The Nambu Jona-Lasinio model with density dependent $U_A(1)$ anomaly

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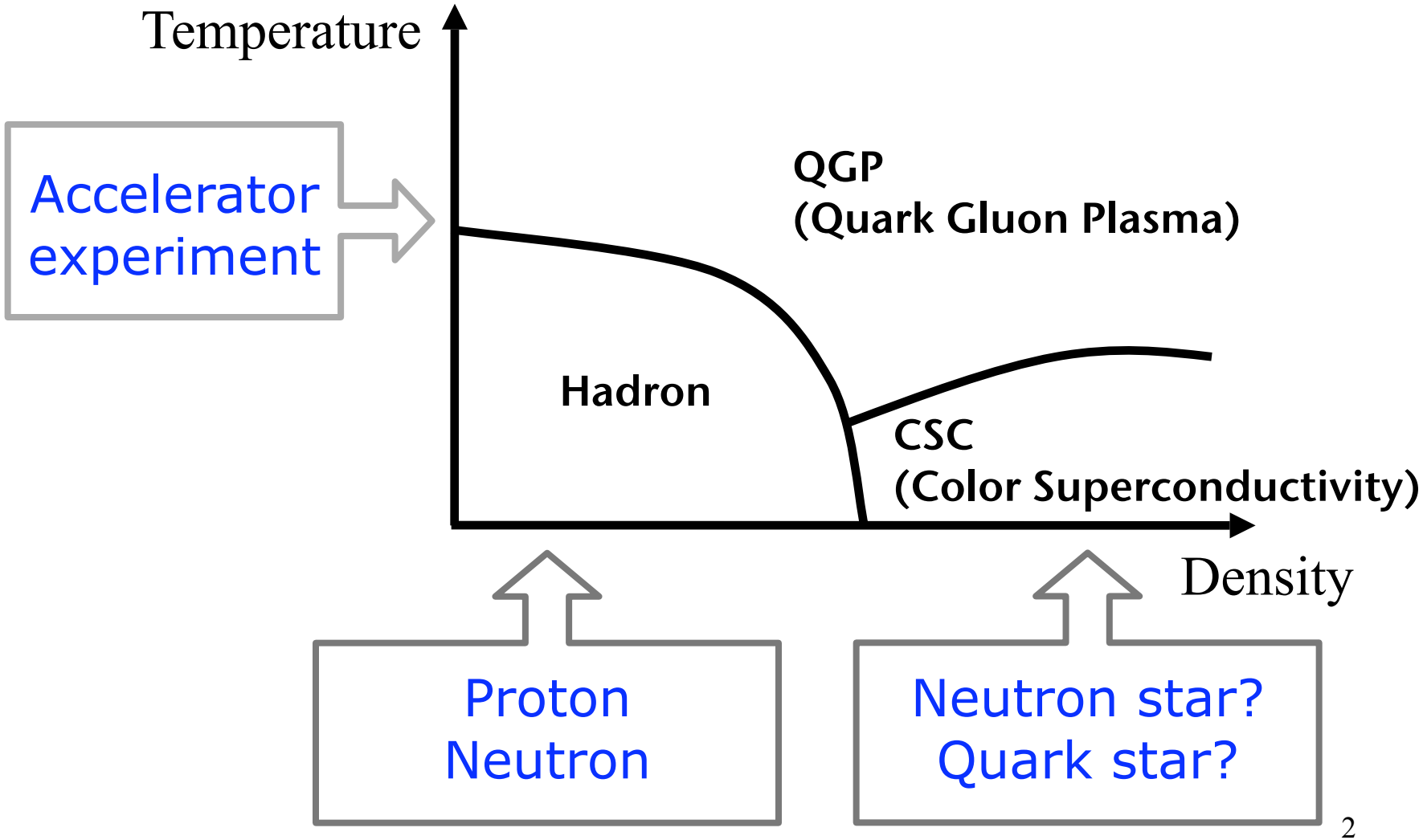
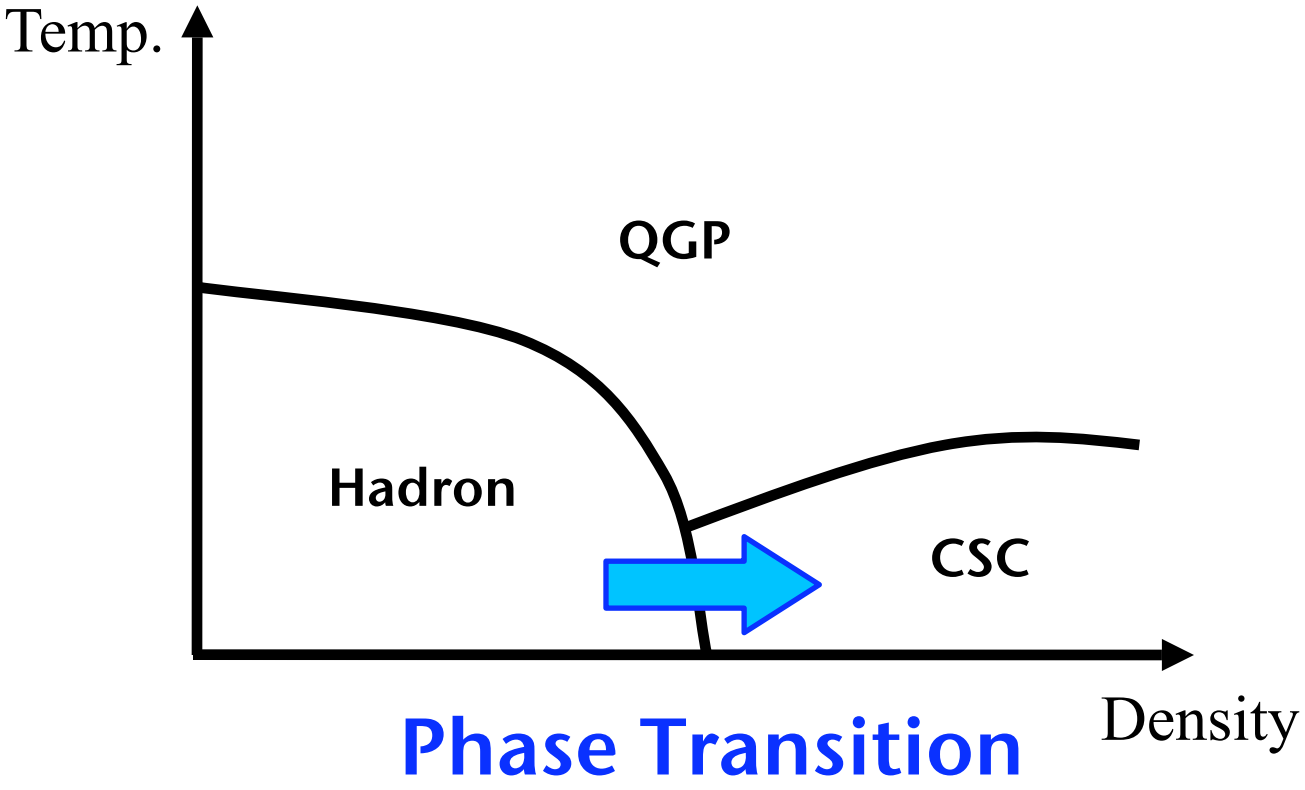
QCD phase diagram



Order of the phase transition



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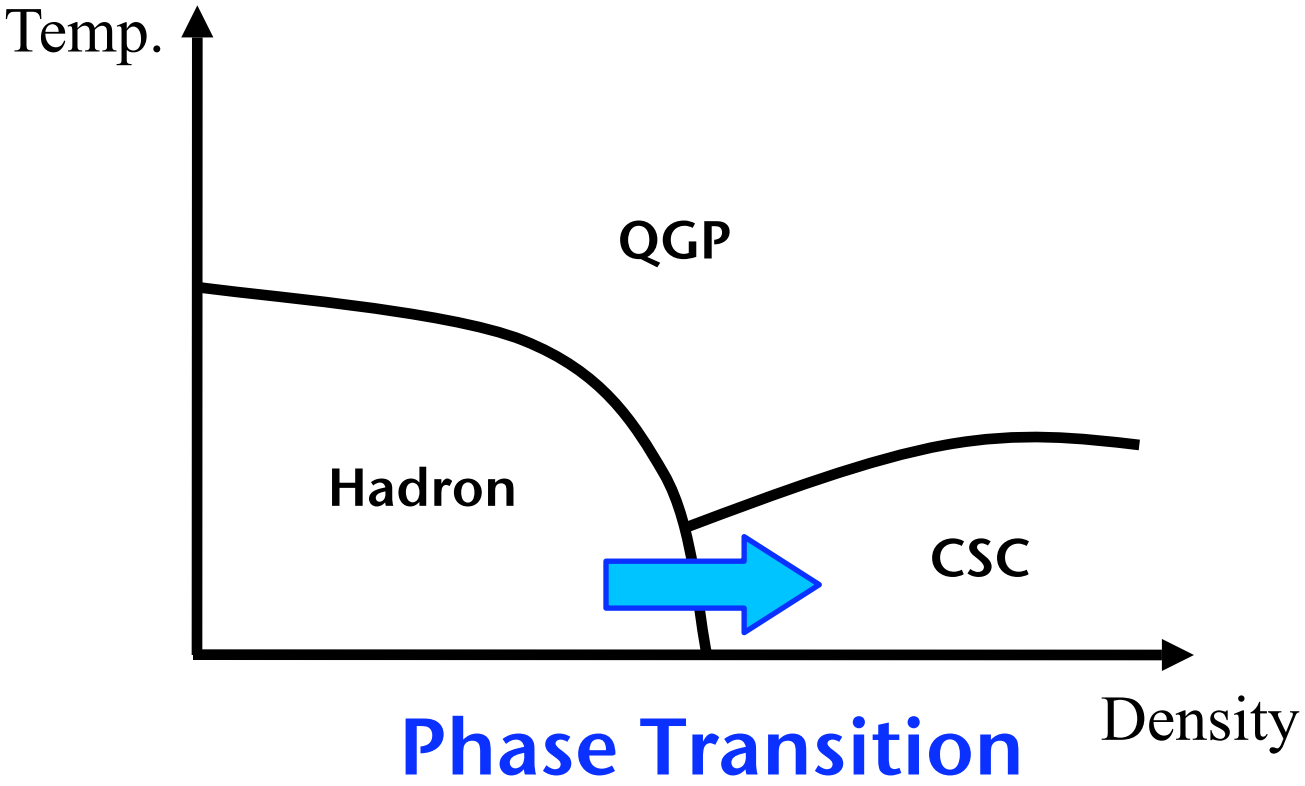


Phase Transition

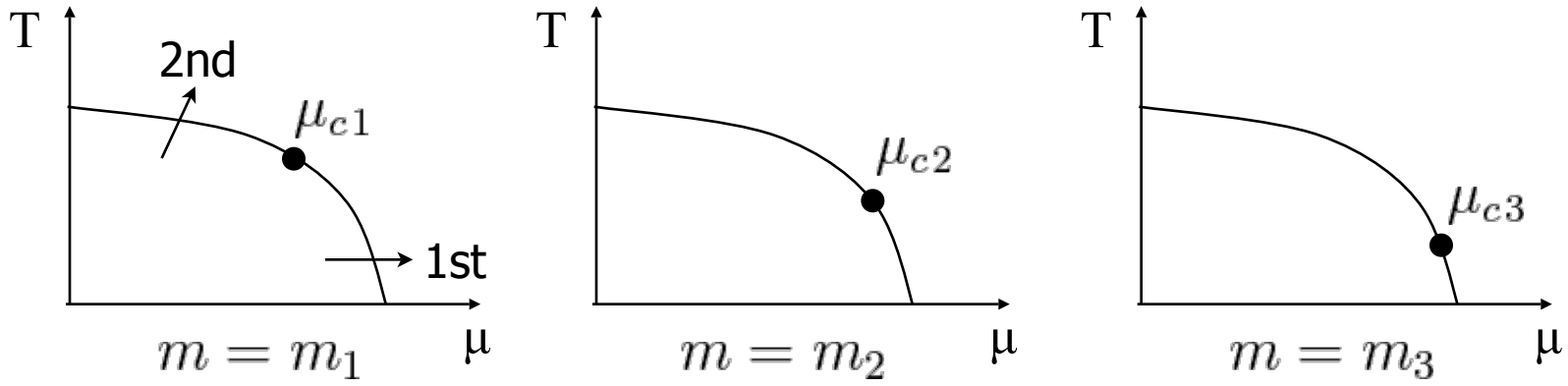
1st or 2nd?

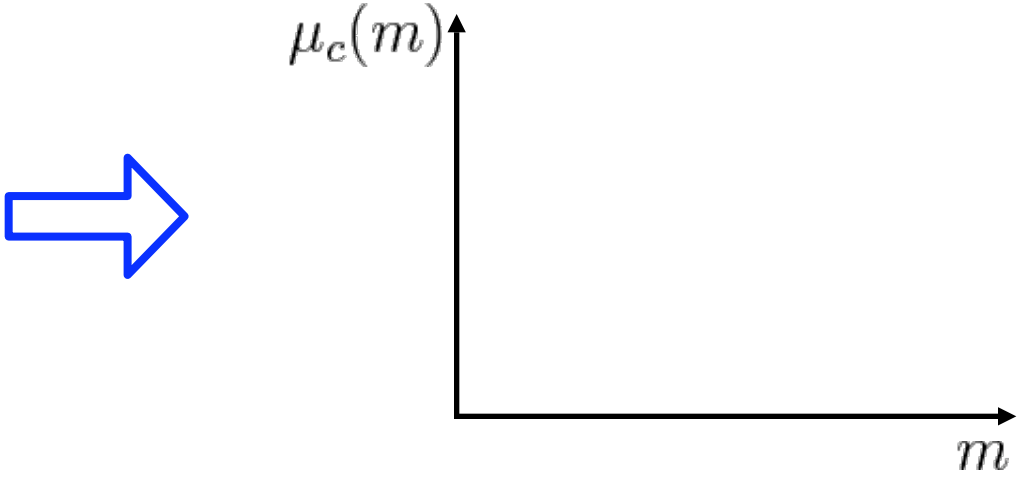
Critical Surface

From now on, we shall treat
current quark mass as a parameter.

Critical curve (2 flavor case)

For simplicity : $m_u = m_d = m$





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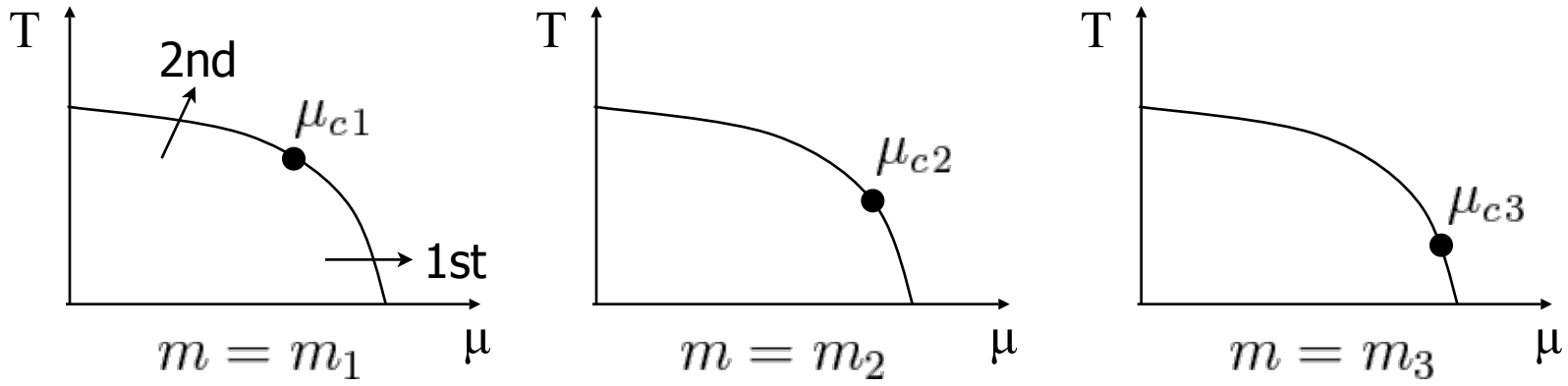

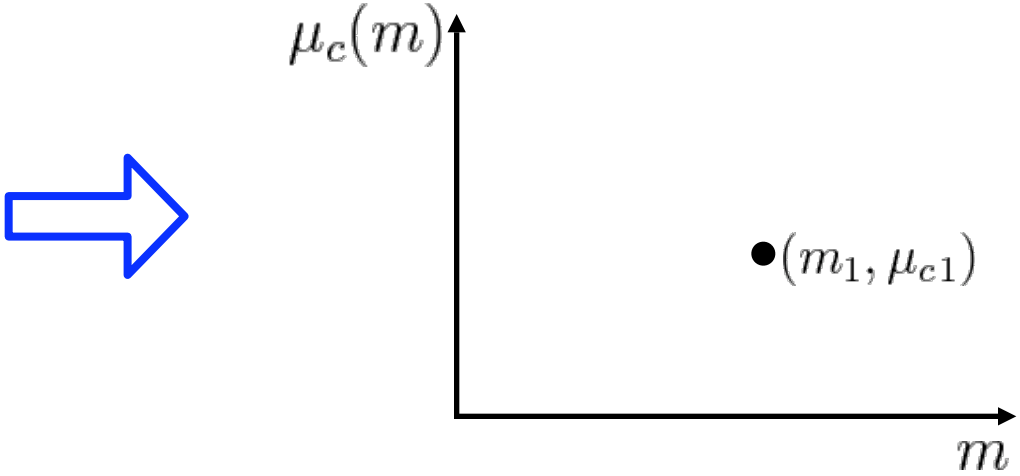

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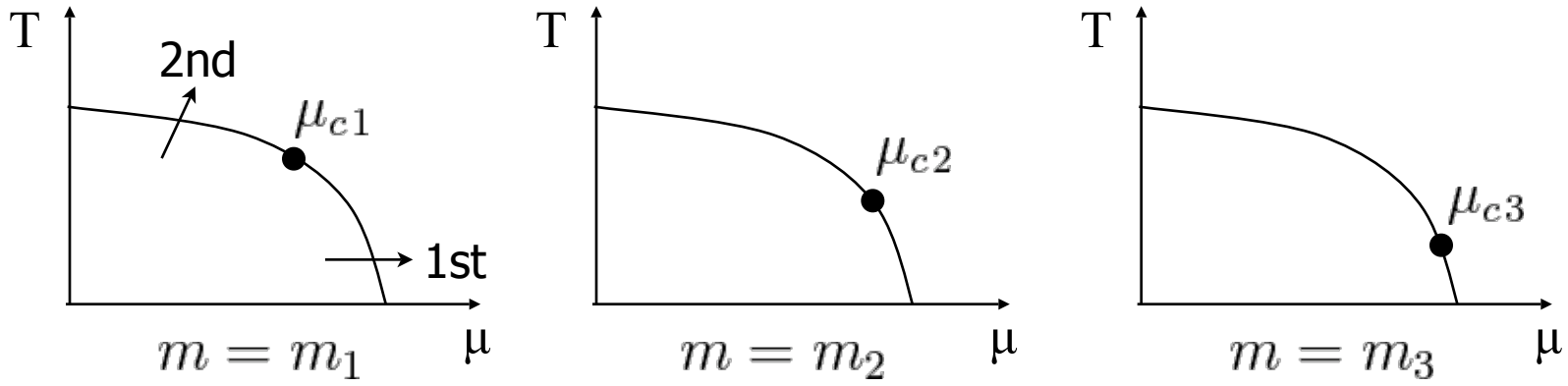

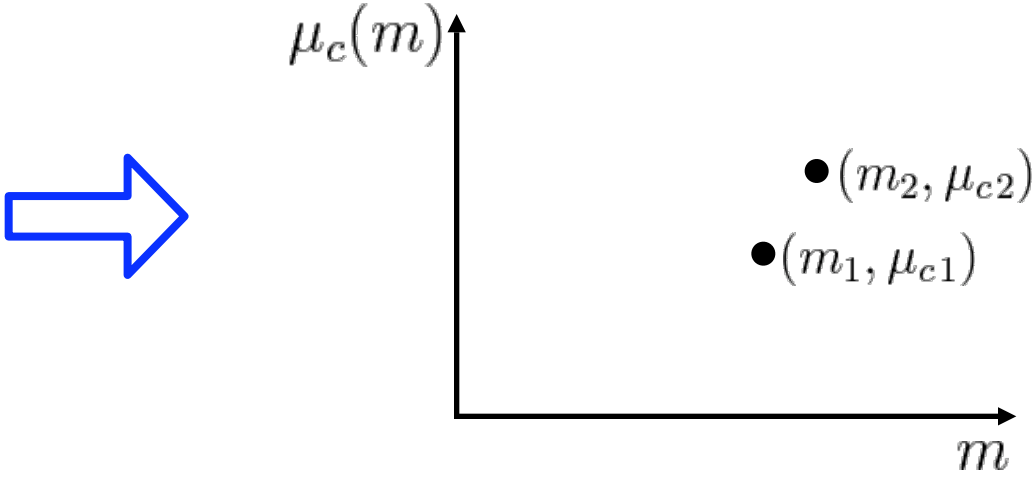

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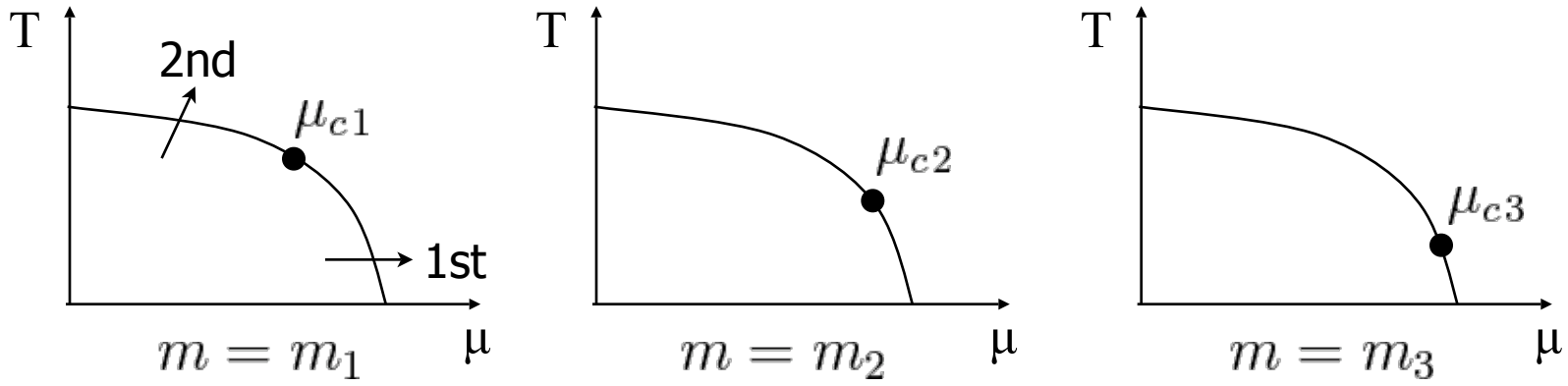

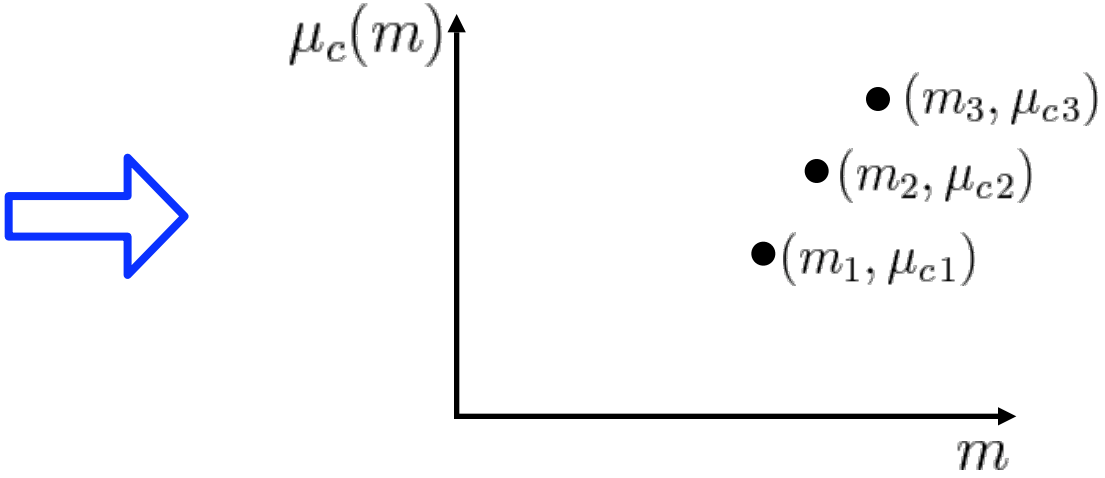

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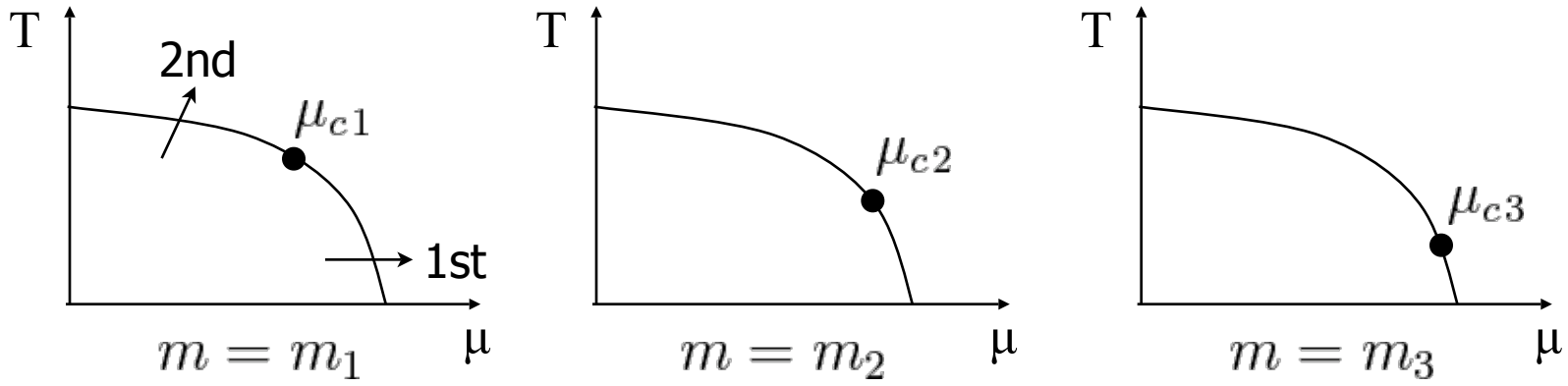
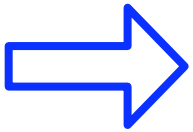
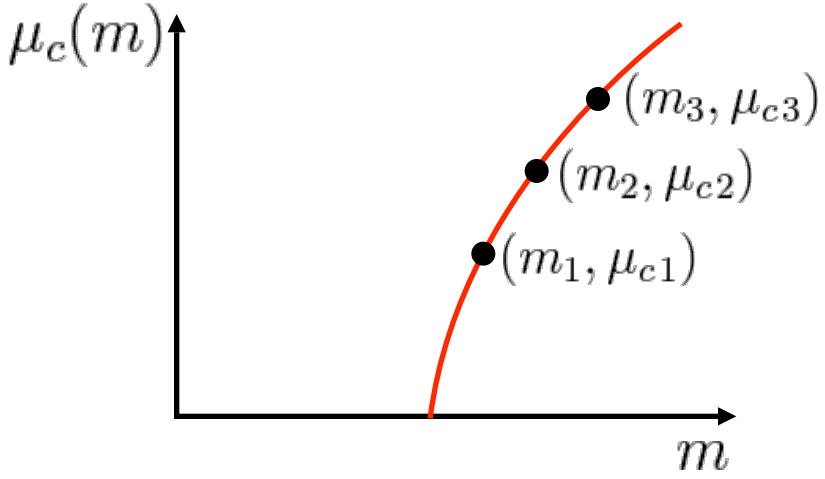
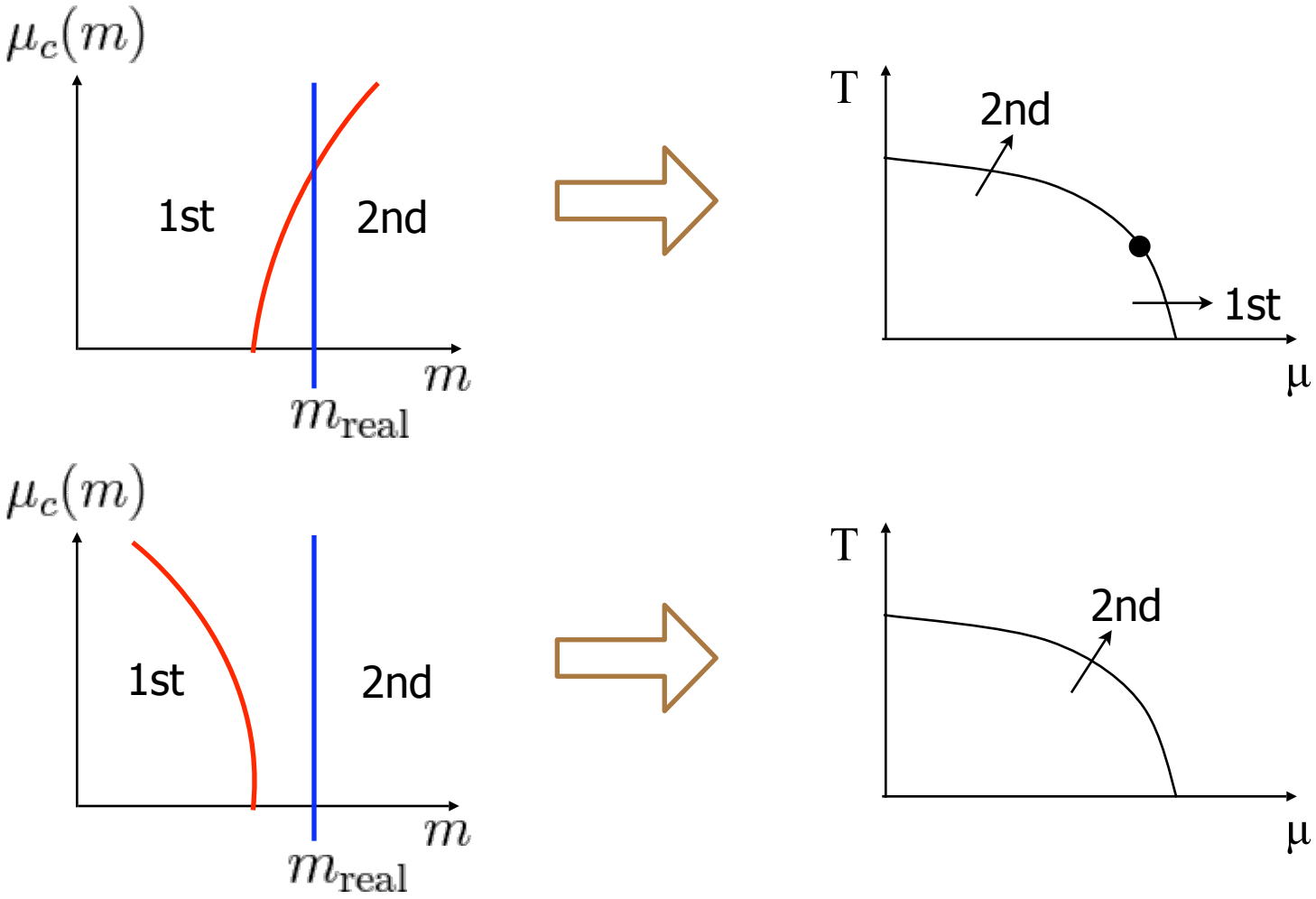






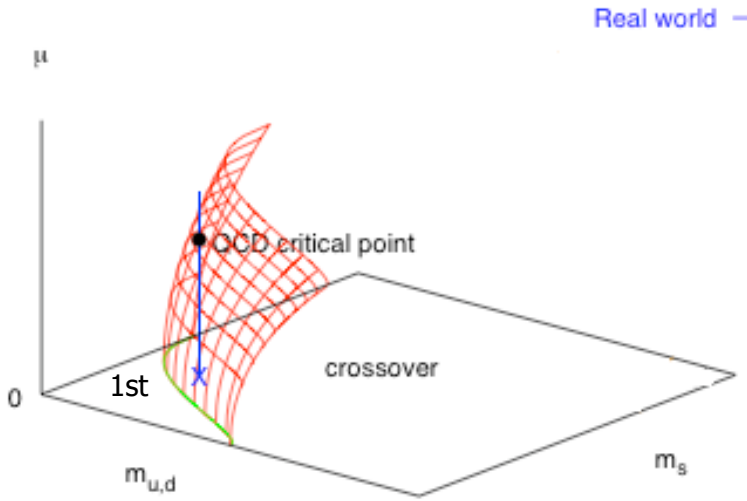
Critical curve

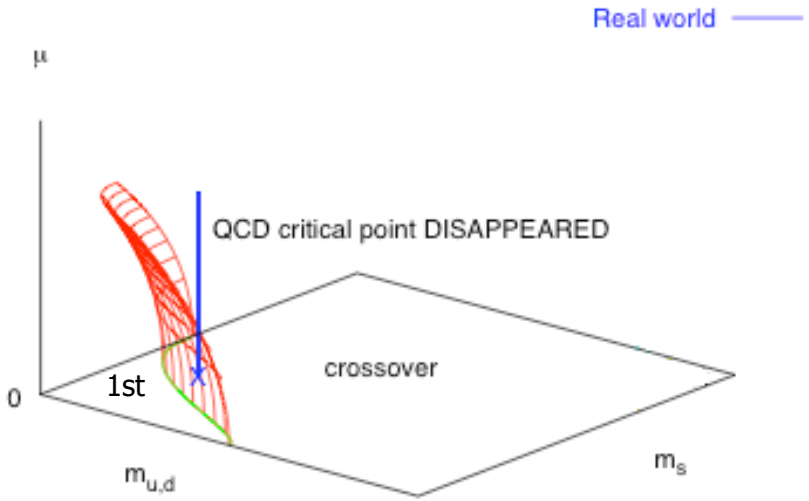
$m_u = m_{\rm real} \simeq 5.5{\rm MeV}$ (Real current mass)

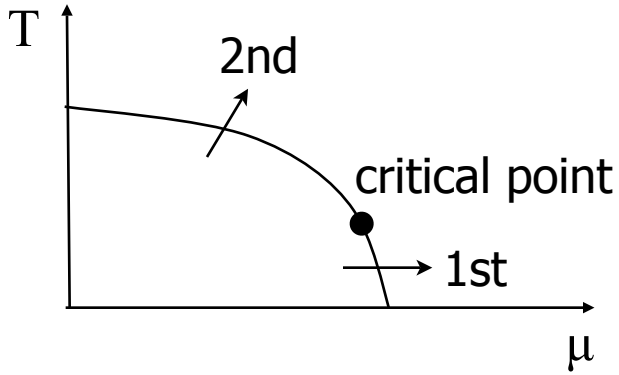


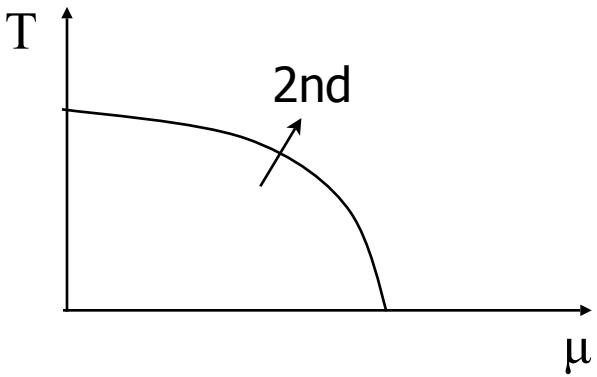
Critical surface

Real mass: $m_u = m_d = 5.5\text{MeV}$, $m_s = 135\text{MeV}$

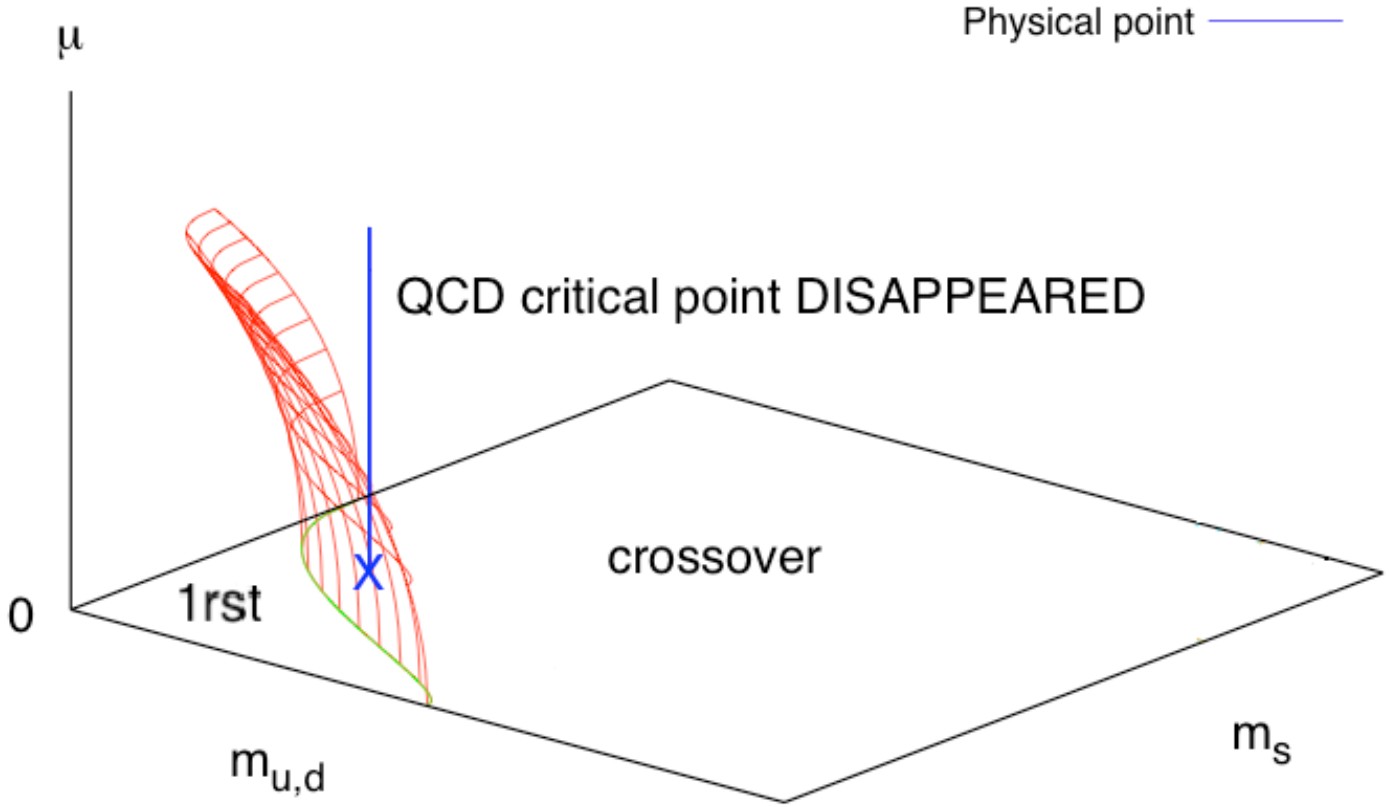






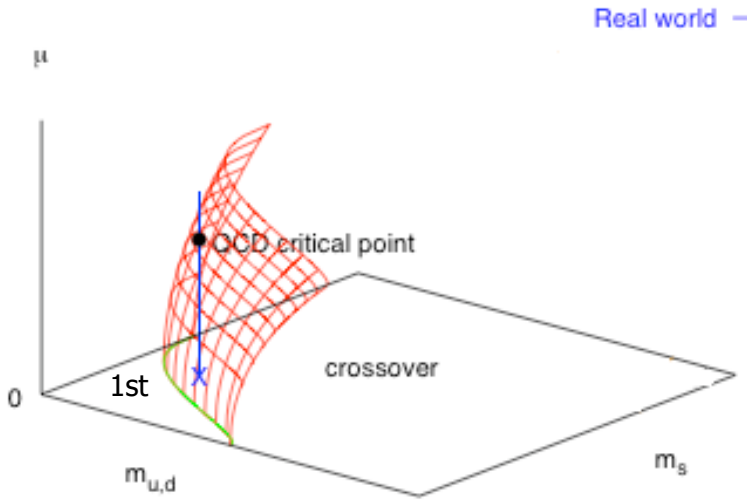


Lattice at finite chemical potential

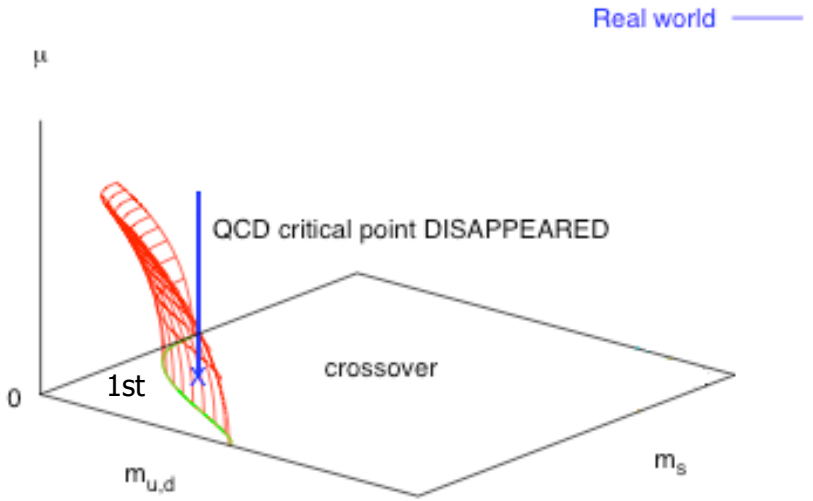


airXiv:0808.1096, Forcrand and Philipsen

Effective theory and Lattice study



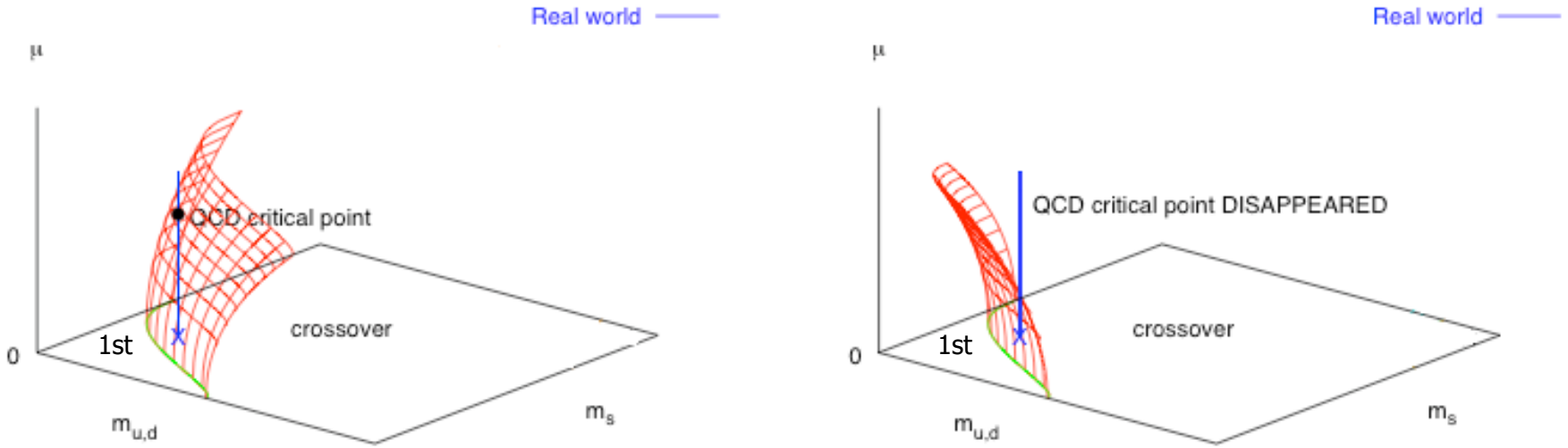
Nambu Jona-Lasinio



Recent Lattice study

by Forcrand & Philipsen(2008)

Effective theory and Lattice study



Nambu Jona-Lasinio ⟺ Recent Lattice study

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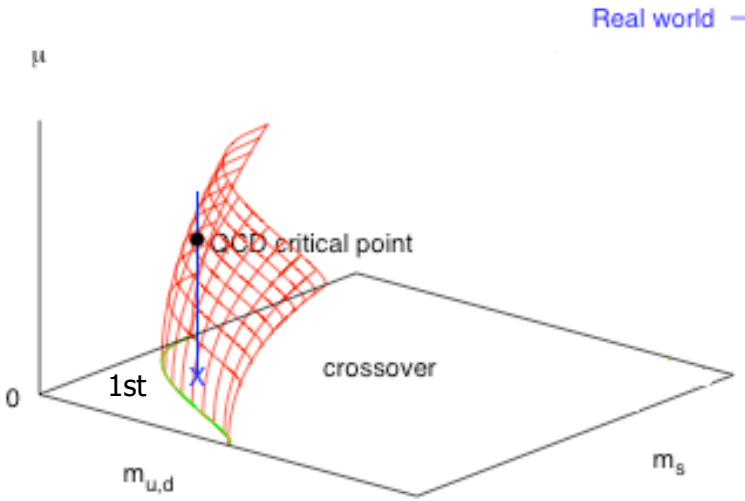
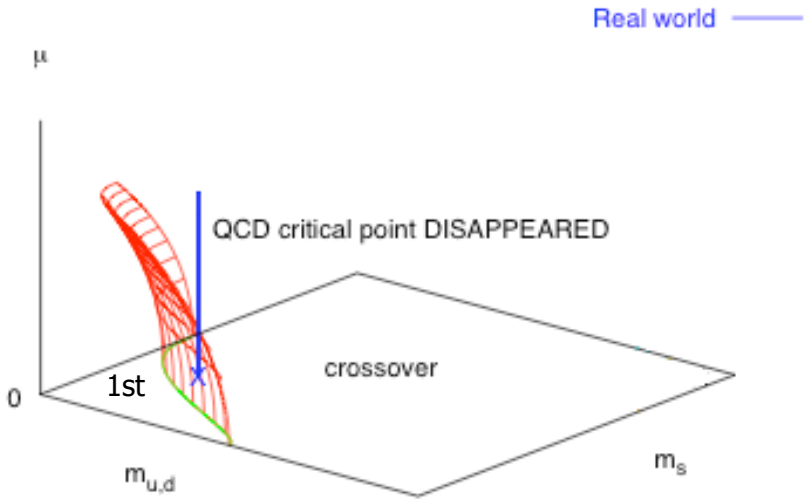
Effective theory and Lattice study





Nambu Jona-Lasinio

Recent Lattice study

by Forcrand & Philipsen(2008)

Not consistent!

Nambu Jona-Lasinio model

NJL Lagrangian

$$\mathcal{L} = \bar{q}(i\gamma\cdot\partial - \hat{m})q + \frac{g_S}{2}\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right] + g_D[\det \bar{q}_i(1-\gamma_5)q_j + h.c.]$$

$U_A(1)$ anomaly

$U_A(1)$ anomaly: Large ⟶ 1st order transition

Small ⟶ 2nd order transition

Our model

Assumption: $g_D(\mu) = g_{D0}\, e^{-(\mu/\mu_0)^2}$

μ_0: free parameter

This form is motivated by

ref. A. Abrikosov, Yad. Fiz. 37, 772 (1983),
T. Schafer, E. Shuryak, Rev. Mod. Phys. 70, 323 (1998).

Our model

Our Lagrangian

$$\mathcal{L} = \bar{q}(i\gamma\cdot\partial - \hat{m})q + \frac{g_S}{2}\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right]$$
$$+ g_{D0}\, e^{-(\mu/\mu_0)^2}\left[\det \bar{q}_i(1-\gamma_5)q_j + h.c.\right]$$

Density dependent $U_A(1)$ anomaly

Parameters

$\{g_s, g_{D0}, \Lambda\}$ determined by $\{m_\pi, f_\pi, m_{\eta'}\}$

Outline of the calculation

Lagrangian ⟶ Effective potential

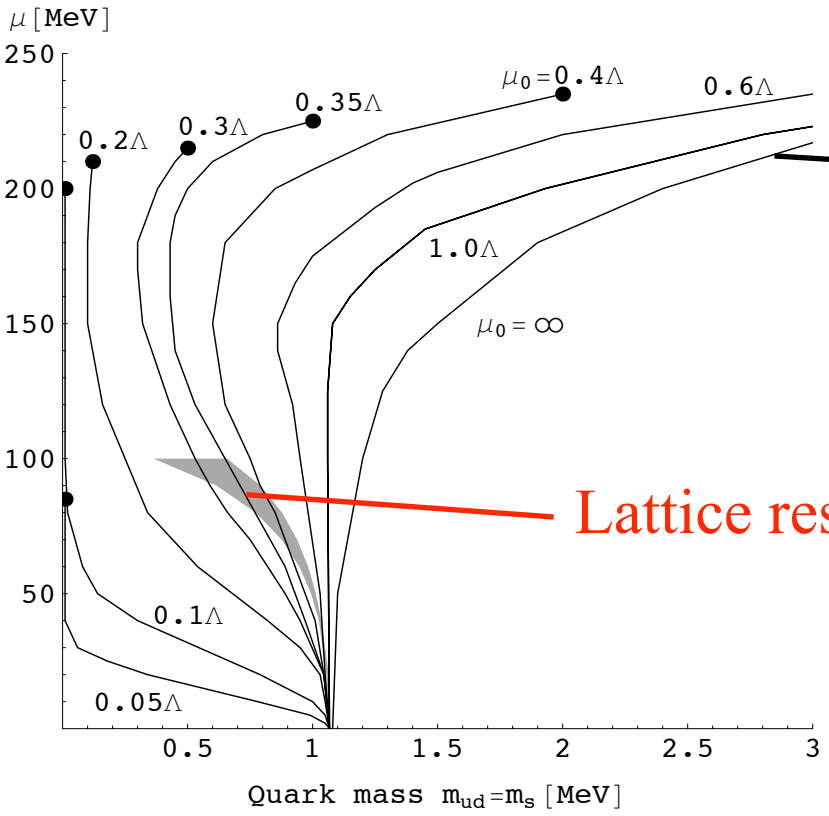
⟶ Gap equation

⟶ Critical point

Results (2D)

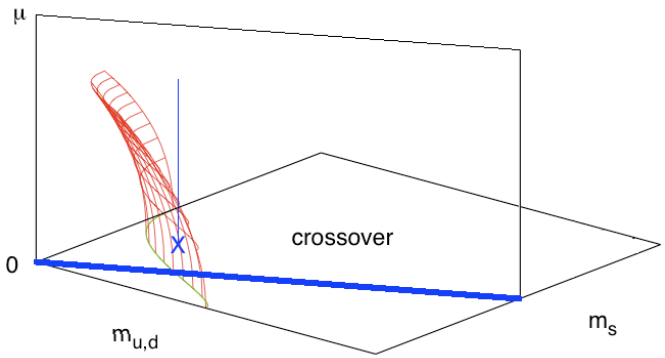
Free parameter : μ_0 (in $g_D(\mu) = g_{D0}\, e^{-(\mu/\mu_0)^2}$)

$\Lambda = 631\mathrm{MeV}$



$\mu_0 = \infty$ means constant g_D (i.e., traditional NJL)

Lattice result

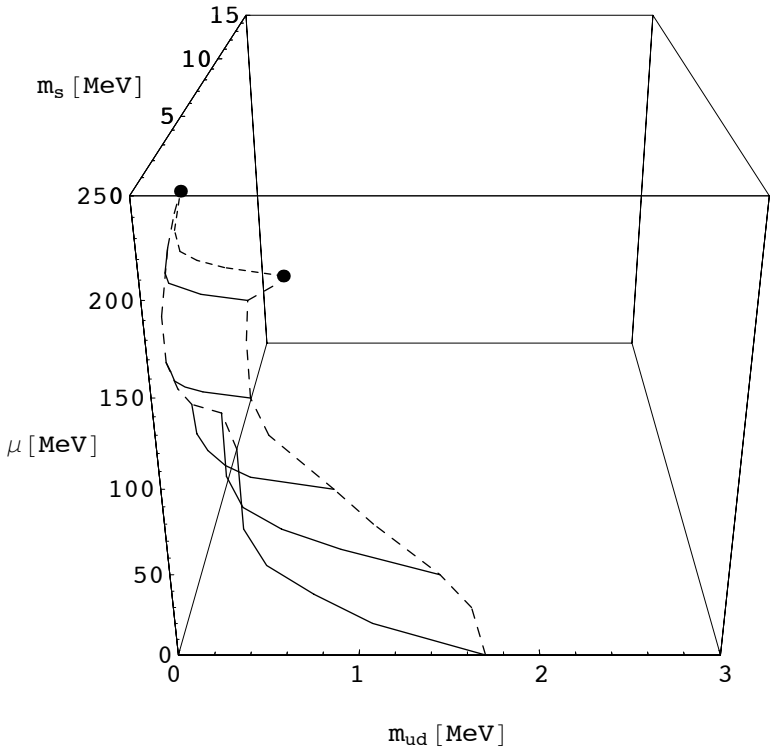


(along $m_u = m_d = m_s$ line)

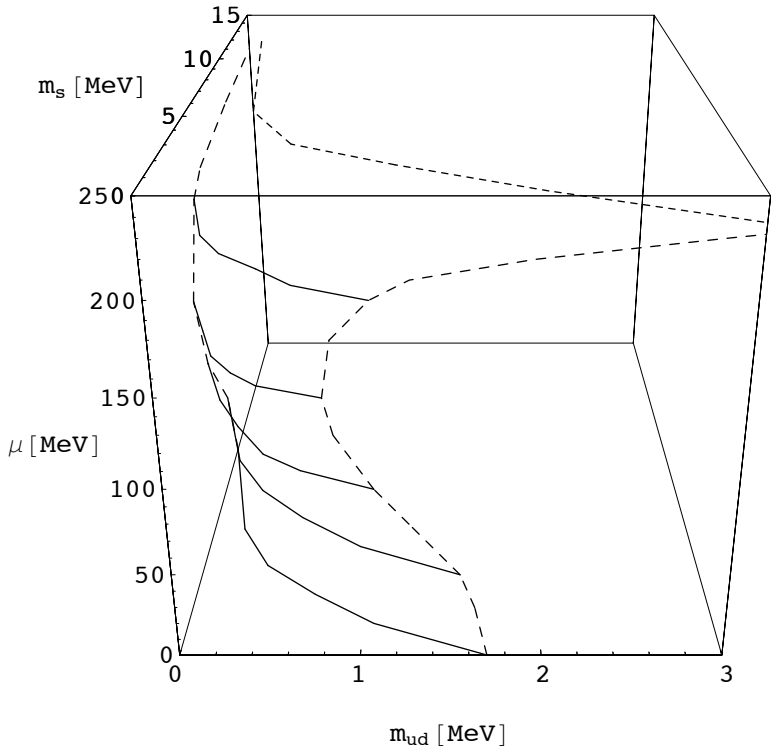
Results (3D)

Parameter : $\mu_0 (= 189 \sim 253 \mathrm{MeV})$

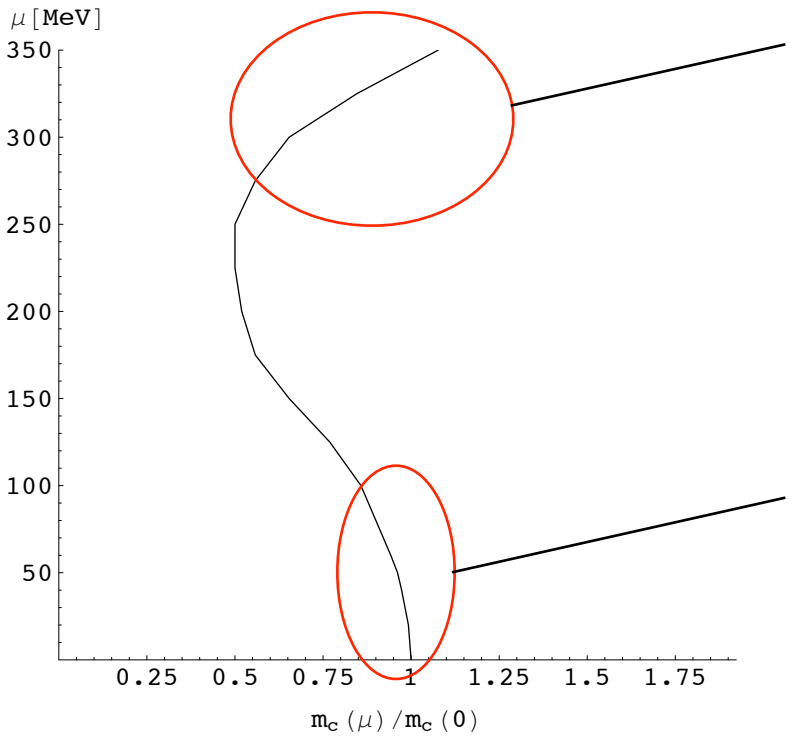
$\mu_0 = 189\mathrm{MeV}$



$\mu_0 = 253\mathrm{MeV}$



Consideration (1)

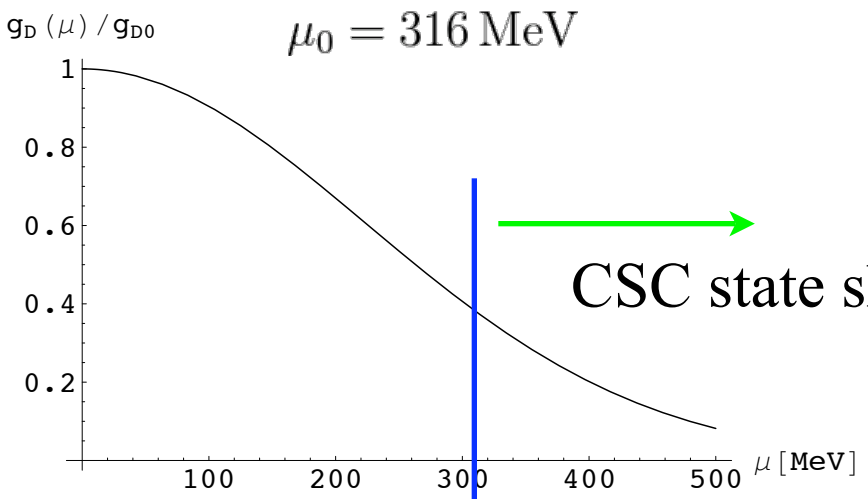


Backbend
due to density effect

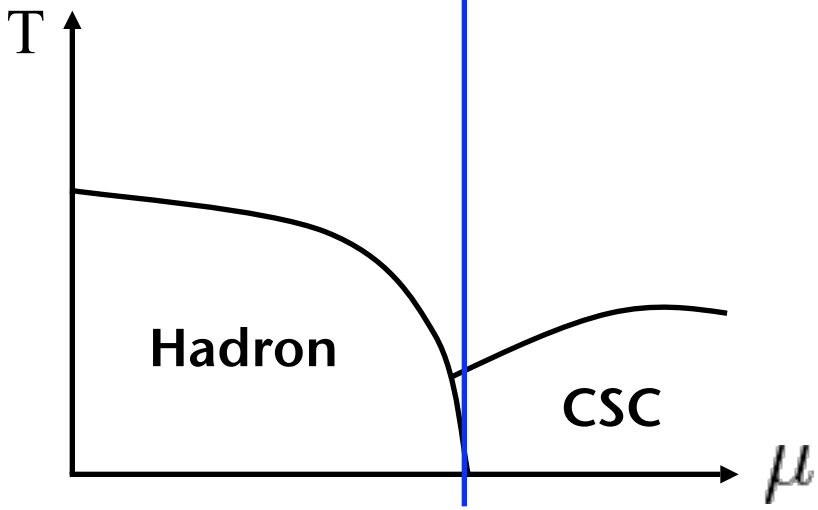
(consistent with former works,
K. Fukushima, arXiv:0809.3080.)

Shrinkage
consistent with Lattice!

Consideration (2)

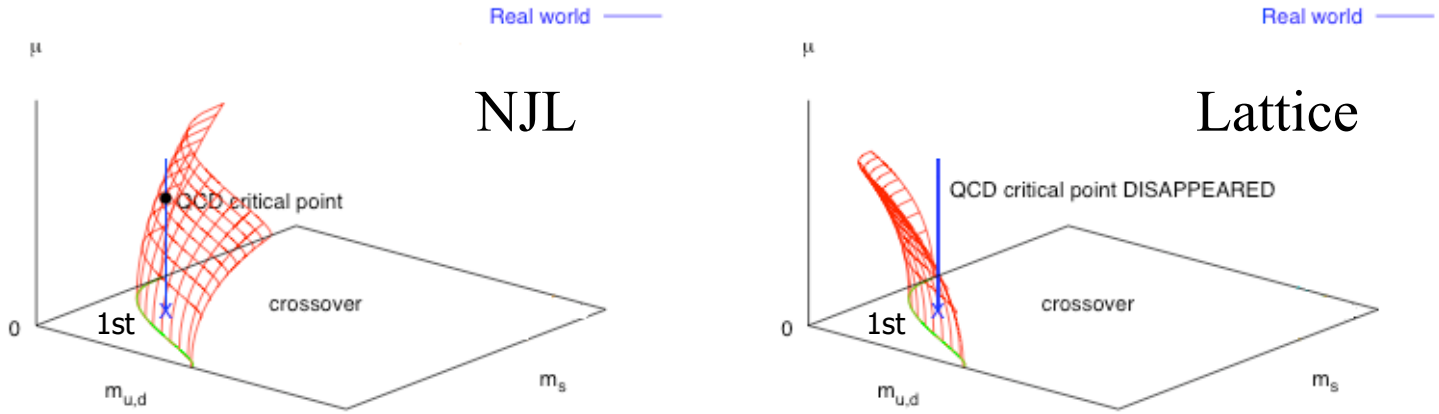


CSC state should dominate

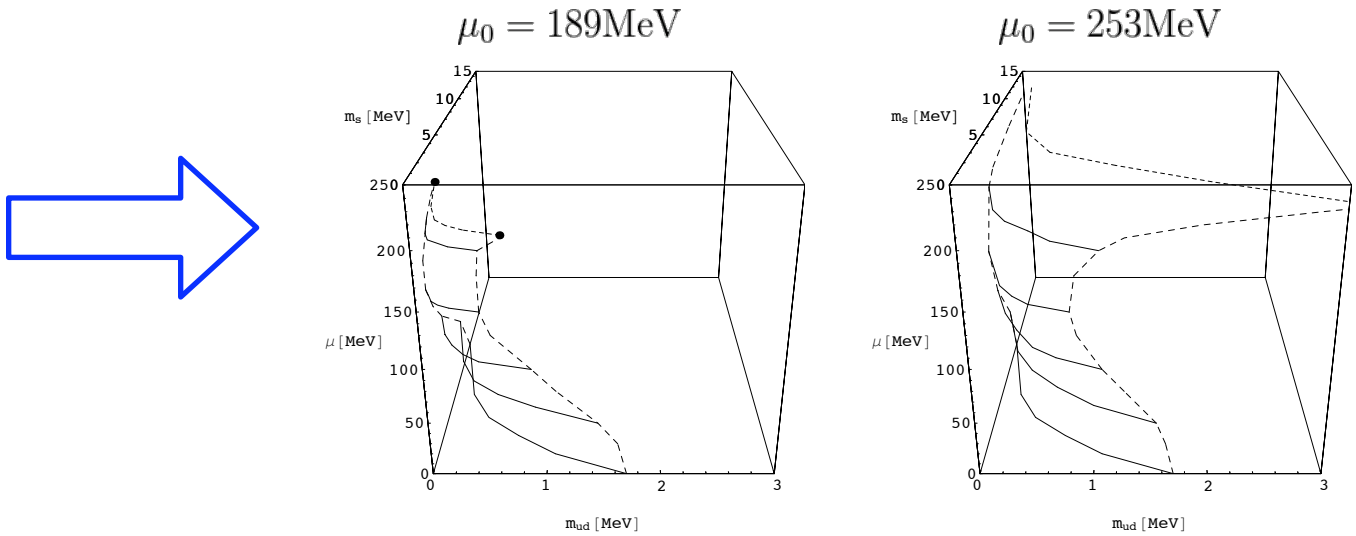


Summary

Inconsistency between traditional NJL and Lattice



We introduce density dependent $U_A(1)$ anomaly.



Discussion

Our model does show the shrinkage of the critical surface.

⟶ Consistent with lattice results!

Fitted parameter indicates the restoration of UA(1) anomaly at moderate baryon density.

⟶ Physically reasonable!

We can not make a strong statement on the order of transition.

⟶ Limitation of effective theory... 

Important Point:

We have introduced density dependence to $U_A(1)$ anomaly, and which seems to work really well!!

Future works: Try different parameter sets