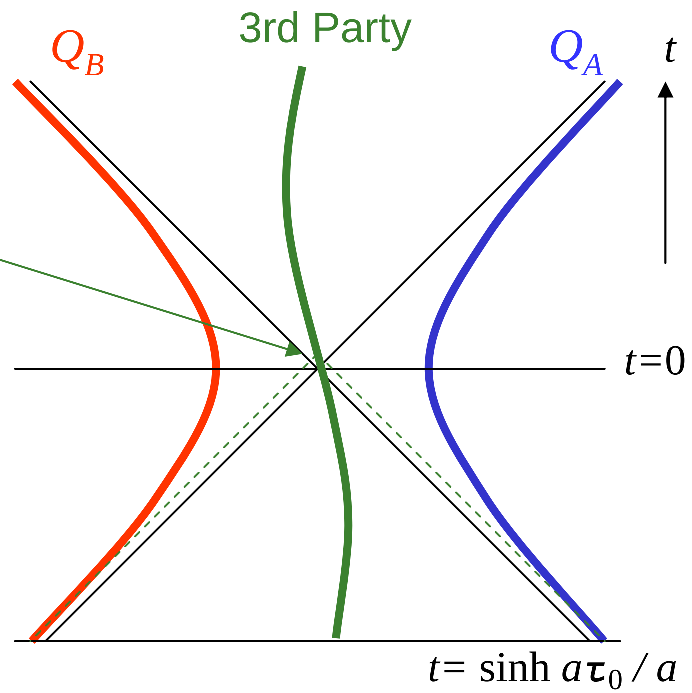


With measurement (on spontaneously emitted photons),
entanglement between two atoms can be created
after the third party have causal contact with both atoms,
while the atoms could be outside the light cones
of each other then.

II. The Model

Conjecture

Entanglement is created after t = (exp $a\tau_0$)/a.



The entanglement creation could not happen any earlier than the earliest possible moment that the third party, which is also local-in-space, is able to have causal contact with both quantum objects.

III. Concluding Remarks

III. Concluding Remark (1)

- RDM truncated up to the 1st excited state contains complete information about the separability of two detectors.

$$\rho^{R}_{n_A n_B, n'_A n'_B} \approx \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{cc} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} & \\ \begin{pmatrix} g & 0 & 0 & gK^2 \mathcal{J}^{A'B'} \\ 0 & gK^2 \mathcal{J}^{BB'} & gK^2 \mathcal{J}^{A'B} & 0 \\ 0 & gK^2 \mathcal{J}^{AB'} & gK^2 \mathcal{J}^{AA'} & 0 \\ gK^2 \mathcal{J}^{AB} & 0 & 0 & gK^4 \left[\mathcal{J}^{AB} \mathcal{J}^{A'B'} + \mathcal{J}^{AA'} \mathcal{J}^{BB'} + \mathcal{J}^{AB'} \mathcal{J}^{A'B} \right] \end{pmatrix} & \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \end{array}$$

III. Concluding Remark (1)

- RDM truncated up to the 1st excited state contains <u>complete</u> information about the separability of two detectors.

$$\rho^{R\ \mathbf{PT}}_{n_A n_B, n'_A n'_B} \approx \begin{array}{cc} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} & \\ \left(\begin{array}{cccc} g & 0 & 0 & gK^2 \mathcal{J}^{A'B'} \\ 0 & gK^2 \mathcal{J}^{BB'} & gK^2 \mathcal{J}^{A'B} & 0 \\ 0 & gK^2 \mathcal{J}^{AB'} & gK^2 \mathcal{J}^{AA'} & 0 \\ gK^2 \mathcal{J}^{AB} & 0 & 0 & gK^4 \left[\mathcal{J}^{AB} \mathcal{J}^{A'B'} + \mathcal{J}^{AA'} \mathcal{J}^{BB'} + \mathcal{J}^{AB'} \mathcal{J}^{A'B} \right] \end{array} \right) & \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \end{array}$$

Partially Transposed $\rho^{R}_{n_A n_B, n'_A n'_B}$ has a negative eigenvalue if

$$0 > \mathcal{J}^{AA'} \mathcal{J}^{BB'} - \mathcal{J}^{AB} \mathcal{J}^{A'B'} = \frac{\Sigma \Omega^2 \det(V + V_0)}{16 \hbar^{10} \left[\langle Q_A^2 \rangle \langle Q_B^2 \rangle - \langle Q_A, Q_B \rangle^2 \right]^2}$$

$$\text{or} \quad 0 > \rho^R_{00,00} \rho^R_{11,11} - \rho^R_{10,01} \rho^R_{01,10}$$

Σ < 0 iff the two detectors are entangled !!

III. Concluding Remark (2)

- A replacement of Wightman function in textbook actually changes the interpretation of the cutoff.

$$\rho^R_{10,10} \approx \frac{\lambda_0^2}{2\hbar\Omega}\int_{\tau_0}^{\tau} ds \int_{\tau_0}^{\tau} ds' e^{-i\Omega(s-s')} D^+\left(z_A^\mu(s), z_A^\nu(s')\right)$$

Positive frequency Wightman function

$$\begin{aligned} D^+(z_j^\mu, z_{j'}^\nu) &\equiv \langle\, 0_M \,|\, \phi\left(z_j^\mu\right)\phi\left(z_{j'}^\nu\right) |\, 0_M \,\rangle \\ &= \int \frac{\hbar d^3k}{(2\pi)^3 2\omega} e^{-\omega\epsilon} e^{-i\omega(z_j^0 - z_{j'}^0) + i\mathbf{k}\cdot(\mathbf{z}_j - \mathbf{z}_{j'})} \\ &= \frac{\hbar/4\pi^2}{|\mathbf{z}_j - \mathbf{z}_{j'}|^2 - \left(z_j^0 - z_{j'}^0 - i\epsilon\right)^2} \end{aligned}$$

$$z_A^\mu = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$$

$$D^+(z_A^\mu(s), z_A^\nu(s')) = \frac{\hbar/4\pi^2}{-\frac{4}{a^2}\sinh\frac{a}{2}\Delta\left(\sinh\frac{a}{2}\Delta - i\epsilon a\cosh aT\right) + \epsilon^2}$$

$$T \equiv (s+s')/2$$
$$\Delta \equiv s - s'$$

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- A replacement of Wightman function in textbook actually changes the interpretation of the cutoff.

$$\rho^R_{10,10} \approx \frac{\lambda_0^2}{2\hbar\Omega}\int_{\tau_0}^{\tau} ds \int_{\tau_0}^{\tau} ds' e^{-i\Omega(s-s')} D^+\left(z_A^\mu(s), z_A^\nu(s')\right)$$

$$D^+(z_A^\mu(s), z_A^\nu(s')) = \frac{\hbar/4\pi^2}{-\frac{4}{a^2}\sinh\frac{a}{2}\Delta\left(\sinh\frac{a}{2}\Delta - i\epsilon a\cosh aT\right) + \epsilon^2}$$

$$T \equiv (s+s')/2$$
$$\Delta \equiv s - s'$$

In textbook this is replaced by (since ϵ is extremely small)

$$D'^+(z_A^\mu(s), z_A^\nu(s')) = \frac{\hbar/4\pi^2}{-\frac{4}{a^2}\sinh^2\frac{a}{2}(\Delta - i\epsilon')}$$

Alternatively, one can use the original D^+ but put the time-resolution of the detector (physical cutoffs)

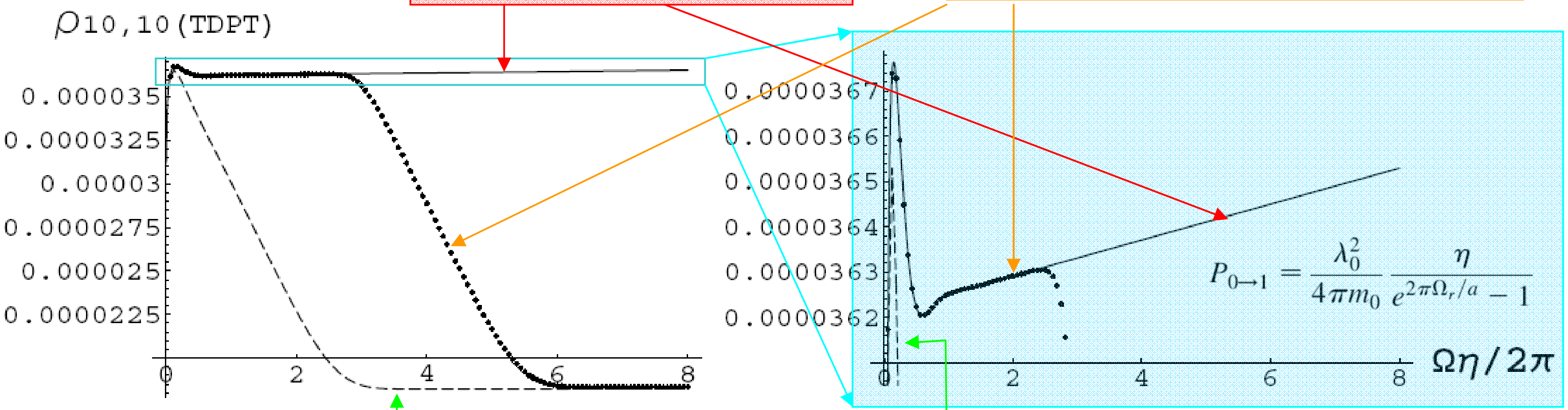
$$\rho^R_{10,10} \approx \frac{\lambda_0^2}{2\hbar\Omega}\int_{\tau_0}^{\tau+\epsilon_1} ds \int_{\tau_0+\epsilon_0}^{\tau} ds' e^{-i\Omega(s-s')} D^+\left(z_A^\mu(s), z_A^\nu(s')\right)$$

III. Concluding Remark (2)

- A replacement of Wightman function in textbook actually changes the interpretation of the cutoff.

modified D'^{+} with $\epsilon' = \epsilon$
$\epsilon_0, \epsilon_1 = 0$

original D^{+} with a new, positive
$\epsilon \ll \epsilon'$, and $\epsilon_0 = \epsilon_1 = \epsilon'$

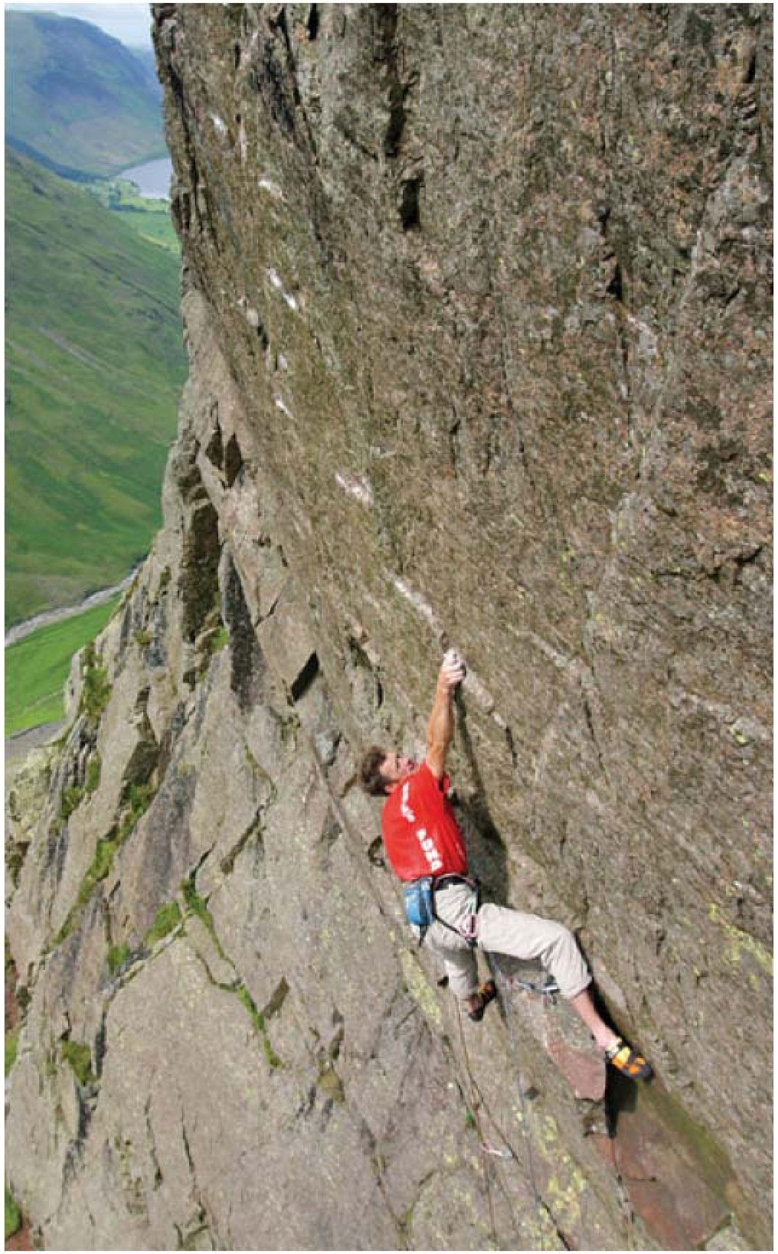


Replacing D^{+} by D'^{+} is equivalent to taking ϵ to 0 and setting $\epsilon_0 = \epsilon_1 = \epsilon'$

original D^{+} with ϵ
$\epsilon_0, \epsilon_1 = 0$

exotic behavior?!

So ϵ' is physical but ϵ is not.



Thank you!