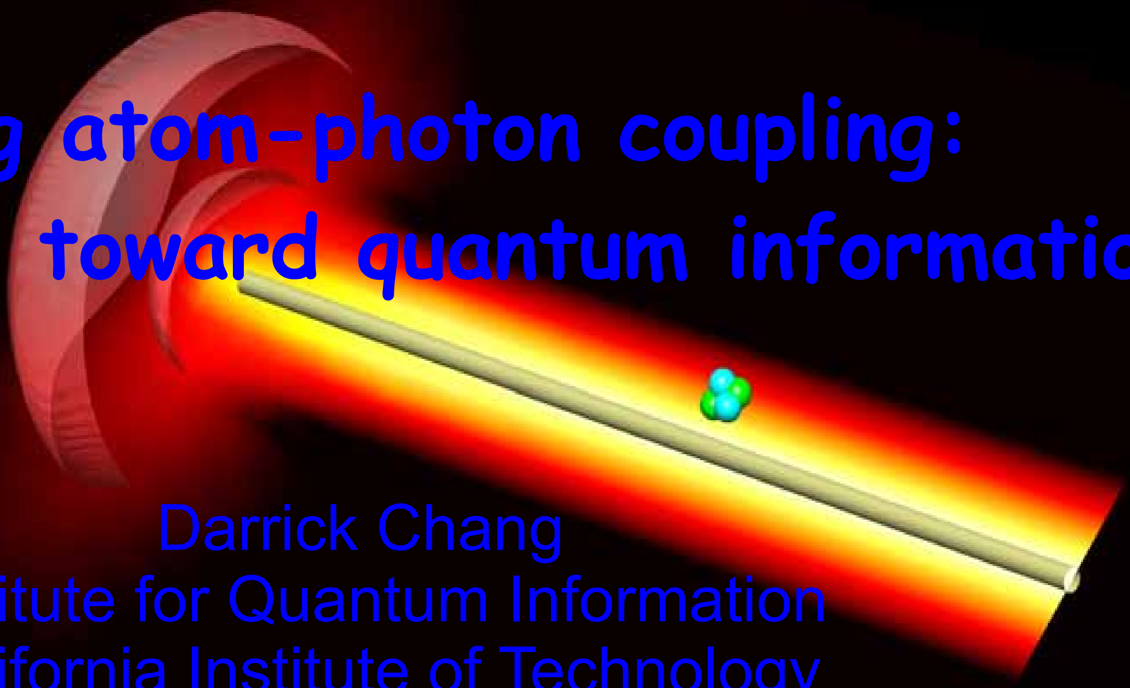

Strong atom-photon coupling: applications toward quantum information

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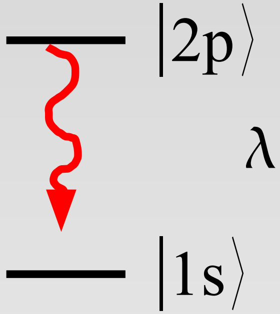
Yilan, Taiwan

Motivation

- Single atoms and single particles of light (photons) are very simple systems whose interactions are well-understood
  - Example: hydrogen atom



$$\lambda = 122\,\mathrm{nm}\,,\ \tau = 1.6\,\mathrm{ns}$$

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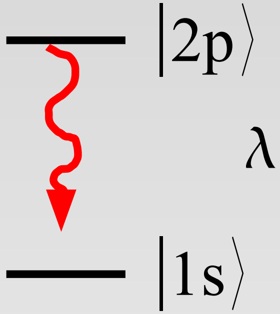
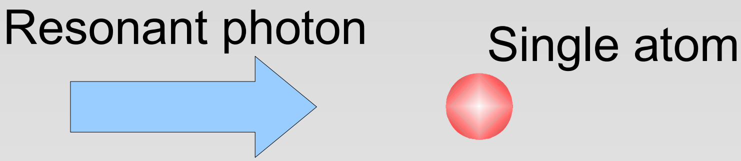


$$\lambda = 122\,\mathrm{nm}\,,\ \tau = 1.6\,\mathrm{ns}$$

- The simplicity of this system allows for fundamental studies and tests of quantum mechanical principles
    - Superposition and entanglement
    - Open quantum systems

Motivation

- One major obstacle: Single atoms and single photons interact *very weakly*
  - Example: scattering problem



$$\sigma = \frac{3\lambda^2}{2\pi},\ A_{beam} > \lambda^2,\ P_{sc} = \frac{\sigma}{A_{beam}}$$

Motivation

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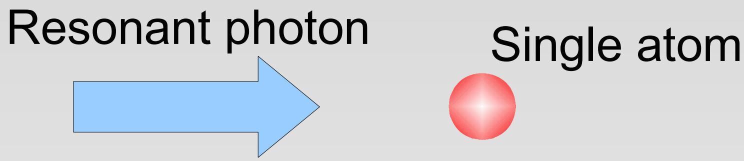
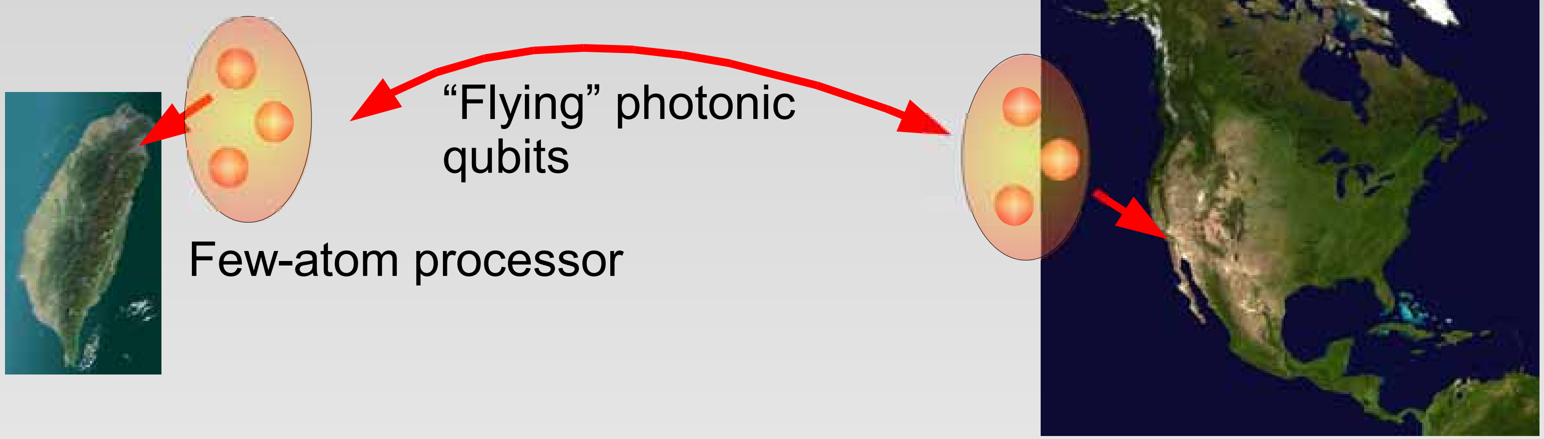


$$\sigma = \frac{3\lambda^2}{2\pi},\ A_{beam} > \lambda^{2,}\ P_{sc} = \frac{\sigma}{A_{beam}}$$

- It is therefore critical to develop techniques to enhance (and control) atom-light interactions
- If developed, these tools not allow for realization of fundamental quantum mechanics, but are an *extremely powerful resource* in many applications (both quantum and classical)

Motivation

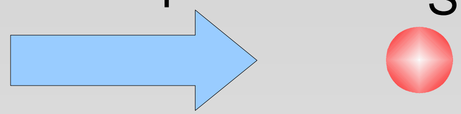
- Some possible applications:
  - Quantum networks



  - Single-photon nonlinear optics
    - Pulses of light do not directly interact with each other, but can be made to via common interaction with matter
    - Allows quantum gates for photons, low-power optical switches and transistors, etc.
  - And much more...

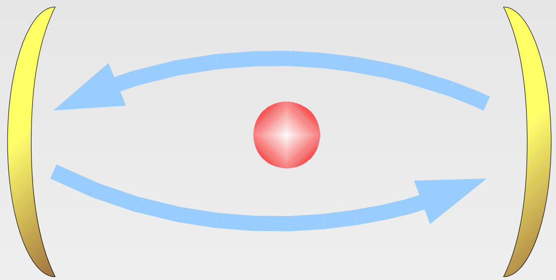
The strategy...

- How do we get around inherently weak coupling?

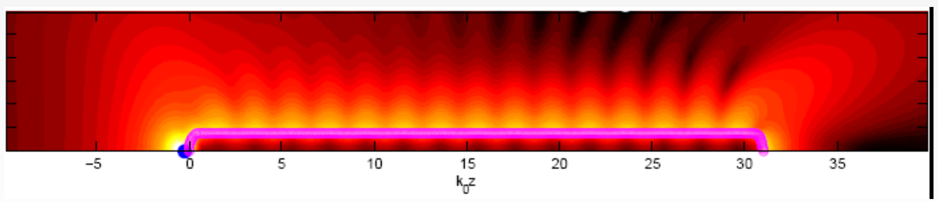


$$\sigma = \frac{3\lambda^2}{2\pi},\ A_{beam} > \lambda^2,\ P_{sc} = \frac{\sigma}{A_{beam}}$$

- Approach #1: Cavity quantum electrodynamics (QED)
  - Put the atom between two mirrors and enhance the interaction by the number of round trips the photon makes



- Approach #2: Plasmonics – Electrodynamics in 1D
  - Circumvent the diffraction limit, $A_{beam} \ll \lambda^2$



Outline

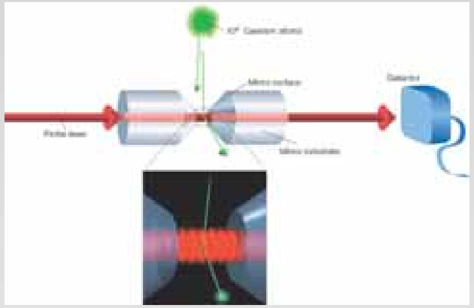
- Lecture 1 – Cavity QED
  - Physical implementations of cavity QED
  - Jaynes-Cummings model Hamiltonian of atom-photon interactions
    - Application: single-photon blockade
  - Cavity QED as an open quantum system
  - The strong-coupling regime
  - A few applications for quantum information:
    - Single-photon generation on demand
    - Quantum state transfer across distant nodes
    - Hybrid quantum networks

Outline

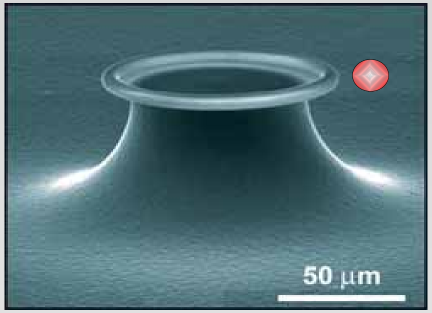
- Lecture 2 – "quantum plasmonics"
  - Introduction: quantum electrodynamics in 1D
  - Physical implementation: surface plasmons on nanowires
  - The strong-coupling regime
  - Experimental observation of strong coupling
  - Integration with conventional photonics
  - An application: a single-photon transistor
  - Outlook: condensed matter physics with photons

Cavity QED: physical implementations

- Many different cavity QED systems are being actively explored
  - Atoms and optical photons



Fabry-Perot system: Kimble (Caltech), Chapman (Georgia Tech), Rempe (Munich), many more...



Microtoroidal resonators: Kimble (Caltech)

Cavity QED: physical implementations

- Many different cavity QED systems are being actively explored
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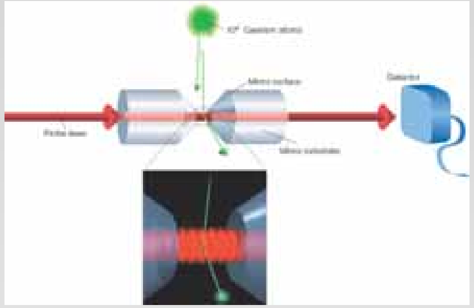

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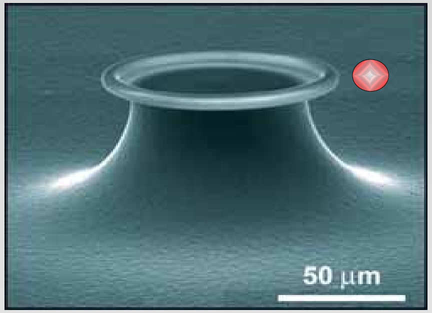
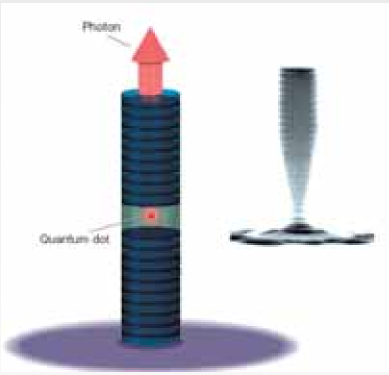
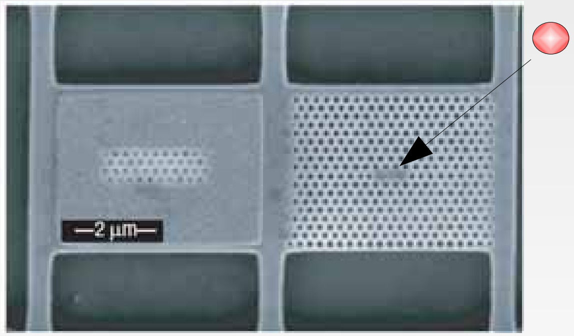
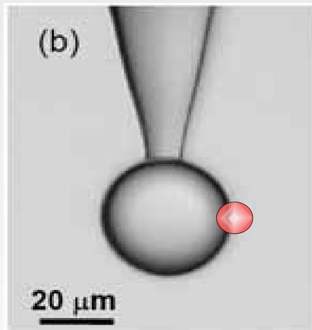


Microtoroidal resonators: Kimble (Caltech)

  - Solid-state “artificial atoms” and optical photons



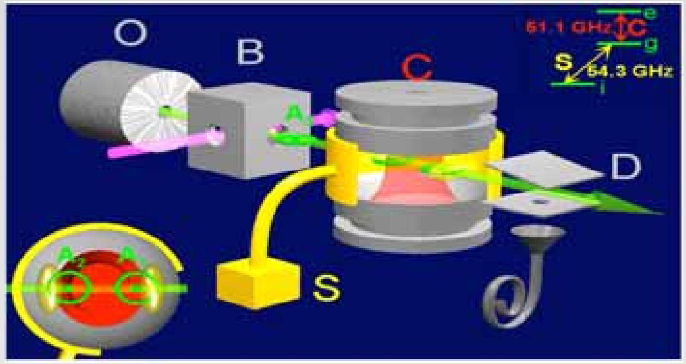




Imamoglu (ETH), Yamamoto (Stanford), Vuckovic (Stanford), Painter (Caltech), Wang (Oregon), many more...

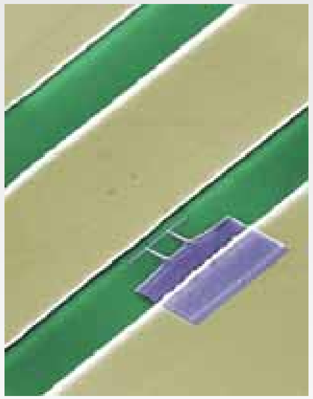
Cavity QED: physical implementations

- Also cavity QED systems appear in many other settings
  - Microwave cavities and Rydberg atoms



Haroche (ENS)

  - Superconducting transmission line cavities and Cooper pair box



Schoelkopf (Yale), Wallraff (ETH)

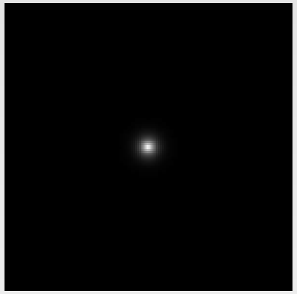
Atoms and photons – a quick quantization

- Interaction Hamiltonian between an electric dipole and electric field:

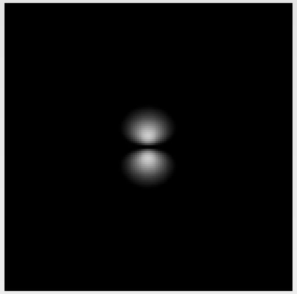
$$V = -\hat{d}\cdot\hat{E}(\vec{r}_{atom})$$

- Quantizing the dipole operator:
  - Suppose our atom has only two states of interest, |g> and |e>

Example: hydrogen atom



$|g\rangle = 1\mathrm{s\ state}$



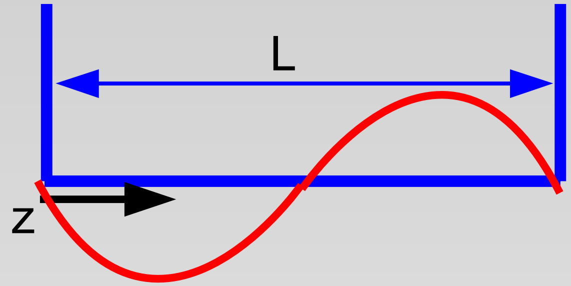
$|e\rangle = 2\mathrm{p\ state}$

$$\begin{aligned}\hat{d} &= q\hat{r}\\ &= \left(|g\rangle\langle g| + |e\rangle\langle e|\right) q\hat{r}\left(|g\rangle\langle g| + |e\rangle\langle e|\right)\\ &= q|g\rangle\langle g|\hat{r}|e\rangle\langle e| + q|e\rangle\langle e|\hat{r}|g\rangle\langle g|\end{aligned}$$

$$\boxed{\begin{aligned}&\hat{d} = d_0\left(\sigma_{ge} + \sigma_{eg}\right)\\ &\quad\sigma_{ij} \equiv |i\rangle\langle j|,\ d_0 \equiv q\langle g|\hat{r}|e\rangle\end{aligned}}$$

Atoms and photons – a quick quantization

- Quantizing the electromagnetic field
  - Simple case: field in a box
  - Classical solutions:



$$E(r,t) = -\sum_n \frac{1}{\sqrt{\epsilon_0}} p_n(t) E_n(z), \quad B(r,t) = \sum_n \sqrt{\mu_0}\, \omega_n q_n(t) B_n(z), \quad \omega_n = 2\pi nc/L$$

  - Mode profiles: $E_n(z) = \hat{y}\sqrt{\frac{2}{V}} \sin\frac{\omega_n z}{c}, \quad B_n(z) = \hat{x}\sqrt{\frac{2}{V}} \cos\frac{\omega_n z}{c}$

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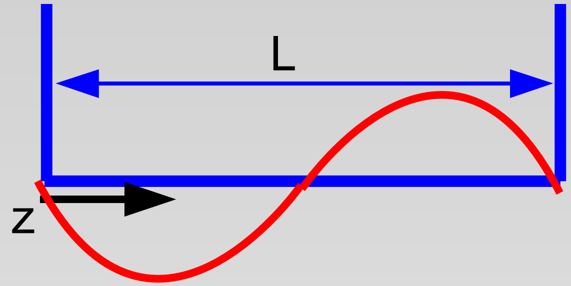

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  - Energy of system: $H = \frac{1}{2}\int d^3 r\, \epsilon_0 E^2 + B^2/\mu_0 = \sum_n p_n^2/2 + \omega_n^2 q_n^2/2$
  - It's the energy of a harmonic oscillator!

Electromagnetic field as a harmonic oscillator

- Can re-write p, q in terms of creation and annihilation operators:

$$\hat{a}_n = \frac{1}{\sqrt{2\hbar\omega_n}}\left(\omega_n q_n + i p_n\right), \quad \hat{a}_n^{\dagger} = \frac{1}{\sqrt{2\hbar\omega_n}}\left(\omega_n q_n - i p_n\right)$$

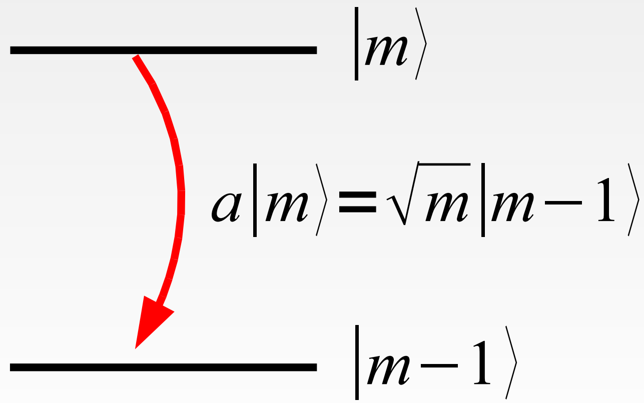
- Hamiltonian:

$$H = \sum_n p_n^2/2 + \omega_n^2 q_n^2/2 = \boxed{\sum_n \hbar\omega_n \hat{a}_n^{\dagger}\hat{a}_n}$$

- In the following we will only care about one mode: $H = \hbar\omega\hat{a}^{\dagger}\hat{a}$

- $\hat{a}^{\dagger}(\hat{a})$ adds (subtracts) one quantum of energy to the system (*i.e.*, a single photon)

Energy eigenstates: $H|m\rangle = \hbar m\omega|m\rangle$



Electric field quantization

- Electric field quantization

$$E(r) = -\sum_n \frac{1}{\sqrt{\epsilon_0}} p_n E_n(z) = \boxed{\hat{y}\sum_n \sqrt{\frac{\hbar\omega_n}{\epsilon_0 V}}\left(\hat{a}_n^{\dagger}+\hat{a}_n\right)\sin k_n z}$$

Electric field operator creates or destroys a single photon

Electric field quantization

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$$E(r) = -\sum_n \frac{1}{\sqrt{\epsilon_0}} p_n E_n(z) = \boxed{\hat{y} \sum_n \sqrt{\frac{\hbar\omega_n}{\epsilon_0 V}} \left(\hat{a}_n^\dagger + \hat{a}_n\right) \sin k_n z}$$

- Single-mode picture: $E(r) = \sqrt{\frac{\hbar\omega}{\epsilon_0 V}} \left(\hat{a}^\dagger + \hat{a}\right) \sin k_n z$

"Electric field per photon"

Electric field quantization

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"Electric field per photon"

- Physical picture: Confining a quantum ($\hbar\omega$) of energy into a smaller box increases the energy density and field intensity

Jaynes-Cummings model

- Combining everything together:

$$V = -\hat{d}\cdot\hat{E}(\vec{r}_{atom})$$

ignore

$$\hat{d} = d_0\left(\sigma_{\mathrm{ge}} + \sigma_{eg}\right) \qquad E(r) = \sqrt{\frac{\hbar\omega}{\epsilon_0 V}}\left(\hat{a}^\dagger + \hat{a}\right)\cancel{\sin \kappa_n z}$$

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- Keep only energy “conserving” terms:

$$V = \frac{\hbar g}{2}(\sigma_{eg}\hat{a} + \sigma_{ge}\hat{a}^\dagger), \qquad g = -2\mathrm{d}_0\sqrt{\frac{\hbar\omega}{\epsilon_0 V}}$$

Excite the atom &
destroy photon

“Single-photon Rabi frequency”

Jaynes-Cummings model

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Excite the atom &
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"Single-photon Rabi frequency"

- Full Hamiltonian (Jaynes-Cummings model)

$$H = \hbar\omega_{atom}\sigma_{ee} + \hbar\omega_{cavity}\hat{a}^\dagger\hat{a} + \frac{\hbar g}{2}\left(\sigma_{eg}\hat{a} + \sigma_{ge}\hat{a}^\dagger\right)$$

Jaynes-Cummings model

- The Jaynes-Cummings Hamiltonian describes the coherent dynamics of cavity QED (and many other systems) very accurately

$$H = \hbar\omega_{atom}\sigma_{ee} + \hbar\omega_{cavity}\hat{a}^{\dagger}\hat{a} + \frac{\hbar g}{2}\left(\sigma_{eg}\hat{a} + \sigma_{ge}\hat{a}^{\dagger}\right)$$

  - Typical atomic cavity QED experiment: g ~ 100 MHz, $\omega_{\text{atom}} \sim \omega_{\text{cavity}} \sim 10^{15}$ Hz
  - Convenient to work in a “rotating frame”

$$H = -\hbar\delta\sigma_{ee} + \frac{\hbar g}{2}\left(\sigma_{eg}\hat{a} + \sigma_{ge}\hat{a}^{\dagger}\right), \quad \delta = \omega_{cavity} - \omega_{atom}$$

A fundamental prediction: Rabi oscillations

- Consider the resonant case, $\delta = 0$

$$H = \frac{\hbar g}{2}\left(\sigma_{eg}\hat{a} + \sigma_{\mathrm{ge}}\hat{a}^{\dagger}\right)$$

- Suppose the system starts with the atom in the excited state:

$$\left|\psi(0)\right\rangle = \left|e, 0_{photon}\right\rangle$$

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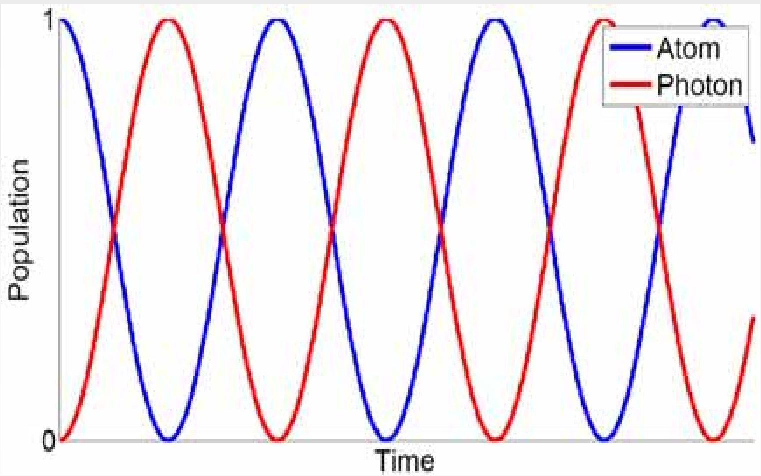
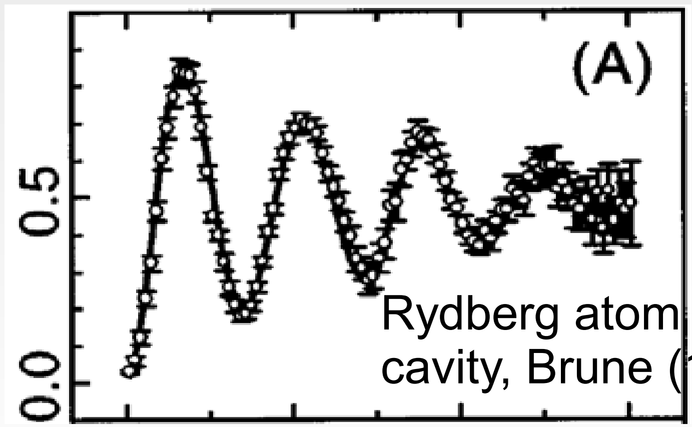
$$\left|\psi(0)\right\rangle = \left|e, 0_{photon}\right\rangle$$

- General solution: $\left|\psi(0)\right\rangle = \cos\frac{\mathrm{gt}}{2}\left|e, 0\right\rangle - i\sin\frac{\mathrm{gt}}{2}\left|g, 1\right\rangle$

- The atom emits and re-absorbs its own photon at a rate *g*!
  - Coherent transfer between light and matter

Ideal





Rydberg atom in microwave cavity, Brune (1996)

Energy levels of system

- Again, take resonant case $H = \frac{\hbar g}{2}\left(\sigma_{eg}\hat{a} + \sigma_{ge}\hat{a}^{\dagger}\right)$
- Diagonalize subspace consisting of n total excitations (|g,n> and |e,n-1>)

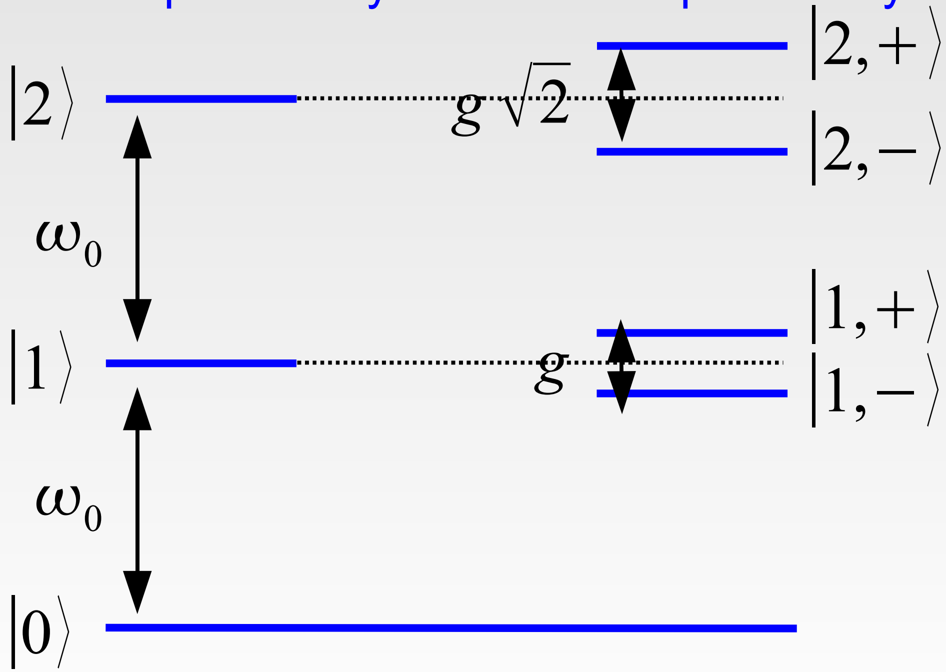
$$|n,-\rangle = |g,n\rangle - |e,n-1\rangle,\ E_{n,-} = -\frac{\hbar g}{2}\sqrt{n}$$

$$|n,+\rangle = |g,n\rangle + |e,n-1\rangle,\ E_{n,+} = \frac{\hbar g}{2}\sqrt{n}$$

- A simple picture of energy levels

Uncoupled cavity

Coupled cavity

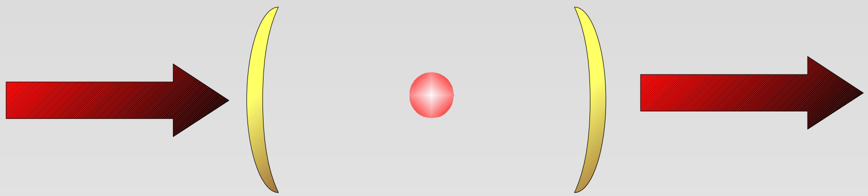


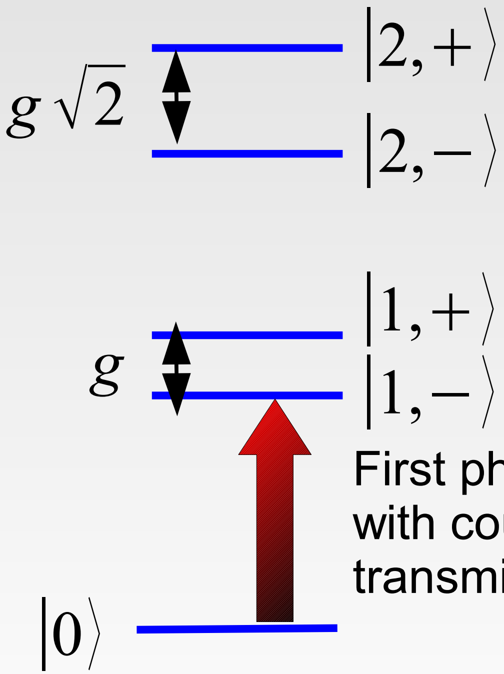
The coupling of the atom to the cavity adds anharmonicity to the system!

Application: single-photon blockade

- The strong anharmonicity allows the atom to mediate *strong interactions* between single photons

- Transmission of single photon





First photon is resonant with coupled system and transmits through

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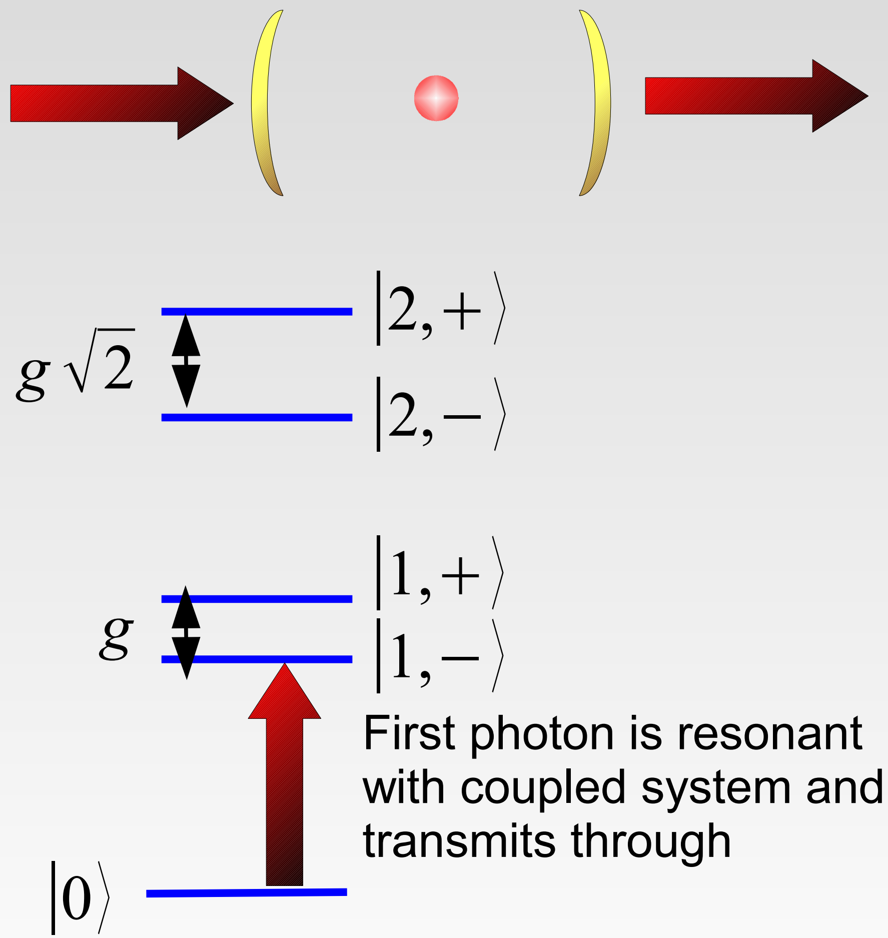
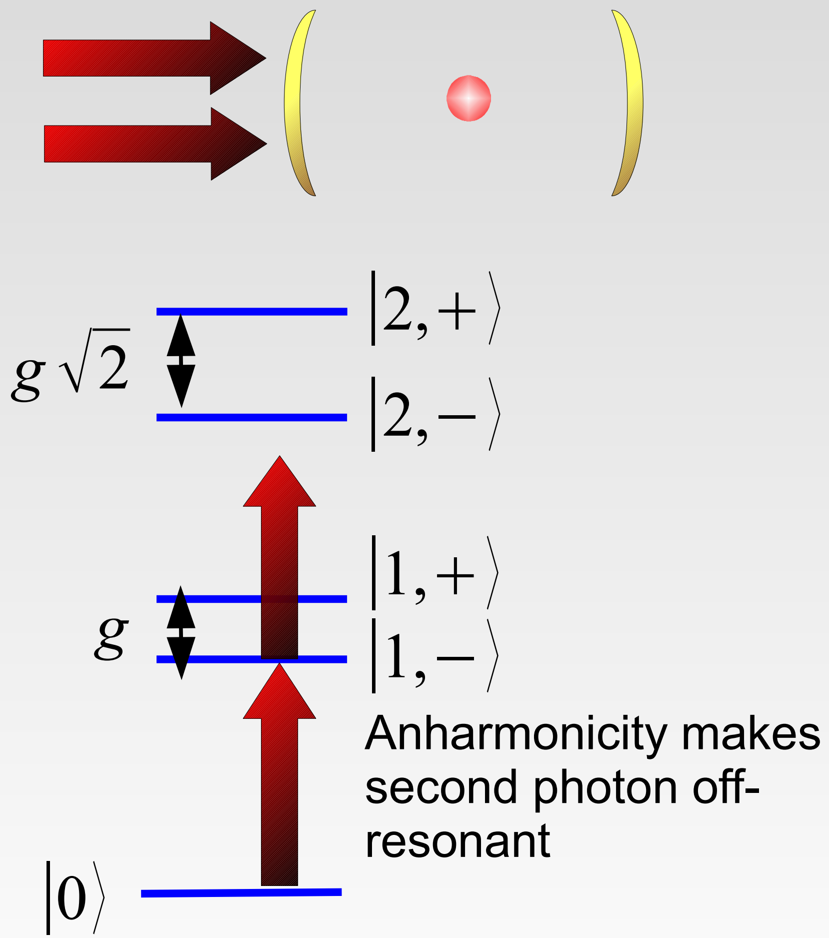
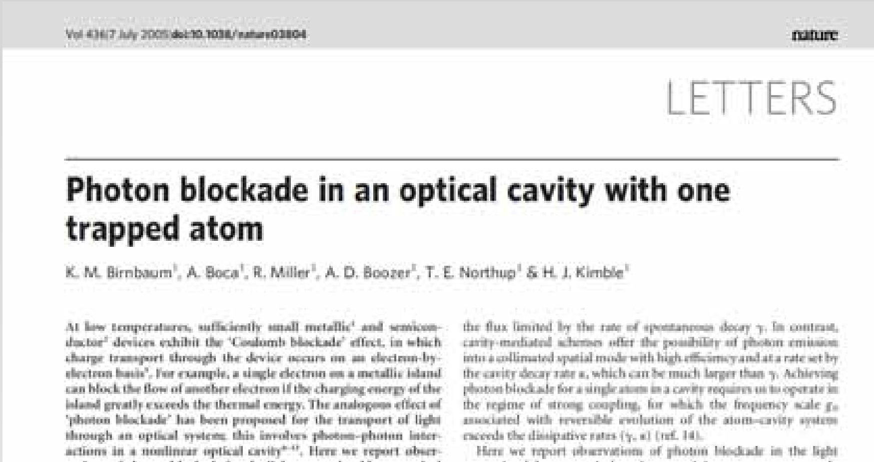


- Blockade of two photons



Application: single-photon blockade

- Experimental observation: Kimble (Caltech), 2005



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LETTERS

Photon blockade in an optical cavity with one trapped atom

K. M. Birnbaum[1], A. Boca[1], R. Miller[1], A. D. Boozer[1], T. E. Northup[1] & H. J. Kimble[1]

At low temperatures, sufficiently small metallic[1] and semiconductor[2] devices exhibit the 'Coulomb blockade' effect, in which charge transport through the device occurs on an electron-by-electron basis[3]. For example, a single electron on a metallic island can block the flow of another electron if the charging energy of the island greatly exceeds the thermal energy. The analogous effect of 'photon blockade' has been proposed for the transport of light through an optical system; this involves photon–photon interactions in a nonlinear optical cavity[4–11]. Here we report obser-

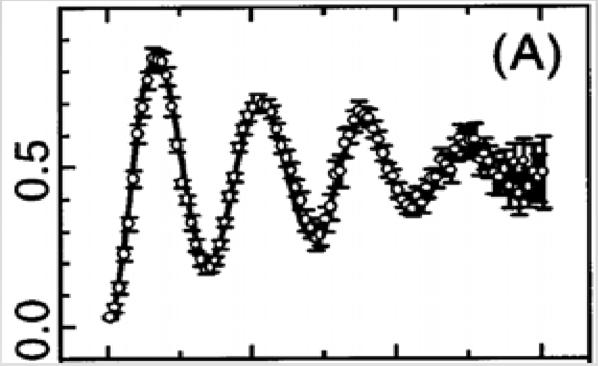
the flux limited by the rate of spontaneous decay γ. In contrast, cavity-mediated schemes offer the possibility of photon emission into a collimated spatial mode with high efficiency and at a rate set by the cavity decay rate κ, which can be much larger than γ. Achieving photon blockade for a single atom in a cavity requires us to operate in the regime of strong coupling, for which the frequency scale g_0 associated with reversible evolution of the atom–cavity system exceeds the dissipative rates (γ, κ) (ref. 14).

Here we report observations of photon blockade in the light

- Single-photon blockade is perhaps simplest example of *single-photon nonlinear optics*
  - *One photon behaves much differently than two*
  - Use strong interactions with a single atom to make individual photons interact with each other
  - More elaborate schemes to realize quantum logic gates involving photons, single-photon optical switches, etc...

Dissipation in cavity QED

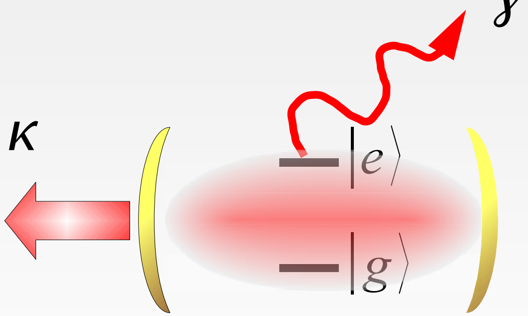
- Phenomena such as Rabi oscillations exhibit decay



- Atoms in cavities are not perfect systems – they leak information to the environment
- This coupling to environment is well-understood, and makes cavity QED a simple open quantum system
- Key mechanisms:

Rate of photon leakage out of cavity κ

$$\left(Q = \frac{\omega}{2\kappa}\right)$$



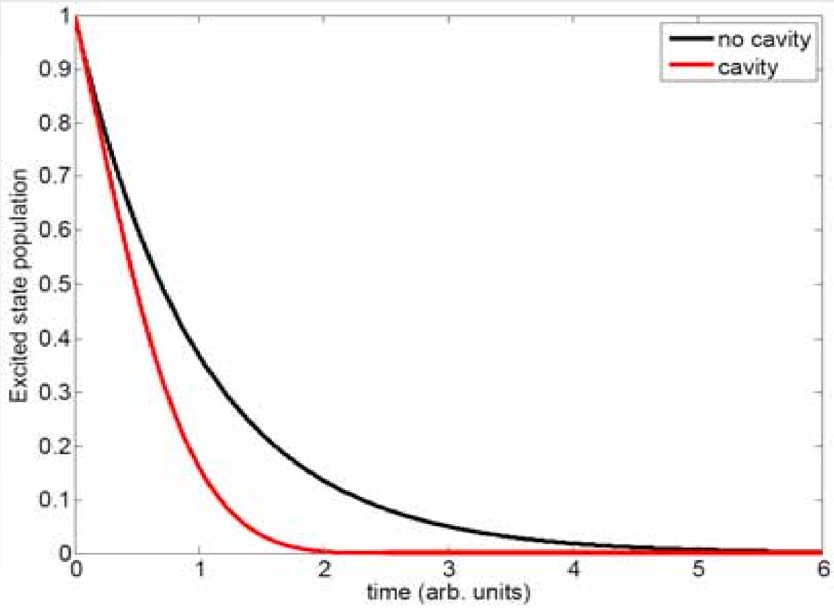
Rate of spontaneous emission of photon out of the cavity

Good cavity / bad cavity

- There are two different regimes of behavior
  - "Good cavity" limit: $g > \kappa, \gamma$
    - Rabi oscillations appear, which decay in time at a rate given by ~ max (κ,γ)

Good cavity / bad cavity

- There are two different regimes of behavior
  - "Good cavity" limit: $g > \kappa, \gamma$
    - Rabi oscillations appear, which decay in time at a rate given by ~ max (κ,γ)
  - "Bad cavity" limit: $g < \kappa$ and/or $g < \gamma$
    - Decay occurs faster than Rabi oscillations can occur
    - Cavity enhances the decay rate of the atom (shortens its lifetime) – the "Purcell effect"



Purcell enhancement and cooperativity

- A resonant cavity enhances the spontaneous emission rate of an atom inside (Purcell effect)
  - Classical interpretation: an oscillating dipole radiates energy and simultaneously decays because it sees its own field. In a cavity, the dipole (atom) sees its own field many times due to reflection.

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Evolution equations

$$\dot{c}_e(t) = \boxed{-\frac{ig}{2}c_g} - \frac{\gamma}{2}c_e$$

$$\dot{c}_g(t) = \boxed{-\frac{ig}{2}c_e} - \frac{\kappa}{2}c_g$$

Coherent evolution

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Dissipation

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$$\dot{c}_g(t) = -\frac{ig}{2}c_e - \frac{\kappa}{2}c_g$$

0

$$c_g(t) \approx -\frac{ig}{\kappa}c_e(t)$$

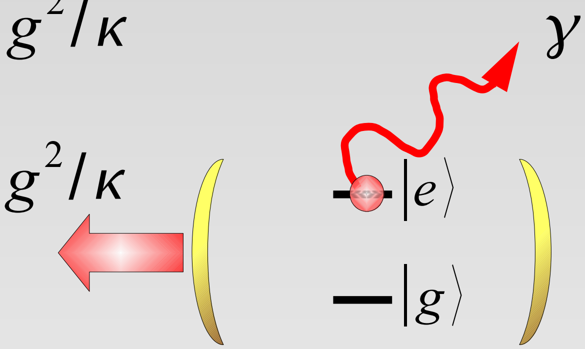
$$\dot{c}_e(t) = -\frac{1}{2}\left(\frac{g^2}{\kappa} + \gamma\right)c_e(t)$$

"Adiabatic elimination" -- If $\kappa >> g$, the amplitude c_g reaches a pseudo-equilibrium

Purcell enhancement and cooperativity

$$\dot{c}_e(t) = -\frac{1}{2}\left(\frac{g^2}{\kappa} + \gamma\right) c_e(t)$$

- Resonant coupling to cavity creates an additional decay rate to the excited state, g^2/κ



  - Cavity-induced decay represents a "good decay" -- light leaking out of the cavity can be collimated or coupled into an optical fiber (and delivered to another atom, for example)
  - Spontaneous emission out of the cavity into 4π represents a "bad decay"

- A useful figure of merit is the "cooperativity" factor – the ratio of good to bad decay

$$C = \frac{g^2}{\kappa\gamma}$$

The strong-coupling regime

- The regime $C = \frac{g^2}{\kappa\gamma} \gg 1$ is known as the strong-coupling regime
  - Note that (coherent evolution rate *g*) >> (decay rates κ,γ) is sufficient but not necessary to reach strong coupling
  - We derived C assuming κ > *g*, but C is an important figure of merit regardless of relative sizes of parameters
  - C is a *fundamental parameter* that determines the fidelity of many quantum information protocols involving cavity QED
- State-of-the-art in cavity QED with atoms:

Kimble, Fabry-Perot cavity:

$$\begin{aligned} g/2\pi &\sim 34\ MHz \\ \gamma/2\pi &\sim 2.6\ MHz \\ \kappa/2\pi &\sim 4.1\ MHz \\ C &\sim 110 \end{aligned}$$

Kimble, microtoroidal cavity:

$$\begin{aligned} g/2\pi &\sim 70\ MHz \\ \gamma/2\pi &\sim 1\ MHz \\ \kappa/2\pi &\sim 5\ MHz \\ C &\sim 1000 \end{aligned}$$

Relation to fundamental cavity properties

- The cooperativity can be related to fundamental cavity properties:

$$C = \frac{g^2}{\kappa \gamma}$$

$$g = 2\mathrm{d}_0 \sqrt{\frac{\hbar \omega}{\epsilon_0 V}}, \quad \kappa = \frac{\omega}{2\mathrm{Q}}, \quad \gamma = \frac{\omega^3 d_0^2}{3\pi \epsilon_0 \hbar c^3}$$

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Spontaneous emission rate of atom in free space (usually not significantly changed by cavity)

$$\boxed{C = \frac{3}{2\pi^2}\frac{Q\lambda^3}{V}}$$

- A high cooperativity can be obtained by achieving very high quality factors and small mode volumes

Cavity input-output relations

- It is not easy to directly measure the light inside the cavity
- Instead, we can only measure the light that leaks out
  - Need some prescription to relate the light inside to what we can measure – an “input-output” relation
- A simple model of cavity leakage:

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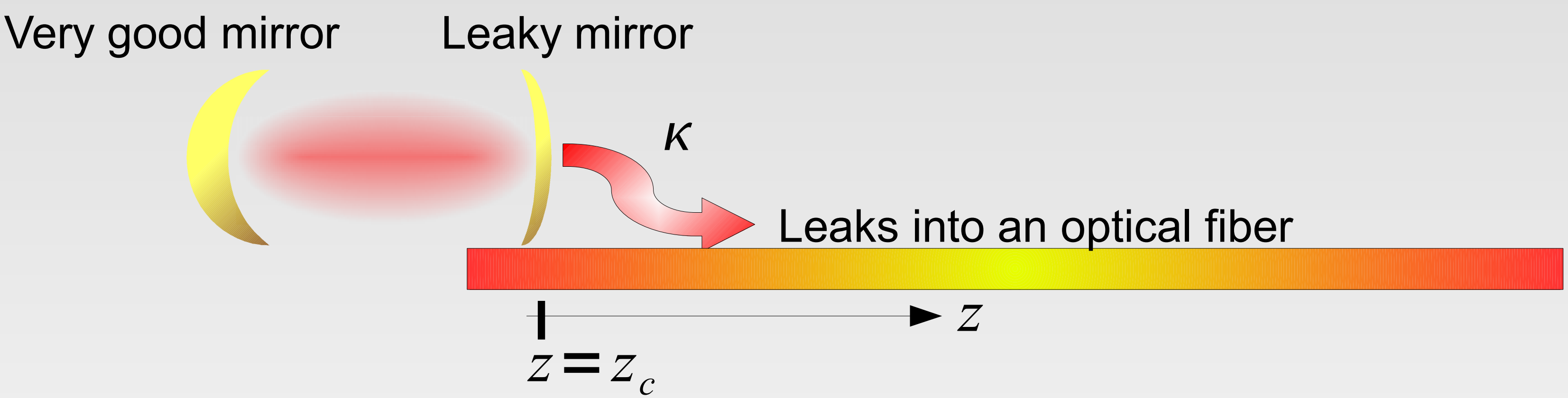

- Waveguide: 1D continuum of modes with wavevectors k, frequencies ω=ck
  - Model Hamiltonian:

$$H=\int dk\,\hbar ck\,\hat{b}_k^\dagger \hat{b}_k+\hbar\omega_{cavity}\hat{a}^\dagger\hat{a}+\hbar\beta\int dk\left(\hat{b}_k e^{ikz_c}\hat{a}_k^\dagger+h.c.\right)$$

Waveguide modes — Cavity — Linear waveguide-cavity coupling

Field propagation equations

$$H = \int dk\, \hbar ck\, \hat{b}_k^\dagger \hat{b}_k + \hbar \omega_{cavity} \hat{a}^\dagger \hat{a} - \hbar \beta \int dk \left(\hat{b}_k e^{ikz_c} \hat{a}_k^\dagger + h.c. \right)$$

- Waveguide field operator:

$$\hat{E}(z) = \frac{1}{\sqrt{2\pi}} \int dk\, \hat{b}_k e^{ikz}$$

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- Waveguide field operator:

$$\hat{E}(z)=\frac{1}{\sqrt{2\pi}}\int dk\,\hat{b}_k e^{ikz}$$

- Heisenberg equation of motion of field

$$\frac{\partial}{\partial t}\hat{E}=\frac{i}{\hbar}\left[H,\hat{E}\right]$$

$$=-\frac{1}{c}\frac{\partial}{\partial z}\hat{E}+\frac{\sqrt{2\pi}\,i\beta}{c}\delta(z-z_c)\hat{a}(t)$$

$$\boxed{\hat{E}(z,t)=\hat{E}_{free}(z,t)+\frac{\sqrt{2\pi}\,i\beta}{c}\Theta(z-z_c)\hat{a}(t-(z-z_c)/c)}$$

- Relates field inside the cavity to what leaks into the waveguide

Cavity evolution

- Also need to determine how coupling to waveguide affects the cavity leakage
- Heisenberg equation of motion of cavity

$$\frac{\partial}{\partial t}\hat{a} = -i\omega_{cavity}\hat{a} + i\beta\int dk\,\hat{b}_k e^{ikz_c}$$

$$= -i\omega_{cavity}\hat{a} + i\beta\sqrt{2\pi}\,\hat{E}(z_c)$$

$$= -i\omega_{cavity}\hat{a} + i\beta\sqrt{2\pi}\left(\hat{E}_{free}(z_c,t) + \frac{\sqrt{2\pi}\,i\beta}{2c}\hat{a}\right)$$

$$= -i\omega_{cavity}\hat{a} - \frac{1}{2}\frac{2\pi\beta^2}{c}\hat{a} + \hat{F}_{noise}(t)$$

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$$= -i\omega_{cavity}\hat{a} - \frac{1}{2}\underbrace{\frac{2\pi\beta^2}{c}}_{=\kappa}\hat{a} + \hat{F}_{noise}(t)$$

Derived dissipation starting from a microscopic model!

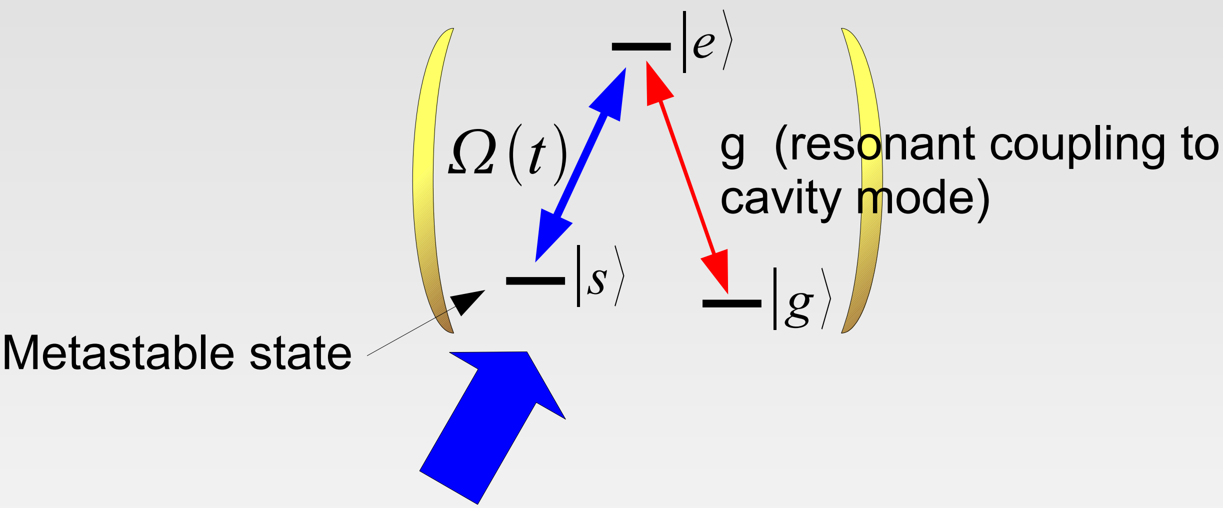
Dissipation is accompanied by noise (fluctuation-dissipation theorem)

- Can now relate cavity dynamics with some measurable quantity (light coming out of cavity)

$$\hat{E}(z_c,t) = \hat{E}_{free}(z_c,t) + i\sqrt{\frac{\kappa}{c}}\,\hat{a}(t)$$

Coherent control in cavity QED: the three-level atom

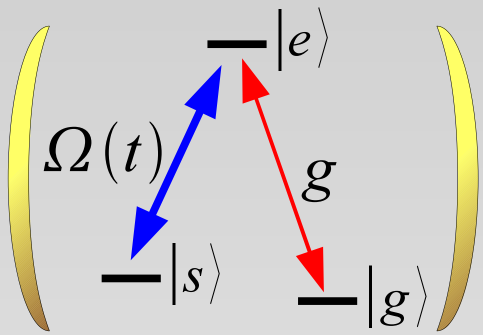
- With a two-level atom, all the rates g, κ, γ are fixed
  - No way of controlling the dynamics or interactions
- Simplest fix: use an atomic system with three internal levels
  - This "simple" system is a *powerful tool* that forms the basis for many cavity QED-based quantum information protocols



External laser couples states |s> and |e> with Rabi frequency Ω(t) – can be tuned by adjusting laser intensity

|s>-|e> is not coupled to cavity mode (*e.g.*, far off resonance with cavity)

Hamiltonian for three-level system



- System Hamiltonian:

$$H = -\hbar\delta\sigma_{ee} + \frac{\hbar g}{2}\left(\sigma_{eg}\hat{a} + \sigma_{ge}\hat{a}^{\dagger}\right) + \frac{\hbar}{2}\underbrace{\left(\Omega(t)\sigma_{es} + \Omega^{*}(t)\sigma_{se}\right)}_{\text{New term: coupling to classical field}}, \qquad \delta = \omega_{cavity} - \omega_{eg}$$

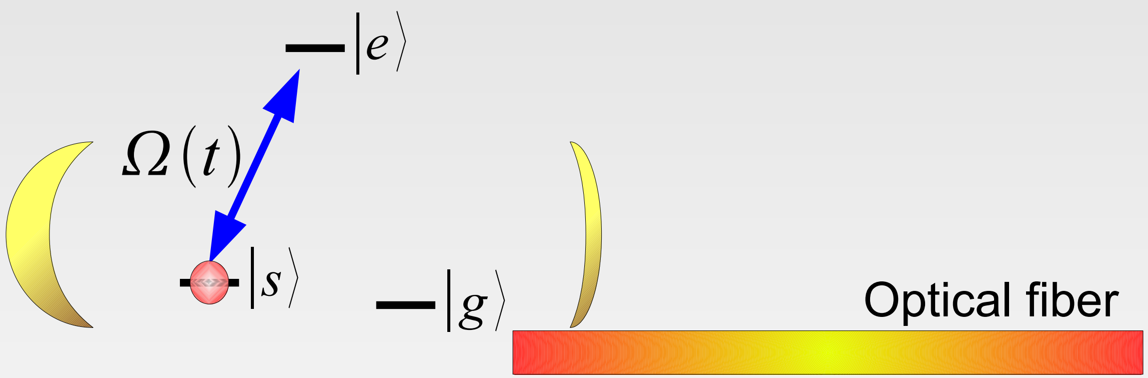
- Plus losses:

$$H_{loss} = -i\hbar\frac{\gamma}{2}\sigma_{ee} - i\hbar\frac{\kappa}{2}\hat{a}^{\dagger}\hat{a}$$

- When the external control laser $\Omega(t)$ is turned off, |s> is de-coupled from rest of system
  - $\Omega(t)$ allows one (roughly speaking) to transfer population into and out of cavity QED system as one pleases

Single photon generation on demand

- Single photons are a key resource in quantum information (*e.g.*, quantum cryptography)
- Cavity QED for generating single photons on demand and controlling the photon pulse shape
  - Related experiments: Kimble (Caltech), Imamoglu (ETH), Yamamoto (Stanford), Vuckovic (Stanford), Rempe (MPQ), ...
- General idea:



Initialize atom in state |s> and drive system to |e> with Ω(t)

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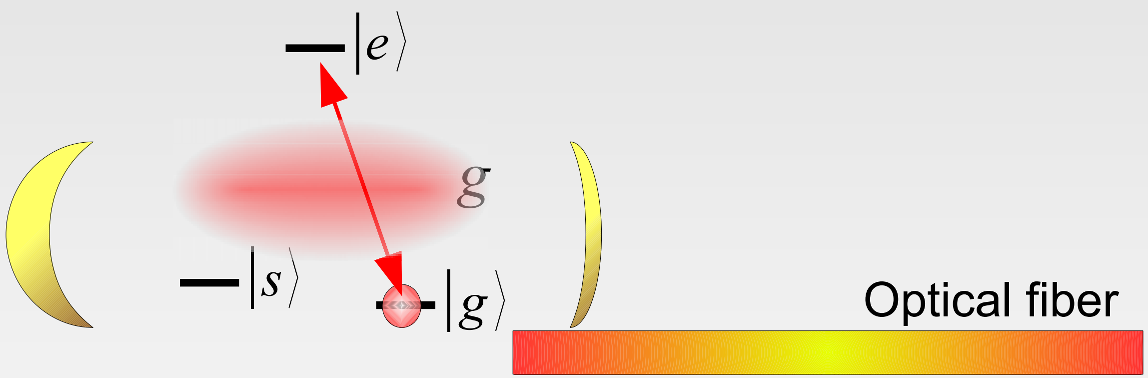

|e> decays into |g> and emits photon into cavity (strong coupling)

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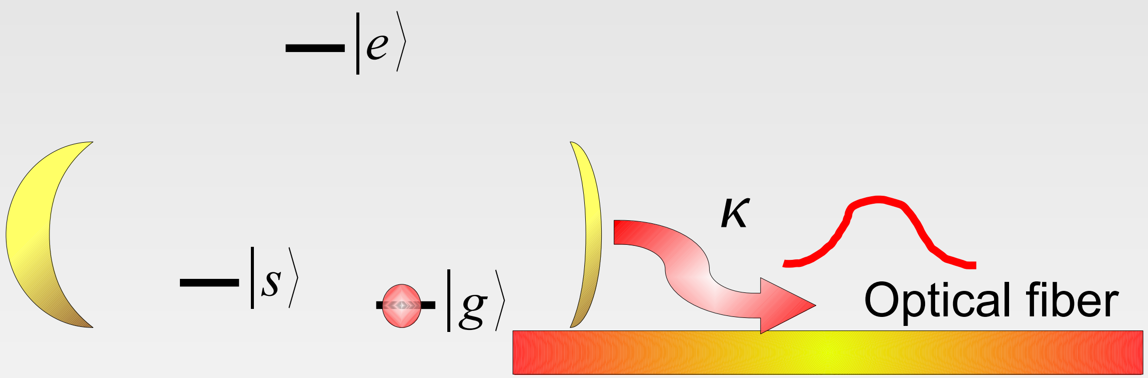
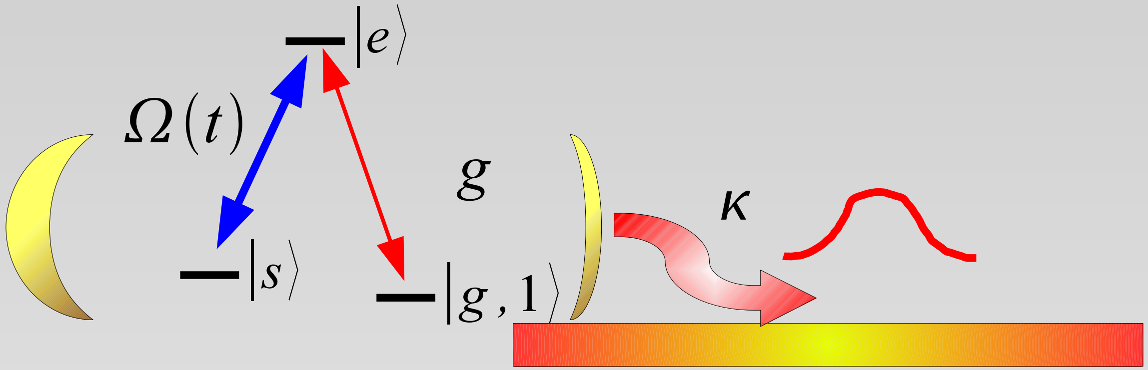


Photon leaks out of cavity, creating an outgoing single photon in optical fiber

Single photon generation on demand



- Effective wave-function of system:

$$|\psi(t)\rangle = c_s(t)|s,0\rangle + c_e(t)|e,0\rangle + c_g(t)|g,1\rangle$$

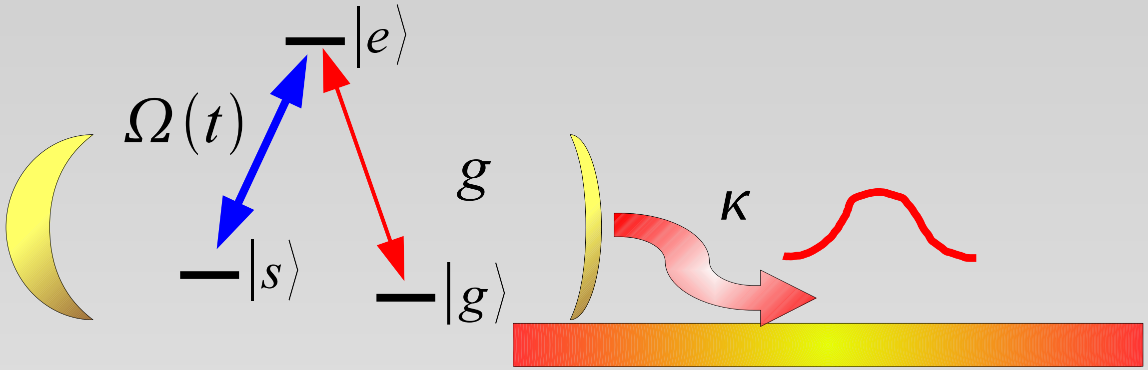
- Equations of motion:

$$\dot{c}_s = -i\frac{\Omega(t)}{2}c_e$$

$$\dot{c}_e = -i\frac{\Omega(t)}{2}c_s - i\frac{g}{2}c_g - \frac{\gamma}{2}c_e$$

$$\dot{c}_g = -i\frac{g}{2}c_e - \frac{\kappa}{2}c_g$$

Single photon generation on demand



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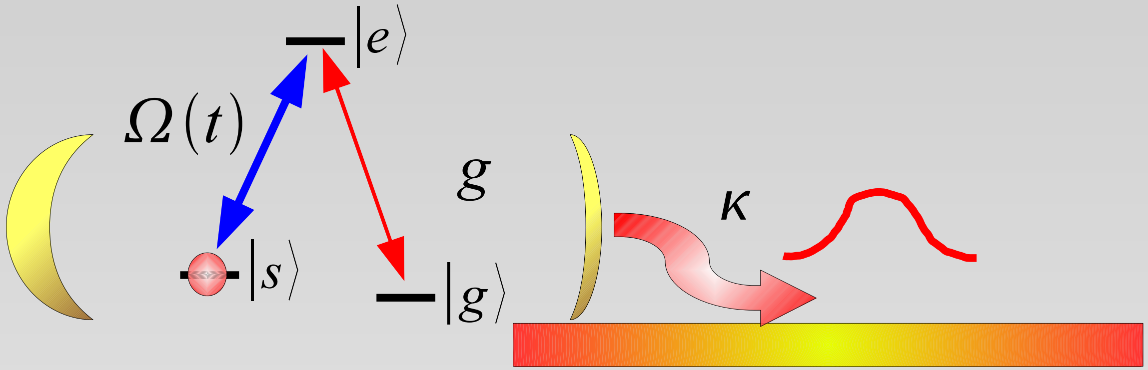
$$\dot{c}_s = -i\frac{\Omega(t)}{2}c_e \quad \Longrightarrow \quad \dot{c}_s = -\frac{1}{2}\frac{\Omega^2(t)\kappa}{g^2+\gamma\kappa}c_s(t)$$

$$\cancelto{0}{\dot{c}_e} = -i\frac{\Omega(t)}{2}c_s - i\frac{g}{2}c_g - \frac{\gamma}{2}c_e \quad \Longrightarrow \quad c_e(t) \approx -i\,c_s(t)\frac{\Omega(t)\kappa}{g^2+\gamma\kappa}$$

$$\cancelto{0}{\dot{c}_g} = -i\frac{g}{2}c_e - \frac{\kappa}{2}c_g \qquad\qquad c_g(t) \approx -c_s(t)\frac{\Omega(t)g}{g^2+\gamma\kappa}$$

Adiabatic elimination – valid when $\Omega(t)$ pumps population at rate << κ

Single photon generation on demand



$$\dot{c}_s \approx -\frac{1}{2}\frac{\Omega^2(t)\kappa}{g^2+\gamma\kappa}c_s(t)$$

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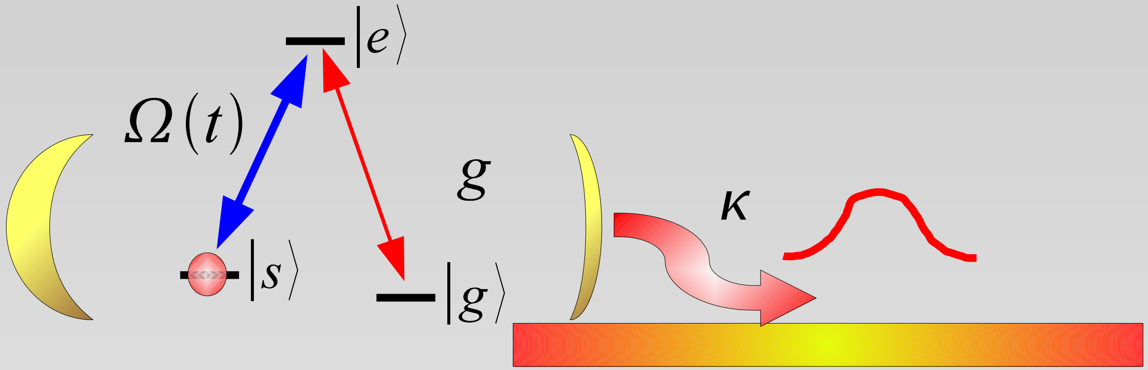
Control field Ω(t) allows us to pump population out of |s> and into cavity in a nearly arbitrary way

$$c_s(t) = \exp\left(-\int_0^t d\tau \frac{1}{2}\frac{\Omega^2(\tau)\kappa}{g^2+\gamma\kappa}\right)$$

- Also recall our output relation: $\hat{E}(z_c,t) = \hat{E}_{free}(z_c,t) + i\sqrt{\frac{\kappa}{c}}\hat{a}(t)$
  - Allows us to determine what comes out of the cavity

Single photon generation on demand



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$$c_s(t) = \exp\left(-\int_0^t d\tau \frac{1}{2}\frac{\Omega^2(\tau)\kappa}{g^2+\gamma\kappa}\right)$$

$$c_e(t) \approx -i c_s(t)\frac{\Omega(t)\kappa}{g^2+\gamma\kappa}$$

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- Also recall our output relation: $\hat{E}(z_c,t) = \hat{E}_{free}(z_c,t) + i\sqrt{\frac{\kappa}{c}}\hat{a}(t)$
  - Allows us to determine what comes out of the cavity
- Single-photon wavefunction $\langle 0|\hat{E}(z_c,t)|\psi(t)\rangle = i\sqrt{\frac{\kappa}{c}}c_g(t)$

Photon wavepacket depends on $\Omega(t)$!
Can shape it however we want just by solving this integral equation.

$$= i\sqrt{\frac{\kappa}{c}}\frac{\Omega(t)g}{g^2+\gamma\kappa}\exp\left(-\int_0^t d\tau \frac{1}{2}\frac{\Omega^2(\tau)\kappa}{g^2+\gamma\kappa}\right)$$

Single photon generation on demand

- Probability of generating single photon: intuitively,

Emission into cavity

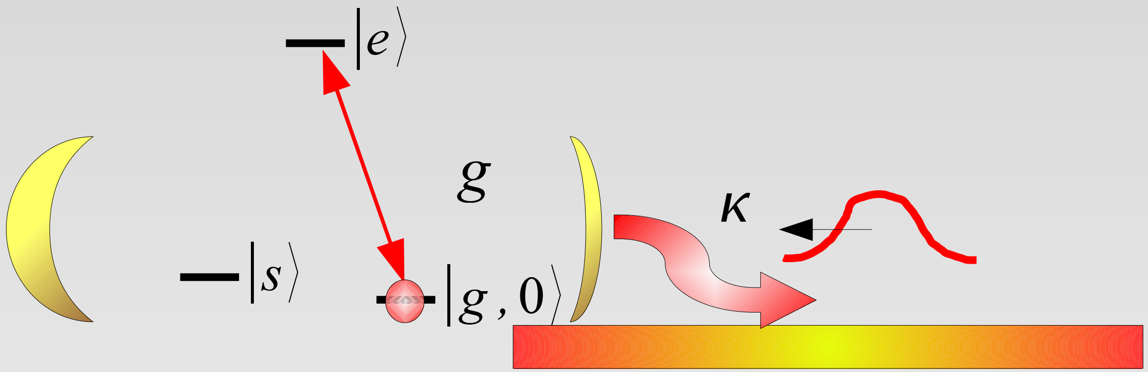
$$P = \frac{g^2/\kappa}{\gamma + g^2/\kappa} = \frac{C}{1+C}$$

Emission out of cavity

  - Illustrates importance of cooperativity as a fundamental parameter
  - The probability P can also be derived by integrating the single-photon wavefunction density, $\left|\langle 0|\hat{E}(z_c,t)|\psi(t)\rangle\right|^2$

Coherent single-photon storage

- One can also consider the reverse process of generation: take an incoming photon and *coherently absorb* it with the atom (by flipping its internal state)



Incident single photon from waveguide

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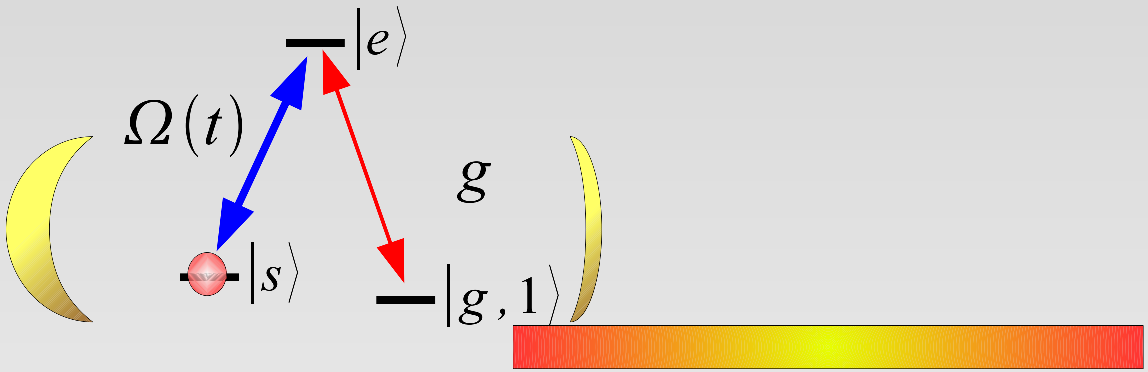


Turn on control field. Two-photon process absorbs incoming photon and *flips the internal atomic state*

- Have effectively converted photonic information into atomic information
- Fulfills basic requirement of a node of quantum network – should be able to "pass *and* catch" photons

Coherent single-photon storage

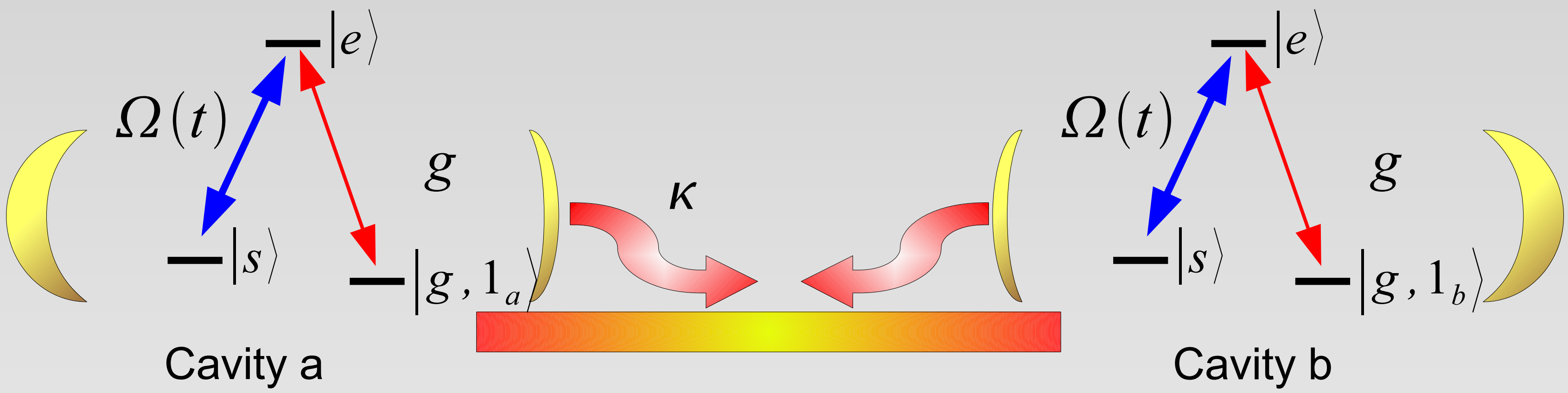
- What is the probability of single-photon storage?
- The single-photon generation process is *entirely quantum mechanical*, thus time-reversal arguments hold
- By time reversal, the maximum probability of storage is the same as generation!

$$P = \frac{C}{1+C}$$

- Also by time reversal, maximum probability is achieved only the proper $\Omega(t)$ is chosen
  - An "impedance-matching" condition

Quantum state transfer

- Connecting two single-photon sources allows for quantum state transfer over long distances



- Atom in cavity *a* is prepared in some arbitrary superposition of internal states, $|\psi_{init}\rangle = (\alpha|s_a\rangle + \beta|g_a\rangle)|g_b\rangle$
- Goal: transfer the quantum state over to atom in cavity *b* by passing and catching photons, $|\psi_{final}\rangle = |g_a\rangle(\alpha|s_b\rangle + \beta|g_b\rangle)$
- One maps atomic information into photonic information and back!

Quantum state transfer

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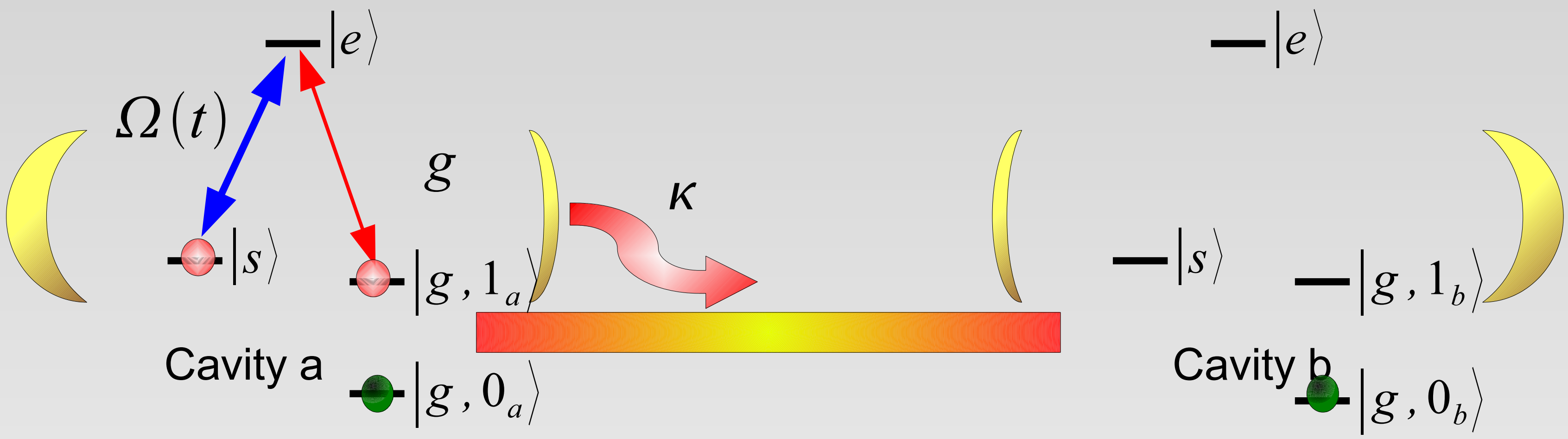

- Protocol:
  - 1. Turn on Ω(t) in cavity *a*. Maps atomic superposition into photonic superposition in cavity.

$$|\psi_{init}\rangle = \left(\alpha|s_a\rangle + \beta|g_a\rangle\right)|g_b\rangle \rightarrow \left(\alpha|g_a, 1_a\rangle + \beta|g_a\rangle\right)|g_b\rangle$$

Quantum state transfer

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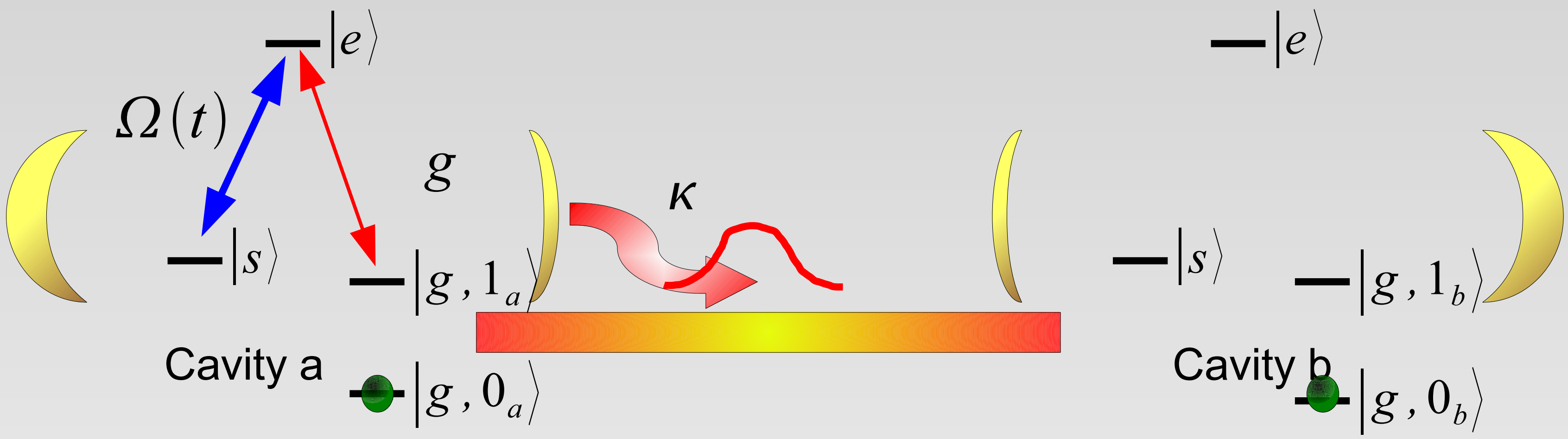

- Protocol:
  - 2. Photon in cavity *a* leaks into waveguide.

$$|\psi_{init}\rangle = (\alpha|s_a\rangle + \beta|g_a\rangle)|g_b\rangle \rightarrow (\alpha|g_a,1_a\rangle + \beta|g_a\rangle)|g_b\rangle \rightarrow |g_a\rangle(\alpha|1_{waveguide}\rangle + \beta|0_{waveguide}\rangle)|g_b\rangle$$

Quantum state transfer

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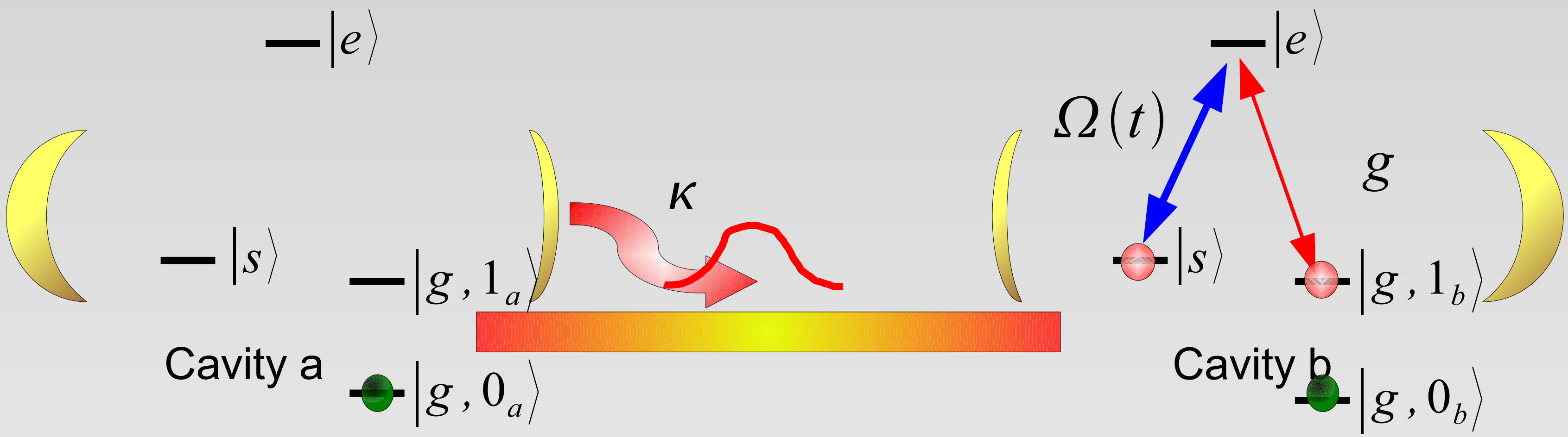
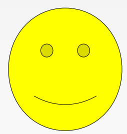


- Protocol:
  - 3. Turn on Ω(t) in cavity b. *If* a photon is present in the waveguide, it enters cavity *b* and gets absorbed by the atom.

$$|\psi_{init}\rangle = (\alpha|s_a\rangle + \beta|g_a\rangle)|g_b\rangle \rightarrow (\alpha|g_a, 1_a\rangle + \beta|g_a\rangle)|g_b\rangle \rightarrow |g_a\rangle(\alpha|1_{waveguide}\rangle + \beta|0_{waveguide}\rangle)|g_b\rangle$$

$$\rightarrow |g_a\rangle(\alpha|s_b\rangle + \beta|g_b\rangle) = |\psi_{final}\rangle$$



Quantum state transfer

- Probability of success:

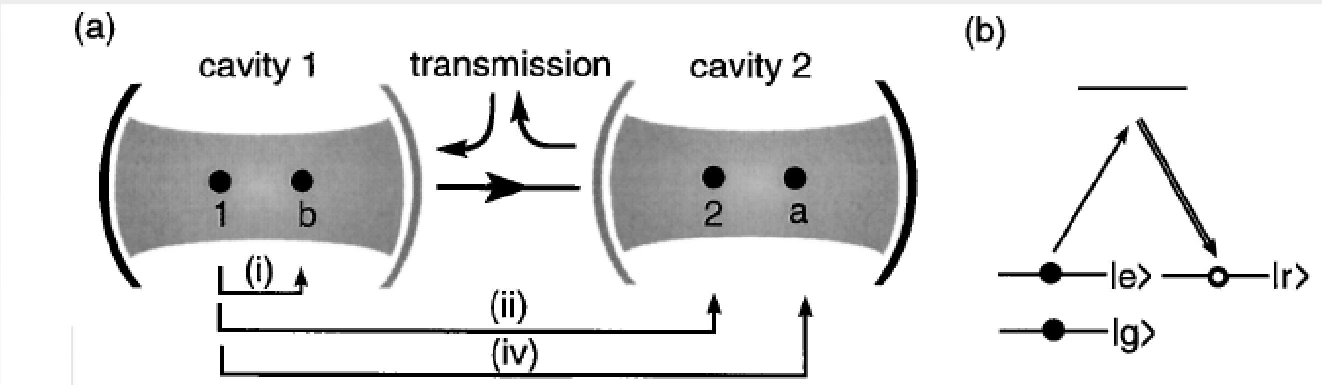
$$P \sim \frac{C}{1+C} \times \frac{C}{1+C} \times \exp\left(\frac{-L_{waveguide}}{L_{loss}}\right)$$

Single-photon generation

Single-photon storage

Losses in waveguide

- Probability can be improved by using two atoms in each cavity to redundantly encode information
  - Protocol: “If at first you don't succeed, try, try again!”



Van Enk, Cirac, Zoller, PRL (1997)

Field correlation functions

- In an experiment, what can we measure to determine whether the output really consists of a single photon?
  - Very difficult to resolve photon number directly

Field correlation functions

- In an experiment, what can we measure to determine whether the output really consists of a single photon?
  - Very difficult to resolve photon number directly
- Measure field correlation functions
  - A simple example: field intensity in fiber

$$I(t) = \left\langle \hat{E}^{\dagger}(z_c, t)\, \hat{E}(z_c, t) \right\rangle$$

  - A single-photon pulse has a weak intensity, but we can't truly distinguish it from very weak classical light... intensity measurements not enough

Field correlation functions

- In an experiment, what can we measure to determine whether the output really consists of a single photon?
  - Very difficult to resolve photon number directly
- Measure field correlation functions
  - A simple example: field intensity in fiber

$$I(t) = \langle \hat{E}^{\dagger}(z_c, t)\hat{E}(z_c, t)\rangle$$

  - A single-photon pulse has a weak intensity, but we can't truly distinguish it from very weak classical light... intensity measurements not enough
- Solution: look at higher-order field correlation functions
  - In particular, look at "normalized second-order correlation function"

$$g^{(2)}(t, t') = \frac{\langle \hat{E}^{\dagger}(z_c, t)\hat{E}^{\dagger}(z_c, t')\hat{E}(z_c, t')\hat{E}(z_c, t)\rangle}{I(t)I(t')}$$

Meaning of $g^{(2)}$

- Physical significance of $g^{(2)}$:

$$g^{(2)}(t,t')=\frac{\left\langle \hat{E}^{\dagger}(z_c,t)\hat{E}^{\dagger}(z_c,t')\hat{E}(z_c,t')\hat{E}(z_c,t)\right\rangle}{I(t)I(t')}$$

normalization

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normalization

- Consider numerator for a pure state:

$$\langle\psi|\hat{E}^{\dagger}(z_c,t)\hat{E}^{\dagger}(z_c,t')\hat{E}(z_c,t')\hat{E}(z_c,t)|\psi\rangle=\langle\tilde{\psi}|\hat{E}^{\dagger}(z_c,t')\hat{E}(z_c,t')|\tilde{\psi}\rangle$$

$$\text{where } |\tilde{\psi}\rangle=\hat{E}(z_c,t)|\psi\rangle$$

"Given my original state |ψ>, destroy a photon at time t"

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"Given my original state |ψ>, destroy a photon at time t"

$$\boxed{\langle\tilde{\psi}|\hat{E}^{\dagger}(z_c,t')\hat{E}(z_c,t')|\tilde{\psi}\rangle=\langle I(t')\rangle_{|\tilde{\psi}\rangle}}$$

"After I destroy a photon at time t, what is the field intensity I measure at some following time t' ?"

$g^{(2)}$ for a single photon source

- For a single photon source, once a photon is destroyed at time t, there are no photons left!

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  - Compare with laser (classical light): $g^{(2)}(t,t')=1$

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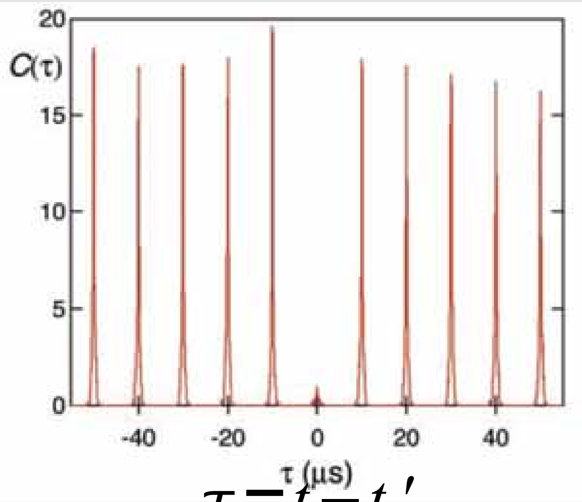
- Single-photon experiment (Kimble, 2004):



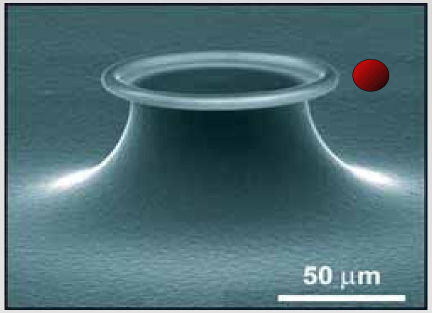
$\tau = t - t'$

  - Peaks are due to repetitions of photon generation experiment
  - *Suppression of peak at $\tau = 0$* indicates very good single-photon source (residual due to dark counts and stray light)

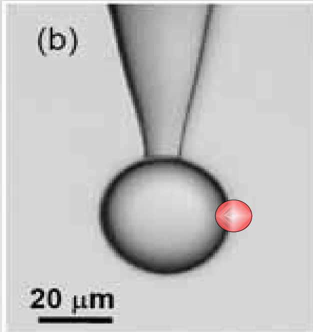
- Photon correlations are widely used to determine non-classicality of light

Hybrid quantum networks

- Many different kinds of systems are being used for cavity QED and quantum optics
  - Examples:



Cs atom coupled to microtoroid



N-V center coupled to microsphere

Rb atoms in vapor cell for photon storage

  - Each system has distinct benefits and drawbacks

Hybrid quantum networks

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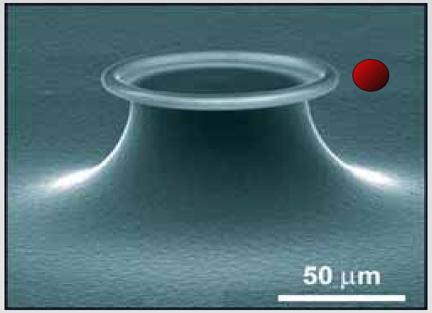

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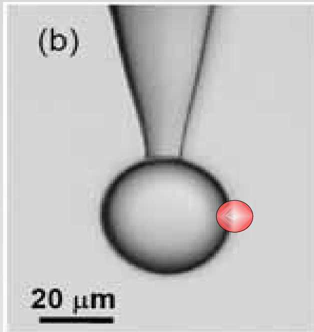

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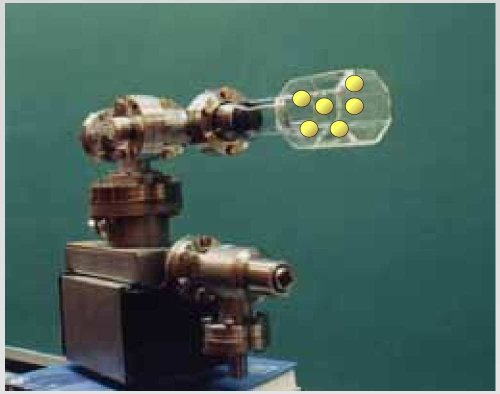
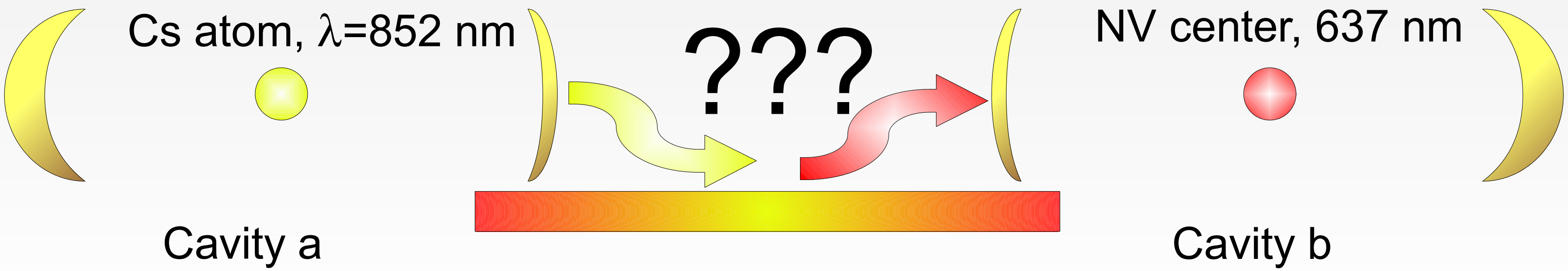


Rb atoms in vapor cell for photon storage

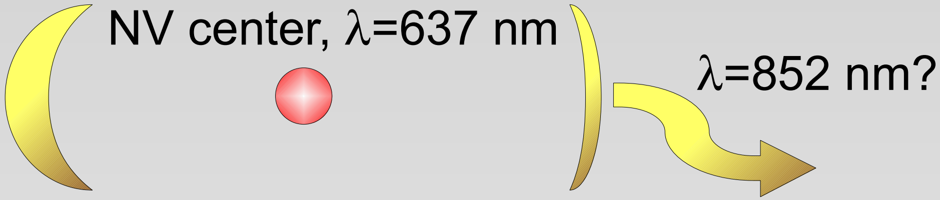
  - Each system has distinct benefits and drawbacks

- Can we create a *hybrid quantum network* where we can mix and match the best attributes?
  - Challenge: different kinds of emitters have different optical frequencies



Spectral control of single photon generation

- Goal: can we control the frequency of a photon emitted by an atom?

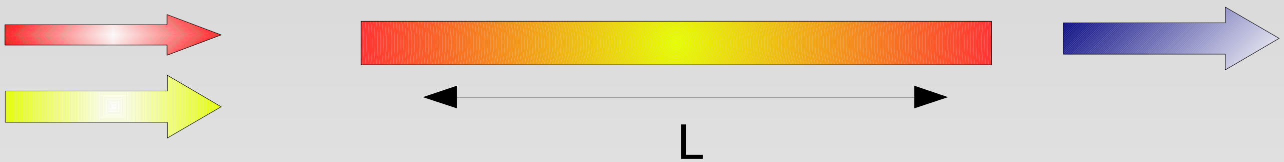


- If possible, it enables:
  - Hybrid quantum networks
  - Conversion of single photons into telecom bands for long-distance propagation
  - Shifting single photons into wavelengths where high-efficiency detectors are available

Sum/difference frequency generation

- Frequency conversion is an old technique in nonlinear optics
  - e.g., nonlinear fiber:

$$P_i e^{-i(\omega_j+\omega_k)t} = \epsilon_0 \chi_{ijk}^{(2)} E_j e^{-i\omega_j t} E_k e^{-i\omega_k t}$$



- Efficient conversion requires:
  - Long interaction lengths L (nonlinearities are usually weak)
  - Energy *and* momentum conservation (phase-matching)
    - Automatically satisfied in free space, but not in dispersive material

Vacuum

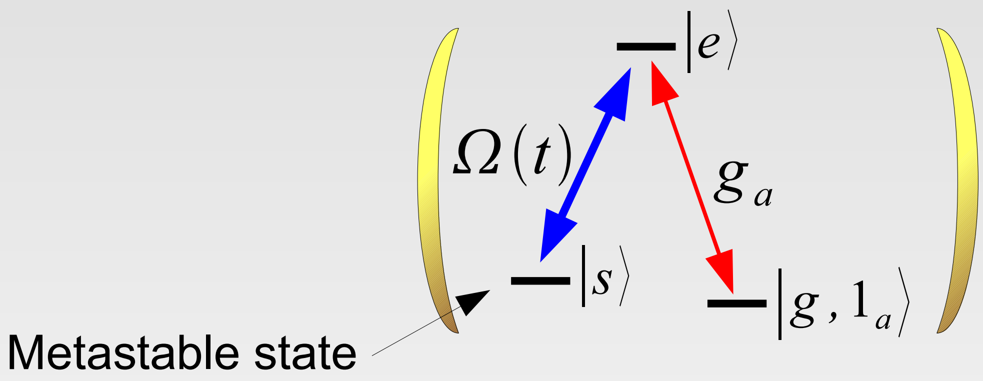
$$\omega_1+\omega_2=\omega_3, \quad \omega_i = ck_i$$
$$k_1+k_2=k_3$$

Fiber

$$\omega_1+\omega_2=\omega_3, \quad \omega_i = n_i ck_i$$
$$k_1+k_2\neq k_3$$

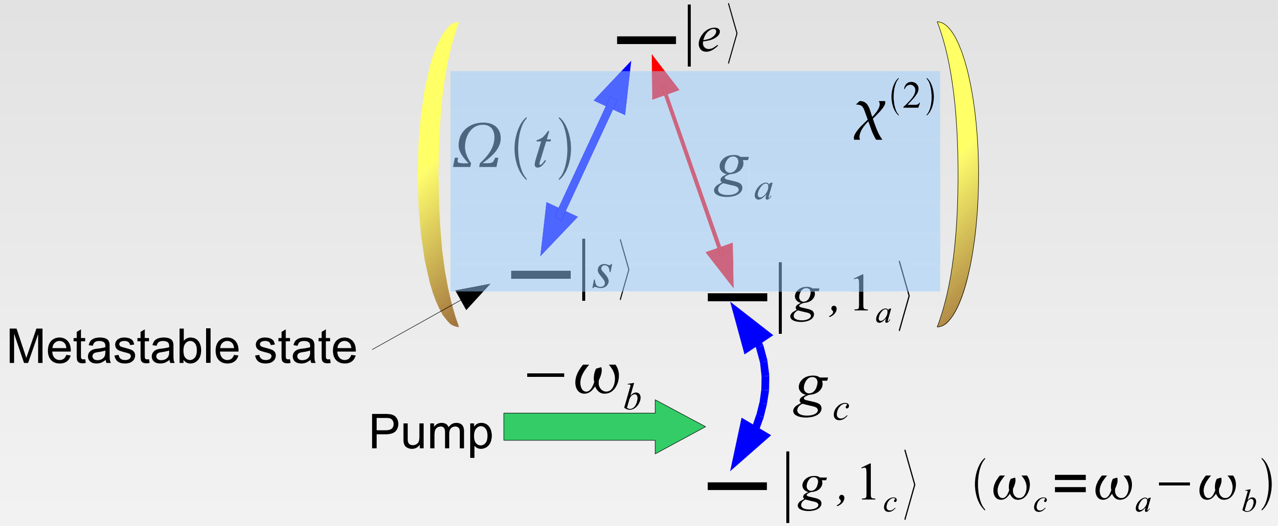
Frequency conversion in a nonlinear cavity

- Idea: use an optical cavity made out of a nonlinear material to accomplish frequency conversion
  - Short interaction length, can integrate on chip
  - No explicit phase-matching requirement
- Similar to previous scheme of single-photon generation, but we now consider two cavity modes



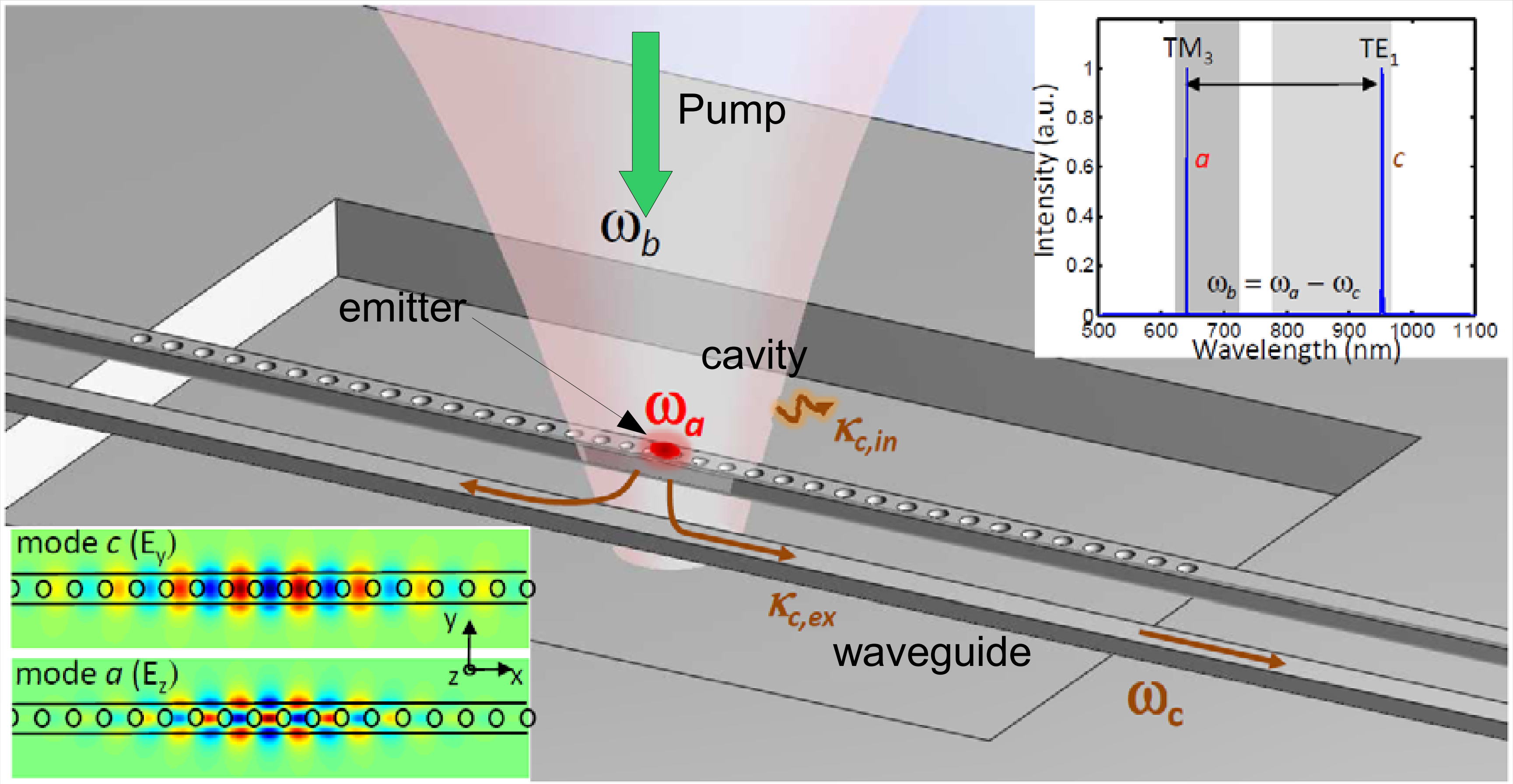
Frequency conversion in a nonlinear cavity

- Proposal: use an optical cavity made out of a nonlinear material to accomplish frequency conversion
  - Short interaction length, can integrate on chip
  - No explicit phase-matching requirement
- Similar to previous scheme of single-photon generation, but we now consider two cavity modes



- Goal: get a single photon to leak out of cavity at frequency ω_c instead of ω_a

Example cavity design



- III-V semiconductor materials offer reasonable optical nonlinearity strengths (*e.g.*, GaP)

Model for spectral control of single photons

- Hamiltonian of two-mode cavity:

$$H = -\hbar\delta\sigma_{ee} + \frac{\hbar g_a}{2}\left(\sigma_{eg}\hat{a}_a + h.c.\right) + \frac{\hbar}{2}\left(\Omega(t)\sigma_{es} + \Omega^*(t)\sigma_{se}\right) + \frac{\hbar g_c}{2}\left(\hat{a}_a\hat{a}_c^\dagger + h.c.\right)$$

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- Nonlinear coupling strength:

$$g_c = -\frac{\epsilon_0}{\hbar}\int dr\,\epsilon\,\chi_{ijk}^{(2)} E_{a,i}^{photon,*}\left(E_{b,j}E_{c,k}^{photon} + E_{c,j}^{photon}E_{b,k}\right)$$

single-photon electric field amplitude

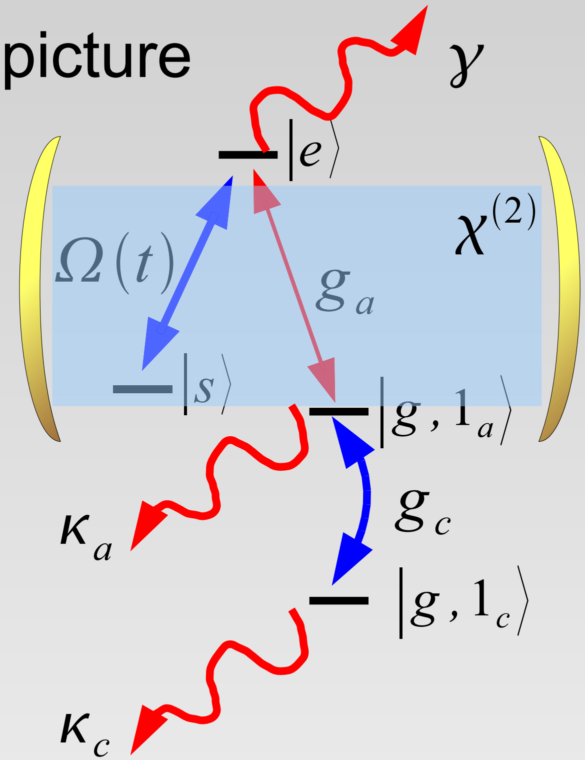
classical pump amplitude

Nonlinear coupling depends on field overlaps and strength of optical nonlinearity, but *can be tuned* by varying the pump field amplitude to reach a desired value

- Circumvents an explicit phase-matching condition
- Turns out an optimal value of g_c exists to maximize frequency conversion process

Optimization of frequency conversion

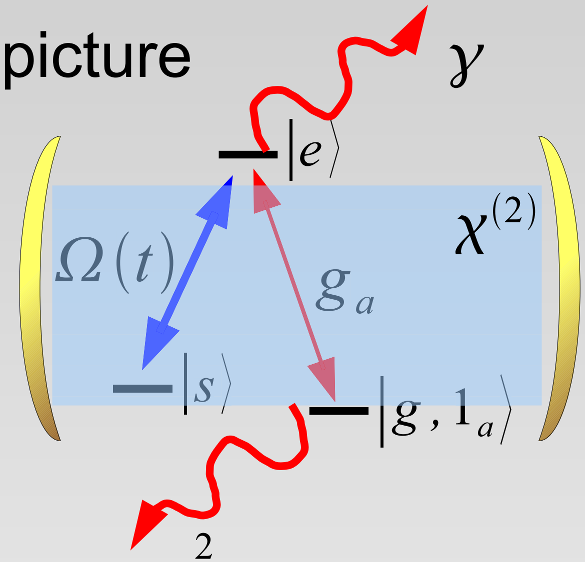
- A simple picture



- Coupling of mode a to c provides an additional effective "leakage" channel for mode a

Optimization of frequency conversion

- A simple picture



$$\kappa_{a,eff} = \kappa_a + \frac{g_c^2}{\kappa_c}$$

Analogous to Purcell effect for excited state

- Probability of frequency conversion (outgoing photon at ω_c)

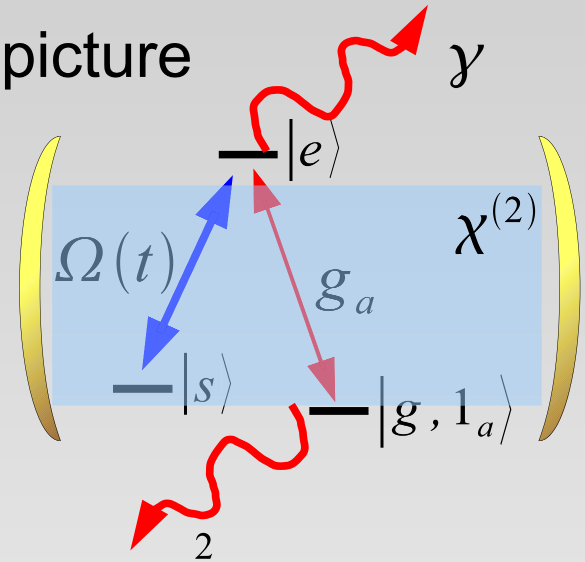
$$P = \boxed{\frac{g_a^2/\kappa_{a,eff}}{\gamma + g_a^2/\kappa_{a,eff}}} \times \frac{g_c^2/\kappa_c}{\kappa_{a,eff}}$$

Emission of excited state into mode a versus total emission

Small g_c is good

Optimization of frequency conversion

- A simple picture



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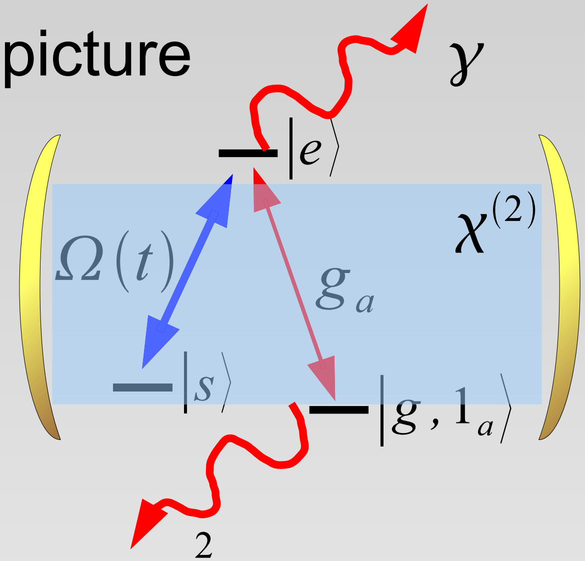
Leakage of mode a into c (good channel)
versus total leakage of mode a

Large g_c is good

Optimization of frequency conversion

- A simple picture



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$$P = \frac{g_a^2/\kappa_{a,eff}}{\gamma + g_a^2/\kappa_{a,eff}} \times \frac{g_c^2/\kappa_c}{\kappa_{a,eff}}$$

- Optimization of g_c (by tuning pump field amplitude) yields

$$P_{max} \approx \left(1 - \frac{2}{\sqrt{C_a}}\right)$$

Again, cooperativity appears as important figure of merit!

Summary of cavity QED

- Cavity QED enhances atom-photon coupling due to large number of round trips
  - A key figure of merit: cavity cooperativity $C = \dfrac{g^2}{\kappa\gamma}$
- Strong coupling allows
  - A single atom to mediate strong nonlinear interactions between single photons
  - Efficient mapping of atomic information to photonic information and vice versa
- The non-classical properties of light can be determined by measuring field correlation functions
- These tools for state manipulation, control, and measurement make cavity QED a powerful tool for quantum information science and study of quantum phenomena

Outlook

- The potential applications of cavity QED are too numerous to describe in a single lecture
- Would like to briefly highlight a few interesting avenues beyond what was described in detail here

Quantum logic gates for photon pairs

- One possible implementation of quantum computing encodes bits in polarization of single photons

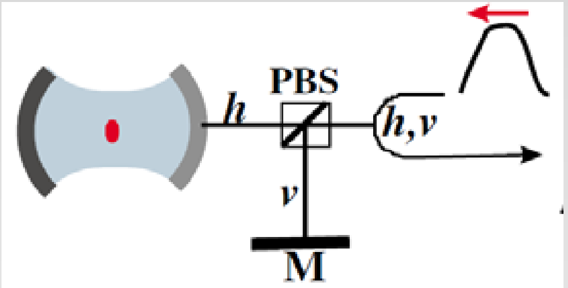
$$|0\rangle = |H\rangle, \; |1\rangle = |V\rangle$$

- Universal quantum computation requires
  - Single qubit rotations (easy with linear optics!)
  - Non-trivial two-qubit interaction (hard: requires strong optical nonlinearities!)
    - Example: C-Phase gate

$$\begin{aligned} |H_1 H_2\rangle &\rightarrow |H_1 H_2\rangle \qquad & |H_1 V_2\rangle &\rightarrow |H_1 V_2\rangle \\ |V_1 H_2\rangle &\rightarrow |V_1 H_2\rangle \qquad & |V_1 V_2\rangle &\rightarrow -|V_1 V_2\rangle \end{aligned}$$

Quantum logic gates for photon pairs

- An implementation using cavity QED
  - Successively bounce each photon off a cavity containing a single atom to achieve controlled atom-photon gate

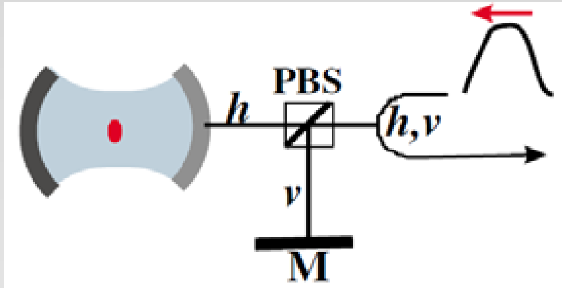


Photon 1

Atom-photon 1 C-Phase gate

+

Rotate atomic state

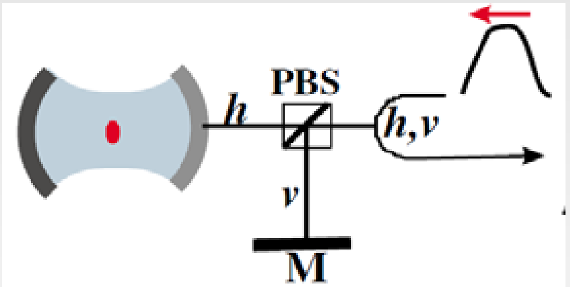


Photon 2

Atom-photon 2 C-Phase gate

+

Rotate atomic state



Photon 1

Atom-photon 1 C-Phase gate

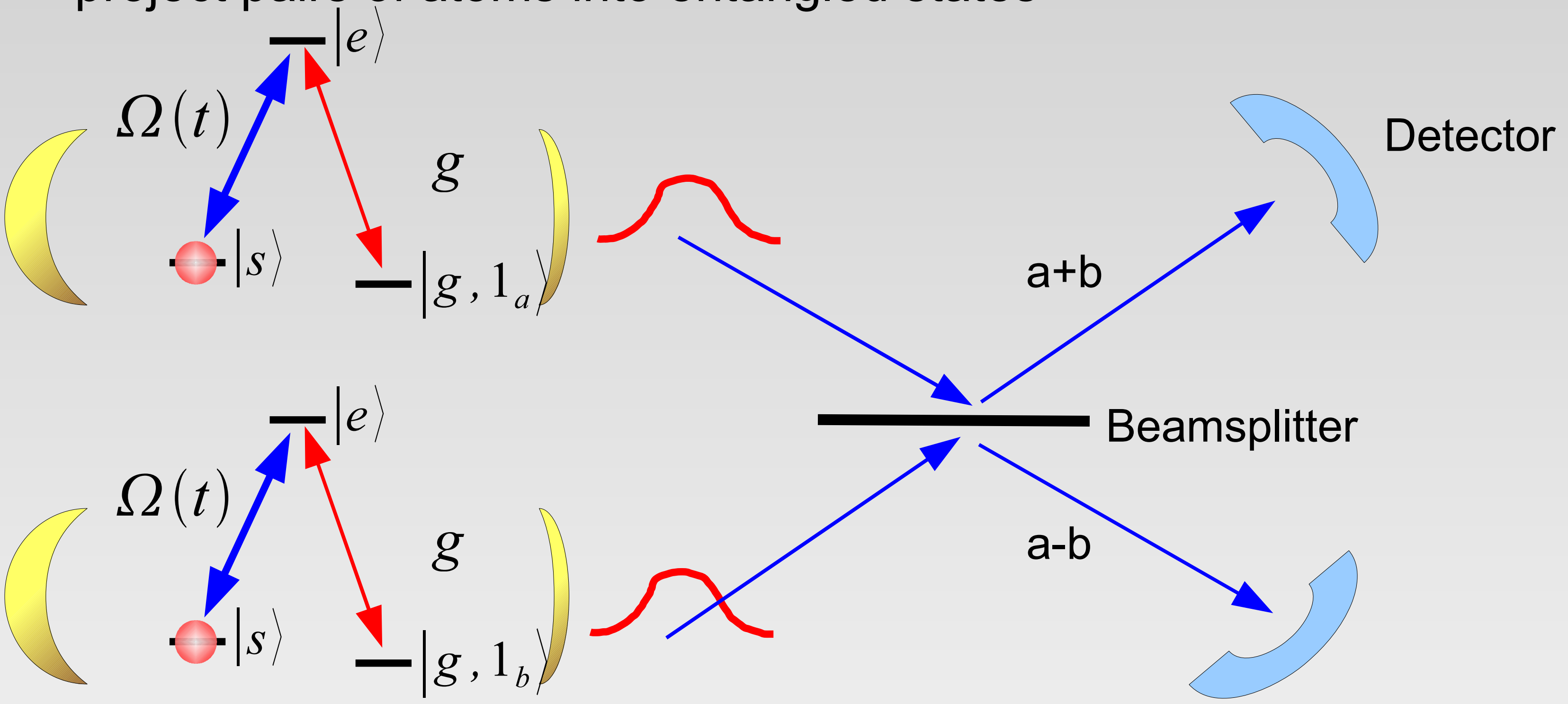
=

Photon 1 – Photon 2 C-Phase gate

Duan, Kimble, PRL 92, 127902 (2004)

Remote atom entanglement

- Use ideas based on single-photon generation and *photon measurement* to project pairs of atoms into entangled states



Two single-photon sources

Browne, Plenio, Huelga, PRL 91, 067901 (2003),
Duan, Kimble, PRL 90, 253601 (2003)

Weak driving:

$$|\psi\rangle \approx |s,0\rangle_a |s,0\rangle_b + \underbrace{\epsilon\left(|g,1\rangle_a |s,0\rangle_b + |s,0\rangle_a |g,1\rangle_b\right)}_{\text{Produces single detector click}} + \epsilon^2 |g,1\rangle_a |g,1\rangle_b, \quad \epsilon \ll 1$$

Remote atom entanglement

- The beamsplitter mixes the output from the two cavities before detection
  - Lose "which-path" information – don't know which cavity the photon click came from!
  - The detection *projects* the system into a state consistent with single click

$$P = \left(|1_a\rangle + |1_b\rangle \right) \left(\langle 1_a| + \langle 1_b| \right)$$

$$|\psi\rangle \approx |s,0\rangle_a |s,0\rangle_b + \epsilon \left(|g,1\rangle_a |s,0\rangle_b + |s,0\rangle_a |g,1\rangle_b \right) + \epsilon^2 |g,1\rangle_a |g,1\rangle_b, \quad \epsilon \ll 1$$

$$P|\psi\rangle \approx |g_a, s_b\rangle + |g_b, s_a\rangle$$

- Atoms in two distant cavities become entangled upon projection!