4th Winter School on Quantum Information Science, Tamkang University, 2009

Introduction to superconducting qubits

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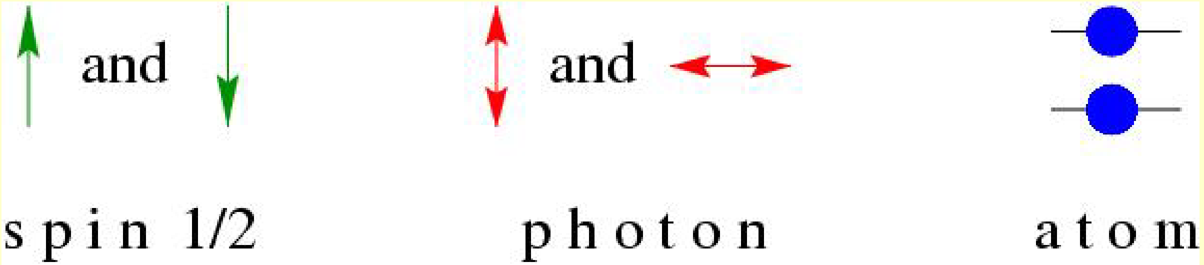
- **Quantum computation: concept + requirements to realizations**
- **Superconducting qubits: basic principles**
- **Coherence and fluctuations**
- **Read-out: Quantum measurement**
- **Quantum-coherent phenomena in sc-qubits**

Requirements to realizations of quantum computers

(D.DiVincenzo 1997)

- **2-level quantum systems — qubits**



superpositions: $a\,|0\rangle + b\,|1\rangle$

- w/ well-defined physical parameters (Hamiltonian, ...)

- no leakage to higher states (3-rd, 4-th, ...)

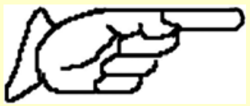
- scalability (N qubits)

Requirements to realizations of quantum computers

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{H}_i(t)\hat{\sigma}_i(t)$$

controlled (on / off)

- 2-level quantum systems — N qubits
- controlled dynamics



- 1-qubit (logic) gates

- spin rotation: $\mathcal{H} = -H_x^i \sigma_x^i$ for time τ

$$U_x^i = \exp(-iH_x^i\hat{\sigma}_x^i\tau/\hbar) = \begin{pmatrix} \cos\alpha & i\sin\alpha \\ i\sin\alpha & \cos\alpha \end{pmatrix}, \qquad \alpha = \frac{H_x^i\tau}{\hbar}$$

- phase shift: $\mathcal{H} = -H_z^i \sigma_z^i$ for time τ

$$U_z^i = \exp(iH_z^i\hat{\sigma}_z^i\tau/\hbar) = \begin{pmatrix} e^{i\beta} & 0 \\ 0 & e^{-i\beta} \end{pmatrix}, \qquad \beta = \frac{H_z^i\tau}{\hbar}$$

Requirements to realizations of quantum computers

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{H}_i(t)\hat{\sigma}_i(t) + \sum_{i<j} J^{ij}(t)(\hat{\sigma}_x^i\hat{\sigma}_x^j + \hat{\sigma}_y^i\hat{\sigma}_y^j)$$

controlled — controlled

- 2-level quantum systems — N qubits
- controlled dynamics
  - 1-qubit gates
  - 2-qubit gates

} universal set

$$\mathcal{H} = J_{ij}(\hat{\sigma}_x^i\hat{\sigma}_x^j + \hat{\sigma}_y^i\hat{\sigma}_y^j) \qquad \text{for time } \tau$$

$$U_{2-\text{bit}}^{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & -i\sin\gamma & 0 \\ 0 & -i\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{matrix} |\uparrow_i\uparrow_j\rangle \\ |\uparrow_i\downarrow_j\rangle \\ |\downarrow_i\uparrow_j\rangle \\ |\downarrow_i\downarrow_j\rangle \end{matrix}, \quad \gamma = \frac{2J^{ij}\tau}{\hbar}$$

$$U_{2-\text{bit}}(\gamma = \pi/4) : |\uparrow\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - i\,|\downarrow\uparrow\rangle)$$ — **entangled state**

Requirements to realizations of quantum computers

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{H}_i(t)\hat{\sigma}_i(t) + \sum_{i<j} J^{ij}(t)(\hat{\sigma}_x^i\hat{\sigma}_x^j + \hat{\sigma}_y^i\hat{\sigma}_y^j) + \mathcal{H}_{\text{meas}}(t) + \mathcal{H}_{\text{diss}}$$

controlled controlled controlled weak

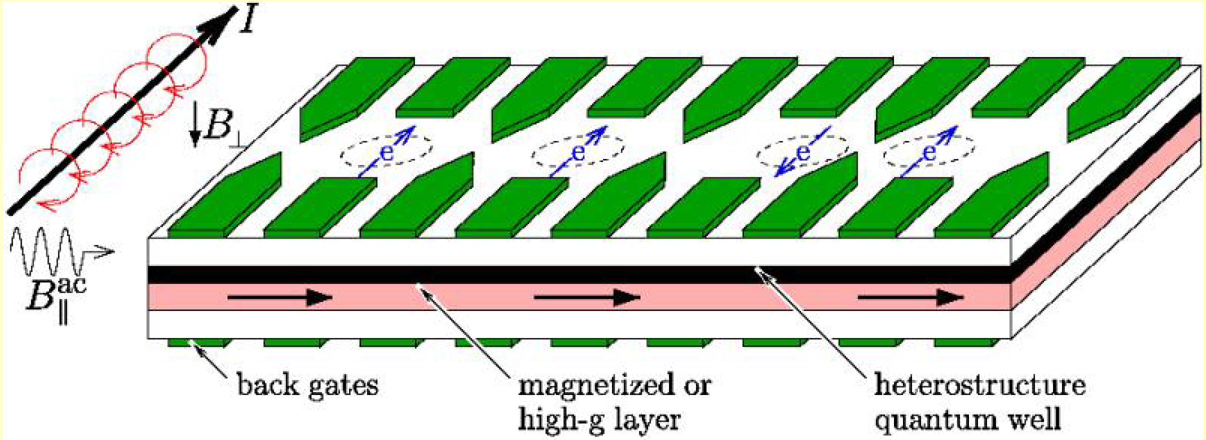
- **2-level quantum systems — N qubits**
- **controlled dynamics**
  - **1-qubit gates**
  - **2-qubit gates**
- **initialization - cooling or read-out**
- **coherence:** $\tau_\varphi > 10^4 \tau_{\text{op}}$ enough for error correction
- **read-out: quantum measurement**

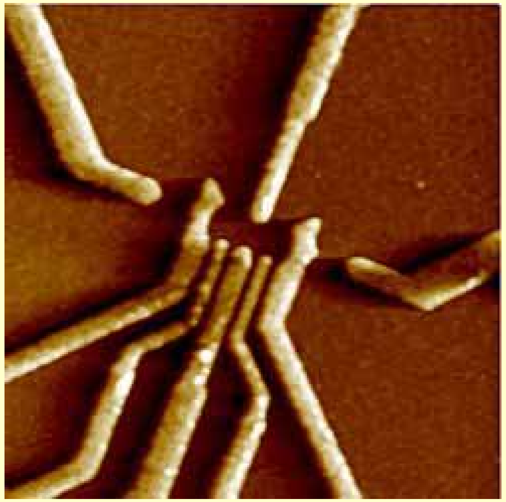
Electron spins in semiconductors

Loss & DiVincenzo, 1998

+ τ_φ spins > τ_φ charges

+ exactly 2 states



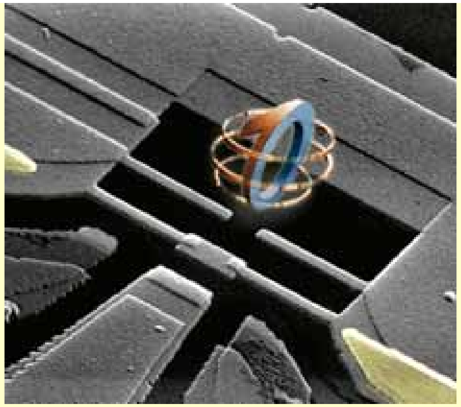


Kouwenhoven et al. (Delft)

Josephson quantum bits

combine:

- coherence of superconducting state
- advanced control techniques for single-charge and SQUID systems



Quantronium

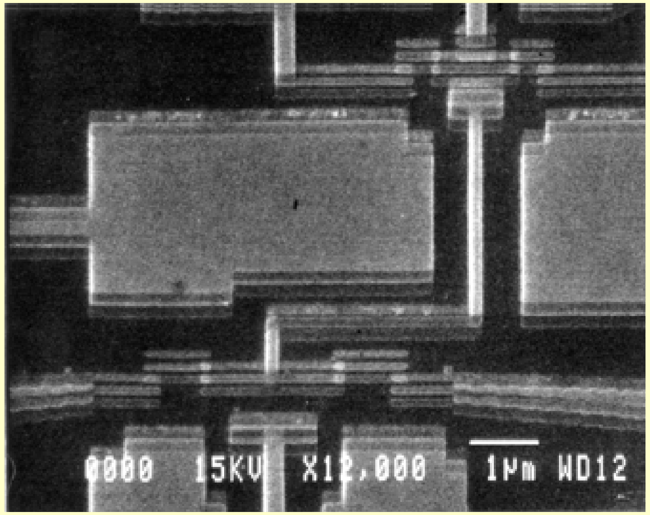
(Saclay)

no excitations at low T

quantum degree of freedom: **charge** or **phase** (magn. flux)

- **macroscopic quantum physics** (unlike in ion traps, NMR, optical resonators)
- artificial atoms: flexibility in fabrication
- **scalability** (many qubits)
- easy to **integrate** in el. structures

Single-electron effects

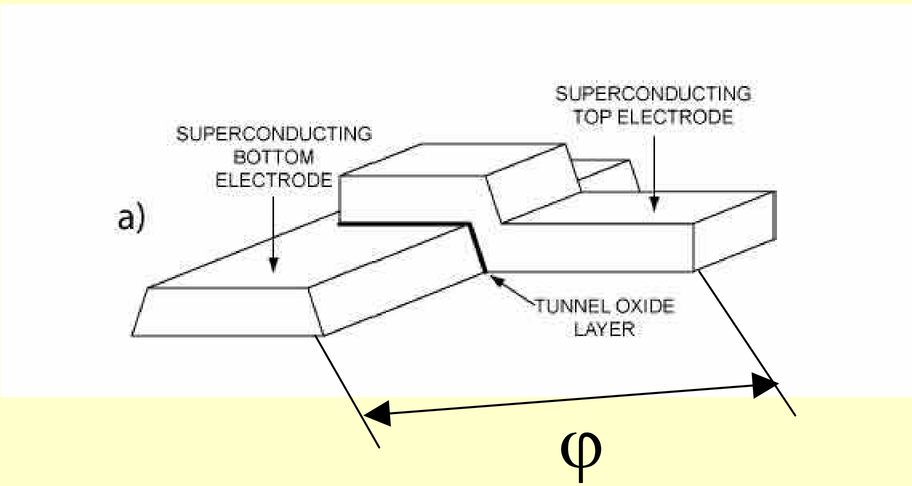


V. Bouchiat et al. (1996)

junctions w/ small area 10nm x 10nm

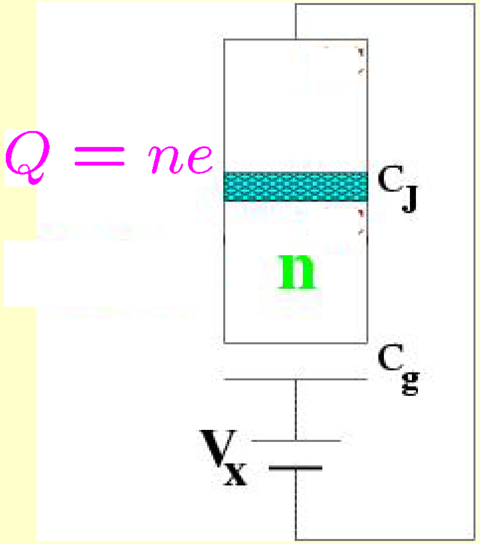
typical capacitance $C \approx 10^{-15}$ F

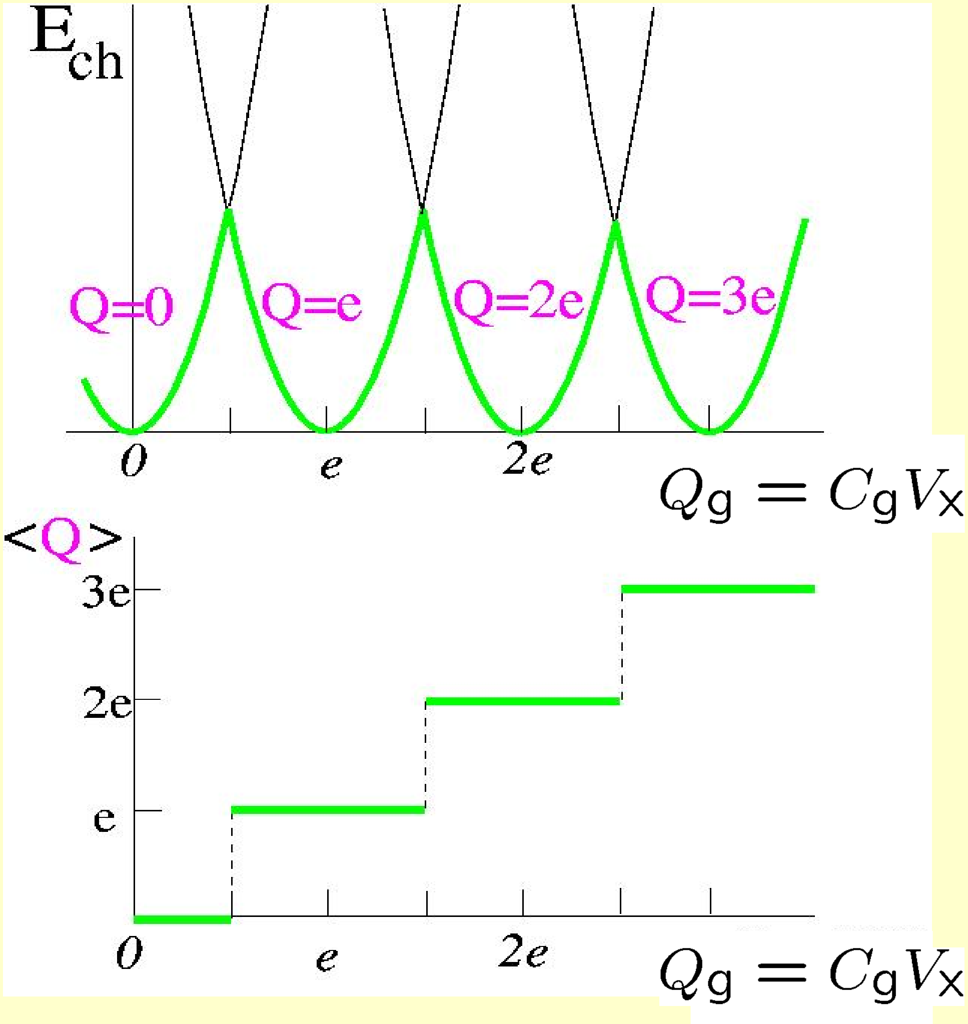
typical energies $E_C = e^2/2C \approx 1$ K



Charging effects in a single-electron box

charging energy: $$E_{\mathrm{ch}}(n, V_{\mathrm{x}}) = \frac{(ne - C_{\mathrm{g}} V_{\mathrm{x}})^2}{2(C_{\mathrm{g}} + C_{\mathrm{J}})}$$

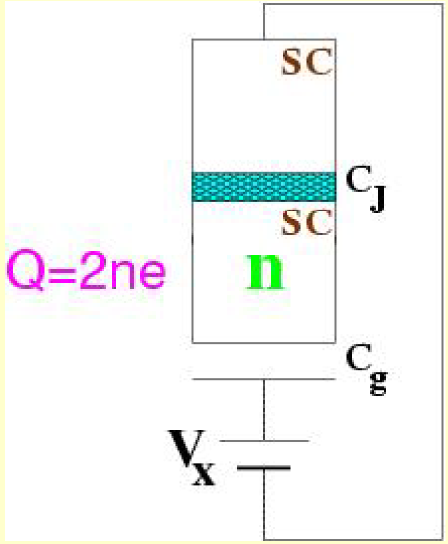


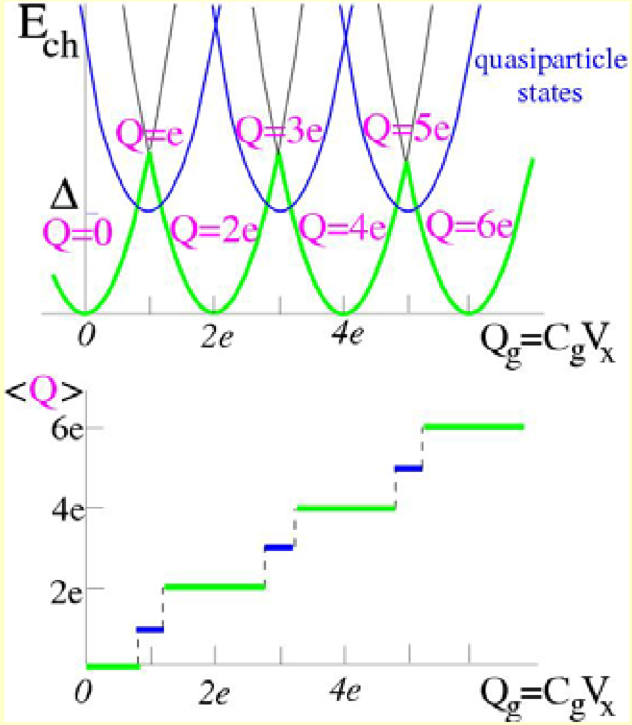


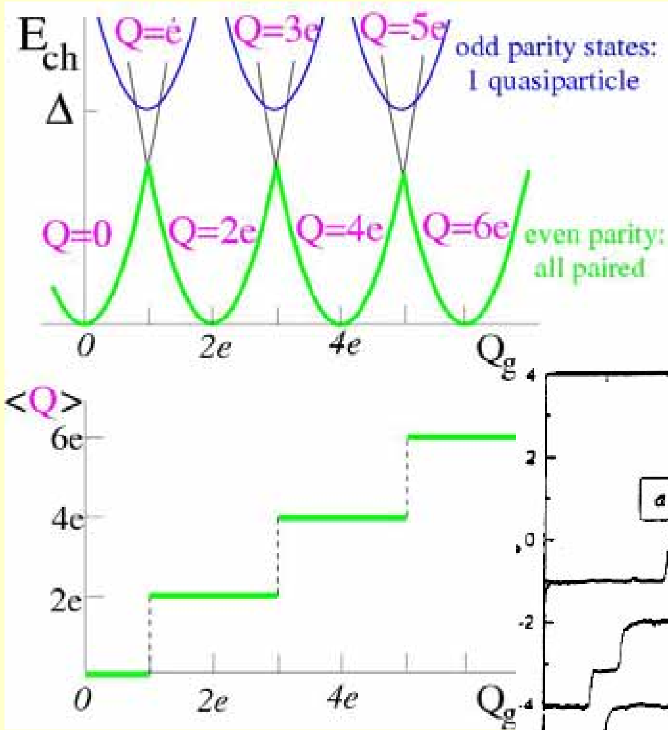
Charging effects in a single-Cooper-pair box

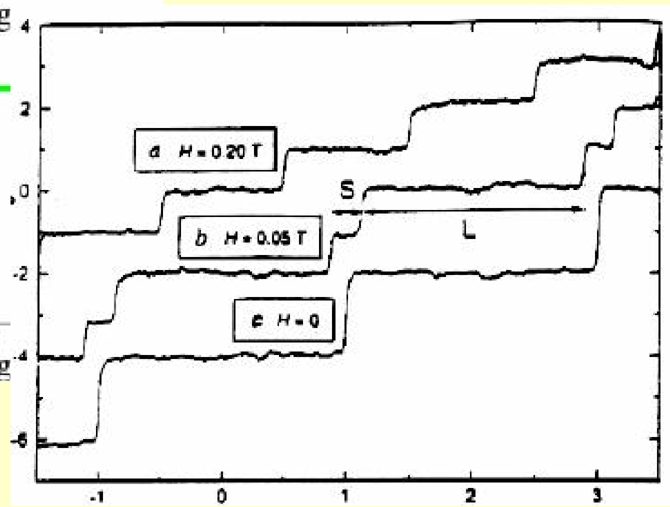
charging energy:

$$E_{\mathrm{ch}}(n, V_{\mathrm{x}}) = \frac{(2ne - C_{\mathrm{g}} V_{\mathrm{x}})^2}{2(C_{\mathrm{g}} + C_{\mathrm{J}})}$$



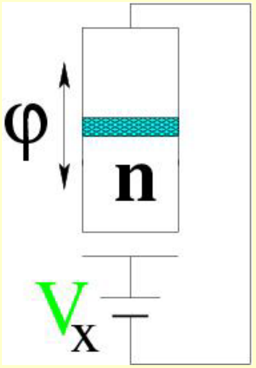




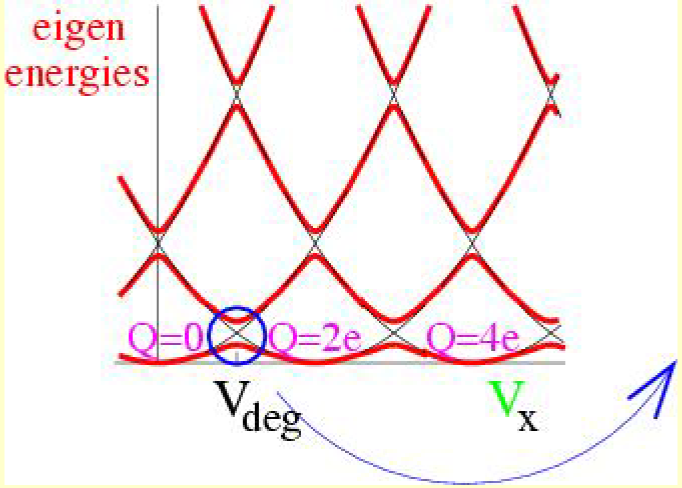


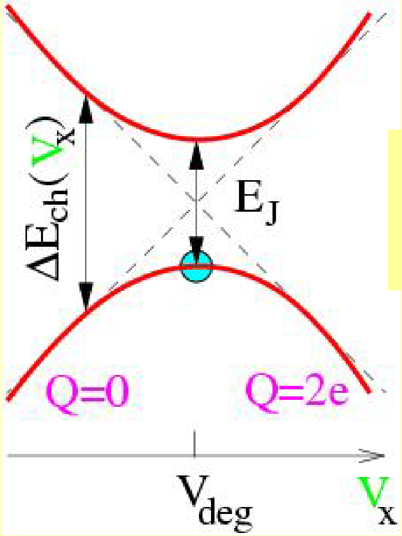
Coherent charge dynamics

$$E_C = e^2/(2C_\Sigma) \gg E_J$$



charging energy + Josephson energy



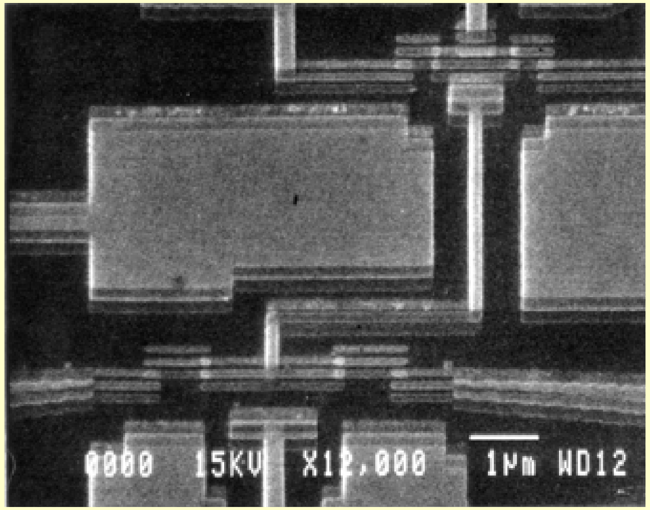


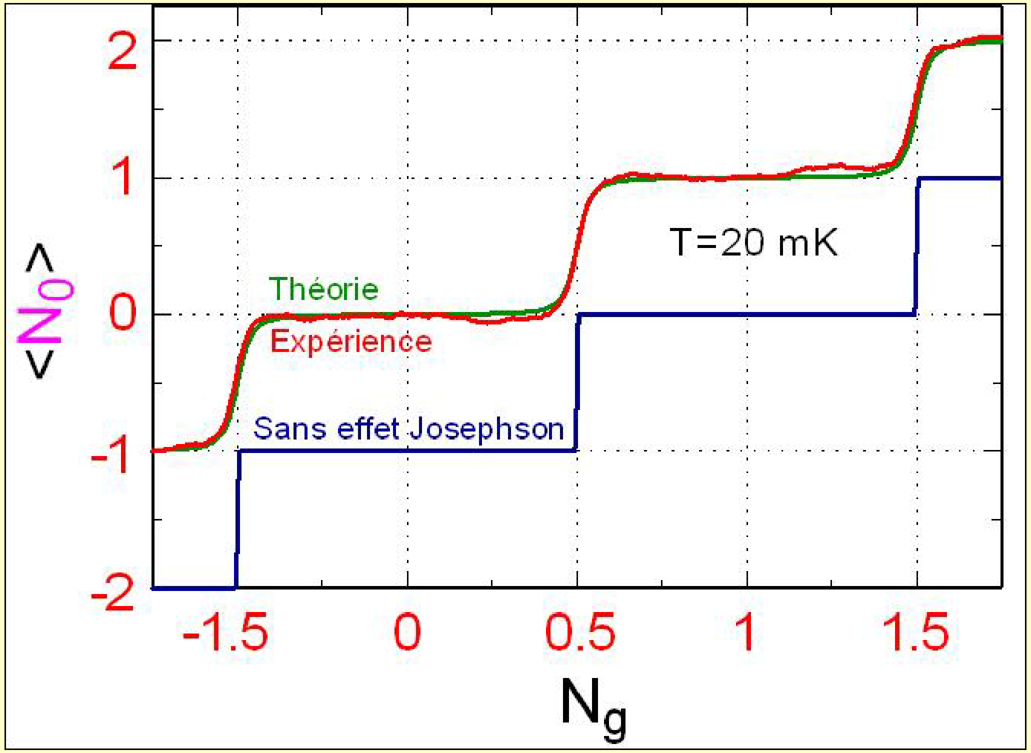
2-level system = qubit

Shnirman et al. 1997
Makhlin et al. 1999

$$\mathcal{H} = -\frac{1}{2}\begin{pmatrix} \Delta E_{ch}(V_x) & E_J \\ E_J & -\Delta E_{ch}(V_x) \end{pmatrix} \quad \begin{matrix} |Q=0\rangle \\ |Q=2e\rangle \end{matrix}$$

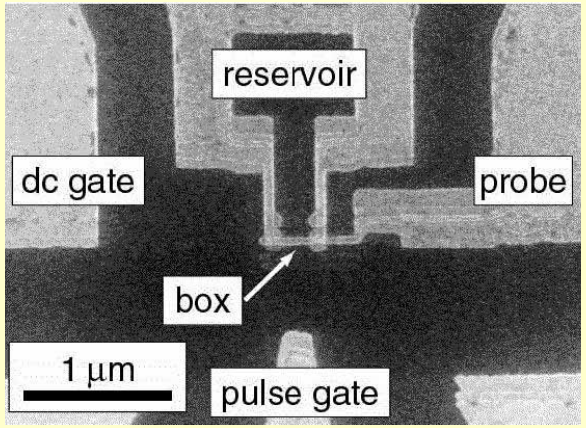
Bouchiat et al., Saclay 1997

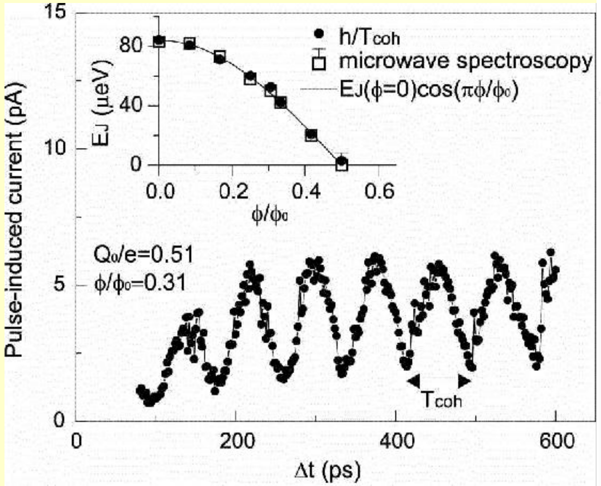


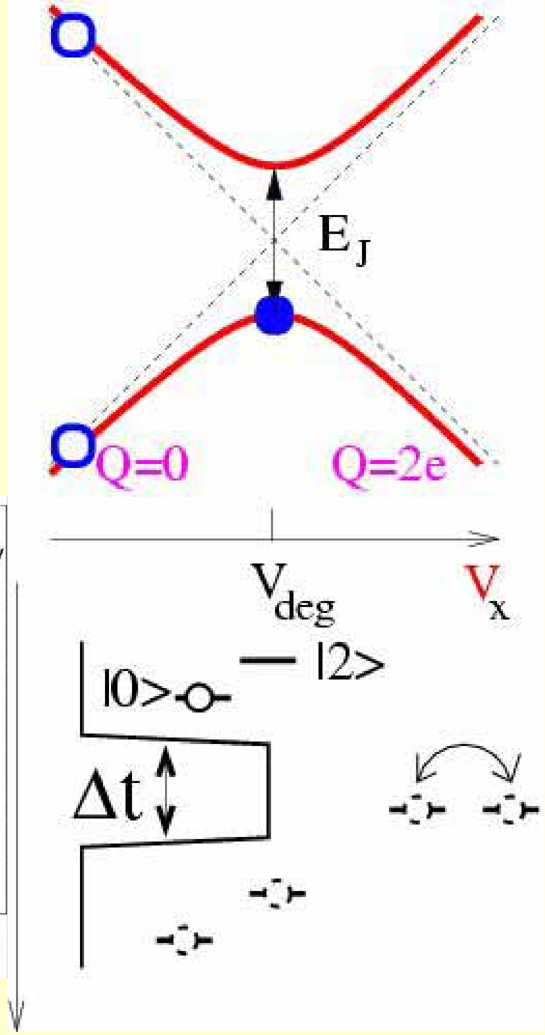


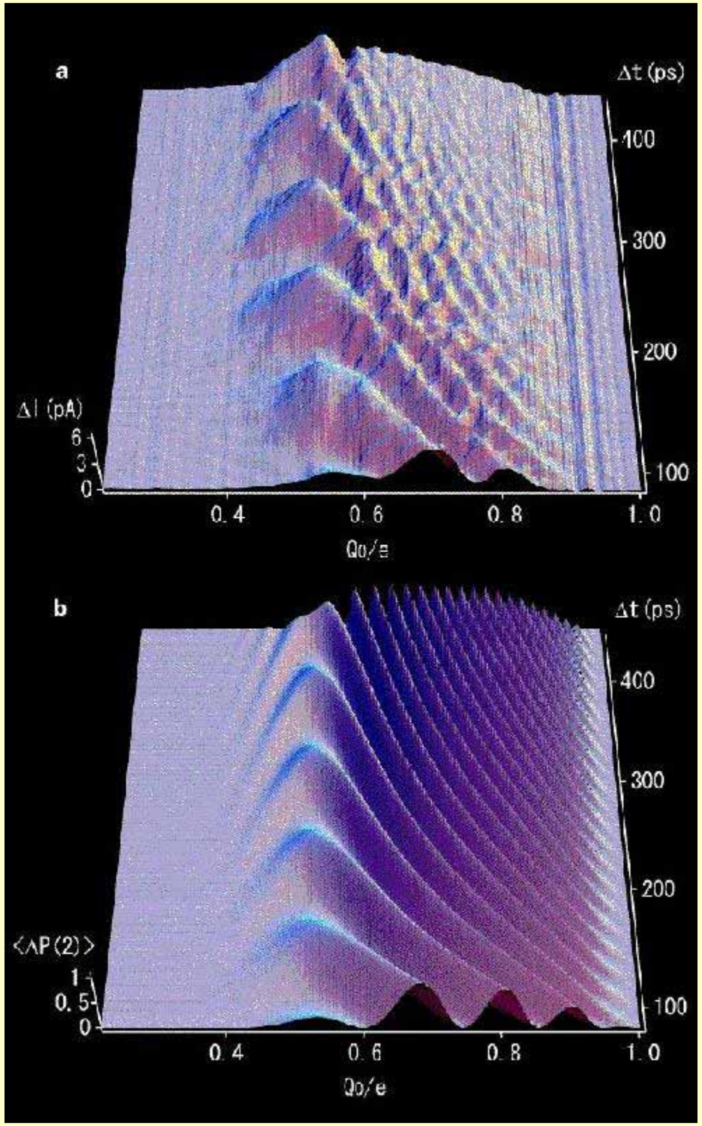
Experiment: coherent oscillations

Y.Nakamura et al., 1999



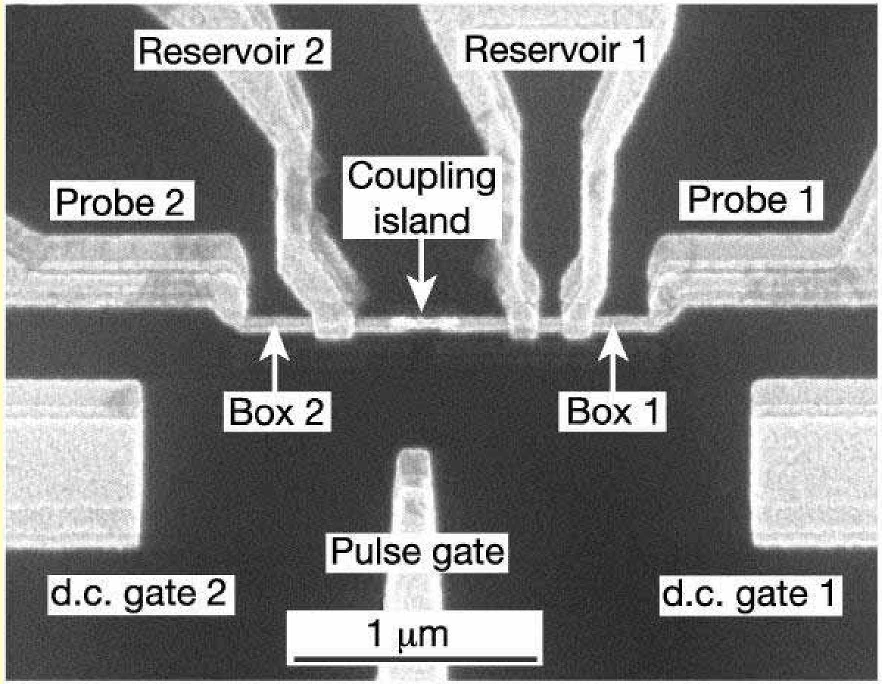


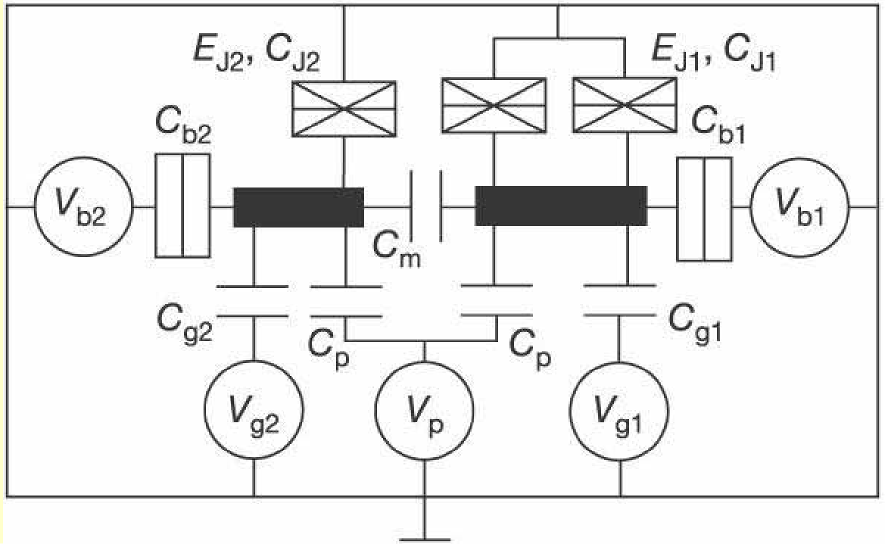




2 qubits: coherent oscillations

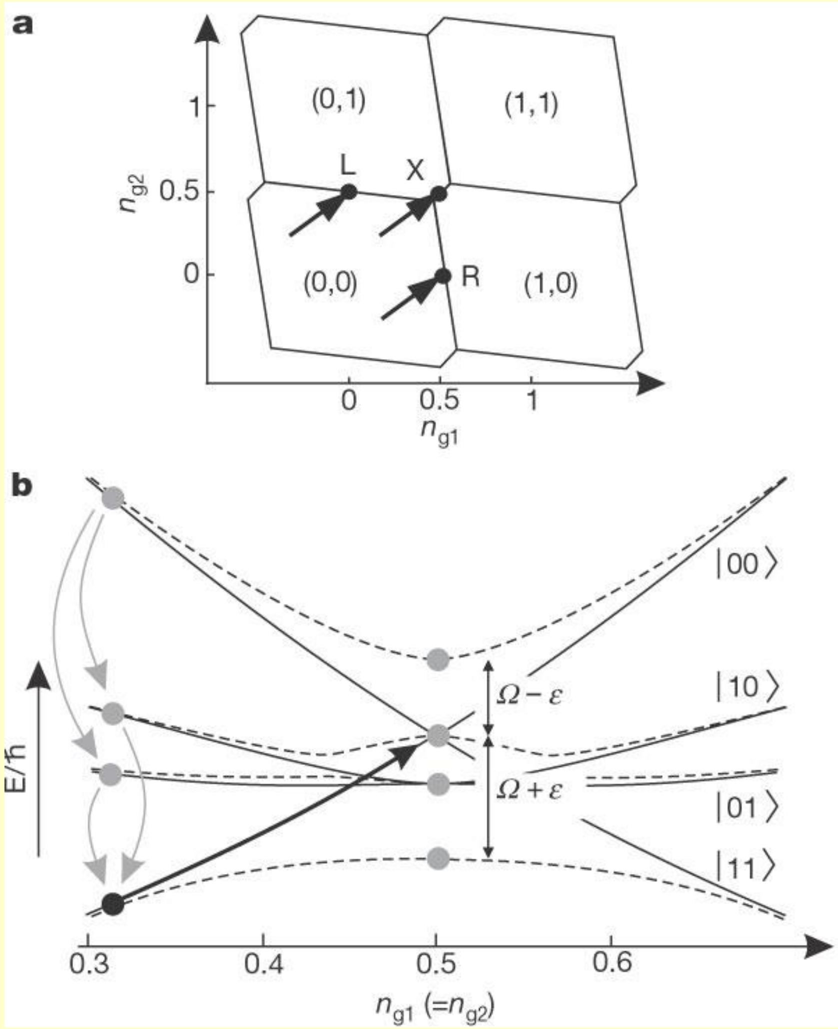
Yu.Pashkin et al., 2002

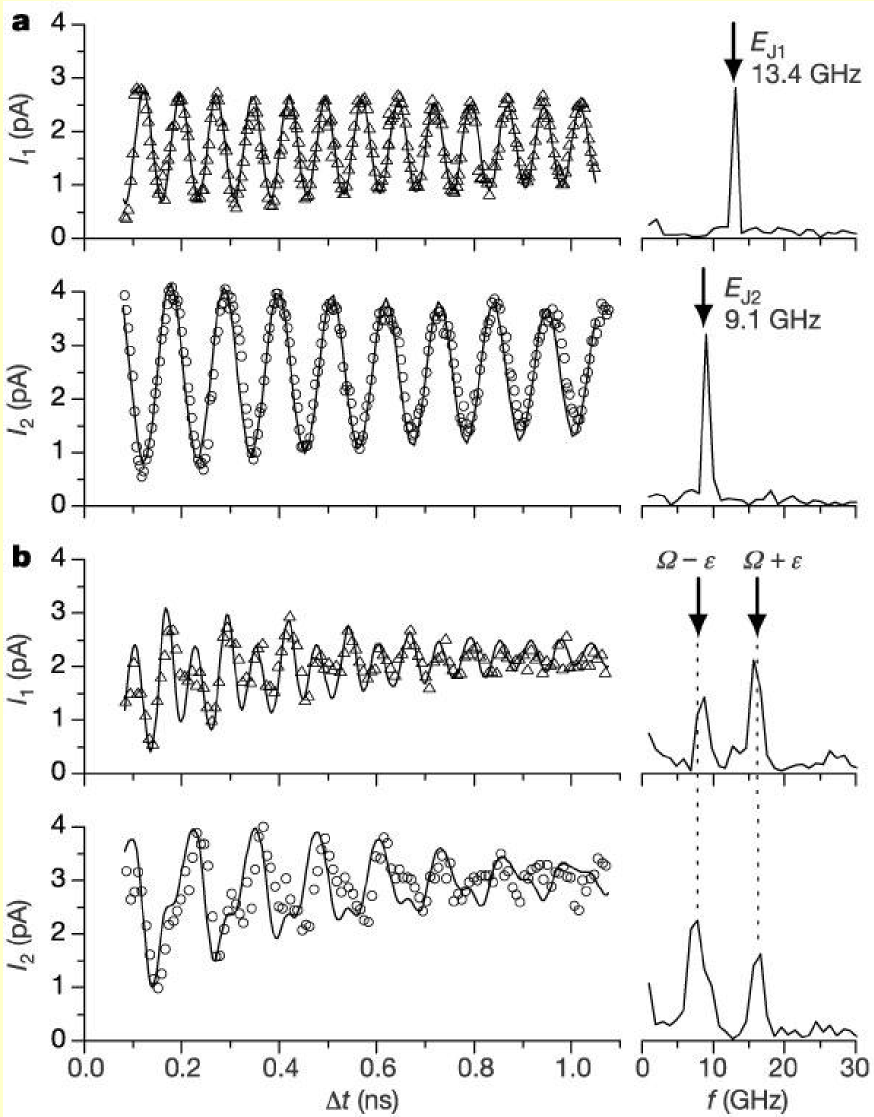




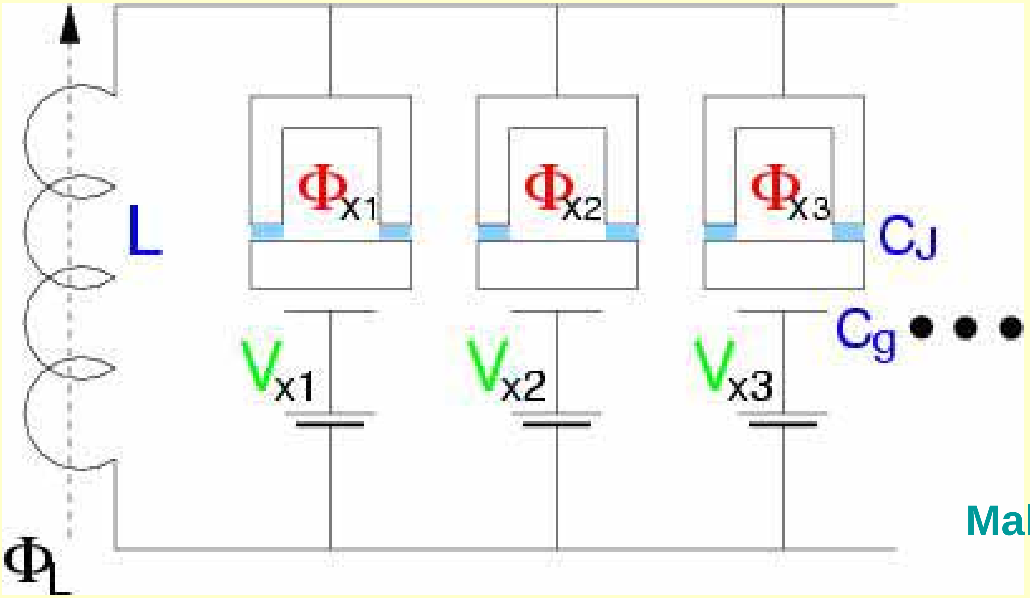
2 qubits: coherent oscillations

Yu.Pashkin et al., 2002





Interaction via an LC-circuit



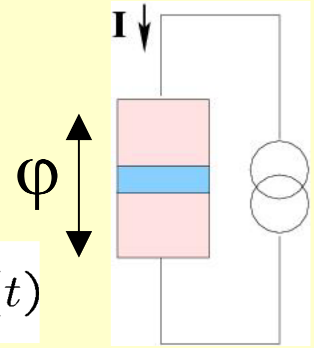
Makhlin et al. 1999

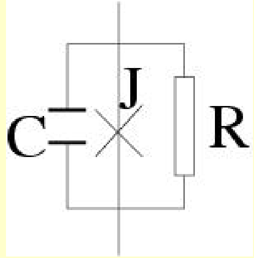
$$\mathcal{H} = -\frac{1}{2}\sum_{i\,1}^{N}\left[\Delta E_{\mathrm{ch}}(V_{\mathrm{x}i})\hat{\sigma}_z^i + E_{\mathrm{J}}(\Phi_{\mathrm{x}i})\hat{\sigma}_x^i\right] + \sum_{i<j}\pi^2\left(\frac{C_{\mathrm{g}}}{C_{\mathrm{J}}}\right)^2\frac{E_{\mathrm{J}}(\Phi_{\mathrm{x}i})E_{\mathrm{J}}(\Phi_{\mathrm{x}j})}{\Phi_0^2/L}\,\hat{\sigma}_y^i\,\hat{\sigma}_y^j$$

controlled

Phase dynamics, SQUIDs

$$E_C = e^2/(2C) \ll E_J$$



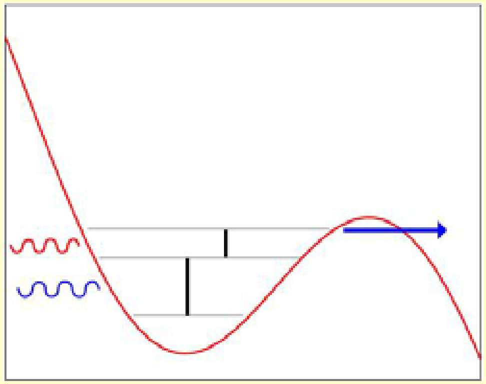


- **classical evolution, RCSJ model**

$$C\frac{\hbar\ddot{\varphi}}{2e} + \frac{1}{R}\frac{\hbar\dot{\varphi}}{2e} + I_c \sin\varphi = I_{ext} + \delta I(t)$$

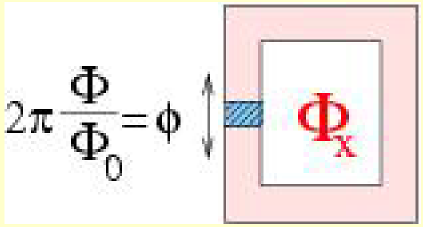
- **macroscopic quantum phase evolution**

$$\mathcal{H} = \frac{Q^2}{2C} - E_J \cos\varphi - I_{ext}\varphi + \mathcal{H}_{diss}; \qquad Q = -i\hbar\frac{\partial}{\partial(\hbar\varphi/2e)}$$

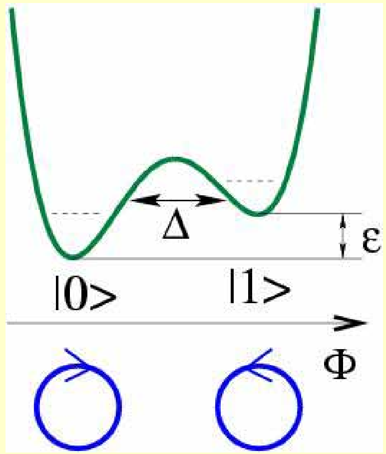


<u>macroscopic</u> & resonant <u>quantum tunneling</u>

- **rf-SQUID**



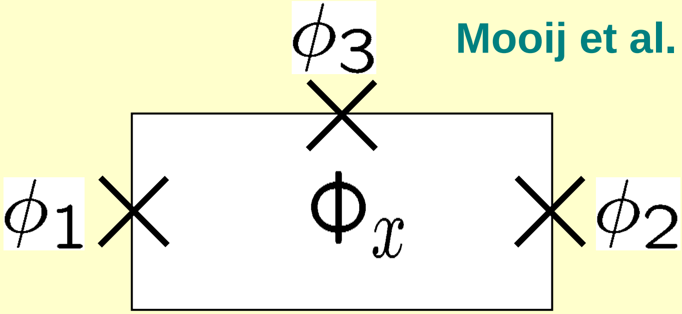
$$\mathcal{H} = \frac{Q^2}{2C} - E_J \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L}$$



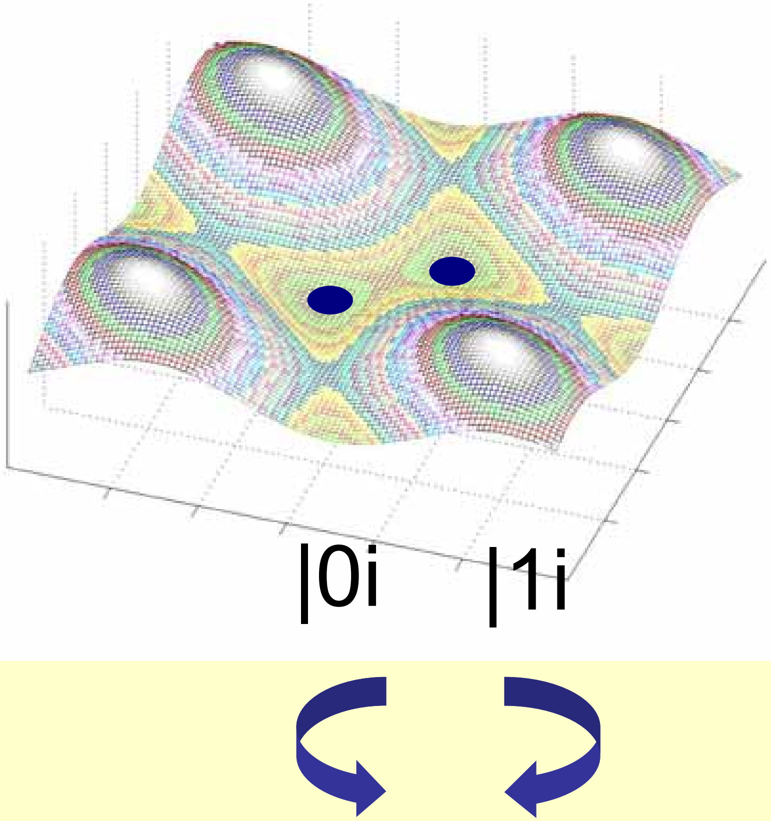
Macroscopic quantum coherence (Leggett)

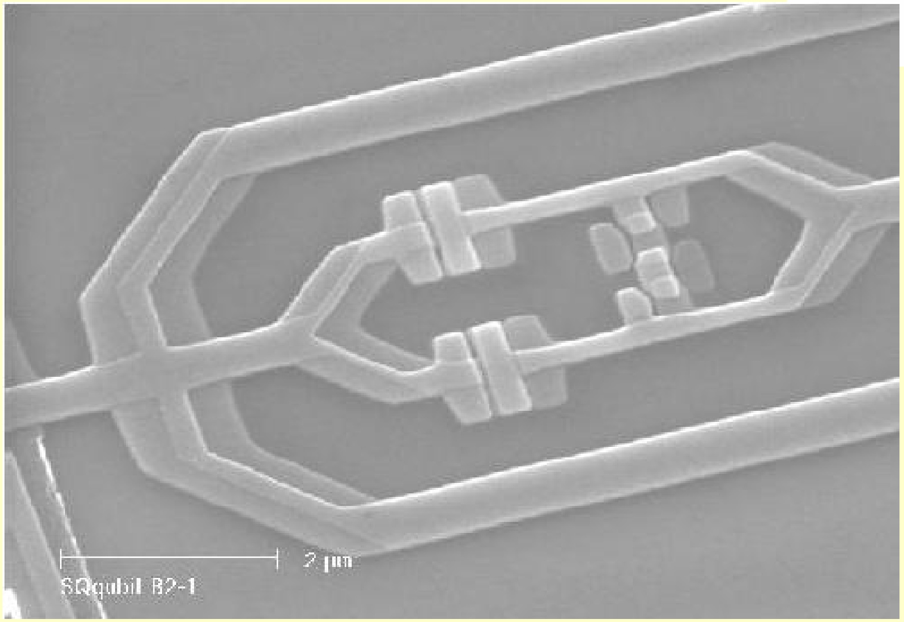
Superconducting flux qubits

Mooij et al. 1999



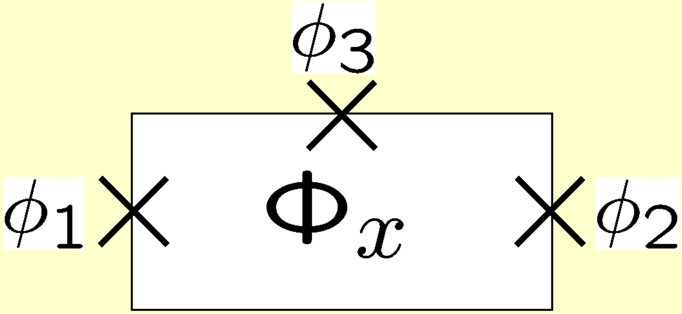
$$\phi_3 = \Phi_x - \phi_1 - \phi_2$$





$$H \approx -\frac{1}{2}\varepsilon(\Phi_x)\,\sigma_z - \frac{1}{2}\Delta\,\sigma_x$$

Chiorescu et al. 2002



$$\phi_3 = \frac{2\pi}{\Phi_0}\Phi_x - \phi_1 - \phi_2$$

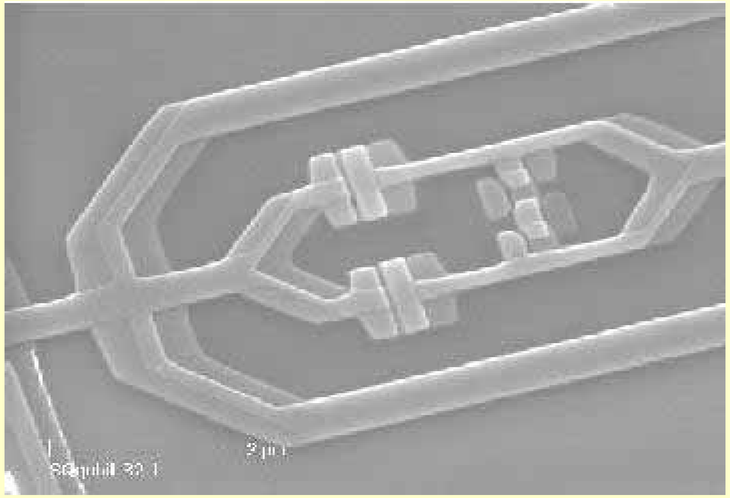
$$U(\phi_1, \phi_2) = -E_J \cos\phi_1 - E_J \cos\phi_2 - \tilde{E}_J \cos(\tfrac{2\pi}{\Phi_0}\Phi_x - \phi_1 - \phi_2)$$

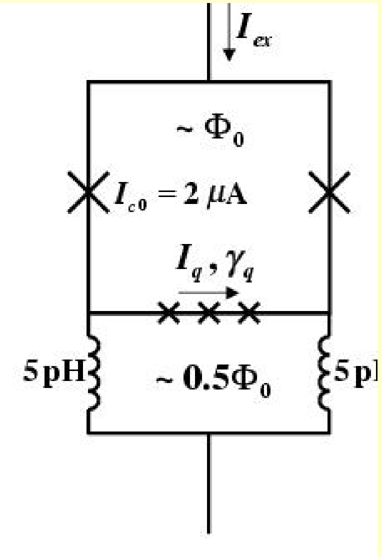
for $\phi_1 = \phi_2, \qquad \Phi_x = \Phi_0$

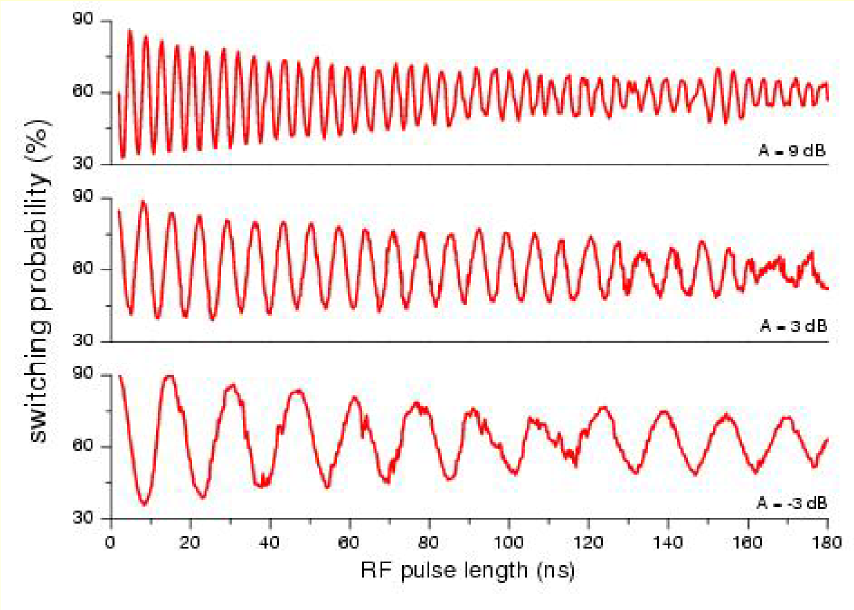
$$U = -2E_J \cos\phi_1 + \tilde{E}_J \cos(2\phi_1)$$

$$\tilde{E}_J \geq 0.5 E_J$$

Coherent oscillations: Delft (2002)

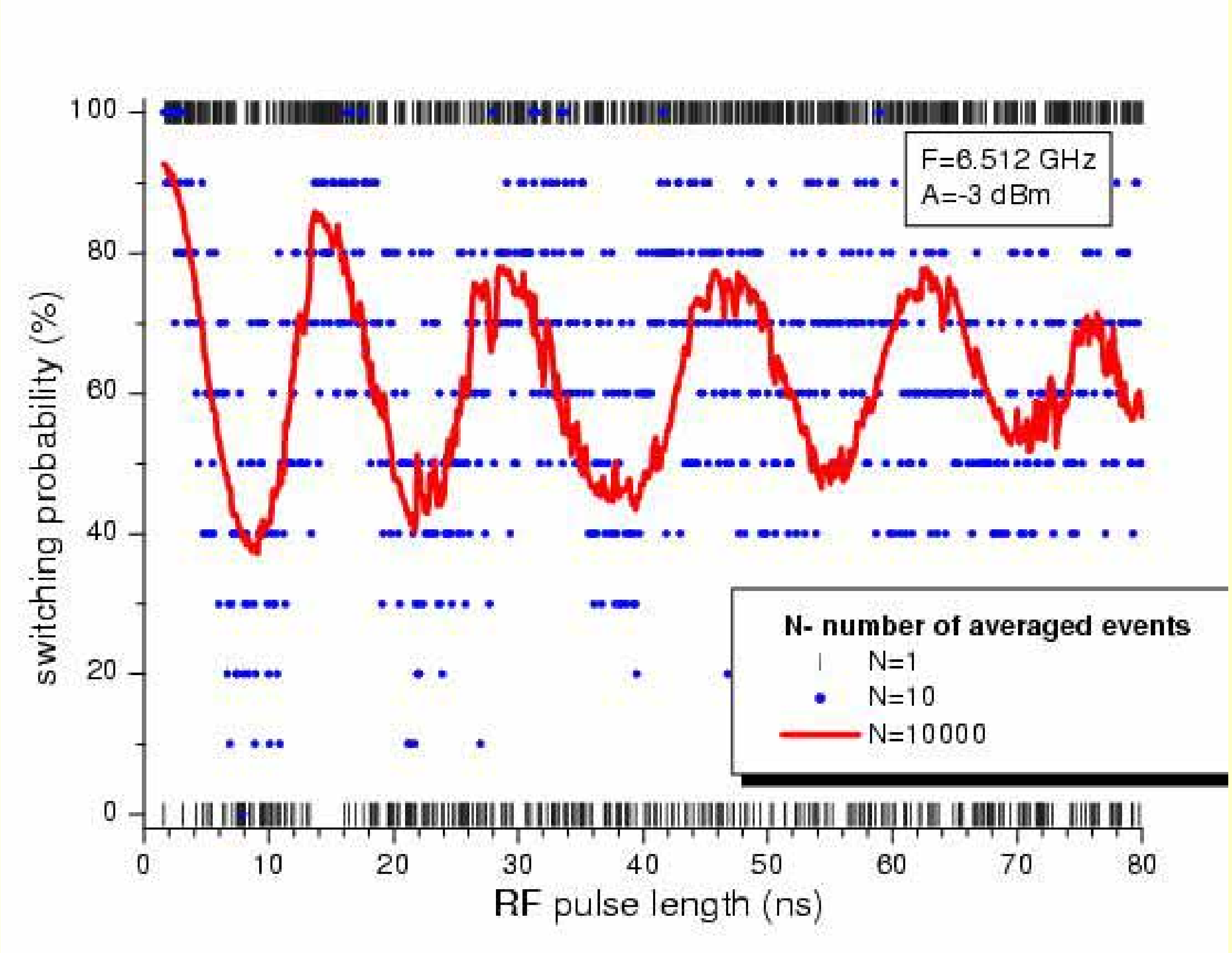






Chiorescu et al. (Delft) 2002

Quantum measurement: magnetic qubit + dc-SQUID



strong threshold measurement