Quantum Dynamics of the Duffing Model for Qubit Readout

Shu-Hao Yeh (葉書豪), Dong-Bang Tsai (蔡東邦),
and Alec Maassen van den Brink (馬尙德)

RCAS, Academia Sinica, Taipei, Taiwan

Background: The Duffing model is an oscillator with weak near-resonant driving, damping, and nonlinearity, which has often been studied in classical mechanics. For certain parameters, the stationary amplitude and phase bifurcate depending on the initial condition, and vary widely between the stable branches. Due to this sensitivity, the system can be used as a detection device. Recently, a superconducting implementation—the *Josephson bifurcation amplifier* (JBA)—has been used for qubit readout, approximately realizing quantum non-demolition measurement (QND) [A. Lupaşcu *et al.*, *Nature Physics* **3**, 119 (2007)]. In the experimental literature, the JBA is often treated classically. However, properly understanding how, say, tunneling modifies the stability of the stationary states, requires a full quantum analysis.

Methods: The Duffing oscillator is described by a quantum master equation in the rotating-wave approximation (RWA). After truncating Hilbert space, we use both real-time simulation and spectral analysis of the evolution superoperator.

Results: A plot of the expectation value for the stationary amplitude versus detuning shows a narrow transition between two branches, located close to the classical prediction. Going beyond averages and plotting the full photon-number distribution (diagonal part of the density matrix) reveals the quantum counterpart of genuine bistability. Finally, the real-time evolution features tunneling between the two classical branches.

Conclusion: Tunneling opens up a *third*, ultra-long time scale, besides the fast "harmonic" oscillations and the slow relaxation to the stationary amplitudes on the dissipative scale (the last two scales are also present in the classical case; in the RWA, the fast scale is eliminated in both the classical and quantum analysis). Tunneling is an error process from the point of view of qubit readout: if the final state no longer depends on the initial conditions, no detection has taken place.