Lecture I. Quantum contextuality: State-independent violation of Bell-like inequalities

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4th Winter School on Quantum Information Science, Tamkang University Lanyang Campus, Yilan, Taiwan, February 11, 2009.

Plan

- EPR argument
- Bell's theorem
    - Without inequalities (AVN proofs)
    - Bell inequalities
- Kochen-Specker theorem
- State-independent violation of Bell-like inequalities

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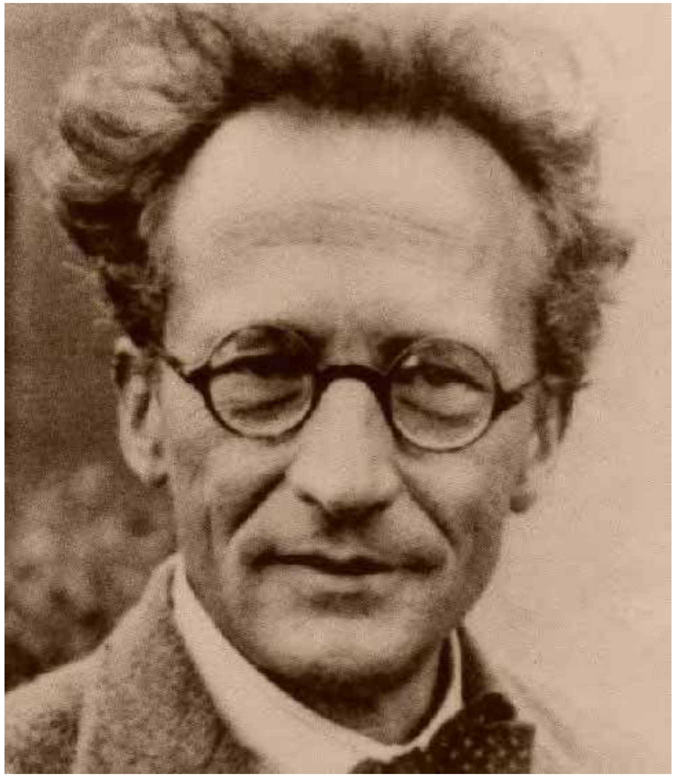
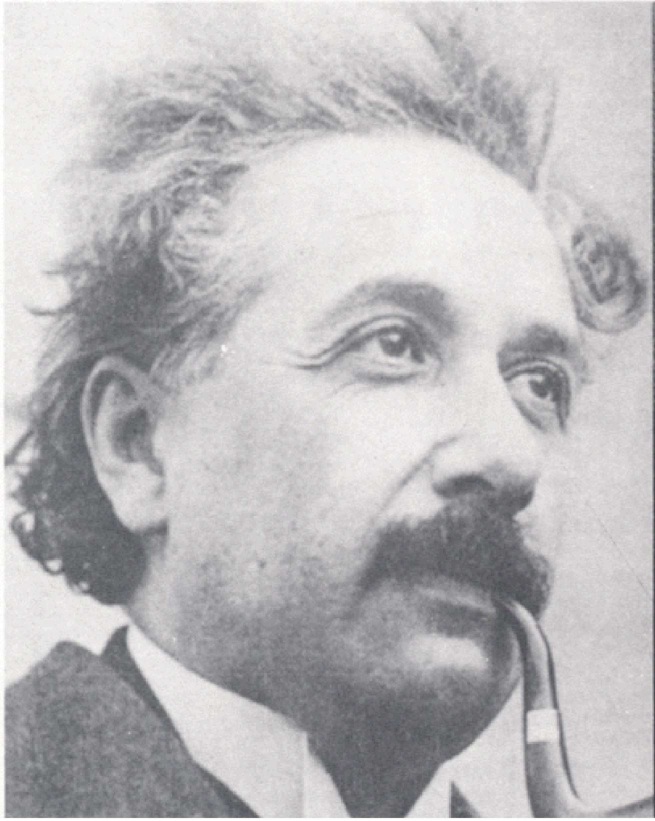
Is entanglement *the* characteristic trait of QM?

"Entanglement is *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought".



E. Schrödinger, Proc. Cambridge Philos. Soc. **31**, 555 (1935).

Is QM complete?



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

The first paper on "quantum nonlocality"

THE NEW YORK TIMES, SATURDAY, MAY 4, 1935.

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

PRINCETON, N. J., May 3.—Professor Albert Einstein will attack science's important theory of quantum mechanics, a theory of which he was a sort of grandfather. He concludes that while it is "correct" it is not "complete."

With two colleagues at the Institute for Advanced Study here, the noted scientist is about to report to the American Physical Society what is wrong with the theory of quantum mechanics, it has been learned exclusively by Science Service.

The quantum theory, with which science predicts with some success inter-atomic happenings, does not meet the requirements for a satisfactory physical theory, Professor Einstein will report in a joint paper with Dr. Boris Podolsky and Dr. N. Rosen.

In the quantum theory as now used, the latest Einstein paper will point out that where two physical quantities such as the position of a particle and its velocity interact, a knowledge of one quantity precludes knowledge about the other. This is the famous principle of uncertainty put forward by Professor Werner Heisenberg and incorporated in the quantum theory. This very fact, Professor Einstein feels, makes the quantum theory fail in the requirements necessary for a satisfactory physical theory.

Two Requirements Listed.

These two requirements are:

1. The theory should make possible a calculation of the facts of nature and predict results which can be accurately checked by experiment; the theory should be, in other words, correct.

2. Moreover, a satisfactory theory should, as a good image of the objective world, contain a counterpart for things found in the objective world; that is, it must be a complete theory.

Quantum theory, Professor Einstein and his colleagues will report, fulfills the correctness requirement but fails in the completeness requirement.

While proving that present quantum theory does not give a complete description of physical reality, Professor Einstein believes some later, still undeveloped, theory will make this possible. His conclusion is:

"While we have thus shown that the wave function [of quantum theory] does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

The development of quantum mechanics has proved very useful in exploring the atom. Six Nobel Prizes in physics, including one to Einstein, have been awarded for various phases of the researches leading up to quantum mechanics. The names of Planck, Bohr, de Broglie, Heisenberg, Dirac and Schroedinger, as well as Einstein, are linked with quantum mechanics.

The exact title of the Einstein-Podolsky-Rosen paper is: "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?"

Explanation by Podolsky.

In explaining the latest view of the physical world as revealed in their researches Dr. Podolsky, one of the authors, said:

"Physicists believe that there exist real material things independent of our minds and our theories. We construct theories and invent words (such as electron, positron, &c.) in an attempt to explain to ourselves what we know about our external world and to help us to obtain further knowledge of it. Before a theory can be considered to be satisfactory it must pass two very severe tests. First, the theory must enable us to calculate facts of nature, and these calculations must agree very accurately with observations and experiments. Second, we expect a satisfactory theory, as a good image of objective reality, to contain a counterpart for every element of the physical world. A theory satisfying the first requirement may be called a correct theory while, if it satisfies the second requirement, it may be called a complete theory.

"Hundreds of thousands of experiments and measurements have shown that, at least in cases when matter moves much slower than light, the theory of Planck, Einstein, Bohr, Heisenberg and Schroedinger known as quantum mechanics is a correct theory. Einstein, Podolsky and Rosen now discuss the question of the completeness of quantum mechanics. They arrive at the conclusion that quantum mechanics, in its present form, is not complete.

"In quantum mechanics the condition of any physical system, such as an electron, an atom, &c., is supposed to be completely described by a formula known as a 'wave function.' Suppose that we know the wave function for each of two physical systems, and that these two systems come together, interact, and again separate (as when two particles collide and move apart). Quantum mechanics, although giving us considerable information about such a process, does not enable us to calculate the wave function of each physical system after the separation. This fact is made use of in showing that the wave function does not give a complete description of physical reality. Since, however, description of physical systems by wave functions is an essential step of quantum mechanics, this means that quantum mechanics is not a complete theory."

Raises Point of Doubt.

Special to THE NEW YORK TIMES.

PRINCETON, N. J., May 3.—Asked to comment on the new ideas of Professor Einstein and his collaborators, Professor Edward U. Condon, mathematical physicist of Princeton University, said tonight:

"Of course, a great deal of the argument hinges on just what meaning is to be attached to the word 'reality' in connection with physics. They have certainly discussed an interesting point in connection with the theory. Dr. Einstein has never been satisfied with the statistical causality which in the new theories replaces the strict causality of the old physics.

"It is reported that when he first learned of the work of Schroedinger and Dirac, he said, 'Der lieber Gott wuerfelt nicht, [the good Lord does not throw dice]. For the last five years he has subjected the quantum mechanical theories to very searching criticism from this standpoint. But I am afraid that thus far the statistical theories have withstood criticism."

EPR: QM is "incomplete"

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

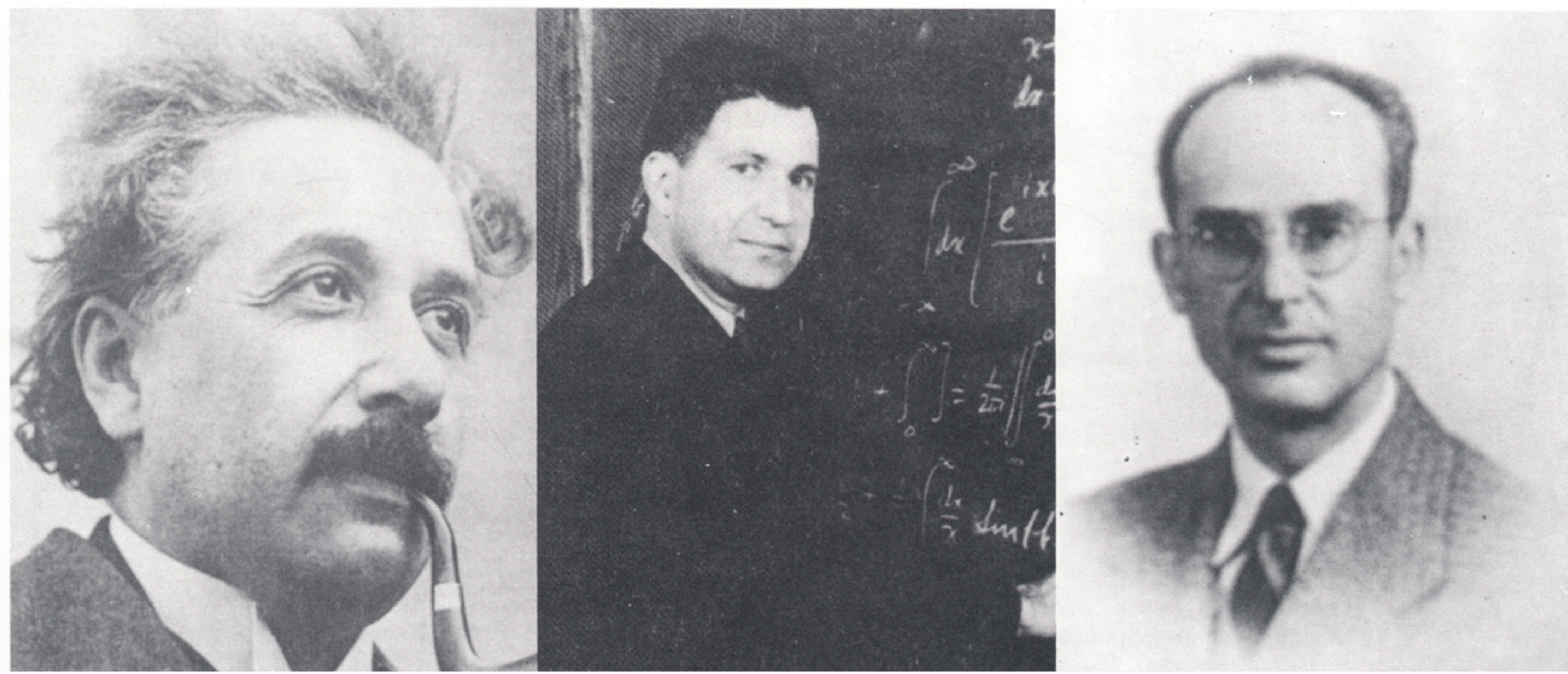
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

According to EPR, any satisfactory physical theory must be:

(1) Correct.

(2) "Complete".

EPR's elements of reality



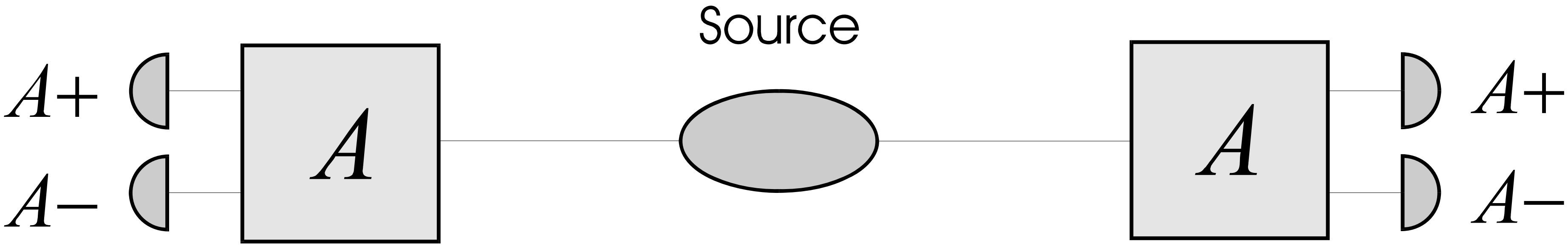
"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

EPR’s elements of reality

“Without in any way disturbing a system” = Spacelike separation.

“Predict with certainty” = Perfect correlations.

Bohm's version of EPR's argument



$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

Bohm's version of EPR's argument

$$X_1 X_2 = -1$$

$$Y_1 Y_2 = -1$$

- X_2 and Y_2 are both "elements of reality".

- In QM, X_2 and Y_2 are incompatible observables (Heisenberg's uncertainty principle).

→ QM is incomplete (according to EPR).

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- EPR argument
- Bell's theorem
  - Without inequalities (AVN proofs)
  - Bell inequalities
- Kochen-Specker theorem
- State-independent violation of Bell-like inequalities

Bell's theorem

No theory of local hidden variables (LHV) can reproduce QM.

It is proven either by the violation of a Bell inequality or by a GHZ-like example.

Any proof of Bell's theorem is state-dependent: It is valid only for entangled states.

Bell inequalities prove the impossibility of local realism

A (loophole-free) violation of a Bell inequality would prove the impossibility of local realism (R. Gill's definition):

(i) Realism: measurement outcomes of nonperformed measurements can be introduced alongside of those of the actually performed measurements.

(ii) Locality: the measurement outcome at Alice's station does not depend on Bob's choice of setting [because they are space-like separated].

(iii) Freedom: Alice and Bob can perform either measurement.

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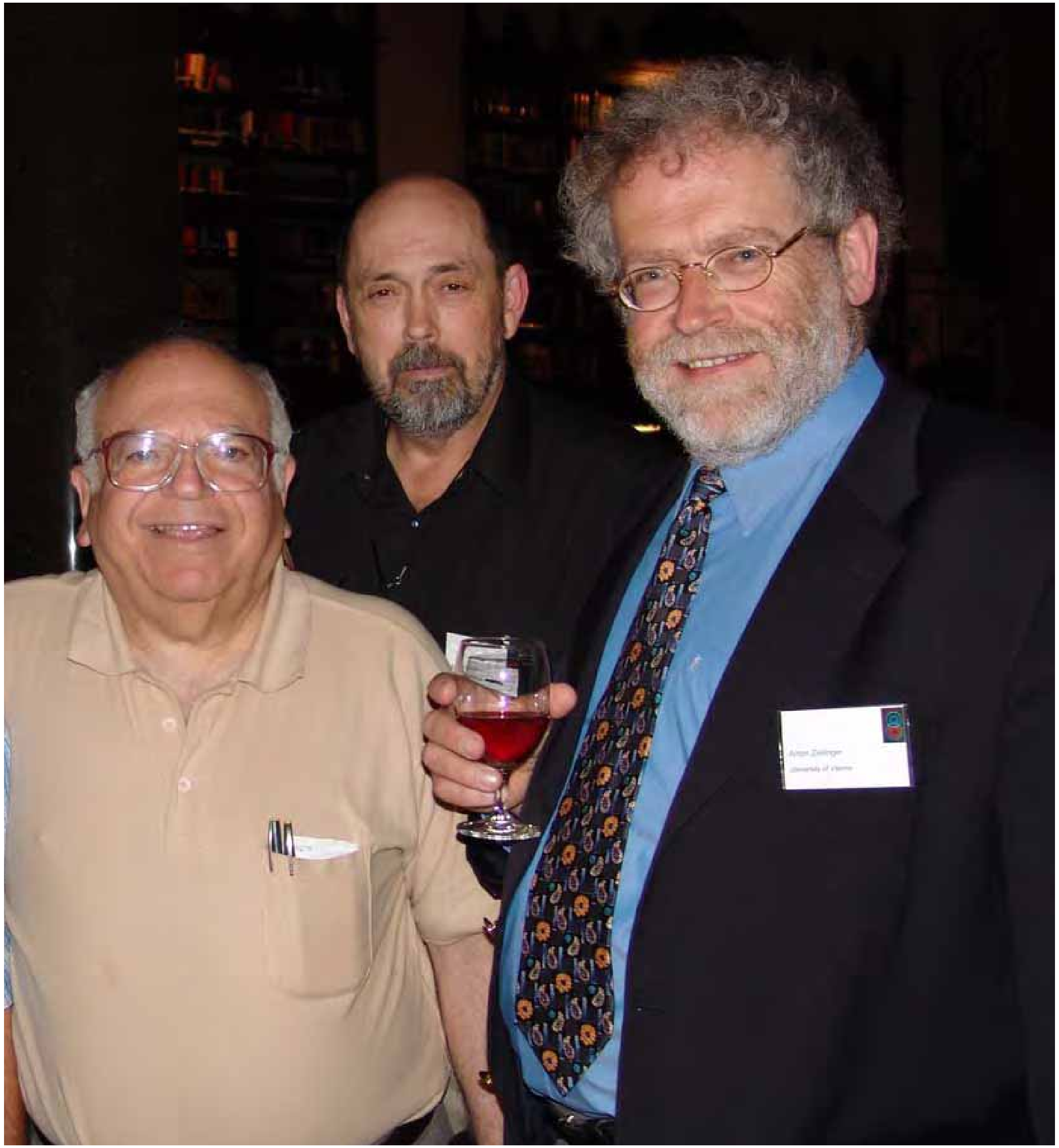
Greenberger, Horne and Zeilinger



GHZ's proof of Bell's theorem

$$|GHZ\rangle = |HHH\rangle - |VVV\rangle$$

$$X_1 Y_2 Y_3 |GHZ\rangle = |GHZ\rangle$$

$$Y_1 X_2 Y_3 |GHZ\rangle = |GHZ\rangle$$

$$Y_1 Y_2 X_3 |GHZ\rangle = |GHZ\rangle$$

$$X_1 X_2 X_3 |GHZ\rangle = -|GHZ\rangle$$

Notation for single photon observables

Polarization observables:

$$X = |H\rangle\langle V| + |V\rangle\langle H|$$

$$Y = i\,(|V\rangle\langle H| - |H\rangle\langle V|)$$

$$Z = |H\rangle\langle H| - |V\rangle\langle V|$$

GHZ's proof of Bell's theorem: X_i and Y_i are ER

GHZ:

$$|HHH\rangle - |VVV\rangle$$

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -v(X_3)$$

GHZ's proof of Bell's theorem: Contradiction!

GHZ:

$$|HHH\rangle - |VVV\rangle$$

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -v(X_3)$$

Why “all-versus-nothing”?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

Why “all-versus-nothing”?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = +v(X_3)$$

Why "all-versus-nothing"?

$$v(X_1)\,v(Y_2) = v(Y_3)$$

$$v(Y_1)\,v(X_2) = v(Y_3)$$

$$v(Y_1)\,v(Y_2) = v(X_3)$$

$$v(X_1)\,v(X_2) = -\,v(X_3)$$

Experimental GHZ

letters to nature

Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement

Jian-Wei Pan*, Dik Bouwmeester†, Matthew Daniell*, Harald Weinfurter‡ & Anton Zeilinger*

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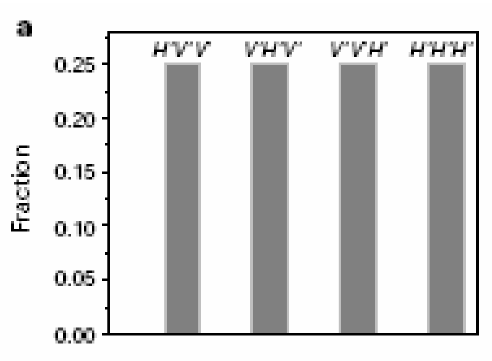
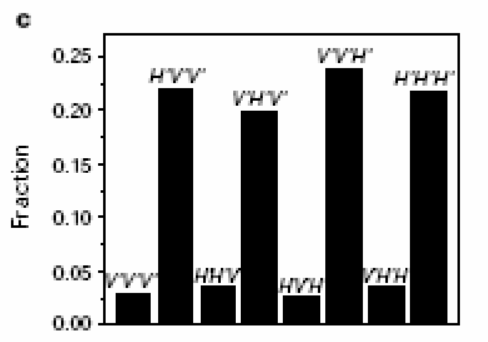




NATURE | VOL 403 | 3 FEBRUARY 2000 |

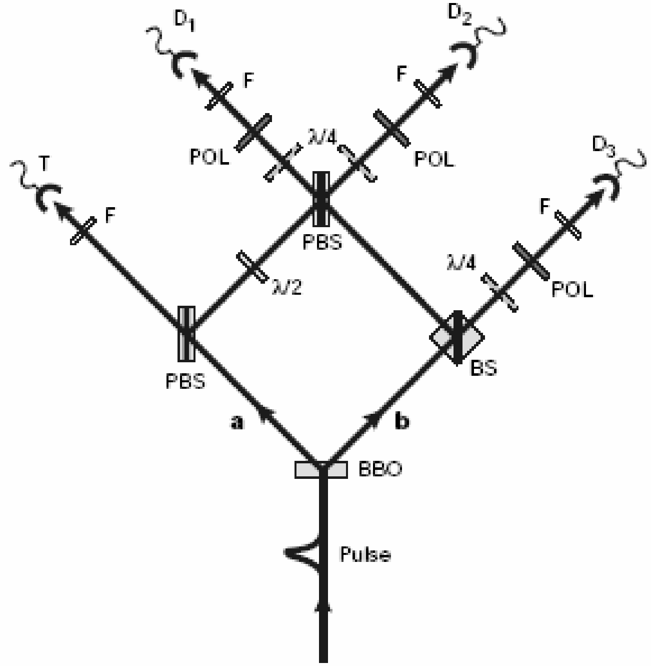
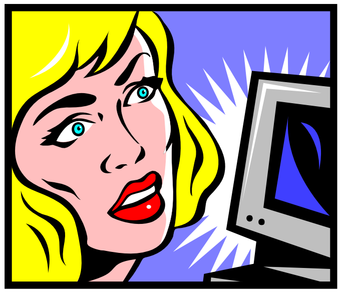


Figure 1 Experimental set-up for Greenberger–Horne–Zeilinger (GHZ) tests of quantum nonlocality. Pairs of polarization-entangled photons[28] (one photon *H* polarized and the other *V*) are generated by a short pulse of ultraviolet light (~ 200 fs, $\lambda = 394$ nm). Observation of the desired GHZ correlations requires fourfold coincidence and therefore two pairs[29]. The photon registered at T is always *H* and thus its partner in **b** must be *V*. The photon reflected at the polarizing beam-splitter (PBS) in arm **a** is always *V*, being turned into equal superposition of *V* and *H* by the $\lambda/2$ plate, and its partner in arm **b** must be *H*. Thus if all four detectors register at the same time, the two photons in D_1 and D_2 must either both have been *VV* and reflected by the last PBS or *HH* and transmitted. The photon at D_3 was therefore *H* or *V*, respectively. Both possibilities are made indistinguishable by having equal path lengths via **a** and **b** to D_1 (D_2) and by using narrow bandwidth filters (*F* ≈ 4 nm) to stretch the coherence time to about 500 fs, substantially larger than the pulse length[30]. This effectively erases the prior correlation information and, owing to indistinguishability, the three photons registered at D_1, D_2 and D_3 exhibit the desired GHZ correlations predicted by the state of equation (1), where for simplicity we assume the polarizations at D_3 to be defined at right angles relative to the others. Polarizers oriented at 45° and $\lambda/4$ plates in front of the detectors allow measurement of linear *H'*/*V'* (circular *R*/*L*) polarization.

Problem

- Two-observer AVN proofs?



The first two-observer AVN proof

Double Bell:

$$(|HV\rangle - |VH\rangle)(|ud\rangle - |du\rangle)$$

$$v(Z_1) = -v(Z_2)$$

$$v(z_1) = -v(z_2)$$

$$v(X_1) = -v(X_2)$$

$$v(x_1) = -v(x_2)$$

$$v(Z_1 z_1) = v(Z_2) v(z_2)$$

$$v(X_1 x_1) = v(X_2) v(x_2)$$

$$v(Z_1) v(x_1) = v(Z_2 x_2)$$

$$v(X_1) v(z_1) = v(X_2 z_2)$$

$$v(Z_1 z_1) v(X_1 x_1) = -v(Z_2 x_2) v(X_2 z_2)$$

AC, PRL **86**, 1911 (2001); **87**, 010403 (2001).

Notation for single photon observables

Polarization observables:

$$X = |H\rangle\langle V| + |V\rangle\langle H|$$

$$Y = i\,(\,|V\rangle\langle H| - |H\rangle\langle V|\,)$$

$$Z = |H\rangle\langle H| - |V\rangle\langle V|$$

Path observables:

$$x = |u\rangle\langle d| + |d\rangle\langle u|$$

$$y = i\,(\,|d\rangle\langle u| - |u\rangle\langle d|\,)$$

$$z = |u\rangle\langle u| - |d\rangle\langle d|$$

Four qubits in two photons

VOLUME 90, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 25 APRIL 2003

All-Versus-Nothing Violation of Local Realism for Two Entangled Photons

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(Received 18 November 2002; published 24 April 2003)

It is shown that the Greenberger-Horne-Zeilinger theorem can be generalized to the case with only two entangled particles. The reasoning makes use of two photons which are maximally entangled both in polarization and in spatial degrees of freedom. In contrast to Cabello's argument of "all versus nothing" nonlocality with four photons [Phys. Rev. Lett. **87**, 010403 (2001)], our proposal to test the theorem can be implemented with linear optics and thus is well within the reach of current experimental technology.

$$z_1 \cdot z_2|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \qquad z'_1 \cdot z'_2|\Psi\rangle_{12} = -|\Psi\rangle_{12},$$

$$x_1 \cdot x_2|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \qquad x'_1 \cdot x'_2|\Psi\rangle_{12} = -|\Psi\rangle_{12},$$

$$z_1 z'_1 \cdot z_2 \cdot z'_2|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$x_1 x'_1 \cdot x_2 \cdot x'_2|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$z_1 \cdot x'_1 \cdot z_2 x'_2|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$x_1 \cdot z'_1 \cdot x_2 z'_2|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2|\Psi\rangle_{12} = -|\Psi\rangle_{12}.$$

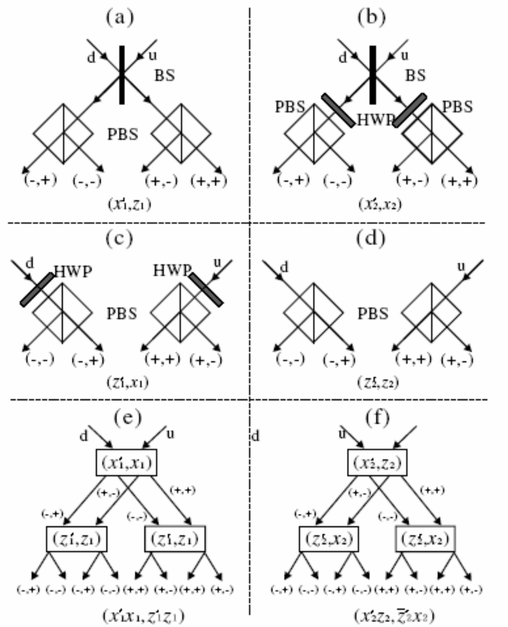

FIG. 2. Six apparatuses for measuring z_1, x'_1, and $z_1 \cdot x'_1$ (a); x_2, x'_2, and $x_2 \cdot x'_2$ (b); z'_1, x_1, and $x_1 \cdot z'_1$ (c); z_2, z'_2, and $z_2 \cdot z'_2$ (d); $z_1 z'_1$, $x_1 x'_1$, and $z_1 z'_1 \cdot x_1 x'_1$ (e); $z_2 x'_2$, $x_2 z'_2$, and $z_2 x'_2 \cdot x_2 z'_2$ (f). By $\pm$, we mean ± 1.

Rome and Hefei experiments

PRL **95,** 240405 (2005) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2005

All-Versus-Nothing Nonlocality Test of Quantum Mechanics by Two-Photon Hyperentanglement

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(Received 27 April 2005; published 9 December 2005)

We report the experimental realization and the characterization of polarization and momentum hyperentangled two-photon states, generated by a new parametric source of correlated photon pairs. By adoption of these states an "all-versus-nothing" test of quantum mechanics was performed. The two-photon hyperentangled states are expected to find at an increasing rate a widespread application in state engineering and quantum information.

PRL **95,** 240406 (2005) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2005

All-Versus-Nothing Violation of Local Realism by Two-Photon, Four-Dimensional Entanglement

Tao Yang,[1] Qiang Zhang,[1] Jun Zhang,[1] Juan Yin,[1] Zhi Zhao,[1,2] Marek Żukowski,[3]
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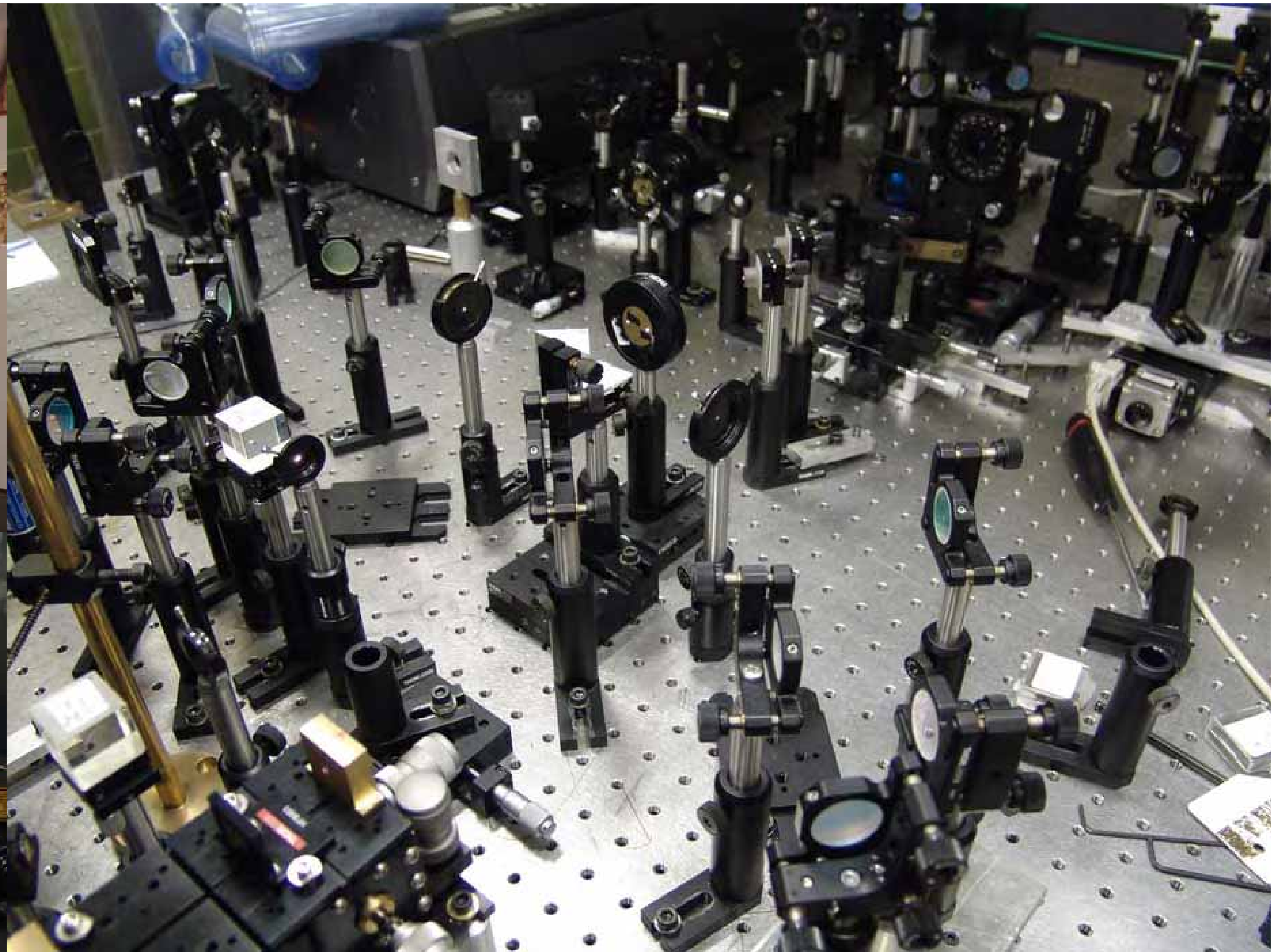
(Received 4 June 2005; published 9 December 2005)

We develop and exploit a source of two-photon, four-dimensional entanglement to report the first two-particle all-versus-nothing test of local realism with a linear optics setup, but without resorting to a noncontextuality assumption. Our experimental results are in good agreement with quantum mechanics while in extreme contradiction to local realism. Potential applications of our experiment are briefly discussed.

Rome experiment 2005



Rome experiment 2005

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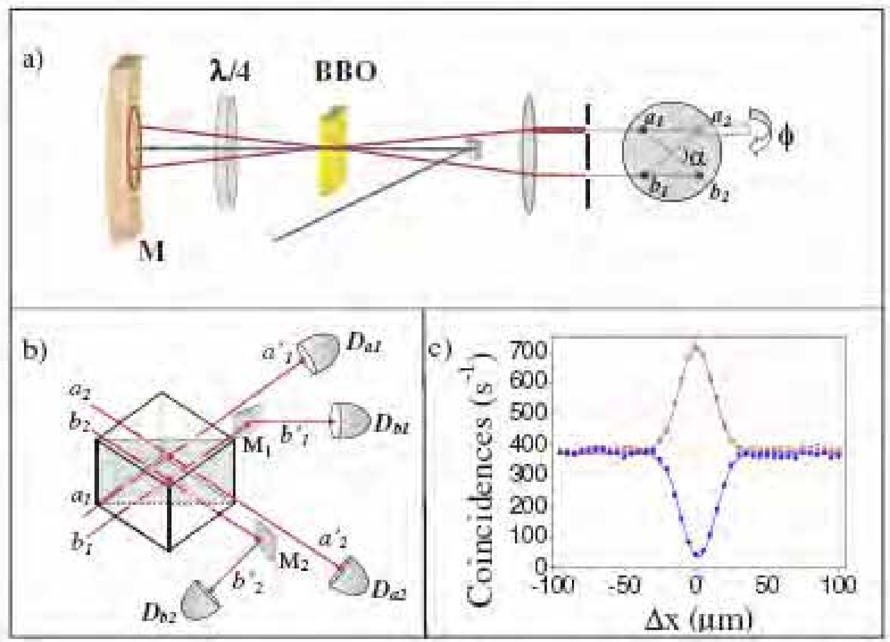
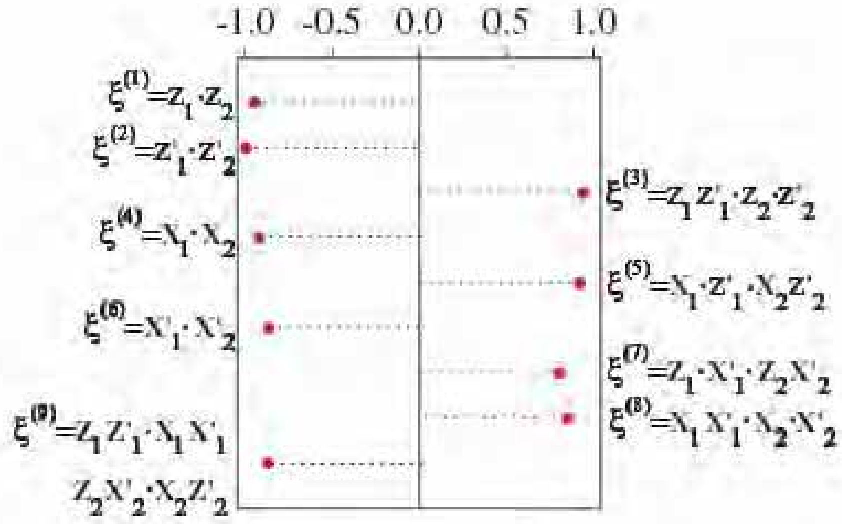




$$7 < \sum_{i=1}^{9} |\xi^{(i)}| = (8.114 \pm 0.011) < 9$$

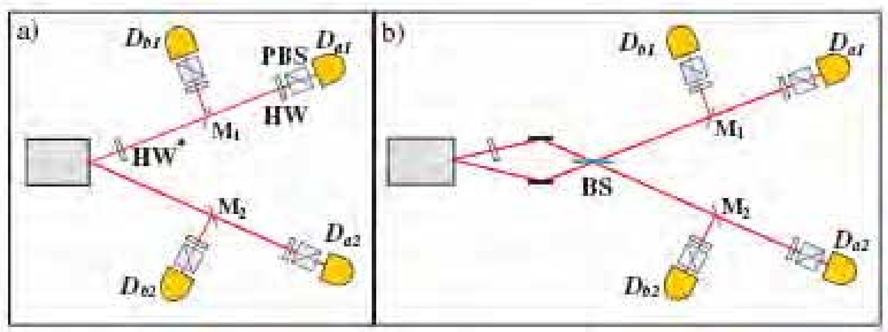
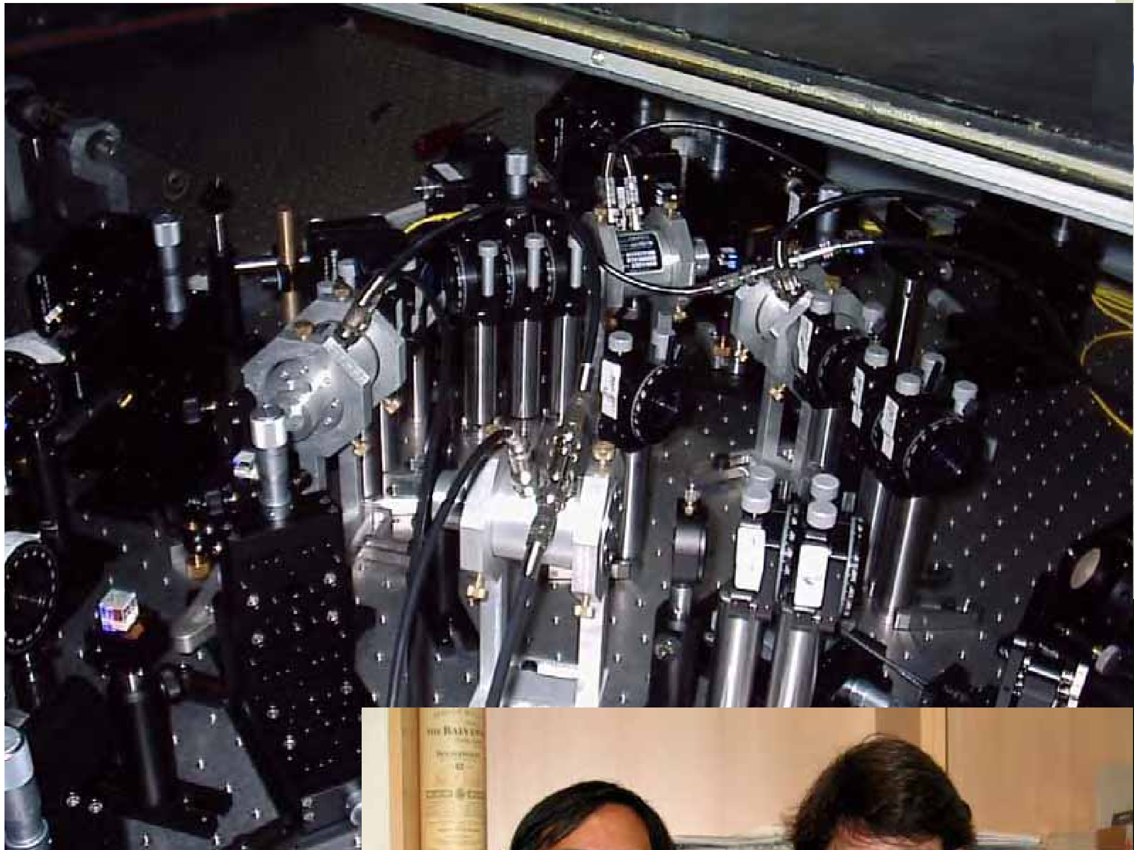
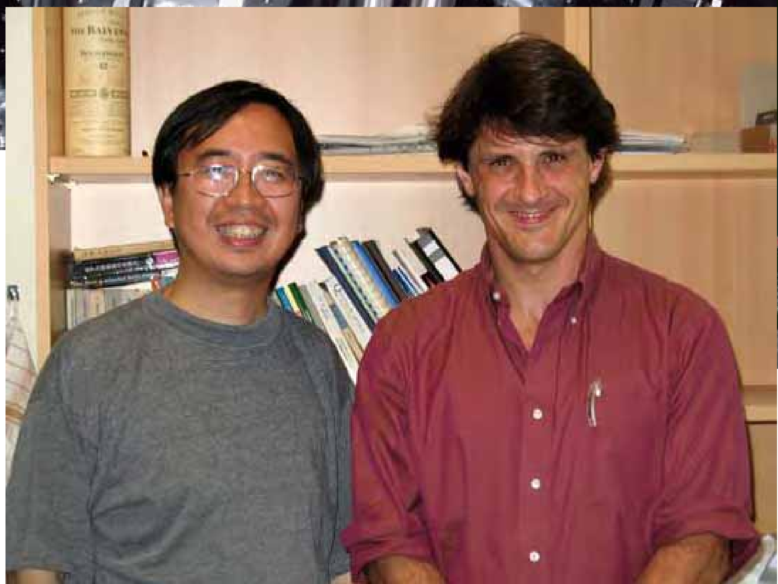


FIG. 3 (color online). Barchart of expectation values for the nine operators involved in the experiment. The following results have been obtained: $z_1 \cdot z_2 = -0.9428 \pm 0.0030$, $z_1' \cdot z_2' = -0.9953 \pm 0.0033$, $z_1 z_1' \cdot z_2 \cdot z_2' = 0.9424 \pm 0.0030$, $x_1 \cdot x_2 = -0.9215 \pm 0.0033$, $x_1 \cdot z_1' \cdot x_2 z_2' = 0.9217 \pm 0.0033$, $x_1' \cdot x_2' = -0.8642 \pm 0.0043$ $z_1 \cdot x_1' \cdot z_2 x_2' = 0.8039 \pm 0.0040$, $x_1 x_1' \cdot x_2 \cdot x_2' = 0.8542 \pm 0.0040$, $z_1 z_1' \cdot x_1 x_1' \cdot z_2 x_2' \cdot x_2 z_2' = -0.8678 \pm 0.0043$.

Hefei experiment 2005





Hefei experiment 2005

All-Versus-Nothing Violation of Local Realism by Two-Photon, Four-Dimensional Entanglement

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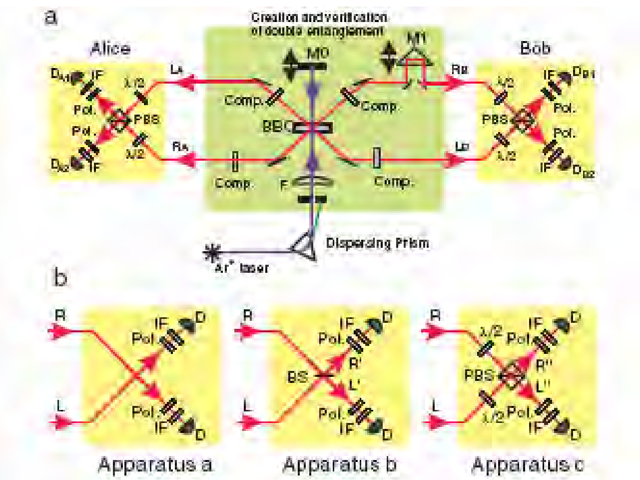

FIG. 1 (color online). Experimental setups. (a) An ultraviolet beam from Argon ion laser (351.1 nm, 120 mW) is directed into the BBO crystal from opposite directions, and thus can create photon pairs (with wavelength 702.2 nm) in $|\Psi\rangle$. Four compensators (Comp.) are used to offset the birefringent effect caused by the BBO crystal during parametric down-conversion. The reflection mirrors M0 and M1 are mounted on translation stages, to balance each arm of the interferometer and to optimize the entanglement in path. (b) Apparatuses to measure all necessary observables of doubly entangled states. D is single-photon count module, with collection and detection efficiency 26%; IF is interference filter with a bandwidth of 2.88 nm and a center wavelength of 702.2 nm; Pol. is polarizer. Apparatus c has been included in (a) at the locations of Alice and Bob.

$$z_A \cdot z_B|\Psi\rangle = -|\Psi\rangle, \qquad z'_A \cdot z'_B|\Psi\rangle = -|\Psi\rangle, \tag{1}$$

$$x_A \cdot x_B|\Psi\rangle = -|\Psi\rangle, \qquad x'_A \cdot x'_B|\Psi\rangle = -|\Psi\rangle, \tag{2}$$

$$z_A z'_A \cdot z_B \cdot z'_B|\Psi\rangle = |\Psi\rangle, \qquad x_A x'_A \cdot x_B \cdot x'_B|\Psi\rangle = |\Psi\rangle, \tag{3}$$

$$z_A \cdot x'_A \cdot z_B x'_B|\Psi\rangle = |\Psi\rangle, \qquad x_A \cdot z'_A \cdot x_B z'_B|\Psi\rangle = |\Psi\rangle, \tag{4}$$

$$z_A z'_A \cdot x_A x'_A \cdot z_B x'_B \cdot x_B z'_B|\Psi\rangle = -|\Psi\rangle. \tag{5}$$

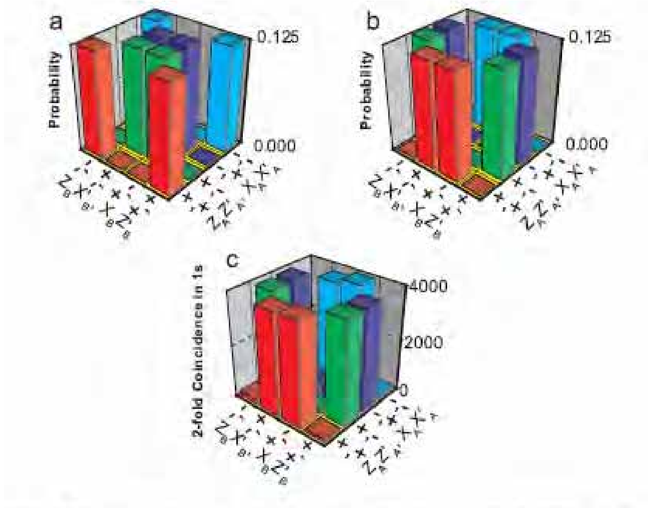

FIG. 3 (color online). Predictions of LR (a) and of QM (b), and observed results (c) for the $z_A z'_A \cdot x_A x'_A \cdot z_B x'_B \cdot x_B z'_B$ experiment.

Requires two-qubit measurements!

$$v(Z_1) = -v(Z_2)$$

$$v(z_1) = -v(z_2)$$

$$v(X_1) = -v(X_2)$$

$$v(x_1) = -v(x_2)$$

$$v(Z_1 z_1) = v(Z_2)v(z_2)$$

$$v(X_1 x_1) = v(X_2)v(x_2)$$

$$v(Z_1)v(x_1) = v(Z_2 x_2)$$

$$v(X_1)v(z_1) = v(X_2 z_2)$$

$$v(Z_1 z_1)v(X_1 x_1) = -v(Z_2 x_2)v(X_2 z_2)$$

Requires two-qubit measurements!

$$v(Z_1) = -v(Z_2)$$

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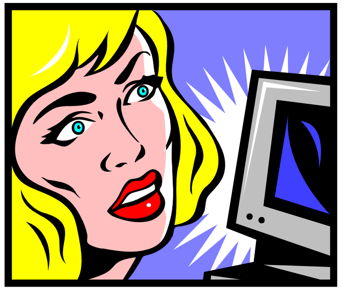
Problem

- Two-observer AVN proof with single-qubit observables?



Two-observer AVN proof with single qubit observables

Hyperentangled cluster:

$$|HuHu\rangle + |HdHd\rangle + |VuVu\rangle - |VdVd\rangle$$

$$v(Z_1) = v(Z_2)$$

$$v(z_1) = v(z_2)$$

$$v(X_1) = v(X_2)v(z_2)$$

$$v(x_1) = v(Z_2)v(x_2)$$

$$v(Y_1) = -v(Y_2)v(z_2)$$

$$v(y_1) = -v(Z_2)v(y_2)$$

$$v(X_2) = v(X_1)v(z_1)$$

$$v(x_2) = v(Z_1)v(x_1)$$

$$v(Y_2) = -v(Y_1)v(z_1)$$

$$v(y_2) = -v(Z_1)v(y_1)$$

Two-observer AVN proof with single qubit observables

$$v(X_1) = v(X_2)\,v(z_2)$$

$$v(Y_1) = -\,v(Y_2)\,v(z_2)$$

$$v(X_1)\,v(x_1) = v(Y_2)\,v(y_2)$$

$$v(Y_1)\,v(x_1) = v(X_2)\,v(y_2)$$

AC, PRL **95**, 210401 (2005).

Rome experiment 2007

Realization and characterization of a 2-photon 4-qubit linear cluster state

Giuseppe Vallone[1,*], Enrico Pomarico[1,*], Paolo Mataloni[1,*], Francesco De Martini[1,*], Vincenzo Berardi[2]
[1]Dipartimento di Fisica dell'Universitá "La Sapienza" and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma, 00185 Italy
[2]Dipartimento Interateneo di Fisica, Università e Politecnico di Bari and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Bari, 70126 Italy

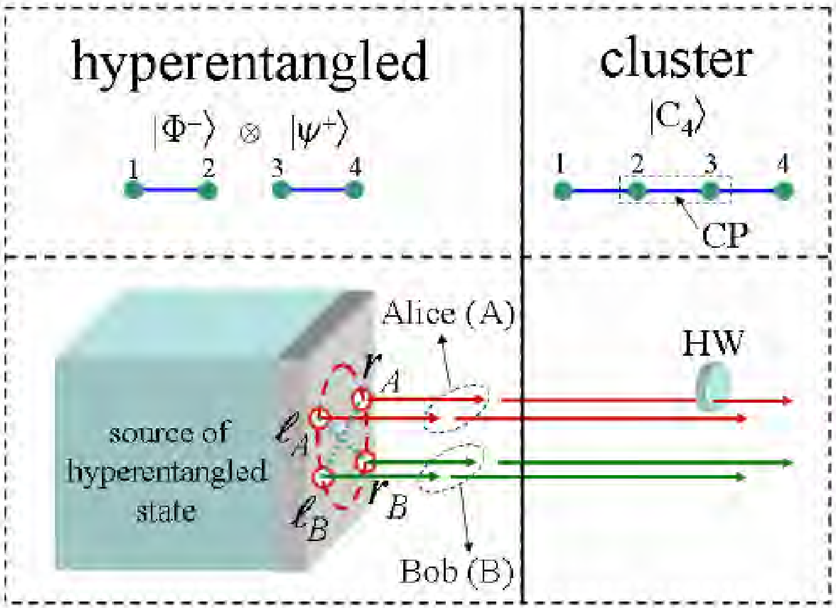


FIG. 1: Generation of the linear cluster state by a source of polarization-momentum hyperentangled 2-photon state. The state $|\Xi\rangle = |\Phi^-\rangle \otimes |\psi^+\rangle$ corresponds to two separate 2-qubit clusters. The *HW* acts as a Controlled-Phase (CP) thus generating the 4-qubit linear cluster $|C_4\rangle$.

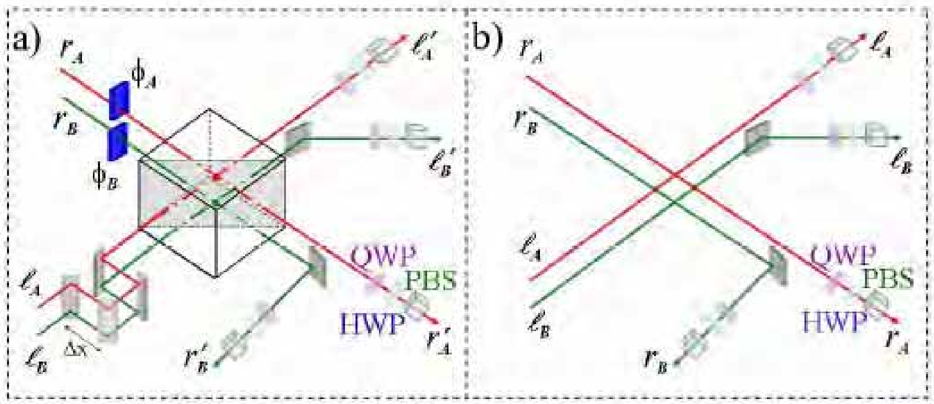


FIG. 2: Interferometer and measurement apparatus. a) The mode pairs r_A-ℓ_B and ℓ_A-r_B are matched on the BS. The phase shifters ϕ_A and ϕ_B (thin glass plates) are used for the measurement of momentum observables. The polarization analyzers on each of BS output modes are shown (QWP/HWP=Quarter/Half-Wave Plate, PBS=Polarized Beam Splitter). b) Same configuration as in a) with BS and glasses removed.

PRL **98**, 180502 (2007).

Rome experiment 2007

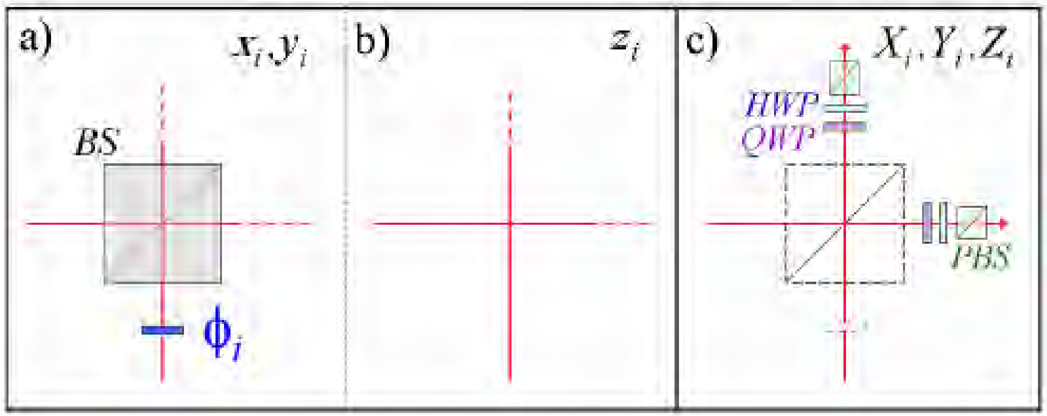


FIG. 4: Measurement setup for momentum (a),b)) and polarization (c)) observables for photon i (i=A, B). By the a) setup we measure x_i ($\phi_i = 0$) and y_i ($\phi_i = \frac{\pi}{2}$), while the b) setup is used for measuring z_i. By the c) setup we measure X_i ($\theta_Q = \frac{\pi}{4}$; $\theta_H = \frac{1}{8}\pi, \frac{3}{8}\pi$), Y_i ($\theta_Q = 0$; $\theta_H = \frac{1}{8}\pi, \frac{3}{8}\pi$) and Z_i ($\theta_Q = 0$; $\theta_H = 0, \frac{\pi}{4}$), where $\theta_{H(Q)}$ is the angle between the $HWP(QWP)$ optical axis and the vertical direction. The polarization analysis is performed contextually to x_i, y_i (i.e. with BS and glass) or z_i (without BS and glass), as shown by the dotted lines for BS and glass in c).

Observable	Value	$\mathcal{W}$	S	C
$Z_A Z_B$	$+0.9283 \pm 0.0032$	✓		
$Z_A x_A x_B$	$+0.8194 \pm 0.0049$	✓		
$X_A z_A X_B$	-0.9074 ± 0.0037	✓		✓
$z_A z_B$	-0.9951 ± 0.0009	✓		✓
$x_A Z_B x_B$	$+0.8110 \pm 0.0050$	✓		✓
$Z_A y_A y_B$	$+0.8071 \pm 0.0050$			✓
$Y_A z_A Y_B$	$+0.8948 \pm 0.0040$			✓
$X_A X_B z_B$	$+0.9074 \pm 0.0037$	✓	✓	✓
$Y_A Y_B z_B$	-0.8936 ± 0.0041		✓	✓
$X_A x_A Y_B y_B$	$+0.8177 \pm 0.0055$		✓	
$Y_A x_A X_B y_B$	$+0.7959 \pm 0.0055$		✓	

TABLE I: Experimental values of the observables used for measuring the entanglement witness $\mathcal{W}$ and the expectation value of S on the cluster state $|C_4\rangle$. The third column (C) refers to the control measurements needed to verify that X_A, Y_A, x_A, X_B, Y_B, y_B and z_B can be considered as elements of reality. Each experimental value corresponds to a measure lasting an average time of 10 sec. In the experimental errors we considered the poissonian statistic and the uncertainties due to the manual setting of the polarization analysis wave plates.

$$\mathrm{Tr}[S\rho_{exp}] = 3.4145 \pm 0.0095$$

PRL **98**, 180502 (2007).

Plan

- EPR argument
- Bell's theorem
  - Without inequalities (AVN proofs)
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Bell-CHSH inequality: Derivation

$$\left|\langle A_0B_0\rangle+\langle A_0B_1\rangle+\langle A_1B_0\rangle-\langle A_1B_1\rangle\right|\leq 2$$

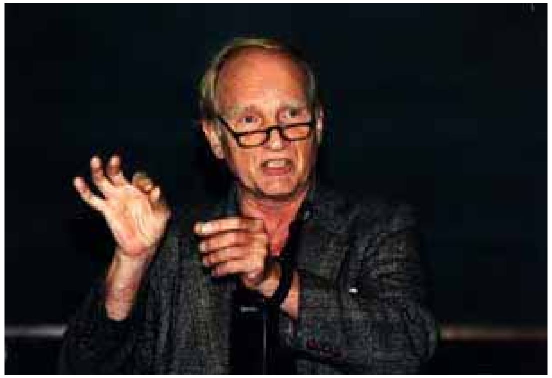
$$A_0, A_1, B_0, B_1 \in \{-1,1\}$$

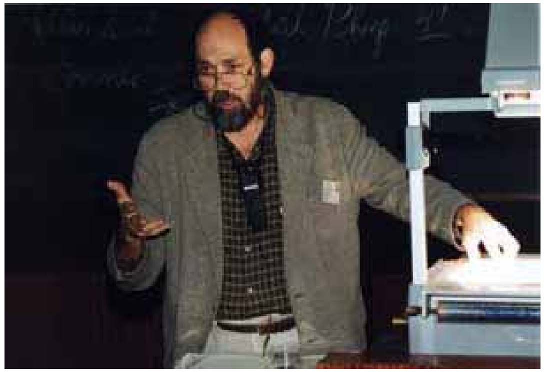
$$\left(A_0+A_1, A_0-A_1\right)\in\{(\pm 2,0),(0,\pm 2)\}$$

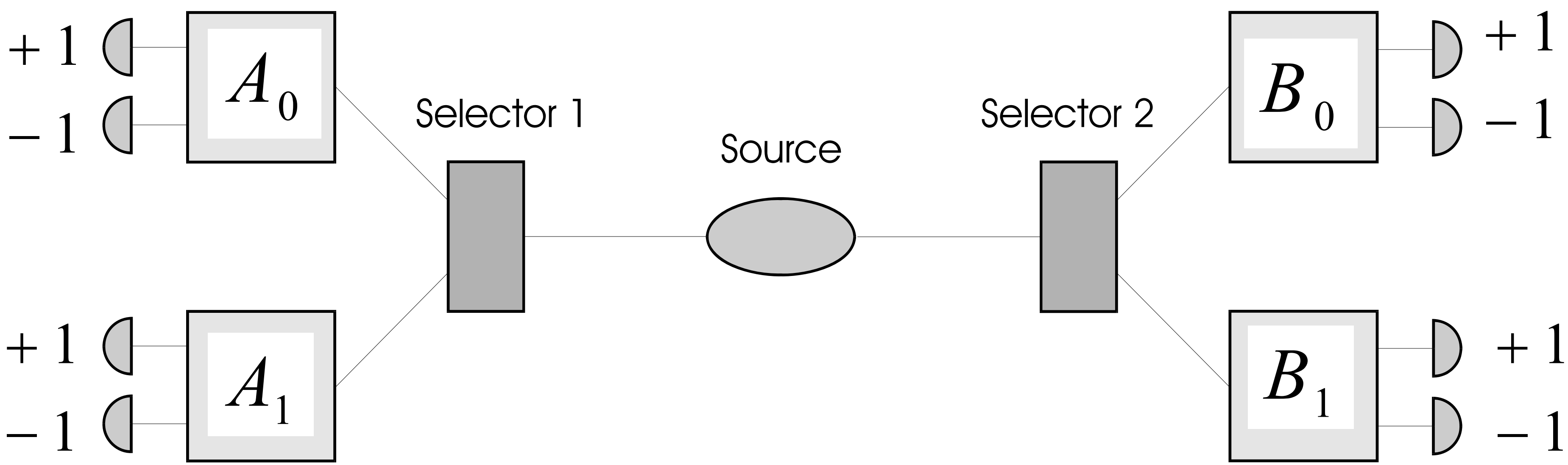
$$\left(A_0+A_1\right)B_0+\left(A_0-A_1\right)B_1\in\{-2,2\}$$

$$-2\leq\langle A_0B_0+A_0B_1+A_1B_0-A_1B_1\rangle\leq 2$$

Scenario for the Bell-CHSH inequality







QM violates the Bell-CHSH inequality

$$\left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}\left(\left|01\right\rangle - \left|10\right\rangle\right)$$

$$\begin{aligned} A_0 &= \sigma_x, \\ A_1 &= \sigma_z, \\ B_0 &= (\sigma_x + \sigma_z)/\sqrt{2}, \\ B_1 &= (\sigma_x - \sigma_z)/\sqrt{2}. \end{aligned}$$

$$\beta_{\mathrm{QM}} = 2\sqrt{2} > 2!!!$$

Bell inequalities

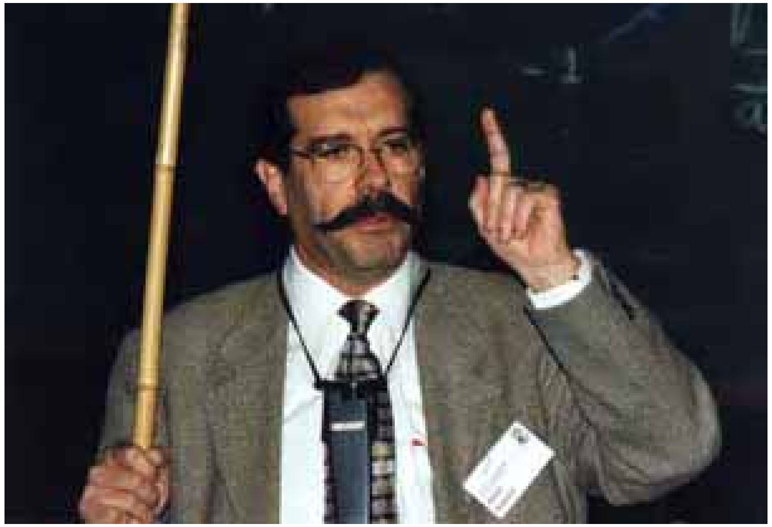
The use of Bell inequalities has some advantages:

(i) Independence of QM. Follow from the assumption of locality (the results of local measurements are independent of spacelike separated events).

(ii) Provide a testable method to experimentally exclude LHV.

(iii) Applications in communication complexity, entanglement detection, security of key distribution, state discrimination…

Aspect's experiments



VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France
(Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.

VOLUME 49, NUMBER 2 PHYSICAL REVIEW LETTERS 12 JULY 1982

Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique, Université Paris-Sud, F-91406 Orsay, France
(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

VOLUME 49, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1982

Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard,[a] and Gérard Roger
Institut d'Optique Théorique et Appliquée, F-91406 Orsay Cédex, France
(Received 27 September 1982)

Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.

Loophole-free Bell experiments

So far, the results of any performed Bell experiment <u>admit</u> an interpretation in terms of local realistic theories.

A loophole-free experiment would require:

- Spacelike separation between Alice's measurement choice and Bob's measurement in order to exclude the possibility that Alice's measurement choice influences the result of Bob's measurement (locality loophole).

- Sufficiently large number of detections of the prepared particles in order to exclude the possibility that the nondetections correspond to local hidden-variable instructions (detection loophole).

Photons, ions… the good news

• Photons are the best candidates for closing the locality loophole. For instance, one can do a Bell experiment with pairs of polarization-entangled photons separated d = 400 m, which is not subject to the locality loophole (Innsbruck 98).

• Ions are the best candidates for closing the detection loophole. For instance, one can do a Bell experiment with pairs of trapped ions with a detection efficiency η = 1 (Boulder 01, Maryland 08).

Photons, ions… the bad news

- Photo-detection efficiency (η = 0.05-0.33) is not high enough to close the detection loophole (η > 0.83 is required for the CHSH inequality).

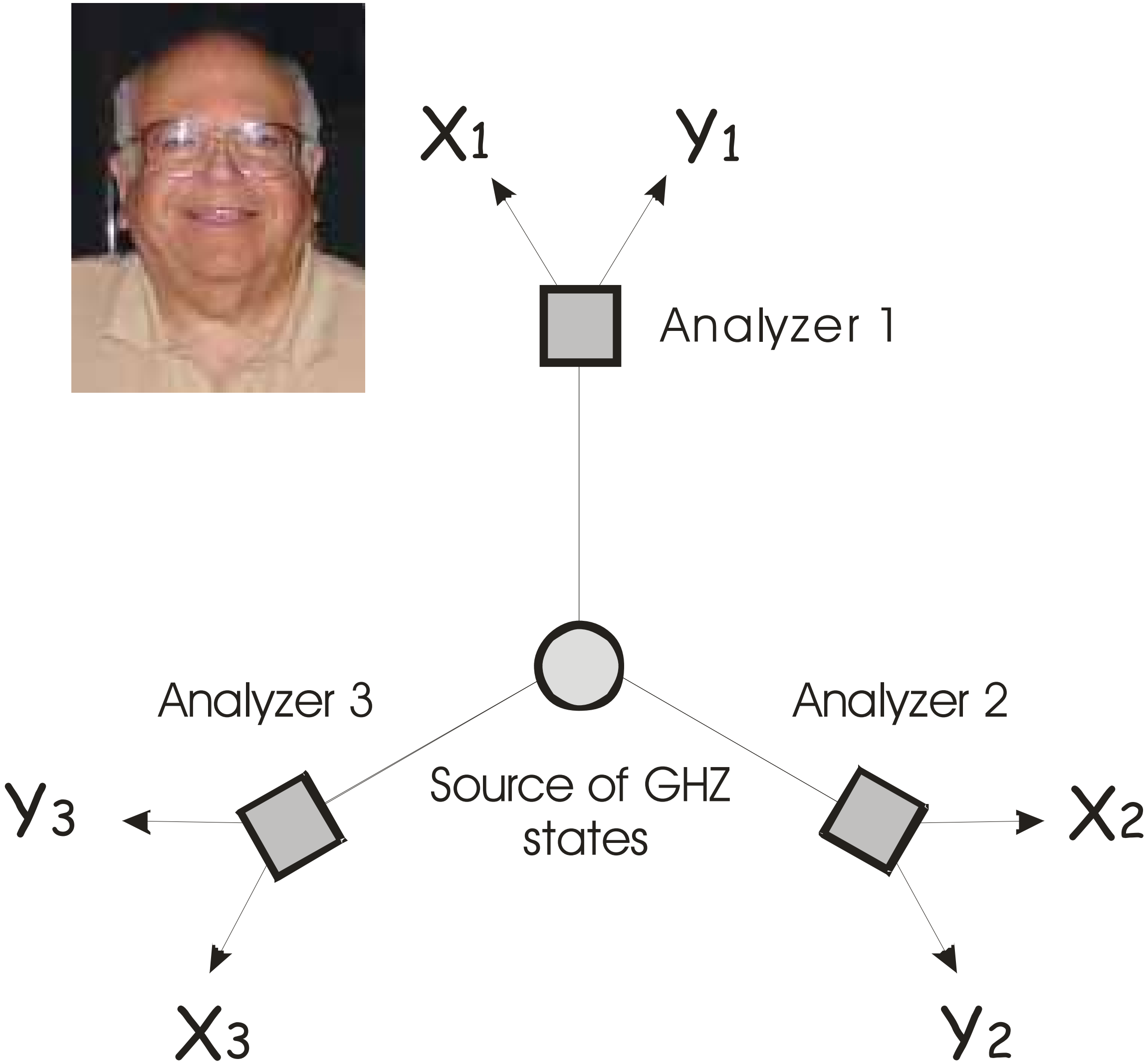
- Separation between trapped ions (*d* = 1 m in the Maryland 08 experiment) is not enough to close the locality loophole (*d* > 15 km is required for the Maryland 08 experiment).

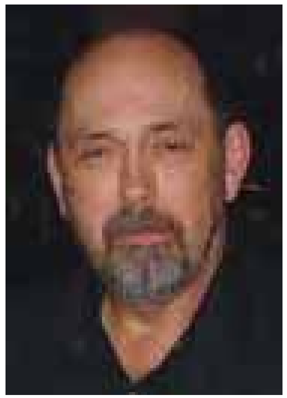
The Mermin inequality

$$\left|\langle A_1 B_0 C_0 \rangle + \langle A_0 B_1 C_0 \rangle + \langle A_0 B_0 C_1 \rangle - \langle A_1 B_1 C_1 \rangle\right| \leq 2$$

$$\beta_{\mathrm{QM}} = 4$$

Scenario for the Mermin inequality





Plan

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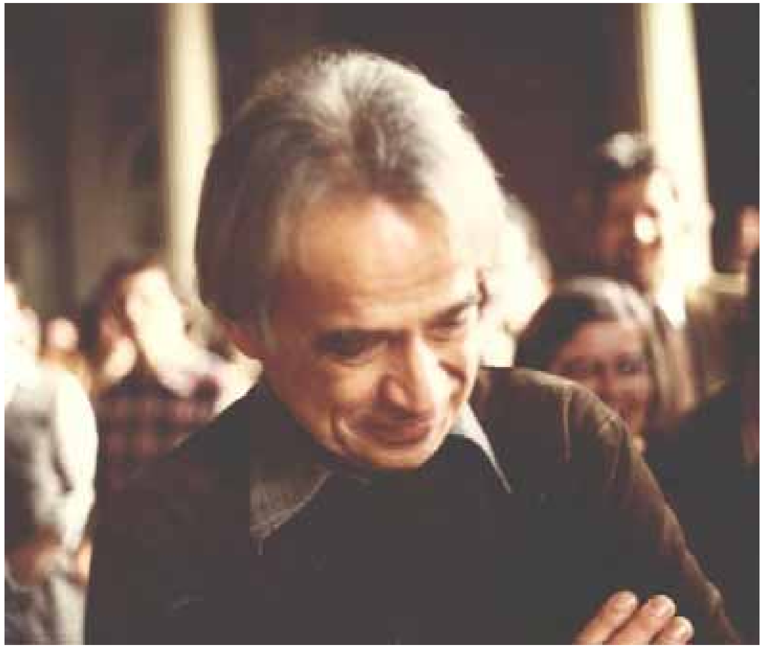
The Kochen-Specker theorem

No theory of noncontextual hidden variables (NCHV) can reproduce QM.

NCHV theories are those based on the assumption of noncontextuality, according to which the result of a measurement is independent of which other compatible observables are jointly measured



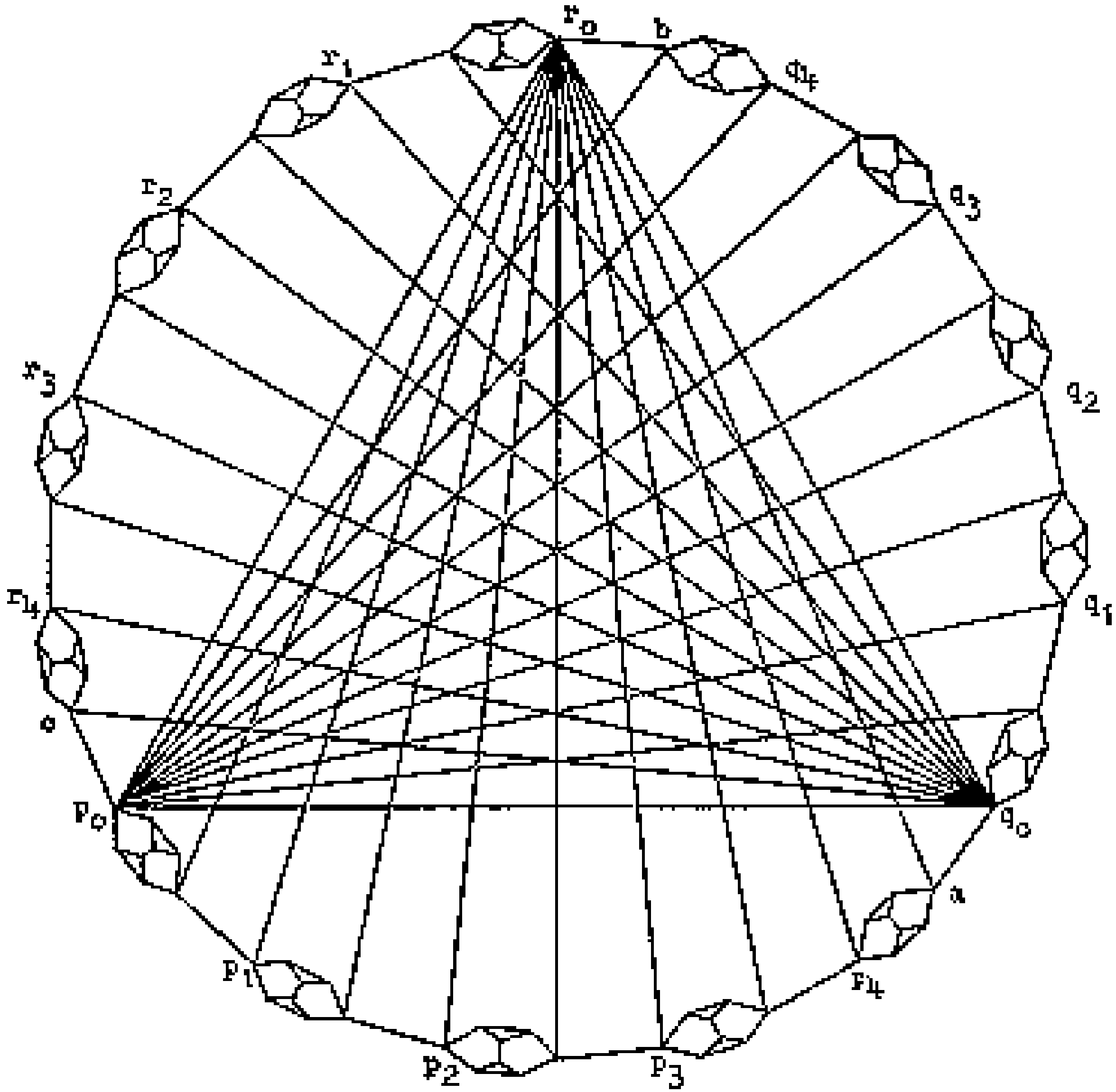
The KS theorem prove the impossibility of…

(i) Realism: measurement outcomes of nonperformed measurements can be introduced alongside of those of the actually performed measurements.

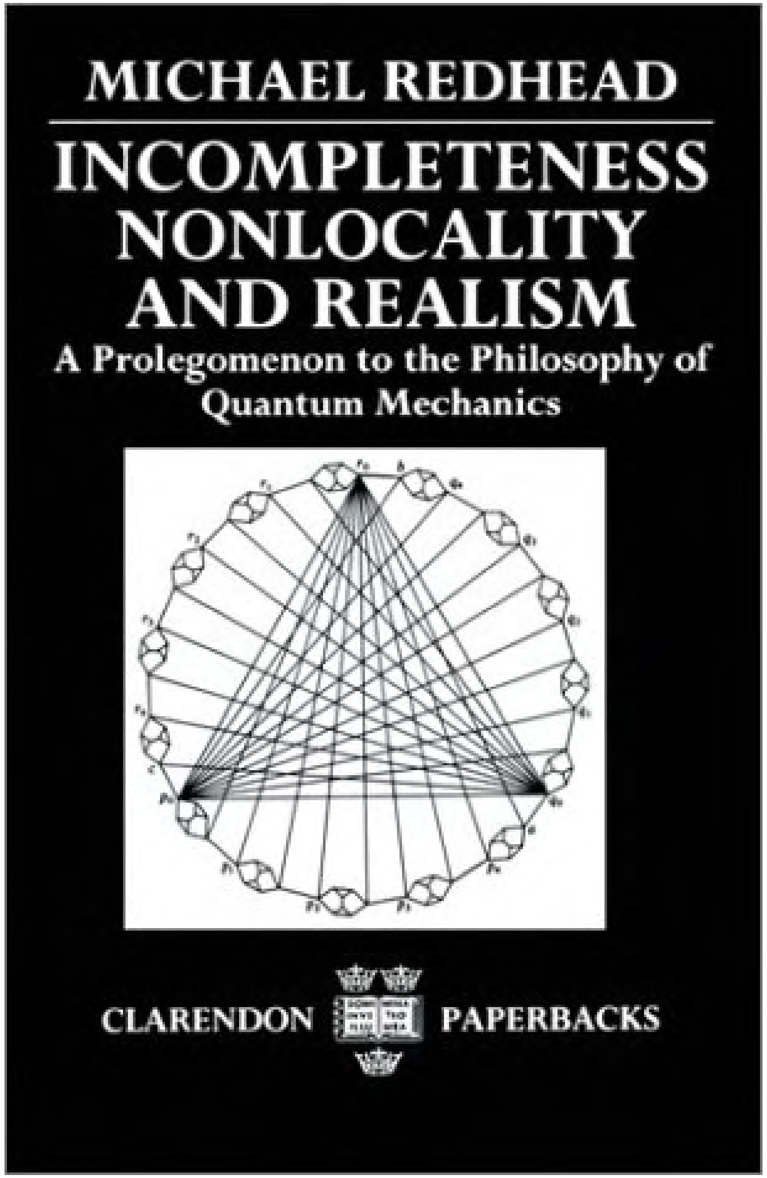
(ii) Noncontextuality: Alice’s measurement outcome does not depend on Bob’s choice of measurement [assuming they measure compatible observables].

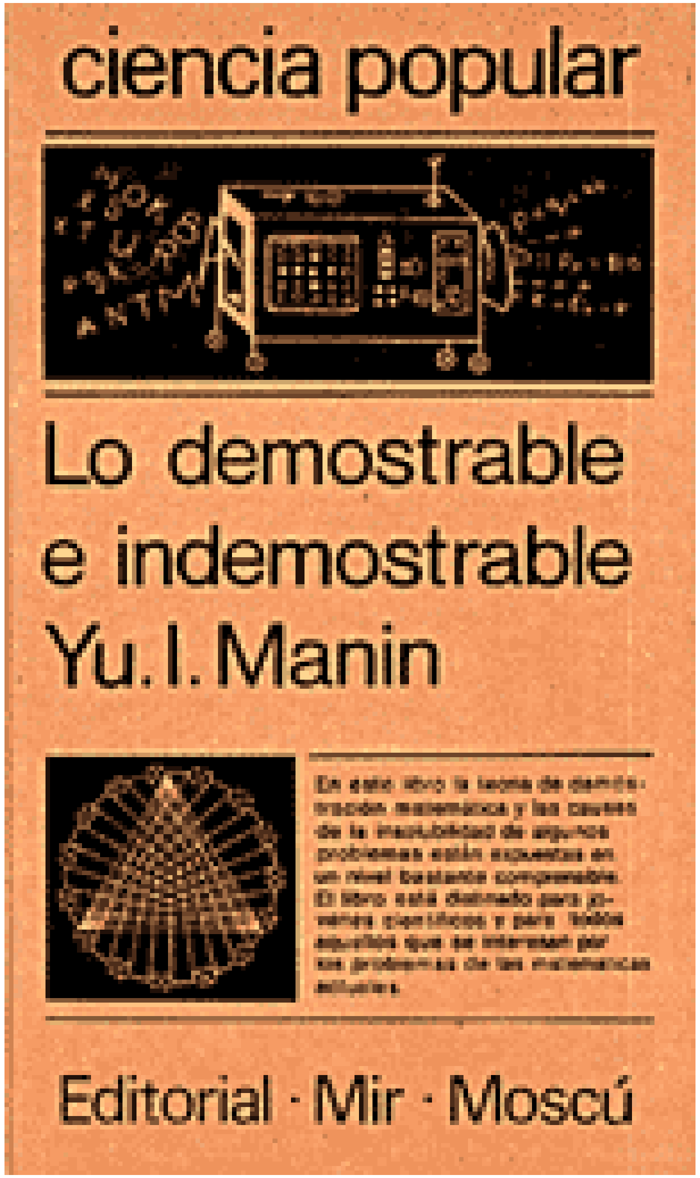
(iii) Freedom: Alice and Bob can perform either measurement.

The Kochen-Specker theorem



The Kochen-Specker theorem





The 18-vector proof of the KS theorem

1000	1111	1111	1000	1001	1001	111-1	111-1	100-1
0100	11-1-1	1-11-1	0010	0100	1-11-1	1-100	0101	0110
0011	1-100	10-10	0101	0010	11-1-1	0011	10-10	11-11
001-1	001-1	010-1	010-1	100-1	0110	11-11	1-111	1-111

- Each vector represents the projection operator onto the corresponding normalized vector. For instance, 111-1 represents the projector onto the vector (1,1,1,-1)/2.
- Each column contains four mutually orthogonal vectors, so that the corresponding projectors sum the identity.
- In any NCHV theory, each column must have assigned the answer "yes" to one and only one vector.
- But such an assignment is impossible, since each vector appears in two columns, so the total number of "yes" answers must be an even number. However, the number of columns is an odd number.

A. Cabello, J. M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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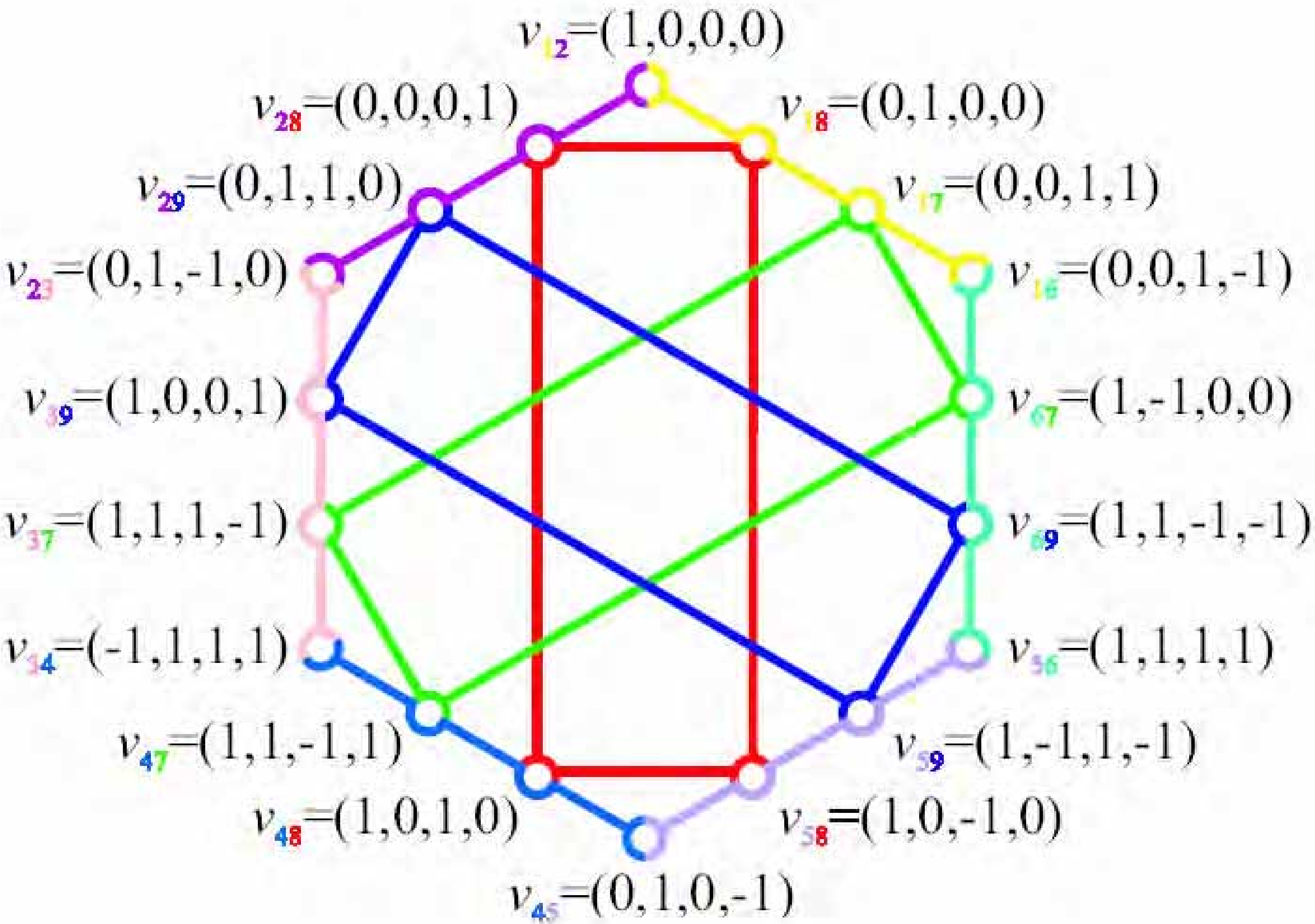

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State-dependent inequalities for NCHV

There are inequalities that are based *only* on the assumption of noncontextuality, in the same way that the Bell inequalities are based only on the assumption of locality.

These inequalities have the advantage of providing a testable method to experimentally exclude any alternative description based on NCHV.

A. Cabello, S. Filipp, H. Rauch, and Y. Hasegawa, Phys. Rev. Lett. **100**, 130404 (2008).

A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Phys. Rev. Lett. **101**, 020403 (2008).

Y. Nambu, e-print arXiv:0805.3398 [quant-ph].

State-independent inequalities?

However, the fact that all these new inequalities are state-dependent, while the proofs of the KS theorem are state-independent, has been recently described as "a drawback".

A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Phys. Rev. Lett. **101**, 020403 (2008).

A natural question is the following: Given a physical system described in QM by a Hilbert space of dimension *d* (i.e., a physical system admitting *d* compatible dichotomic observables), is it possible to derive <u>experimentally testable</u> <u>Bell-like correlation inequalities</u> <u>using only the assumption of noncontextuality</u>, such that <u>any quantum state violates them</u>?

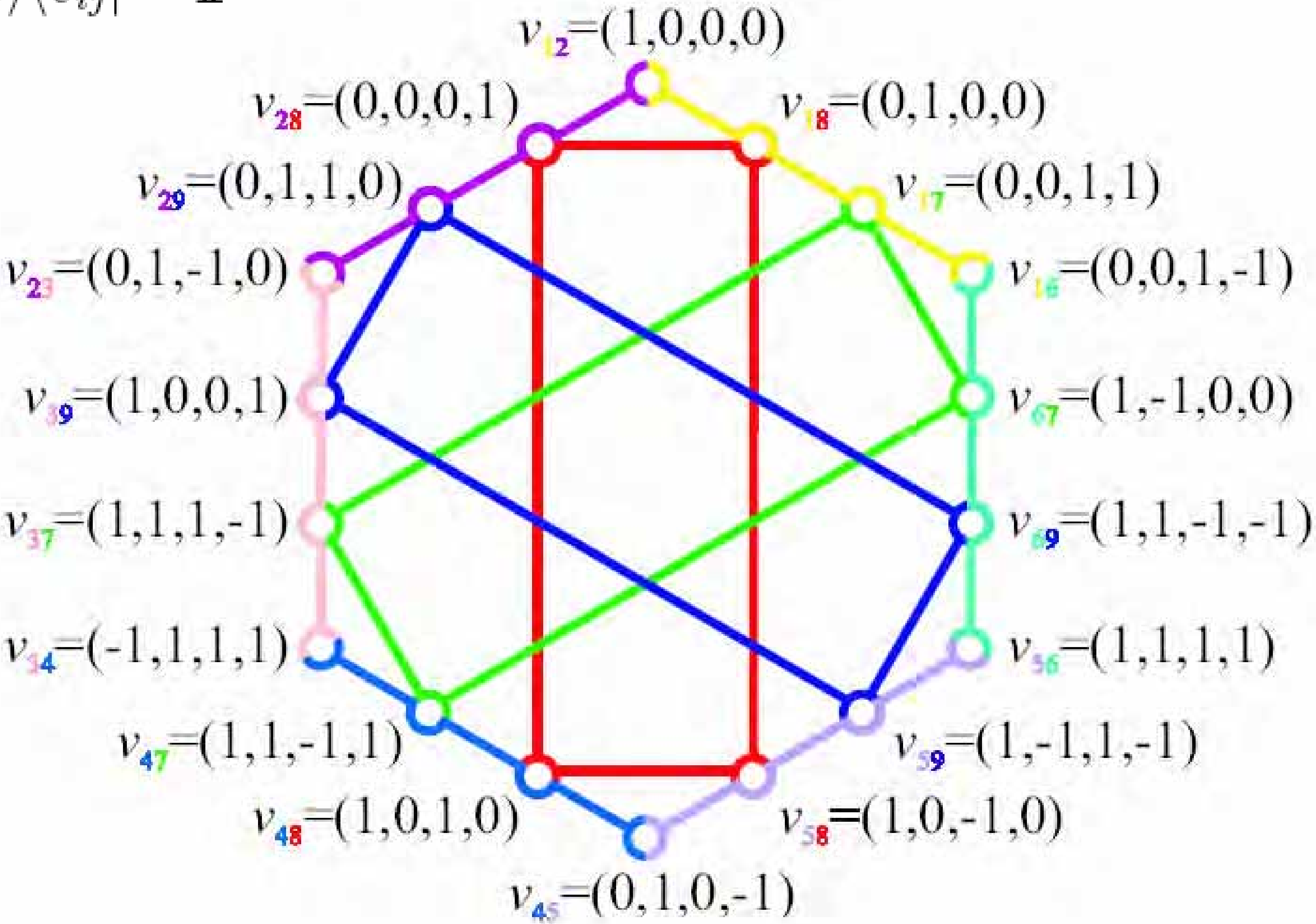
First inequality

$$
\begin{aligned}
&-\langle A_{12}A_{16}A_{17}A_{18}\rangle - \langle A_{12}A_{23}A_{28}A_{29}\rangle - \langle A_{23}A_{34}A_{37}A_{39}\rangle \\
&-\langle A_{34}A_{45}A_{47}A_{48}\rangle - \langle A_{45}A_{56}A_{58}A_{59}\rangle - \langle A_{16}A_{56}A_{67}A_{69}\rangle \\
&-\langle A_{17}A_{37}A_{47}A_{67}\rangle - \langle A_{18}A_{28}A_{48}A_{58}\rangle - \langle A_{29}A_{39}A_{59}A_{69}\rangle \leq 7
\end{aligned}
$$

A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008).

State-independent violation

$$A_{ij} = 2|v_{ij}\rangle\langle v_{ij}| - \mathbb{1}$$



A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008).

First inequality

$$-\langle A_{12}A_{16}A_{17}A_{18}\rangle - \langle A_{12}A_{23}A_{28}A_{29}\rangle - \langle A_{23}A_{34}A_{37}A_{39}\rangle$$
$$-\langle A_{34}A_{45}A_{47}A_{48}\rangle - \langle A_{45}A_{56}A_{58}A_{59}\rangle - \langle A_{16}A_{56}A_{67}A_{69}\rangle$$
$$-\langle A_{17}A_{37}A_{47}A_{67}\rangle - \langle A_{18}A_{28}A_{48}A_{58}\rangle - \langle A_{29}A_{39}A_{59}A_{69}\rangle \leq 7$$

$$\beta_{\mathrm{QM}} = 9$$

A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008).

Second inequality

$$\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4$$

$$A = \sigma_z^{(1)}, \qquad B = \sigma_z^{(2)}, \qquad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)},$$
$$a = \sigma_x^{(2)}, \qquad b = \sigma_x^{(1)}, \qquad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)},$$
$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)}, \qquad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)}, \qquad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}.$$

$$\beta_{\mathrm{QM}} = 6$$

A. Peres, Phys. Lett. A **151**, 107 (1990).

N. D. Mermin, Phys. Rev. Lett. **65**, 3373 (1990).

Particular cases of the second inequality

$$-\langle AB\rangle - \langle ab\rangle - \langle \alpha\beta\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle \leq 3$$

A. Cabello, S. Filipp, H. Rauch, and Y. Hasegawa, Phys. Rev. Lett. **100**, 130404 (2008).

$$\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\rangle \leq 4$$

Y. Nambu, e-print arXiv:0805.3398 [quant-ph].

Third inequality

$$\langle \mathcal{A}_1\mathcal{B}_1\mathcal{B}_2\prod_{i=3}^{n}\mathcal{B}_i\rangle + \langle \mathcal{A}_2\mathcal{B}_1\mathcal{C}_2\prod_{i=3}^{n}\mathcal{C}_i\rangle + \langle \mathcal{A}_3\mathcal{C}_1\mathcal{B}_2\prod_{i=3}^{n}\mathcal{C}_i\rangle$$
$$+\langle \mathcal{A}_4\mathcal{C}_1\mathcal{C}_2\prod_{i=3}^{n}\mathcal{B}_i\rangle - \langle \mathcal{A}_1\mathcal{A}_2\mathcal{A}_3\mathcal{A}_4\rangle \leq 3$$

$$\mathcal{A}_1 = Z_1 \otimes Z_2 \otimes Z_3 \otimes \ldots \otimes Z_n,$$
$$\mathcal{A}_2 = Z_1 \otimes X_2 \otimes X_3 \otimes \ldots \otimes X_n,$$
$$\mathcal{A}_3 = X_1 \otimes Z_2 \otimes X_3 \otimes \ldots \otimes X_n,$$
$$\mathcal{A}_4 = X_1 \otimes X_2 \otimes Z_3 \otimes \ldots \otimes Z_n,$$
$$\mathcal{B}_i = Z_i,$$
$$\mathcal{C}_i = X_i.$$

$$\beta_{\mathrm{QM}} = 5$$

N. D. Mermin, Phys. Rev. Lett. **65**, 3373 (1990).

Particular case of the third inequality

$$\langle \mathcal{B}_1 \mathcal{B}_2 \prod_{i=3}^{n} \mathcal{B}_i \rangle + \langle \mathcal{B}_1 \mathcal{C}_2 \prod_{i=3}^{n} \mathcal{C}_i \rangle + \langle \mathcal{C}_1 \mathcal{B}_2 \prod_{i=3}^{n} \mathcal{C}_i \rangle - \langle \mathcal{C}_1 \mathcal{C}_2 \prod_{i=3}^{n} \mathcal{B}_i \rangle \leq 2$$

N. D. Mermin, Phys. Rev. Lett. **65**, 1838 (1990).

Is it universal?

Can *any* physical system which, in principle, admits a non-contextual description show a state-independent violation of one of these inequalities?

Non-contextual descriptions are possible whenever $d \geq 3$. If $d = 2$, there are no 3 observables $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, such that $\mathcal{A}$ is compatible with $\mathcal{B}$ and $\mathcal{C}$, but $\mathcal{B}$ is incompatible with $\mathcal{C}$; thus, non-contextual descriptions of these systems are meaningless. $d = 3$ is the first case in which non-contextuality is a non-trivial property.

P. Badziąg, I. Bengtsson, A. Cabello, and I. Pitowsky, e-print arXiv:0809.0430 [quant-ph].

Conclusions

(i) “classical” states are impossible in quantum mechanics, and this impossibility can be tested by experiments.

(ii) Bell inequalities are particular cases of more general inequalities in which neither space-like separation nor entanglement play a fundamental role.

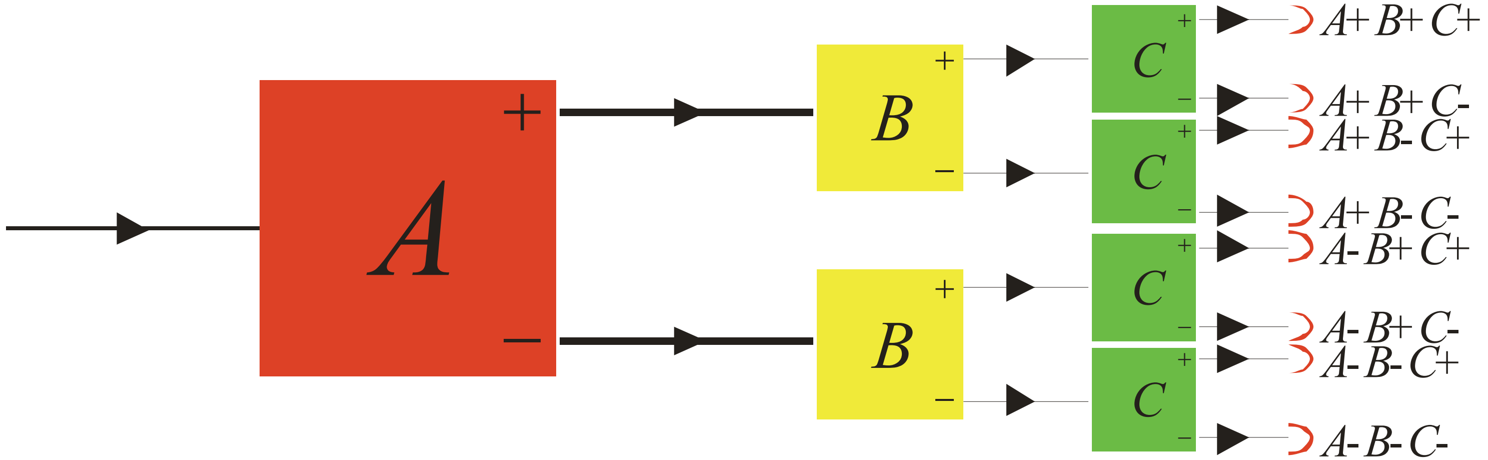
P. Badziąg, I. Bengtsson, A. Cabello, and I. Pitowsky, e-print arXiv:0809.0430 [quant-ph].

How can these inequalities be tested?

Experimental state-independent violation

$$\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \le 4$$

$$A = \sigma_z^{(1)}, \quad B = \sigma_z^{(2)}, \quad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)},$$
$$a = \sigma_x^{(2)}, \quad b = \sigma_x^{(1)}, \quad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)},$$
$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)}, \quad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)}, \quad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}.$$



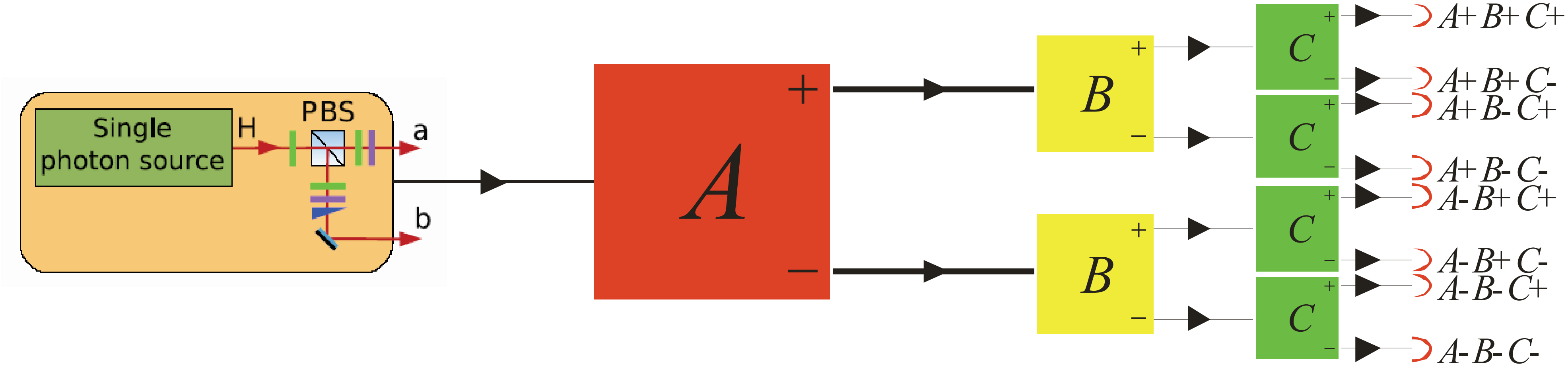
Experimental state-independent violation

$$\langle ABC \rangle + \langle abc \rangle + \langle \alpha\beta\gamma \rangle + \langle Aa\alpha \rangle + \langle Bb\beta \rangle - \langle Cc\gamma \rangle \leq 4$$

$$A = \sigma_z^{(1)}, \quad B = \sigma_z^{(2)}, \quad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)},$$

$$a = \sigma_x^{(2)}, \quad b = \sigma_x^{(1)}, \quad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)},$$

$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)}, \quad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)}, \quad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}.$$



E. Amselem, M. Rådmark, M. Bourennane, and A. Cabello.

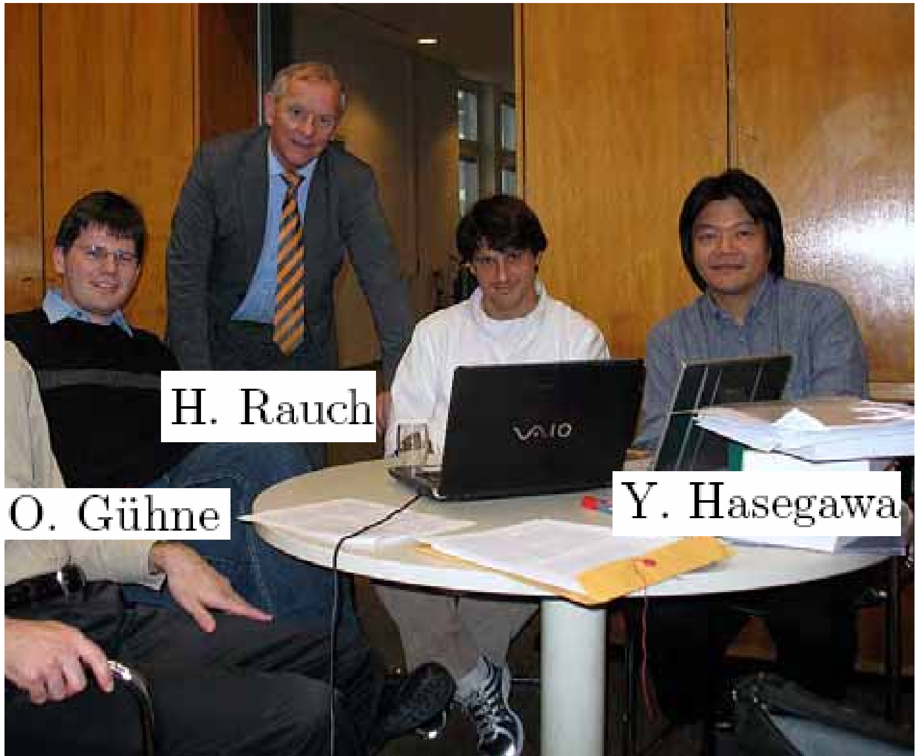
Can it be tested in different physical systems?

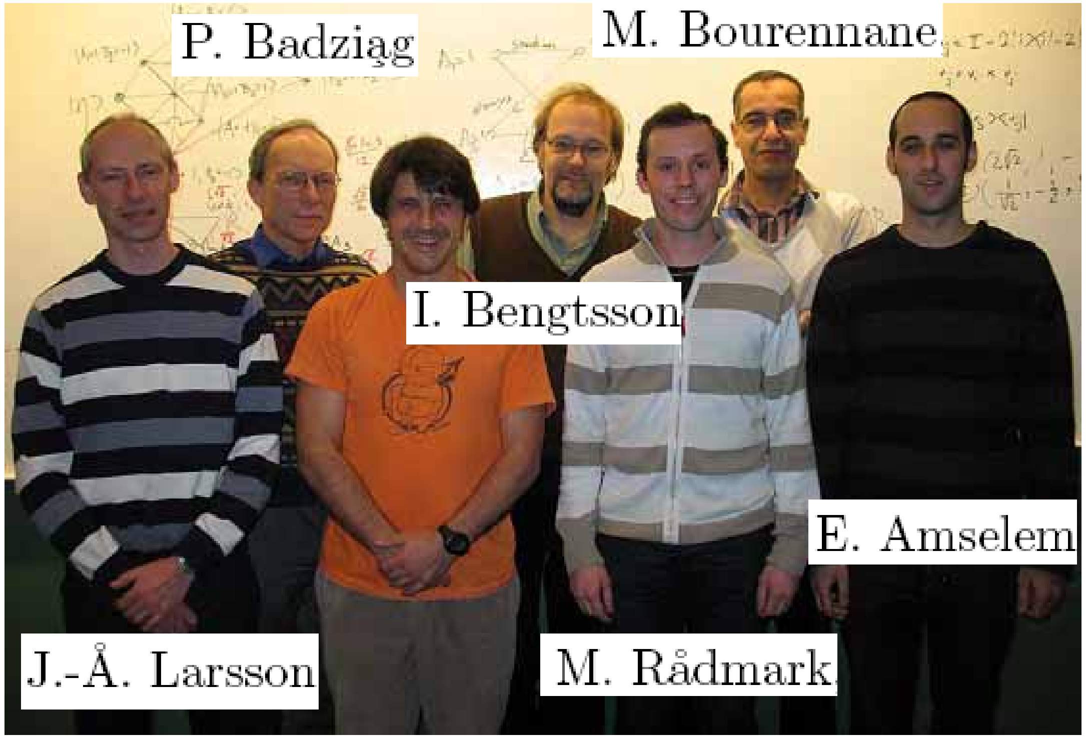
- Polarization and path of single photons? Yes
- Spin and path of single neutrons? Partially, H. Bartosik *et al.*
- Ions?
- Other systems?

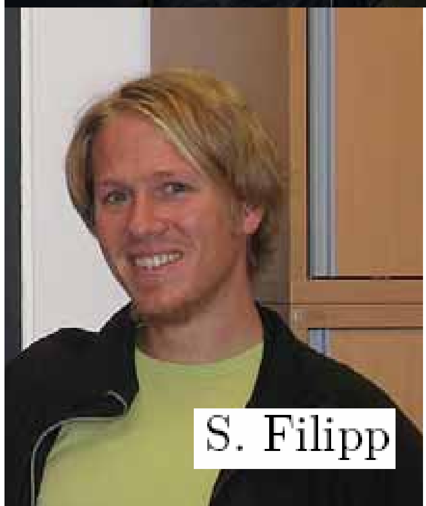
Loopholes?

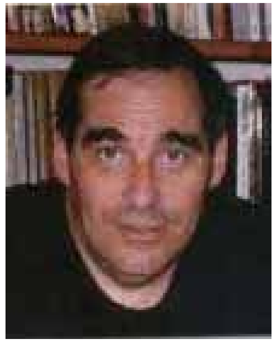
- Locality loophole? No
- Detection loophole? Yes
- New loopholes? Yes: Compatibility loophole

Collaborators









I. Pitowsky