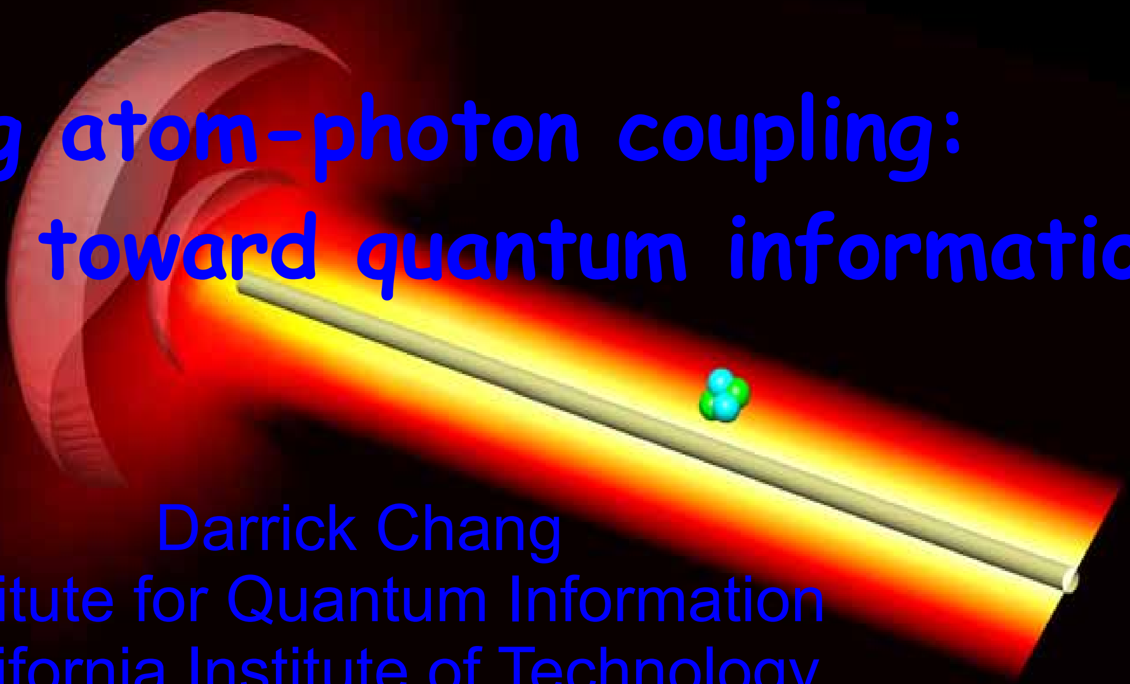
Strong atom-photon coupling: applications toward quantum information



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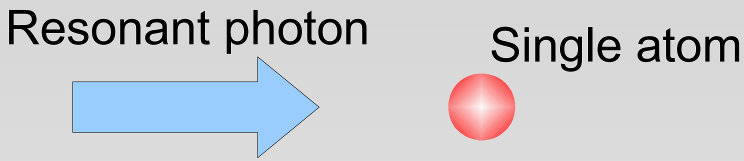
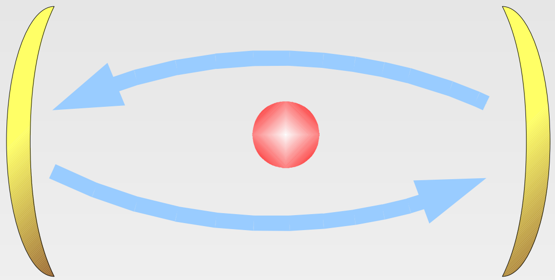
Strong coupling of atoms and photons

- How do we get around inherently weak coupling between atoms and photons?

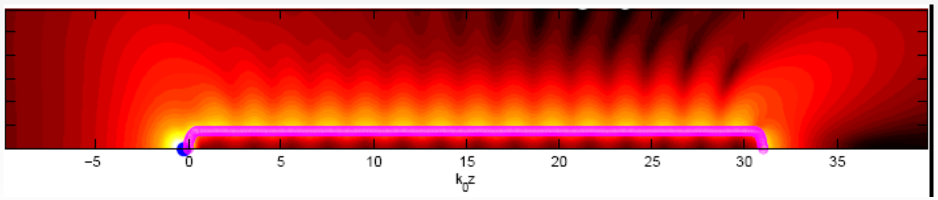


$$\sigma = \frac{3\lambda^2}{2\pi},\ A_{beam} > \lambda^2,\ P_{sc} = \frac{\sigma}{A_{beam}}$$

- Approach #1: Cavity quantum electrodynamics (QED)
  - Put the atom between two mirrors and enhance the interaction by the number of round trips the photon makes



- Approach #2: Plasmonics (“Electrodynamics in 1D”)
  - Circumvent the diffraction limit, $A_{beam} \ll \lambda^2$

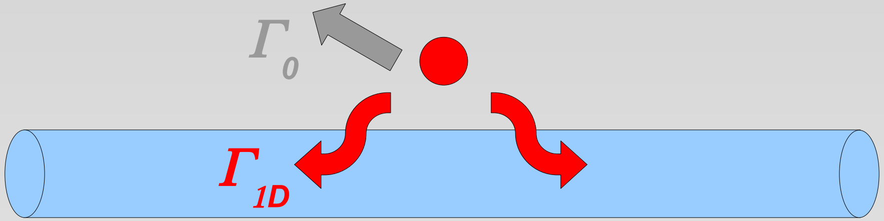


Outline

- Motivation: what's special about strongly confined photons in 1D?
- Strong atom-photon coupling using nanoscale surface plasmons
  - “Cavity QED,” but without a cavity!
- Single-photon nonlinear optics using surface plasmons
  - Single-photon transistor
- “Extreme” nonlinear optics – strongly correlated, many-body photon states
  - Crystallization of photons in a nonlinear fiber
- Outlook on quantum optics at the nanoscale

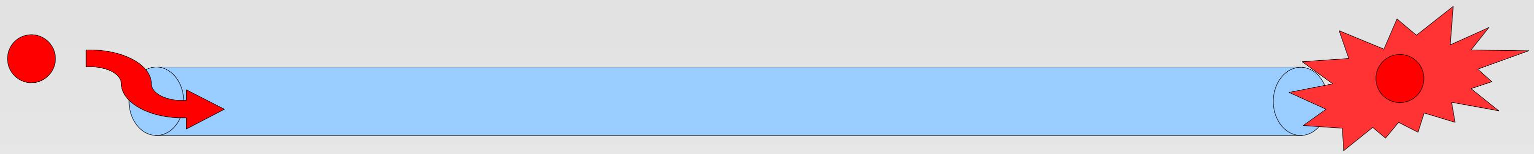
Novel physics in lower dimensions

- Examples of few-particle behavior:
  - Enhancement of spontaneous emission and efficient photon collection

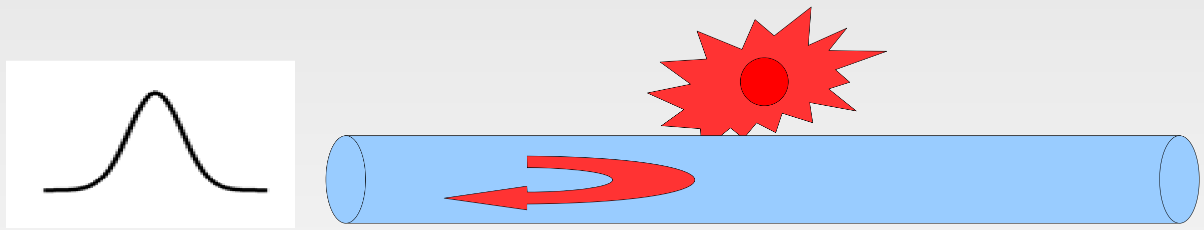


$$\frac{\Gamma_{1D}}{\Gamma_0} \sim \frac{\lambda^2}{A}\frac{c}{v_g}$$

  - Infinite-range interactions (never lose a photon in pure 1D system)



  - Single-photon blockade



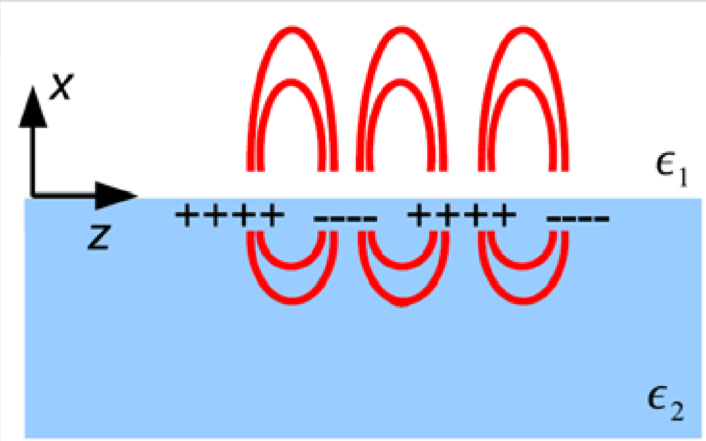
- Physical systems of implementation?
- Use tools of quantum optics to gain control over these processes!
- Even more exotic behavior to learn about?

Surface plasmons on a flat interface

- Surface plasmons – coupled excitations of EM field and ***free-charge density waves*** guided along a conductor-dielectric interface

- Simplest example: SPs on a flat surface

Electric field / charge distribution



$D_i = \epsilon_i E_i$

$\epsilon_1 > 0$ (normal dielectric)

e.g., Drude model $\epsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$, $\omega < \omega_p$

$\epsilon_2 < 0$ (conductor)

- How to derive the surface plasmon modes:
  - Guess solutions

$$E_j = e^{ik_\parallel z - \kappa_j |x| - i\omega t} E_0 (\hat{z} + \alpha_j \hat{x})$$

$\kappa_j \equiv i k_{j,\perp}$

$k_\parallel^2 + k_{j,\perp}^2 = \epsilon_j (\omega/c)^2$

→ Maxwell's Eqns.

$$B_j = e^{ik_\parallel z - \kappa_j |x| - i\omega t} (E_0 / i\omega)(ik_\parallel \alpha_j + \kappa_j \, sign(x)) \hat{y}$$

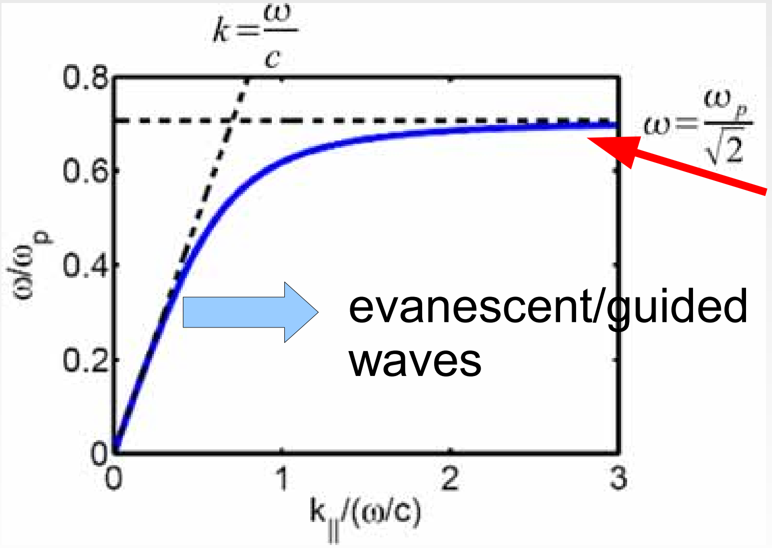
Surface plasmon dispersion relation

- Solve for coefficients α_j and wavevector $k_{\|}$ by enforcing boundary conditions:
  - Continuity of $D_{\perp}$
  - Continuity of $B_{\|}$
- Solution:

$$E_j = e^{ik_{\|} z - \kappa_j |x| - i\omega t} E_0 \left(\hat{z} + i \frac{k_{\|}}{\kappa_j} \hat{x} \right)$$

$$k_{\|}^2 = \left(\frac{\omega}{c} \right)^2 \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}$$

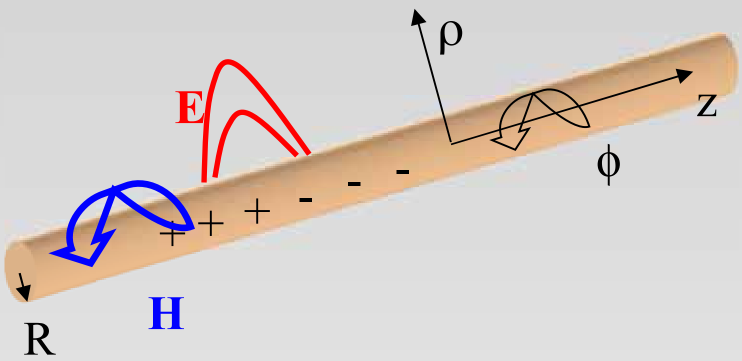
- Dispersion relation for Drude model $\epsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \; , \omega < \omega_p$



- "X-ray wavelengths at optical frequencies"
- Small wavelengths and tight confinement only at frequencies around $\omega_p / \sqrt{2}$
  - Very material dependent

Surface plasmons on a nanowire

- Surface plasmon modes of a nanowire:



- Like optical fiber modes, but charge leads to unique properties!

- Finding modes of nanowire: use separation of variables

$$E_j = \alpha_j e^{ik_\parallel z - i\omega t} e^{im\phi} F_j(k_{j\perp}\rho),\ \text{similar expression for } H_j$$

$$j=1(\text{outside wire}),\ j=2(\text{inside wire}),\ F_j\ \text{Bessel/Hankel functions}$$

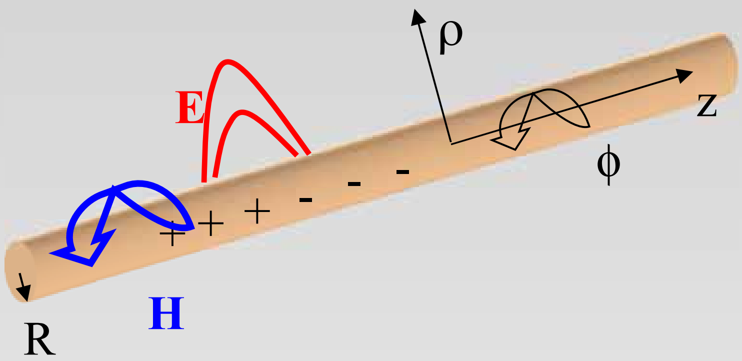
- Solve for coefficients α_j, $k_{||}$ by enforcing proper boundary conditions at the interface ρ=R, for example,

$$\epsilon_1 E_{1,\perp}(R) = \epsilon_2(\omega) E_{2,\perp}(R)$$

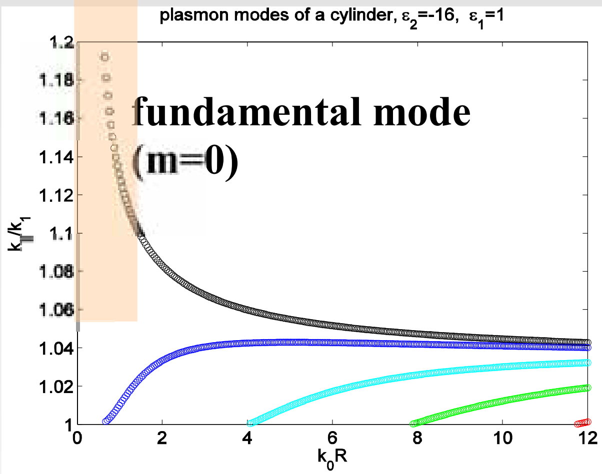
Unique feature: ε(ω)<0 for a conductor, *e.g.*, Drude model $\epsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$, $\omega < \omega_p$

Surface plasmons on a nanowire

- Surface plasmons of a nanowire

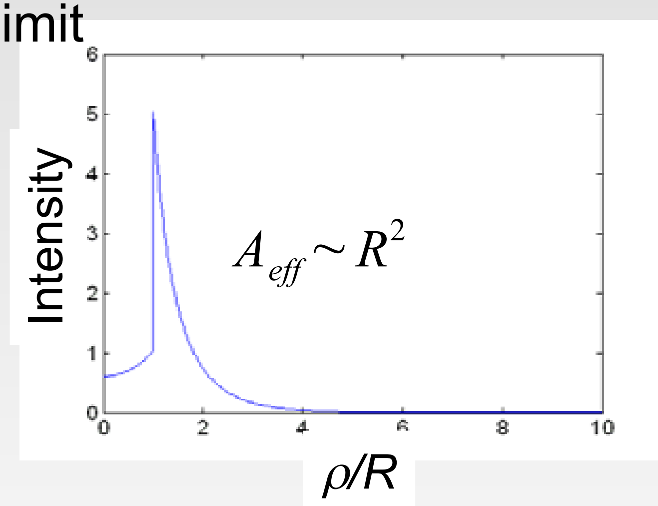


- Slow phase and group velocities ($k \sim 1/R$)



$k_{\|}$ vs. R

- Confinement below diffraction limit



- Sub-wavelength wire ➔ system becomes a single-mode fiber
- Single mode behaves "***electrostatically***" -- R becomes only length scale

A more mathematical explanation

- To solve a full electrodynamics problem, need to find the E&M Green's function, *e.g.*,

$$G(r,r')=\frac{e^{ik|r-r'|}}{4\pi|r-r'|}\ \text{in free space}$$

- For small systems (like a nanowire),

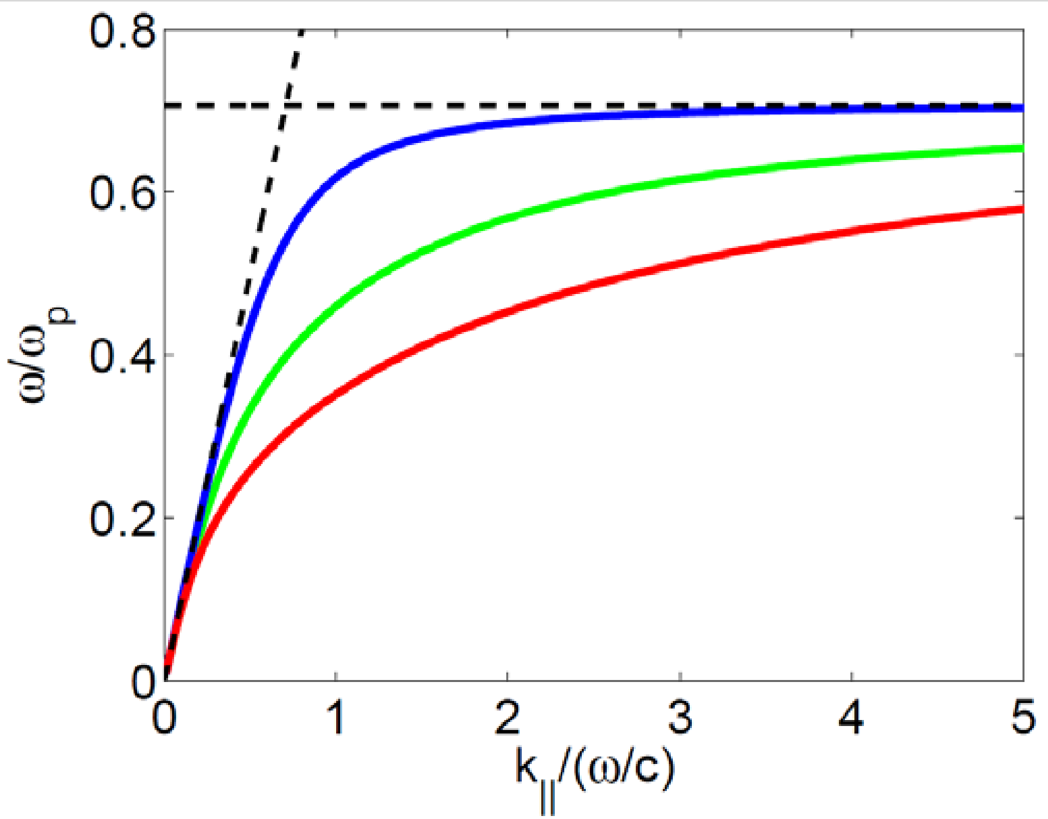
$$G(r,r')\sim\frac{1}{4\pi|r-r'|}+O(k)=\text{Electrostatic Green's function!}$$

  - For small systems, can just solve Laplace's Equation, $\nabla^2\Phi=0$, to find electric field and charge distribution

- In electrostatics, the wire radius R is the only length scale ($\lambda=2\pi/k$ cannot appear in the solution)
  - R should determine all of the electric field and charge distribution properties

A geometrical effect

- Compare the surface plasmon dispersion relations of a flat interface and nanowire (Drude model):

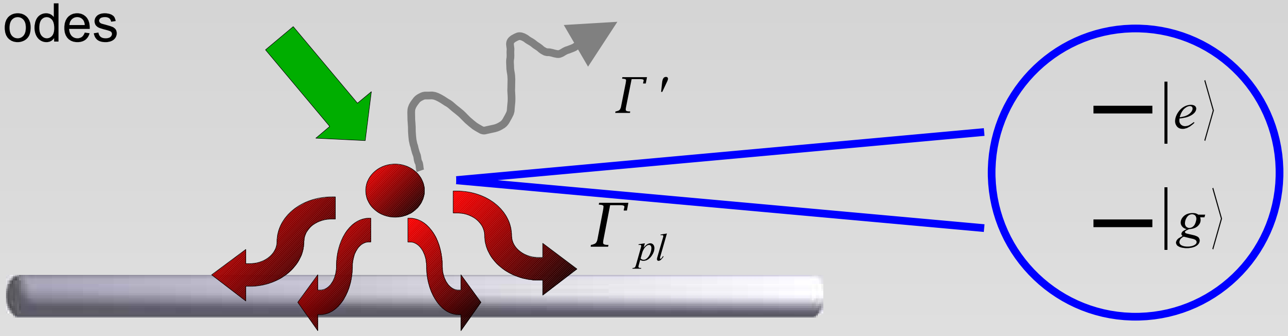


Blue: flat interface
Green: $k_0R = 0.5$
Red: $k_0R = 0.2$

- Small wires pull dispersion relation to the right
  - Can achieve "x-ray wavelengths" at any frequency! Not just near $\omega_p/\sqrt{2}$
  - Tight confinement now achieved by wire size (geometry) instead of material properties

Strong coupling to surface plasmons

- Slow velocity + tight confinement = ***strong emission*** into surface plasmon modes



- Hamiltonian for system resembles a multi-mode Jaynes-Cummings model

$$H=\hbar\omega_0\sigma_{ee}+\int dk\,\hbar v|k|\hat{a}_k^{\dagger}\hat{a}_k+\int dk\,\frac{\hbar g_k}{2}\left(\sigma_{eg}\hat{a}_k e^{ikz_a}+h.c.\right)$$

Waveguide modes: k>0 (k<0) for right (left) propagating modes, $v=d\omega/dk$ is group velocity of surface plasmons

$$v\propto\omega R$$

SP-atom coupling: atom can be excited and destroy a photon at position z_a, coupling strength

$$g_k\sim d_0\sqrt{\frac{\hbar\omega_k}{\epsilon_0 A_{eff}}}f(k_{1,\perp}\rho)$$

$$g_k\propto\frac{d_0}{R}\sqrt{\frac{\hbar\omega_k}{\epsilon_0}}$$

Spontaneous emission into SPs

- Spontaneous emission rate into SPs can be calculated using Fermi's Golden Rule
  - Valid for a discrete system coupled to a continuum of modes
  - Also can be calculated using an *"input-output"* formalism similar to cavity coupled to waveguide

- Fermi's Golden Rule:

$$\Gamma_{pl} \sim g(\omega_0)^2 D(\omega_0)$$
$$\sim g(\omega_0)^2 (d\omega / dk)^{-1}$$

$$g_k \propto \frac{d_0}{R}\sqrt{\frac{\hbar\omega_k}{\epsilon_0}}$$

$$d\omega / dk = v \propto \omega R$$

- Putting everything together,

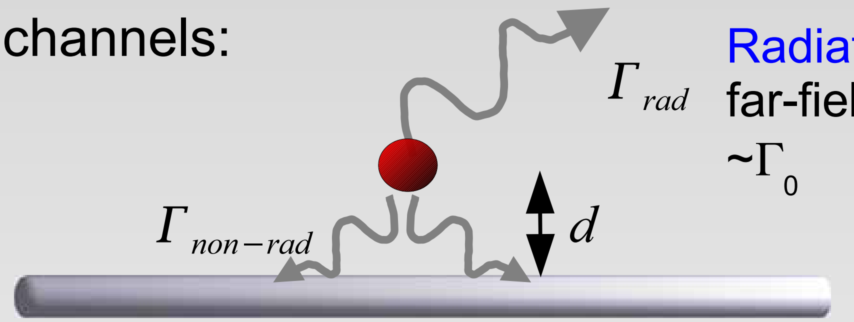
$$\boxed{\Gamma_{pl} \sim \frac{\Gamma_0}{(k_0 R)^3}}\text{, where } \Gamma_0 = \text{spontaneous emission rate of atom in free space}, \; k_0 = 2\frac{\pi}{\lambda_0}$$

- **A broadband, geometrical effect (occurs just by using small wires!)**

The effective "Purcell" factor

- Like in cavity QED, the rate Γ_{pl} is not always of direct importance itself
  - Need to compare this "good" with "bad" decay rates

- The bad decay channels:



Radiative emission: atoms emits into far-field radiative modes at a rate $\sim\Gamma_0$

Non-radiative emission: atomic near-field induces local "currents" that are dissipated by the conductor (causing heating)

$$\Gamma_{non-rad} \sim \frac{\Gamma_0}{(k_0 d)^3} \boxed{\frac{\mathrm{Im}\,\epsilon_2}{(\mathrm{Re}\,\epsilon_2)^2}}$$

Electric near field of dipole (atom) scales like $1/d^3$

Is proportional to material losses/absorption (imaginary part of ε)

$$\epsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \approx 1 - \frac{\omega_p^2}{\omega^2} + i\frac{\omega_p^2\gamma}{\omega^3}$$

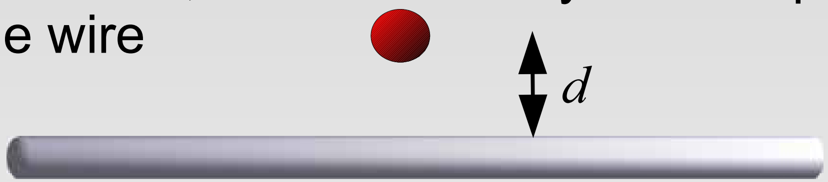
Effective Purcell factor

- Like cooperativity parameter C in cavity QED, we can define a branching ratio between good and bad decay channels

$$P(r)=\frac{\Gamma_{pl}}{\Gamma_{rad}+\Gamma_{non-rad}}$$

Effective "Purcell factor" for surface plasmons

- P is position-dependent, since the decay rates depend on how close the emitter sits to the wire



$$\Gamma_{pl}\sim\frac{\Gamma_0}{(k_0 R)^3}e^{-k_{1,\perp}d}$$

decays exponentially away from wire edge

$$\Gamma_{non-rad}\sim\frac{\Gamma_0}{(k_0 d)^3}\frac{\mathrm{Im}\,\epsilon_2}{(\mathrm{Re}\,\epsilon_2)^2}$$

diverges as one approaches wire edge

- To maximize P, one wants the emitter to sit within the evanescent field of the SPs, but not so close to the wire edge that non-radiative emission dominates

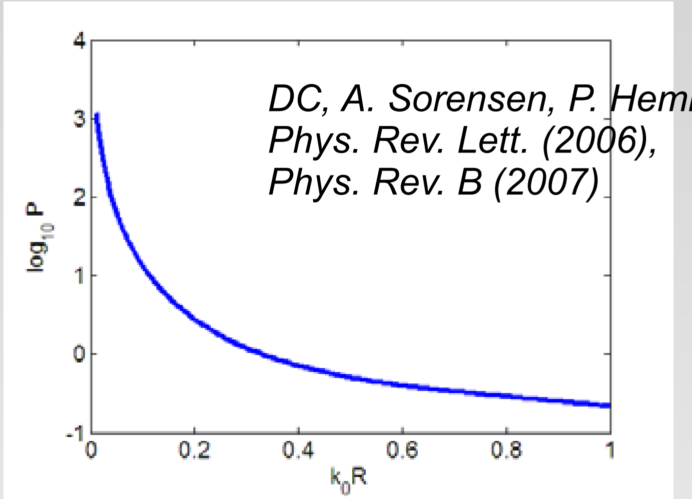
- Also better to use smaller wires to increase Γ_{pl}

Optimization of Purcell factor

- Purcell factor as a function of wire size (optimized over position of emitter)

Effective Purcell factor $P = \frac{\Gamma_{pl}}{\Gamma_{other}}$

approaching 10^3 in realistic systems

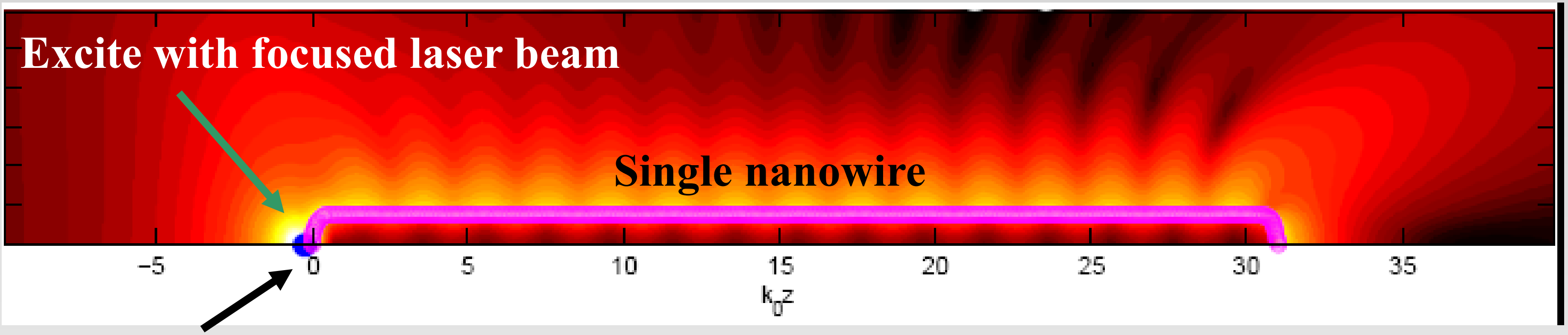


- The regime P>1 is the strong-coupling regime for SP systems (just like C>1 for cavity QED)
  - Many important protocols for quantum information have efficiencies depending only on P (or C)

- Nanowire acts as a "***super lens***" with extraordinary numerical aperture
  - Emission almost completely directed into the nanowire
- ***A broadband, geometrical effect!***

Probing the strong coupling regime

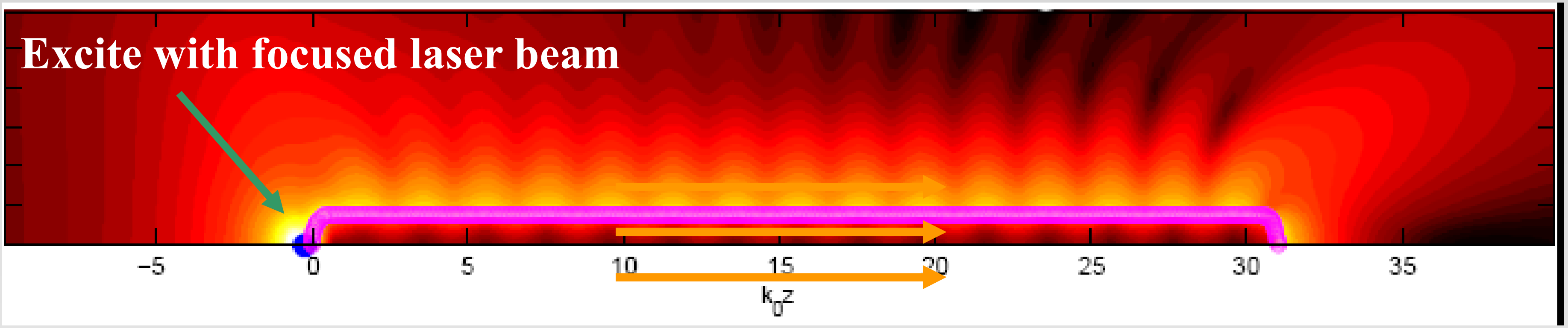
- A first experiment to detect strong coupling:



Single emitter (two-level atom) coupled to nanowire

Probing the strong coupling regime

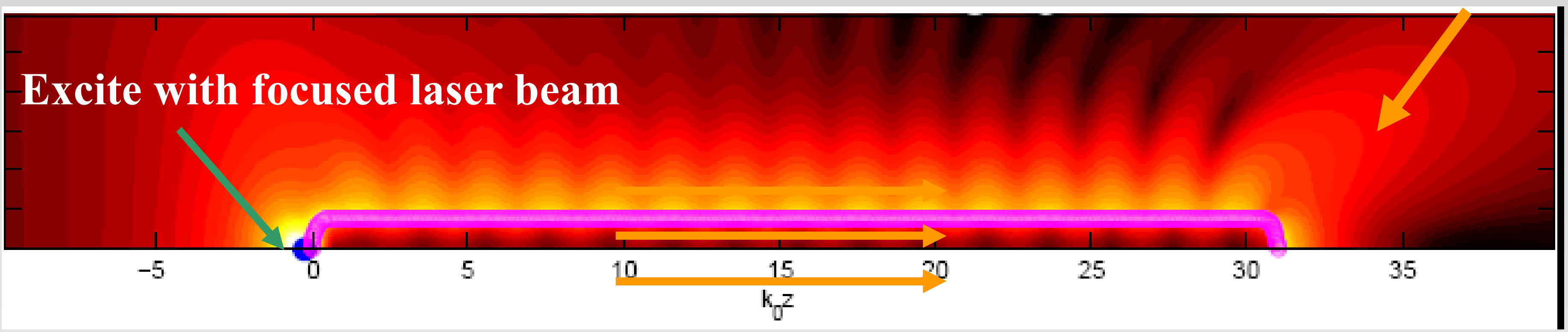
- A first experiment to detect strong coupling:



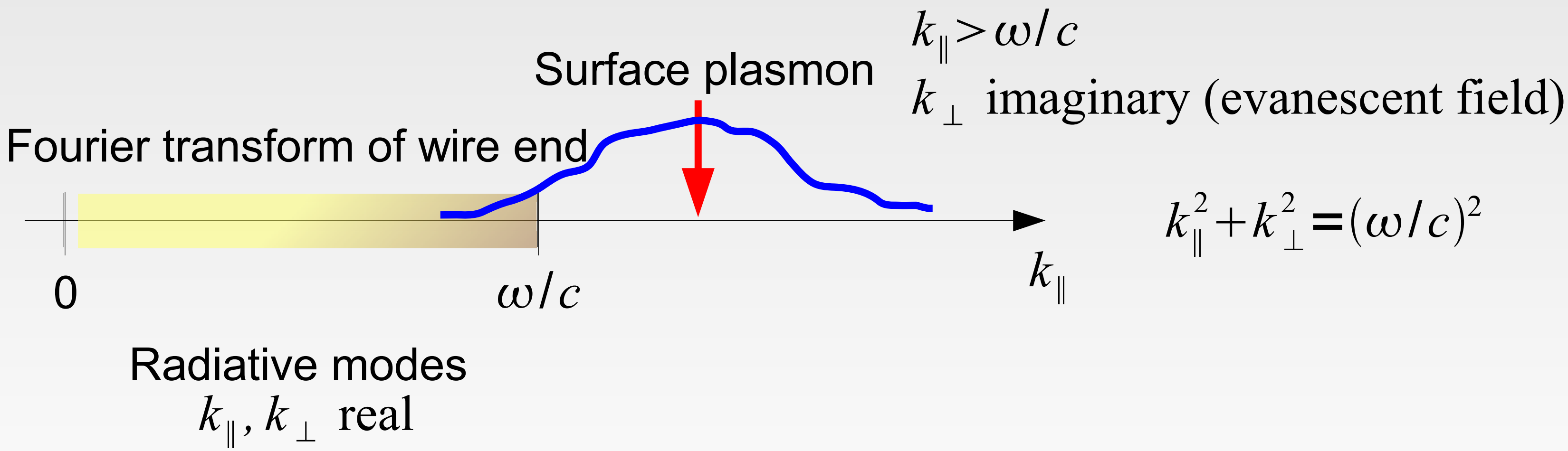
Probing the strong coupling regime

- A first experiment to detect strong coupling:

Collect light scattered off wire end into the far field

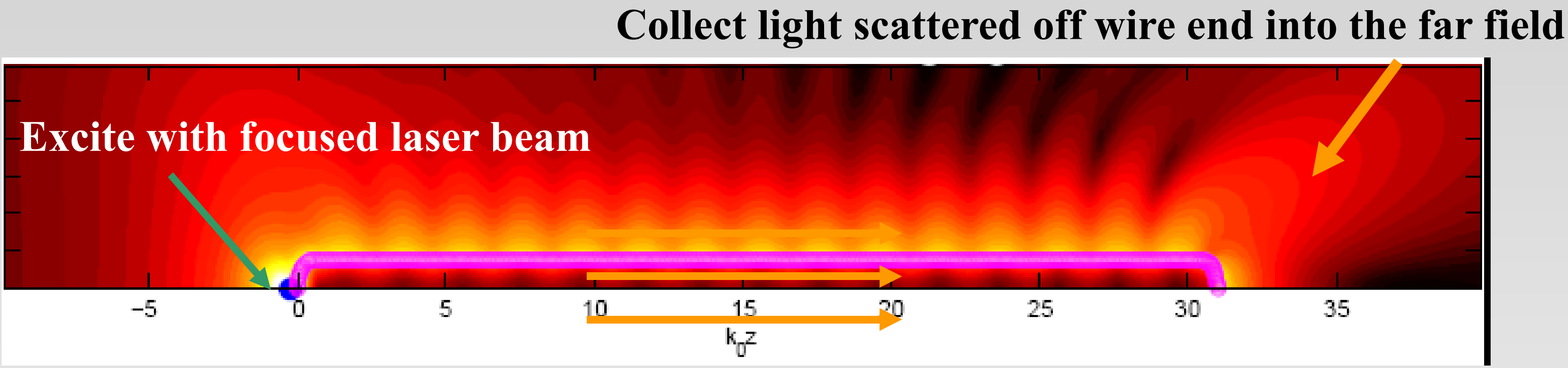


- Scattering process as momentum transfer:



Probing the strong coupling regime

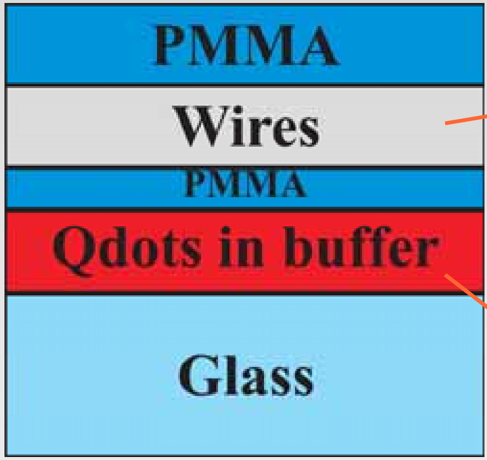
- A first experiment to detect strong coupling:

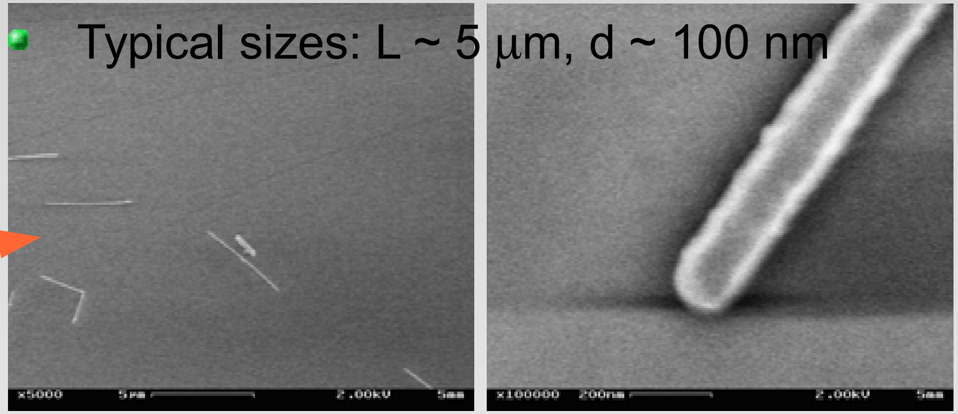


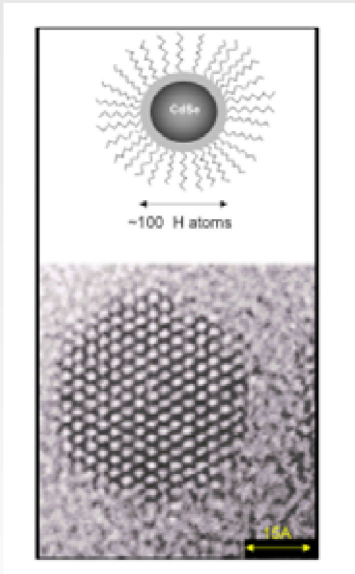
- Collected light should consist of single photons ➔ *measurable by field correlations*

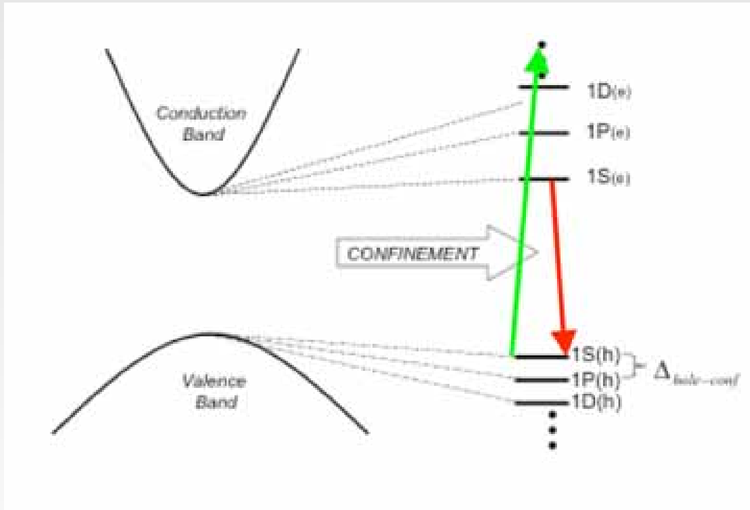
Sample preparation

- Chemically synthesized crystalline silver nanowires
- Typical sizes: L ~ 5 μm, d ~ 100 nm





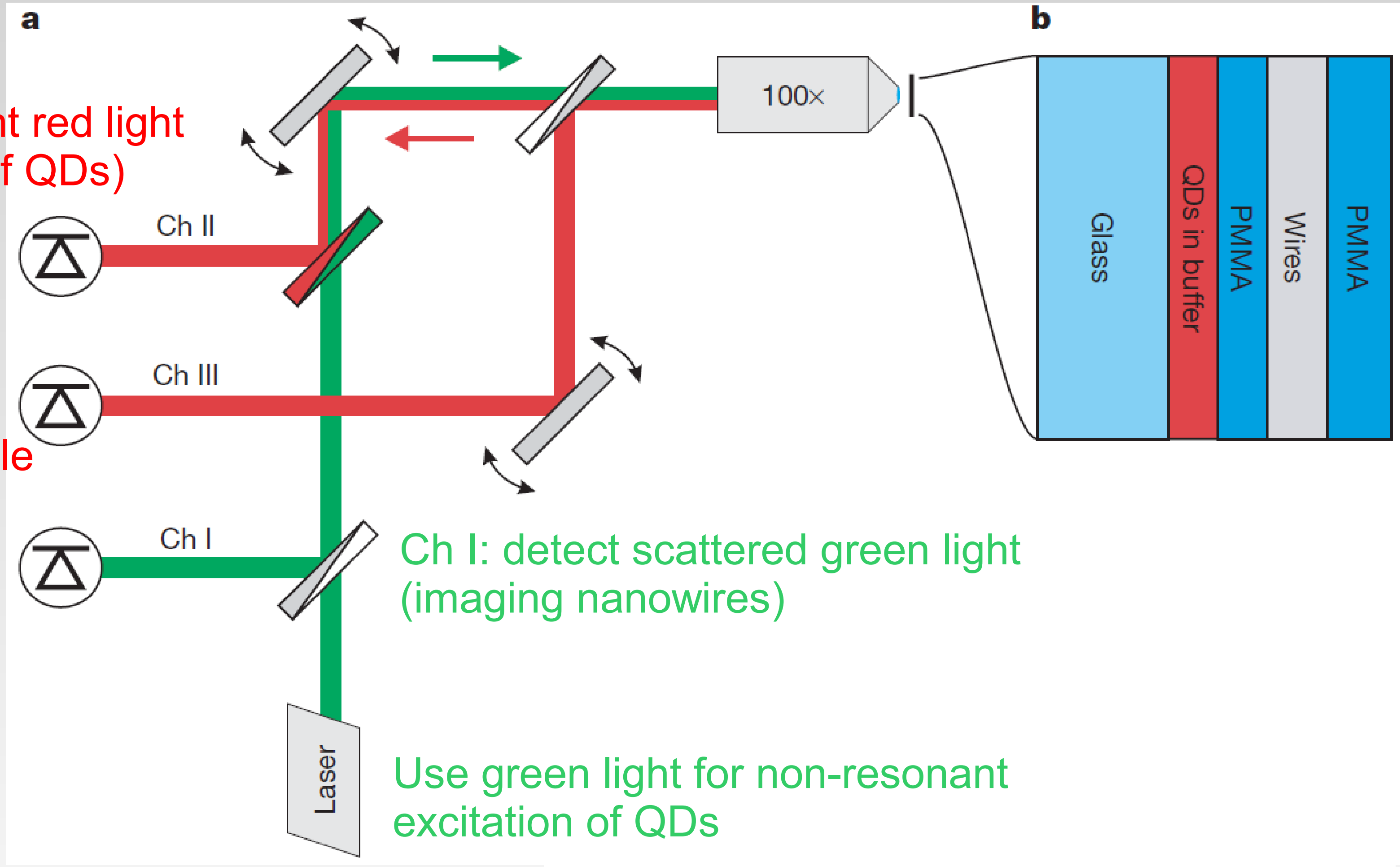




- Good for imaging: excite w/ green, collect in red

Imaging setup

- Modified confocal microscopy



Ch II: detect fluorescent red light (free-space emission of QDs)

Ch III: independent red scan (scattering of single photons from wire end)

Ch I: detect scattered green light (imaging nanowires)

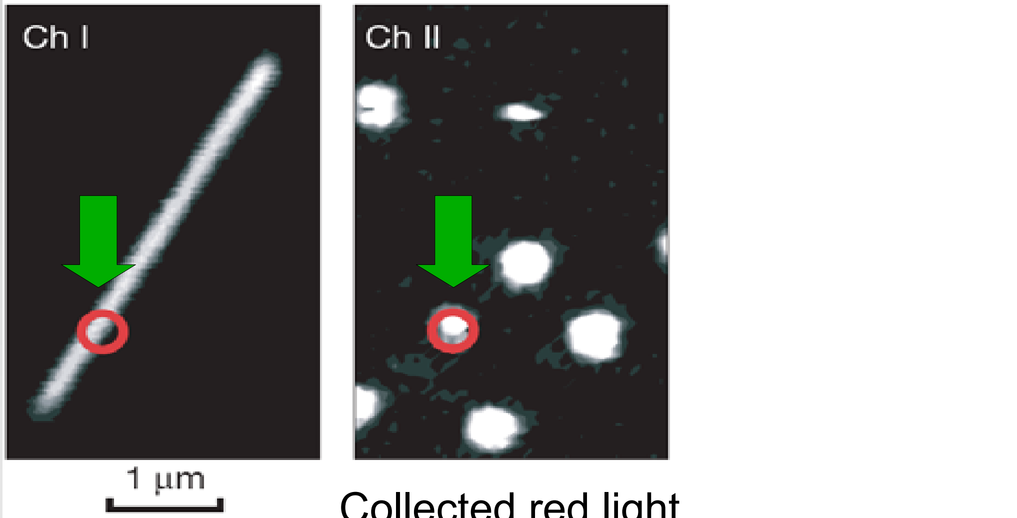
Use green light for non-resonant excitation of QDs

Observation of strong coupling

- Typical image (green excitation):

Wire end lights up!

Scattered green light

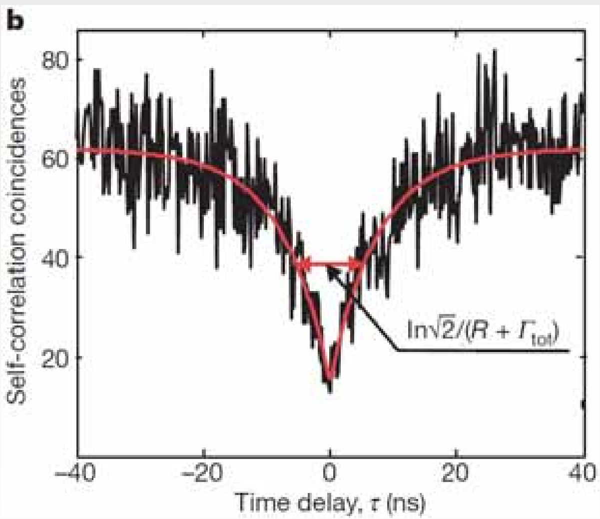


Collected red light

Independent red scan

- Check that the green arrow is really a single QD: measure second-order field correlation function of its direct fluorescence

$G^{(2)}(\tau)$



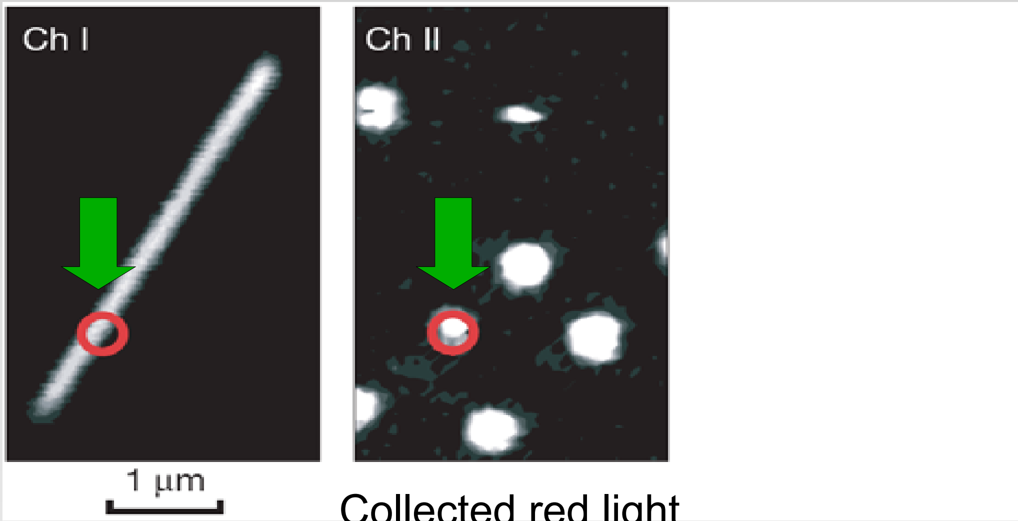
- If my light detector clicks at τ=0, what is the likelihood it clicks again at time τ?
- Observe anti-bunching: a single two-level atom!

Observation of strong coupling

- Typical image (green excitation):

Wire end lights up!

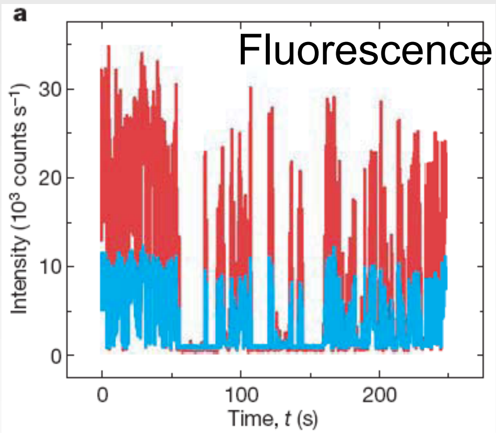
Scattered green light



Collected red light

Independent red scan

- What about light scattered off the wire end?



Fluorescence time trace

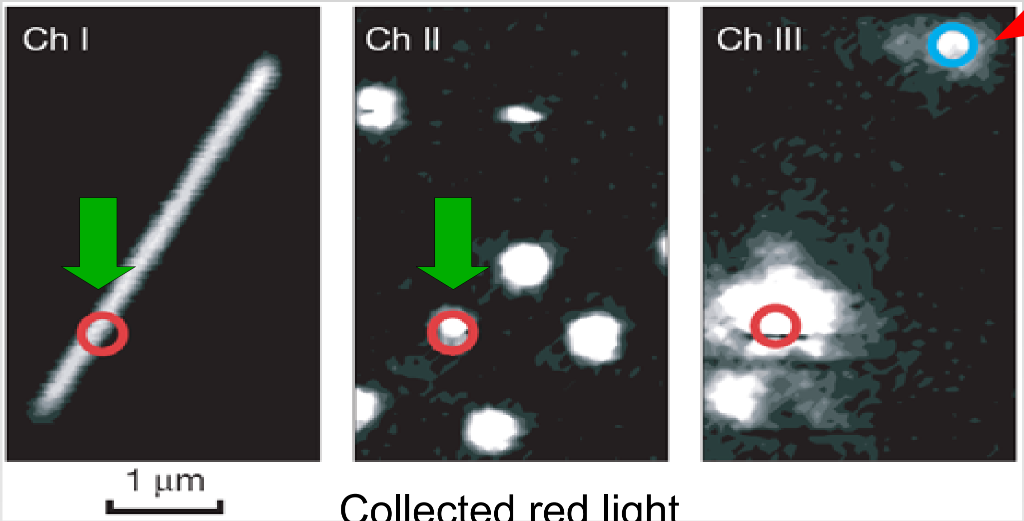
- High degree of correlation indicates light at wire end originates from QD

Observation of strong coupling

- Typical image (green excitation):

Wire end lights up!

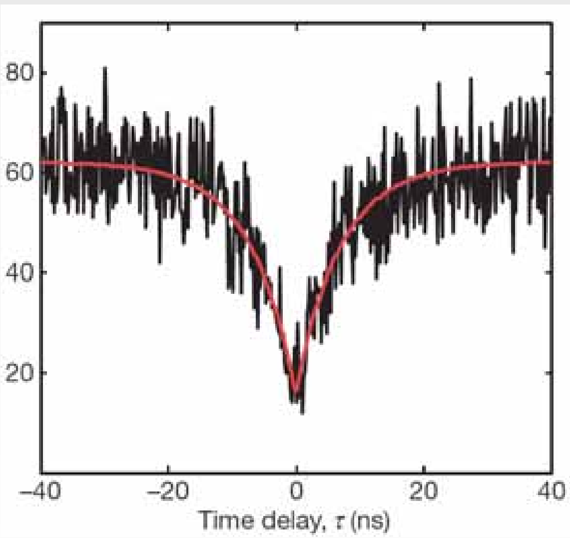
Scattered green light



Independent red scan

Collected red light

- What about light scattered off the wire end?



- Cross-correlation function: if I measure light from direct emission, what is the likelihood I also measure light at wire end?

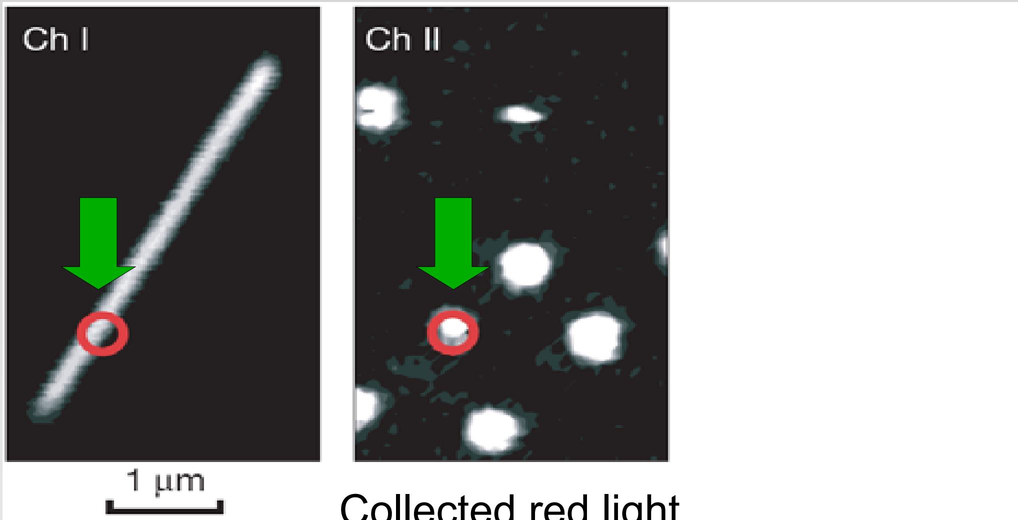
- Anti-bunching: the QD emits a single photon either into free space or into SPs, but never both at the same time!

Observation of strong coupling

- Typical image (green excitation):

Wire end lights up!

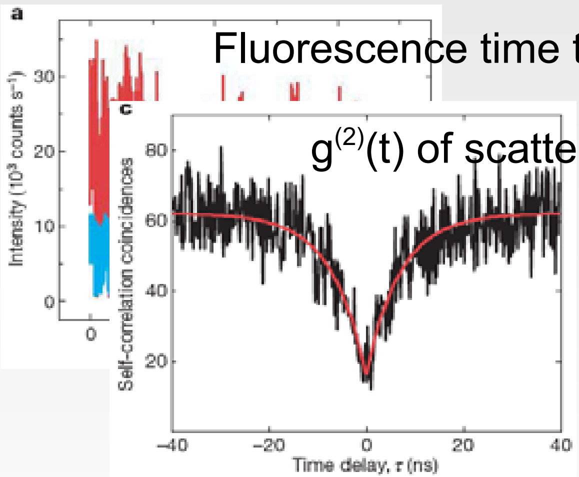
Scattered green light



Independent red scan

Collected red light

- What about light scattered off the wire end?



Fluorescence time trace

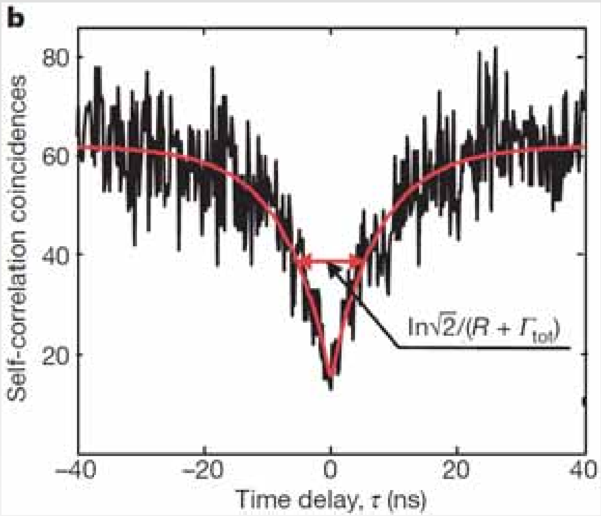
$g^{(2)}(t)$ of scattered light

- Efficient single photon emission into nanowire surface plasmons
- ***Broadband coupling*** – does not depend on QD inhomogenities, spectral diffusion, etc.

What can we conclude?

- Efficient coupling between a single quantum emitter (QD) and single, quantized surface plasmons (single photons)
- Broadband coupling – effect is observed despite large inhomogeneity of QDs, spectral diffusion, etc., and is observed at room temperature

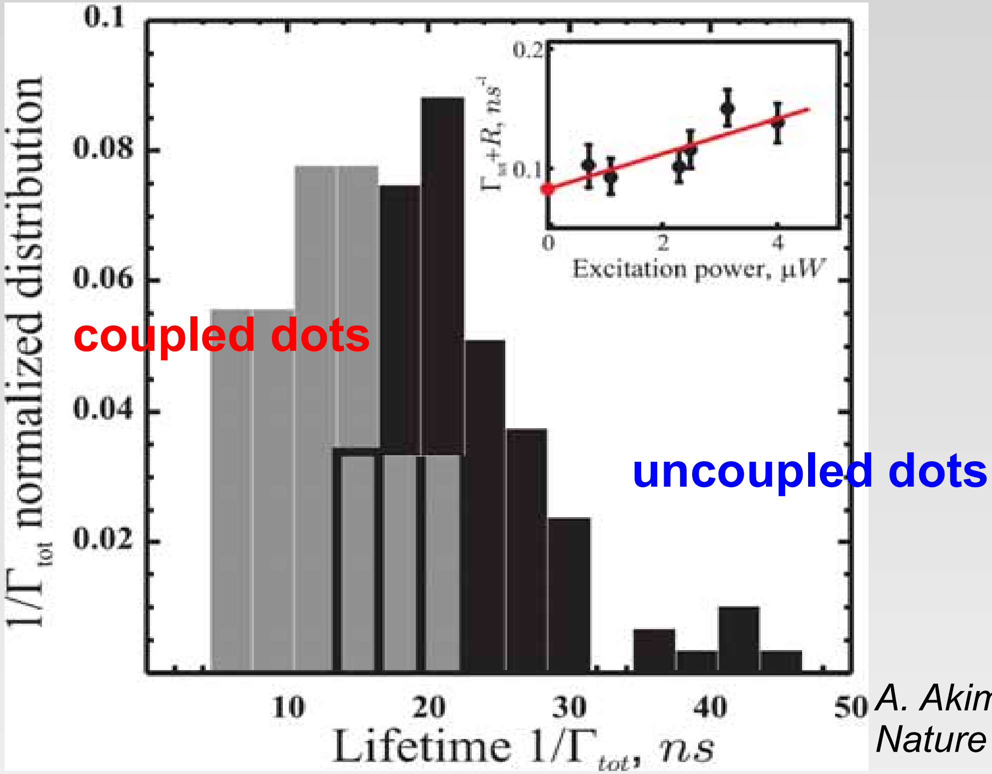
- We can also determine the Purcell factor by looking at field correlation functions



The *width* of the anti-bunching dip tells us how long it takes for the system to get ready to emit another photon (*i.e.*, spontaneous emission time)

Enhancement and coupling efficiency

- QDs and samples are very inhomogeneous, but can build up distributions

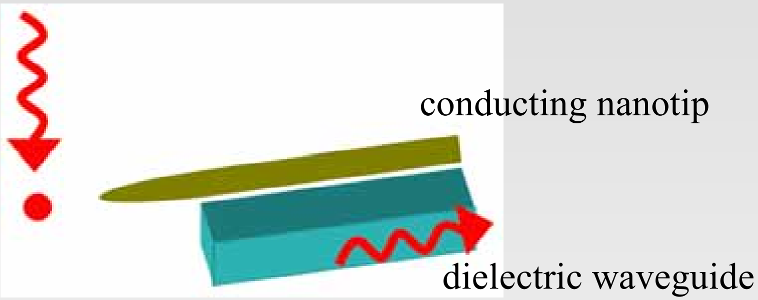


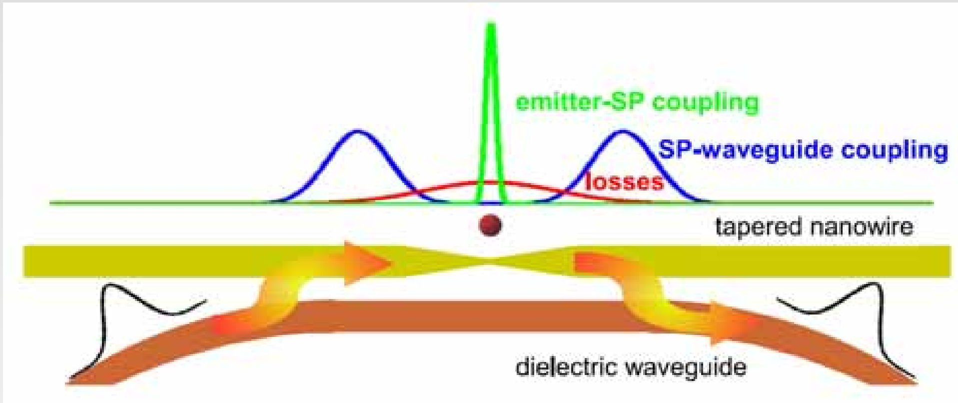
A. Akimov, A. Mukherjee, C. Yu et al., Nature (2007)

- **2.5x enhancement of spontaneous emission, 60% coupling efficiency into nanowire**
- Can do better with smaller wires, but harder to see scattered SPs

Integrated systems

- Previous approach:
  - Propagation distance limited by losses in conductor
  - Scattering at wire ends is not very efficient and is omnidirectional
- Solution: optimized nano-structure geometries and evanescent out-coupling to dielectric waveguides



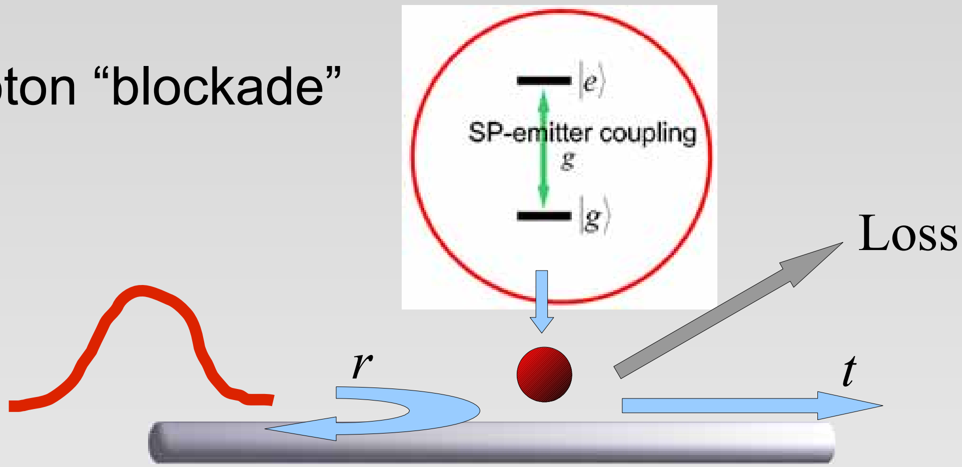


- Theory: **95%** emitter to waveguide coupling is possible when optimized
- Enables many exciting opportunities
  - Single photons on demand
  - Coupling of distant qubits involving "passing and catching" photons
  - Large-scale integrated systems

Single-photon nonlinear optics

- Tight confinement of plasmons ➔ strong nonlinear interactions between photons mediated by emitters

- Single-photon "blockade"



$$|r|^2+|t|^2+\text{Loss}=1$$

- Let's build up some intuition
  - P $\longrightarrow\infty$ (true 1D system): there can be no losses, since the atom always re-scatters back into the 1D waveguide. One never loses the photon!

$$|r|^2+|t|^2=1$$

  - P>>1 (occurs when A_{eff} << λ^2): the atom cross section (~ λ^2) is larger than the pipe carrying the photons. It "clogs" the pipe and the photon *must be reflected* most of the time!

$$|r|\approx 1$$

Reflection and transmission coefficients

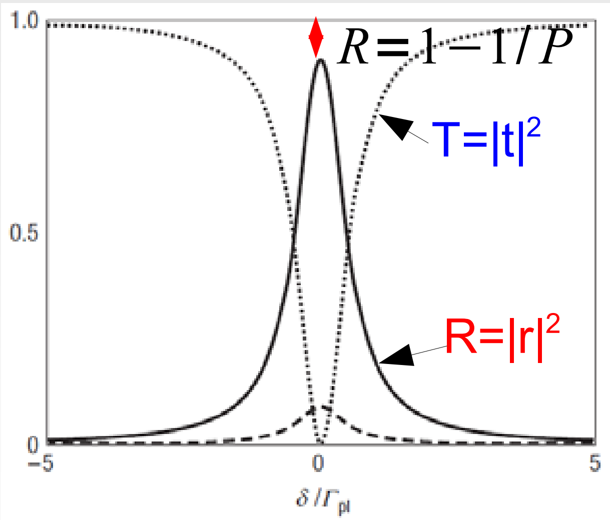
- r,t can be calculated using an input-output formalism
  - Relates light scattered by the atom to light in the nanowire

Cavity coupled to waveguide ⟷ Atom coupled to waveguide

$$\text{cavity: } \hat{E}(z,t)=\hat{E}_{free}(z-vt)+i\sqrt{\frac{\kappa}{v}}\Theta(z-z_c)\hat{a}(t-(z-z_c)/v)$$

$$\text{atom: } \hat{E}(z,t)=\hat{E}_{free}(z-vt)+i\sqrt{\frac{\Gamma_{pl}}{v}}\Theta(z-z_a)\sigma_{ge}(t-(z-z_a)/v)$$

- Reflectance and transmittance spectrum

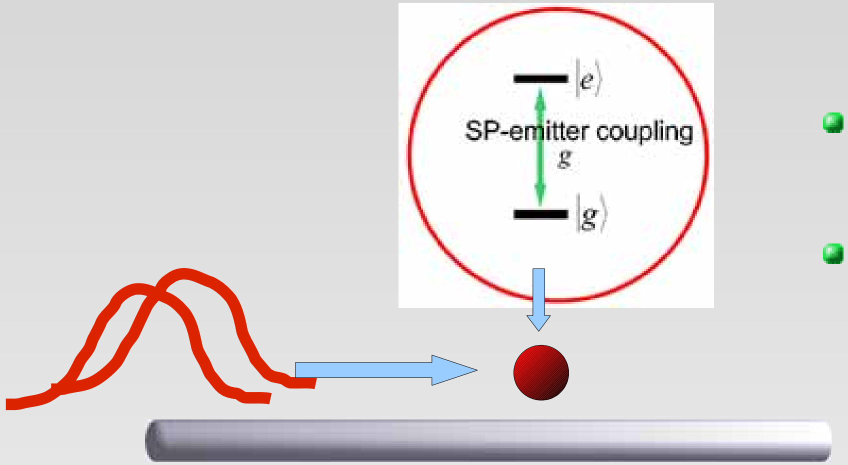


$$\delta=\omega_{photon}-\omega_{eg}$$

- On resonance, a single atom is optically dense to single photons
- A near-perfect mirror for single photons
- In contrast to cavity QED, strong atom-photon coupling is *achieved on a single pass*

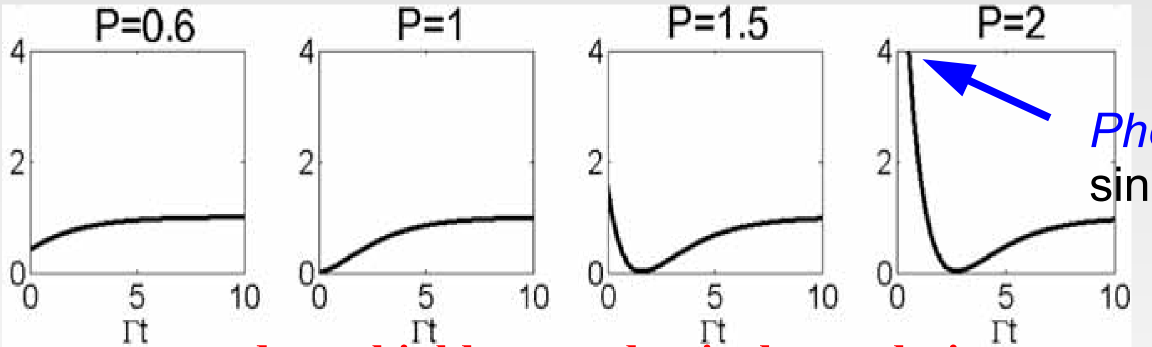
Single-photon nonlinear optics

- Resonant single photons are blocked, but what about photon pairs?
  - The two-level atom is anharmonic – can't absorb two photons at once



- Intuition: photon pairs should tend to be transmitted past the atom
- Can look for signatures of pair transmission in correlation functions!

DEC, A. Sorensen, E. Demler, M. Lukin, *Nature Physics (2007)*



Photon "bunching": highly unlikely a single photon is found alone

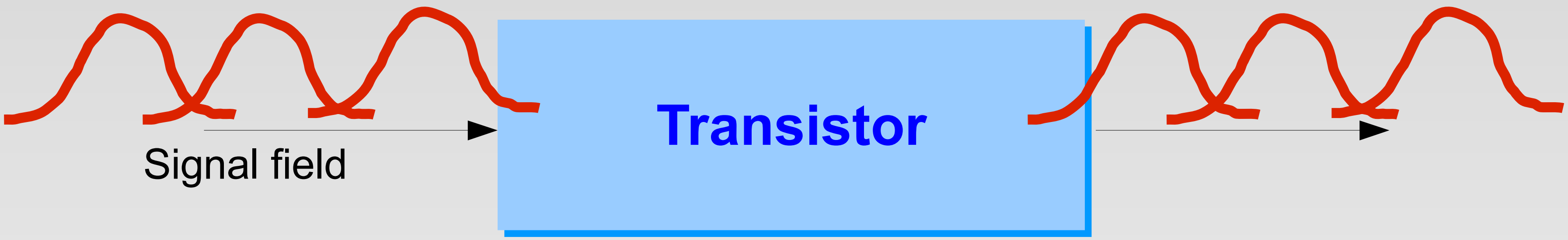
$$\left(g^{(2)}(t)=1 \ \text{ for classical laser}\right)$$

- **produces highly non-classical correlations**

- Two-level emitter acts as a single-photon switch
  - Single photons are reflected, pairs are transmitted
- Use quantum optical techniques to gain even more control of this process!

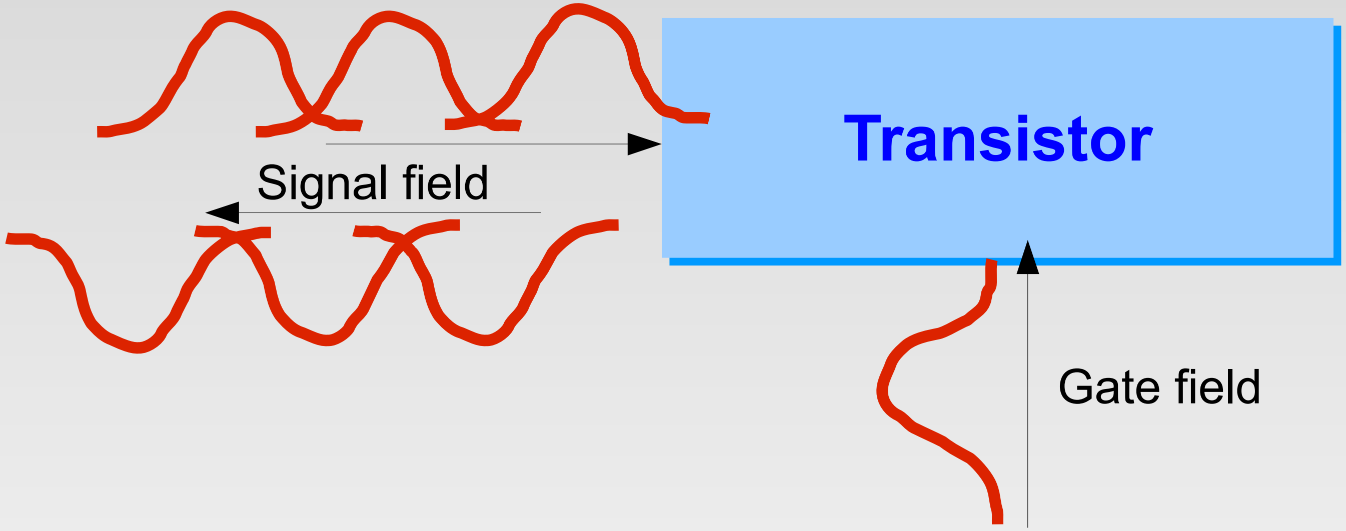
Single-photon transistor

- A single photon in a “gate” field controls the propagation of a stream of “signal” photons



Single-photon transistor

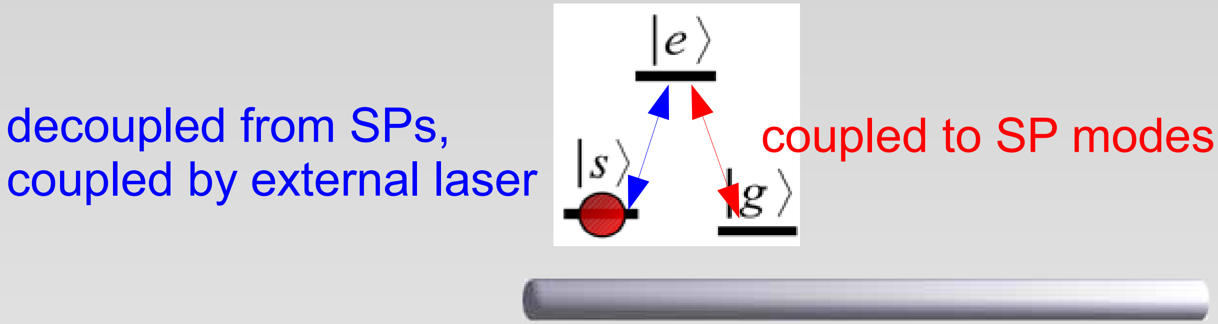
- A single photon in a “gate” field controls the propagation of a stream of “signal” photons



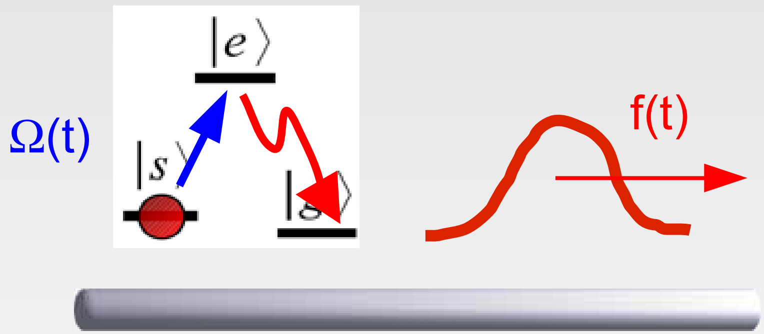
- Possible applications: single-photon detection, optical computing, generating Schrodinger cat states
  - Can use many signal photons to determine whether a single gate photon is present

System: three-level atom

- Three-level atom allows coherent control over interactions



- Single-photon generation and photon shaping on demand

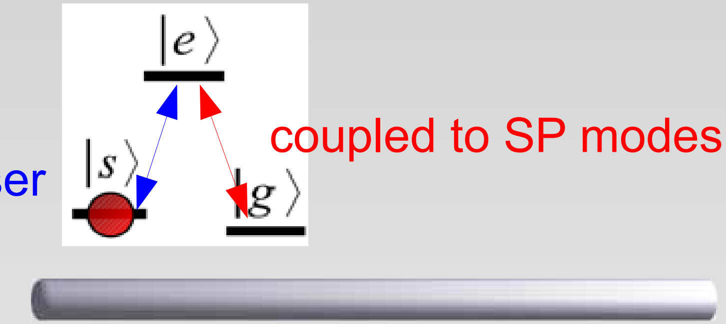


- One-to-one map between control field shape $\Omega(t)$ and shape f(t) of out-going single photon wavepacket (same as cavity QED)

System: three-level atom

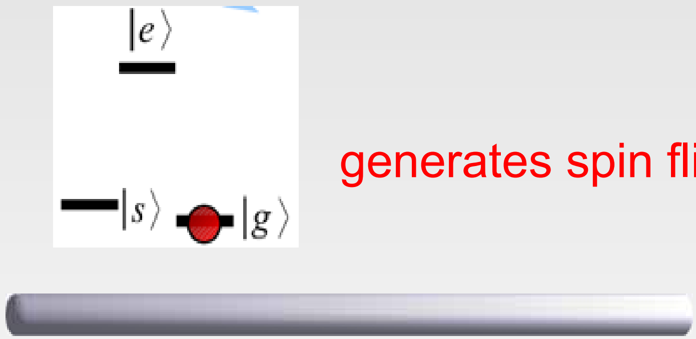
- Three-level atom allows coherent control over interactions

decoupled from SPs,
coupled by external laser



coupled to SP modes

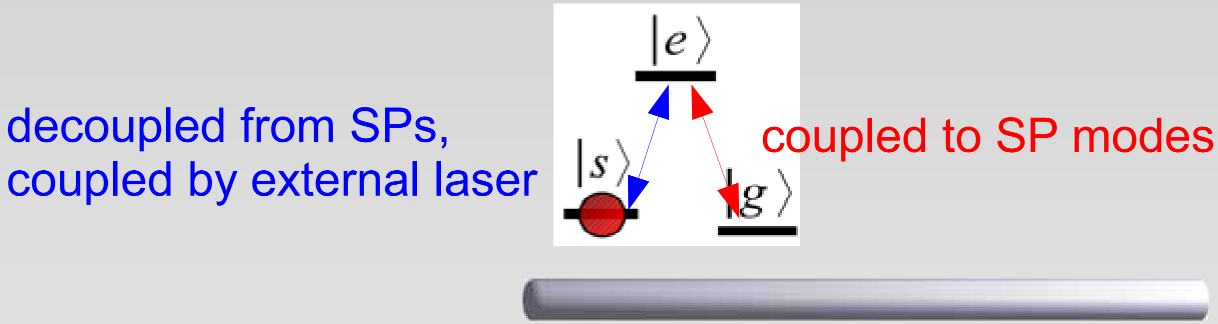
- Single-photon generation and photon shaping on demand



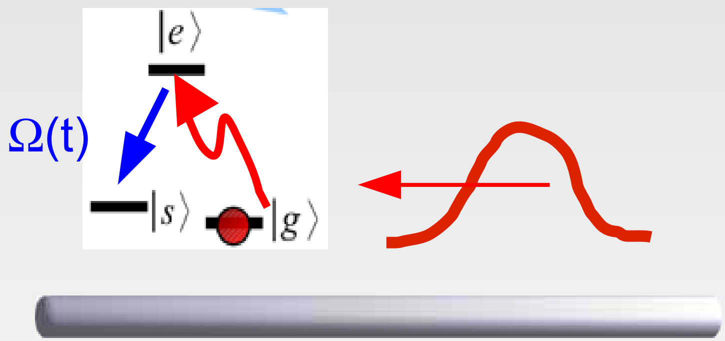
generates spin flip

System: three-level atom

- Three-level atom allows coherent control over interactions



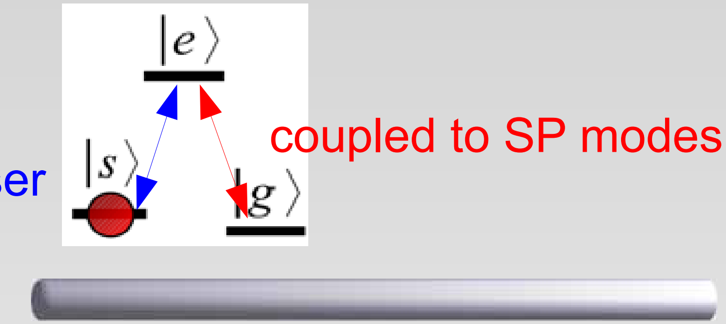
- Time-reversal: coherent photon storage



System: three-level atom

- Three-level atom allows coherent control over interactions

decoupled from SPs,
coupled by external laser



coupled to SP modes

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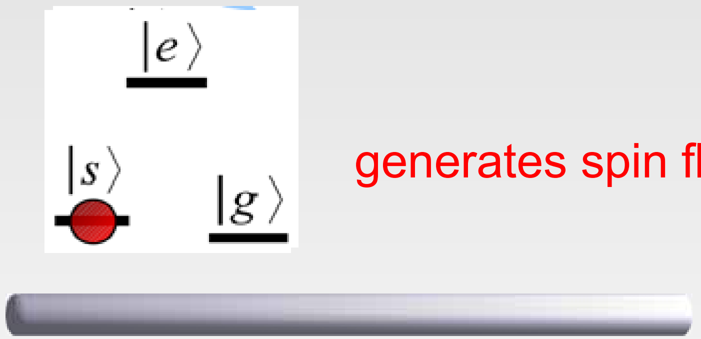
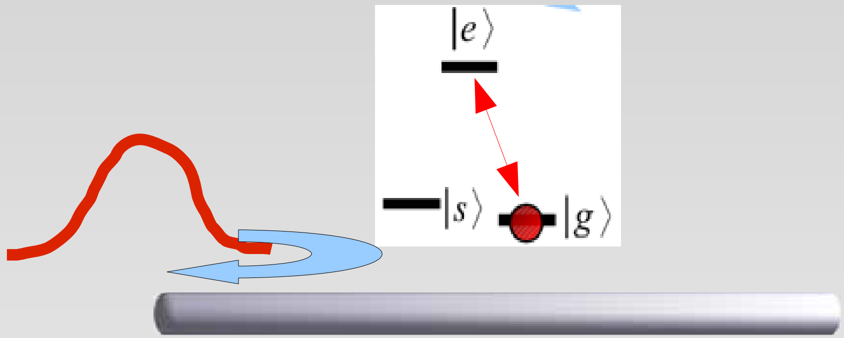


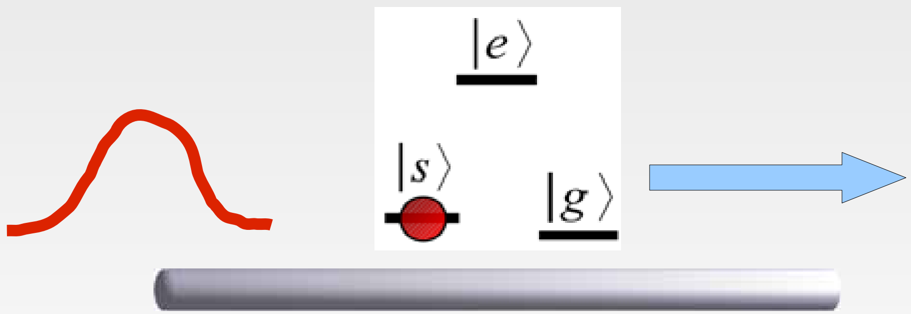
generates spin flip

Photon transport for three-level atom

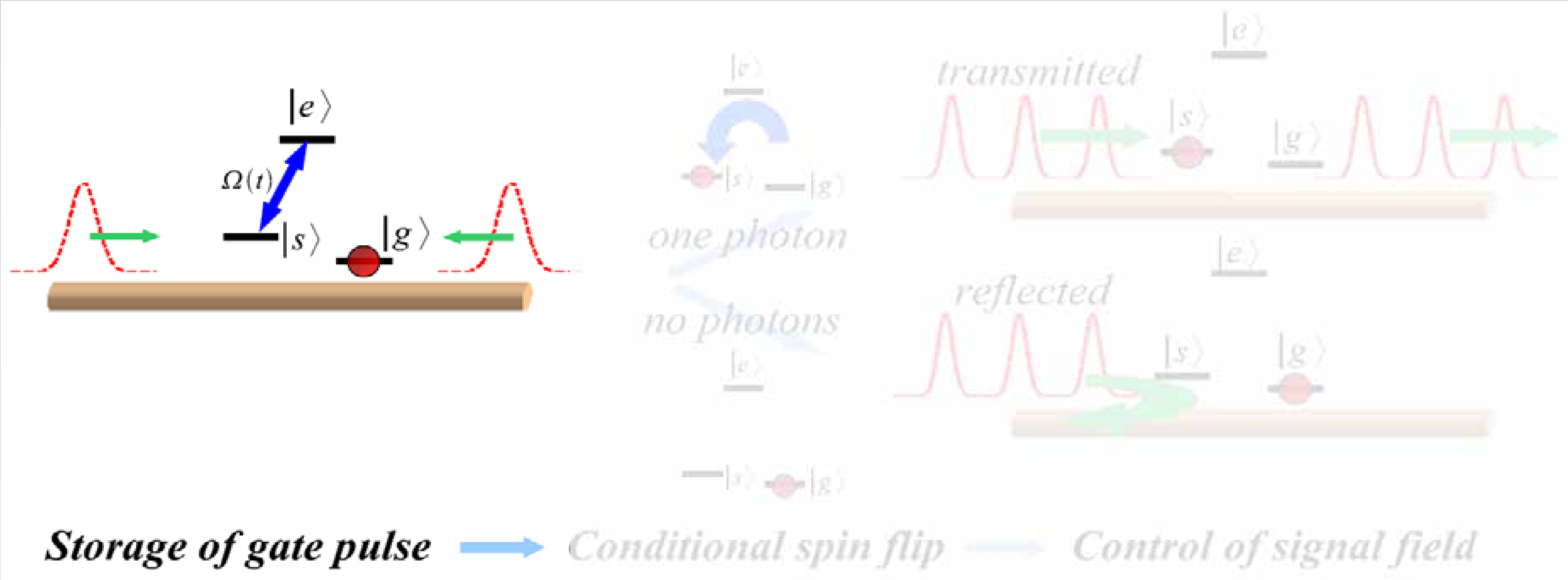
- When the atom is in state |g>, it is strongly coupled to the SPs
  - Highly reflecting for resonant single photons



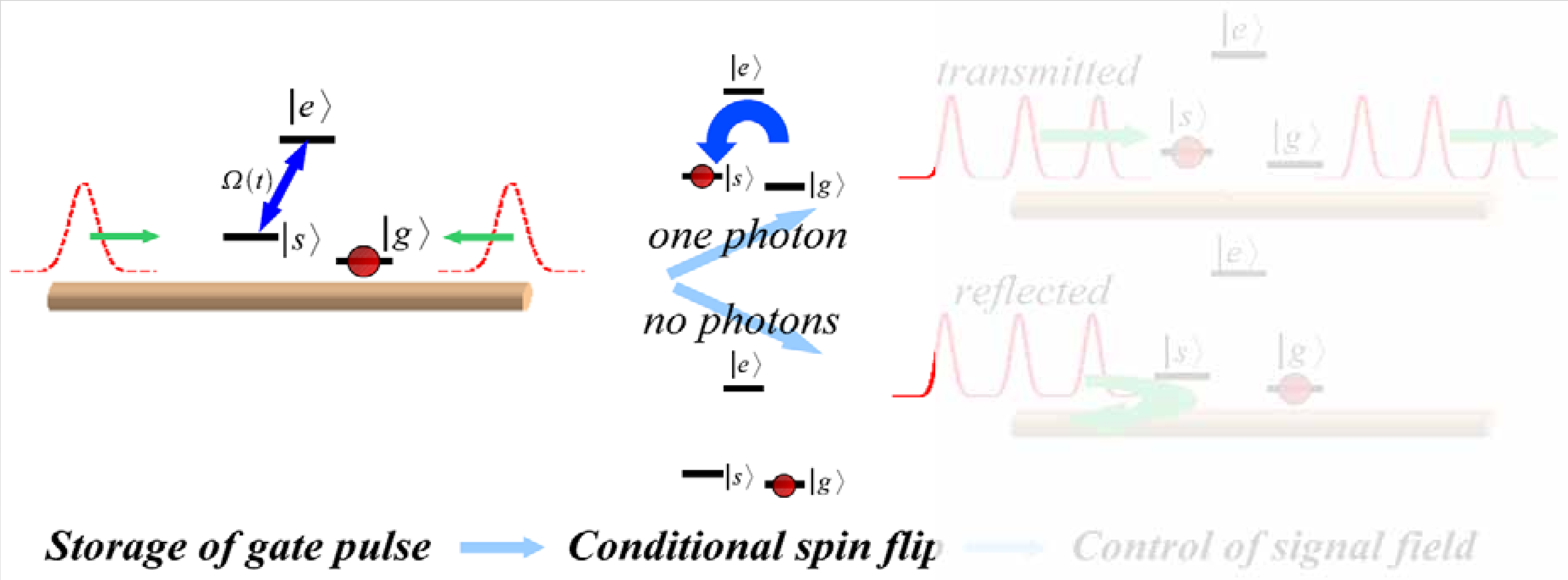
- When the atom is in state |s>, it is de-coupled from the SPs
  - The atom doesn't see SPs at all



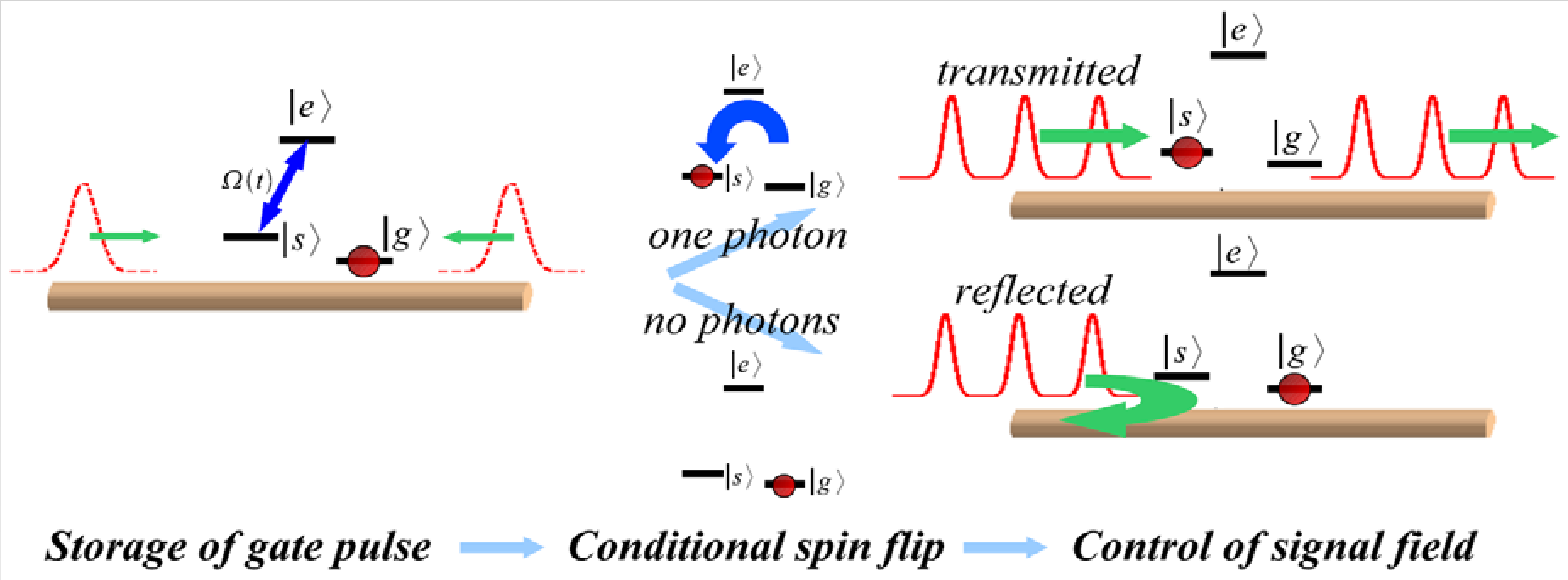
Single-photon transistor



Single-photon transistor

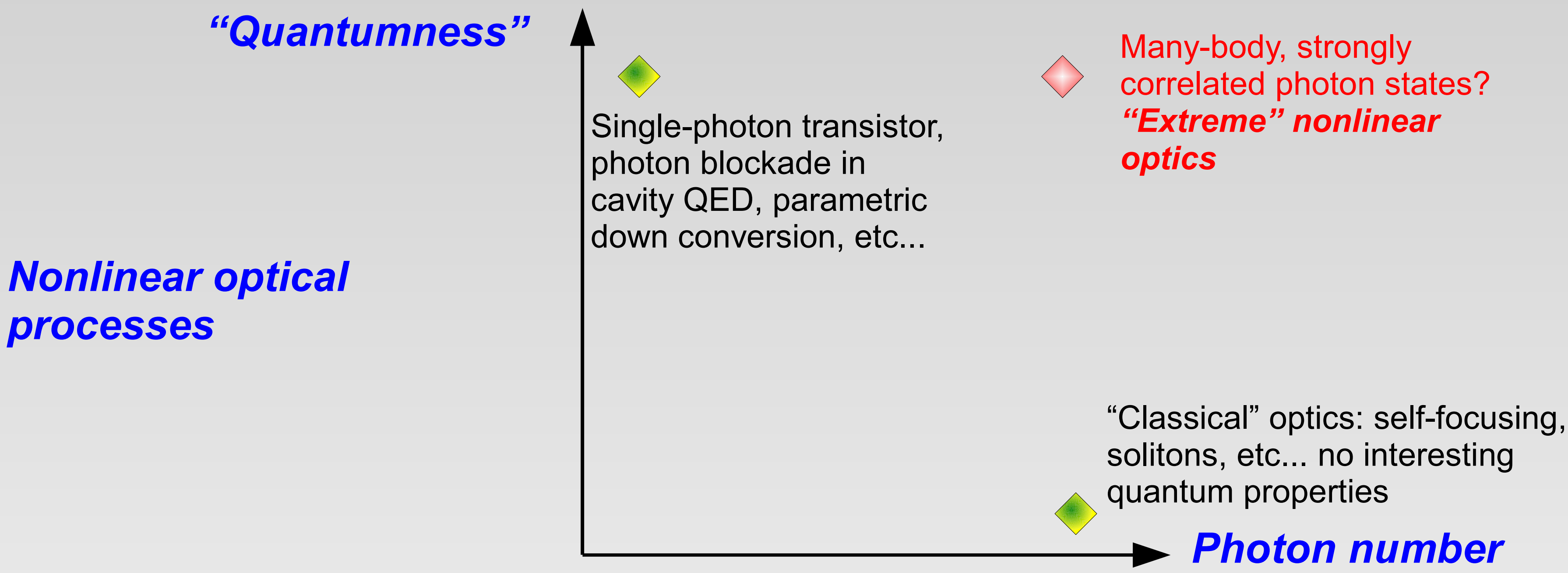


Single-photon transistor



- Transistor "gain" is determined by Purcell factor of system
  - # signal photons / gate photon > P
- Other schemes are possible to loosen timing requirements, etc...

Strongly correlated, many-body physics



- Strongly correlated, many-body photonic states:
  - System no longer well-described by the properties of the underlying individual photons
- Other strongly correlated, many-body phenomena:
  - Fractional and integer quantum hall effect
  - Confinement in quantum chromodynamics

Recent work on phase transitions

Strongly Correlated Photons in a Two-Dimensional Array of Photonic Crystal Microcavities

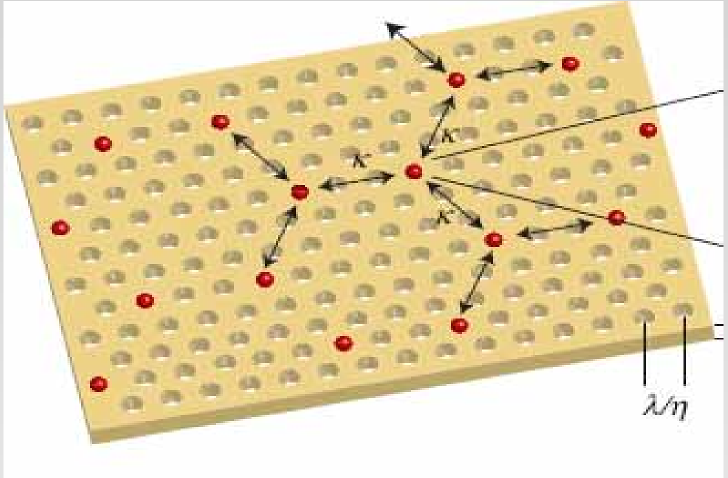
Y.C. Neil Na,[1] Shoko Utsunomiya,[2] Lin Tian,[1] and Yoshihisa Yamamoto[1,2]
[1]*E. L. Ginzton Laboratory, Stanford University, Stanford, CA 94305, USA*
[2]*National Institute of Informatics, Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan*

Strongly interacting polaritons in coupled arrays of cavities

MICHAEL J. HARTMANN[1,2]*, FERNANDO G. S. L. BRANDÃO[1,2] AND MARTIN B. PLENIO[1,2]*
[1]Institute for Mathematical Sciences, Imperial College London, 53 Exhibition Road, SW7 2PG, UK
[2]QOLS, The Blackett Laboratory, Imperial College London, Prince Consort Road, SW7 2BW, UK
*e-mail: m.hartmann@imperial.ac.uk; m.plenio@imperial.ac.uk

Quantum phase transitions of light

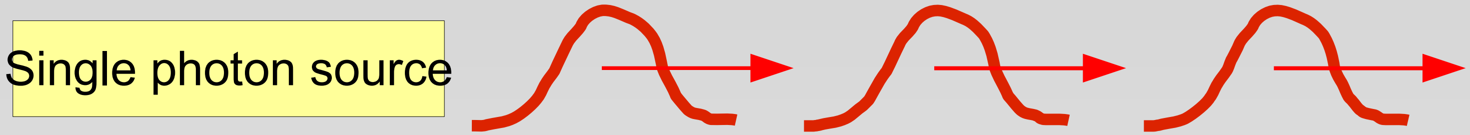
ANDREW D. GREENTREE[1]*, CHARLES TAHAN[1,2], JARED H. COLE[1] AND LLOYD C. L. HOLLENBERG[1]
[1]Centre for Quantum Computer Technology, School of Physics, The University of Melbourne, Victoria 3010, Australia
[2]Cavendish Laboratory, University of Cambridge, JJ Thomson Ave, Cambridge CB3 0HE, UK
*e-mail: andrew.greentree@ph.unimelb.edu.au



- Large system of identical emitters in coupled cavities is technically difficult
  - *e.g.*, disorder could ruin the quantum phases
- Ground state of such system has interesting properties, but photons don't thermalize, so how to reach it?

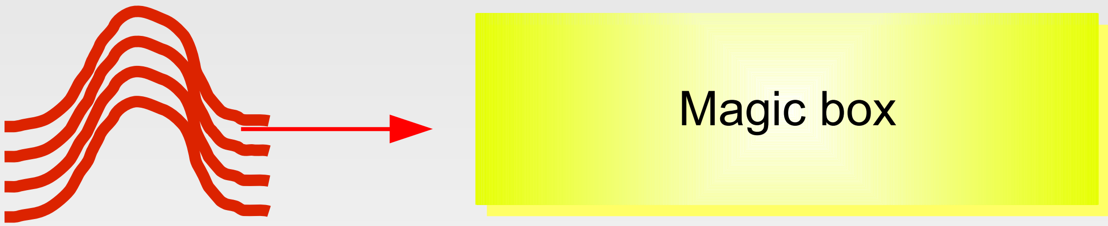
Motivation: “Crystallization” of photons

- How do we create a periodic train (***a crystal***) of single photons?



- Technical challenges: limited efficiency, inhomogeneous wavepackets, multiple photons, etc...

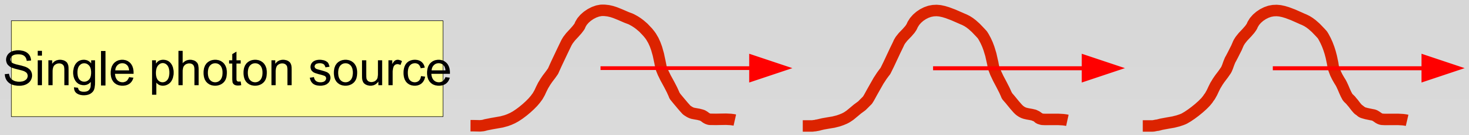
- ***A different paradigm***: classical light in, photon crystal out



Coherent (laser) pulse
– overlapping photons

Crystallization of photons

- How do we create a periodic train (***a crystal***) of single photons?



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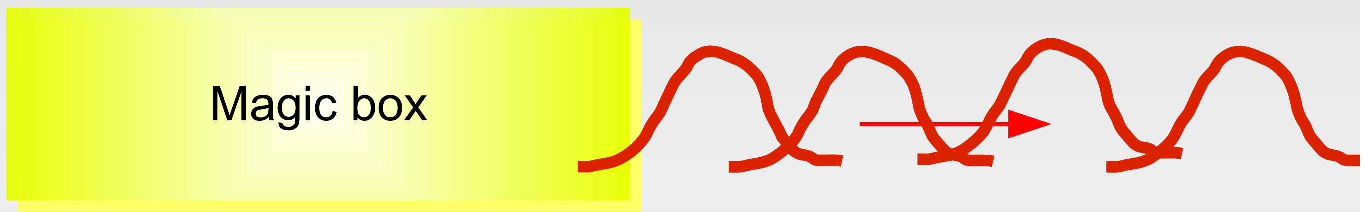


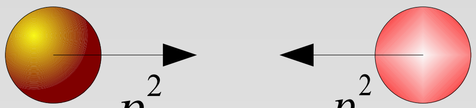
Crystal of single photons!

- It's not totally crazy... let's look to an example from condensed matter physics

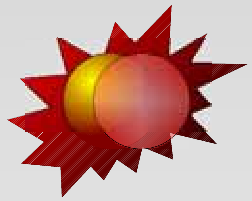
Another problem: interacting hard-core bosons

- Consider a 1D system of bosons with hard-core repulsive interactions

$$H=\sum_{i=1}^{N}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_i^2}+\sum_{i\neq j} g\,\delta(z_i-z_j),\qquad g\rightarrow\infty$$



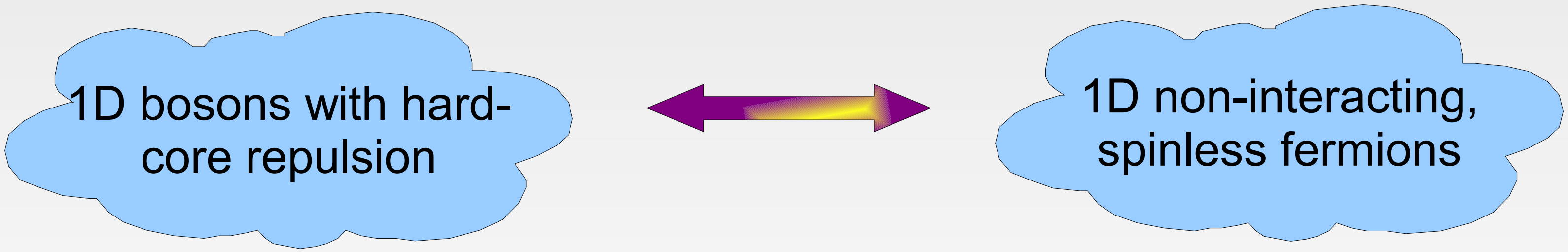
$$K=\frac{p_1^2}{2m}\qquad K=\frac{p_2^2}{2m}$$



$$I\propto g$$

- Two particles cannot sit on top of each other (takes infinite energy)... this resembles the Pauli exclusion principle for fermions

- In fact, a one-to-one map exists:



- Leads to a very interesting ground state for bosons! (***Tonks-Girardeau gas***)
- More realistic definition of TG regime: $\gamma\equiv\frac{I}{K}=\frac{gm}{n_z}\gg 1$

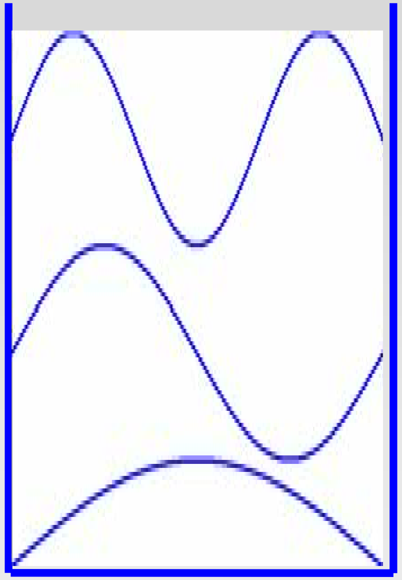
Tonks-Girardeau gas

- The relationship between hard-core bosons and non-interacting fermions in 1D:
- Consider N non-interacting particles in a box

$\psi_2(z)$

$\psi_1(z)$

$\psi_0(z)$



- Fermions obey Pauli exclusion: no two particles can occupy the same state
  - N particle ground state:

$$\Psi_n(z_{1,}..,z_n) = \mathcal{A}\ \psi_0(z_1)\psi_1(z_2)\ldots\psi_{n-1}(z_n)$$

anti-symmetrize

- Wave function vanishes when any two position arguments are equal

$$\Psi_n(z,z,z_{3,}..,z_n)=0$$

- Hard-core bosons:
  - Wave function also must vanish when any two positions are equal (infinite interaction energy)
  - N particle ground state:

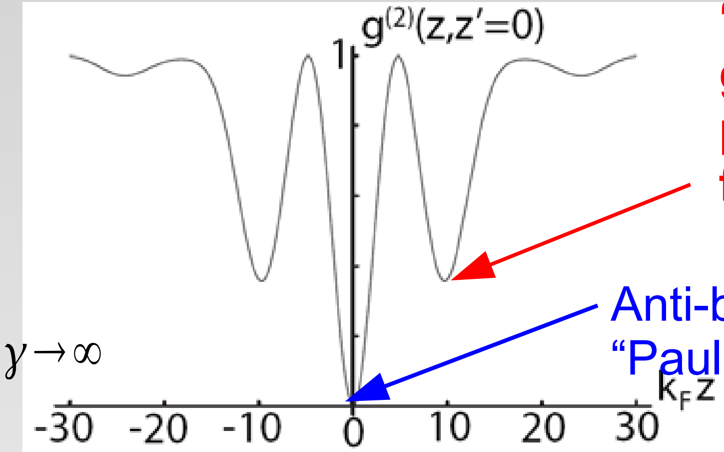
$$\Psi_n(z_{1,}..,z_n) = \mathcal{S}\ \psi_0(z_1)\psi_1(z_2)\ldots\psi_{n-1}(z_n)$$

(versus BEC for non-interacting)

symmetrize

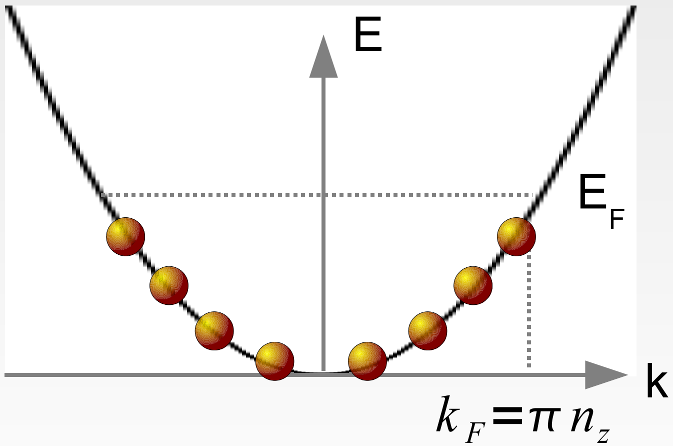
Correlations of a Tonks-Girardeau gas

- Density-density correlation function for TG gas



- How do these oscillations arise?

Ground state of fermions: filled Fermi sea



Correlations of a Tonks-Girardeau gas

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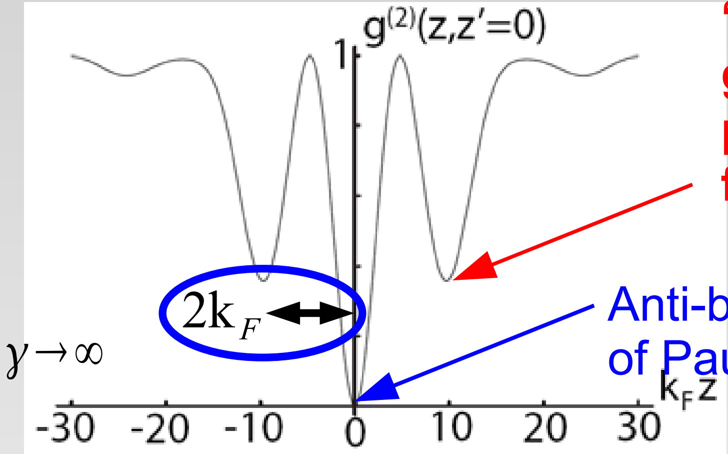
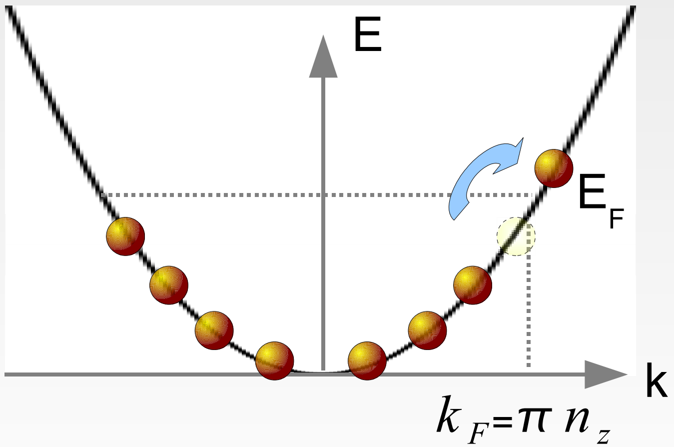


"crystal" correlations – gain knowledge about positions of other particles from knowing that of one

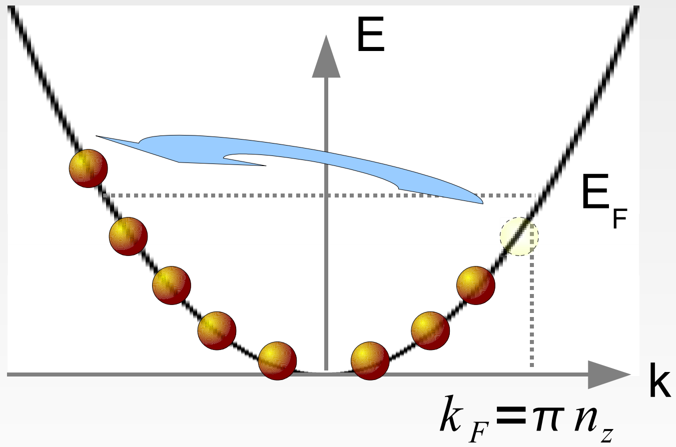
Anti-bunching – signature of Pauli exclusion

- How do these oscillations arise?

Two types of low-energy excitations:
Small wavevector (long wavelength)



Large wavevector $2k_F$ (short wavelength)

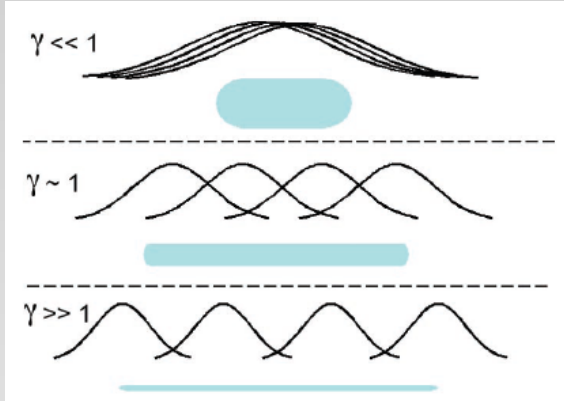


Leads to oscillations in correlation function!

TG gas of ultracold atoms

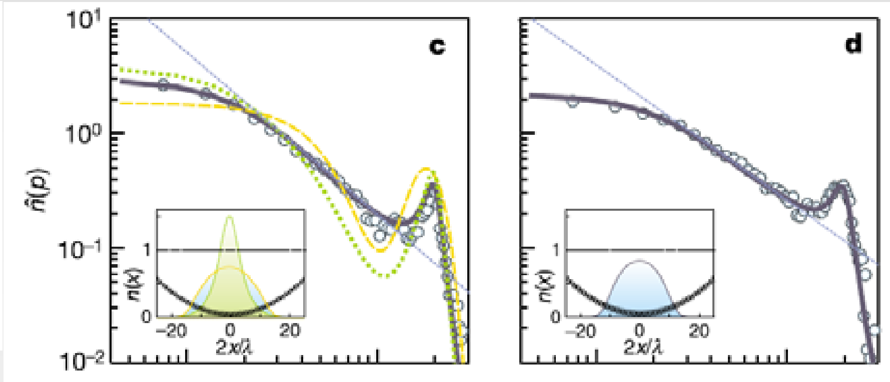
Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*



Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]



Forming a TG gas of photons

- ***Three requirements:***

- Need a physical implementation
  - 1D waveguide, strong interactions between photons (mediated by atoms)

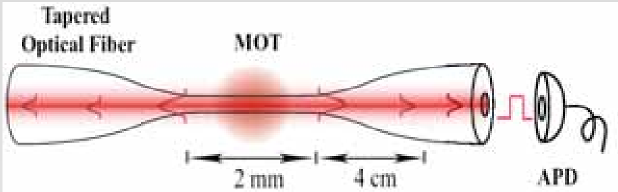
- Need some prescription to prepare the TG gas ground state

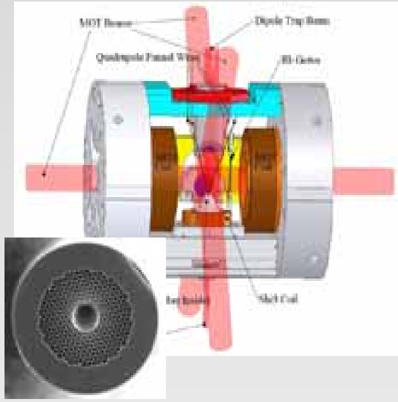
- Need a method to detect strongly correlated states
  - Done! Can look at field correlation functions of light leaving our waveguide.

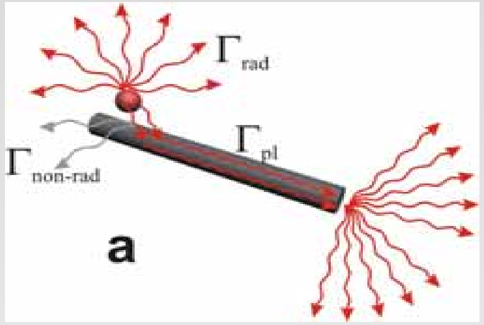
Possible physical realizations

- 1D waveguides with loaded cold atoms that mediate interactions

- Tapered optical fibers
- Photonic crystal fibers
- Plasmonics







	Hakuta (Tokyo), Rauschenbeutel (Bonn)	Lukin (Harvard) & Vuletic (MIT)	Lukin & Park (Harvard)
Typical core diameter	~ 100 nm	~ 5 μm	~ 100 nm
Best possible confinement	$\sim (\lambda / n)^2$	$\sim \lambda^2$	$\sim R^2$
Max coupling efficiency	< 50%	< 50%	>99% ☺
Propagation losses	☺	☺	😐
Loading cold atoms	☺	☺	???

Engineering coherent atom-photon interactions

- We need photons to obey a non-trivial evolution:

$$H=\sum_{i=1}^{N}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_i^2}+\sum_{i\neq j} g\,\delta(z_i-z_j)$$

Second quantization:

$$i\frac{\partial\Psi}{\partial t}=-\frac{1}{2m}\frac{\partial^2\Psi}{\partial z^2}+2g\Psi^\dagger\Psi\Psi\ +0\times v_g\frac{\partial\Psi}{\partial z}$$

No velocity (photon trapping)

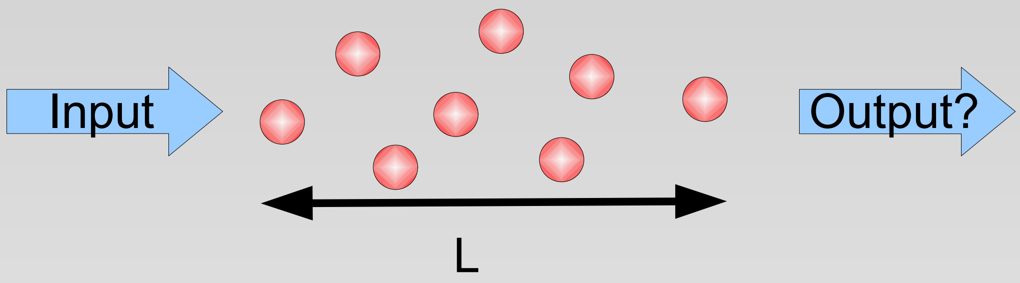
effective mass

Kerr nonlinearity

- Quantum optical techniques ➔ manipulate propagation and interaction of photons
- Ideas based on Electromagnetically Induced Transparency (EIT) provide widely tunable system

Field propagation in an atomic medium

- Consider the linear propagation of light through an atomic system

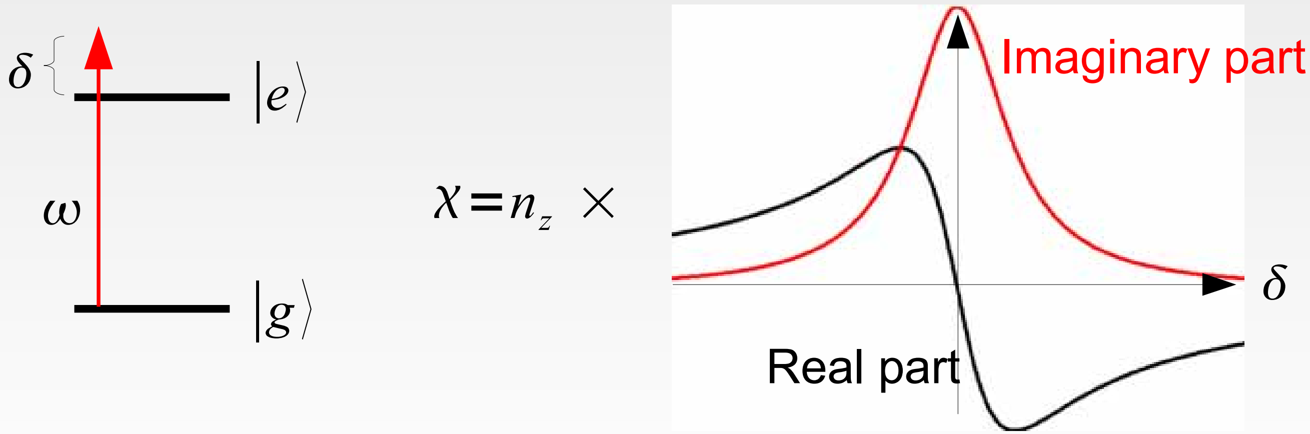


- Output is related to input through a susceptibility:

$$E_{out}(\omega, L) = E_{\text{in}}(\omega) e^{i\omega\chi L/c}$$

  - Real part of χ yields a phase shift (dispersion)
  - Imaginary part of χ yields absorption

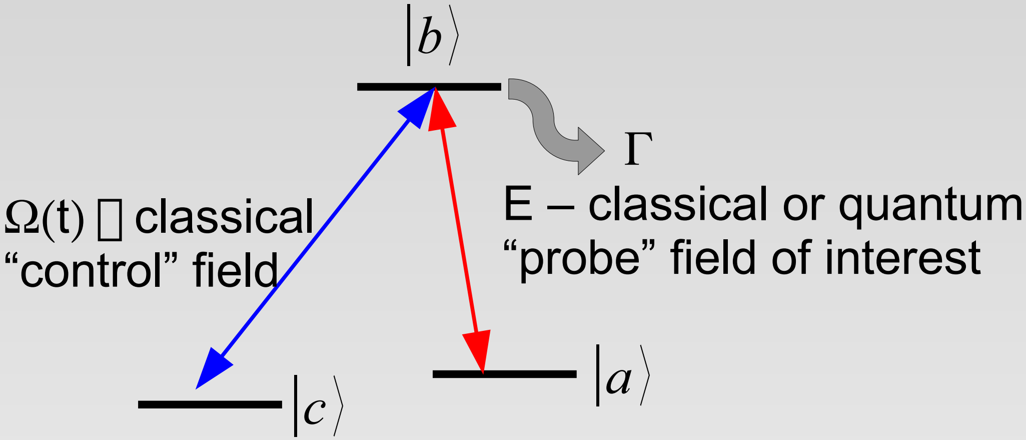
- Susceptibility for two-level atoms:



- Response of harmonic oscillator near resonance
- Large absorption on resonance

An introduction to EIT

- Three-level atom:

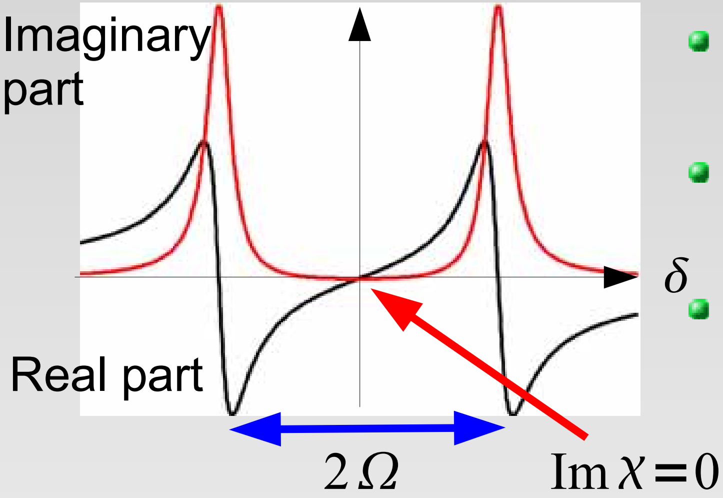


$$H = \Omega|b\rangle\langle c| + E|b\rangle\langle a| + h.c.$$
$$|D\rangle \sim E|c\rangle - \Omega|a\rangle$$
$$H|D\rangle = 0$$

- Dark state D decoupled from H and excited state – no absorption of E on resonance
- Effect of quantum coherence and interference

Susceptibility for three-level atom

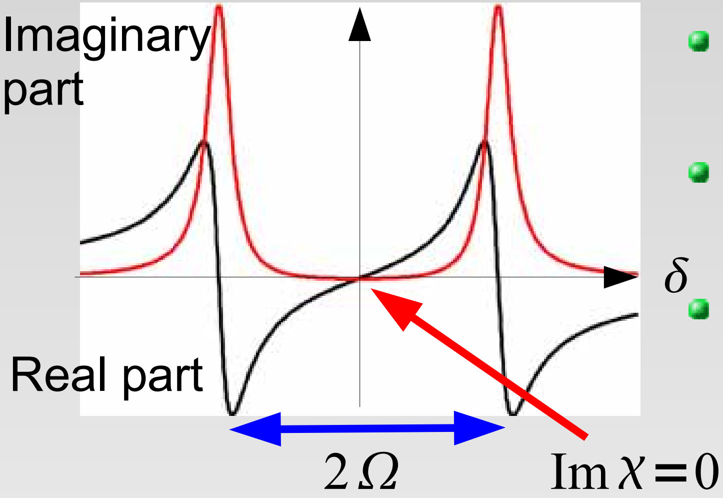
- Susceptibility for three-level atom



- Looks like two separate harmonic oscillator resonances
- Control field creates two resonances separated by frequency 2Ω
- No absorption on resonance! A "transparency window" is created.

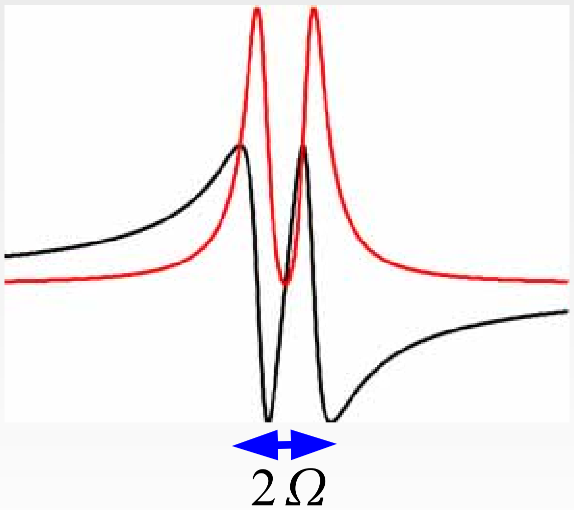
Susceptibility for three-level atom

- Susceptibility for three-level atom



- Looks like two separate harmonic oscillator resonances
- Control field creates two resonances separated by frequency 2Ω
- No absorption on resonance! A “transparency window” is created.

- Case of small control field (small Ω)



- Can create steep variations in real part of susceptibility around resonance
- Small change in frequency leads to large change in propagation phase
  - Small group velocity!

$$v_g \sim \frac{\Omega^2(t)}{\Gamma\, n_z}$$

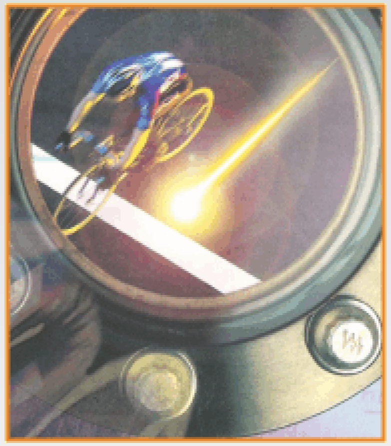
Slow light

- Speed of light in EIT can be made arbitrarily small in principle!
  - Use high atomic densities and small control fields

$$v_g \sim \frac{\Omega^2(t)}{\Gamma n_z}$$

  - In practice limited just by atomic decoherence

Experimental demonstration:



Reduction of speed of light to 17 m/s

Hau (Harvard), Nature (1999)

Engineering coherent atom-photon interactions

- We need photons to obey a non-trivial evolution:

$$H = \sum_{i=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_i^2} + \sum_{i\neq j} g\,\delta(z_i - z_j)$$

Second quantization:

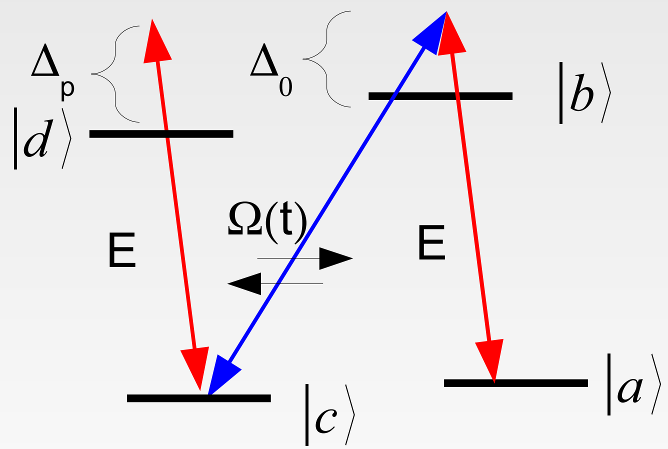
$$i\frac{\partial \Psi}{\partial t} = -\frac{1}{2m}\frac{\partial^2 \Psi}{\partial z^2} + 2g\Psi^\dagger \Psi \Psi \;\; + 0\times v_g \frac{\partial \Psi}{\partial z}$$

No velocity (photon trapping)

effective mass

Kerr nonlinearity

- How does EIT help us achieve this evolution?

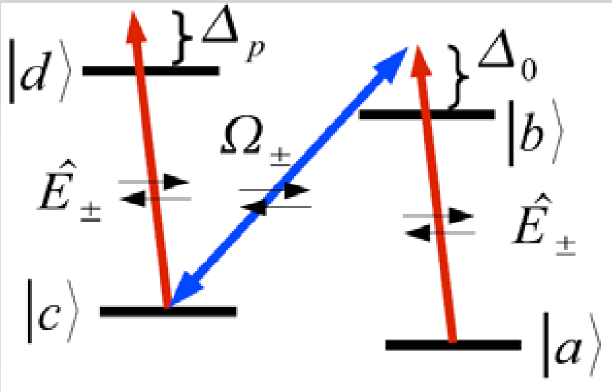


- Use the versatility of atomic level structure and tunability of frequencies to add more terms into EIT evolution equations

- This level scheme allows one to realize the nonlinear Schrodinger equation (NLSE) for photons

Nonlinear Schrodinger equation for photons

- Full dynamics of system given by 1-D nonlinear Schrodinger equation



$$i\partial_t \Psi(z,t) = -\frac{1}{2m_{\text{eff}}}\partial_z^2 \Psi(z,t) + 2\tilde{g}\Psi^\dagger(z,t)\Psi^2(z,t),$$

$$\Psi = \frac{(\Psi_+ + \Psi_-)}{2}, \qquad m_{\text{eff}} = -\frac{\Gamma_{1D} n_z}{4\Delta_0 v_g}, \qquad 2\tilde{g} = \frac{\Gamma_{1D} v_g}{\Delta_p}$$

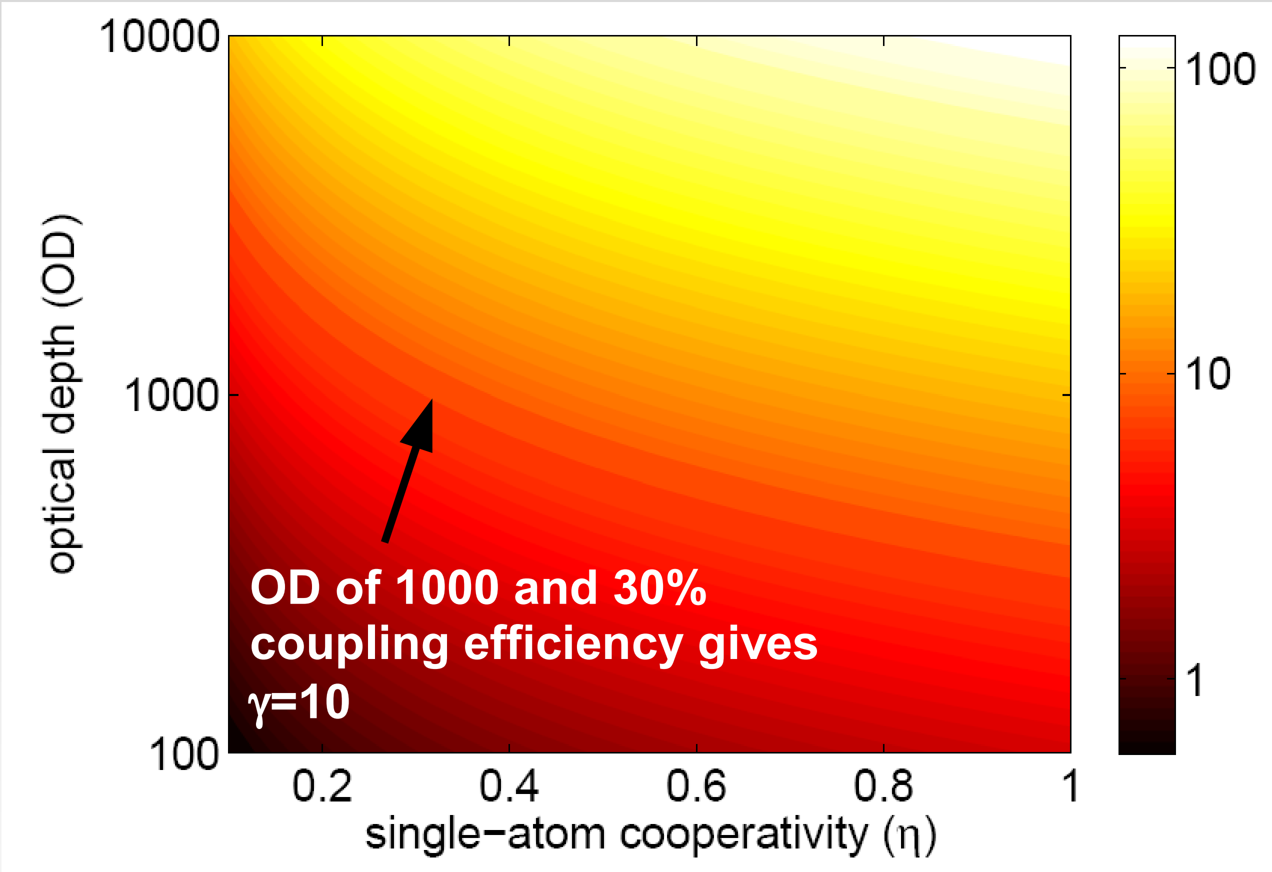
- This is the equation that leads to a TG gas of photons when γ is large
  - γ is dynamically tunable in our system by changing detunings!

$$\gamma(t) = \frac{\tilde{g}(t)\, m_{eff}(t)}{n_{ph}}$$

- The value of γ we can achieve depends on our physical resources:
  - The number of atoms we have (atom-photon coupling enhanced by large atom number)
  - Strong coupling per single atom $\eta = \dfrac{\Gamma_{1D}}{\Gamma_{total}}$

Experimental parameters

- Requirements for observing a TG gas and crystal correlations
  - N=10 photons

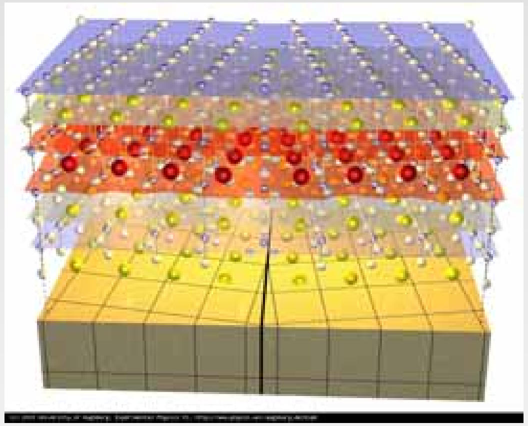


DEC, V. Gritsev, et al., submitted (2008),
also see cond-mat/0712.1817

A TG gas of photons and beyond

- Quantum optical techniques + novel technologies = strongly correlated, many-body photon gases
- Applications in areas such as quantum information and metrology
  - Sub-shot noise photon number fluctuations!

- Novel connections between optical and condensed matter physics
- TG gas is solvable, but many other many-body systems are not



High-Tc superconductivity

- Use light to simulate quantum Hamiltonians of interest and learn about fundamental phenomena

- Interesting open challenges – non-equilibrium physics, photon absorption & noise, externally driven systems

Outlook

Integrated photonics: plasmonics + optical waveguides

New physics with strongly correlated photons: simulation of quantum matter

New applications: *e.g.*, electrical SP detection

AMO physics + nano-optics

Nonlinear optics with single photons

?????

Atom-nanoscale interface: single atom trapping, manipulation and readout

Efficient single-photon manipulation: new tools for quantum information

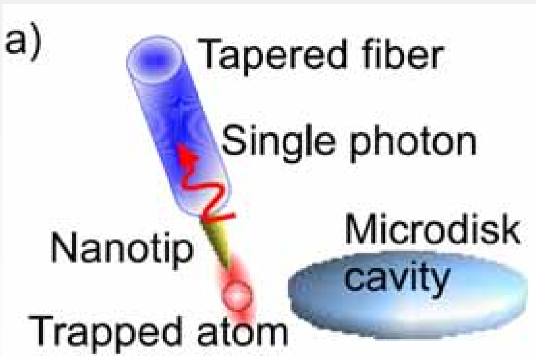
New regimes of operation: carbon nanotubes @ THz

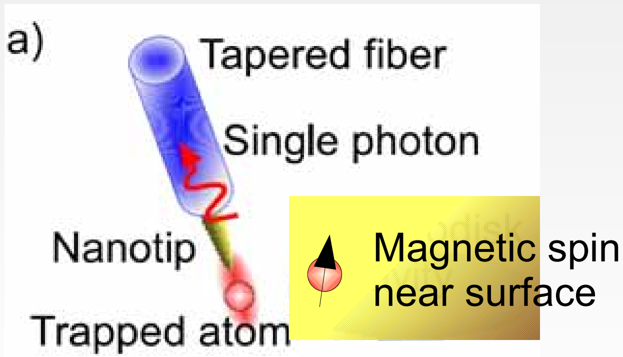
Nanoscale traps for atoms using SPs

- Dilemma: in quantum information and other applications, use atoms or "artificial atoms"?

	Atoms	Artificial
Homogeneity	☺	😐
"Clean" transitions (e.g., three-level system)	☺	😐
Simple environments	☺	😐
Robust nanoscale positioning/trapping	😐	☺
Can bring into nanoscale proximity of other systems	😐	☺

- Nanoscale trapping schemes for neutral atoms would allow:
  - Realization of similar functionalities as artificial atoms
  - While maintaining benefits of neutral atoms





Challenges of optical trapping