New non-perturbative scheme for the running coupling constant

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Based on: arXiv:0902.3768.

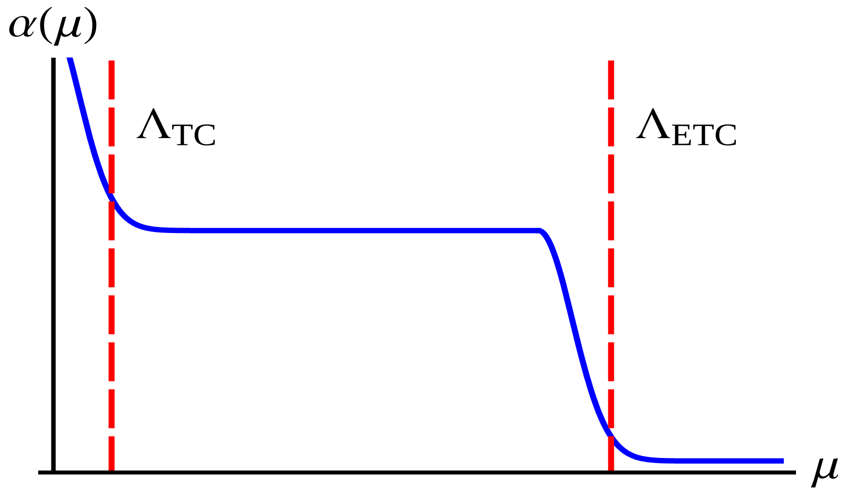
Outline

- Motivation
- The step-scaling method
- A new non-perturbative scheme

  $\longrightarrow$ The finite-volume Wilson-loop scheme

  $\longrightarrow$ Feasibility numerical study in quenched QCD
- Conclusion and outlook

Motivation
Walking technicolour



- Generates large anomalous dimension for $\bar{\psi}\psi$ to solve the FCNC problem.

- Modifies the relevant spectral function to elude the S-paramter criticism *a'la* Peskin and Takeuchi.

- $\Lambda_{\mathrm{ETC}}/\Lambda_{\mathrm{TC}} \sim 10^2$.

  $\longrightarrow$ Compared to the typical lattice size $L/a \sim 20$ in each direction.

Motivation
Technicolour in the twenty-first century

- Except for "TeV-QCD", technicolour has not been ruled out.
- Serious lattice calculations to support/kill technicolour
  - Hadron spectrum.

    L. Del Debbio, A. Patella and C. Pica
    S. Catterall and F. Sannino
    A. Hietanen *et al.*

  - Pheses of candidate walking theories.

    S. Catterall *at al.*
    T. DeGrand, B. Svetitsky and Y. Shamir

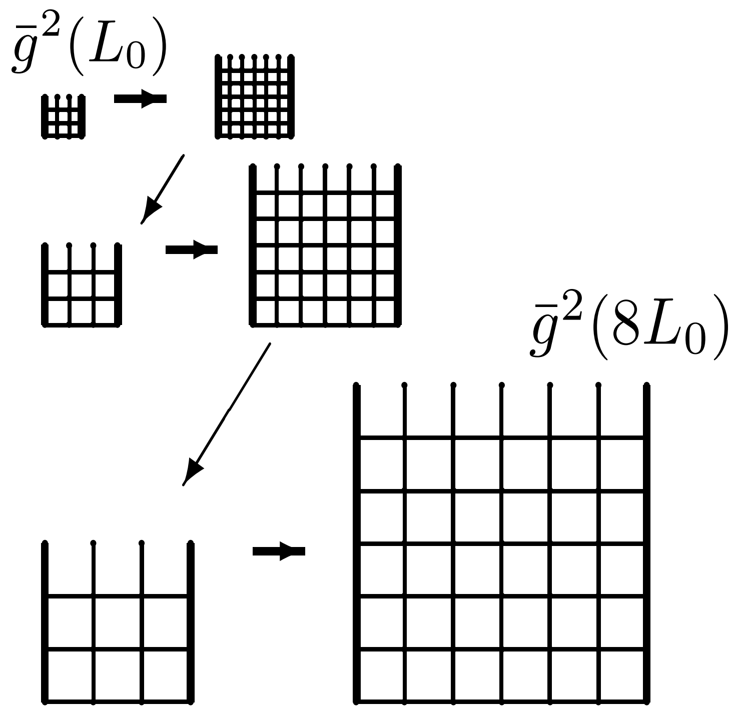
  - Spectrum of the Dirac operator.

    Z. Fodor *et al.*

  - The step-scaling method for the coupling constant.

    T. Appelquist, G. Fleming and E. Neil
    This work

The step-scaling method
The idea



- Extrapolate to the continuum limit at every step.
- Vary the scale by changing the dimensionful lattice size L_0.

The step-scaling method
Implementation: An example

1. Prepare lattices of size $\widehat{L_0}^4$:
$$\widehat{L_0} = L_0/a = 6, 8, 10, 12,$$
and tune the lattice spacing (via varying the bare coupling).

   $\longrightarrow$ The renormalised coupling $g(L_0)$ is the same on these lattices.

2. Double the lattice size to be
$$\widehat{L_0} = L_0/a = 12, 16, 20, 24,$$
and calculate $g(2L_0, a/L_0)$.

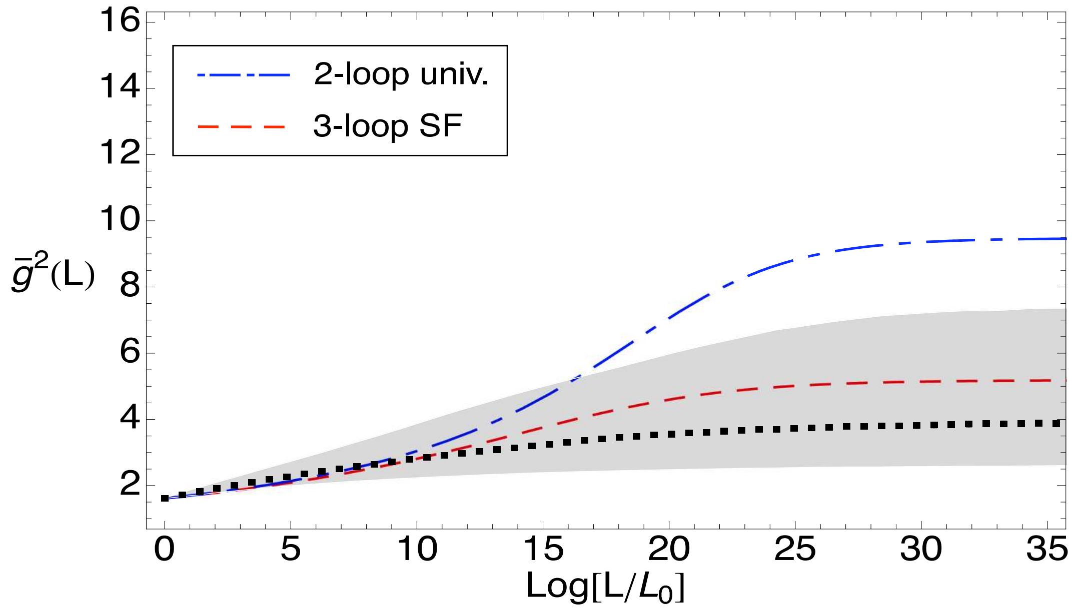
   $\longrightarrow$ Extrapolate to the continuum limit to get $g(2L_0)$.

3. Go back to the lattices in 1., and tune the lattice spacing.

   $\longrightarrow$ The renormalised coupling equals $g(2L_0)$ on all these lattices.

   $\longrightarrow$ Repeat 2. to obtain $g(4L_0)$.

The step-scaling method
Results for the Schödinger Functional scheme



T.Appelquist, G.Fleming and E.Neil, PRL100, 171607 (2008)

- $N_f = 12$.
- Scheme dependence?
- Controversy from the study of the Dirac operator eigenvalue spectrum?

A new non-perturbative scheme
General considreations

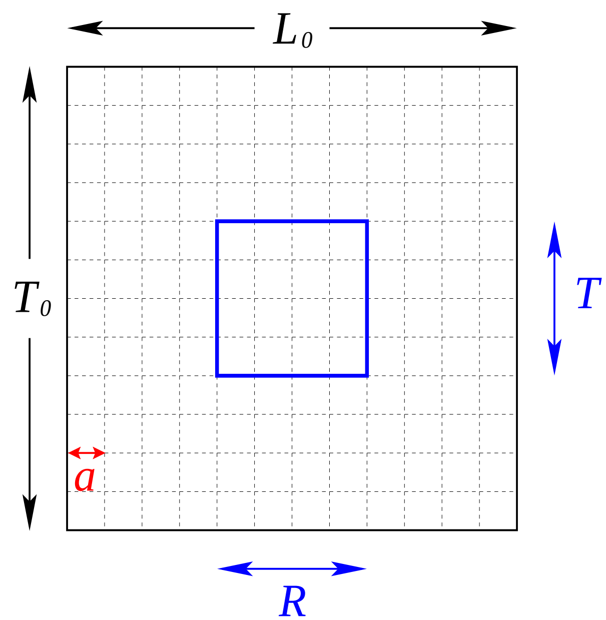
$$A_{\mathrm{LO}} = k \; g_0^2$$

$$A_{\mathrm{NP}}(\mu) = k \; g^2(\mu)$$

$$\Big\downarrow \quad \text{Lattice}$$

$$A_{\mathrm{NP}}(L,a) = k(a) \; g^2(L,a)$$

The finite-volume Wilson-loop scheme
The idea



- Set up $T_0 = L_0$ and $T = R$.

- When $a \to 0$, with fixed $\hat{r} \equiv (R + a/2)/L_0$, there is only one scale.

  $\longrightarrow$ Varying $\hat{r}$ corresponds to changing scheme.

The finite-volume Wilson-loop scheme
More details

- The Creutz Ratio (CR):

$$-R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T;L_0,T_0) \rangle |_{T=R,T_0=L_0} \xrightarrow{\text{LO}} k g_0^2 .$$

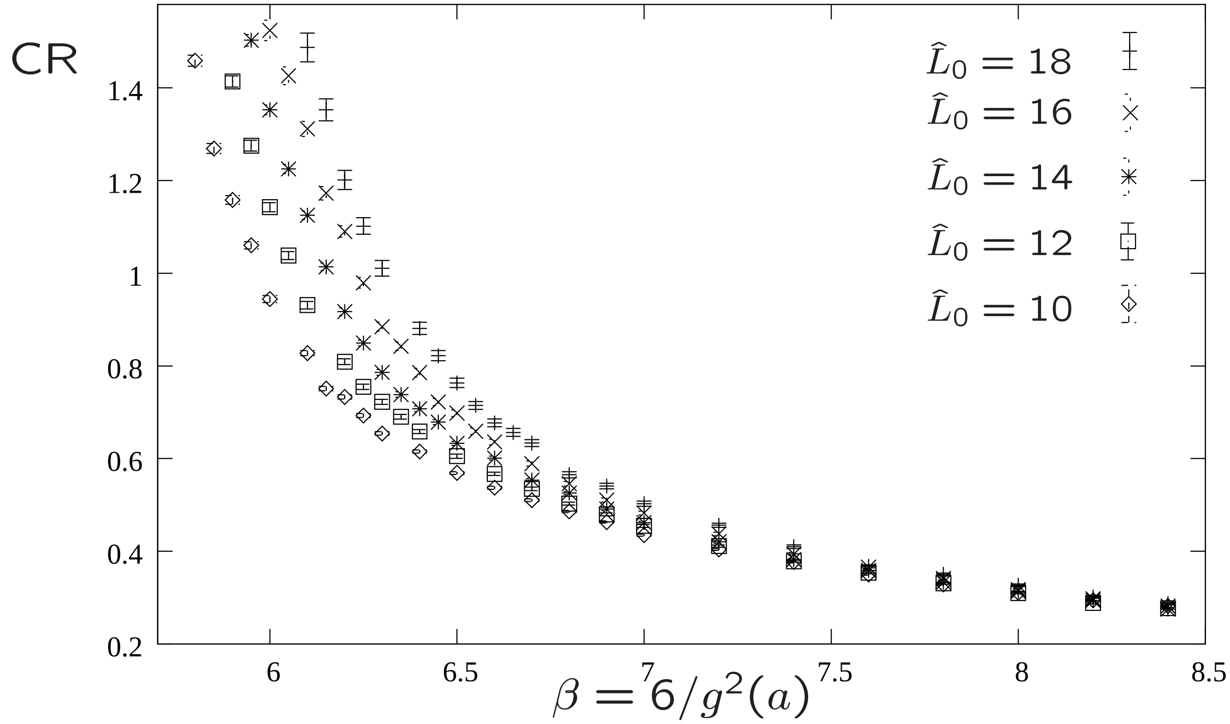
- On the lattice ($\hat{R} = R/a$ and $\hat{L}_0 = L/a$):

$$-(\hat{R}+1/2)\ \chi(\hat{R}+1/2, \hat{T}+1/2; \hat{L}_0) = -(\hat{R}+1/2)\ \ln \left[\frac{W(\hat{R}+1,\hat{T}+1;\hat{L}_0) W(\hat{R},\hat{T};\hat{L}_0)}{W(\hat{R}+1,\hat{T};\hat{L}_0) W(\hat{R},\hat{T}+1;\hat{L}_0)} \right] .$$

- The factor k can be calculated analytically (periodic BC)

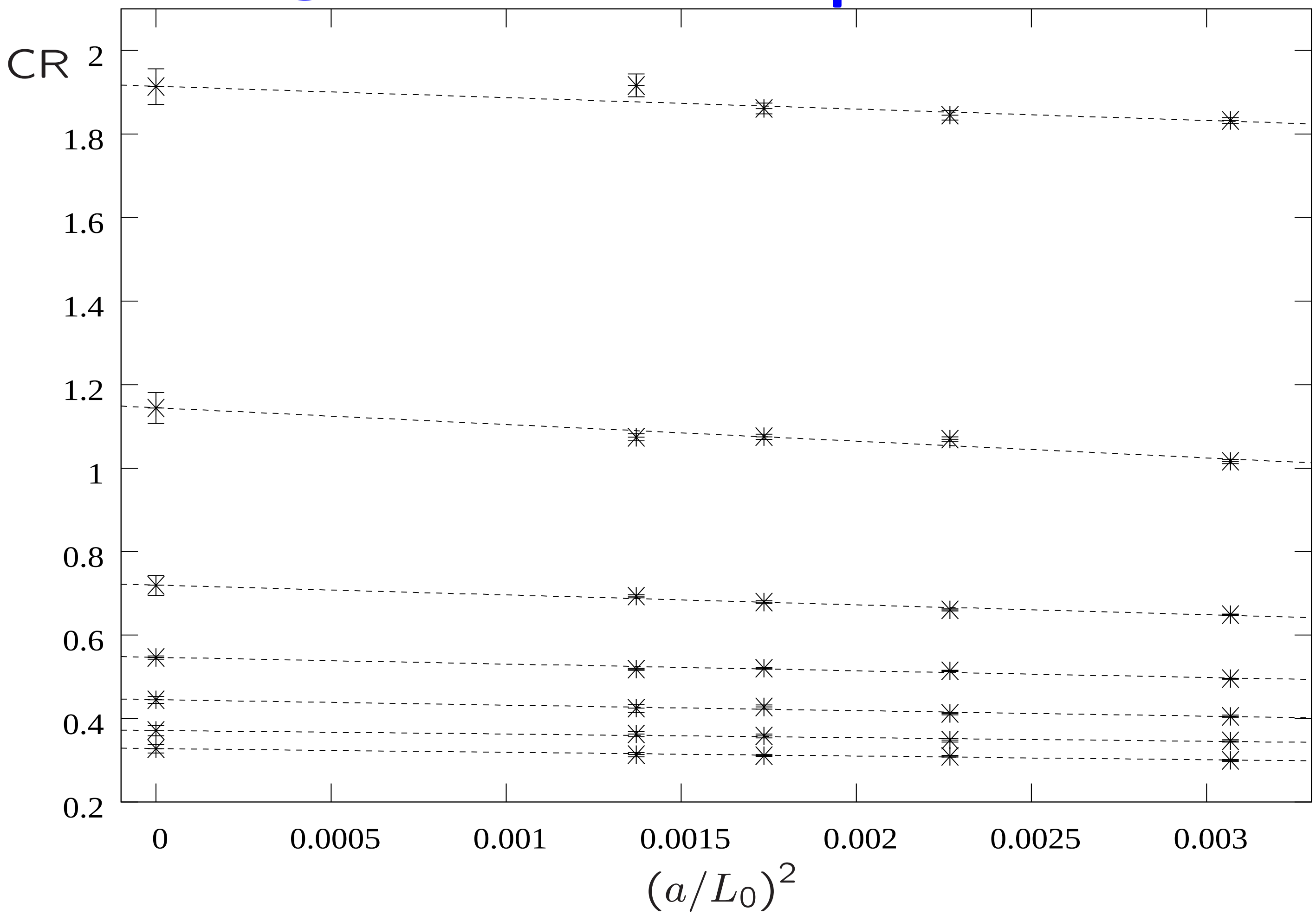
$$k = -R^2 \frac{\partial^2}{\partial R \partial T} \left[\frac{4}{(2\pi)^4} \sum_{n_\mu \neq 0} \left(\frac{\sin\left(\frac{\pi n_0 T}{L_0}\right)}{n_0} \right)^2 \frac{\mathrm{e}^{i \frac{2\pi n_1 R}{L_0}}}{n_0^2 + \vec{n}^2} \right]_{T=R} + \text{zero} - \text{mode contrib.}$$

Numerical test of the FV Wilson-loop scheme
Simulation parameters in quenched QCD

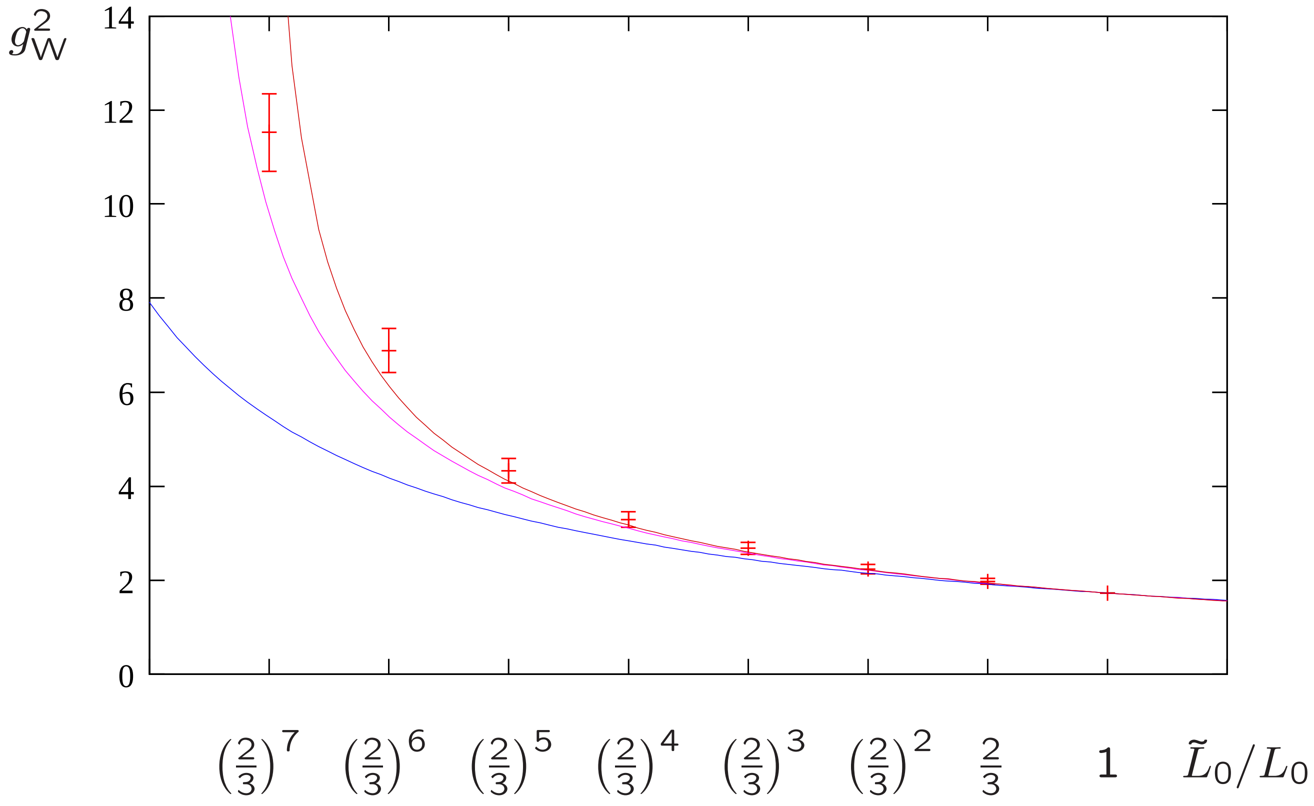


- Take CR $= 0.2871$ as the starting point (step 0, scale $\tilde{L}_0$).
- Take the step size $s = 1.5$.

Numerical test of the FV Wilson-loop scheme
Continuum extrapolation



Numerical test of the FV Wilson-loop scheme
Result in quenched QCD



Concluding remarks and outlook

- We have designed a new non-perturbative scheme for calculating the running coupling constant.

- We have demonstrated it is valid through numerical tests in quenched QCD.

- Setting the (quenched) scale, our preliminary result is:

$$\frac{\Lambda_{\mathrm{SF}}^{2-\mathrm{loop}}}{\Lambda_{\mathrm{W}}^{2-\mathrm{loop}}} \sim 1.78.$$

- Study of large-N_f gauge theories and the $\bar{\psi}\psi$ anomalous dimension is under way.

It is a new lattice research avenue.