

**Environment driven**  
**Superconductor-Insulator phase transition**  
**in One-Dimensional Josephson-Junction Arrays**

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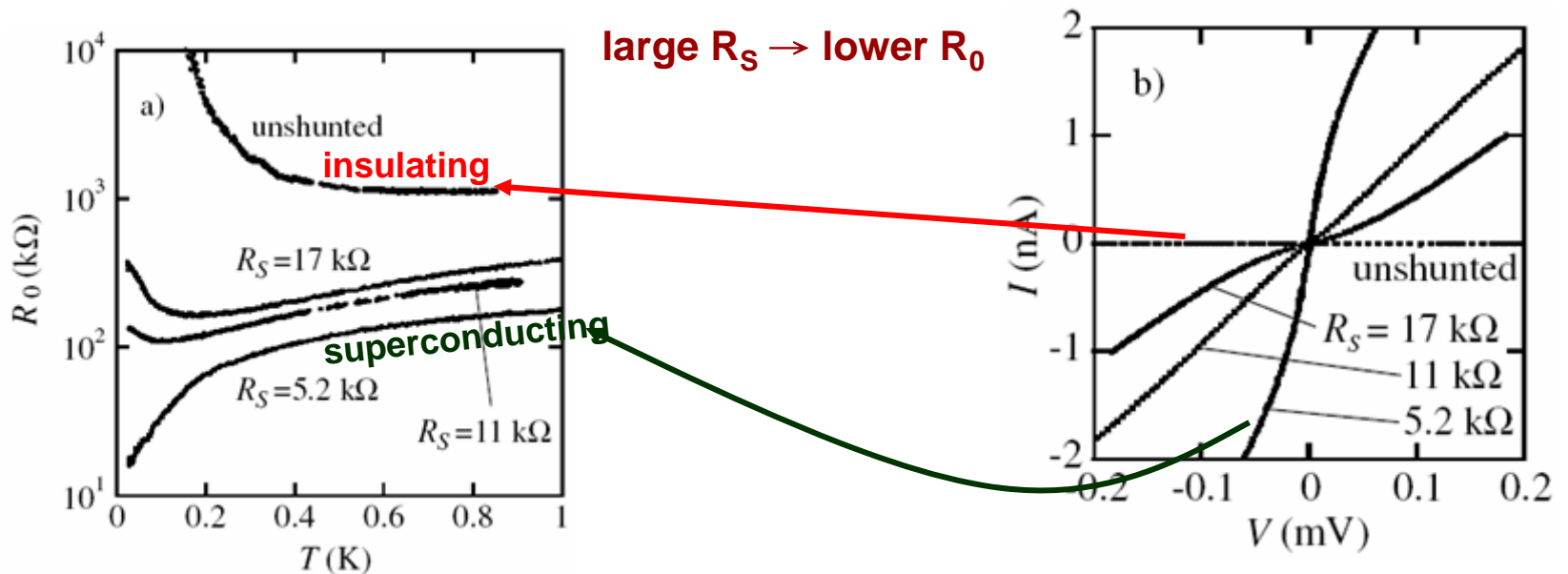
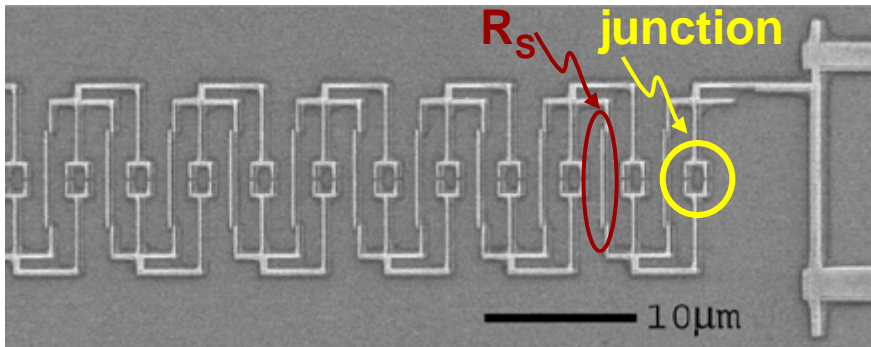
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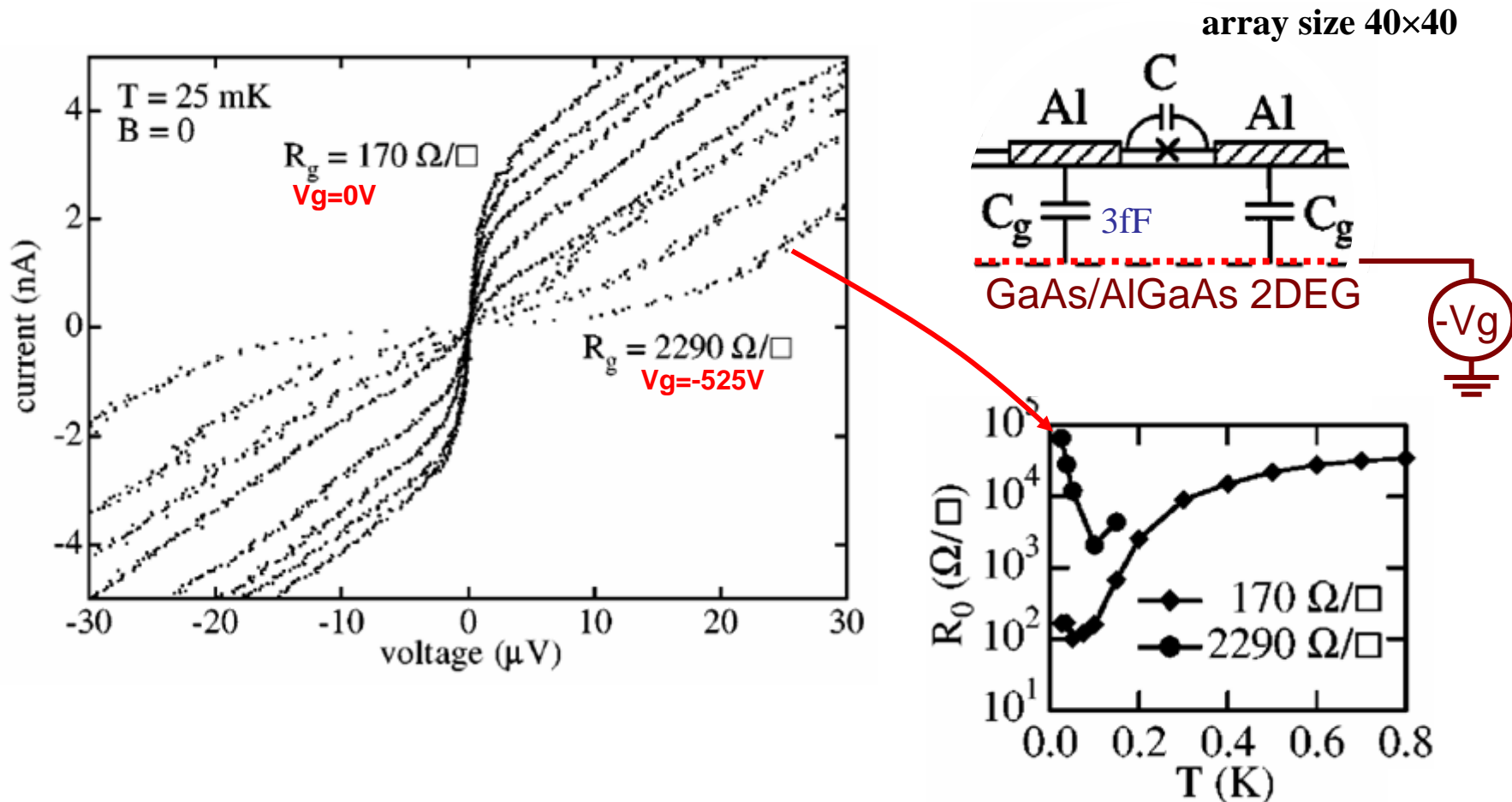
# Quantum Phase Transition in One-Dimensional Arrays of Resistively Shunted Small Josephson Junctions

Hisao Miyazaki, Yamaguchi Takahide, Akinobu Kanda, and Youiti Ootuka, PRL, 89, 197001 (02)



# Dissipation-Driven Superconductor-Insulator Transition in a Two-Dimensional Josephson-Junction Array

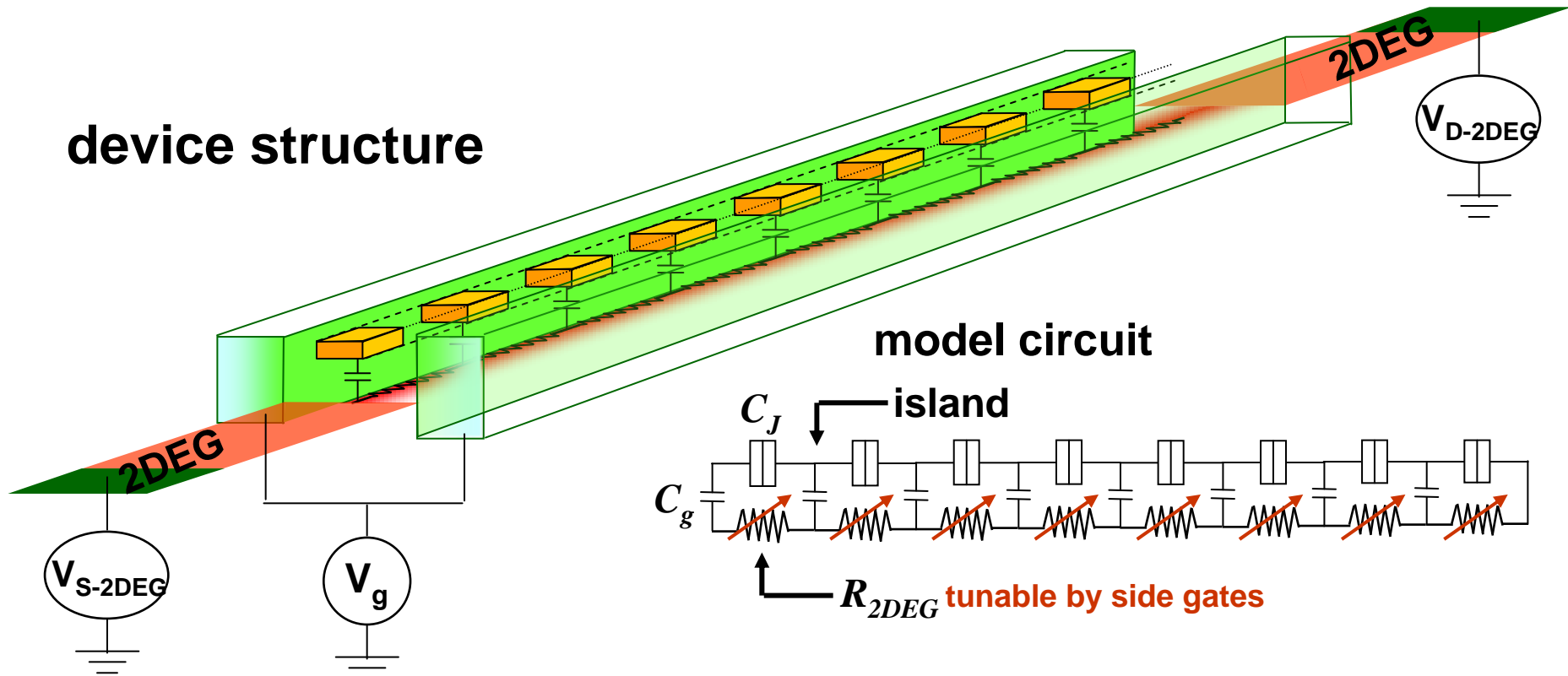
A. J. Rimberg, T. R. Ho, C. Kurdak, and John Clarke, PRL 78, 2632 (97)



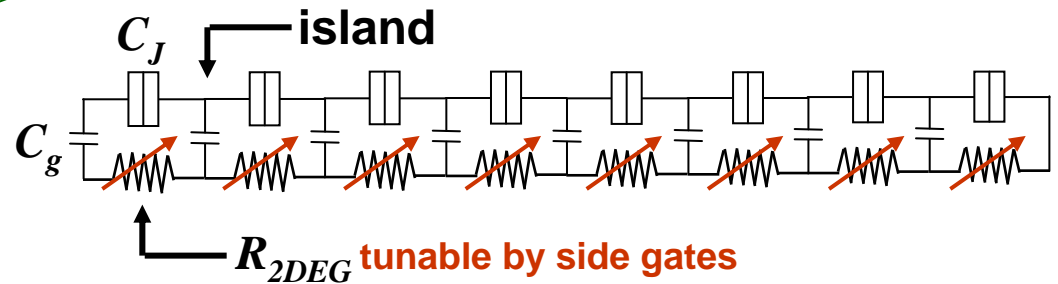


# 1D Josephson junction array with tunable environment

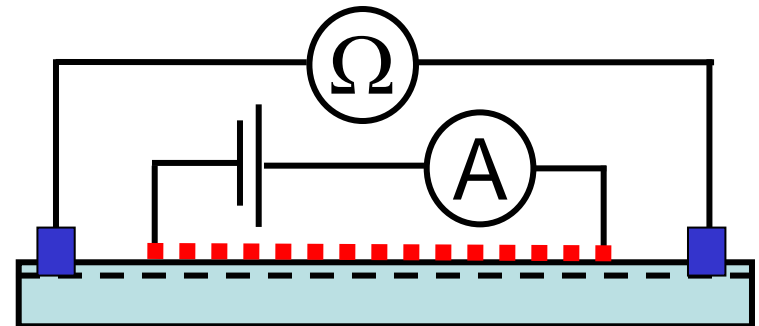
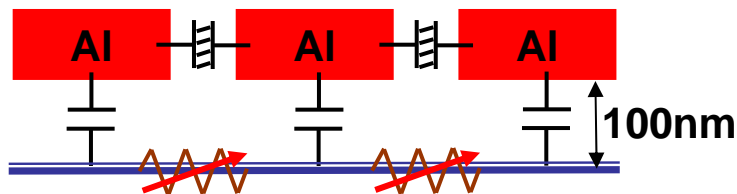
device structure



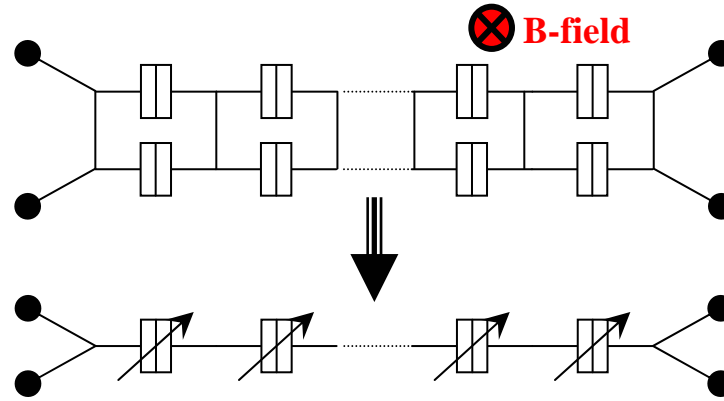
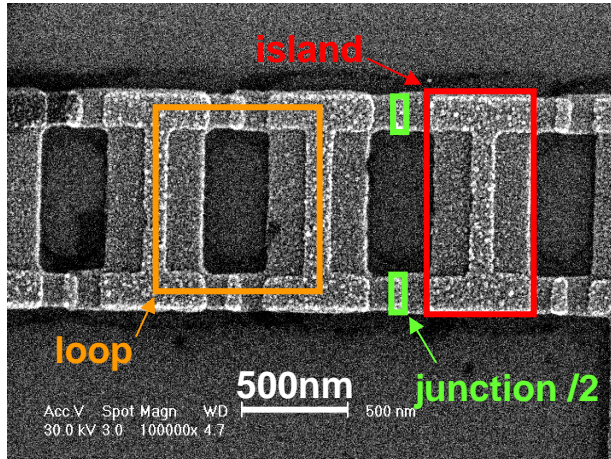
model circuit



measurement circuit



# 1D-Josephson Junction Array with tunable coupling strength



Two relevant energy scales:

Charging energy  
for single electrons

$$E_{CP} = \frac{4e^2}{2(2C)}$$

$$= 212 \mu\text{eV}$$

$$2C = 1.5\text{fF}$$

with  $C_S = 45\text{fF}/\mu\text{m}^2$

Josephson coupling energy

$$E_J^0 = \frac{\Delta}{2} \frac{R_Q}{R_N}$$

$$= 96.3 \mu\text{eV (A1)} \text{ and } 81.3 \mu\text{eV (A2)}$$

$$R_N \text{ (6.75k}\Omega \text{ for A1 and 8.0k}\Omega \text{ for A2), } \Delta = 200\mu\text{eV}$$

modulated coupling energy

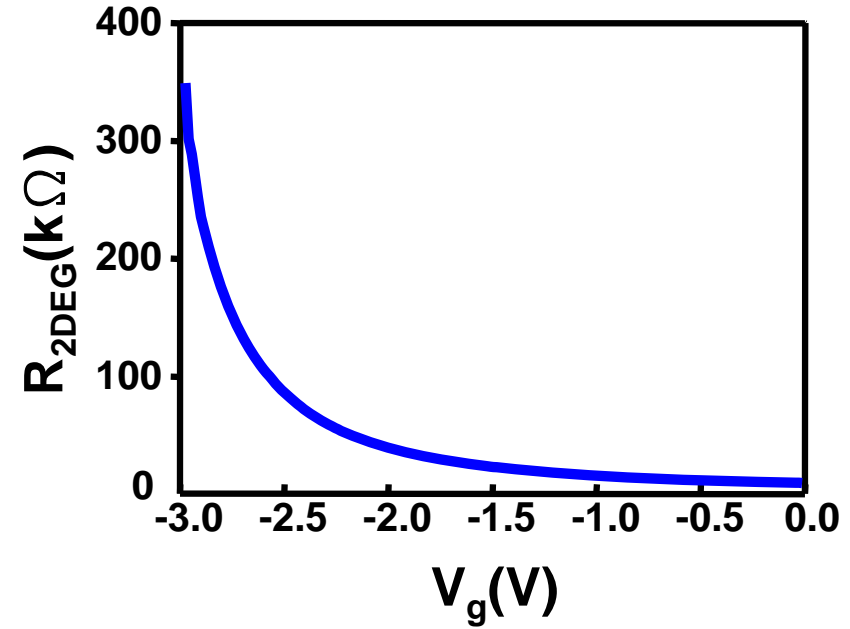
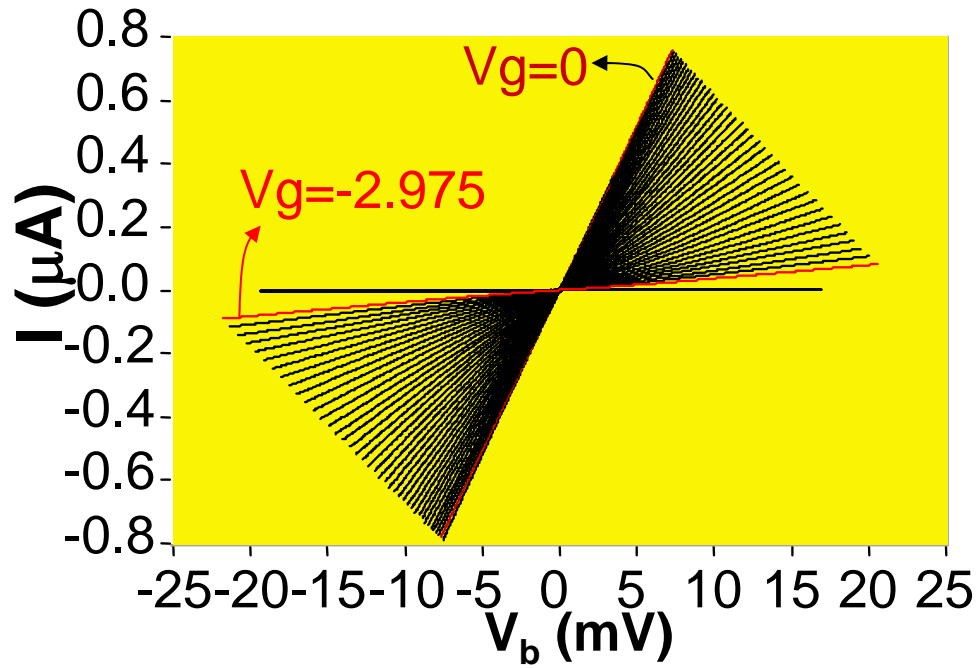
$$E_J = E_J^0 \cos(\pi f)$$

$$f = AB/\Phi_0$$

$$f = B/42.5\text{Gs}$$

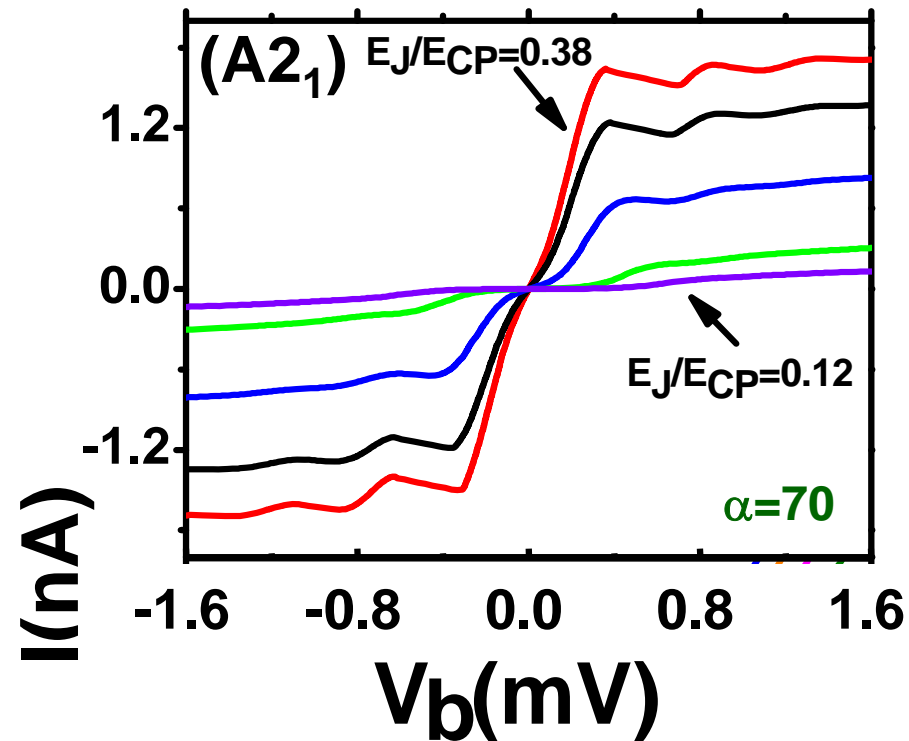
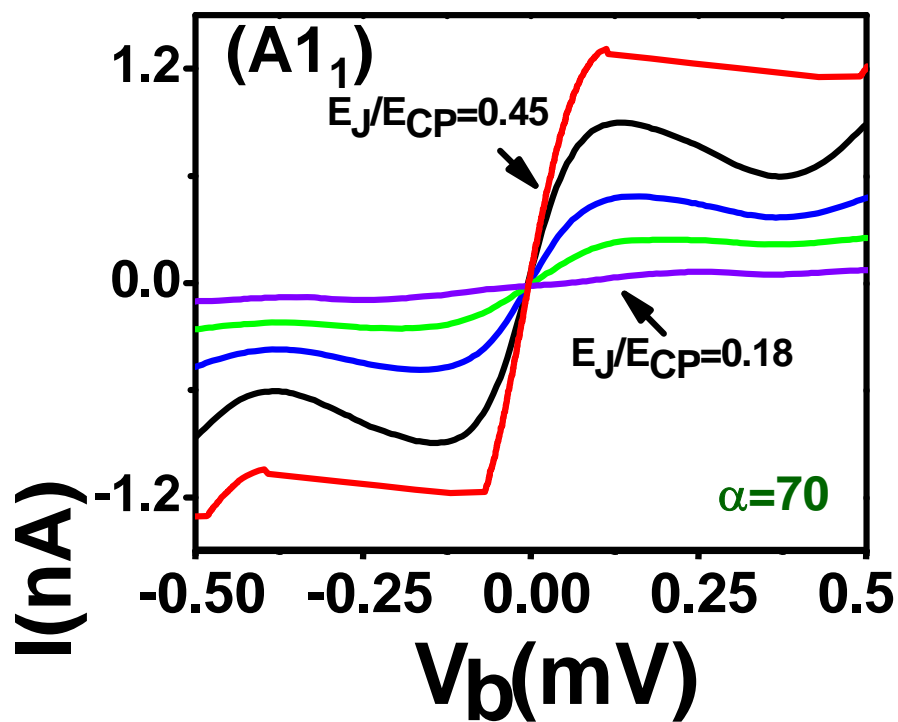
$$\alpha \equiv R_Q/R_{2DEG}$$

# Characteristics of the 2 Dimensional Electron Gas layer



$$\alpha \equiv R_Q / R_{2\text{DEG}}$$

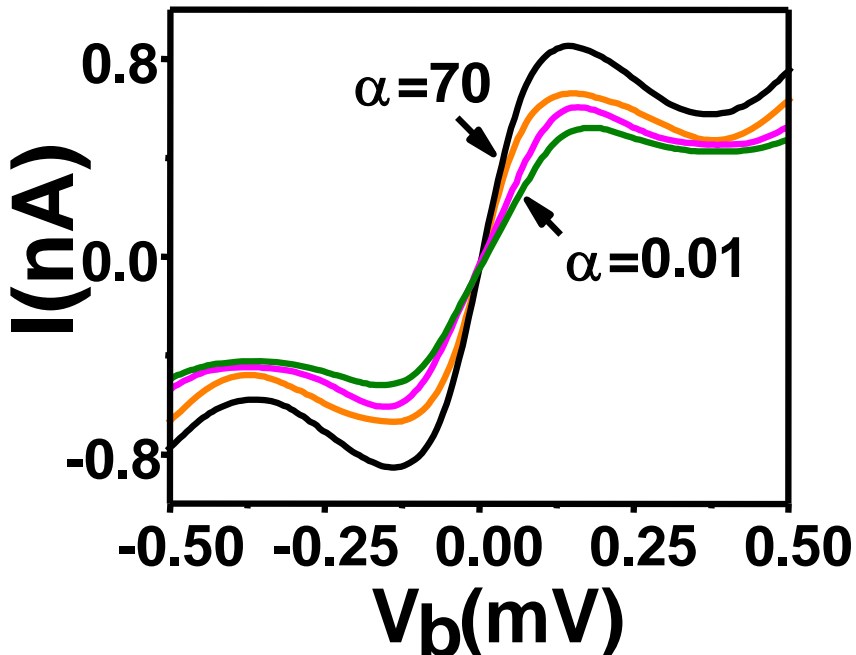
# $E_J/E_{CP}$ and $\alpha$ dependence of $IV_b$ curves



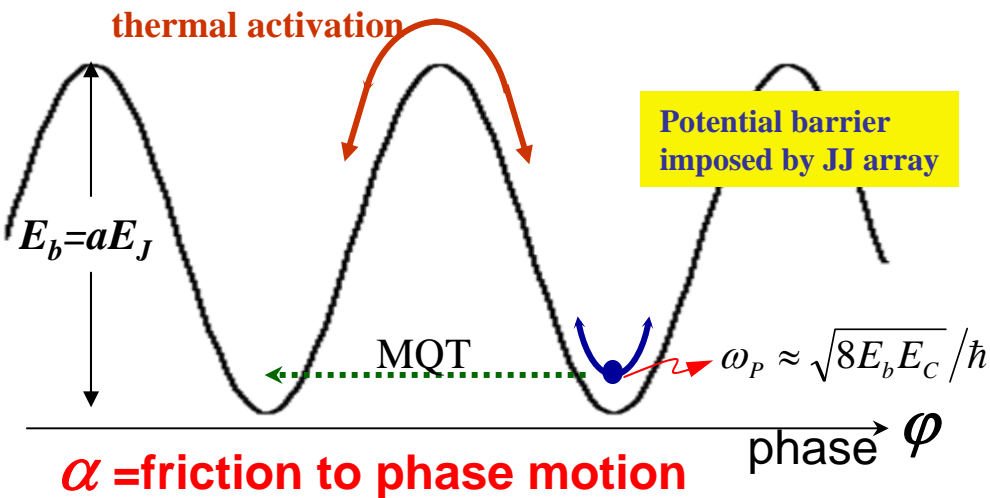
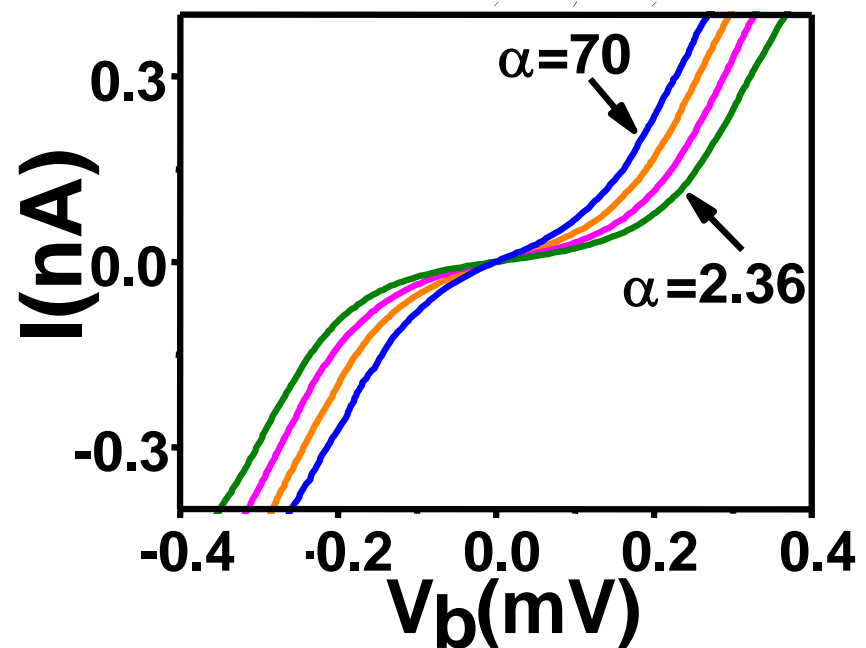


# Effect of $\alpha$ in phase- and charge-order regimes

phase order regime

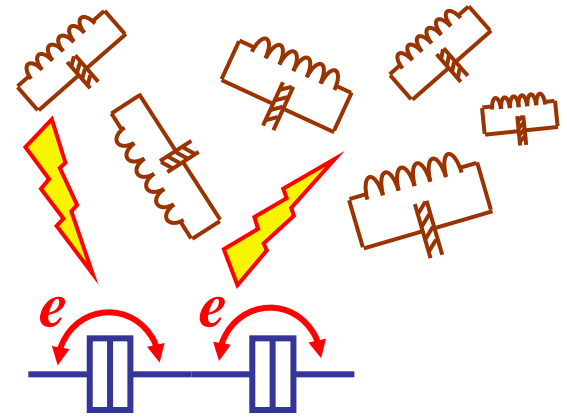


charge order regime



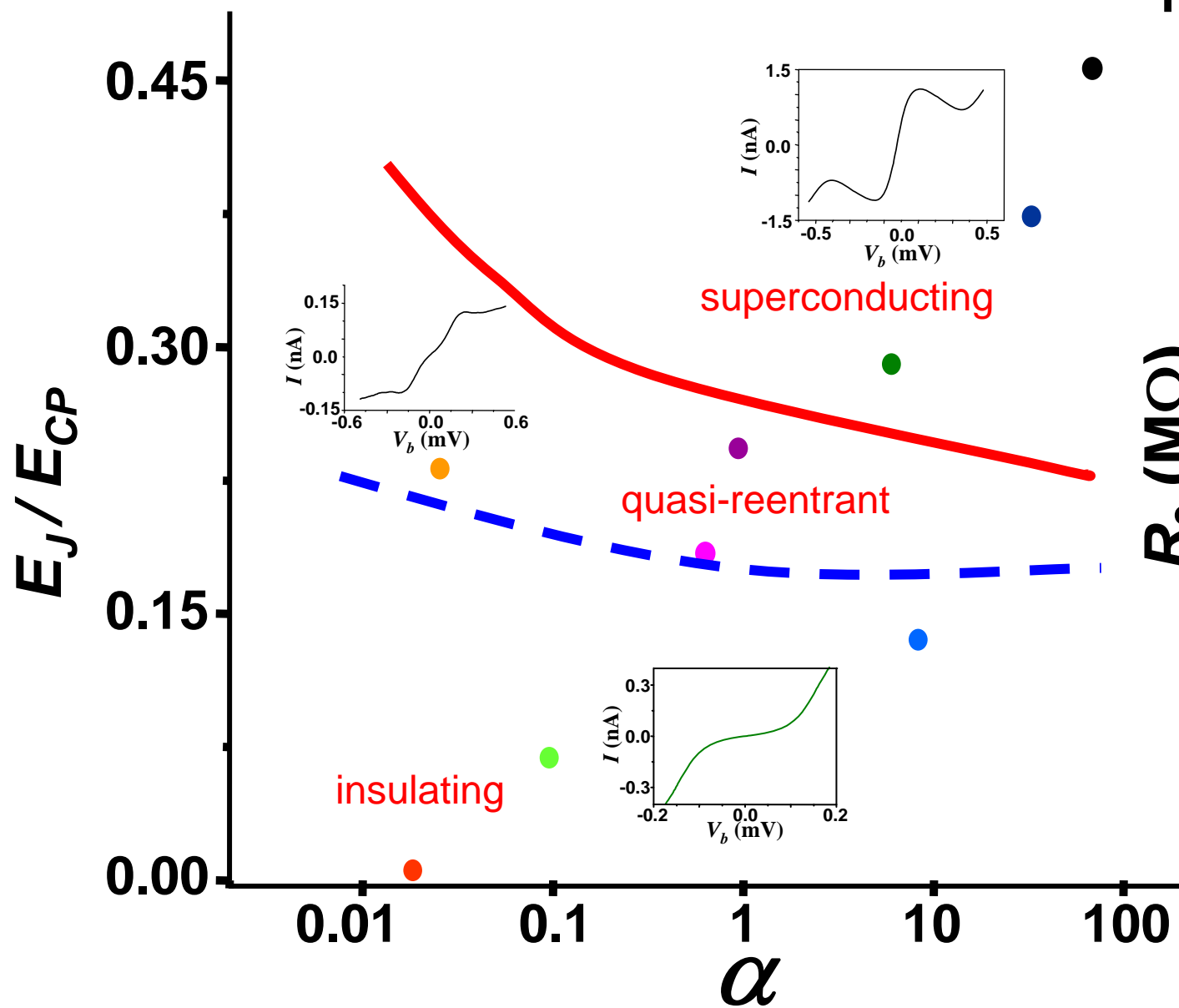
$\alpha$  = friction to phase motion

LC harmonic oscillators

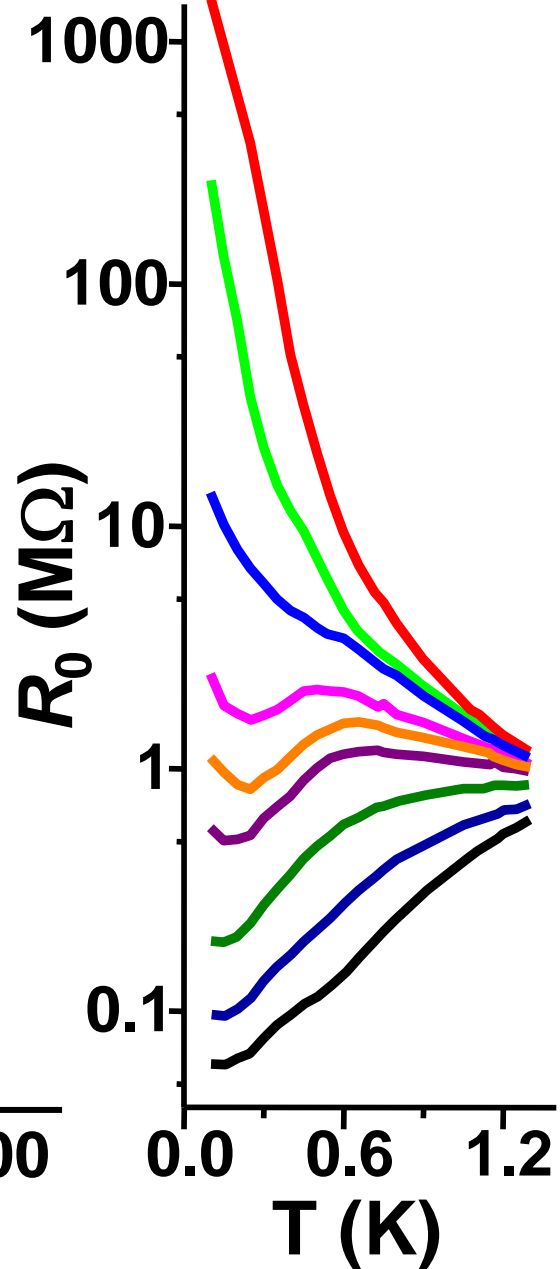


$\alpha$  = promote charge tunneling

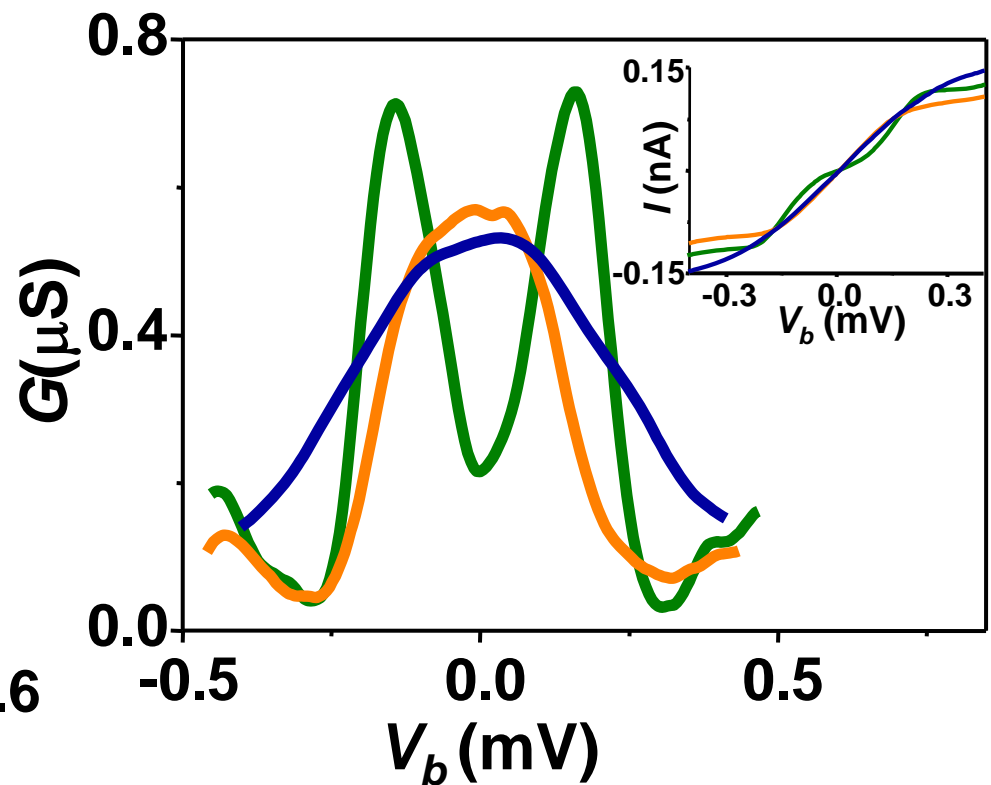
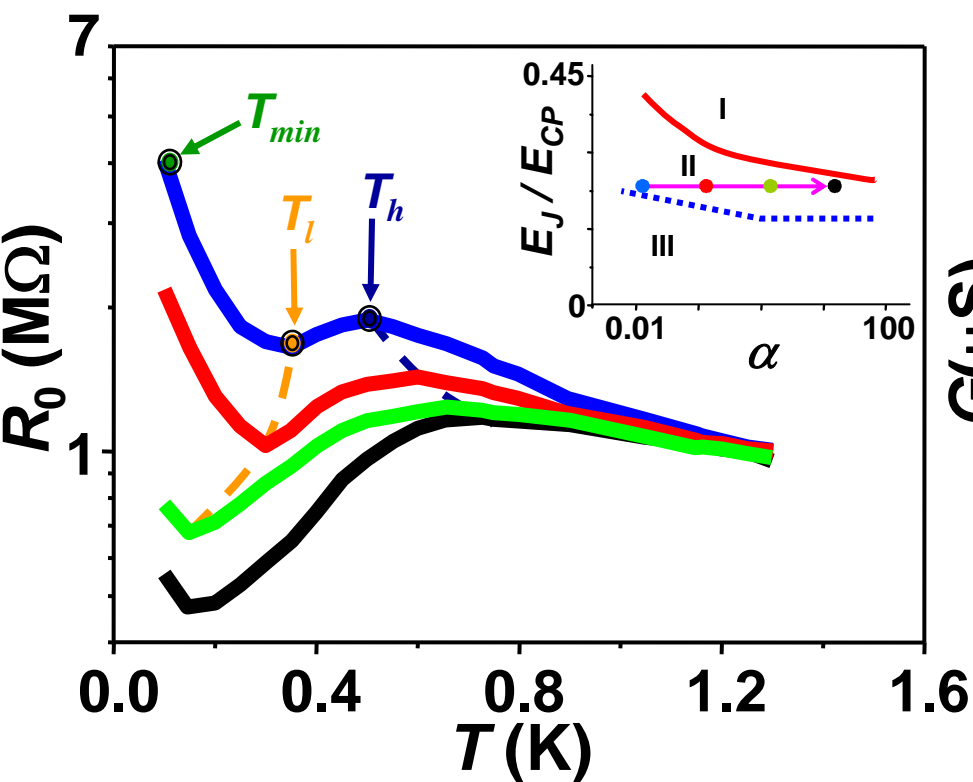
# Low temperature phase diagram



$R_0(T)$  plot



# Effect of $\alpha$ on quasi-reentrance behavior



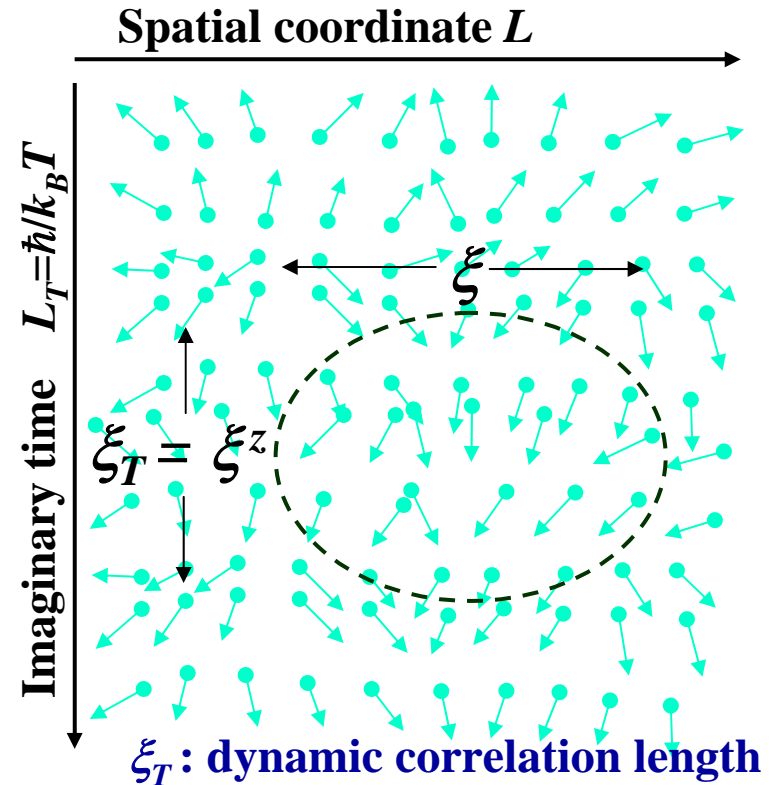
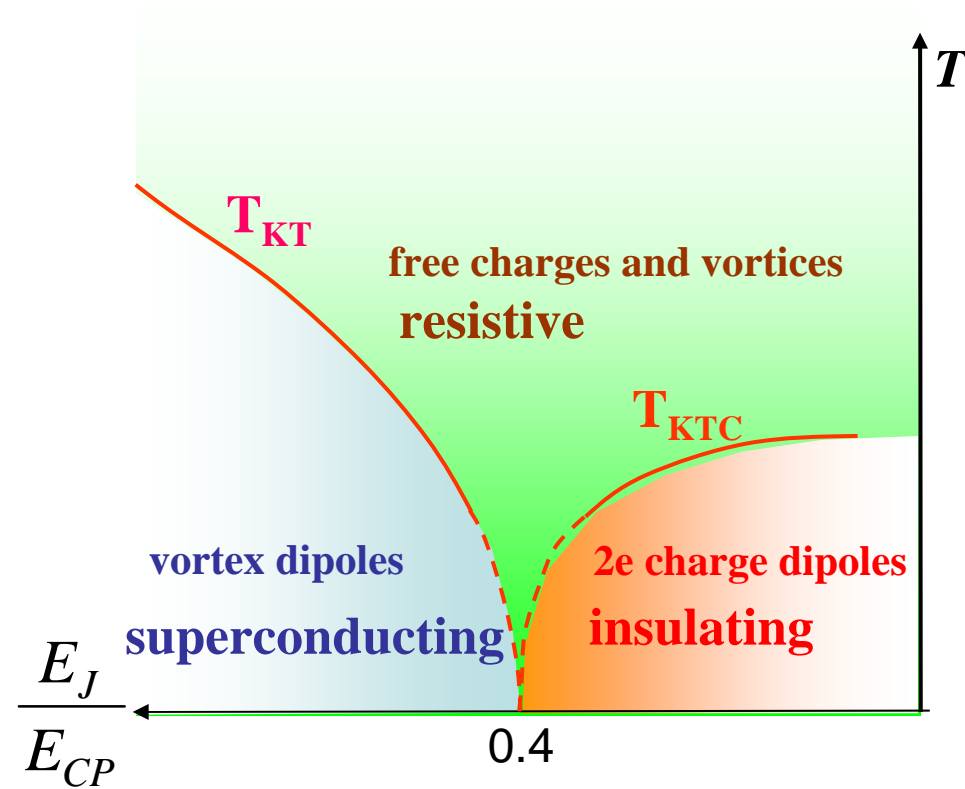
# Phase diagram { **no electromagnetic environment** varying temperature

$$L_{JJA} = \frac{1}{2} \sum_{ij} Q_i (\hat{C}^{-1})_{ij} Q_j + \sum_{\langle i,j \rangle} E_J [1 - \cos(\varphi_i - \varphi_j)]$$

charging energy

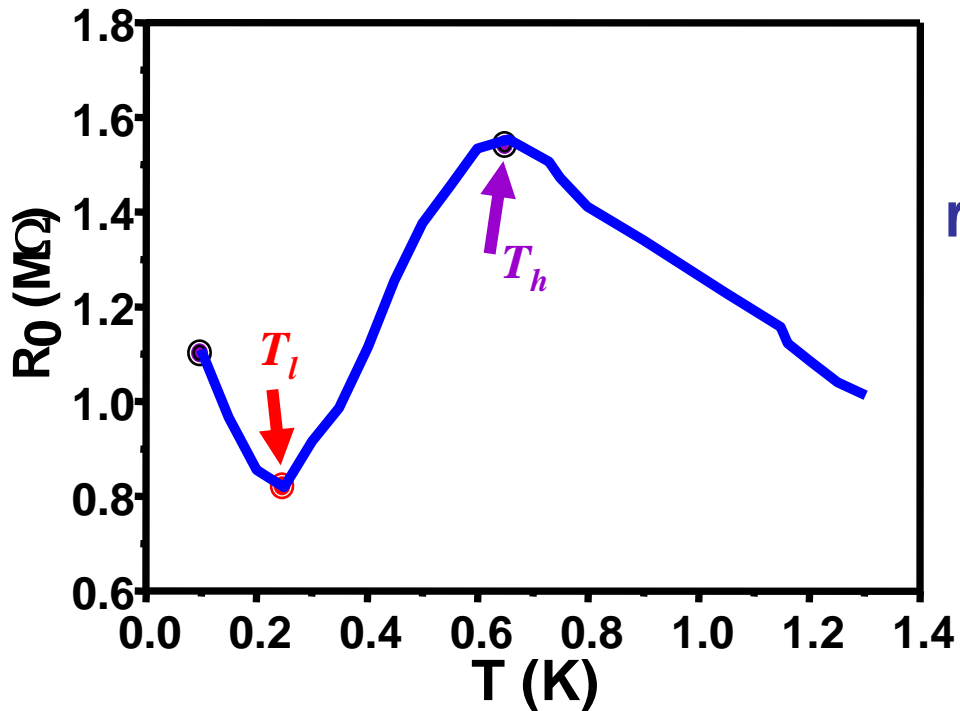
Josephson coupling energy

S. Chakravarty, G. L. Ingold, S. Kivelson, and A. Luther,  
Phys. Rev. Lett. **56**, 2303 (1986)



# Quasi-reentrance behavior:

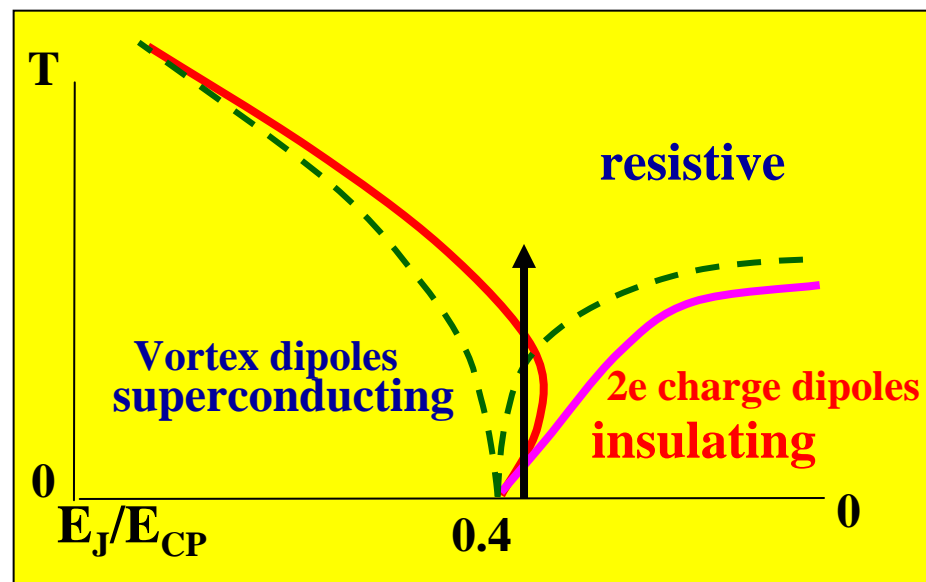
no environment



renormalization and dissipation  
from junction  $R_J$

Capacitance renormalization

$$C^* \rightarrow C + \delta C \quad \delta C = \frac{3\pi\hbar}{32\Delta R_J}$$



# In the presence of an electromagnetic environment

$$L_{total} = \underbrace{\frac{1}{2} \sum_{ij} Q_i (\hat{C}^{-1})_{ij} Q_j + \sum_{\langle i,j \rangle} E_J [1 - \cos(\varphi_i - \varphi_j)]}_{1D \text{ Junction array}} + \underbrace{\frac{1}{2} \sum_n (m_n \dot{x}_n^2 - m_n \omega_n^2 x_n^2)}_{2DEG} - \underbrace{\sum_{in} F_{in}(Q_i, \varphi_i, x_n, \lambda_{in})}_{array-2DEG \text{ interaction}}$$

**2DEG → harmonic oscillators**

resonant frequencies  $\omega_n = 2\pi n k_B T$  Matsubara frequencies

$\lambda_{in}$  = the coupling strength  $\begin{cases} \text{superconducting island } i \\ \text{environment oscillator } n \end{cases}$

**In phase order regime: ( $E_J > 0.2 E_{CP}$ )**

Ohmic environment →

$$\sum_n \frac{\pi \lambda_{in}^2}{2m_n} \delta(\omega - \omega_n) = R_{2DEG}^{-1}$$

**Superconducting - insulating boundary  $\langle E_J \cos \varphi_{ij} \rangle = 0$**

$$1 - E_J \int_0^{1/k_B T} d\tau \exp \left[ -2k_B T \sum_n \frac{E_{CP}}{\sqrt{1 + \alpha E_{CP}/2\pi\omega_n}} \frac{1 - \cos(\omega_n \tau)}{\omega_n^2} \right] \begin{cases} > 0 \rightarrow \text{superconductor} \\ < 0 \rightarrow \text{insulator} \end{cases}$$

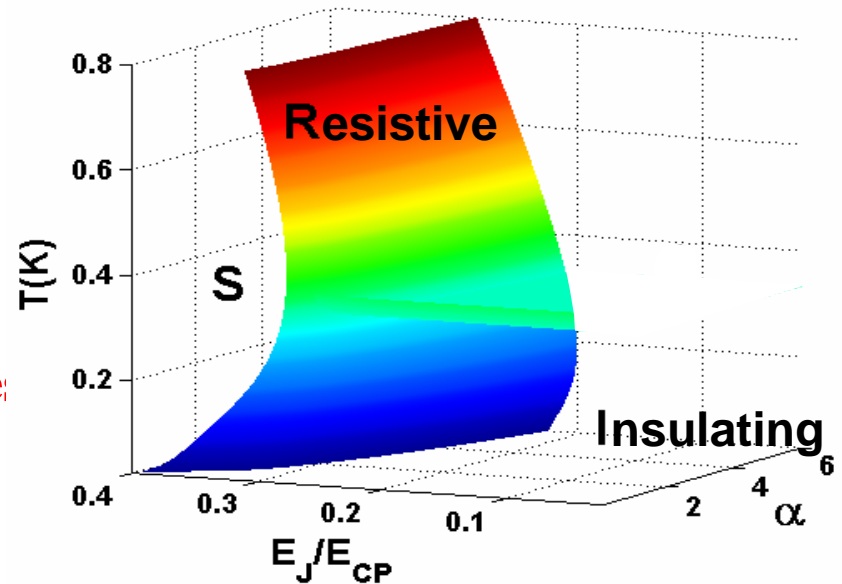
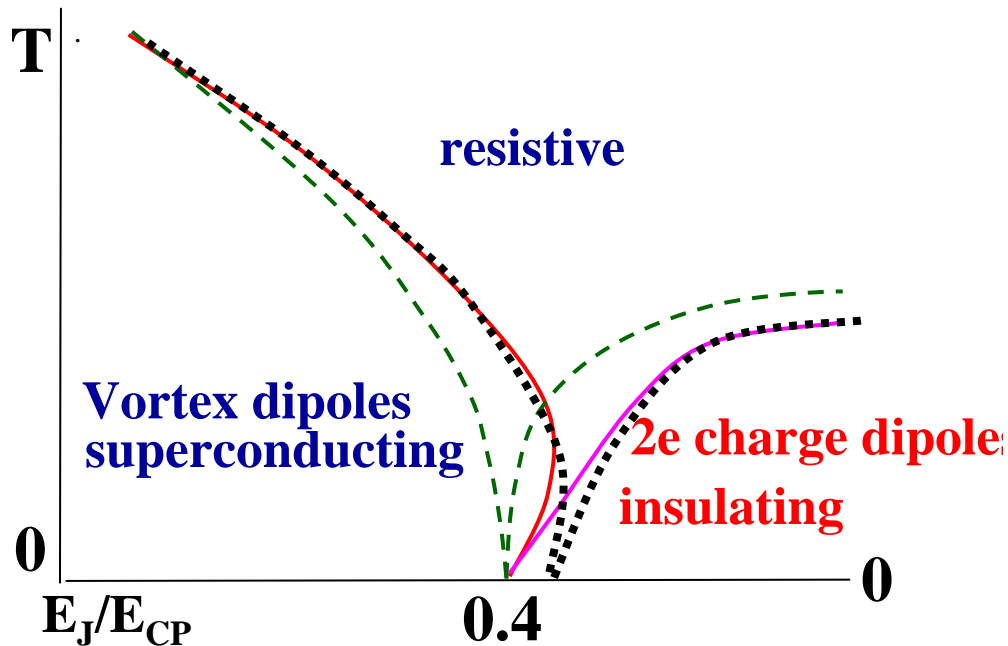
$$1 - E_J \int_0^{1/k_B T} d\tau \exp \left[ -2k_B T \sum_n \frac{E_{CP}}{\sqrt{1 + \alpha E_{CP}/2\pi\omega_n}} \frac{1 - \cos(\omega_n \tau)}{\omega_n^2} \right] > 0 \rightarrow \text{superconductor}$$

$$< 0 \rightarrow \text{insulator}$$

Presence of  $E_{CP}$  ← phase-charge duality

$\alpha \neq 0$  suppresses phase fluctuations → promotes Cooper pair tunneling

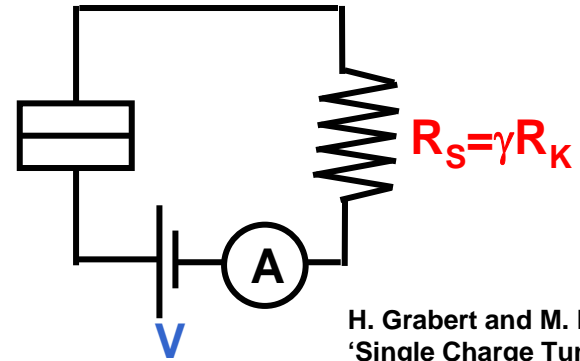
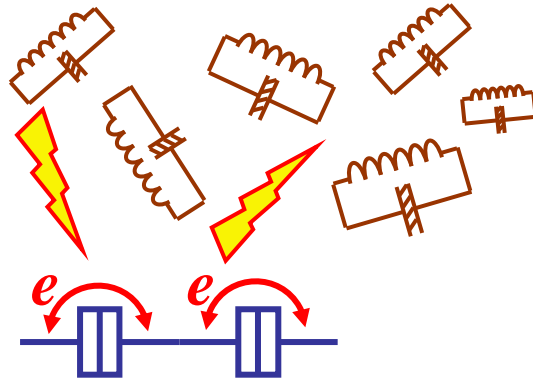
→ an effective reduction of  $E_{CP}$  by a factor of  $\sqrt{1 + \alpha E_{CP}/2\pi\omega_n}$



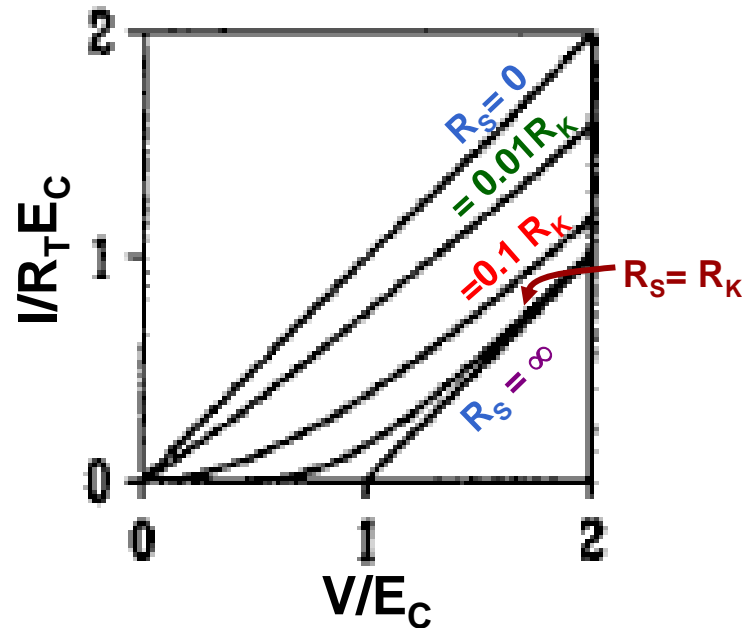
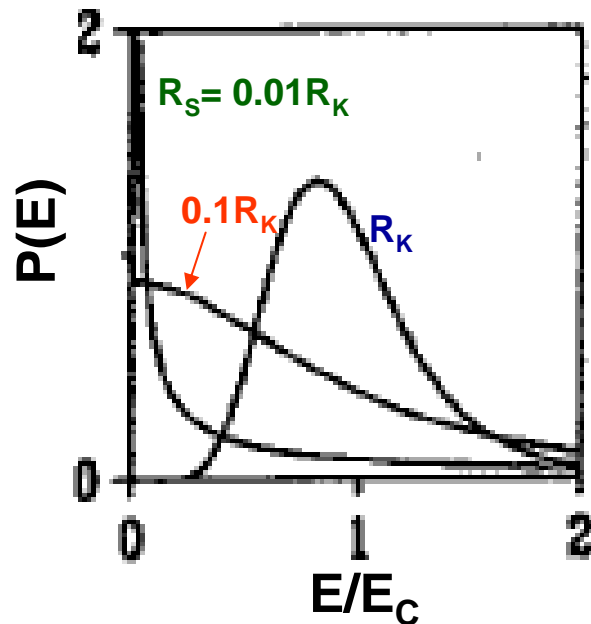
# Phase boundary in charge order regime ( $E_J \ll E_{CP}$ )

Cooper pair tunneling rate:  $\Gamma(E) = \frac{\pi}{2\hbar} E_J^2 P(E)$

$P(E)$  ---- Probability for the tunneling electron to exchange energy  $E$  with the environment.



H. Grabert and M. H. Devoret, 'Single Charge Tunneling', (Plenum Press, New York, 1992)

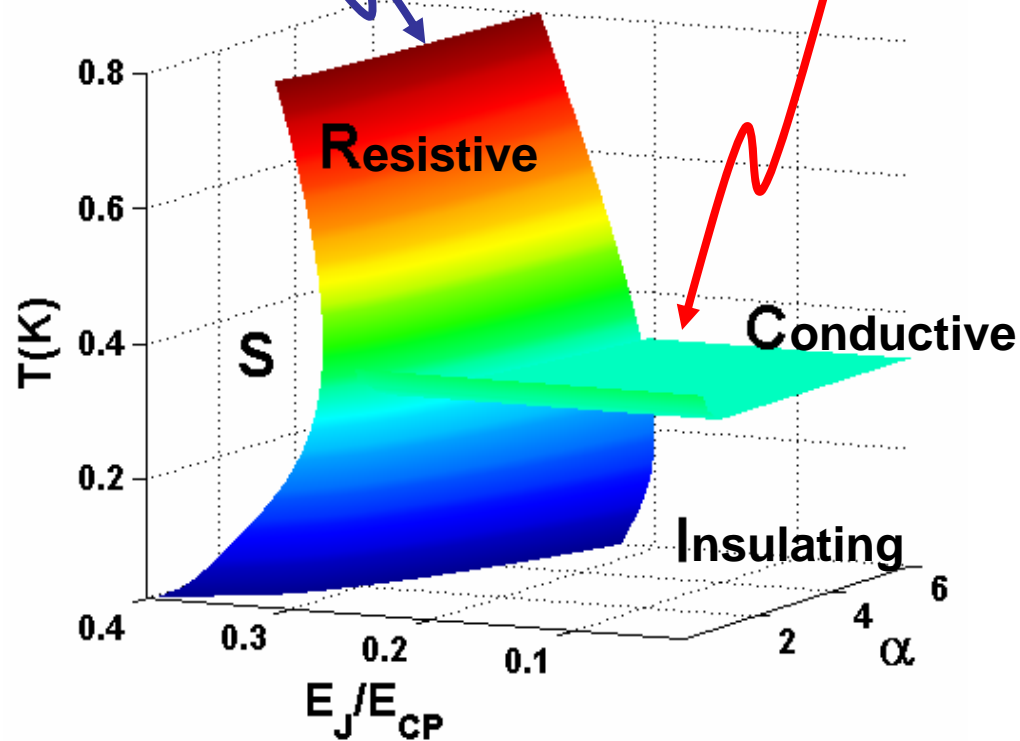




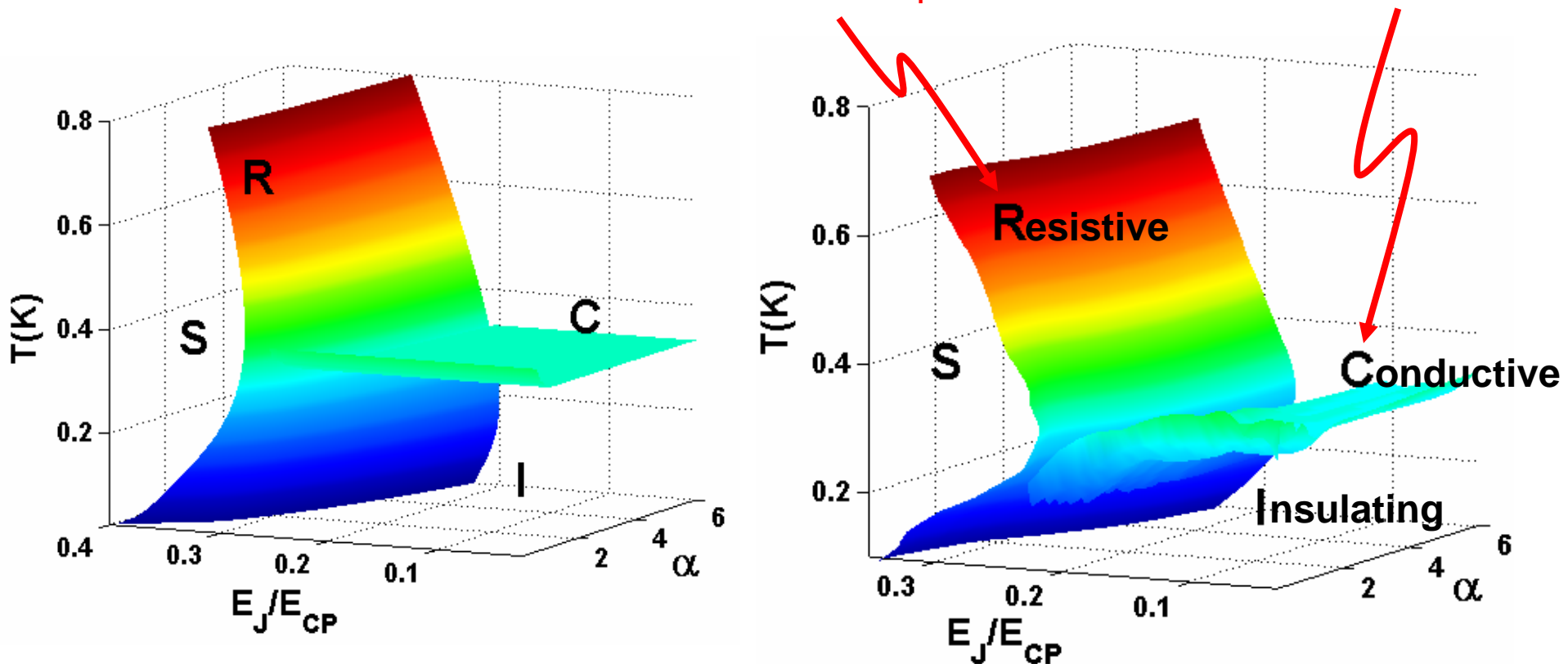
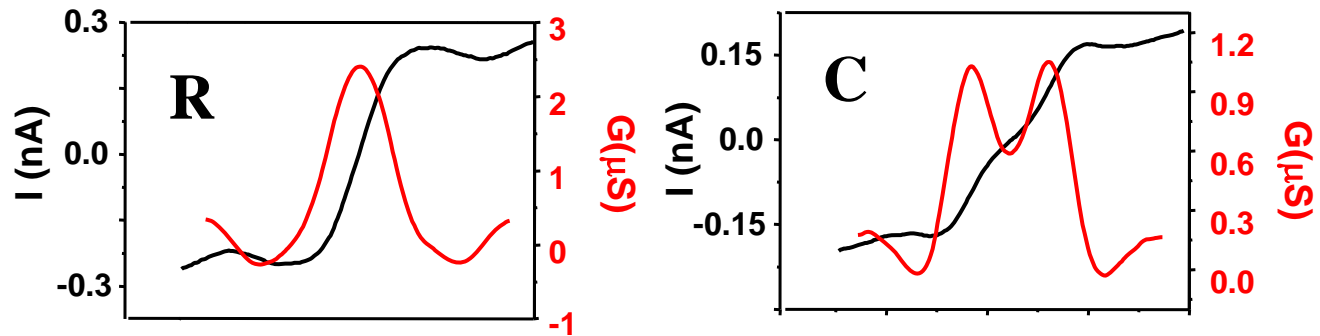
# Calculated phase diagram for both

phase order regime

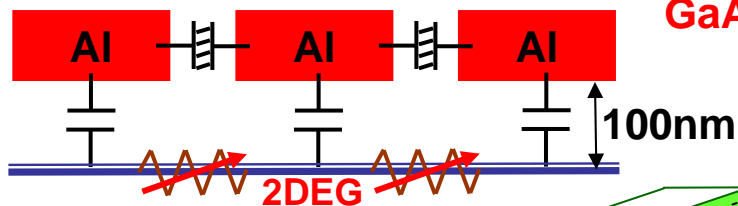
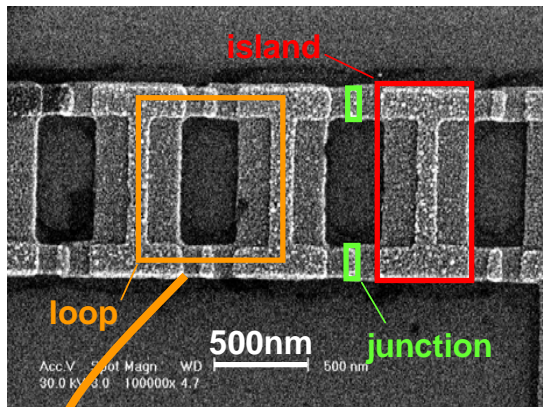
charge order regime



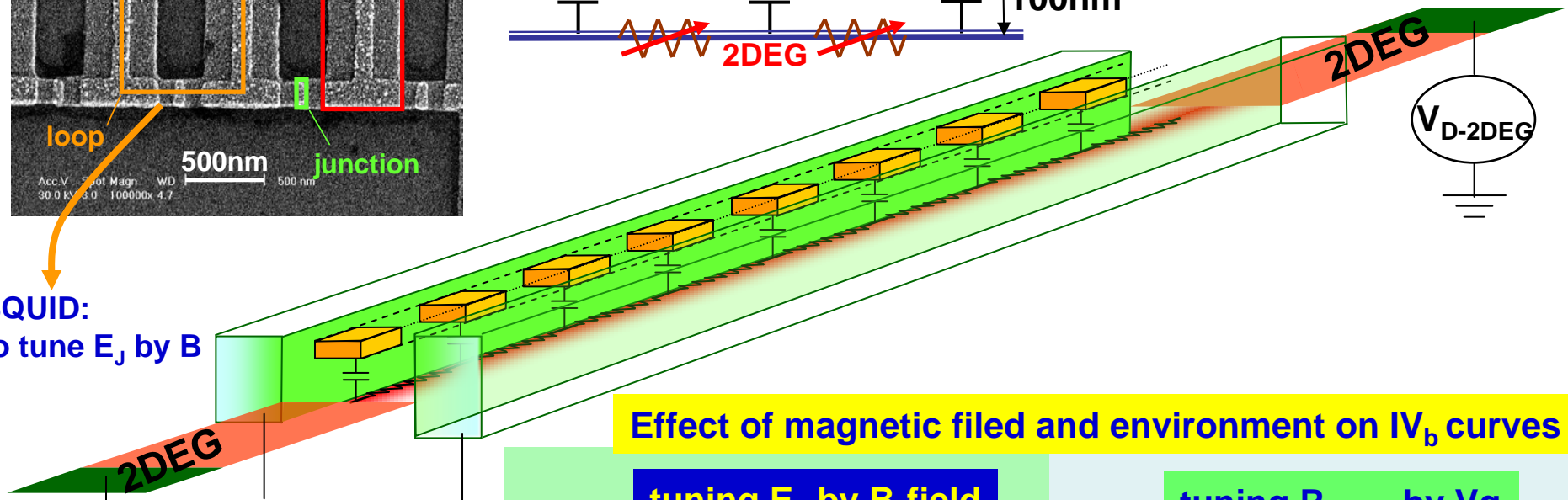
# Comparison between theoretical and experimental phase diagram



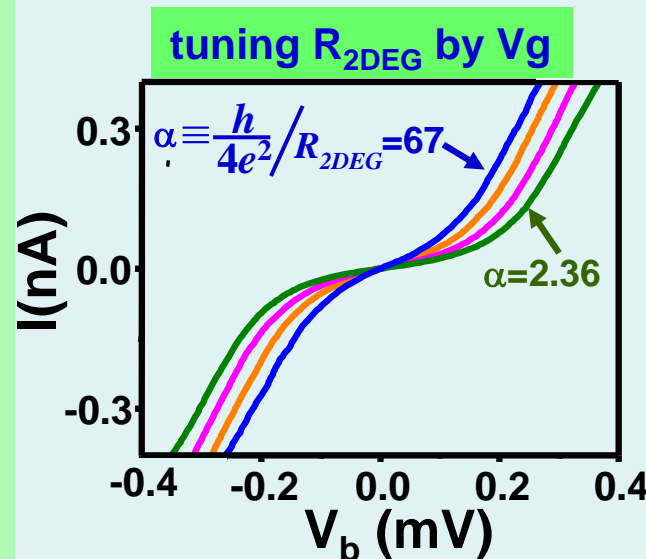
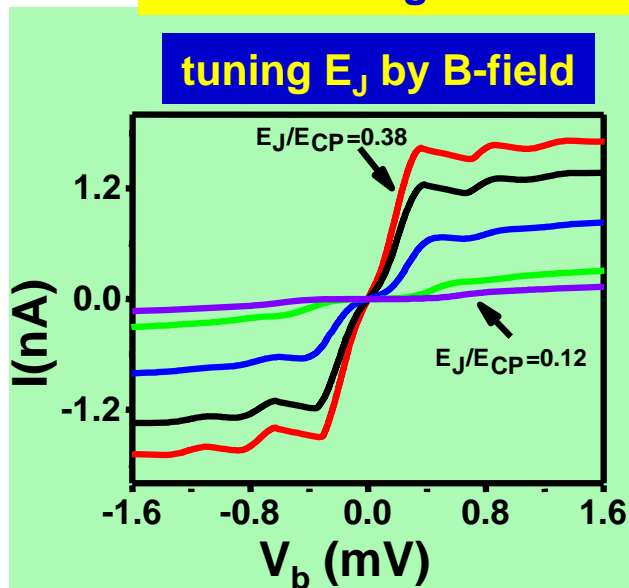
# 1D Josephson junction arrays with a tunable environment



GaAs 2D electron gas (2DEG)



Effect of magnetic field and environment on  $I V_b$  curves

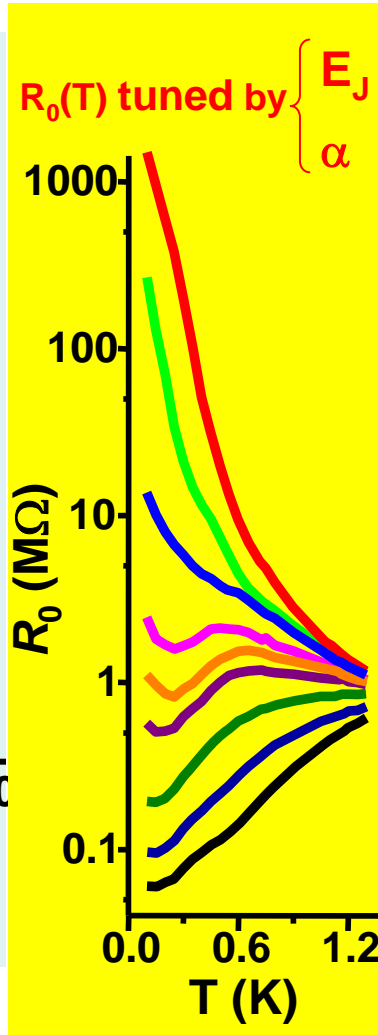
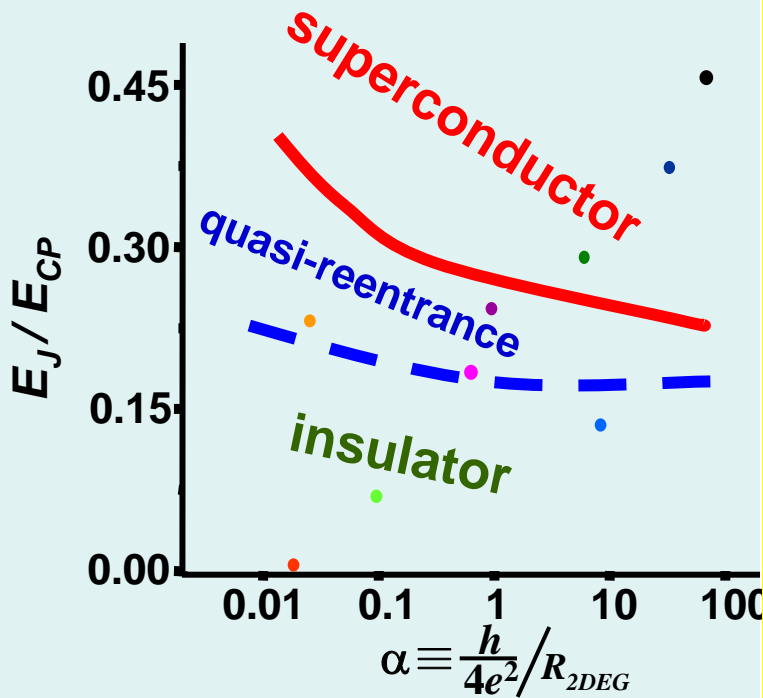


SQUID:  
to tune  $E_J$  by B

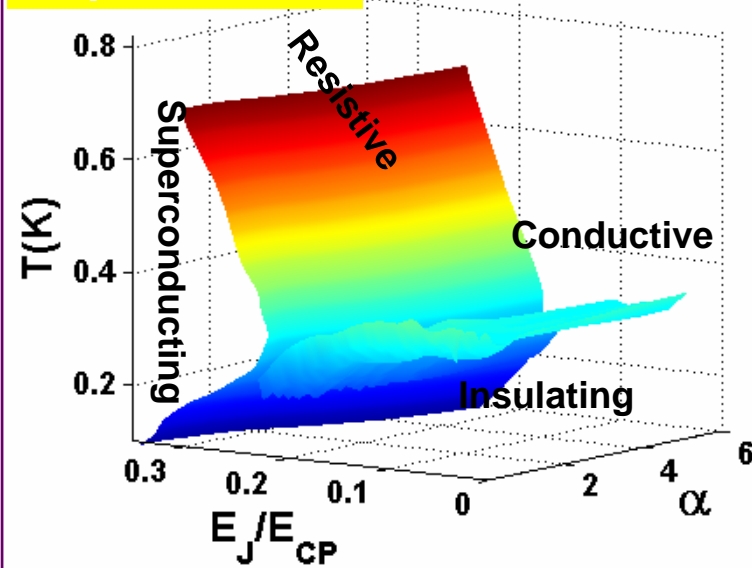
to tune 2DEG resistance

# Phase diagrams

T=0 phase diagram



Experimental:



Theoretical:

