



Eighth Particle Physics Phenomenology Workshop  
May 22, 2009

# PARTIALLY STRONG SCATTERINGS OF WEAK GAUGE BOSONS

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Based mainly on Cheung, CC, Yuan, PRD **78**, 051701 (2008)

# Outline

- Review of EWSB in SM
- Vector boson scatterings as a probe of EWSB
- Effects of an extended Higgs sector
- LHC signals of partially strong scatterings
- Summary



# Missing Piece of SM

- One important mission of LHC experiments is to scrutinize and understand the EWSB in elementary particle physics.
- In the SM, EWSB is achieved through the Higgs mechanism, which predicts the existence of a Higgs boson without further information on its mass.
- Finding the Higgs boson will not only provide us more insight into the origin of particle masses, but also help develop subsequent physics.

# Questions of Interest

- Suppose we detect only a light Higgs boson ( $m_h \approx 200$  GeV) and nothing else at the LHC during the first couple of years.
- Can we claim that we understand EWSB then?
- Is it the SM Higgs boson, or does it belong to a larger Higgs sector?
- In the latter case, what would be clear and direct ways to figure it out in experiments (assuming that we cannot probe and detect the other degrees of freedom directly for some reason)?



# Possible Answers

- Some theorists suggest to study Higgs decays in high precision in order to pin down its couplings to SM fermions (and find out whether it is SM-like).

Giudice, Grojean, Pomarol, Rattazzi 2007  
Mantry, Trott, Wise 2007; Randall 2007

- Another direction is to study its interactions with the weak gauge bosons through gauge boson scatterings at high energies.
- These two approaches are complementary to each other. In fact, the latter is more direct than the former.

# Unitarity Constraints

- General consideration of the  $S$ -matrix theory  
⇒ Scattering amplitude (or cross section) cannot grow with energy.
- Violation of the unitarity indicates the breakdown of an effective theory.  
⇒ A more complete theory must come in to save the situation.  
⇒ The scale of new physics is around or below the unitarity-violating scale.



# Unitarity Problem in Fermi Theory

- Consider the scattering of  $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$ . The cross section to the lowest order is given by

$$\sigma_{tot}(\nu_\mu e^- \rightarrow \nu_e \mu^-) = \frac{G_F^2}{\pi} s$$

where only the lowest partial wave ( $l=0$ ) contributes and unitarity in quantum mechanics requires

$$\frac{G_F^2}{\pi} s \leq \frac{\pi}{2E_{\text{cm}}^2} \Rightarrow E_{\text{cm}} \leq \left( \frac{\pi\sqrt{2}}{4G_F} \right)^{\frac{1}{2}} \simeq 310 \text{ GeV}$$

where  $E_{\text{cm}}$  is the scattering energy in the CM frame.

- This signals a breakdown scale for the theory.  
 $\Rightarrow$  New degrees of freedom (the  $W$  bosons in this case) must emerge at or around this scale to save the trouble.

# Minimal Higgs Model

- In the Higgs sector, the scalar potential is given by

$$\begin{aligned} V(\mathbf{w}, h) &= \frac{\lambda}{4} [(\mathbf{w}^2 + h^2) - v^2]^2 \\ &\rightarrow \frac{\lambda}{4} (\mathbf{w}^2 + h^2)^2 + \lambda v h (\mathbf{w}^2 + h^2) + \frac{m_h^2}{2} h^2 \end{aligned}$$

after  $h$  acquires a VEV, where  $m_h^2 = 2\lambda v^2$ .

[This mimics the pre-QCD  $\sigma$  model ( $\mathbf{w} \rightarrow \boldsymbol{\pi}$  and  $h \rightarrow \sigma$ ).]

- After introducing the  $SU(2)_L \times U(1)_Y$  gauge interactions, the massless Goldstone fields  $\mathbf{w}$  then become the longitudinal modes of the gauge bosons,  $W_L$  and  $Z_L$ , through the Higgs mechanism.



# Equivalence Theorem

Cornwall, Levin, Tiktopoulos 1974; Vayonakis 1976;  
Lee, Quigg, Thacker 1976; Chanowitz and Gaillard 1985;  
Gounaris, Kogerler, Neufeld 1986; He, Kuang, Li 1992, 1994

- At high energies, the longitudinal components of the weak bosons recall their identities as the Goldstone modes and interact according to the Higgs potential.
- At the tree level, the high-energy ( $E \gg M_W$ ) scattering amplitudes for longitudinally polarized  $W$ 's and  $Z$ 's are equivalent to those for the corresponding  $w$  and  $z$  Goldstone bosons:

$$\mathcal{M}(W_L(p_1), W_L(p_2), \dots)_U = \mathcal{M}(w(p_1), w(p_2), \dots)_F + \mathcal{O}(M_W/E) .$$

- The weak gauge boson scatterings at high energies thus reveal the nature of EWSB.

# Longitudinal Polarization

- In the CM frame of the scattering process ( $V = W$  or  $Z$ )

$$V_L(p_1)V_L(p_2) \rightarrow V_L(k_1)V_L(k_2)$$

the polarization of the weak gauge boson of  $V_L(p_1)$ , for example, can be written as (in high-energy expansion)

$$\epsilon_L^\mu(p_1) \simeq \frac{p_1^\mu}{M_W} - \frac{2M_W}{s} p_2^\mu ,$$

where  $s = (p_1+p_2)^2 = 4E^2$ .

$$\sim \frac{E}{M_W} \quad \sim \frac{M_W}{E}$$

- This is good for studying terms of  $O(E^0)$  or above.
- We use the exact expression for polarization vectors in all our numerical calculations.



# Partial-Wave Expansion

- The scattering amplitude can be decomposed into partial waves as

Lee, Quigg, Thacker 1977

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta) ,$$

where the  $J$ -th partial-wave coefficient may be written as

$$a_J = A_4 \left( \frac{E}{M_W} \right)^4 + A_2 \left( \frac{E}{M_W} \right)^2 + A_0 \left( \frac{E}{M_W} \right)^0 ,$$

where  $E$  is the scattering energy in the CM frame.

- The amplitudes do not grow with  $E$  in the SM because:
  - the  $A_4$  terms vanish among purely gauge diagrams; and
  - the cancellations of  $A_2$  terms involve the Higgs boson.
- The situation will become different if the electroweak sector is extended...

# Unitarity Violation

- Even within the SM, the tree-unitarity may be violated if the Higgs mass  $m_h$  is greater than  $\sim 1$  TeV.
  - $\Rightarrow$  large  $\lambda$  and thus quantum corrections;  
i.e., failure of Born approximation
  - $\Rightarrow$  strongly interacting Higgs sector
  - $\Rightarrow$  expect to see a spectrum of resonances as strong interactions in the GeV regime.
- On the other hand, if a light Higgs boson is discovered, it would indicate that weak interactions remain weak at all energies.  $\Leftarrow$  the case we are considering



# Partial Unitarity Violation

- In many extensions of the SM, several Higgs bosons participate in the EWSB together.
- We consider the scenario that only one of them is sufficiently light and detectable at colliders, whereas the others are too heavy to probe.
- The tree-unitarity may be violated at energies between the light Higgs boson and the heavy ones, for the scattering cross sections have effectively  $(E/M_W)^2$  growth in the intermediate scale [no  $(E/M_W)^4$  terms still].

# Example: THDM

- Take as one example the two-Higgs doublet model (THDM) where two scalar doublets are simultaneously involved in the EWSB.
- The  $\Phi VV$  couplings in this case are:

	SM	THDM
$g_{hVV}$	$g_{hVV}^{\text{SM}}$	$g_{hVV}^{\text{SM}} \sin(\beta - \alpha)$
$g_{HVV}$	0	$g_{hVV}^{\text{SM}} \cos(\beta - \alpha)$

- In general, we parameterize the suppressed coupling as
 
$$g_{hVV} = \sqrt{\delta} g_{hVV}^{\text{SM}}, \quad (\delta < 1)$$
 and assume that the other heavy degrees of freedom decouple for the illustration purpose.

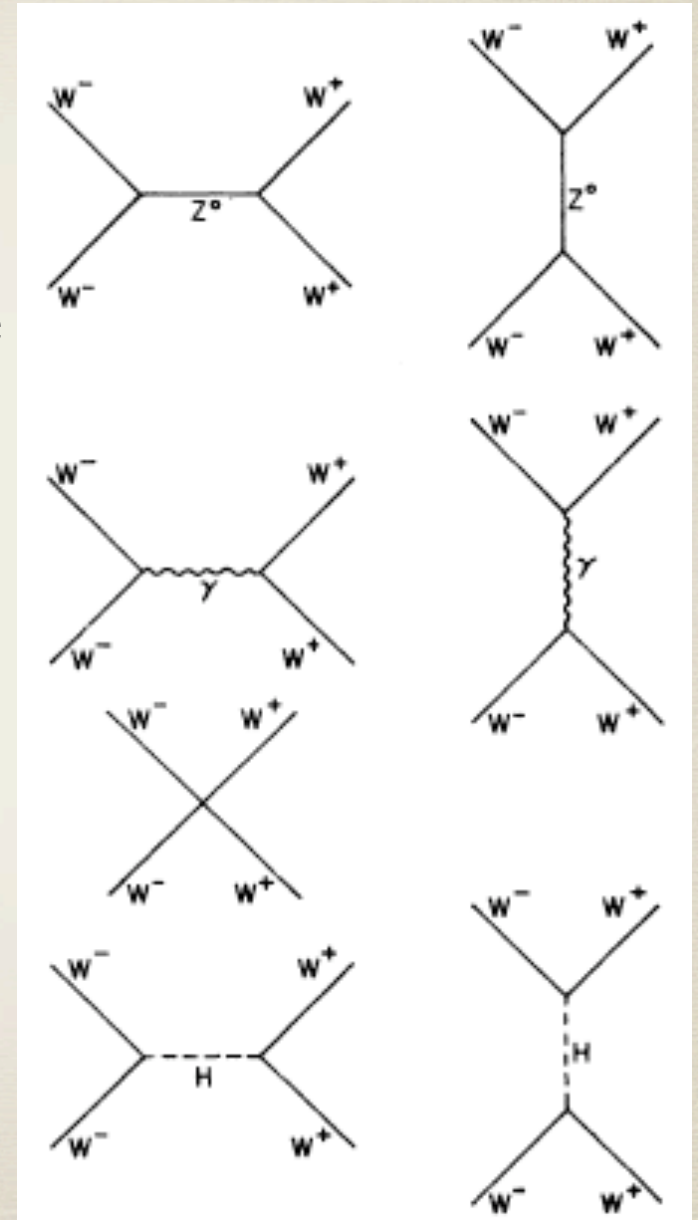


# Remarks

- Using the conventional  $\gamma\gamma$  and  $b\bar{b}$  decay modes of the light Higgs boson to hunt for new physics becomes difficult.
- The  $\gamma\gamma$  mode is suppressed when  $g_{hWW}$  is smaller than its SM value.
- The  $b\bar{b}$  mode also suffers from the reduced  $g_{hWW}$  coupling because the associate production of  $W^\pm h$  gets smaller.
- While these modes become less useful, the  $W_L W_L$  scattering enjoys its partial growth.

$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$$

- As an explicit example, consider this process in the SM in the  $s \gg m_h^2, M_W^2$  limit.
- Tree-level Feynman diagrams in the unitarity gauge:
  - 1 four-point interaction;
  - $Z$  and  $\gamma$  in  $s$  and  $t$  channels; and
  - Higgs boson in  $s$  and  $t$  channels.
- Other  $V_L V_L \rightarrow V_L V_L$  scatterings have similar structures.





$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$$

- Individual amplitudes of gauge diagrams:

$$i\mathcal{M}_4 = i \frac{g^2}{4M_W^4} \left[ s^2 + 4st + t^2 - 4M_W^2(s+t) - \frac{8M_W^2}{s} ut \right]$$

$$i\mathcal{M}_t^{\gamma+Z} = -i \frac{g^2}{4M_W^4} \left[ (s-u)t - 3M_W^2(s-u) + \frac{8M_W^2}{s} u^2 \right]$$

$$i\mathcal{M}_s^{\gamma+Z} = -i \frac{g^2}{4M_W^4} \left[ s(t-u) - 3M_W^2(t-u) \right]$$

- Individual diagrams grow like  $(E/M_W)^4$ !
- The sum of them nicely cancel with each other to remove such a divergence.

$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$$

- However, there is still an  $O((E/M_W)^2)$  divergence in the sum, which needs a sufficiently light Higgs boson to cure:

$$i\mathcal{M}^{\text{gauge}} = -i \frac{g^2}{4M_W^2} u + \mathcal{O}((E/M_W)^0), \quad \sim \left(\frac{E}{M_W}\right)^2$$

$$i\mathcal{M}^{\text{Higgs}} = -i \frac{g^2}{4M_W^2} \left[ \frac{(s - 2M_W^2)^2}{s - m_h^2} + \frac{(t - 2M_W^2)^2}{t - m_h^2} \right]$$

$$\simeq i \frac{g^2}{4M_W^2} u + \mathcal{O}((E/M_W)^0).$$

$\Rightarrow$  complete  $(E/M_W)^2$  cancellation

- The success of SM is thus seen to rely on nice relations among gauge bosons couplings (due to gauge invariance) and a suitable Higgs boson (depending on EWSB structure).



$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$$

- However, the story changes dramatically if  $g_{hVV} = \sqrt{\delta} g_{hVV}^{\text{SM}}$  as assumed:

$$i\mathcal{M}^{\text{gauge}} = -i \frac{g^2}{4M_W^2} u + \mathcal{O}((E/M_W)^0) ,$$

$$i\mathcal{M}^{\text{Higgs}} = -i \delta \frac{g^2}{4M_W^2} \left[ \frac{(s - 2M_W^2)^2}{s - m_h^2} + \frac{(t - 2M_W^2)^2}{t - m_h^2} \right]$$

$$\simeq i \delta \frac{g^2}{4M_W^2} u + \mathcal{O}((E/M_W)^0) .$$

$\Rightarrow$  only *partial*  $(E/M_W)^2$  cancellation

- This gives rise to the “bad” high-energy behavior in the scattering cross section.

# Scattering Channels

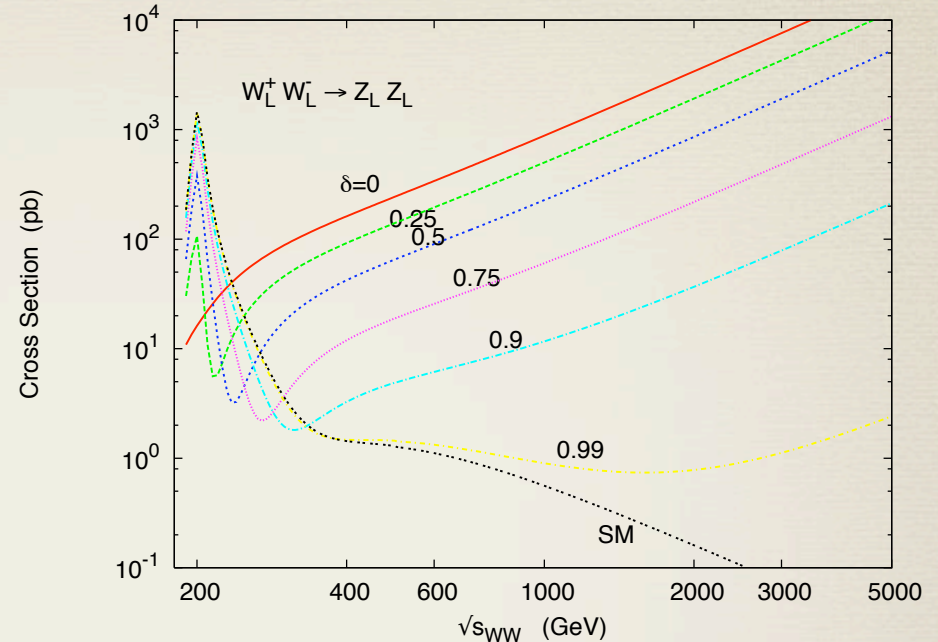
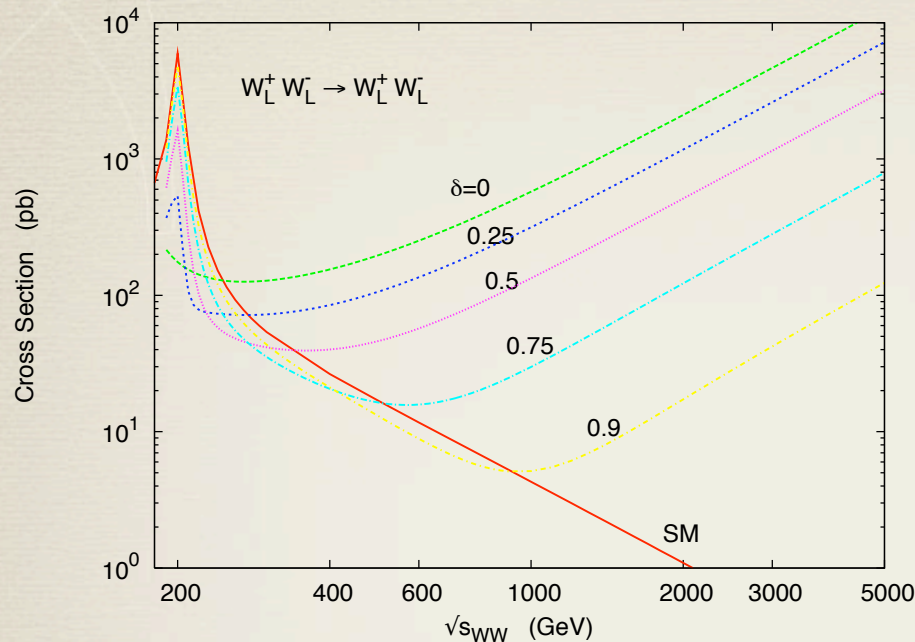
- Channels being studied:

Channel	Gauge	Higgs
$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$	x,s,t	s,t
$W_L^+ W_L^- \rightarrow Z_L Z_L$	x,t,u	s
$Z_L Z_L \rightarrow Z_L Z_L$	—	s,t,u
$W_L^\pm Z_L \rightarrow W_L^\pm Z_L$	x,s,u	t
$W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm$	x,t,u	t,u

- Cross sections of the first two resonant channels grow with energy as it goes above the light Higgs boson mass.
- Cross sections of the last two non-resonant channels increase in a less dramatic way.
- The  $Z_L Z_L \rightarrow Z_L Z_L$  channel is suppressed from SM by  $\delta$  because it is purely Higgs-mediated.



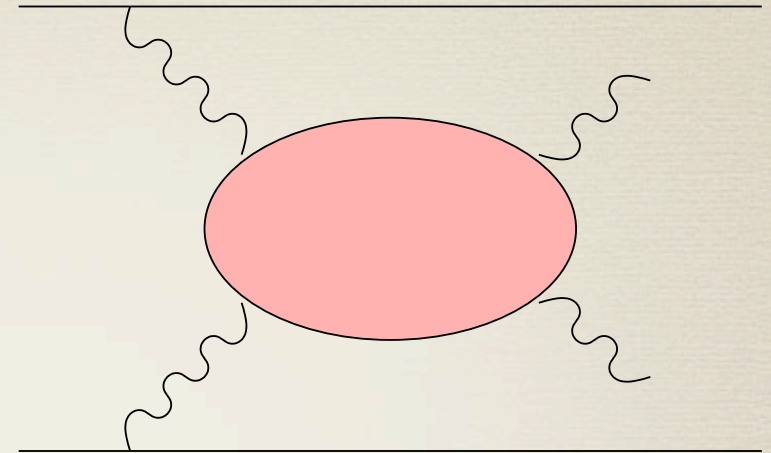
# Cross Sections



- Scattering cross sections of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  (L) and  $W_L^+ W_L^- \rightarrow Z_L Z_L$  (R) as functions of the scattering energy.
- Assume  $m_h = 200$  GeV and an angular cut  $|\cos\theta| \leq 0.8$ .
- The turn-over effect is different from SM both qualitatively and quantitatively, even if effects of heavy Higgs bosons of TeV masses are included.

# Effective $W$ Approximation

- At LHC, the weak gauge bosons in the initial state are radiated off the jets from the colliding protons.
- We employ the so-called effective  $W$  approximation (EWA)



$$f_{q \rightarrow W_L}(x) \sim \frac{1-x}{x}$$

Dawson 1985

up to some coupling factors depending upon the types of the quark and the gauge boson ( $x = \text{energy fraction}$ ).

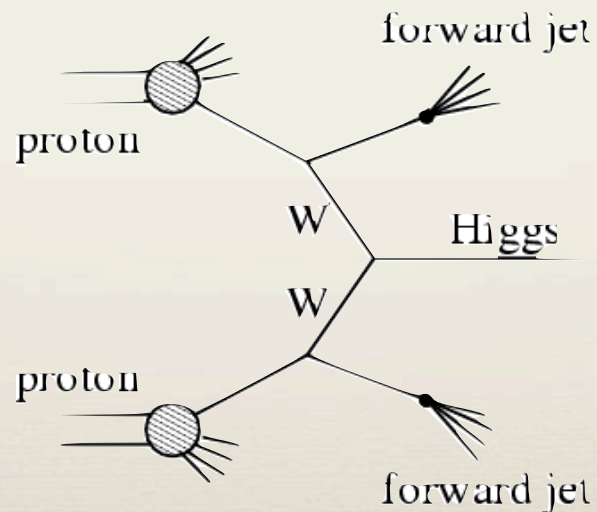
- The same approximation is used for the  $Z$  boson as well.
- The radiation probability peaks when  $x \rightarrow 0$  and vanishes when  $x \rightarrow 1$ .  $\Rightarrow$  most energy stays with jets



# Enhancing S/B

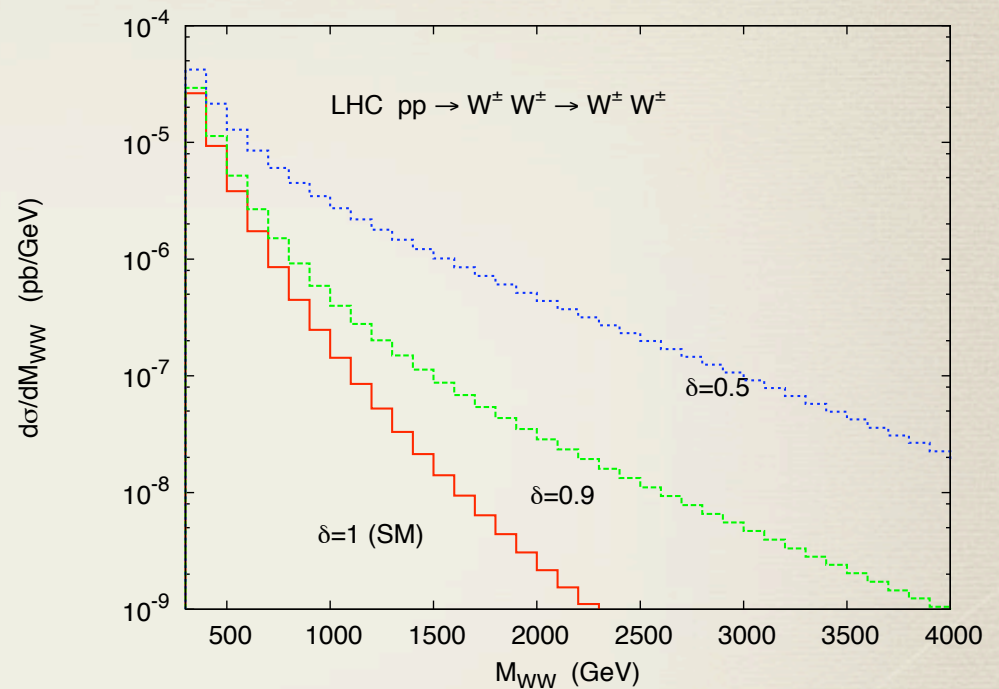
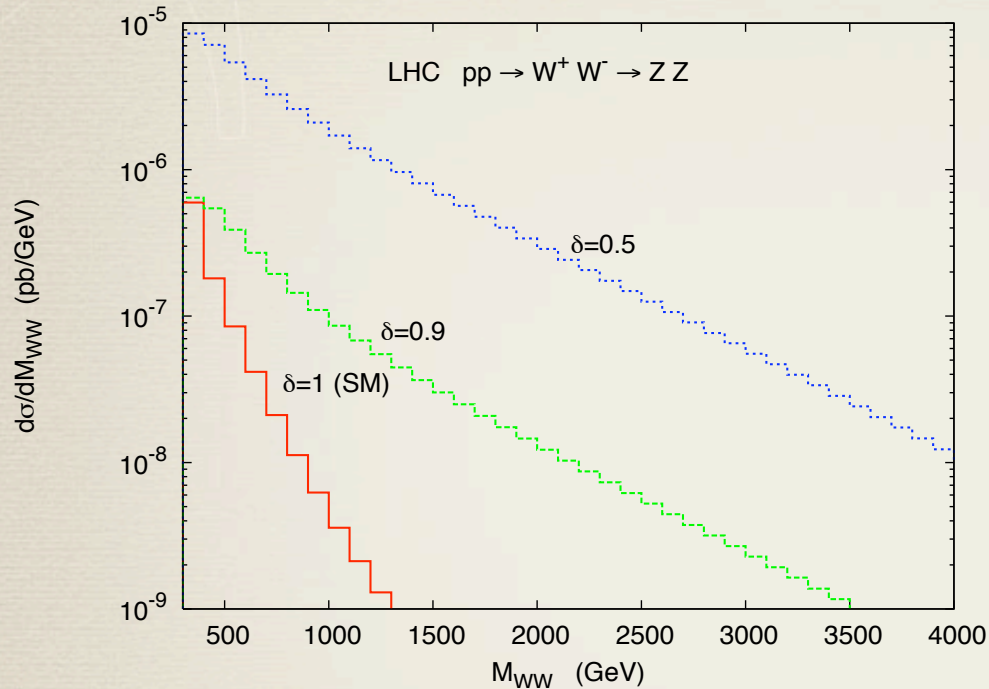
Bagger et al 1994, 1995

- A good part of initial energy is carried away by the jets.
  - ⇒ energetic forward jets
  - ⇒ large rapidity gap ( $|\eta| > 2$ ) [meaning little hadronic activity in central region]
  - ⇒ use central jet veto and forward jet tagging
- A very similar idea had been proposed for Higgs production through VBF.



# Diff. x-sec @ LHC

- Invariant mass distribution using naive EWA:



- The difference from the SM can be significant (even for  $\delta$  as large as 0.9), provided that the UV-completing degrees of freedom are sufficiently heavy.



# LHC Signals

- Study the leptonic final states of the above-mentioned scattering channels to avoid QCD background.
- Focus on  $WW \rightarrow \ell\nu\ell\nu$  and  $ZZ \rightarrow \ell^+\ell^-\ell^+\ell^- + \ell\ell\nu\nu$

Table 1: Event rates for longitudinal weak gauge boson scattering at the LHC with a yearly luminosity of  $100 \text{ fb}^{-1}$  using the EWA for  $\delta = 1$  (SM), 0.9, 0.5 and 0 (No Higgs). Branching ratios for the leptonic final states are summed for  $\ell = e$  and  $\mu$ . We set  $m_h = 200 \text{ GeV}$  and  $M_{WW}^{\min} = 300 \text{ GeV}$ .

Subprocess	Number of Events			
	$\delta = 1$ (SM)	0.9	0.5	0 (No Higgs)
$W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm \rightarrow \ell^\pm \nu \ell^\pm \nu$	21	26	57	118
$W_L^\pm W_L^\mp \rightarrow W_L^\pm W_L^\mp \rightarrow \ell^\pm \nu \ell^\mp \nu$	8	7	17	67
$W_L^\pm Z_L \rightarrow W_L^\pm Z_L \rightarrow \ell^\pm \nu \ell^+ \ell^-$	4	5	13	33
$W_L^+ W_L^- \rightarrow Z_L Z_L \rightarrow \ell^+ \ell^- \ell^+ \ell^-$	0.04	0.12	2	9
$W_L^+ W_L^- \rightarrow Z_L Z_L \rightarrow \ell^+ \ell^- \nu \bar{\nu}$	0.25	0.74	12	50
$Z_L Z_L \rightarrow Z_L Z_L \rightarrow \ell^+ \ell^- \ell^+ \ell^-$	0.4	0.32	0.08	0
$Z_L Z_L \rightarrow Z_L Z_L \rightarrow \ell^+ \ell^- \nu \bar{\nu}$	2.4	2	0.5	0

# Work in Progress

Cheung, CC, Hsiao, and Yuan

- In models with a massive  $Z'$  gauge boson associated with an extra  $U(1)$  symmetry, the  $ZWW$ ,  $ZZWW$ , and  $hZZ$  couplings are necessarily modified due to the  $Z$ - $Z'$  mixing, assuming no fine-tuning.
- In contrast, the  $hWW$  coupling remains intact.
- Partial growth of the longitudinal weak gauge boson scattering at high energies goes not only as  $(E/M_W)^2$ , but even as  $(E/M_W)^4$  !
- Detailed study of the high-energy behavior for all scattering channels can help us determine whether this is the case.

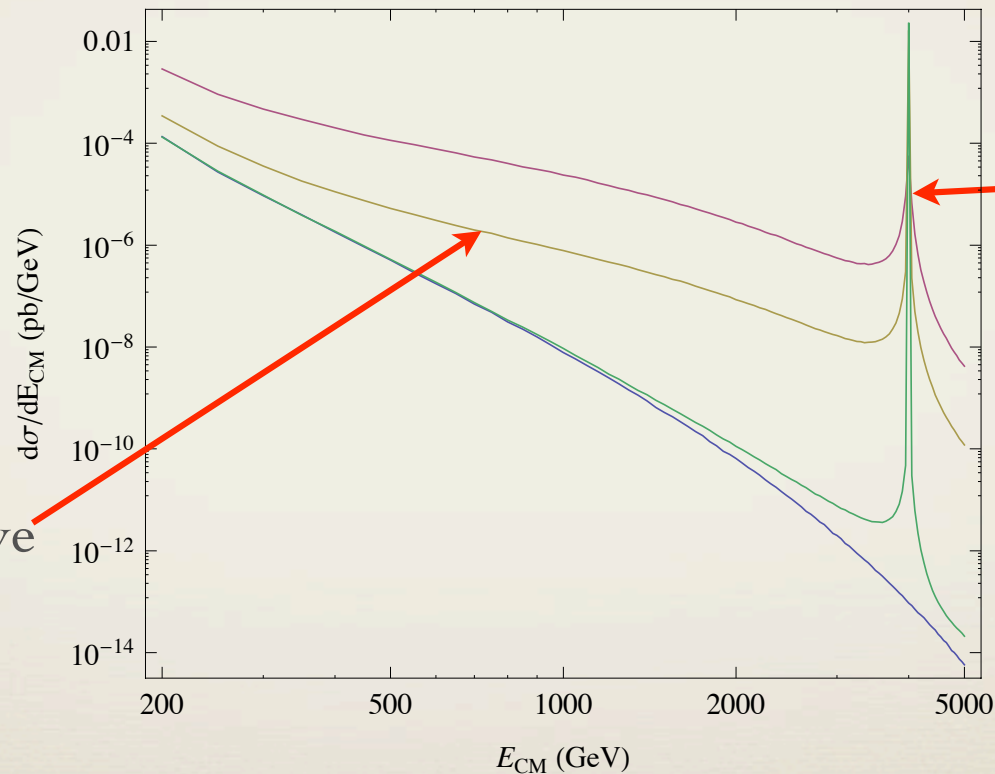


# Preliminary Result

Cheung, CC, Hsiao, and Yuan

- Here is one example:

$$pp \rightarrow W_L^+ W_L^- \rightarrow W_L^+ W_L^-$$



assume  $m_{Z'} = 4$  TeV  
and different mixing  
angles with Z

noticeable effective  
 $E^4$  growth

# Summary

- ☑ Supposing that only a light Higgs boson is detected and it is not fully responsible for the EWSB, we expect discernible energy-growing behavior of  $V_L V_L$  scatterings at the LHC.
- ☑ The cross sections increase until the other heavier Higgs bosons or UV-completing parts come to unitarize the amplitudes.
- ☑ The LHC signature here is excess production of longitudinal gauge boson pairs in the large invariant mass region from their leptonic decays.
- ☑ By careful study of measured event rates, we should be able to deduce how much the  $g_{bVV}$  couplings deviate from the SM values, shedding some light on new physics.