4th Winter School on Quantum Information Science, Tamkang University, 2009

Introduction to superconducting qubits-II

Yuriy Makhlin

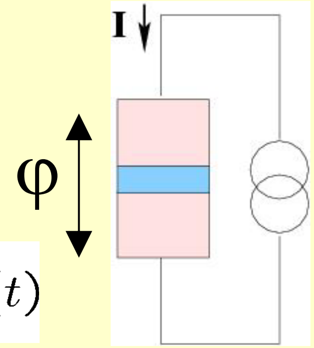
Landau Institute for Theoretical Physics

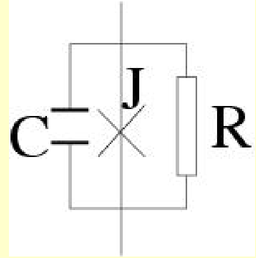
- **Superconducting qubits: basic principles**
- **Coherence and fluctuations**
- **Quantum-coherent phenomena in sc-qubits**

Phase dynamics, SQUIDs

$$E_C = e^2/(2C) \ll E_J$$

- **classical evolution, RCSJ model**

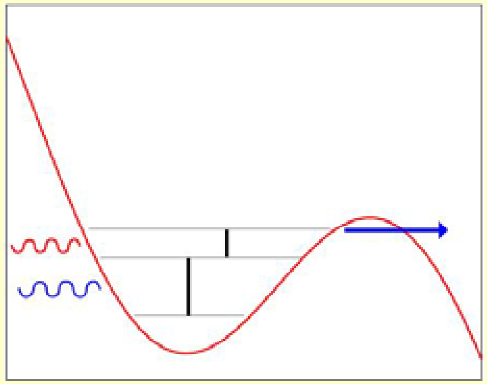




$$C\frac{\hbar\ddot{\varphi}}{2e} + \frac{1}{R}\frac{\hbar\dot{\varphi}}{2e} + I_c \sin\varphi = I_{ext} + \delta I(t)$$

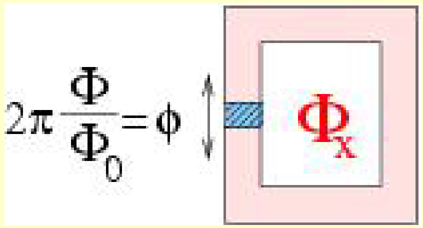
- **macroscopic quantum phase evolution**

$$\mathcal{H} = \frac{Q^2}{2C} - E_J \cos\varphi - I_{ext}\varphi + \mathcal{H}_{diss}; \qquad Q = -i\hbar\frac{\partial}{\partial(\hbar\varphi/2e)}$$

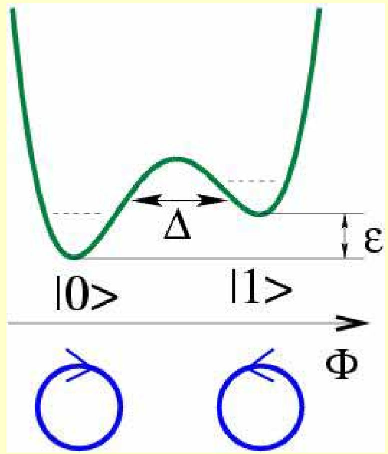


macroscopic & resonant quantum tunneling

- **rf-SQUID**



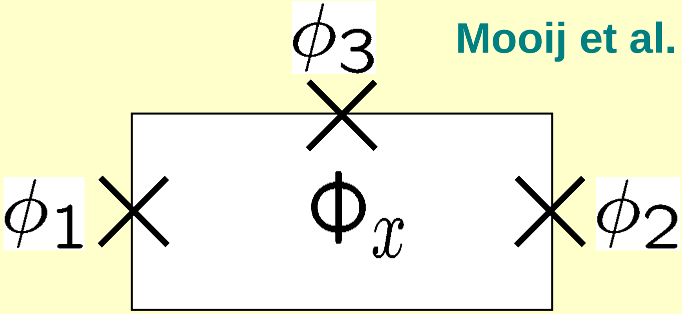
$$\mathcal{H} = \frac{Q^2}{2C} - E_J \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L}$$



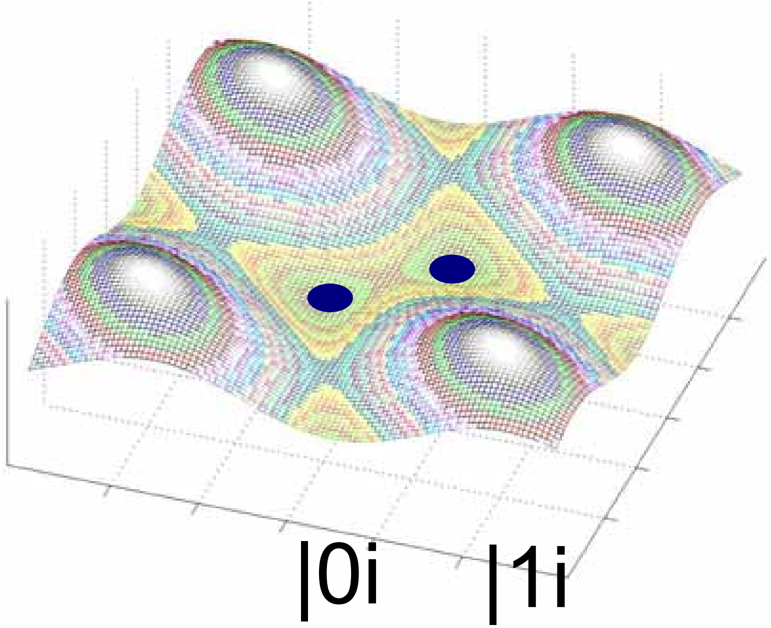
Macroscopic quantum coherence (Leggett)

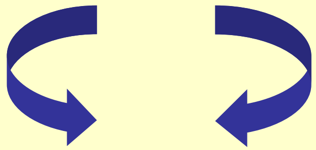
Superconducting flux qubits

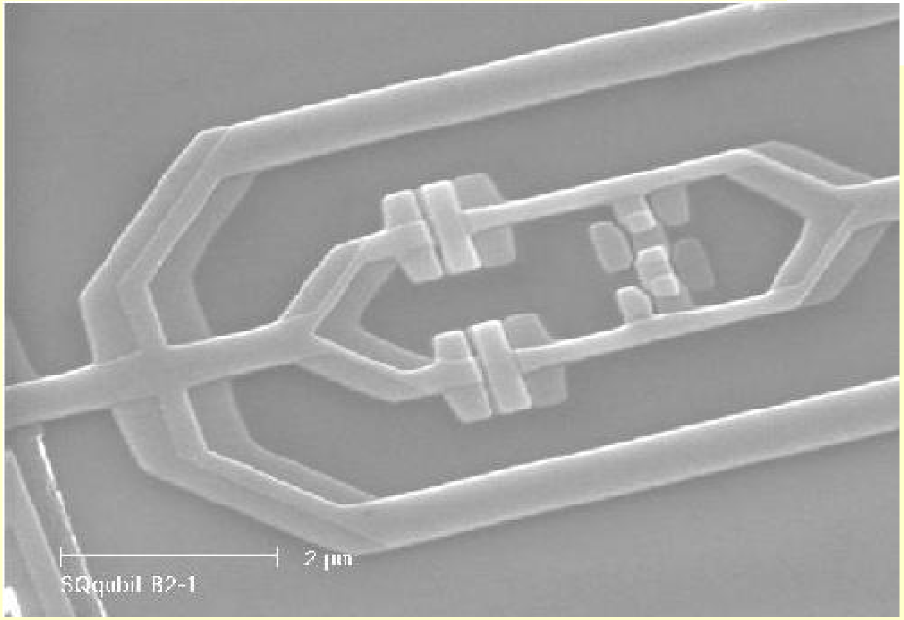
Mooij et al. 1999



$$\phi_3 = \Phi_x - \phi_1 - \phi_2$$

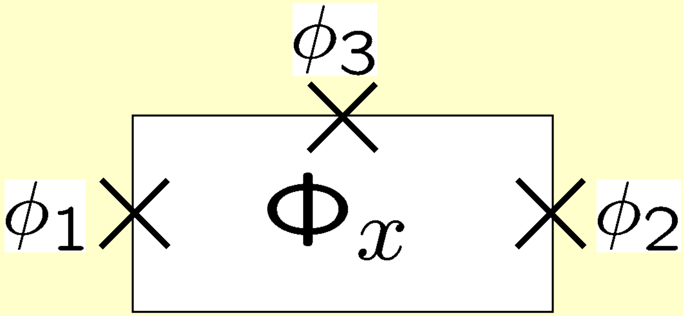






$$H \approx -\frac{1}{2}\varepsilon(\Phi_x)\,\sigma_z - \frac{1}{2}\Delta\,\sigma_x$$

Chiorescu et al. 2002



$$\phi_3 = \frac{2\pi}{\Phi_0}\Phi_x - \phi_1 - \phi_2$$

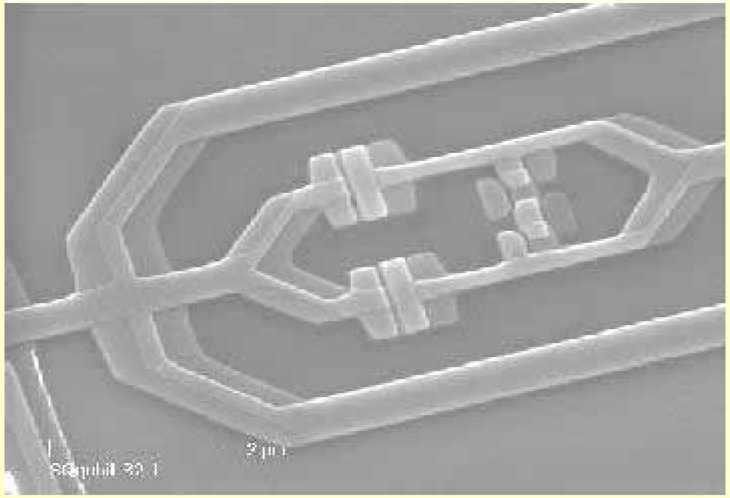
$$U(\phi_1, \phi_2) = -E_J \cos\phi_1 - E_J \cos\phi_2 - \tilde{E}_J \cos(\tfrac{2\pi}{\Phi_0}\Phi_x - \phi_1 - \phi_2)$$

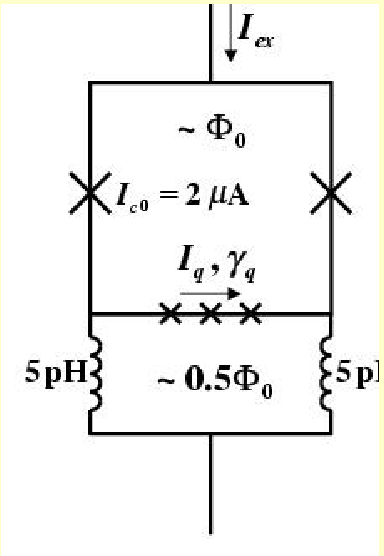
for $\phi_1 = \phi_2, \qquad \Phi_x = \Phi_0$

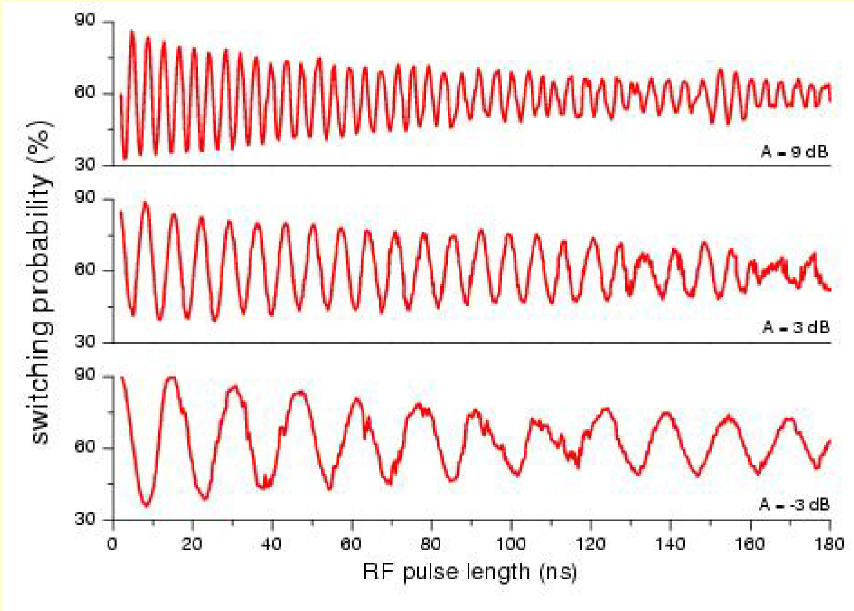
$$U = -2E_J \cos\phi_1 + \tilde{E}_J \cos(2\phi_1)$$

$$\tilde{E}_J \geq 0.5 E_J$$

Coherent oscillations: Delft (2002)

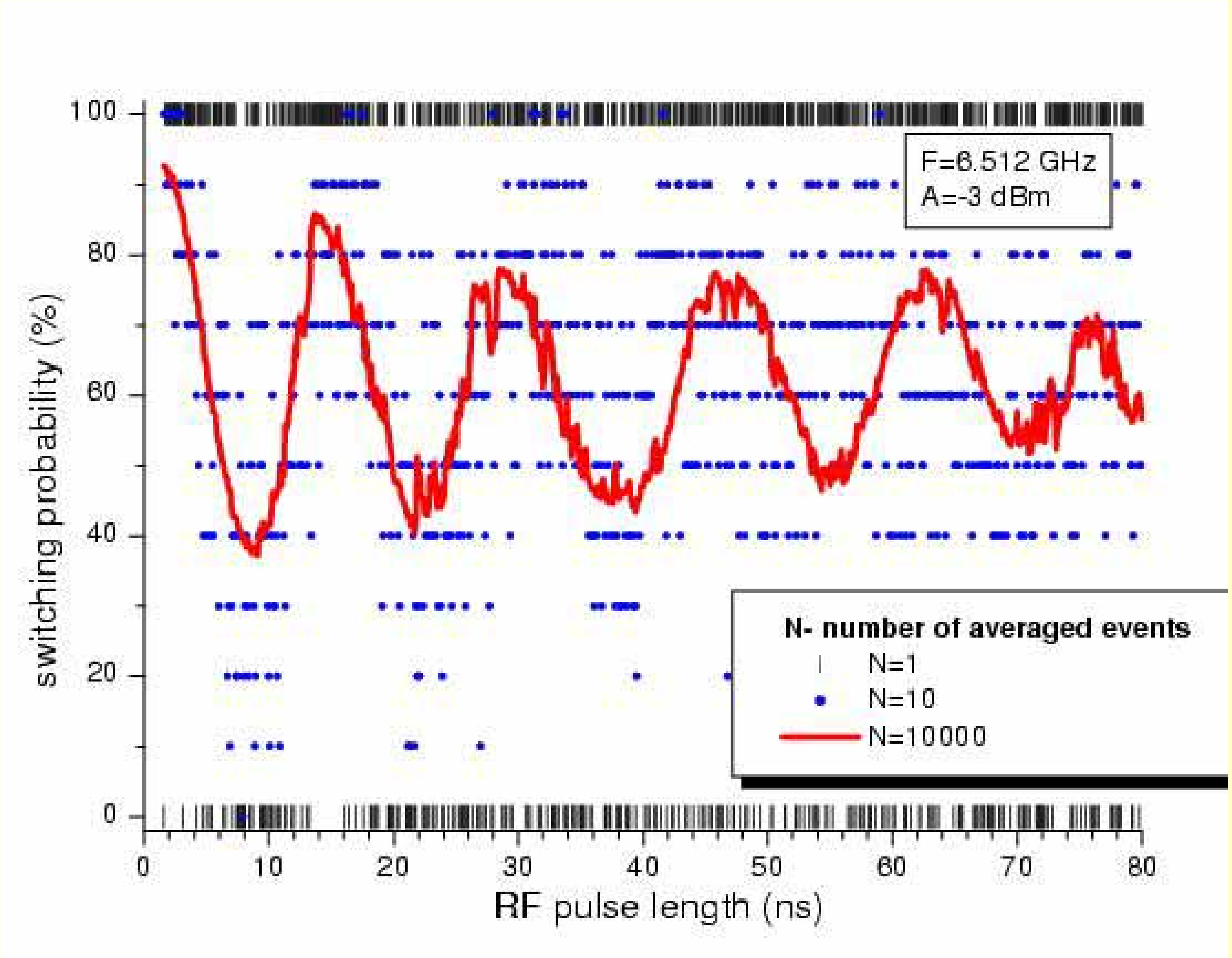






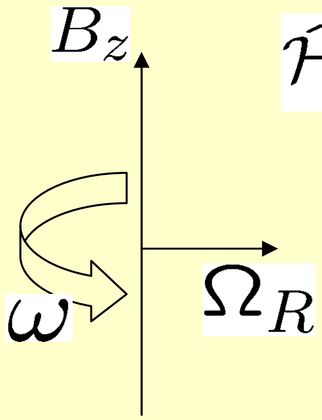
Chiorescu et al. (Delft) 2002

Quantum measurement: magnetic qubit + dc-SQUID



strong threshold measurement

Rabi oscillations

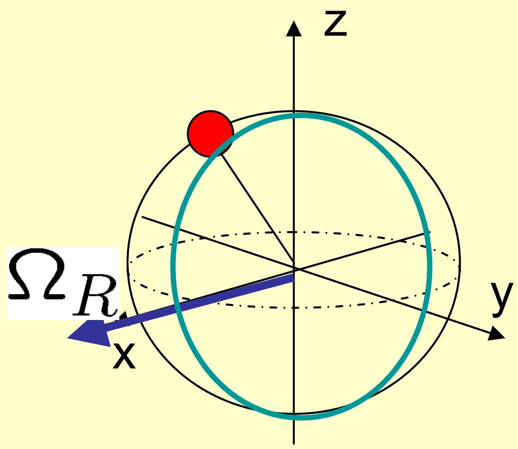


$$\hat{\mathcal{H}} = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}\Omega_R(\cos\omega t\ \hat{\sigma}_x + \sin\omega t\ \hat{\sigma}_y)$$

$$\omega = B_z$$

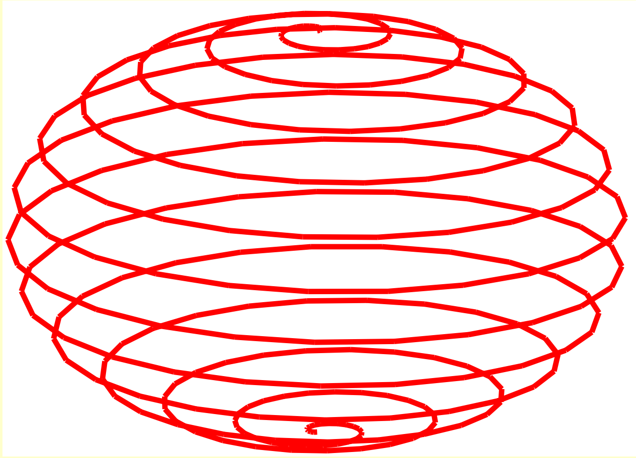
in rotating frame

$$\tilde{\mathcal{H}} = \dot{U}U^\dagger + U\mathcal{H}U^\dagger, \qquad U = \exp\left(-i\omega t\sigma_z/2\right)$$

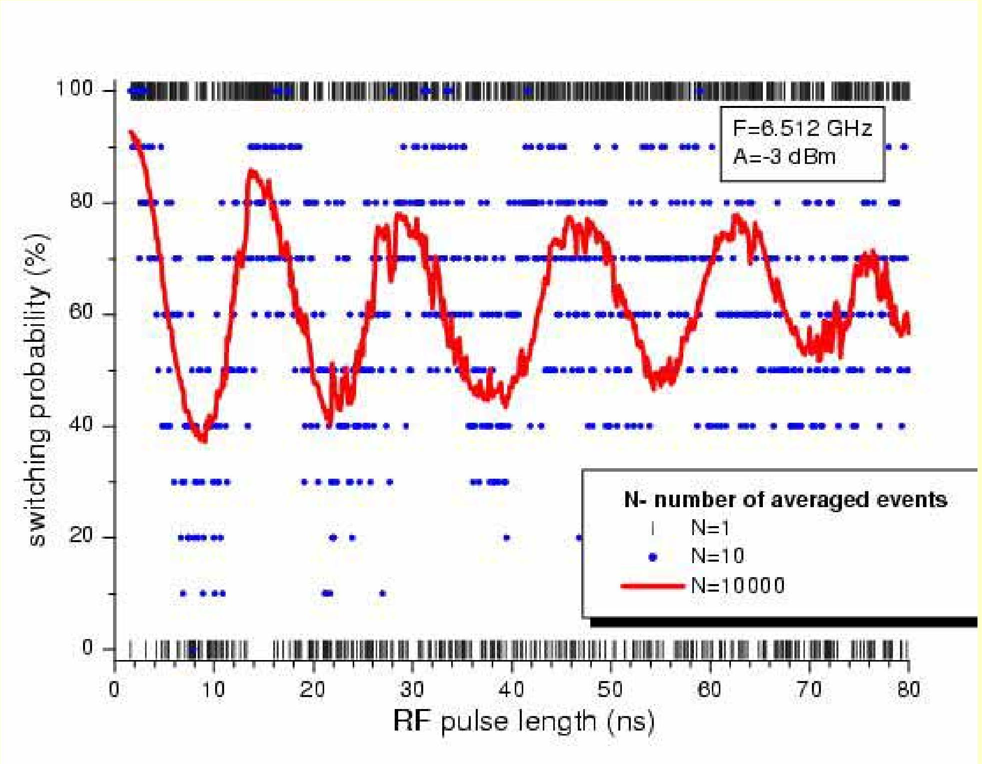


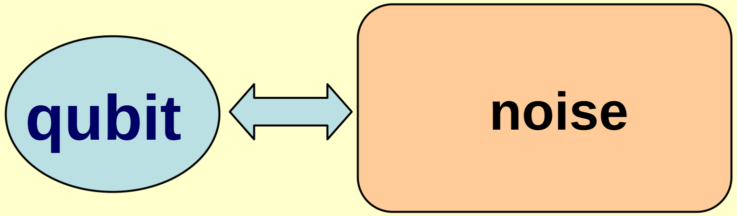
$$\tilde{\mathcal{H}} = -\frac{1}{2}\Omega_R\,\sigma_x$$

in lab frame



Coherent oscillations and noise



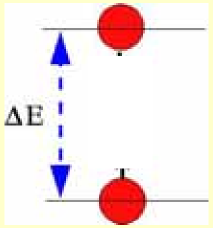


random phase

qubits as spectrometers of quantum noise

Longitudinal coupling ⇒ “pure” dephasing

$$H = -\frac{1}{2}(\Delta E + X)\sigma_z + H_{bath}$$



X – classical or quantum field, Gaussian

$$|\rho_{01}(t)| \propto \left\langle \exp\left(-\frac{i}{\hbar}\int_0^t X(\tau)d\tau\right)\right\rangle = \exp\left(-\frac{1}{2\hbar^2}\int_0^t d\tau_1 \int_0^t d\tau_2 \langle X(\tau_1)X(\tau_2)\rangle\right)$$

$$= \exp\left(-\frac{1}{2\hbar^2}\int \frac{d\omega}{2\pi} S_X(\omega)\frac{\sin^2(\omega t/2)}{(\omega/2)^2}\right)$$

$$\frac{\sin^2(\omega t/2)}{(\omega/2)^2} \approx 2\pi\delta(\omega)t$$

$$\approx \exp\left(-\frac{1}{2\hbar^2}S_X(\omega \approx 0)\cdot t\right) = \exp\left(-\frac{t}{T_2^*}\right)$$

for regular spectrum

$$S_X(\omega) = \frac{E_{1/f}^2}{|\omega|} \to \infty \quad \text{for } \omega \to 0$$

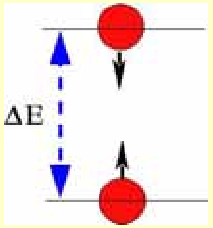
for 1/f noise

$$= \exp\left(-\frac{E_{1/f}^2}{2\pi} t^2 \ln|\omega_{ir} t|\right)$$

e.g., Cottet et al. 01

Transverse coupling ⇒ relaxation

$$H = -\frac{1}{2}\Delta E\sigma_z - \frac{1}{2}X\sigma_x + H_{Bath}$$



Golden Rule:

$$\Gamma_\uparrow = \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_i \left|\langle i \mid X \mid f\rangle\right|^2 \delta\left(E_i + \Delta E - E_f\right)$$

$$= \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_i \langle i \mid X \mid f\rangle\langle f \mid X \mid i\rangle \frac{1}{2\pi\hbar}\int dt \exp\left[i\left(E_i + \Delta E - E_f\right)t/\hbar\right]$$

$$= \frac{1}{4\hbar^2}\int dt \sum_i \rho_i \langle i \mid X(t)X(0) \mid i\rangle \exp\left[i\Delta E t/\hbar\right]$$

$$= \frac{1}{4\hbar^2}\left\langle X(t)X(0)\right\rangle_{\omega=\Delta E/\hbar}$$

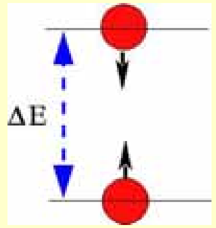
$$\Gamma_\downarrow = \frac{1}{4\hbar^2}\left\langle X(t)X(0)\right\rangle_{\omega=-\Delta E/\hbar}$$

⇨ $$\frac{1}{T_1} \equiv \Gamma_{\mathrm{rel}} = \Gamma_\uparrow + \Gamma_\downarrow = \frac{1}{2\hbar^2}S_X(\omega = \Delta E/\hbar)$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_2^*}$$

Bloch equations, applicability

$$H = -\frac{1}{2}\Delta E\,\sigma_z \;-\frac{1}{2} X\,\sigma_z - \frac{1}{2} X'\,\sigma_x + H_{bath}$$



$$\frac{d}{dt}\,\mathbf{S} = \mathbf{B}\times\mathbf{S} - \frac{1}{T_1}(\mathbf{S}_{\parallel} - \mathbf{S}_0)\hat{\mathbf{z}} - \frac{1}{T_2}\mathbf{S}_{\perp}$$

Bloch (46,57)
Redfield (57)

perturbation theory

works for weak short-correlated noise τ_c¿ T_1, T_2

Beyond Bloch equations

- 1/f noise – long correlation time τ_c
- optimal operation points: X^2 or higher powers
- sharp pulses (state preparation)
- time-dependent field, Berry phase

Bloch equations

$$\frac{d}{dt}\mathbf{S} = \mathbf{B}\times\mathbf{S} - \frac{1}{T_1}(\mathbf{S}_\parallel - \mathbf{S}_0)\hat{\mathbf{z}} - \frac{1}{T_2}\mathbf{S}_\perp$$

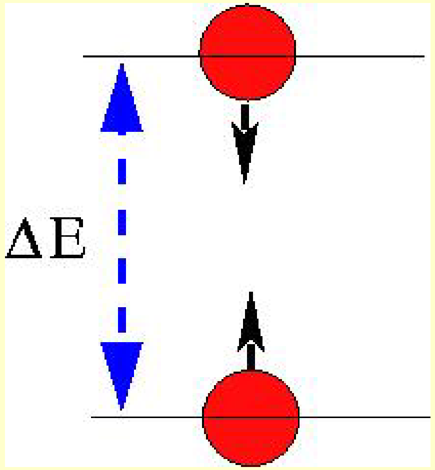
Bloch (1946,1957)
Redfield (1957)

for weak short-correlated noise

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_2^*}$$

energy relaxation

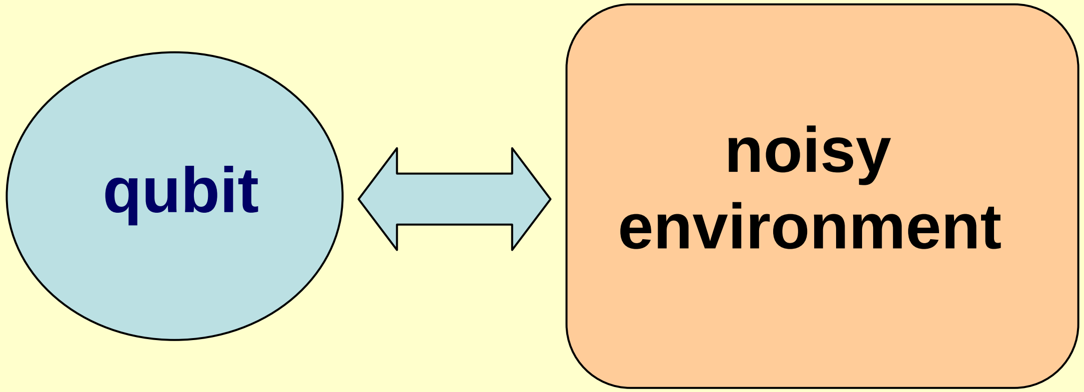
$$\frac{1}{T_1} = \Gamma_\uparrow + \Gamma_\downarrow = \frac{1}{2\hbar^2} S_X(\omega = \Delta E/\hbar)$$



phase relaxation

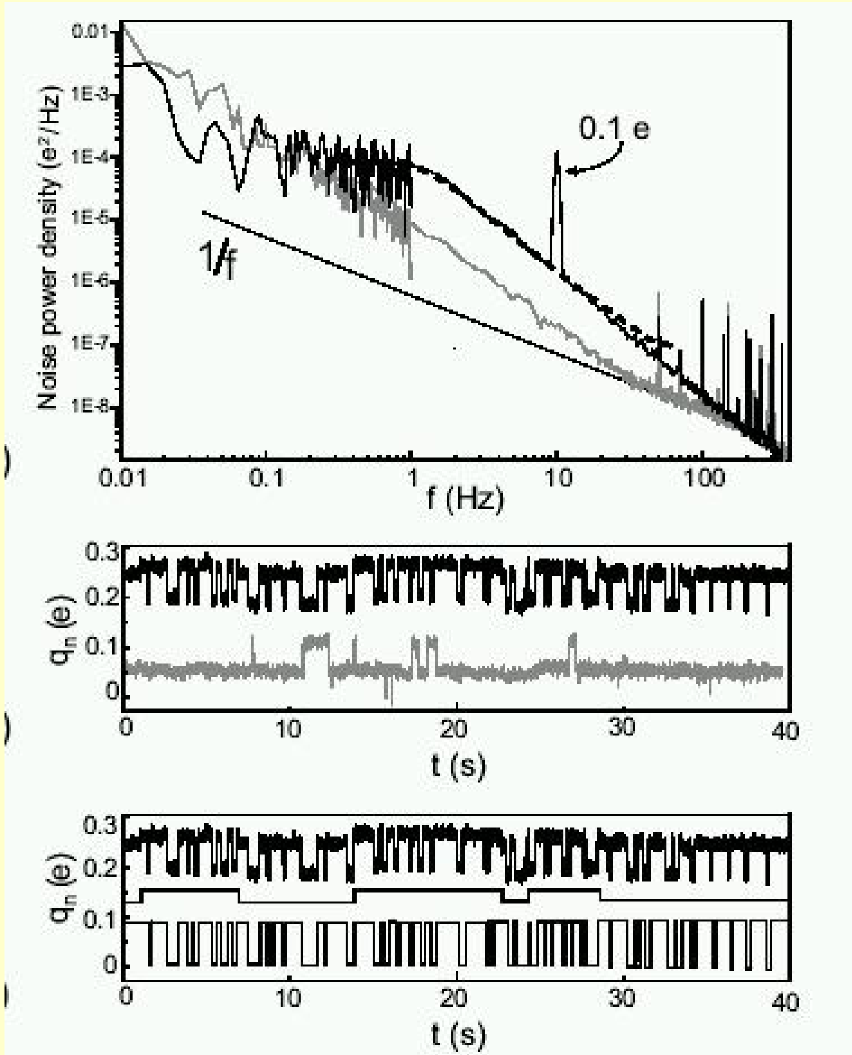
$$\frac{1}{T_2^*} = \frac{1}{2\hbar^2} S_X(\omega = 0)$$

Qubits and environment



- Decoherence induced by noise
- Qubits as spectrometers

Nanoelectronic circuits and 1/f noise

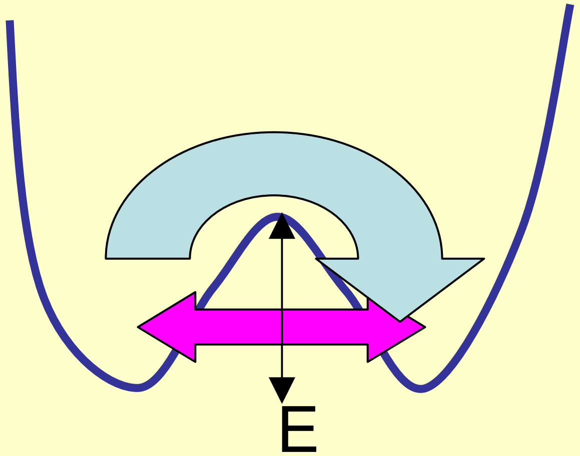


Bouchiat et al. ´97

Charge noise:

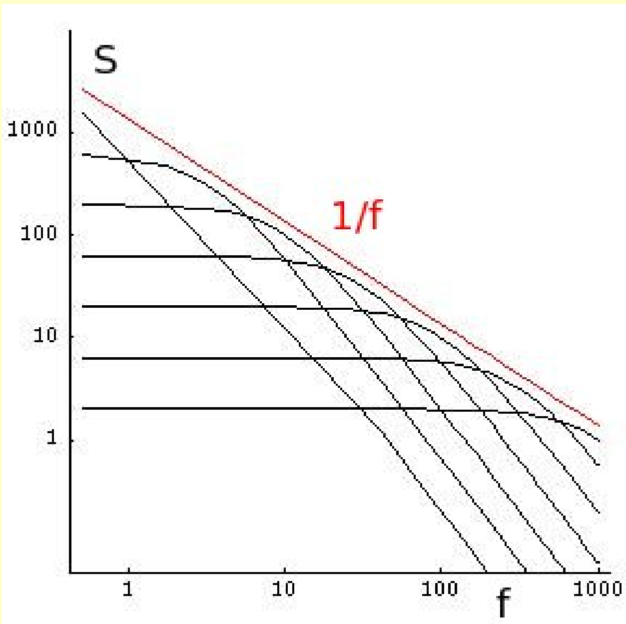
- 1/f noise
- individual Lorentzians – bistable fluctuators
- T-dependence saturated at at low T. 300 mK

1/f noise from bistable fluctuators



$\Gamma / \, e^{-E/kT}$ or $\Gamma / \, e^{-E/h\omega_0}$

$$dw = g(E)dE \propto \begin{Bmatrix} h\omega_0 \\ kT \end{Bmatrix} \frac{d\Gamma}{\Gamma}$$

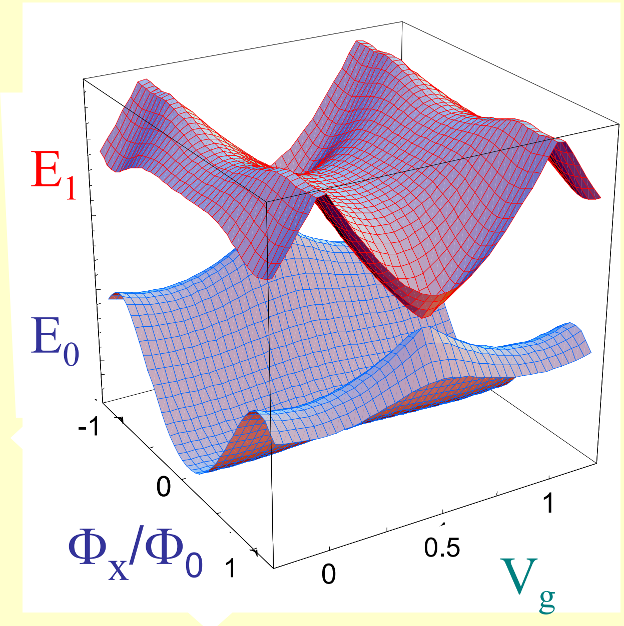


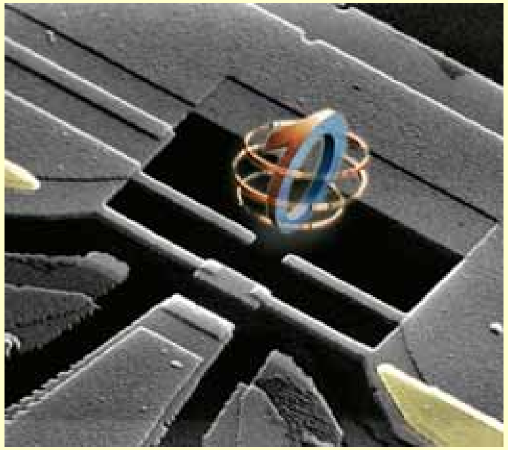
$$S(\omega) \propto \int \frac{d\Gamma}{\Gamma} \frac{\Gamma}{\omega^2 + \Gamma^2} \propto \frac{1}{\omega}$$

McWhorter, 1958; Dutta, Horn, RMP'81

Charge-phase qubit $E_C \approx E_J$

$$H = -\frac{1}{2} E_{ch}(V_g)\,\sigma_z - \frac{1}{2} E_J(\Phi_x)\,\sigma_x$$





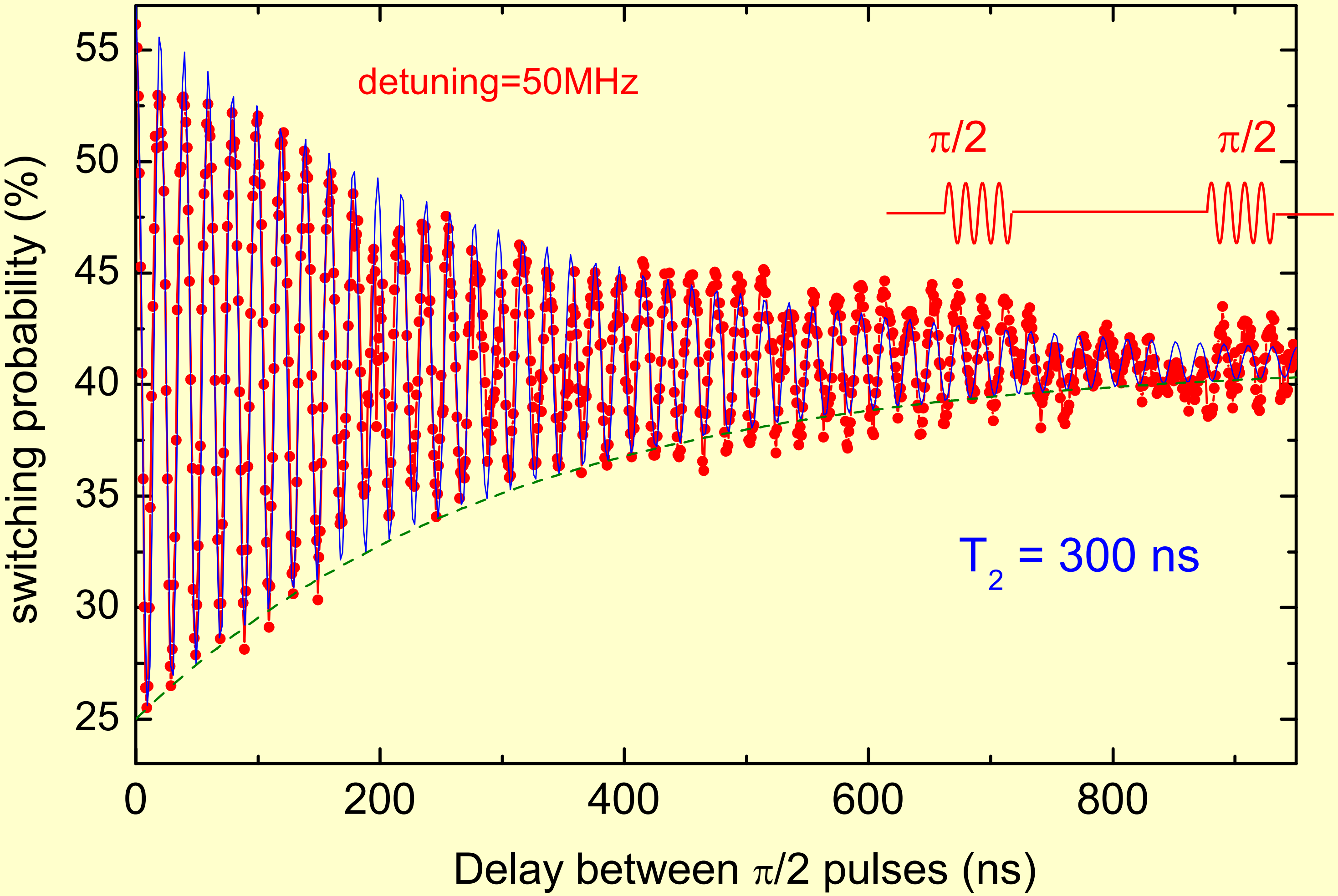
Quantronium (Saclay)

Operation at optimal point (saddle)

- minimizes noise effects
- voltage fluctuations couple transversely
- flux fluctuations couple quadratically

$$H = -\frac{1}{2} E_J(\Phi_{x0})\,\sigma_x \;-\frac{1}{2}\frac{\partial E_{ch}}{\partial V_g}\Big|_{V_{go}} \delta V_g\, \sigma_z - \frac{1}{4}\frac{\partial^2 E_J}{\partial \Phi_x^2}\Big|_{\Phi xo} \delta\Phi_x^2\, \sigma_x$$

Decay of Ramsey fringes at optimal point



Vion et al., Science 02, …

Flux qubit: Bertet et al. ´04

Quadratic longitudinal coupling:

$$H = -\frac{1}{2}\left(\Delta E + \lambda X^2\right)\sigma_z$$

- Spectrum of fluctuations of $X^2(t)$?
- Distribution of fluctuations of $X^2(t)$?

Even if $X(t)$ is distributed Gaussian (central limit theorem), $X^2(t)$ is not!

- **1/f noise**

$$S_X = \frac{E_{1/f}^2}{|\omega|} \quad \Rightarrow \quad S_{X^2} \approx \frac{E_{1/f}^4}{|\omega|}\ln\frac{\omega}{\omega_{ir}}$$

again 1/f noise with different scale

if X^2 is Gaussian $\Rightarrow$

$$|\rho_{01}(t)| = \exp\left(-\frac{1}{\pi}\Gamma_f^2 t^2 \ln^2|\omega_{ir} t|\right)$$

$$\Gamma_f \equiv \lambda E_{1/f}^2$$

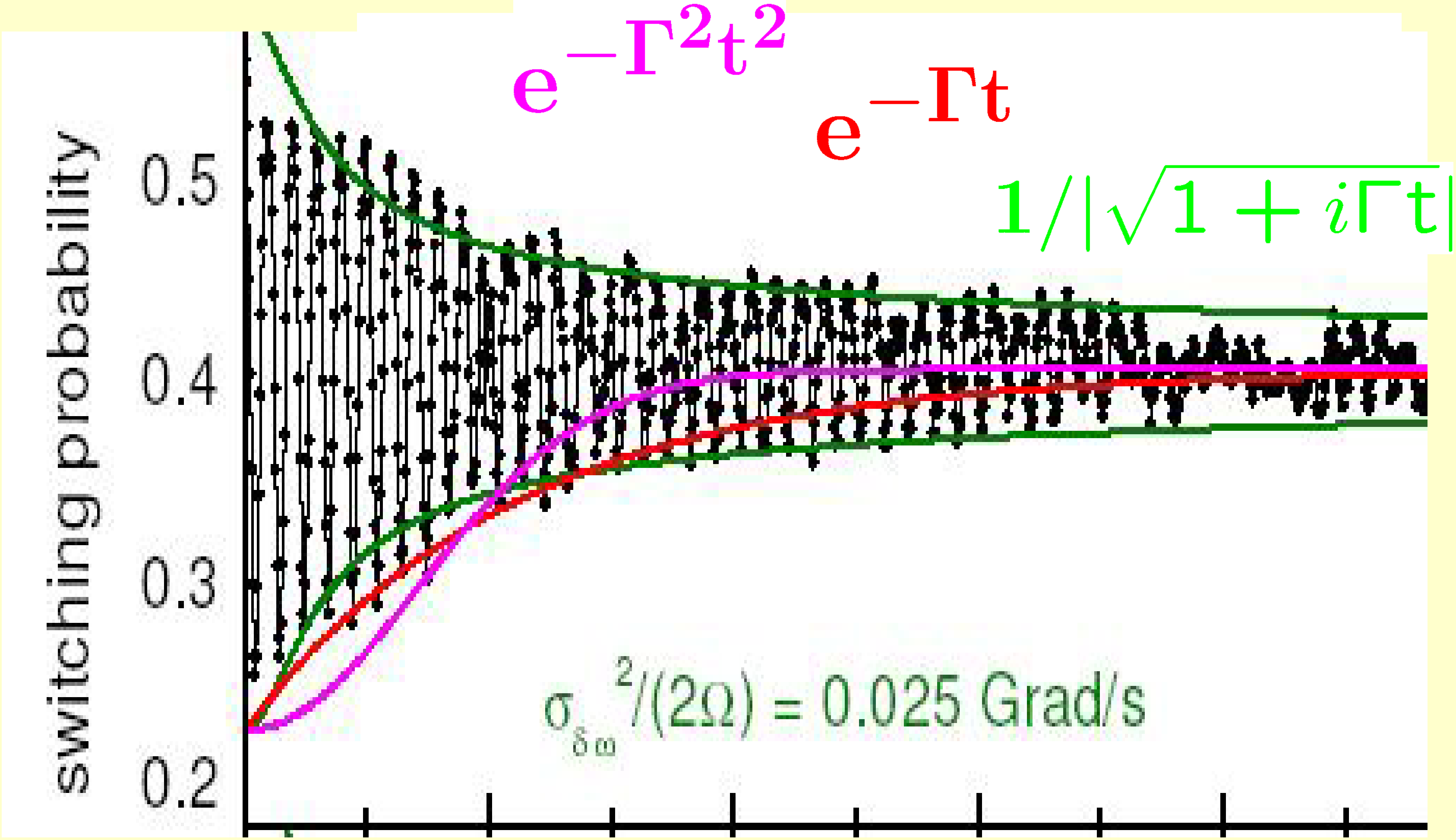
$$\mathbf{p(X)} \propto \exp(-\mathbf{X^2}/\mathbf{2}\sigma_{\mathbf{X}}^{\mathbf{2}})$$

$$P(t)^{static} = \int dX\, p(X)\, e^{i\lambda X^2 t} = \frac{1}{\sqrt{1 - 2i\lambda\sigma_X^2 t}}$$

YM, Shnirman 04
D. Averin et al. 04
E. Paladino et al. 04

$$P(t)^{static} \propto 1/\sqrt{t}$$

Fitting the experiment

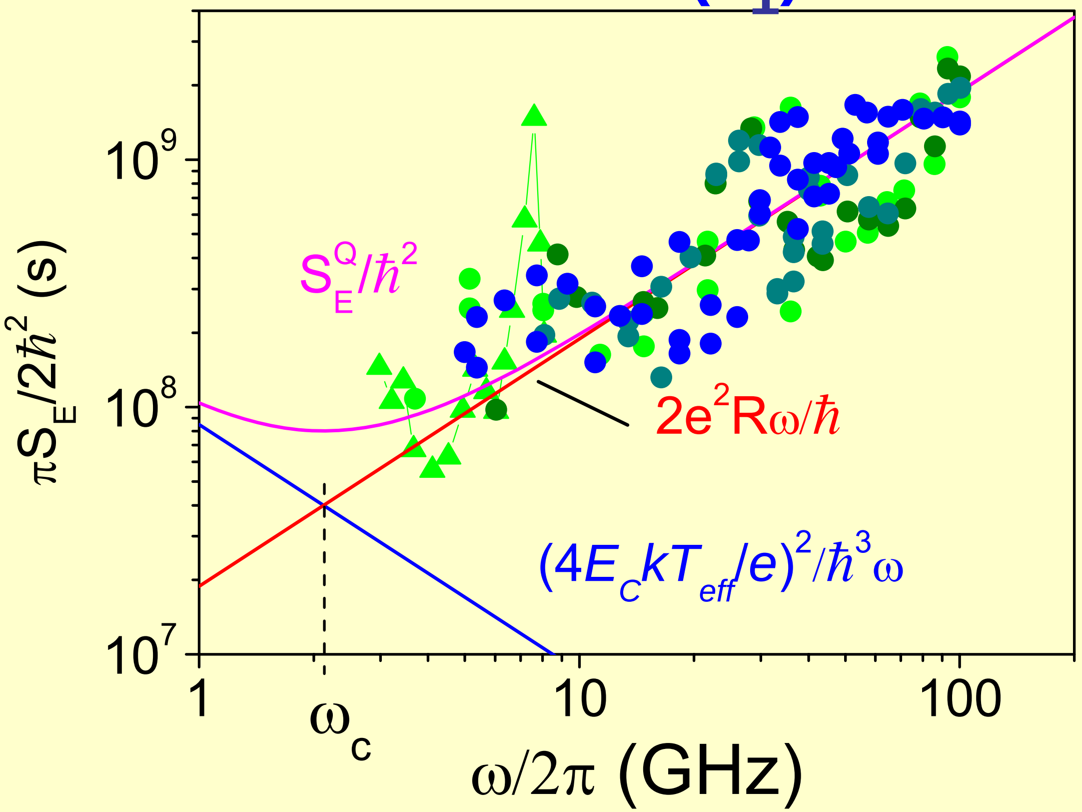


$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_2^*}$$

$$\frac{1}{T_1} = \Gamma_\uparrow + \Gamma_\downarrow = \frac{1}{2\hbar^2} S_X(\omega = \Delta E/\hbar)$$

Johnson-Nyquist noise

Relaxation (T_1) in a charge qubit

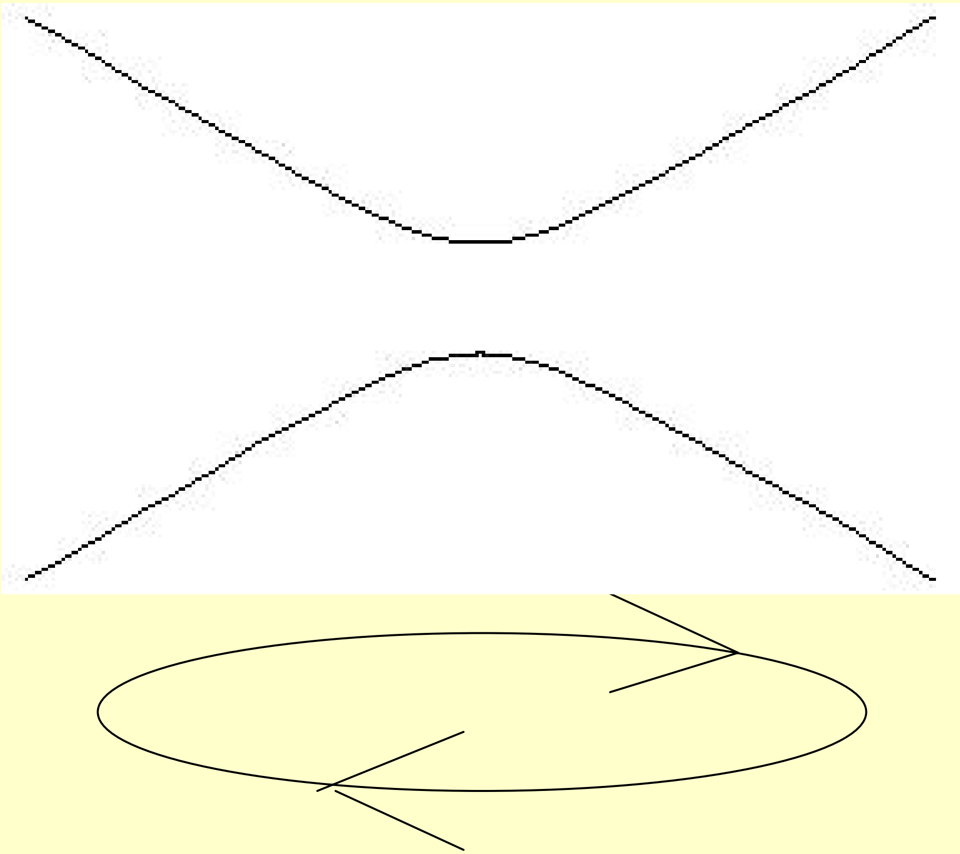


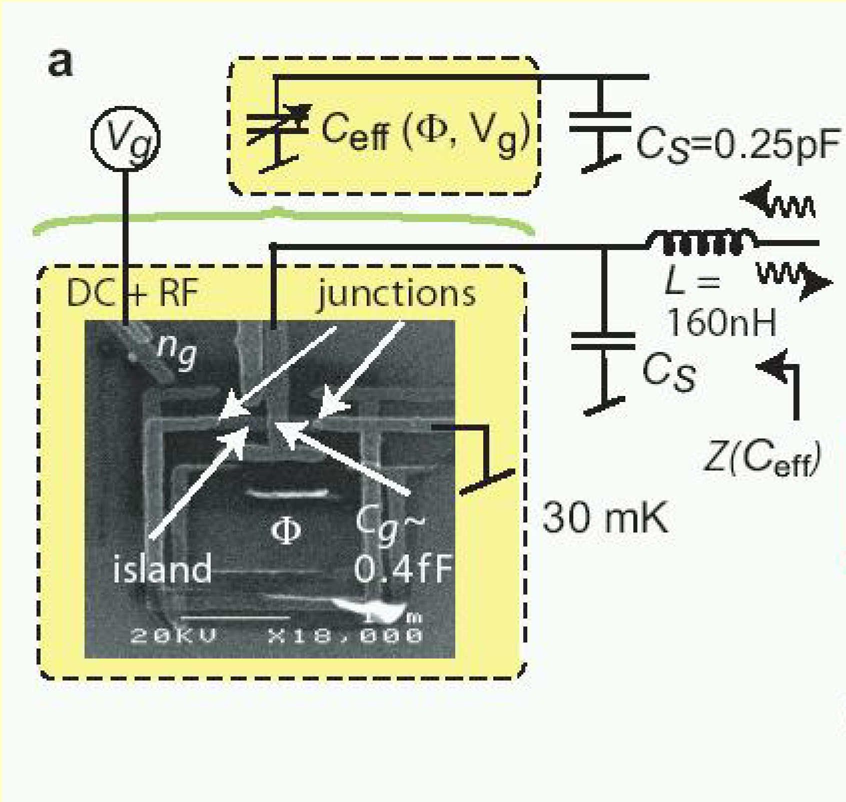
O.Astafiev et al. 2004-06

$$S(\omega) \approx \frac{a\,(k_B T)^2}{\hbar\omega} + a\,\hbar\omega$$

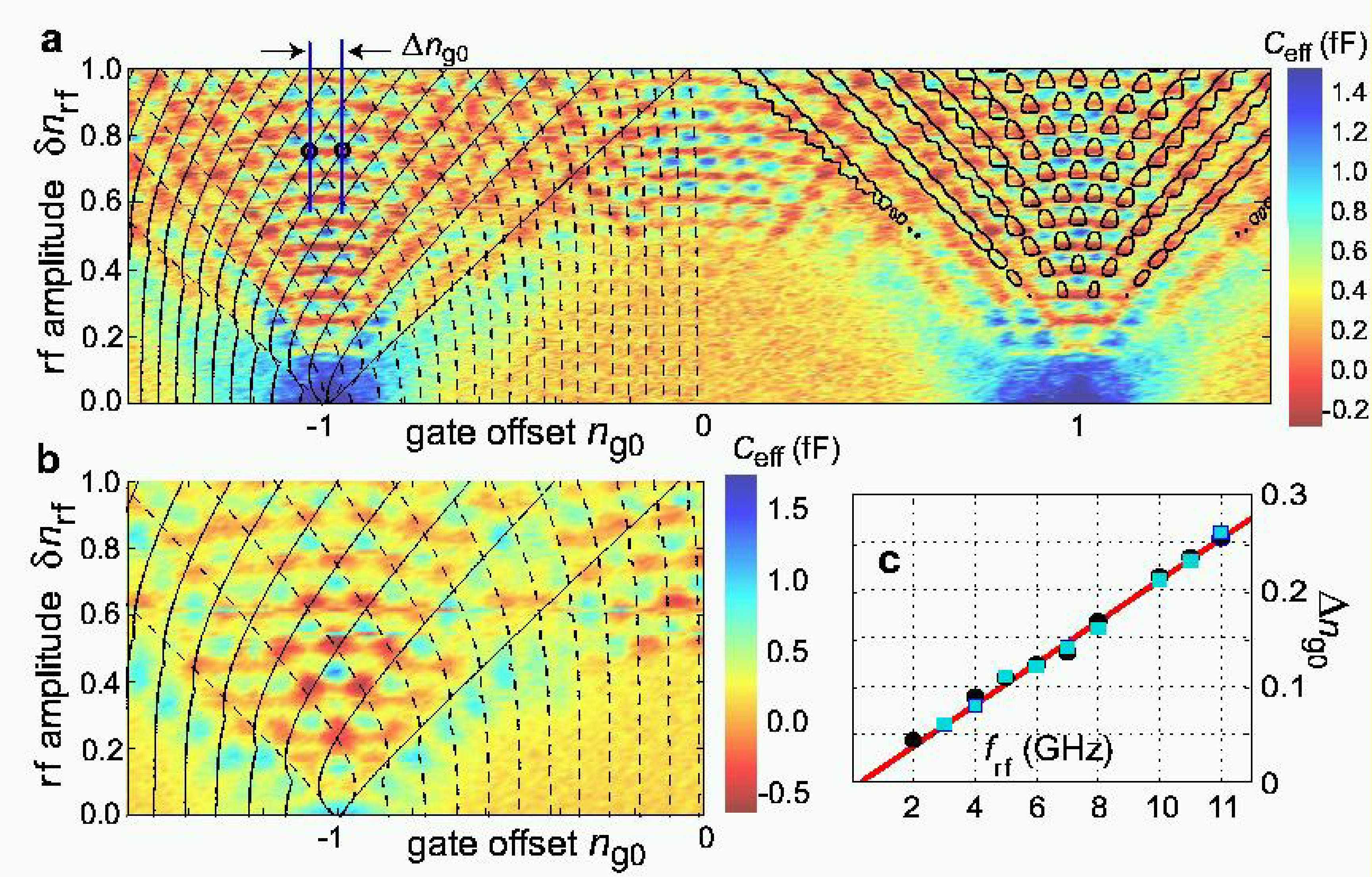
low- and high-frequency noises are connected

Landau-Zener interference



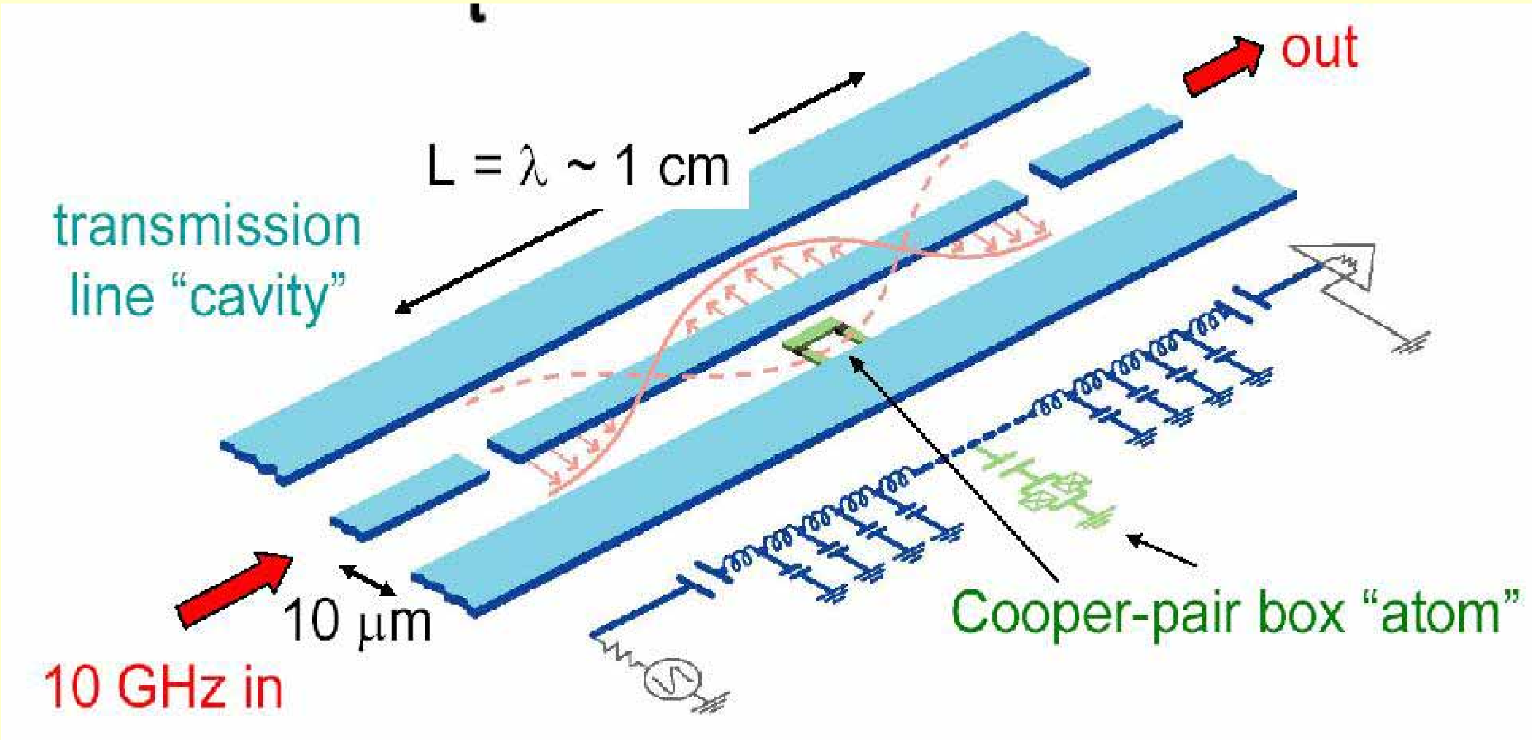


Landau-Zener interference in superconducting qubits



Sillanpää et al. 2005

cQED – circuit QED in superconducting systems

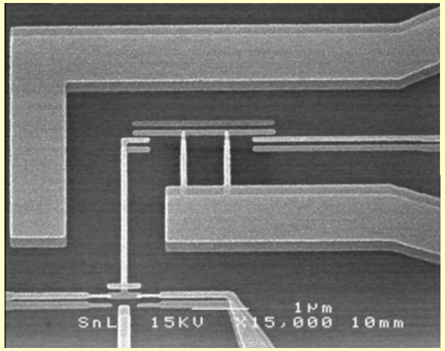


Wallraff et al., Nature 04

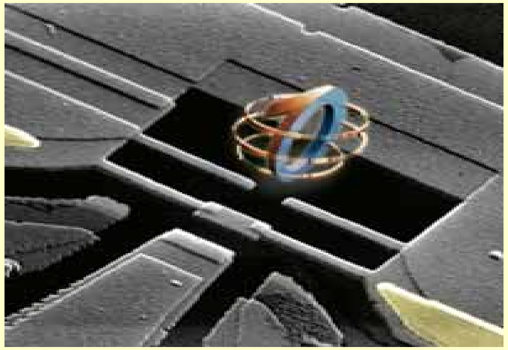
Conclusions

- superconducting qubis are a promising realization of QC
- experiments w/ 1-2 qubits
- precise measurements w/ weak backaction
- applications of qubits: spectrometry

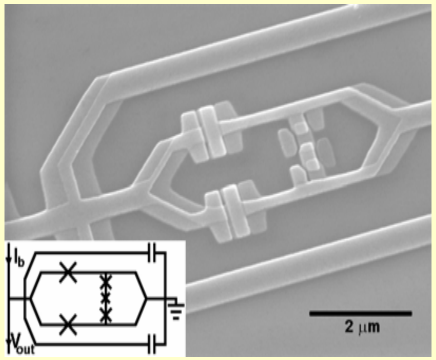
charge



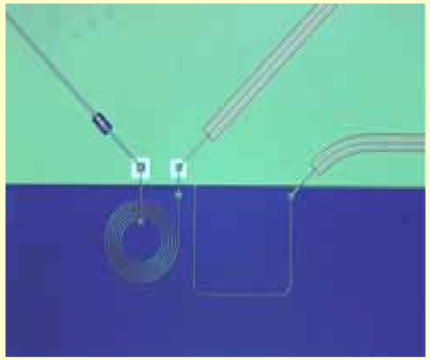
charge-phase



flux



phase



junction size, E_J/E_C →

M.H.Devoret, A.Wallraff, J.Martinis, cond-mat/0411174

D.Esteve, D.Vion, cond-mat/0505676

YM, G.Schoen, A.Shnirman, cond-mat/0011269, RMP 2001

J. Martinis, Superconducting phase qubits, 2008

C.Slichter, Principles of *Magnetic Resonance, Springer, 1996*

G.Ithier et al., cond-mat/0508588

J.A.Schreier et al., arxiv.org:0712.3581