Lecture II. Energy-time entanglement: New interferometer for a genuine Bell experiment

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- The Bell-CHSH inequality
- The Franson Bell-CHSH experiment
- Franson's postselection gives anything
- Proposed Bell-CHSH experiment with energy-time entanglement
- Implementations and future developments



The Bell-CHSH inequality

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The Bell-CHSH inequality





The CHSH inequality

 $A_0, A_1, B_0, B_1 \in \{-1, 1\}$ $(A_0 + A_1, A_0 - A_1) \in \{(\pm 2, 0), (0, \pm 2)\}$ $(A_0 + A_1)B_0 + (A_0 - A_1)B_1 \in \{-2, 2\}$ $-2 \le \langle A_0 B_0 + A_0 B_1 + A_1 B_0 - A_1 B_1 \rangle \le 2$ $\left|\left\langle A_0 B_0 \right\rangle + \left\langle A_0 B_1 \right\rangle + \left\langle A_1 B_0 \right\rangle - \left\langle A_1 B_1 \right\rangle\right| \le 2$

The CHSH inequality is violated

$$\beta_{\text{QM}} = \left| \left\langle A_0 B_0 \right\rangle + \left\langle A_0 B_1 \right\rangle + \left\langle A_1 B_0 \right\rangle - \left\langle A_1 B_1 \right\rangle \right|$$
$$= \left| -\cos \theta_{A_0 B_0} - \cos \theta_{A_0 B_1} - \cos \theta_{A_1 B_0} + \cos \theta_{A_1 B_1} \right|$$

$$\hat{A}_{0} = \sigma_{y} \qquad \qquad \hat{B}_{0} = (\sigma_{y} + \sigma_{x})/\sqrt{2}$$
$$\hat{A}_{1} = \sigma_{x} \qquad \qquad \hat{B}_{1} = (\sigma_{y} - \sigma_{x})/\sqrt{2}$$

$$\beta_{\rm QM} = 2\sqrt{2} > 2!!!$$

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Two particles exhibit energy-time entanglement when they are emitted at the same time (in an energy-conserving process) and the uncertainty in the time of emission makes undistinguishable two alternative paths that the particles can take.

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8 May 1989

Bell Inequality for Position and Time

J. D. Franson

Applied Physics Laboratory, Johns Hopkins University, Laurel, Maryland 20707-6099 (Received 24 October 1988)

The quantum-mechanical uncertainty in the position of a particle or the time of its emission is shown to produce observable effects that are inconsistent with any local hidden-variable theory. A new experimental test of local hidden-variable theories based on optical interference is proposed.

Aerts, Kwiat, Larsson and Zukowski's criticism

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PHYSICAL REVIEW LETTERS

11 October 1999

Two-Photon Franson-Type Experiments and Local Realism

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The two-photon interferometric experiment proposed by J.D. Franson [Phys. Rev. Lett. **62**, 2205 (1989)] is often treated as a "Bell test of local realism." However, it has been suggested that this is incorrect due to the 50% postselection performed even in the ideal gedanken version of the experiment. Here we present a simple local hidden variable model of the experiment that successfully explains the results obtained in usual realizations of the experiment, even with perfect detectors. Furthermore, we also show that there is *no* such model if the switching of the local phase settings is done at a rate determined by the *internal geometry of the interferometers*.

Aerts et al. showed that, even in the ideal case of perfect preparation and perfect detection efficiency, there is a local hidden variable model that simulates the results of quantum mechanics for the Franson experiment. This model proves that:

"The Franson experiment does not and cannot violate local realism".

"The reported violations of local realism from Franson experiments have to be reexamined".

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Long-distance all-in-fibre entanglement experiments at telecom wavelengths

FIG. 13. Geneva and Lake Geneva. The Swisscom optical fiber cable used for quantum cryptography experiments runs under the lake between the town of Nyon, about 23 km north of Geneva, and the center of the city.

Gisin et al.: Quantum cryptography Rev. Mod. Phys., Vol. 74, No. 1, January 2002

Alice

Bob

Alice randomly chooses the phase of the phase shifter ϕ_A between A_0 and A_1 , and records the counts in each of her detectors (labeled a = +1 and a = -1), the detection times, and the phase settings at $t_D - t_I$, where t_D is the detection time and t_I is the time the photon takes to reach the detector from the location of the phase shifter ϕ_A .

Similarly, Bob chooses ϕ_B between B_0 and B_1 , and records the counts in each of his detectors (labeled b = +1and b = -1), the detection times, and the phase settings.

(I) To have two-photon interference, the emission of the two photons must be simultaneous, the moment of emission unpredictable, and both interferometers identical. If the detections of the two photons are coincident, there is no information about whether both photons took the short paths S or both took the long paths L. □ In energy-time experiments, a non-linear crystal is pumped continuously by a monochromatic laser so the moment of emission is unpredictable in a temporal window equal to the coherence time of the pump laser.

In time-bin experiments, a non-linear crystal is pumped by pulses previously passing through an unbalanced interferometer, so it is the uncertainty of the arrival time of the pump pulse to the crystal what causes the uncertainty in the emission time.

(II) To prevent single-photon interference, the difference between paths L and S, i.e., twice the distance between BS1 and M1, $\Delta \mathcal{L} = 2d(BS1, M1)$, must satisfy $\Delta \mathcal{L} > ct_{\rm coh}$, where c is the speed of light and $t_{\rm coh}$ is the coherence time of the photons.

(III) To make distinguishable those events where one photon takes S and the other takes L, $\Delta \mathcal{L}$ must satisfy $\Delta \mathcal{L} > c \Delta t_{\text{coinc}}$, where Δt_{coinc} is the duration of the co-incidence window.

(IV) To prevent that the local phase setting at one side can affect the outcome at the other side, the local phase settings must randomly switch (ϕ_A between A_0 and A_1 , and ϕ_B between B_0 and B_1) with a frequency of the order c/D, where D = d(Source, BS1).

$$P(A_i = +1) = P(A_i = -1) = \frac{1}{2}$$
$$P(B_j = +1) = P(B_j = -1) = \frac{1}{2}$$

25% of two-photon events in which photon 1 is detected a time $\Delta \mathcal{L}/c$ before photon 2,

25% of two-photon events in which photon 1 is detected $\Delta \mathcal{L}/c$ after photon 2,

50% of two-photon events in which both photons are detected simultaneously.

For the coincident events,

$$P(A_i = a, B_j = b) = \frac{1}{4} \left[1 + ab\cos(\phi_{A_i} + \phi_{B_j}) \right],$$

where $a, b \in \{-1, +1\}$.

The observers reject the 50% of two-photon events in which photons are detected at different times, and keep the 50% of two-photon events in which both photons are detected simultaneously.

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 $t_D - t_I$.

For the Bell-CHSH inequality there is no problem if photons make the +1/-1 decision based on the local phase setting. The problem is that the 50% postselection procedure *should be independent on the phase settings*, otherwise the Bell-CHSH inequality is not valid.

If the S/L decision can depend on the phase settings, then, after the 50% postselection procedure, one can formally obtain not only the violations predicted by quantum mechanics, as proven in Aerts *et al.*, but *any* violation, even those forbidden by quantum mechanics.

50% postselection procedure in Franson's experiment allows the subensemble of selected events to depend on the phase settings.

A_0	A_1	B_0	B_1	$\langle A_0 B_0 \rangle$	$\langle A_0 B_1 \rangle$	$\langle A_1 B_0 \rangle$	$\langle A_1 B_1 \rangle$
S+	S+	S+	L+	+1	rejected	+1	rejected

 $\beta_{\rm CHSH} = 4$

A_0	A_1	B_0	B_1	$\langle A_0 B_0 \rangle$	$\langle A_0 B_1 \rangle$	$\langle A_1 B_0 \rangle$	$\langle A_1 B_1 \rangle$
S+	S+	S+	$L\pm$	+1	rejected	+1	rejected
S-	S-	S-	$L\mp$	+1	rejected	+1	rejected
L+	L+	L+	$S\pm$	+1	rejected	+1	rejected
L-	L-	L-	$S\mp$	+1	rejected	+1	rejected
S+	S-	$L\pm$	S+	rejected	+1	rejected	-1
S-	S+	$L\mp$	S-	rejected	+1	rejected	-1
L+	L-	$S\pm$	L+	rejected	+1	rejected	-1
L-	L+	$S\mp$	L-	rejected	+1	rejected	-1
S+	$L\pm$	S+	S+	+1	+1	rejected	rejected
S-	$L\mp$	S-	S-	+1	+1	rejected	rejected
L+	$S\pm$	L+	L+	+1	+1	rejected	rejected
L-	$S\mp$	L-	L-	+1	+1	rejected	rejected
$L\pm$	S+	S+	S-	rejected	rejected	+1	-1
$L\mp$	S-	S-	S+	rejected	rejected	+1	-1
$S\pm$	L+	L+	L-	rejected	rejected	+1	-1
$S\mp$	L-	L-	L+	rejected	rejected	+1	-1

A_0	A_1	B_0	B_1	$\langle A_0 B_0 \rangle$	$\langle A_0 B_1 \rangle$	$\langle A_1 B_0 \rangle$	$\langle A_1 B_1 \rangle$
S+	S+	S-	$L\pm$	-1	rejected	-1	rejected
S-	S-	S+	$L\mp$	-1	rejected	-1	rejected
L+	L+	L-	$S\pm$	-1	rejected	-1	rejected
L-	L-	L+	$S\mp$	-1	rejected	-1	rejected
S+	S-	$L\pm$	S-	rejected	-1	rejected	+1
S-	S+	$L\mp$	S+	rejected	-1	rejected	+1
L+	L-	$S\pm$	L-	rejected	-1	rejected	+1
L-	L+	$S\mp$	L+	rejected	-1	rejected	+1
S-	$L\pm$	S+	S+	-1	-1	rejected	rejected
S+	$L\mp$	S-	S-	-1	-1	rejected	rejected
L-	$S\pm$	L+	L+	-1	-1	rejected	rejected
L+	$S\mp$	L-	L-	-1	-1	rejected	rejected
$L\pm$	S-	S+	S-	rejected	rejected	-1	+1
$L\mp$	S+	S-	S+	rejected	rejected	-1	+1
$S\pm$	L-	L+	L-	rejected	rejected	-1	+1
$S\mp$	L+	L-	L+	rejected	rejected	-1	+1

 $\beta_{\rm CHSH} = -4$

A_0	A_1	B_0	B_1	$\langle A_0 B_0 \rangle$	$\langle A_0 B_1 \rangle$	$\langle A_1 B_0 \rangle$	$\langle A_1 B_1 \rangle$
S+	S+	S+	$L\pm$	$^{+1}$	rejected	+1	rejected
S-	S-	S-	$L\mp$	+1	rejected	+1	rejected
L+	L+	L+	$S\pm$	+1	rejected	+1	rejected
L-	L-	L-	$S\mp$	+1	rejected	+1	rejected
S+	S-	$L\pm$	S+	rejected	+1	rejected	-1
S-	S+	$L\mp$	S-	rejected	+1	rejected	-1
L+	L-	$S\pm$	L+	rejected	+1	rejected	-1
L-	L+	$S\mp$	L-	rejected	+1	rejected	$^{-1}$
S+	$L\pm$	S+	S+	+1	+1	rejected	rejected
S-	$L\mp$	S-	S-	+1	+1	rejected	rejected
L+	$S\pm$	L+	L+	+1	+1	rejected	rejected
L-	$S\mp$	L-	L-	+1	+1	rejected	rejected
$L\pm$	S+	S+	S-	rejected	rejected	+1	-1
$L\mp$	S-	S-	S+	rejected	rejected	+1	-1
$S\pm$	L+	L+	L-	rejected	rejected	+1	-1
S∓	L-	L-	L+	rejected	rejected	+1	-1

A_0	A_1	B_0	B_1	$\langle A_0 B_0 \rangle$	$\langle A_0 B_1 \rangle$	$\langle A_1 B_0 \rangle$	$\langle A_1 B_1 \rangle$
S+	S+	S-	$L\pm$	$^{-1}$	rejected	$^{-1}$	rejected
S-	S-	S+	$L\mp$	$^{-1}$	rejected	$^{-1}$	rejected
L+	L+	L-	$S\pm$	$^{-1}$	rejected	$^{-1}$	rejected
L-	L-	L+	$S\mp$	$^{-1}$	rejected	$^{-1}$	rejected
S+	S-	$L\pm$	S-	rejected	-1	rejected	+1
S-	S+	$L\mp$	S+	rejected	-1	rejected	+1
L+	L-	$S\pm$	L-	rejected	-1	rejected	+1
L-	L+	$S\mp$	L+	rejected	$^{-1}$	rejected	+1
S-	$L\pm$	S+	S+	$^{-1}$	$^{-1}$	rejected	rejected
S+	$L\mp$	S-	S-	-1	-1	rejected	rejected
L-	$S\pm$	L+	L+	-1	-1	rejected	rejected
L+	$S\mp$	L-	L-	$^{-1}$	$^{-1}$	rejected	rejected
$L\pm$	S-	S+	S-	rejected	rejected	$^{-1}$	+1
$L\mp$	S+	S-	S+	rejected	rejected	$^{-1}$	+1
$S\pm$	L-	L+	L-	rejected	rejected	$^{-1}$	$^{+1}$
$S\mp$	L+	L-	L+	rejected	rejected	-1	+1

If each of the 32 sets of instructions in the green table occurs with probability p/32, and each of the 32 sets of instructions in the red table with probability (1-p)/32, then, for any value of $0 \le p \le 1$, the model gives 25% of SL events, 25% of LS events, 50% of SS or LL events, and satisfies (1a) and (1b). If p = 0, the model gives $\beta_{\text{CHSH}} = -4$. If p = 1, the model gives $\beta_{\text{CHSH}} = 4$ (and simulates the outcomes of a Popescu-Rohrlich nonlocal box). The maximal quantum violation $\beta_{\text{CHSH}} = 2\sqrt{2}$, satisfying (3), is obtained when $p = (2 + \sqrt{2})/4$.

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Franson's energy-time Bell experiment

Alice

Bob

The two photons end in different sides only when both are detected in coincidence. If one photon takes S and the other photon takes L, both will end on detectors of the same side.

(I') To have two-photon interference, the emission of the two photons must be simultaneous, the moment of emission unpredictable, and both arms of the setup identical.

(II') Single-photon interference is not possible. The requirements in (II) are no longer necessary.

(III') To temporally distinguish two photons arriving at the same detector at times t and $t + \frac{\Delta \mathcal{L}'}{c}$, where $\Delta \mathcal{L}' = 2[d(\text{Source}, BS2) + d(BS2, M1)]$, the dead time of the detectors must be smaller than $\frac{\Delta \mathcal{L}'}{c}$. The requirements in (III) are no longer necessary.

(IV') To prevent that the local phase setting at one side can affect the outcome at the other side, the local phase settings must randomly switch (ϕ_A between A_0 and A_1 , and ϕ_B between B_0 and B_1) with a frequency of the order c/D', where $D' = d(\text{Source}, \phi_A) \gg \Delta \mathcal{L}'$.

The predictions of quantum mechanics are similar to those in Franson's proposal: Eqs. (1a) and (1b) hold, there is 25% of events in which both photons are detected on the left at times t and $t + \frac{\Delta \mathcal{L}'}{c}$, 25% of events in which both photons are detected on the right, and 50% of coincident events for which (3) holds.

The observers must keep the coincident events and reject those giving two detections on detectors of the same side.

(i) The rejection of events is local and does not require communication between the observers.

(ii) The selection and rejection of events is independent of the local phase settings.

Consider a *selected* event: both photons have been detected at time t_D , one in a detector a on the left, and the other in a detector b on the right. t_I is the time a photon takes from ϕ_A (ϕ_B) to a detector a (b). The phase setting of ϕ_A (ϕ_B) at $t_D - t_I$ is in the backward light cone of the photon detected in a (b), but the point is, could a different value of one or both of the phase settings have caused that this *selected* event would become a *rejected* event?

No. This would require a mechanism to make one detection to "wait" until the information about the setting in other side comes. However, when this information has finally arrived, the phase settings (both of them) have changed, so this information is useless to base a decision on it. There is no physical mechanism preserving locality which can turn a selected event into a rejected event.

Now consider a rejected event. For instance, one in which both photons are detected in the detectors a on the left, one at time $t_D = t$, and the other at $t_D = t + \frac{\Delta \mathcal{L}'}{c}$. Then, the phase settings of ϕ_B at times $t_D - t_I$ are out of the backward light cones of the detected photons. The photons cannot have based their decisions on the phase settings of ϕ_B . Could a different value of ϕ_A have caused that this rejected event would become a selected event?

No. This would require a mechanism to make one detection to wait until the information about the setting arrives to the other side. However, when this information has finally arrived to the other side, the phase setting of ϕ_A has changed so this information is useless. There is no mechanism preserving locality which can turn a rejected event into a selected event.

The selected events are *independent* of the local phase settings. For the selected events, only the +1/-1 decision can depend on the phase settings. This is exactly the assumption under which the Bell-CHSH inequality is valid. Therefore, an experimental violation of the Bell-CHSH inequality using this setup and the postselection procedure provides a conclusive (assuming perfect detectors) test of local realism using energy-time entanglement.

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First implementation

A. Rossi, G. Vallone, F. De Martini, and P. Mataloni, PRA 78, 012345 (2008).

Hyper-entanglement

FIG. 8: Scheme for the generation of two photon polarization/momentum/time-bin entangled states.

A. Rossi, G. Vallone, F. De Martini, and P. Mataloni, PRA 78, 012345 (2008).

Can you do exactly this experiment?

Energy-time entanglement in other physical systems?

- Electronic systems?
- Neutrons?
- Other systems?

Collaborators

A. Cabello. A. Rossi, G. Vallone, F. De Martini, and P. Mataloni, PRL 102, 040401 (2009).

